Title: Dynamical trapping near a quantum critical point

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Abstract: We consider a closed system where the parameter controlling a quantum phase transition is promoted to a dynamical field interacting with the quantum critical theory. In the case that the field has an energy extensive in the volume we can treat its evolution classically. We find that the field can become trapped near the phase transition point due to its interactions with the degrees of freedom of the quantum critical theory. The trapping/untrapping transition can be understood using Kibble-Zurek scaling arguments. We check the general framework numerically in the particular case of the 1D transverse field Ising chain, where the transverse magnetic field is dynamical. This constitutes a dynamical mechanism for tuning a relevant parameter to zero through a non-equilibrium process.

Dynamical trapping near a Quantum Critical Point

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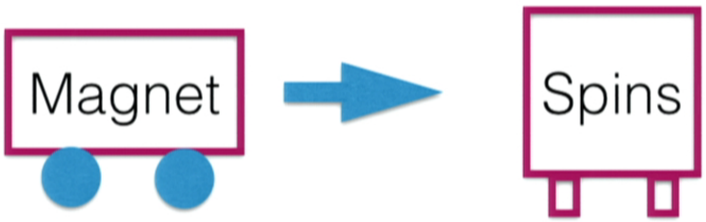
w/ M. Kolodrubetz and A. Polkovnikov.
(in progress, to appear)

Extensive energy for λ is good:

1. Large mass means λ is effectively classical.

Setup possibly realized in a variety of systems

1. Toys: λ related to the motion of a macroscopic object.



Basic observations

As $\lambda(t) \to \lambda_c$ the gap closes & most entropy is generated at the critical point.

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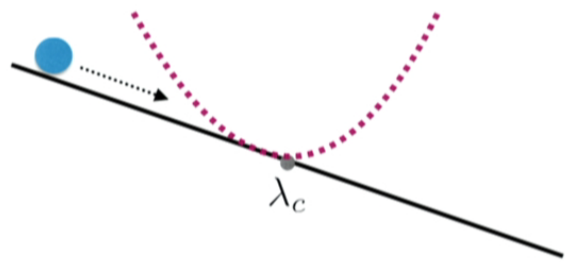
Exciting quantum d.o.f. can lead to a trapping effective potential for lambda at the QCP.



(Similar to temperature modifying the effective potential)

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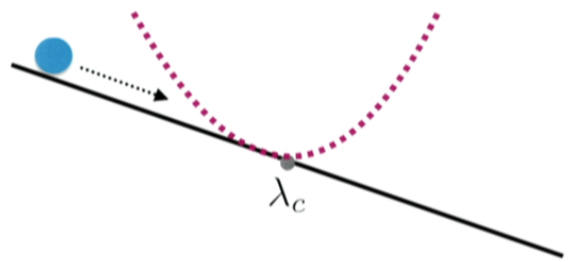

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Trapping regimes using scaling analysis

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Zero total potential for lambda

Away from QCP: $\lambda(t) = vt$

$$K_{in} = \frac{1}{2}(\mu L^d)v^2 \quad \text{(set } \mu = 1 \text{ for simplicity)}$$

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Most heat generated at QCP → use scaling arguments:

1. Relate v to $m_* \sim \frac{1}{\xi}$:

Example - standard mean field: $z = 1,\ \nu = \frac{1}{2}$

$$Q \sim v^{\frac{(d+1)}{3}} \gtrsim v^2$$

Testing scaling with TFI chain

$$H = -\sum_j \left[(1 - \lambda(t)) s_j^z + s_j^x s_{j+1}^x \right]$$

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Dynamics can be solved numerically (wave function factorizes in k-space):

After Jordan-Wigner:

$$H_k = (1 - \lambda - \cos k)(c_k^\dagger c_k + c_{-k}^\dagger c_{-k} - 1) + \sin k (c_k^\dagger c_{-k}^\dagger + c_{-k} c_k)$$

$$H_k(\lambda) = (1 - \lambda - \cos k)\sigma_k^z + (\sin k)\sigma_k^x$$

$$i \frac{d\Psi_k}{dt} = (H_k(\lambda))_{2\times 2} \Psi_k$$

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$$i \frac{d\Psi_k}{dt} = (H_k(\lambda))_{2\times 2}\, \Psi_k$$

$$(\mu L)\ddot{\lambda} = -2 \sum_k \langle \sigma_k^z \rangle(\lambda) + \partial_\lambda V_{GS}(\lambda)$$

