Title: New Vacuum and Representation For Loop Quantum Gravity

Date: May 21, 2014 01:20 PM

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Abstract:

Thanks to Steffen for organizing QG Noons 2014!

(and for giving me a Penrose time slot)

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Overview

Motivation.

What is vacuum?

Loop quantum gravity and Ashtekar-Lewandowski representation

A very short tour.

The new vacuum and the new representation.

Dualization of the above.

More things I will not manage to cover.

Inner product, basis of excitations, inner product of excitations, diffeo symmetry, dynamics

Conclusion and outlook

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What is vacuum?

4

Vacuum

•vacuum = minimal energy state

•however in GR: What is energy?

•vacuum = most typical/ simplest state, easiest to prepare ... sometimes: easiest to write down

•excitations: other states we can prepare (with some minimal effort)

•however, notion of vacuum depends on situation / dynamics:

- •condensate states: for instance scalar field have non-vanishing expectation values
- •usually accommodated by change of representation / Hilbert space, as condensation involves non-normalizable sates with respect to original Hilbert space

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Fock vacuum and Fock space

•vacuum state seemingly simple = state with no particles $|\emptyset\rangle$

(however very entangled ("non-local") state: Reh-Schlieder theorem)

•vacuum is invariant under basic symmetries of theory (here Poincare group)

•excitations: particle states - created by creation operators

•"discrete basis": Fock states $|n_{k_1}, n_{k_2}, n_{k_3}, n_{k_4}, \ldots\rangle$

•Fock Hilbert space can be generated from vacuum via creation operators: vacuum is cyclic

•Fock space carries representation of observable algebra (creation and annihilation operators)

Formalization: GNS construction, vacuum = functional on observable algebra

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further remarks:

•"discrete basis": Fock states $|n_{k_1}, n_{k_2}, n_{k_3}, n_{k_4}, ...\rangle$

actually denote $|n_{k_1}, n_{k_2}, n_{k_3}, n_{k_4}, ...n_k = 0, \quad \forall k \neq k_i\rangle$ ("embedding of Hilbert spaces")

•explicit embedding map/vacuum non-trivial and non-local: depends on the dynamics

$$|n_k = 0\rangle \sim \exp(-\sqrt{k^2+m^2}\,\phi(k)\phi(-k))$$

•this representation is very unpractical if excitations are condensed ...

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Kinematical Loop Quantum Gravity Hilbert space

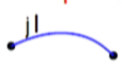
[Ashtekar, Isham, Lewandowski 90's]

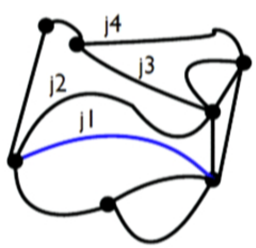
geometric phase space variables: $\{A\,,\,E\} = \delta$

connection

flux: spatial geometry

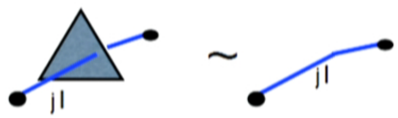
Holonomies
(associated to paths)
act as "creation"
operators





spin network
based on graph

Fluxes
(associated to surfaces)
act as derivatives



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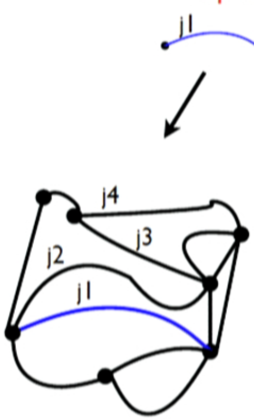

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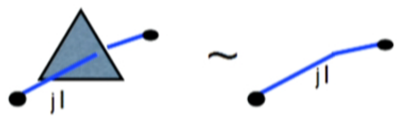

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Kinemtical **Loop Quantum Gravity Hilbert space**

[Ashtekar, Isham, Lewandowski 90's]

- cyclic vacuum state $|\emptyset\rangle =$ state with no spatial geometry
- holonomy operators act as creation operators (also as annihilation operators)
- basis of excitations: labelled by graphs
- embedding of Hilbert spaces based on these graphs:

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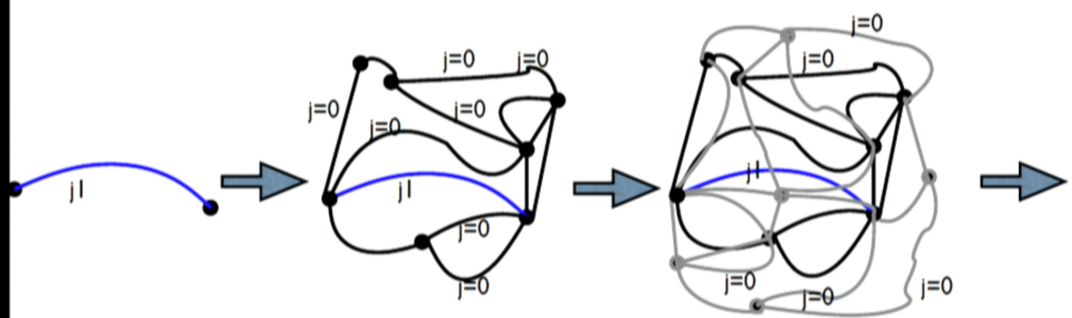
Embedding Hilbert spaces

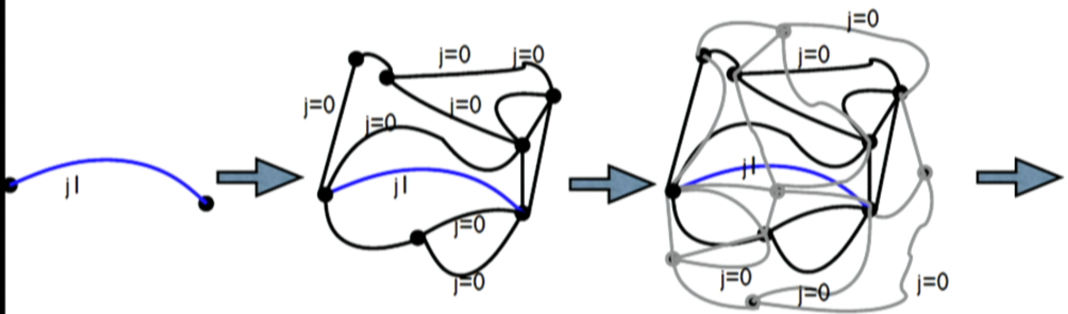


Same state (based on one edge) represented in different graph Hilbert spaces.

"No edge" is equivalent to having an edge with j=0.

Vacuum and refining:
A notion of vacuum tells as also how to refine any state and to embed it into continuum Hilbert space. Defines the continuum Hilbert space.

Embedding Hilbert spaces



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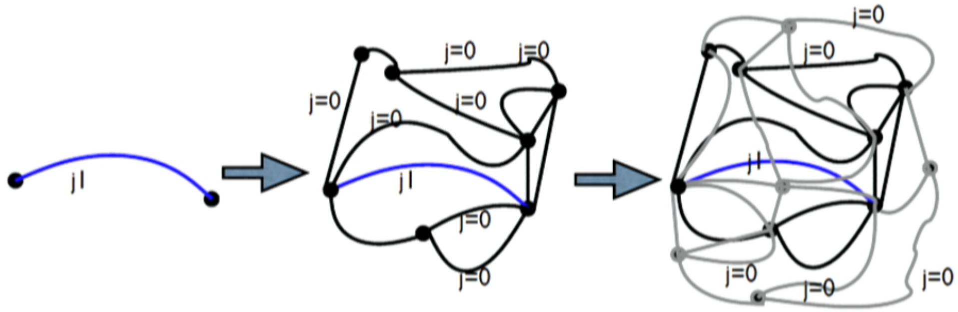
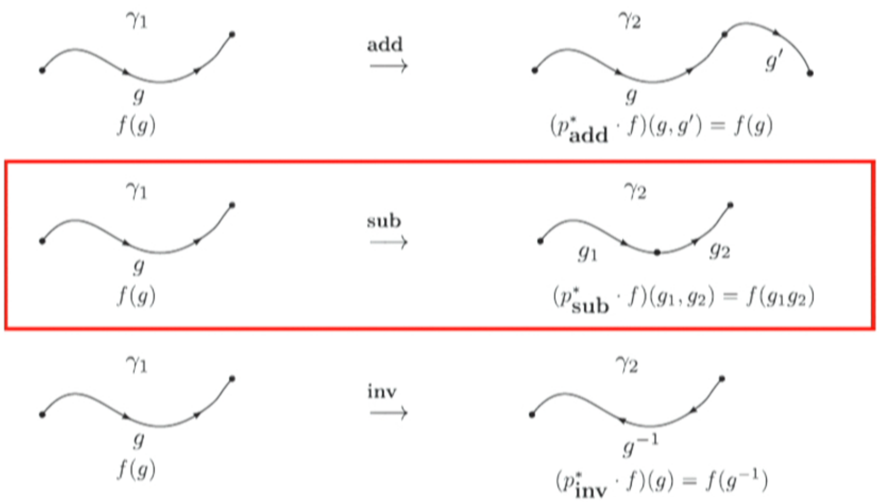
Cylindrical consistency

