Title: Lattice models for anomalous field theories

Date: May 29, 2014 04:00 PM

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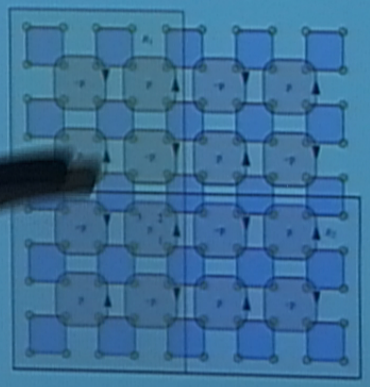
Abstract:

Lattice models for anomalous field theories

John McGreevy, UCSD

based on work with

S. M. Kravec and Brian Swingle



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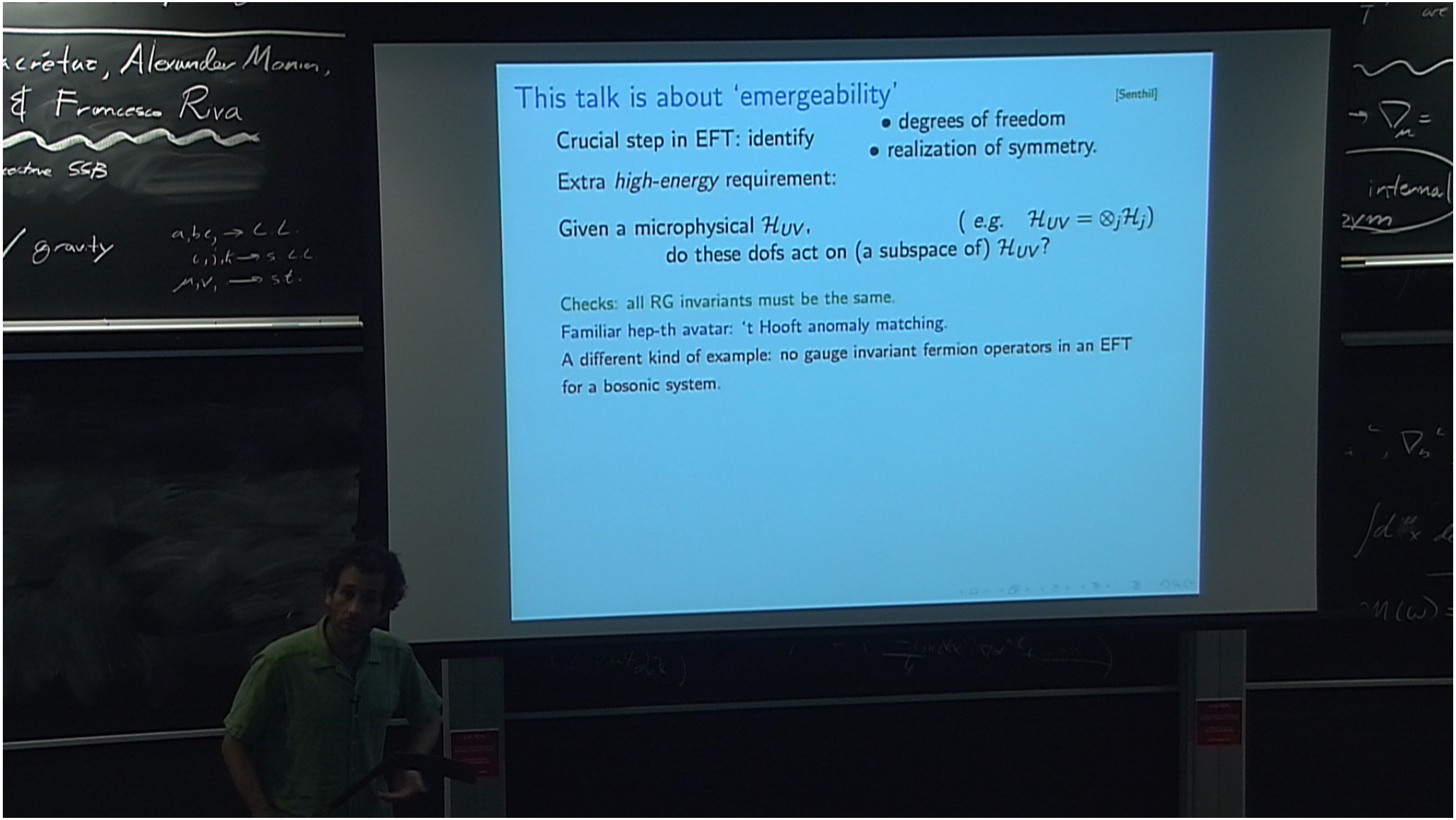
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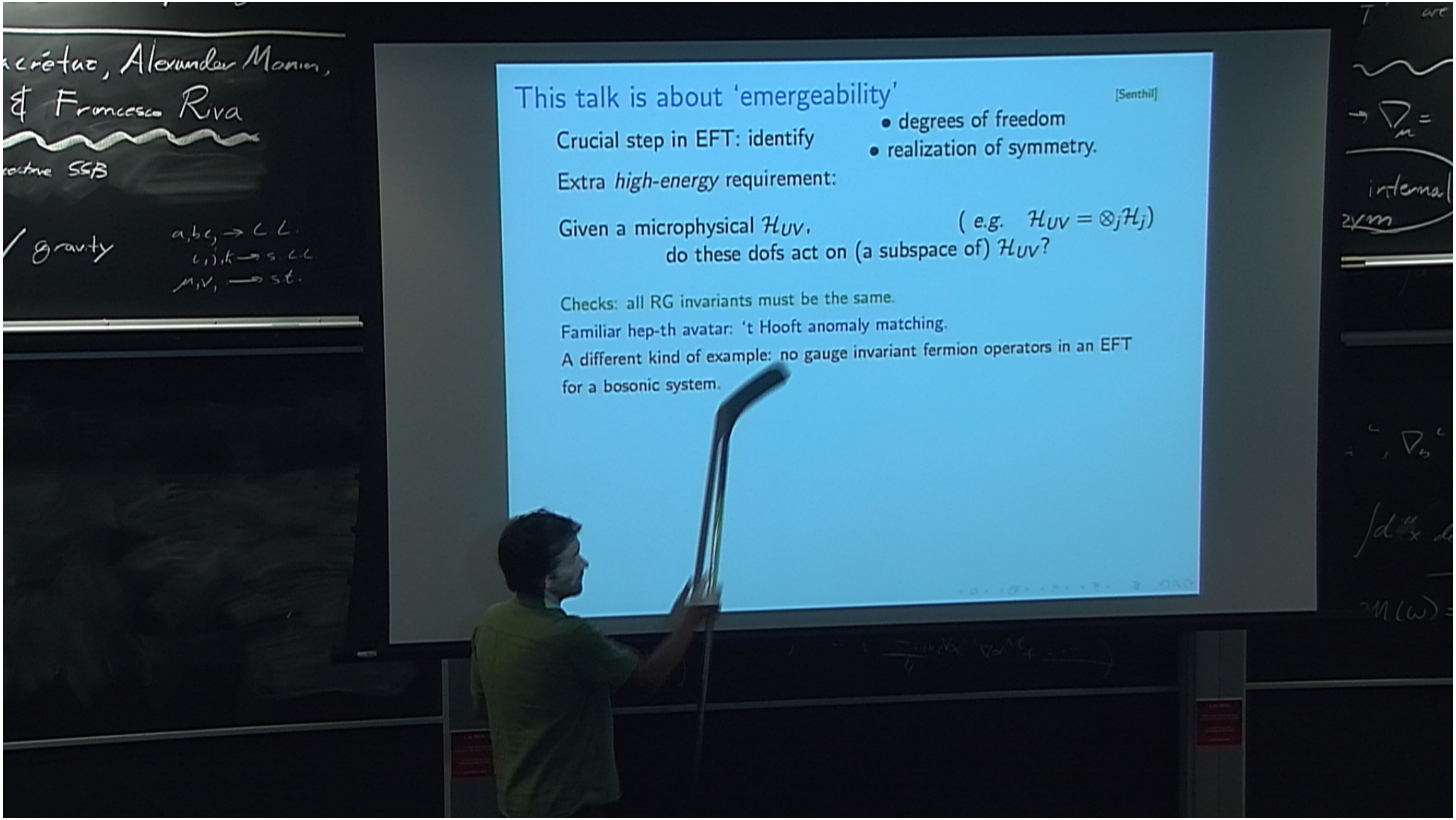