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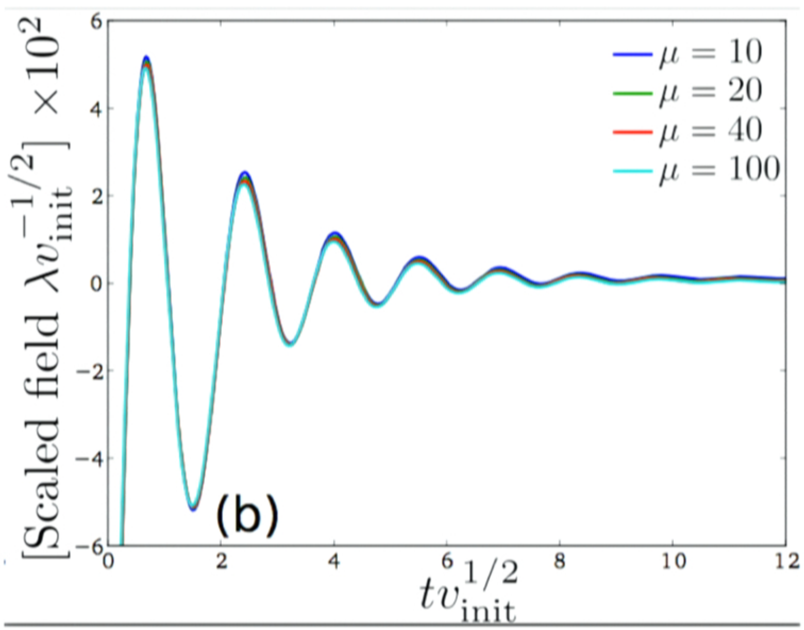
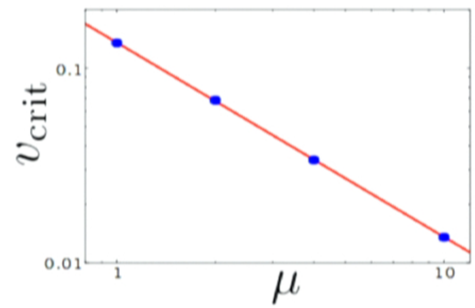
$$\lambda \sim \sqrt{v},\ t \sim 1/\sqrt{v}$$





