

Title: Relativity of non relativistic systems

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Abstract: To the best of our knowledge, the fundamental laws of physics are Lorentz invariant. This means that condensed matter systems at finite density still display full Lorentz symmetry: it is just spontaneously broken (i.e. by state considered) and thus non-linearly realized. This simple observation allows to derive exact results about the spectrum of theories at finite charge density and suggests to classify condensed matter systems according to all the inequivalent ways in which boosts can be spontaneously broken.

Relativity of non-relativistic Systems

Federico Piazza

based on works with
A. Nicolis, R. Penco, R. Rattazzi, R. Rosen



Systems at finite density: a ‘high energy’ remark

* Lorentz is always broken spontaneously * in the real world

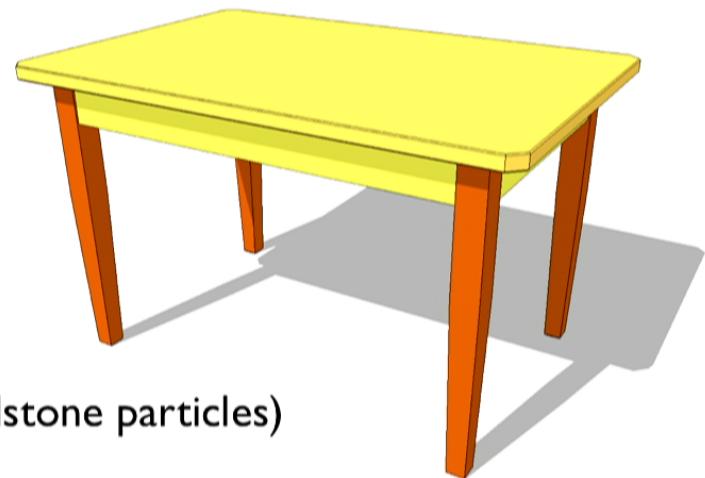


- * Galilei as good as Lorentz
- * Relativity: non linearly realized

Systems at finite density: a ‘high energy’ remark

Lorentz is always broken **spontaneously** in the real world

Outline:



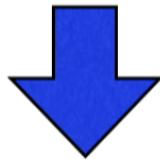
- 1) **Exact** results (massive Nambu-Goldstone particles)
- 2) Classification (and ‘‘definition’’) of **condensed matter systems**
- 3) Patterns non-realized in nature (**Framids etc.**)

Cosmology (or: how I got into this)

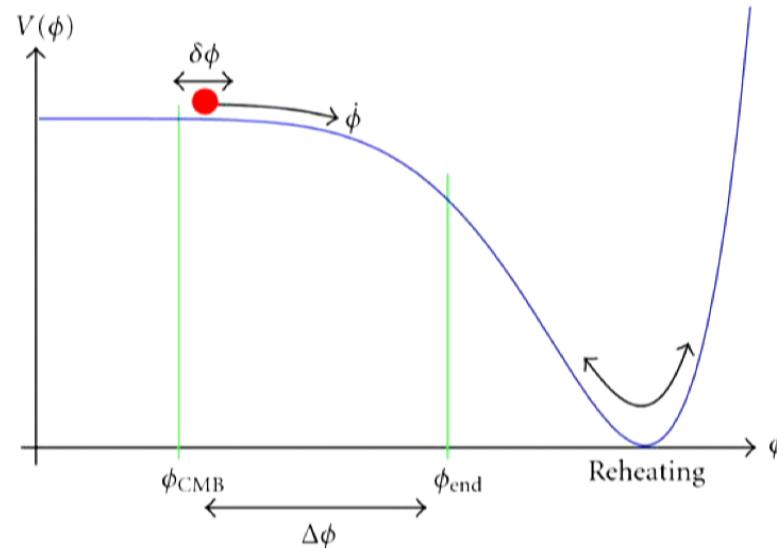
- Strong indication for a primordial **inflation** phase of quasi-de Sitter expansion