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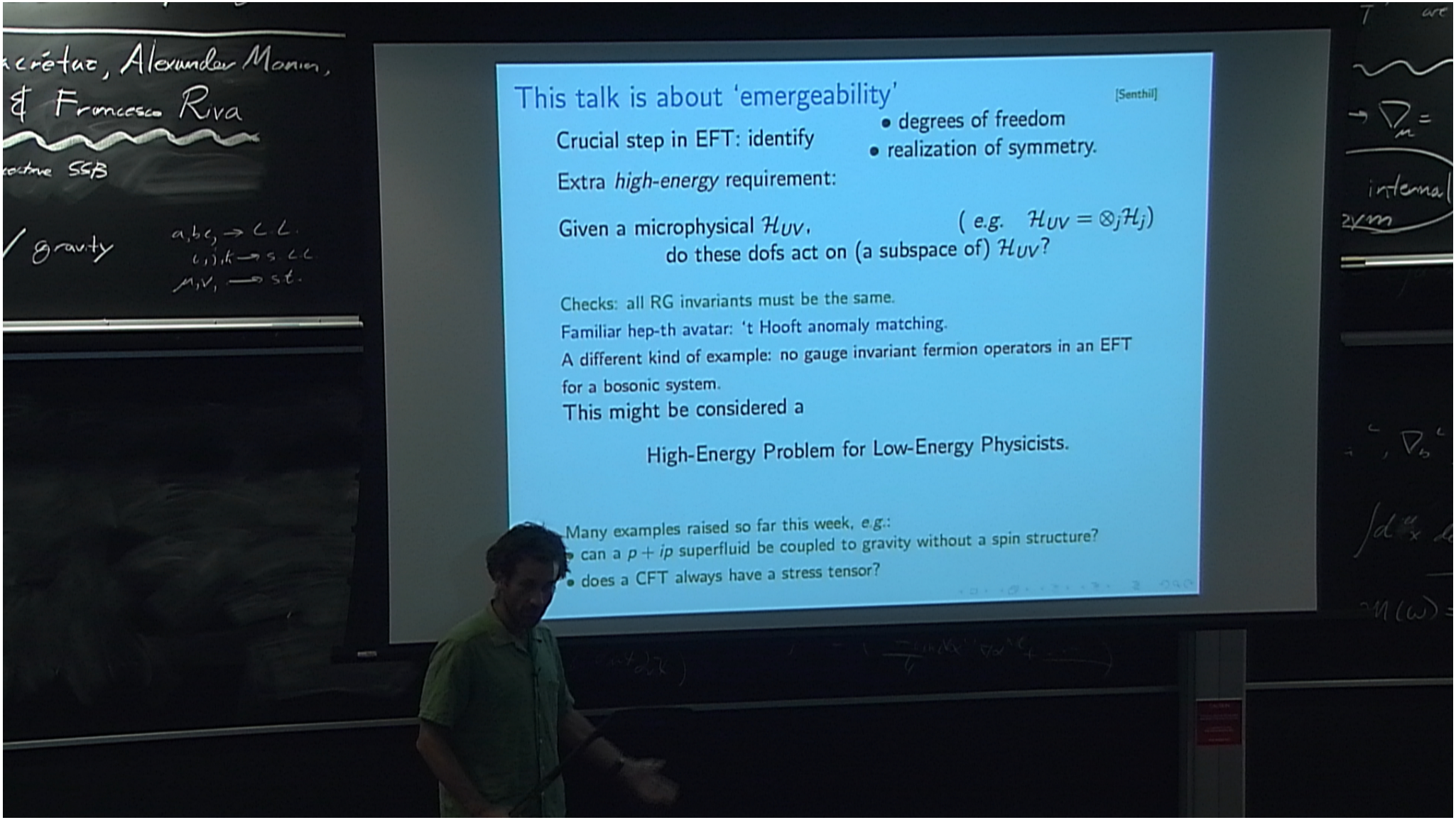
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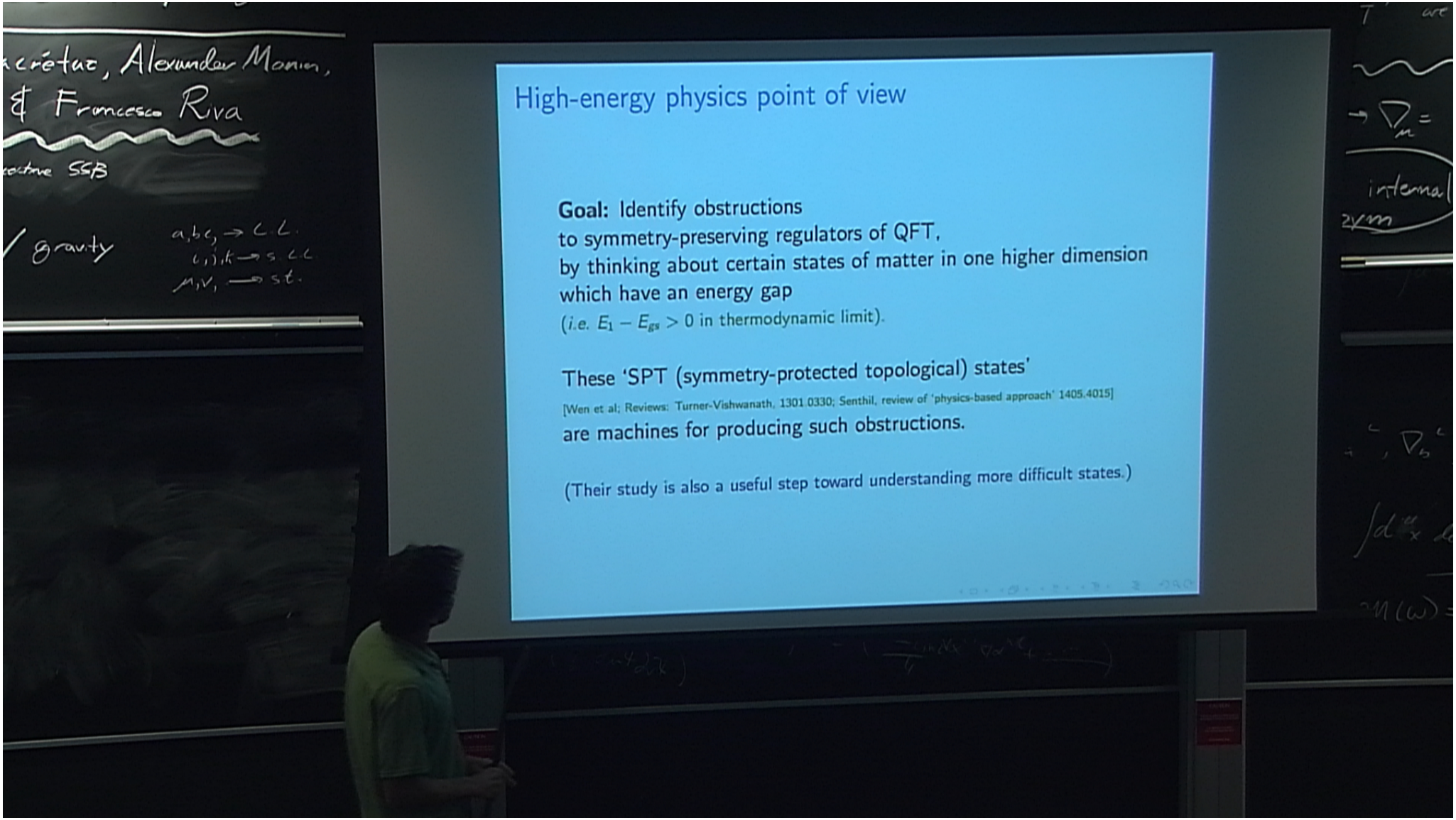
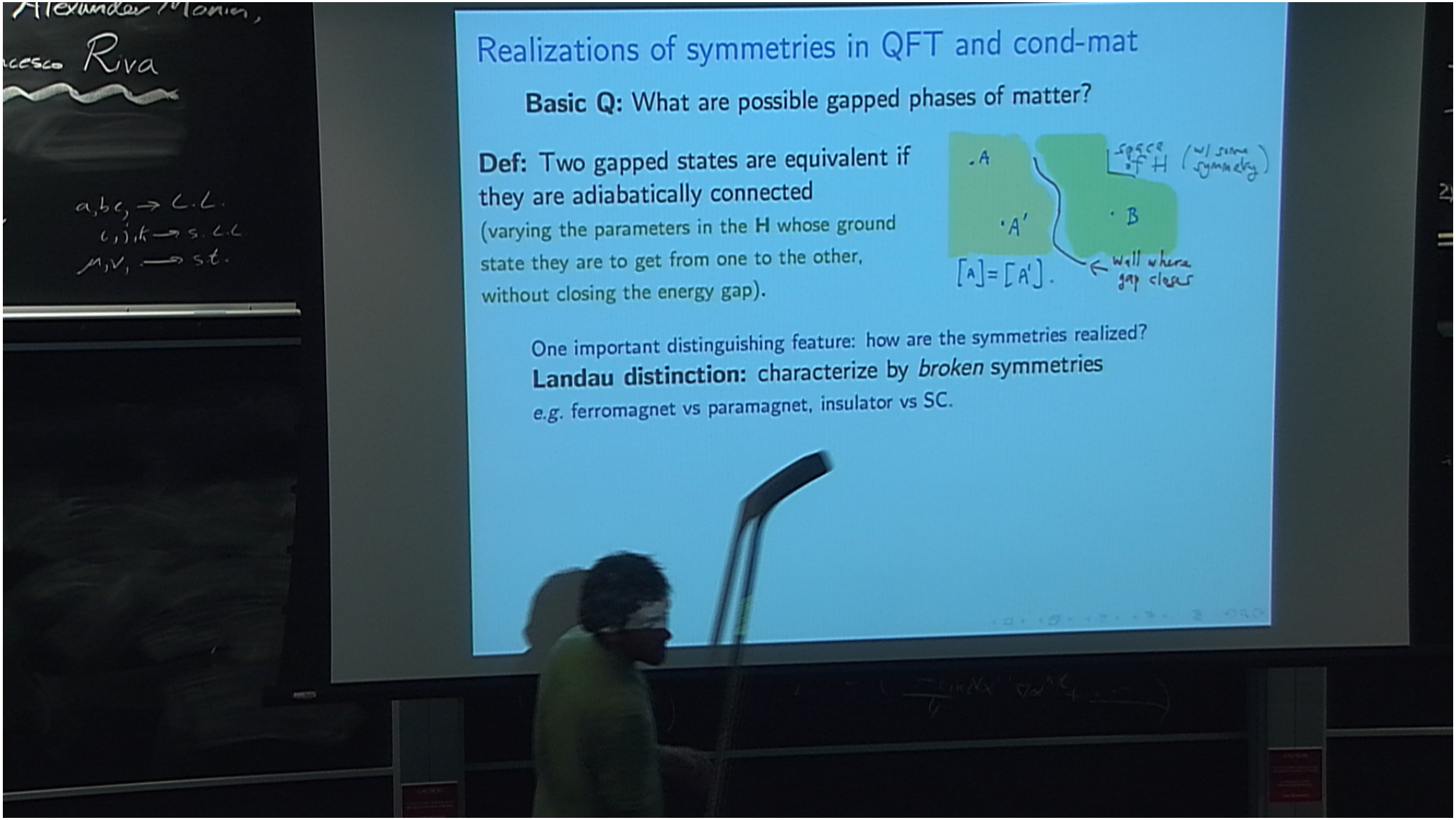
This might be considered a
High-Energy Problem for Low-Energy Physicists.
Many examples raised so far this week, e.g.:
• can a $p + ip$ superfluid be coupled to gravity without a spin structure?
• does a CFT always have a stress tensor?