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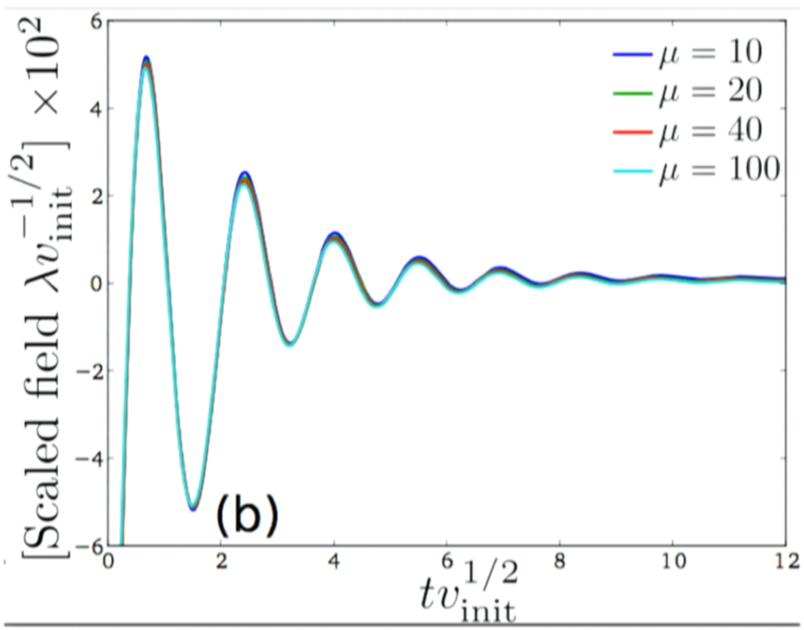
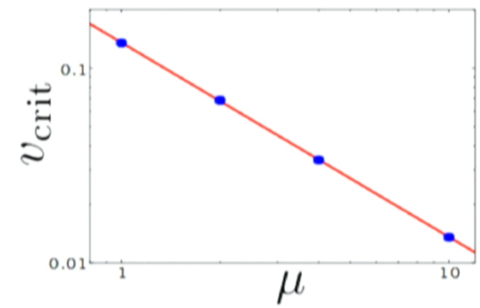
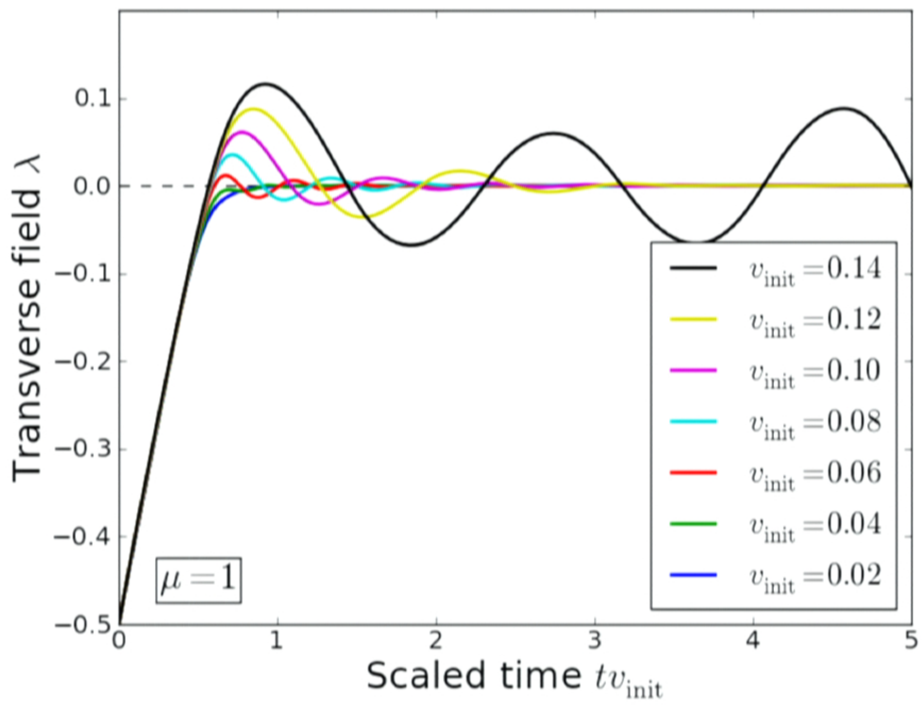