Flat to good approximation

$$\frac{V'}{V} \ll 1, \frac{V''}{V} \ll 1$$



Generalization:
moving in a symmetry
direction



Spontaneous Symmetry Probing

- Time dependent field states in the presence of a continuous symmetry
- In particular:

$$\dot{\phi}_j \propto \delta\phi_j$$

time evolution → ← **symmetry action**

- Equivalently,

$$\bar{H}|\mu\rangle \equiv (H - \mu Q)|\mu\rangle = 0$$

+ $|\mu\rangle$ **breaks** Q

Systems at finite charge density

At first sight: **explicit** breaking of Lorentz and all non-commuting charges

$$\bar{H} = H - \mu Q$$

However: Lorentz clearly broken just **spontaneously**

Spontaneous Symmetry Breaking: Generalities

$$\langle 0 | [Q(t), A(0)] | 0 \rangle = \text{const.} \quad \text{always} \quad \left(\frac{dQ}{dt} = 0 \right)$$

More precisely,

$$\begin{aligned} 0 &= \int d^3x \langle 0 | [\partial_\mu J^\mu(\vec{x}, t), A] | 0 \rangle \\ &= \int d^3x \langle 0 | [j^0(\vec{x}, t), A] | 0 \rangle + \int d^3x \langle 0 | [\partial_i J^i(\vec{x}, t), A] | 0 \rangle \end{aligned}$$

Spontaneous Symmetry Breaking: Generalities

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Because commutator
of local operators

Spontaneous Symmetry Breaking: Generalities

$$\langle 0 | [Q(t), A(0)] | 0 \rangle = \text{const.} \quad \text{always} \quad \left(\frac{dQ}{dt} = 0 \right)$$

$$\neq 0 \quad \text{for some } A$$

after some manipulations...

$$= \sum_n e^{-iE_n(0)t} \langle 0 | J^0(0) | n, 0 \rangle \langle n, 0 | A | 0 \rangle - c.c.$$

Goldstone Theorem: both $J^\mu(x)$ and $A(x)$ interpolate a massless state

$$\langle 0 | J^\mu(x) | \pi(p) \rangle = i v e^{ip_\mu x^\mu} p^\mu$$

Systems at finite charge density

$$\langle \mu | [Q_a(t), A(0)] | \mu \rangle = \text{const.} \neq 0 \quad \text{after some manipulations...}$$

$$= \sum_n e^{-iE_n(0)t} \langle \mu | e^{i\mu Qt} J_a^0(0) e^{-i\mu Qt} | n, 0 \rangle \langle n, 0 | A | \mu \rangle - c.c.$$

say that

$$[Q_a, J_b^0(x)] = i f_{ab}^c J_c^0(x)$$

the interpolator is a time-dependent combination of conserved currents

$$e^{i\mu Qt} J_a^0(0) e^{-i\mu Qt} = (e^{-f_1 \mu t})_a^b J_b^0(0)$$

Take f_{1a}^b in 'normal form': block diagonal with pieces $\begin{pmatrix} 0 & +q_\alpha \\ -q_\alpha & 0 \end{pmatrix}$

Each block: one massive Goldstone state $m = \mu q_\alpha$

see also: Watanabe, Brauner, Murayama 1303.1527, and Tomas' talk at this conference

Example: SO(3) - one triplet

$$\mathcal{L} = -\frac{1}{2}\partial_\mu \vec{\phi} \partial^\mu \vec{\phi} - \frac{1}{2}m^2 \vec{\phi}^2 - \frac{1}{4}\lambda(\vec{\phi}^2)^2$$

radial and angular
coordinates for 1-2

$$\begin{aligned}\mathcal{L} = & -\frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2}\sigma^2 \partial_\mu \theta \partial^\mu \theta - \frac{1}{2}\partial_\mu \phi_3 \partial^\mu \phi_3 \\ & - \frac{1}{2}m^2 (\sigma^2 + \phi_3^2) - \frac{1}{4}\lambda (\sigma^2 + \phi_3^2)^2\end{aligned}$$

SSP solution:

$$\dot{\theta} = c; \quad \sigma^2 = \frac{c^2 - m^2}{\lambda}; \quad \phi_3 = 0$$

Perturbations:

$$\begin{aligned}\mathcal{L}^{(2)} = & -\frac{1}{2}\partial_\mu \delta\sigma \partial^\mu \delta\sigma - \frac{1}{2}\sigma^2 \partial_\mu \pi \partial^\mu \pi - \frac{1}{2}\partial_\mu \phi_3 \partial^\mu \phi_3 \\ & + 2c\sigma\dot{\pi}\delta\sigma - (c^2 - m^2)\delta\sigma^2 - \frac{1}{2}c^2\phi_3^2\end{aligned}$$

However: SO(3) - symmetric traceless rep.