High-energy physics point of view
Goal: Identify obstructions
to symmetry-preserving regulators of QFT,
by thinking about certain states of matter in one higher dimension
which have an energy gap
(i.e. $E_1 - E_{gs} > 0$ in thermodynamic limit).
These 'SPT (symmetry-protected topological) states'
[Wen et al; Reviews: Turner-Vishwanath, 1301.0330; Senthil, review of 'physics-based approach' 1405.4015]
are machines for producing such obstructions.
(Their study is also a useful step toward understanding more difficult states.)

Plan

1. Introduction
2. Ideas about regularizing the Standard Model
   [Wen, J. Wang-Wen, You-BenTov-Xu]
3. Constraints on 3+1d QFT
   from symmetry-protected topological (SPT) states
   In particular, we'll identify constraints on manifest electric-magnetic duality symmetry.
   [Shauna Kravec, JM, 1306.3992, PRL
   work in progress with Brian Swingle]
4. A machine for *explicitly* realizing SPT states
   [Shauna Kravec, JM, Brian Swingle, in progress]


Realizations of symmetries in QFT and cond-mat
Basic Q: What are possible gapped phases of matter?
Def: Two gapped states are equivalent if they are adiabatically connected
(varying the parameters in the H whose ground state they are to get from one to the other, without closing the energy gap).
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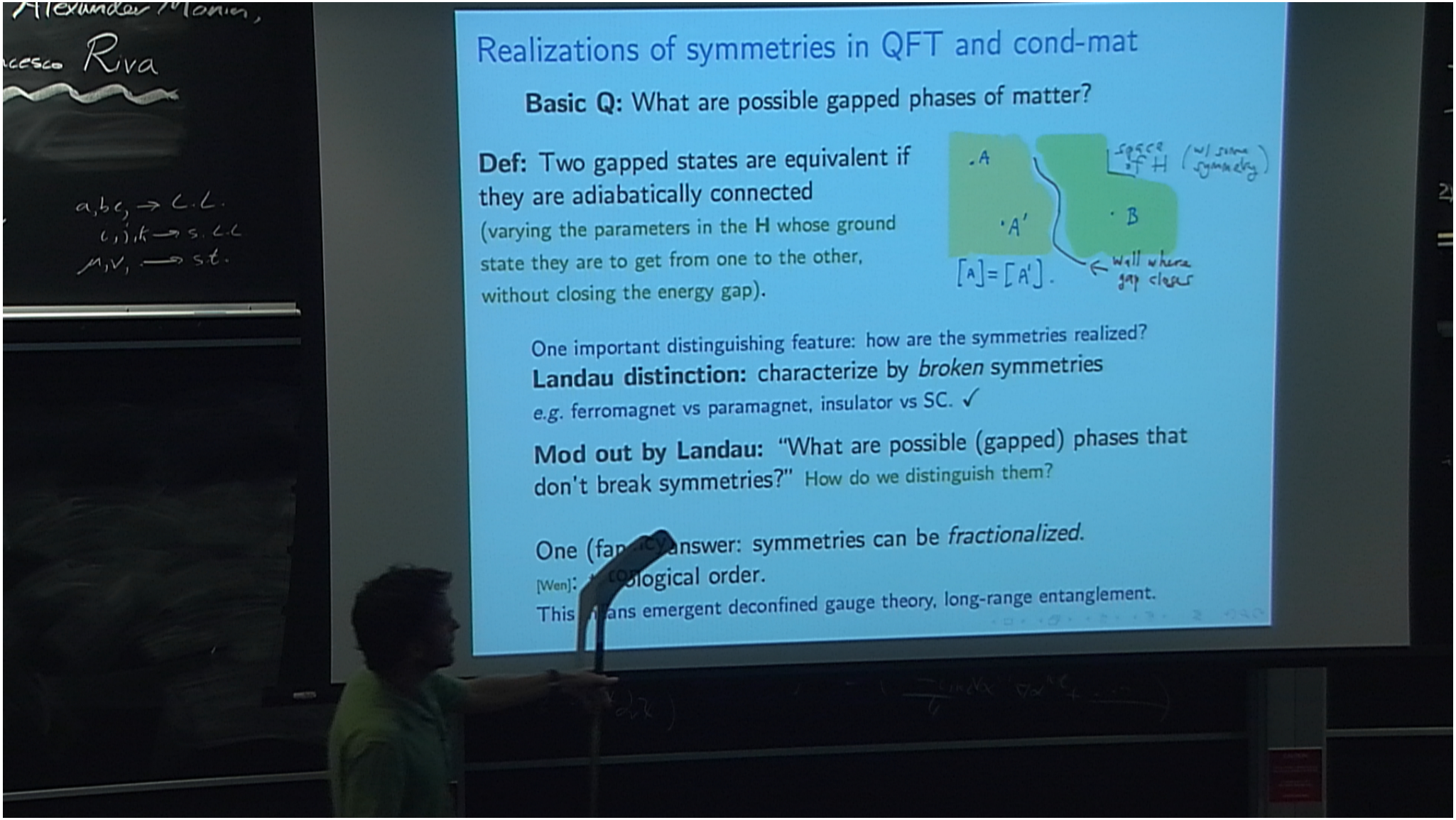
B
space of H (w/ some symmetry)
[A] = [A'].
wall where gap closes
One important distinguishing feature: how are the symmetries realized?
Landau distinction: characterize by broken symmetries
e.g. ferromagnet vs paramagnet, insulator vs SC.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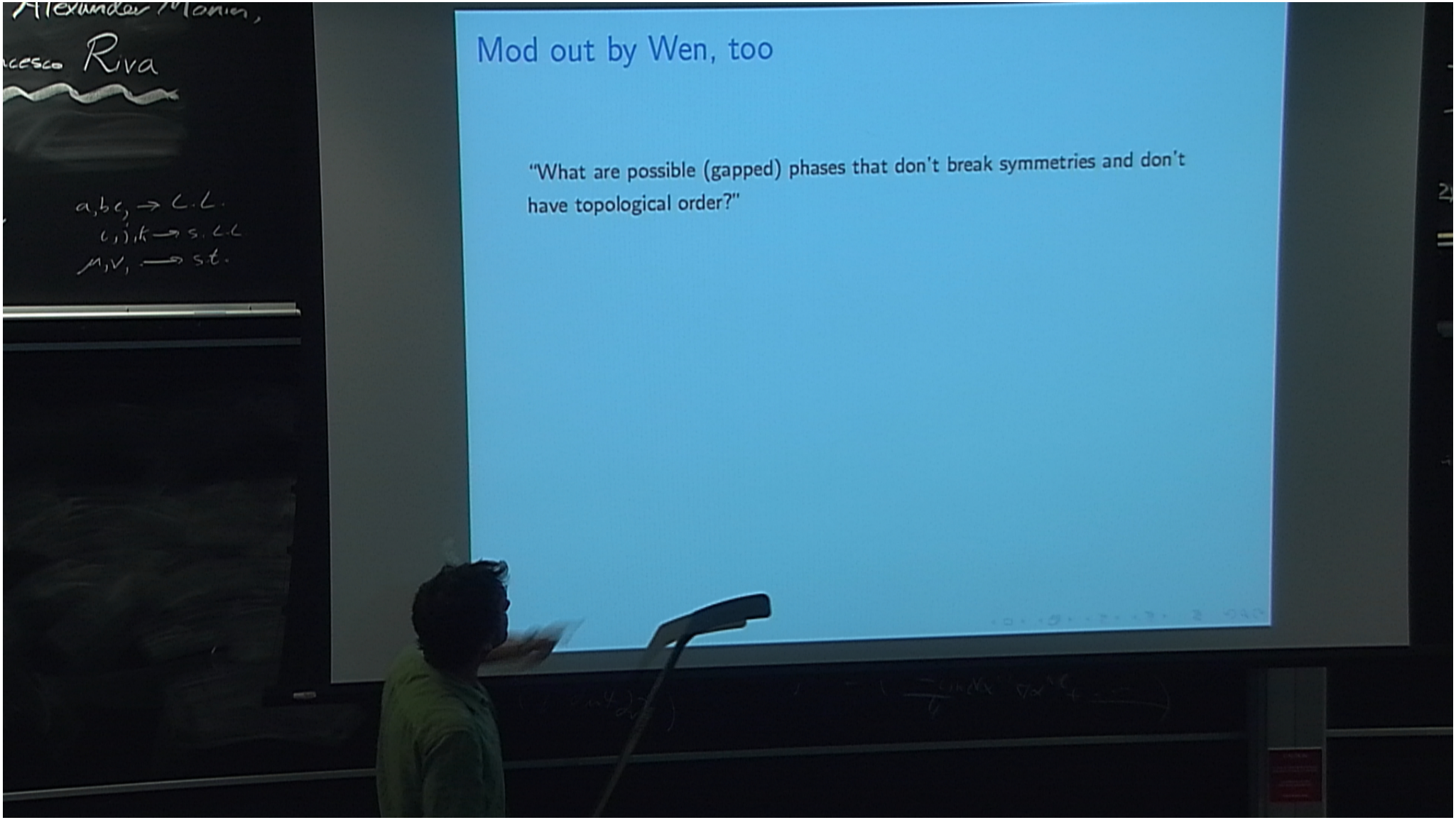
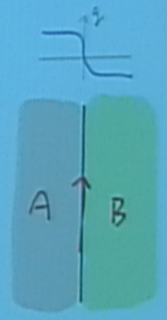
Mod out by Landau: "What are possible (gapped) phases that don't break symmetries?" How do we distinguish them?
One (fa answer: symmetries can be fractionalized.
[Wen]: t pological order.
This means emergent deconfined gauge theory, long-range entanglement.