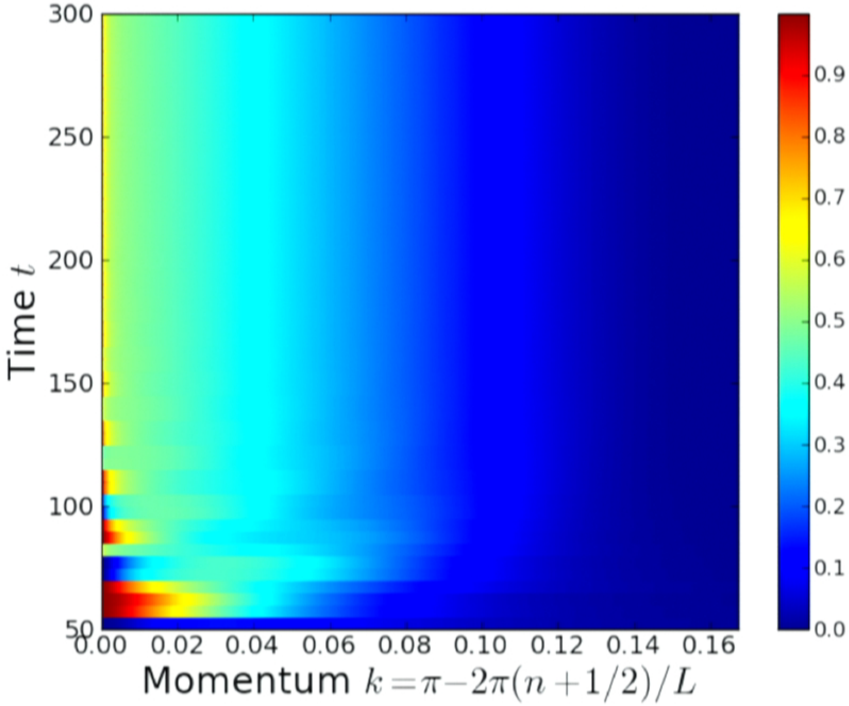


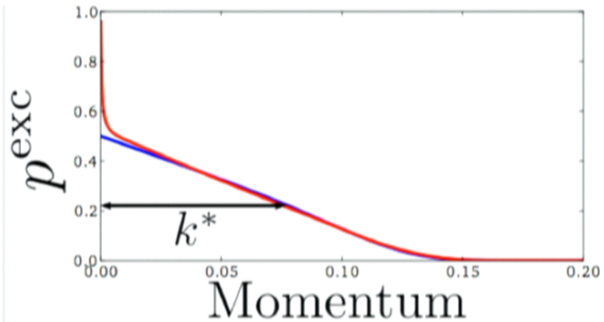
Trapping:



Transverse field λ
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
0
1
2
3
4
5
Scaled time tv_{init}
$\mu = 1$
$v_{\text{init}} = 0.14$
$v_{\text{init}} = 0.12$
$v_{\text{init}} = 0.10$
$v_{\text{init}} = 0.08$
$v_{\text{init}} = 0.06$
$v_{\text{init}} = 0.04$
$v_{\text{init}} = 0.02$

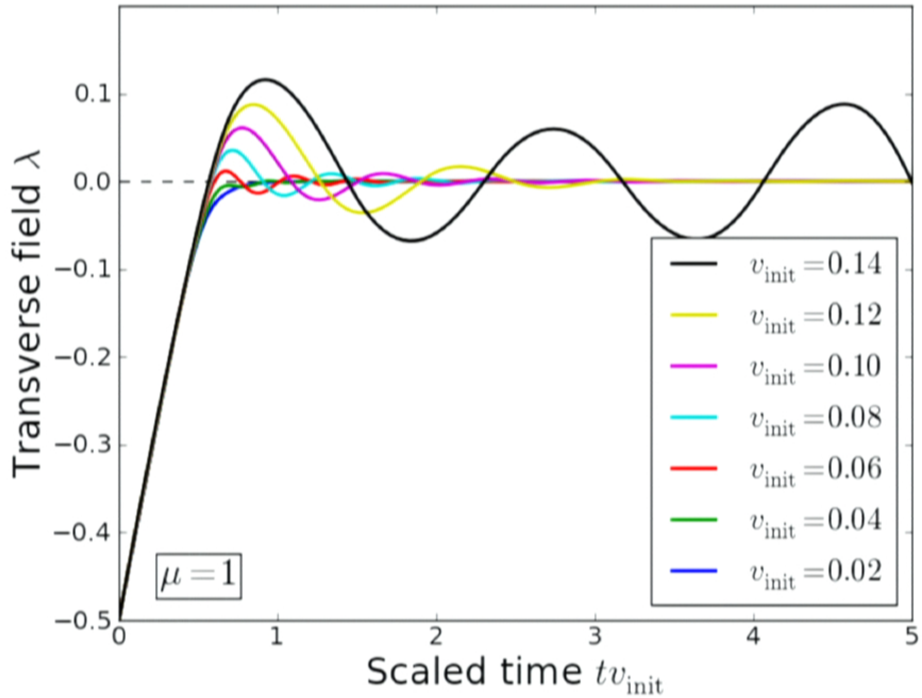
Excitation spectrum:





$$k^* \sim \sqrt{\mu} v$$

Trapping:



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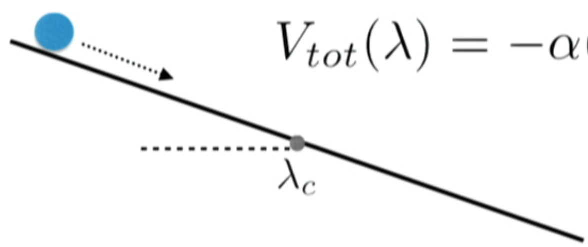
Trapping regimes using scaling analysis II

Potential for lambda with small slope



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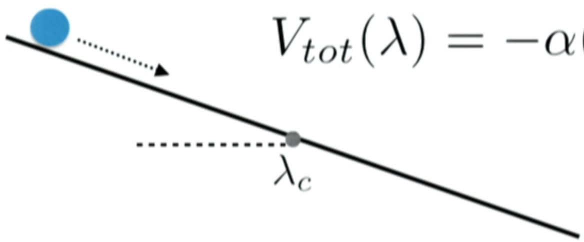