Any result of a computation (observables, inner product) should be independent of which representative one chooses.



Cylindrical consistency of basic observables



Cylindrical consistency condition for holonomies:
reflect composition: g=g1.g2

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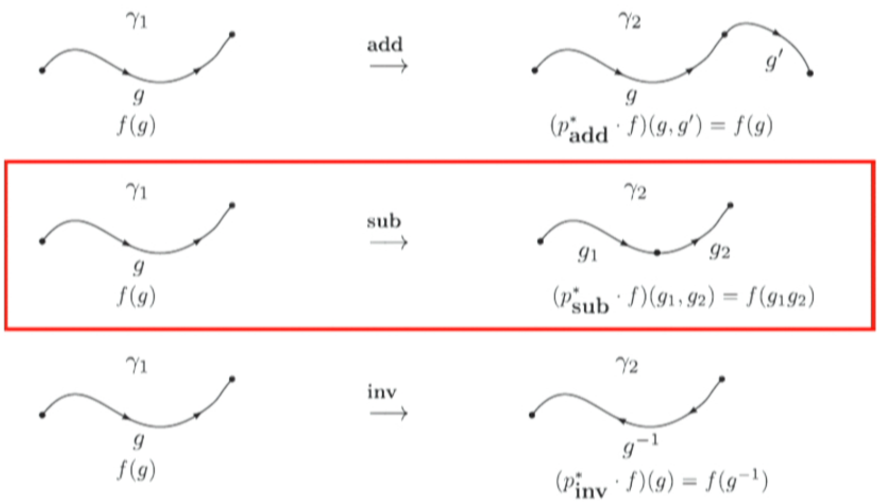

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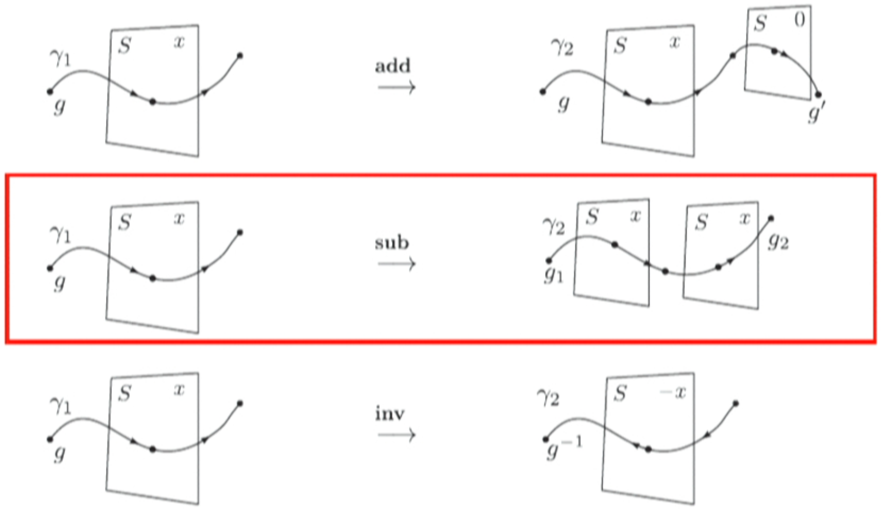
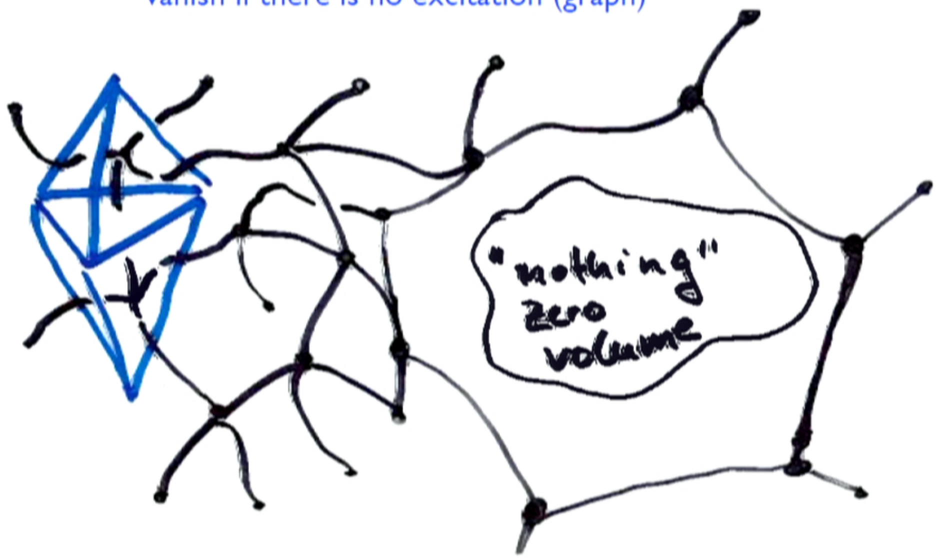