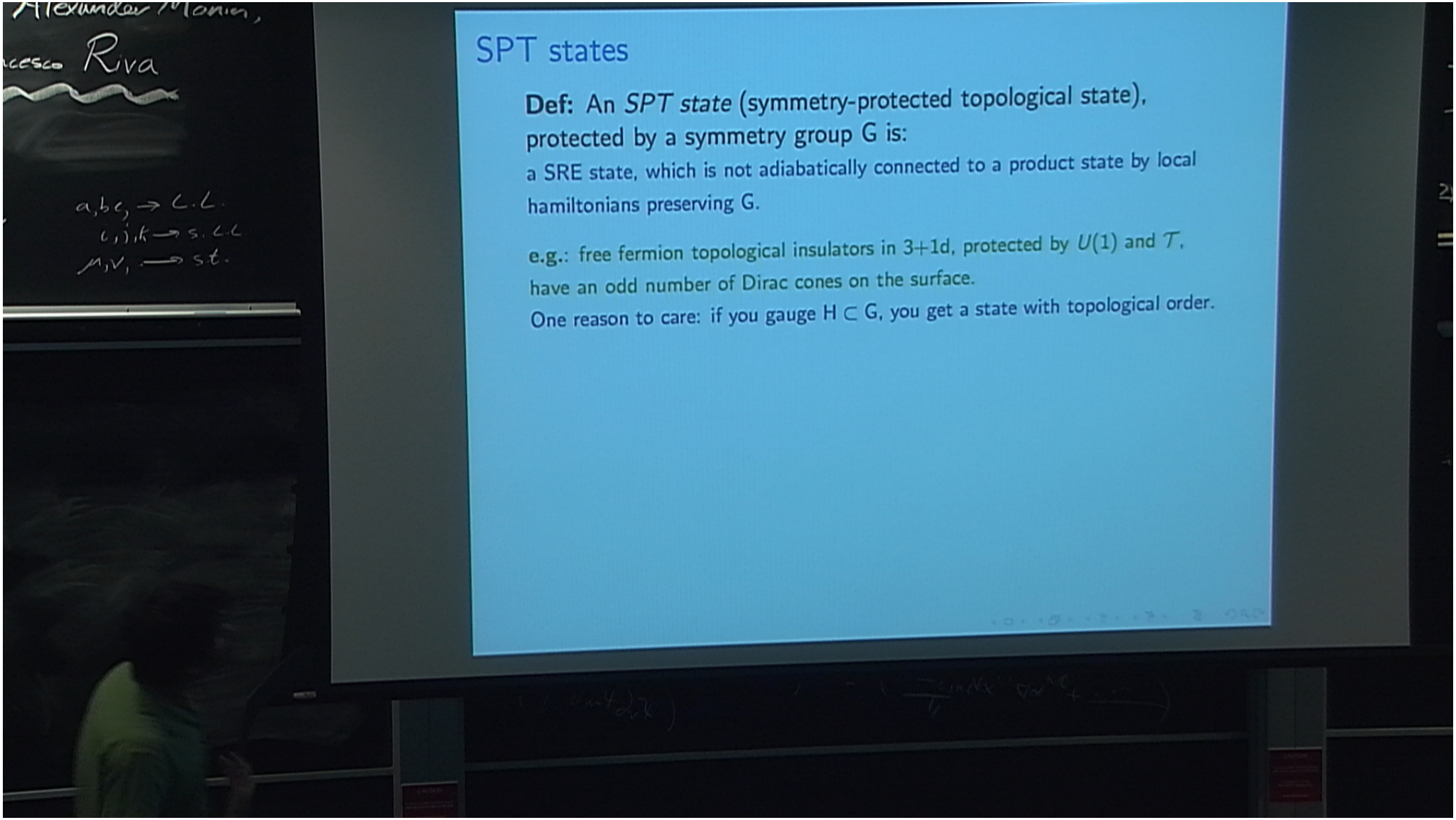

Mod out by Wen, too
"What are possible (gapped) phases that don't break symmetries and don't have topological order?"

SRE states are characterized by their edge states

Rough idea: just like varying the Hamiltonian in time to another phase requires closing the gap $\mathbf{H} = \mathbf{H}_1 + g(t)\mathbf{H}_2$, so does varying the Hamiltonian in space $\mathbf{H} = \mathbf{H}_1 + g(x)\mathbf{H}_2$.



- Important role of SRE assumption: Here we are assuming that the bulk state has short-ranged correlations, so that changes we might make at the surface cannot have effects deep in the bulk.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Def: An SPT state (symmetry-protected topological state), protected by a symmetry group G is:
a SRE state, which is not adiabatically connected to a product state by local hamiltonians preserving G.
e.g.: free fermion topological insulators in 3+1d, protected by U(1) and T, have an odd number of Dirac cones on the surface.
One reason to care: if you gauge H ⊂ G, you get a state with topological order.