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Let's consider starting from rest some distance λ_{in} from QCP.

$$v_c \sim \sqrt{\alpha \lambda_{in}}$$

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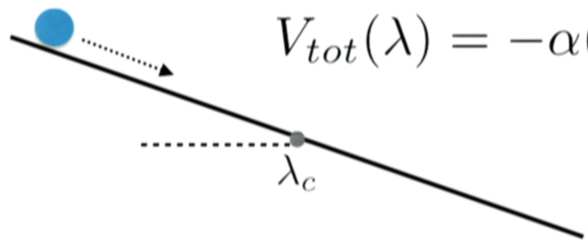

$$V_{tot}(\lambda) = -\alpha(\lambda - \lambda_c), \;\; \alpha \ll 1$$

Let's consider starting from rest some distance λ_{in} from QCP.

$$v_c \sim \sqrt{\alpha \lambda_{in}} \qquad \& \qquad Q \sim v_c^{\frac{\nu(d+z)}{\nu z+1}} \gtrsim \alpha \lambda_{in}$$

Implies trapping for : $\lambda_{in} \lesssim \frac{1}{\alpha}$

However - above not right for low lambda!

Implicit assumption in the scaling argument:

Time to QCP >> Inverse Gap

In this limit:

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Change is effectively instantaneous.

Like sudden quench: λ_{in} only scale.

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$$\Rightarrow \quad Q \sim \lambda_{in}^{\nu(d+z)}$$

& $Q \gtrsim \alpha\lambda_{in}$ gives final trapping range:

Recall: $K_{in} = \frac{1}{2}(\mu L^d)v^2$

$$\mu \sim \frac{\lambda \text{ extensivity}}{\text{quantum dof extensivity}}$$

For example, if QCT has large N: $\mu \sim \frac{1}{N}$

Restoring μ :

$$\alpha^{\frac{1}{\nu(d+z)-1}} \lesssim \lambda_{in} \lesssim \frac{1}{\alpha}\left(\frac{1}{\mu}\right)^{\frac{\nu(d+z)}{2-\nu(d-z)}}$$

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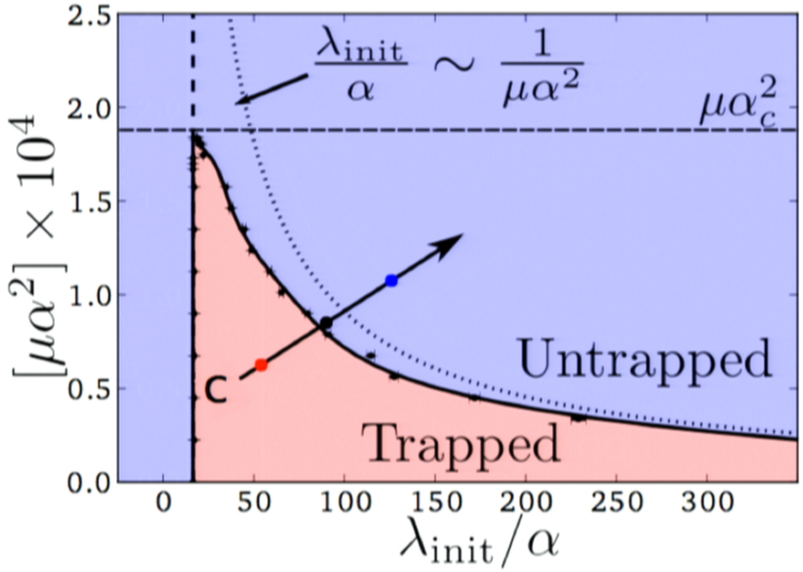
TFI chain: $1 \lesssim \frac{\lambda_{in}}{\alpha} \lesssim \frac{1}{\mu\alpha^2}$