Cylindrical consistency condition for fluxes:
Fluxes of surfaces associated to the same edge coincide.

Kinematical Loop Quantum Gravity Hilbert space

(spatial) geometry operators (area, volume) built from fluxes:

vanish if there is no excitation (graph)



Ashtekar - Lewandowski vacuum (no graph) $\psi_{vac}(A) \equiv 1$, $E \equiv 0$

totally squeezed state
totally degenerate geometry → Very hard to extract low energy physics

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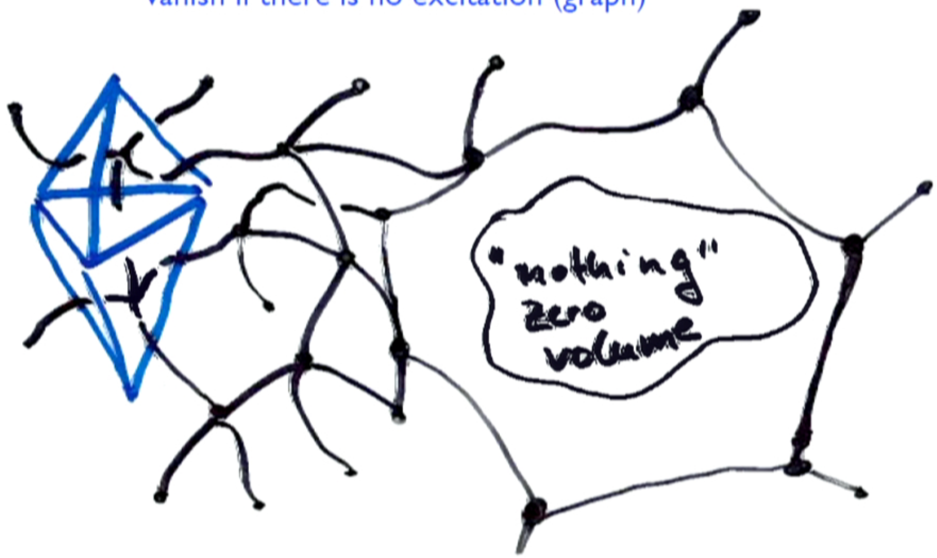