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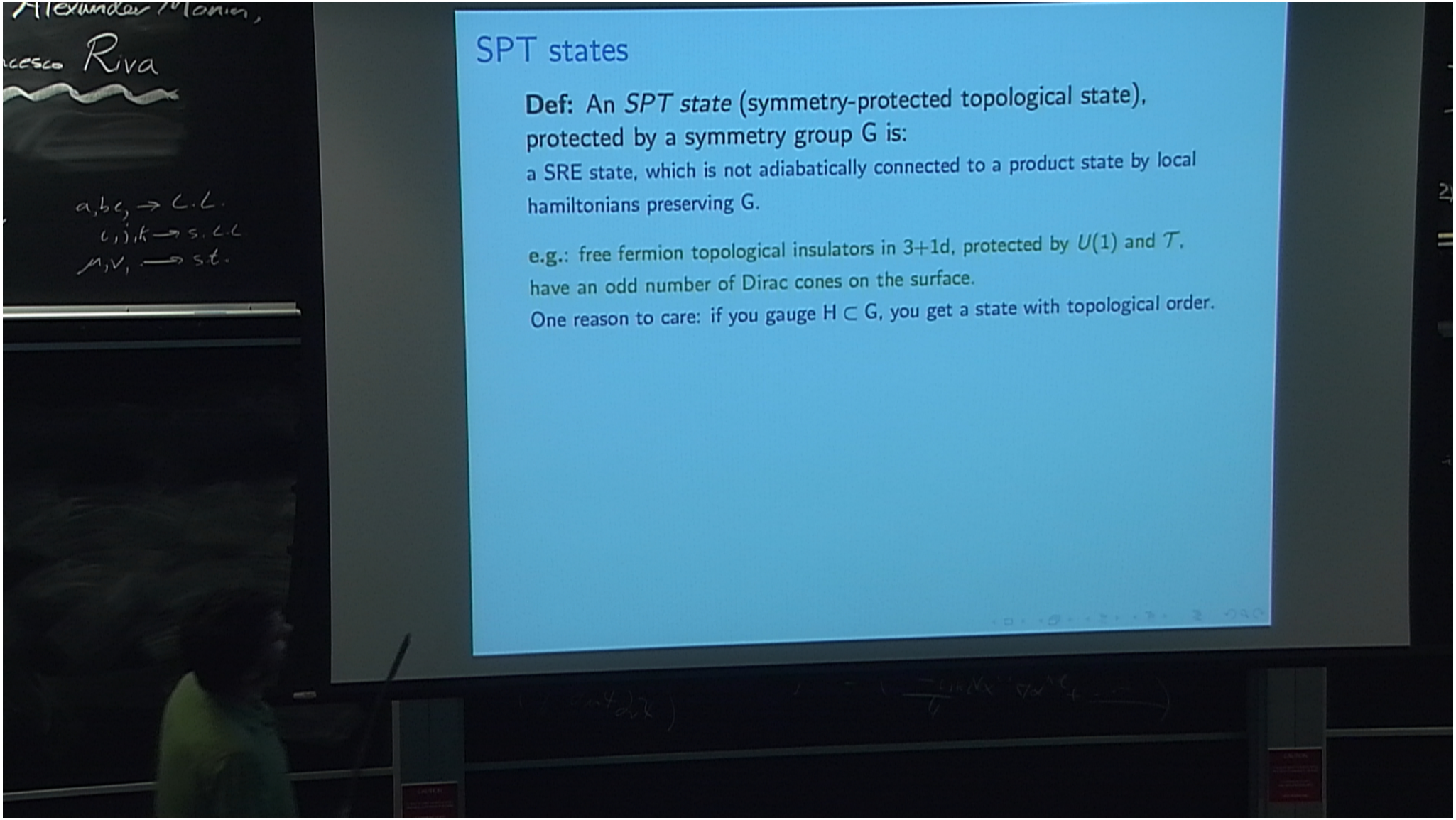
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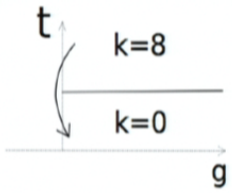
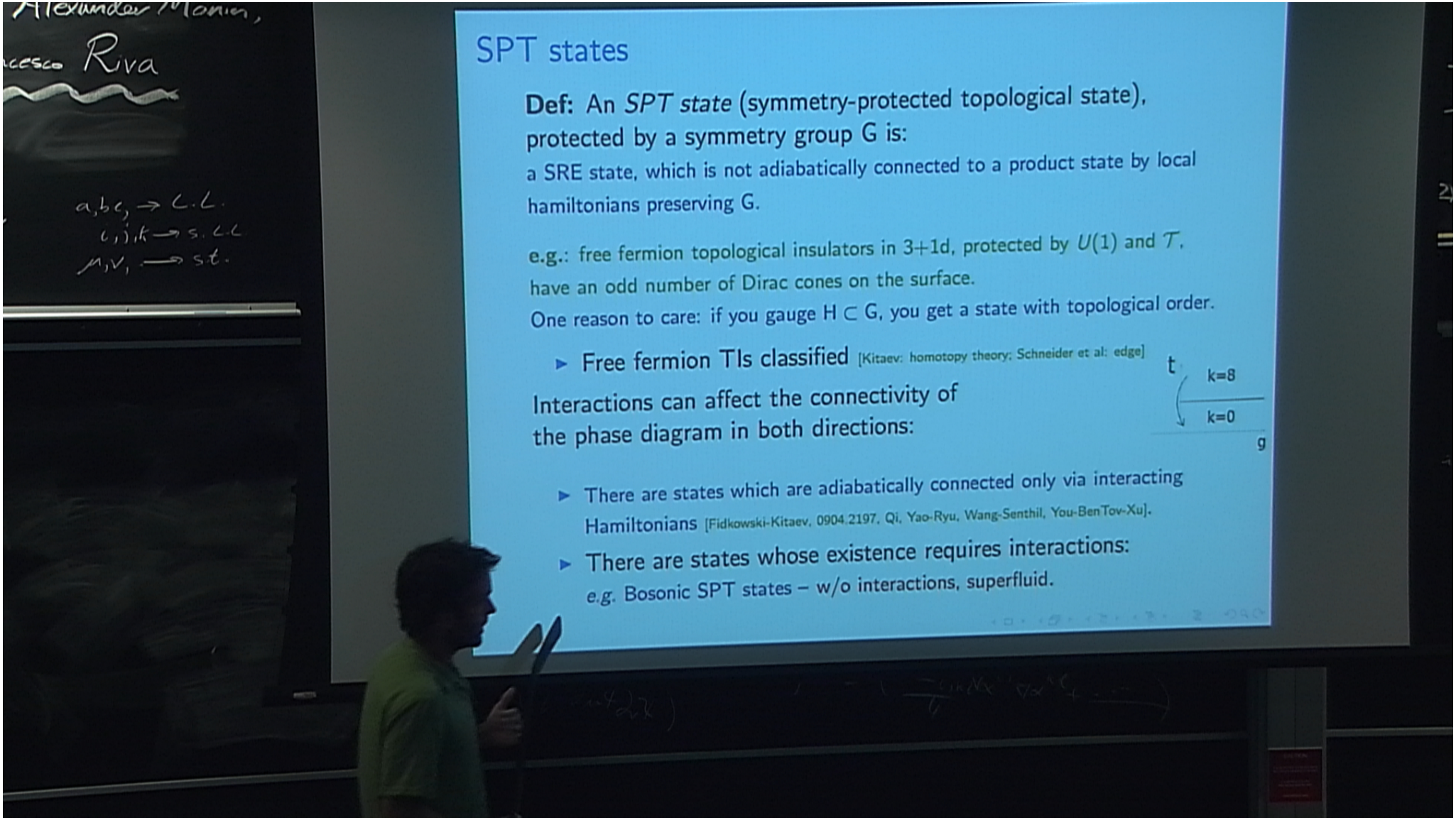
- Free fermion TIs classified [Kitaev: homotopy theory; Schneider et al: edge]

Interactions can affect the connectivity of the phase diagram in both directions:



- There are states which are adiabatically connected only via interacting Hamiltonians [Fidkowski-Kitaev, 0904.2197, Qi, Yao-Ryu, Wang-Senthil, You-BenTov-Xu].
- There are states whose existence requires interactions:
  e.g. Bosonic SPT states – w/o interactions, superfluid.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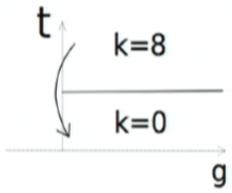