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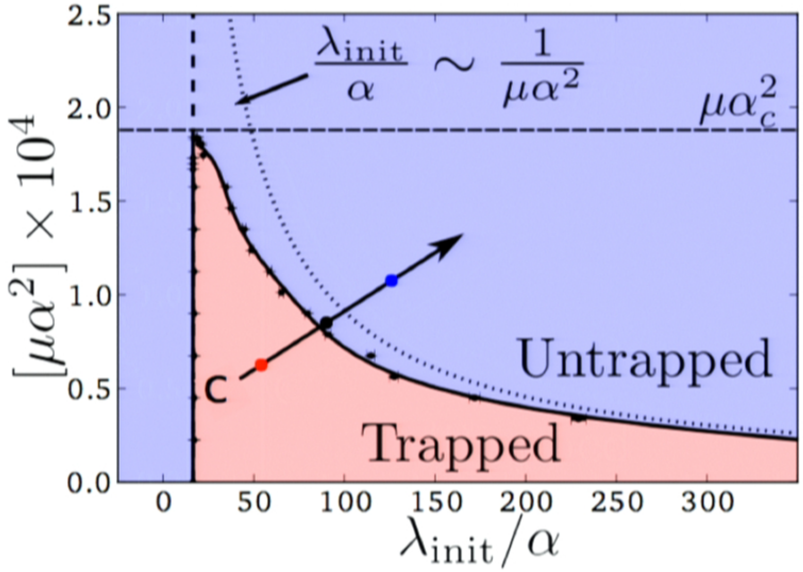

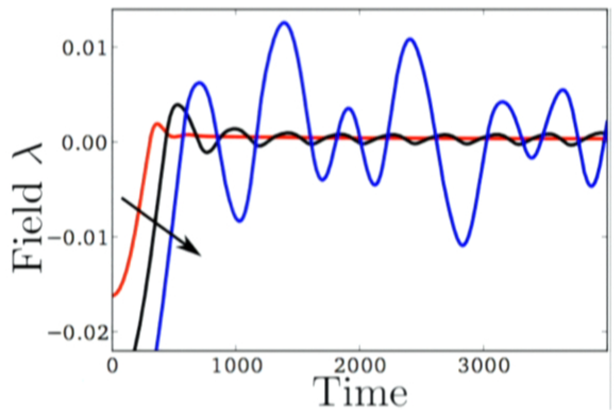

Degree of trapping (speculative)

In the trapped regime - how small is the final λ ?

Effective potential due to excitations in TFI near QCP:

$$V_{tot}(\lambda) \approx \int dk \; p_{exc}(k)\sqrt{k^2+\lambda^2} - \alpha\lambda$$

$$\approx \int^{k*} dk \; \sqrt{k^2+\lambda^2} - \alpha\lambda, \quad (k^*)^2 \sim \alpha\lambda_{in}$$

Assuming: $k* \gg \lambda_f$,

Minimizing gives:

$$\lambda_f \sim \frac{\alpha}{Log(k^*/\alpha)}$$

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$$V_{tot}(\lambda) \approx V_{exc}(\lambda, k^*) - \alpha\lambda$$

Assume excitation potential is analytic in $m^2(\lambda) \sim \lambda^{2\nu}$

$$V_{tot}(\lambda) \approx V_0(k^*) + \lambda^{2\nu} V_1(k^*) - \alpha\lambda + \cdots$$

$$V_1(k^*) \sim (k^*)^{d+z-2}$$

Minimizing gives: $\lambda_f \sim \left(\frac{\alpha}{(k^*)^{d+z-2}} \right)^{\frac{1}{2\nu-1}}$

Now $(k^*)^{d+z} \sim \alpha\lambda_{in}$ & trapping requires : $\lambda_{in} \lesssim \frac{1}{\alpha}$

$$\lambda_f \gtrsim \alpha^{\frac{1}{2\nu-1}}$$

$$\lambda_f \gtrsim \alpha^{\frac{1}{2\nu-1}}$$ Implies super-trapping for mean field!

I.e. for $\nu = \frac{1}{2} + \epsilon, \ \alpha \ll 1$

$$\lambda_f \sim \alpha^{\frac{1}{2\epsilon}} \ll \alpha$$

Conclusions

1. Studied a closed system consisting of a field coupled to a quantum theory as a dynamical parameter controlling criticality.
2. In the case that the field has extensive energy, we where able to use Kibble-Zurek scaling arguments to analyze dynamics.
3. Argued for dynamical trapping for slow velocities/small slopes.
4. Tested general framework numerically with the TFI chain.
5. Interesting future directions:

   Theories with scalars and spontaneous symmetry breaking (e.g. O(N) models) - does anything change?
   Super-trapping with $\nu = \frac{1}{2}$.
   Is there an analogous story for classical phase transitions?
   Can this have interesting implications for the early universe?