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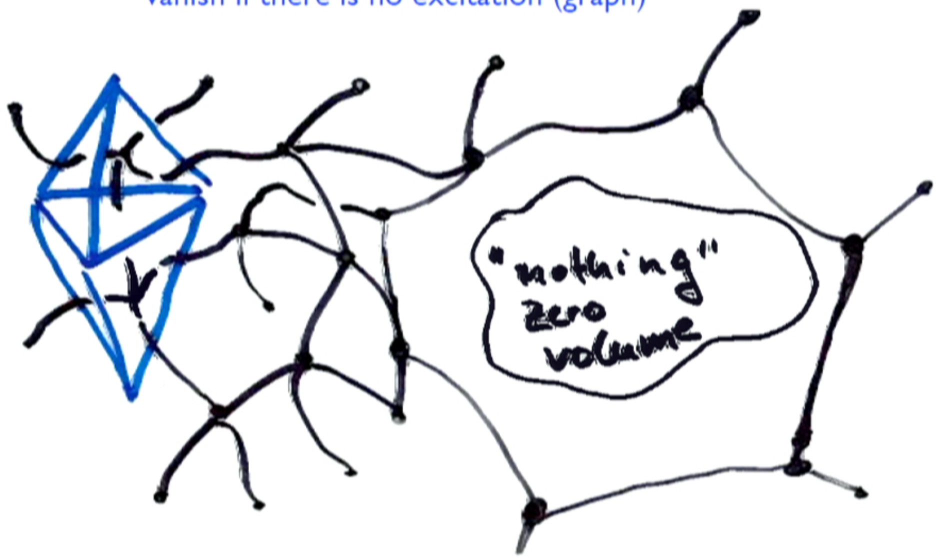

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Are there other representations based on diffeomorphism invariant vacuum?

(**F-LOST**) Uniqueness theorem would suggest: no.

skip to 22

New Loop Quantum Gravity Hilbert space

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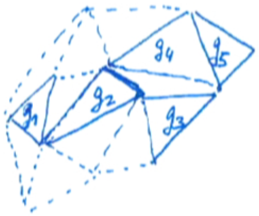
- Wilson surface operators (exponentiated fluxes) generate excitations



- basis of excitations: labelled by "surface network"

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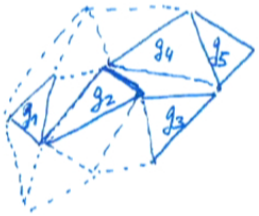

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Set-up

- manifold with auxiliary metric
- set of embedded triangulations
  - embedded vertices: carry coordinate labels
  - edges: geodesics with respect to auxiliary metric (replaces piecewise linear)
  - triangles, tetrahedra: given by minimal surfaces
- dual complex (for instance barycentric, however details do not matter)

  with a root node (fixing a reference frame)

- so far: representation only for gauge invariant operators (modulo root)

- embeddings given by refining Alexander moves: implemented by "gluing BF amplitudes"
- equips the set of triangulations with a partial (directed)* order

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Cylindrical consistency conditions

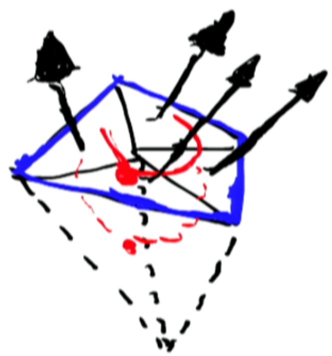
	AL embedding	BF embedding
holonomies	compose	stay constant
fluxes	stay constant	compose

We need to compose fluxes!

Here: Fluxes associated to surfaces (3+1dim) are added.

Integrated simplicial fluxes

In d=3 (need a surface tree for parallel transport):



black arrows: elementary fluxes

blue: piece of a surface

red: bonsai tree for piece of surface

parallel transport (dashed red) takes place

in tetrahedra `below' the surface

For composition of integrated fluxes need to specify a `bridge' edge

(which connects the two surface trees).

New Cylindrical consistency

- construction realizes exponentiated fluxes as cylindrically consistent observables
- cylindrically consistent observables now based on composition of fluxes:
- requires more structure than for holonomies

  (fluxes need to be parallel transported for composition)

- flatness ensures that choices for this structure do not matter in the quantum theory

Cylindrical consistency ensures that the BF representation can be defined as a continuum Hilbert space.

How did we evade F-LOST uniqueness

Given observable algebra
(holonomies for all possible* paths and fluxes for all possible* surfaces)
and assuming

- (spatially) diffeomorphism invariant vacuum
- cyclic vacuum
- irreducible representation
- representation of spatial diffeomorphisms

STILL SATISFIED

- weakly continuous exponentiated fluxes

Not SATISFIED

the Ashtekar-Lewandowski representation is unique.

We changed the observable algebra.