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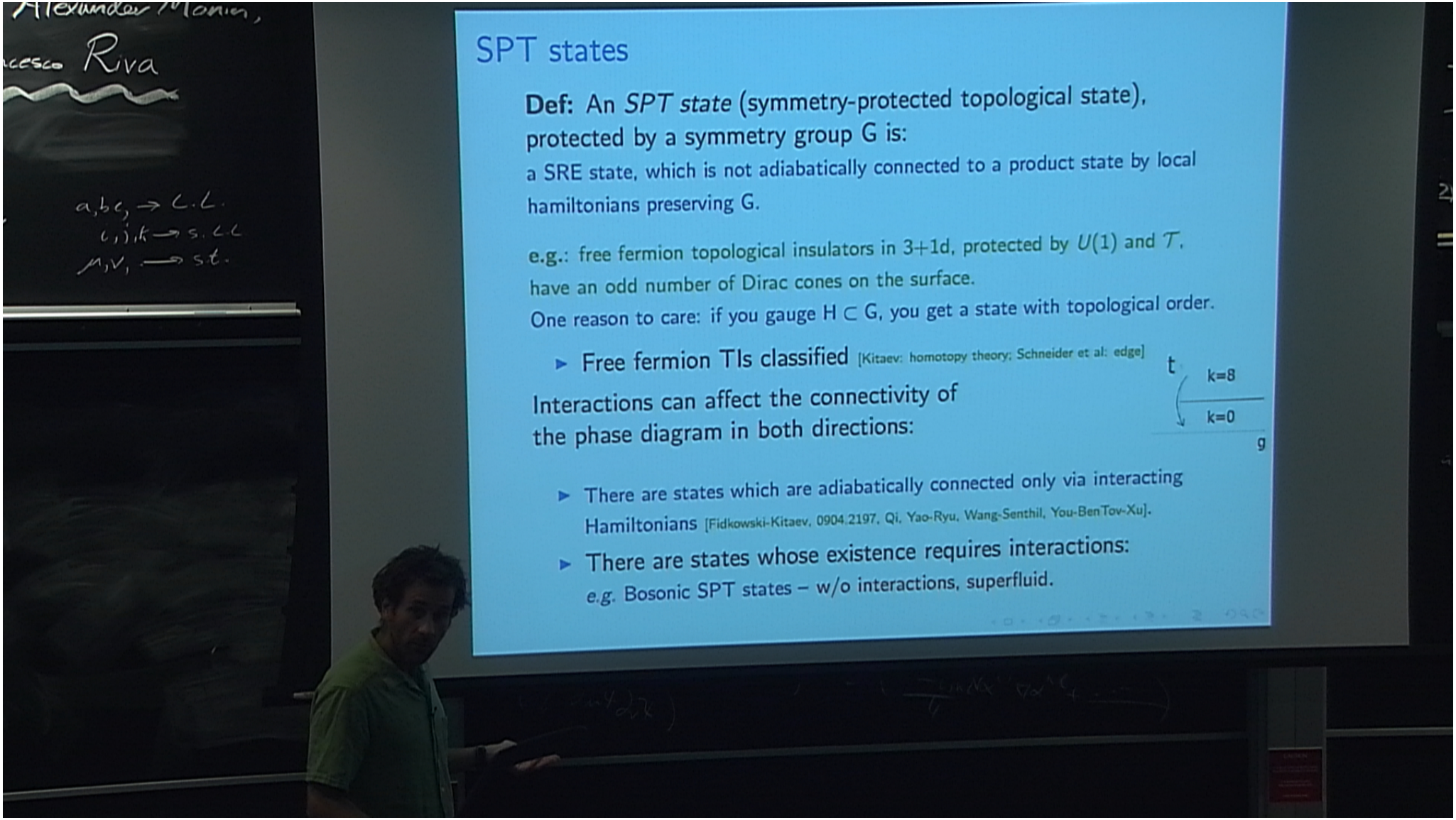
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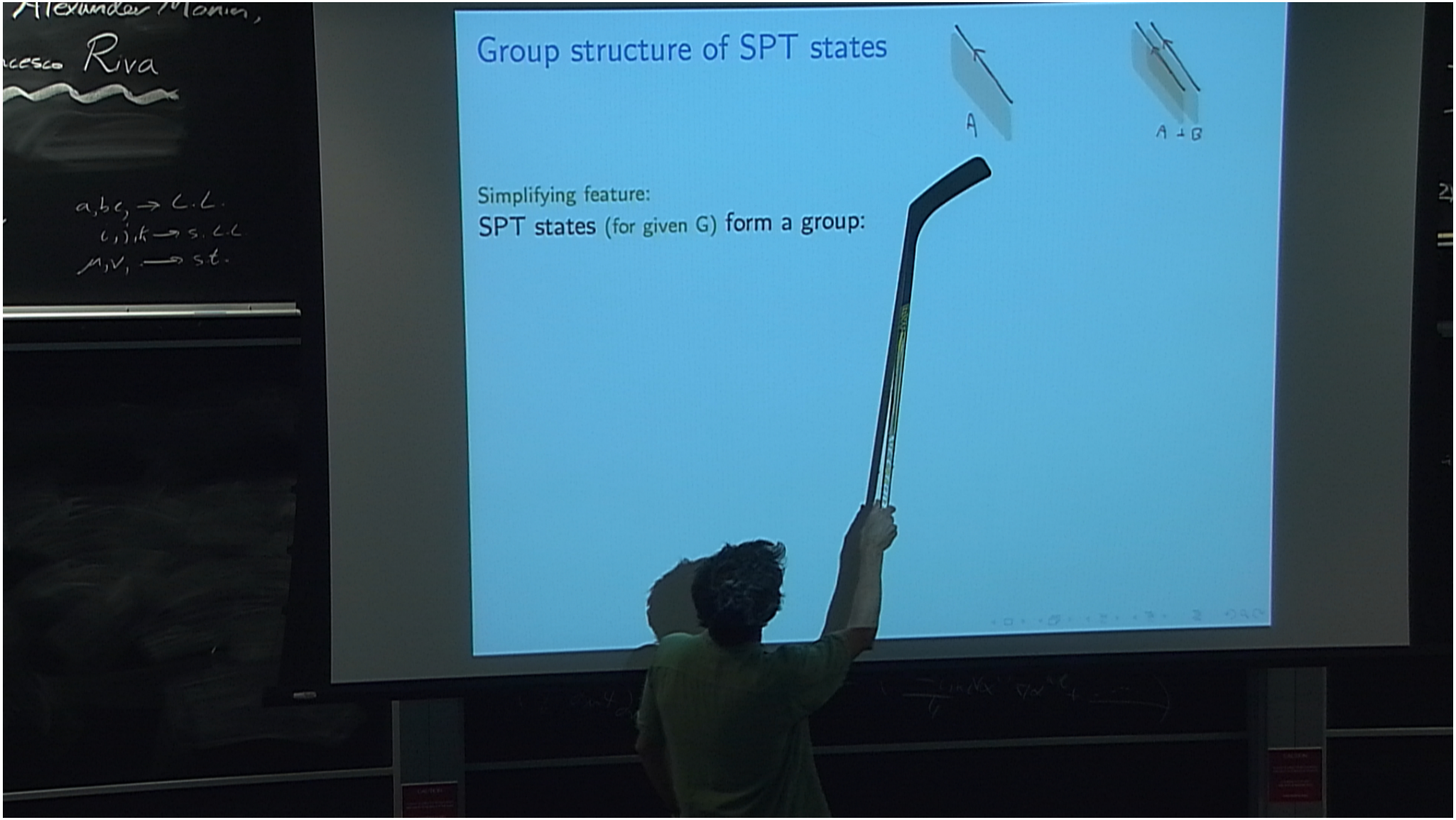
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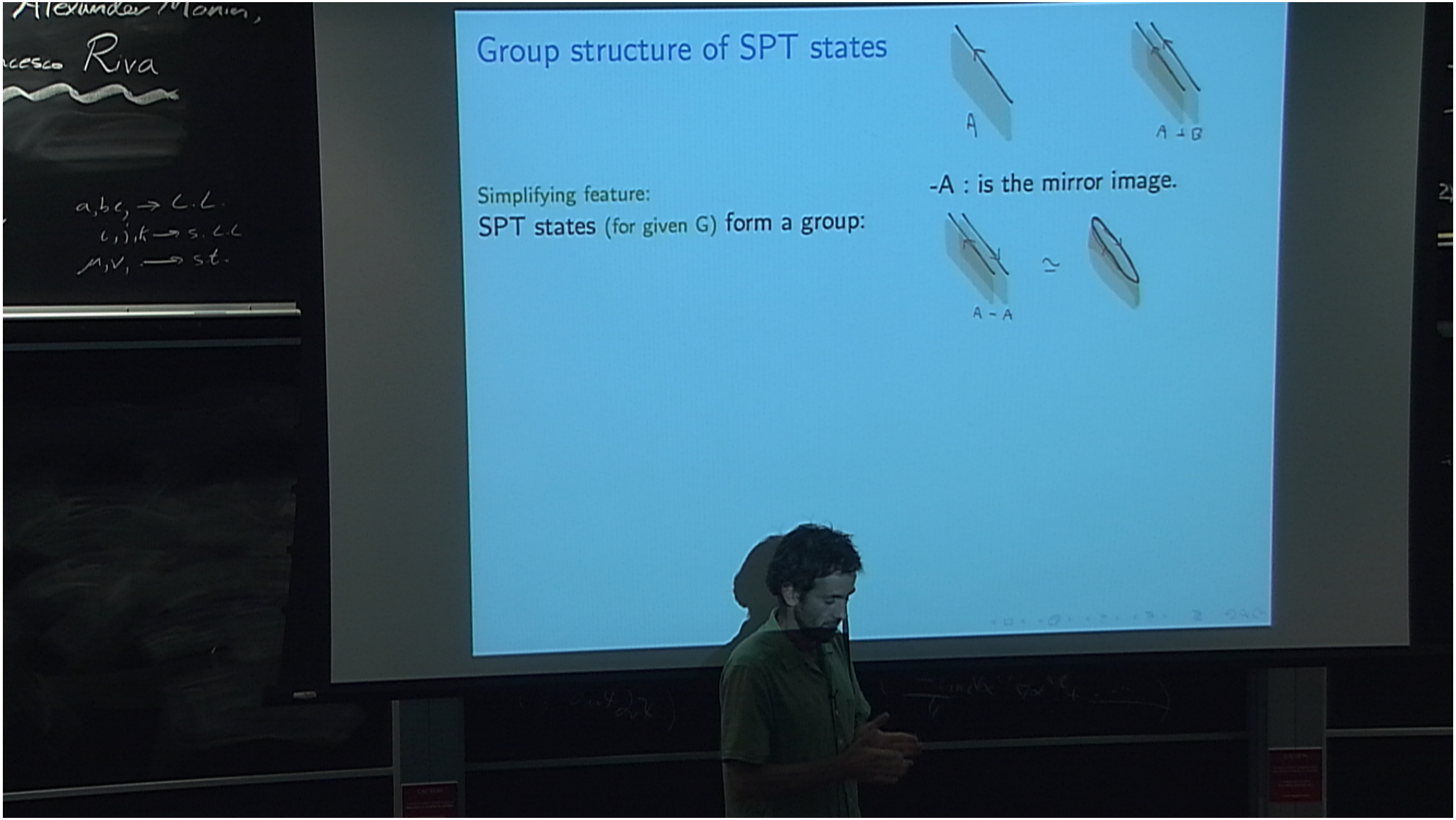
SPT states (for given G) form a group: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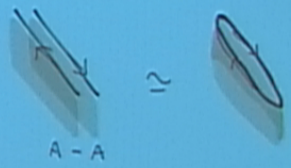

- With bulk topological order, bulk quasiparticles still nontrivial. Not a group.
- There can be many realizations of the edge of A, same 'SPT-ness'.
- The edge of A can be symmetric and gapped but topologically ordered.