$$\mathcal{L} = -\frac{1}{2} \partial_\mu \Phi^i{}_j \partial^\mu \Phi^j{}_i - \lambda (\Phi^i{}_j \Phi^j{}_i - v^2)^2$$

SSP solution:

$$\langle \Phi \rangle = e^{i\mu t L_3} \begin{pmatrix} \Phi_0 & 0 & 0 \\ 0 & -\Phi_0 & 0 \\ 0 & 0 & 0 \end{pmatrix} e^{-i\mu t L_3}$$

1) Fixed gap Goldstone $m = \mu$

2) ``Un-fixed gap Goldstone $m = 3\mu$

Other ex: SO(3) - two triplets. Etc.

1306.1240, A. Nicolis, R. Penco, F.P., R. Rosen

Finite charge density: coset construction

- Full symmetry group:

$$Q_I$$

- Unbroken generators

$$T_A \text{ (subgroup)}$$

- Broken generators

$$X_a$$

- Charge at finite density

$$\mu Q = \mu_X X + \mu_T T$$

- 1) maximum number of unbroken generators
- 2) completely antisymmetric in (X_a, T_A)

Unbroken

$$\left\{ \begin{array}{l} \bar{P}^0 \equiv H - \mu Q \\ \bar{P}^i \equiv P^i \\ J_i \\ T_A \end{array} \right.$$

Broken

$$\left\{ \begin{array}{l} Q \\ X, X_a \\ K_i \end{array} \right.$$

Finite charge density: coset construction

$$\Omega = e^{ix^\mu \bar{P}_\mu} e^{i\pi(x) X} e^{i\pi^a(x) X_a} e^{i\cancel{\eta^i}(x) K_i}$$

- Boost-Goldstones always eliminated by inv. Higgs (see Riccardo's talk)
- Internal Goldstones further classified: commuting vs. non-commuting

$$[Q, X_a] = iM_{ab}X^b \quad M_{ab} = \text{diag} \left\{ 0, \dots, 0, \begin{pmatrix} 0 & q_1 \\ -q_1 & 0 \end{pmatrix}, \dots, \begin{pmatrix} 0 & q_k \\ -q_k & 0 \end{pmatrix} \right\}.$$


Finite charge density: coset construction

$$\Omega = e^{ix^\mu \bar{P}_\mu} e^{i\pi(x)X} e^{i\pi^a(x)X_a} e^{i\gamma^i(x)K_i}$$

- Commuting Goldstones $\pi - \pi^\alpha$ only appear with derivatives
- One derivative mixing is important: $M_{\alpha\beta}\pi^\alpha \dot{\pi}^\beta$

$$M = \text{diag} \left\{ \underbrace{0, \dots, 0}_{\text{Linear dispersion relations}}, \left(\begin{array}{cc} 0 & M_1 \\ -M_1 & 0 \end{array} \right), \dots, \left(\begin{array}{cc} 0 & M_k \\ -M_k & 0 \end{array} \right) \right\}$$

- Linear dispersion relations + massless quadratic \leftrightarrow gapped $m \sim \mu$

Nielsen and Chada '76, Watanabe and Brauner '11. Hitoshi's talk at this workshop

Summary

- n_1 massless linear (from commuting sector)
- n_2 massless quadratic (from commuting sector)
- n_3 fixed gap (from non-commuting sector)
- n_4 unfixed gap (from both sectors)

$$n_2 \leq n_4 \leq n_2 + n_3$$

Classification of “condensed matter”

Full Symmetry group: Poincaré

$$\left\{ \begin{array}{ll} P^\mu & \text{translations} \\ J^i & \text{rotations} \\ K^i & \text{boosts} \end{array} \right.$$

$$[J_i, P_j] = i\epsilon_{ijk}P_k, \quad \text{etc.}$$

Classification of “condensed matter”

Full Symmetry group: Poincaré

$$\left\{ \begin{array}{ll} P^\mu & \text{translations} \\ J^i & \text{rotations} \\ K^i & \text{boosts} \end{array} \right. + \text{internal 'Q' symmetries}$$



Excitations of the c.m. system

$$\left\{ \begin{array}{ll} \bar{P}^\mu & \text{translations} \\ \bar{J}^i & \text{rotations} \end{array} \right.$$

$$[\bar{J}_i, \bar{P}_j] = i\epsilon_{ijk} \bar{P}_k$$
$$[\bar{J}_i, \bar{J}_j] = i\epsilon_{ijk} \bar{J}_k$$