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- simplicial fluxes, associated to triangles in (arbitrarily refined) triangulations
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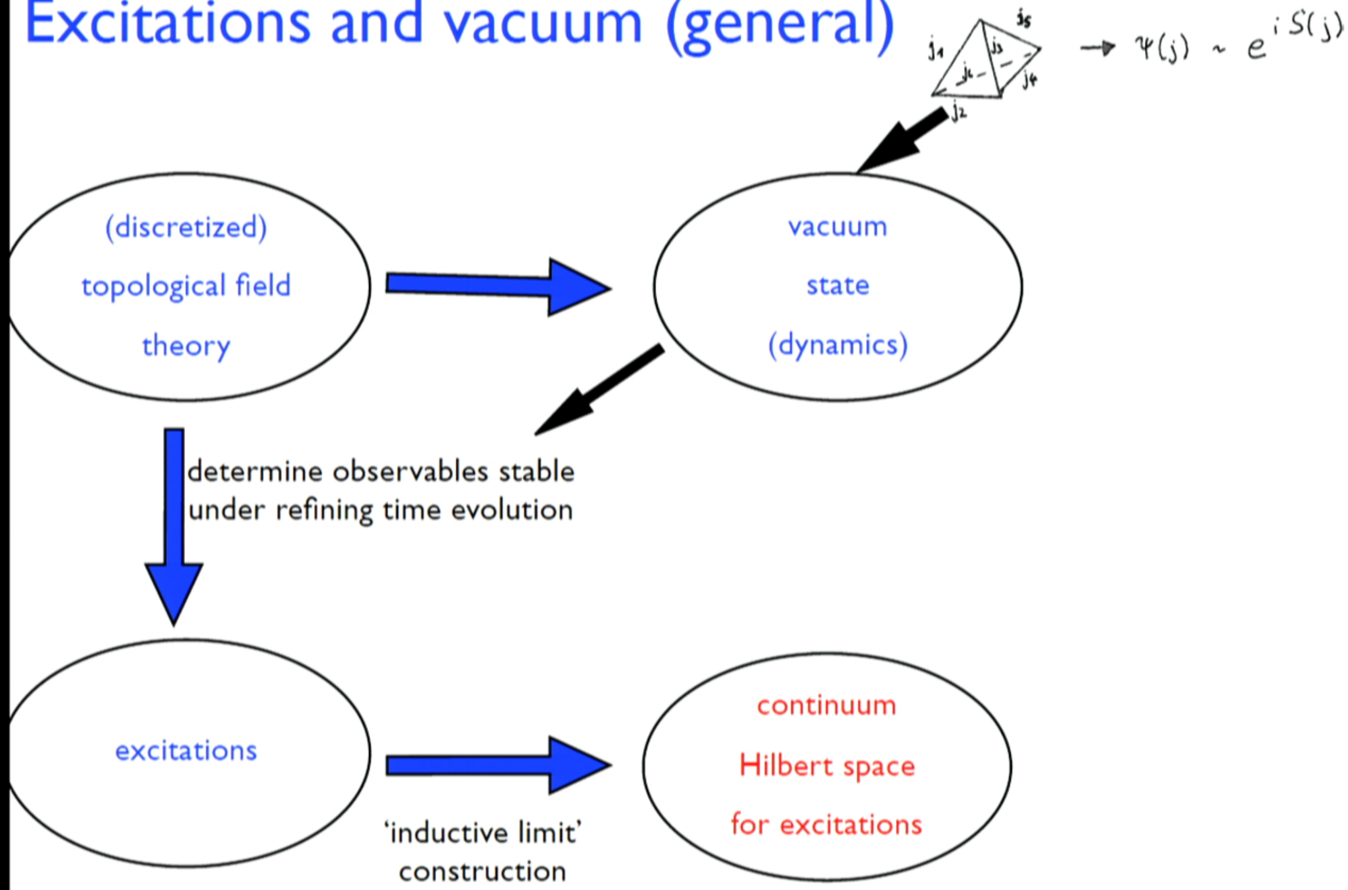
Excitations and vacuum (general)



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;onclusions and outlook

Refining by time evolution:

We have a better understanding of how to construct the physical vacuum as a continuum object, starting from spin foam amplitudes.