Inverse of A cancels the SPT-ness of A's edge.

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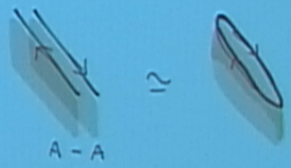

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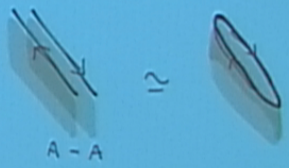

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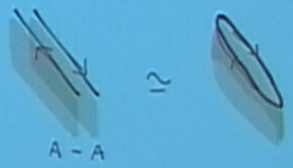

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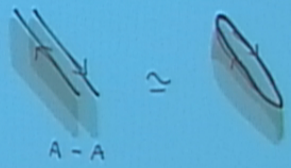

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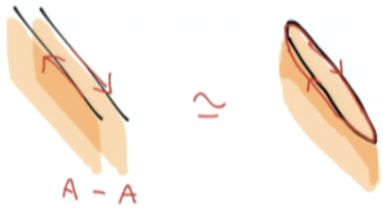

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Surface-only models

Counterfactual:

Suppose the edge theory of an SPT state were realized *otherwise* – intrinsically in D dimensions, with a local hamiltonian.

Then we could paint that the conjugate local theory on the surface without changing anything about the bulk state.

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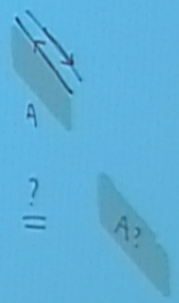
Suppose the edge theory of an SPT state were realized *otherwise* – intrinsically in D dimensions, with a local hamiltonian.

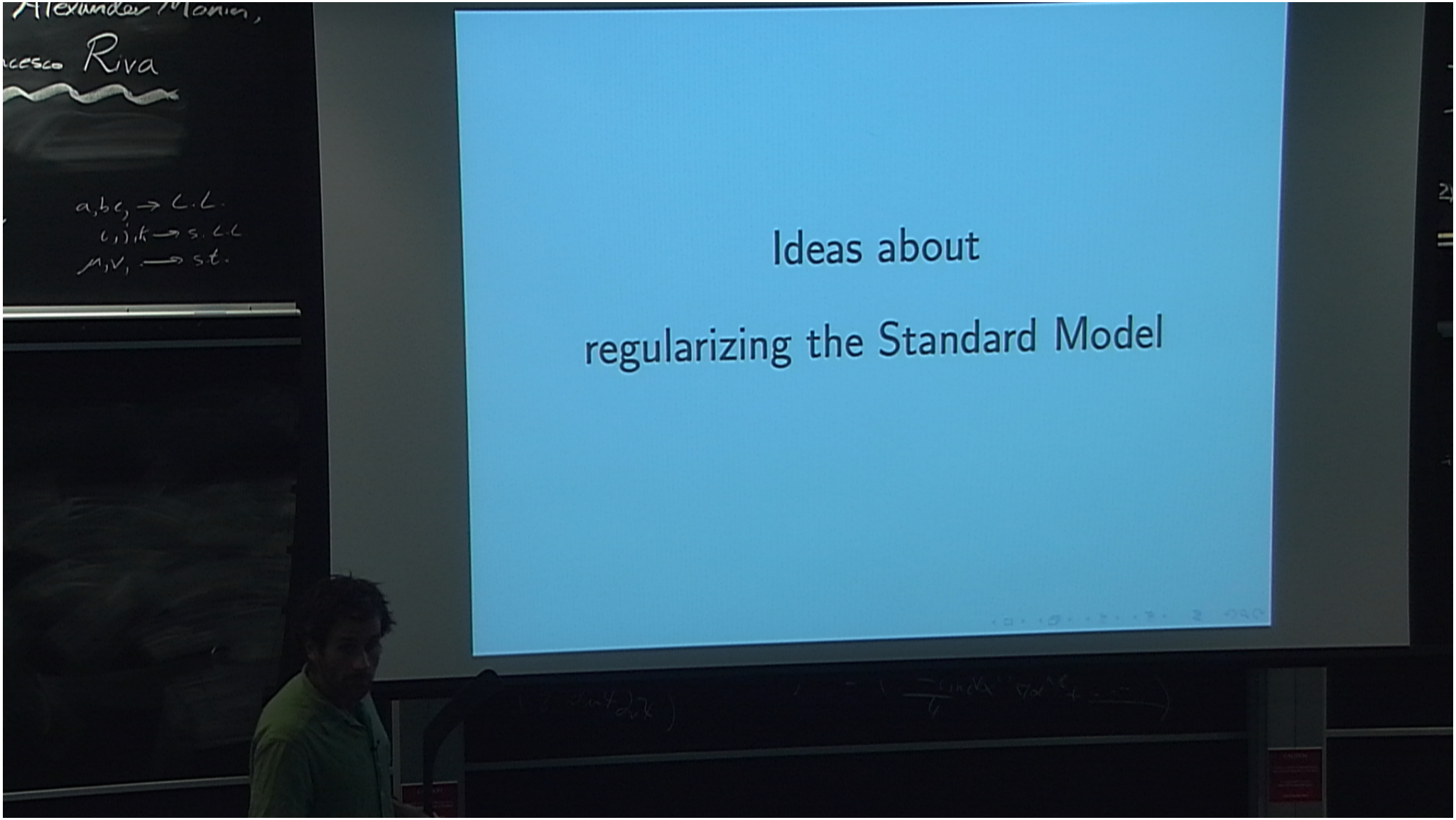
Then we could paint that the conjugate local theory on the surface without changing anything about the bulk state. And then small deformations of the surface Hamiltonian, localized on the surface, consistent with symmetries, can pair up the edge states.