Possible discrete versions: not considered here

Classification of “condensed matter”

$$\left\{ \begin{array}{l} \bar{H} = H \\ \bar{P}^i = P^i + Q^i \\ \bar{J}^i = J^i + q^i \end{array} \right. \quad \text{internal symmetry: Euclidean group}$$

Solids

3 Goldstones

$$\langle \phi^I(x) \rangle = x^I \quad \left(\begin{array}{l} \text{Fluids! if also} \\ \phi^I \rightarrow \xi^I(\phi) \quad \det \frac{\partial \xi^I}{\partial \phi^J} = 1 \end{array} \right)$$

Classification of “condensed matter”

$$\left\{ \begin{array}{l} \bar{H} = H + Q \\ \bar{P}^i = P^i + Q^i \\ \bar{J}^i = J^i + q^i \end{array} \right. \quad \begin{array}{c} \text{internal symmetry: Euclidean group} \\ + \text{U}(1) \end{array}$$

Supersolids

4 Goldstones

$$\langle \phi(x) \rangle = t$$

$$\langle \phi^I(x) \rangle = x^I$$

Classification of “condensed matter”

$$\left\{ \begin{array}{l} \bar{H} = H + Q \\ \bar{P}^i = P^i \\ \bar{J}^i = J^i + Q^i \end{array} \right. \quad \text{internal symmetry: } \text{SO}(3) \times \text{U}(1)$$

“Type II Super-framids” 4 Goldstones

$$\langle A_i^\mu(x) \rangle = t \delta_i^\mu$$

He-3 B-phase?

with A. Nicolis, R. Penco, R. Rattazzi, R. Rosen, to appear

Framids

Naive analysis

$$A^\mu(x) = (e^{i\vec{\eta}(x)\vec{K}})^\mu{}_\nu \delta_0^\nu$$

$$\mathcal{L} \sim (\partial_\mu A^\mu)^2 + (\partial_\mu A^\nu)^2 + (A^\mu \partial_\mu A_\nu)^2$$

Coset construction: same results

$$\mathcal{L}_2 = \frac{f^2}{2} \left[\dot{\vec{\eta}}^2 - c_L^2 (\vec{\partial} \cdot \vec{\eta}_L)^2 - c_T^2 \partial_i \eta_T^j \partial_i \eta_T^j \right]$$

\mathcal{L}_3 and \mathcal{L}_4 completely determined up to two derivatives

Framids: solids in disguise?

No! different scaling:

$$\mathcal{L}_{\text{framid}} \sim (\partial\eta)^2 + \partial^2\eta^3 + \partial^2\eta^4 + \dots \quad \mathcal{M}_{2 \rightarrow 2} \propto E^2$$

$$\mathcal{L}_{\text{solid}} \sim (\partial\pi)^2 + (\partial\pi)^3 + (\partial\pi)^4 + \dots \quad \mathcal{M}_{2 \rightarrow 2} \propto E^4$$

Framids: why they are not around?

$$\begin{cases} \bar{H} &= H \\ \bar{P}^i &= P^i \\ \bar{J}^i &= J^i \end{cases}$$

Type I Framids

- 1) No adjustable ``thermodynamic'' parameter
- 2) Not “compressible” in a standard way (type I)

$$\delta T^{00} = \text{const.}$$

$$\delta T^{0i} = 0$$

$$\delta T^{ij} = \text{const.} \times \delta^{ij}$$

$$\begin{cases} \bar{H} &= H \\ \bar{P}^i &= P^i \\ \bar{J}^i &= J^i + Q_i \end{cases}$$

Type II Framids



not allowed by any configuration $\vec{\eta}(x)$

$$[\bar{P}^{\mu}, Q'] = Q$$

$$\bar{H} = H - \mu Q$$

$$\nabla \pi = \partial \pi + \pi' \langle f_i^n \rangle - e^{int} \delta_i^n$$

$$\phi(x) \rightarrow \phi(x) + C + \sum_n x^n$$

Back to Cosmology (or: how I will get out of this)

Spontaneously breaking boosts is the main occupation of cosmological fields (inflaton, dark energy etc.)

Condensed matter

superfluids
solids
framids



Modified gravity

ghost-condensate
solid inflation
Einstein aether