New Vacuum and Representation:

The construction of the BF vacuum /representation is a nice exercise towards this end:

Realization of a condensate state and representation.

Opens up lots of new avenues for further devolopments:

Very near to spin foam dynamics. Can discuss simplicity constraints [a la Zapata 96]

Might facilitate extraction of low energy physics, cosmology etc.

Many generalizations possible. One is:

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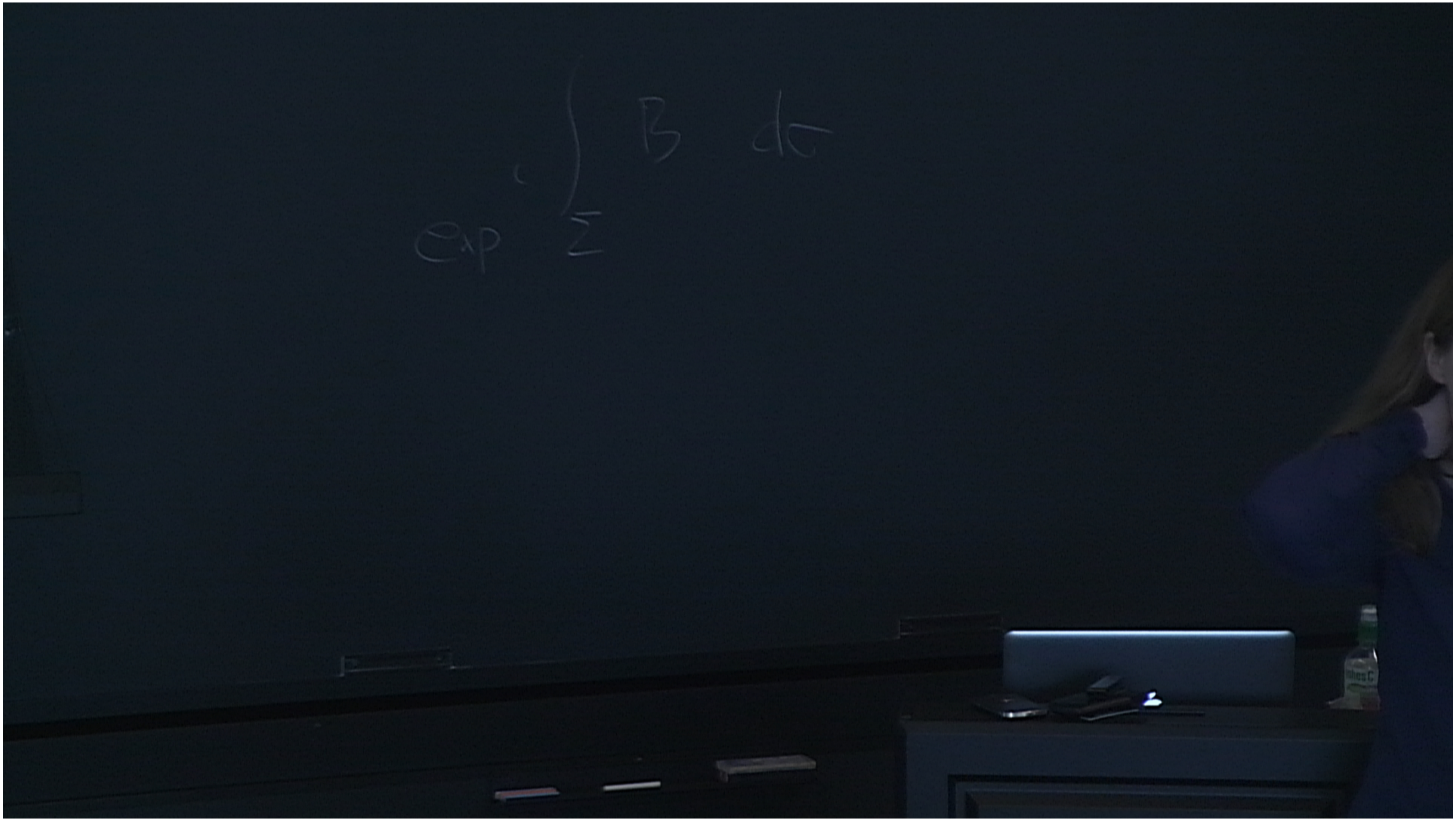
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