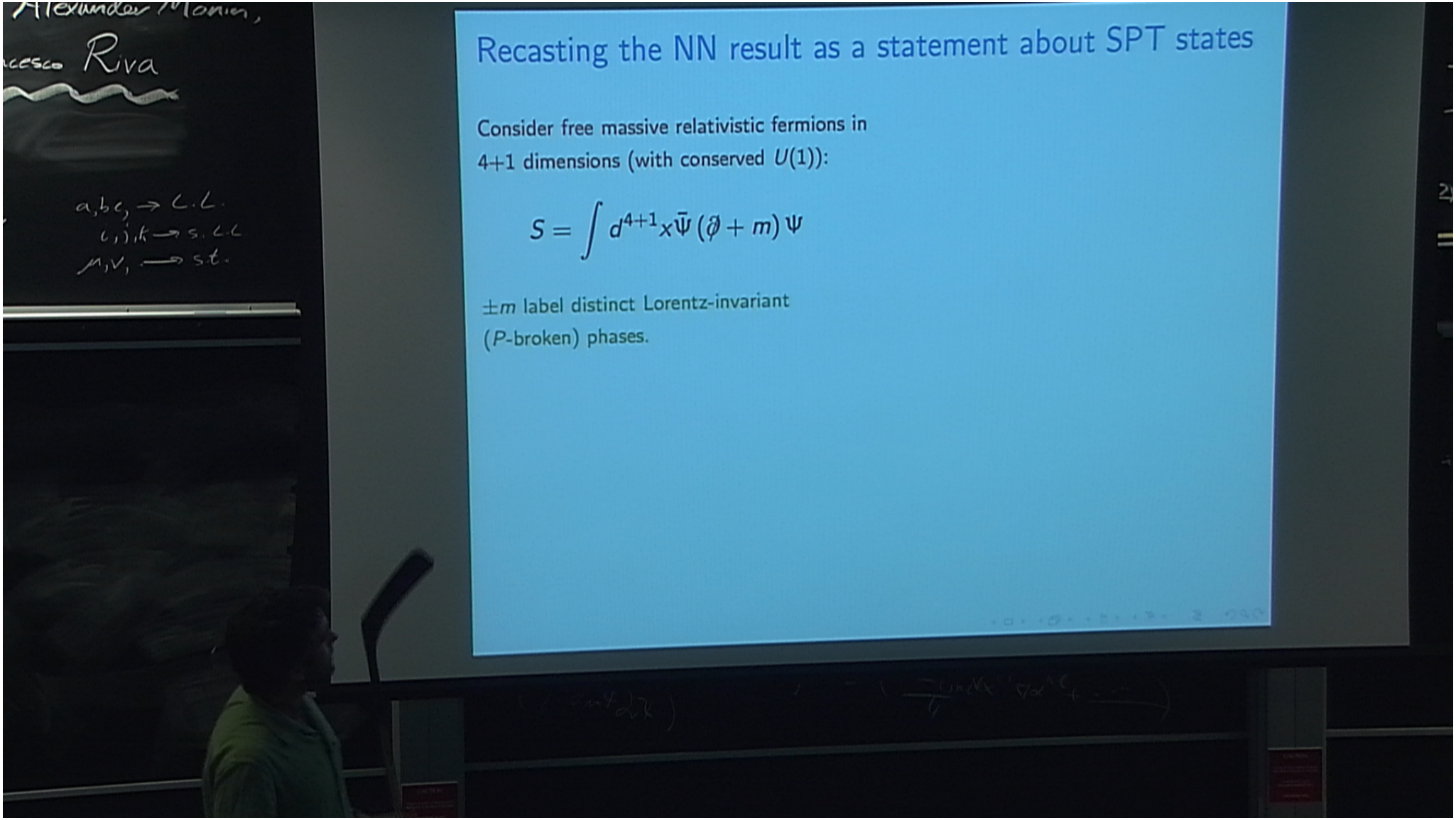


But this contradicts the claim that we could characterize the $D+1$-dimensional SPT state by its edge theory.

Conclusion: Edge theories of SPT_G states cannot be regularized intrinsically in D dims, *exactly* preserving on-site G – "surface-only models" or "not edgeable".

[Wang-Senthil, 1302.6234 – general idea, concrete surprising examples of 2+1 surface-only states
Wen, 1303.1803 – attempt to characterize the underlying mathematical structure, classify *all* such obstructions
Metlitski-Kane-Fisher, 1302.6535; Burnell-Chen-Fidkowski-Vishwanath, 1302.7072]


Ideas about
regularizing the Standard Model


Recasting the NN result as a statement about SPT states
Consider free massive relativistic fermions in 4+1 dimensions (with conserved $U(1)$):
$$S = \int d^{4+1}x \bar{\Psi} \left(\not{\partial} + m \right) \Psi$$
$\pm m$ label distinct Lorentz-invariant (P-broken) phases.

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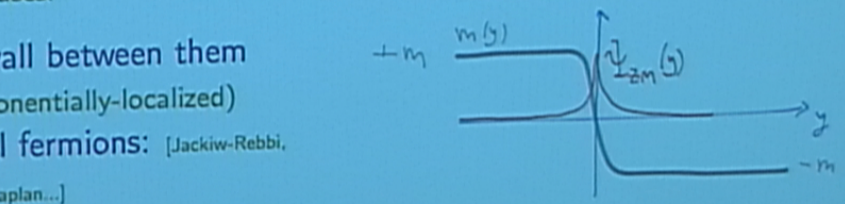
$\pm m$ label distinct Lorentz-invariant (P-broken) phases.

Domain wall between them hosts (exponentially-localized) 3+1 chiral fermions: [Jackiw-Rebbi, Callan-Harvey, Kaplan...]

One proof of this:
Couple to external gauge field
$\Delta S = \int d^5x A^\mu \bar{\Psi} \gamma_\mu \Psi$.

$$\log \int [D\Psi] e^{iS_{4+1}[\Psi, A]} \propto \frac{m}{|m|} \int A \wedge F \wedge F$$



Galling fact: if we want the extra dimension to be finite, there's another domain wall with the antichiral fermions.
...d if we put it too far away, the KK gauge bosons are too light...

Recasting the NN result as a statement about SPT states

Consider free massive relativistic fermions in 4+1 dimensions (with conserved $U(1)$):

$$S = \int d^{4+1}x \bar{\Psi} \left(\not{\partial} + m\right) \Psi$$

$\pm m$ label distinct Lorentz-invariant (P-broken) phases.

Domain wall between them hosts (exponentially-localized) 3+1 chiral fermions: [Jackiw-Rebbi, Callan-Harvey, Kaplan...]

One proof of this:
Couple to external gauge field
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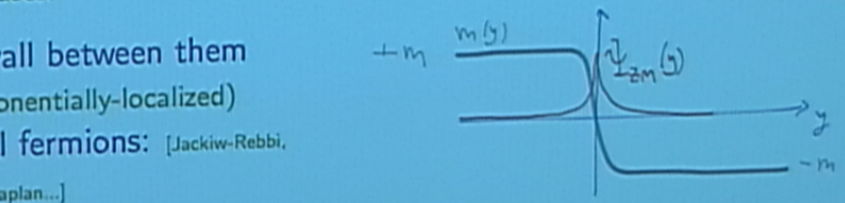

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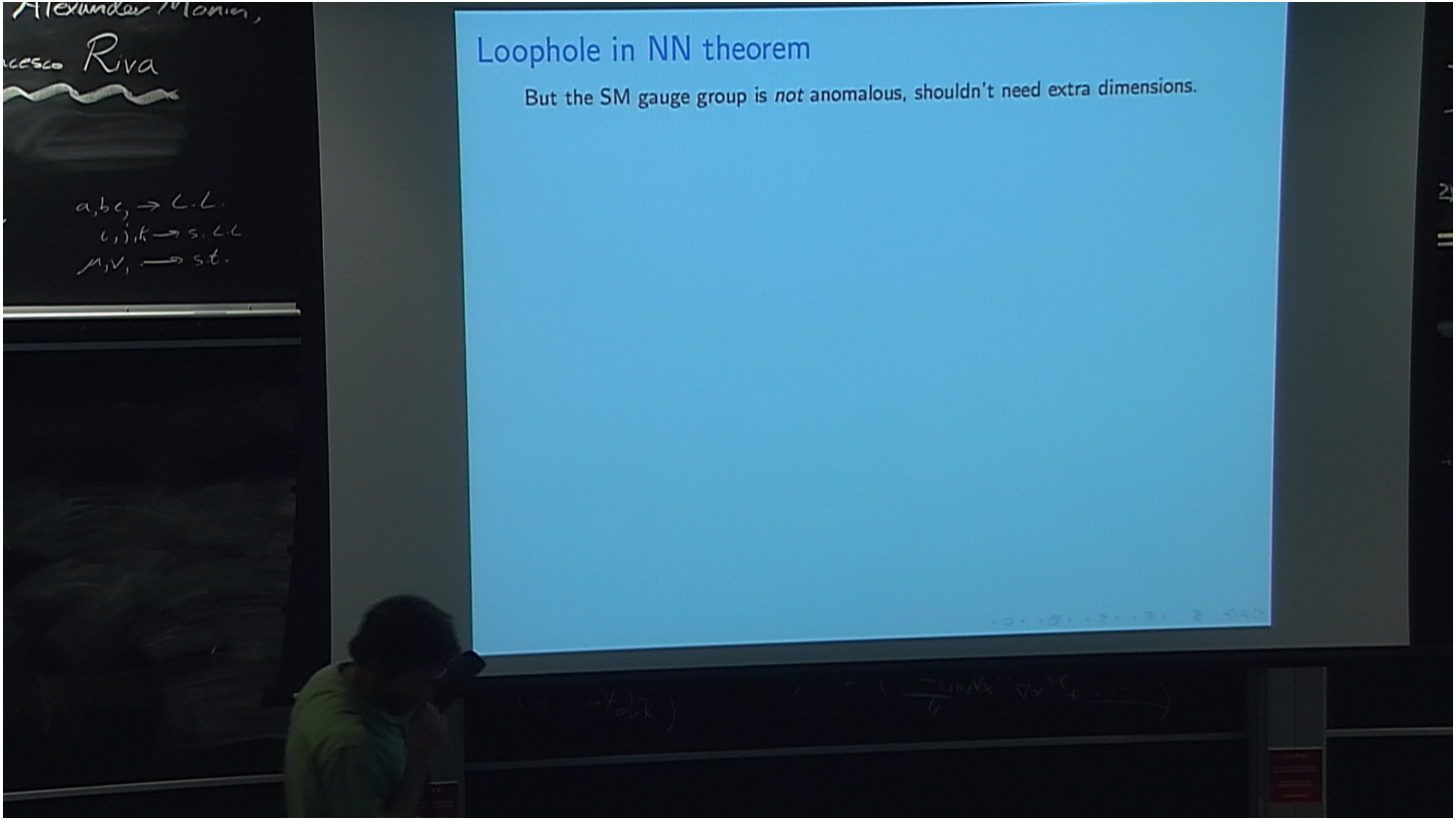
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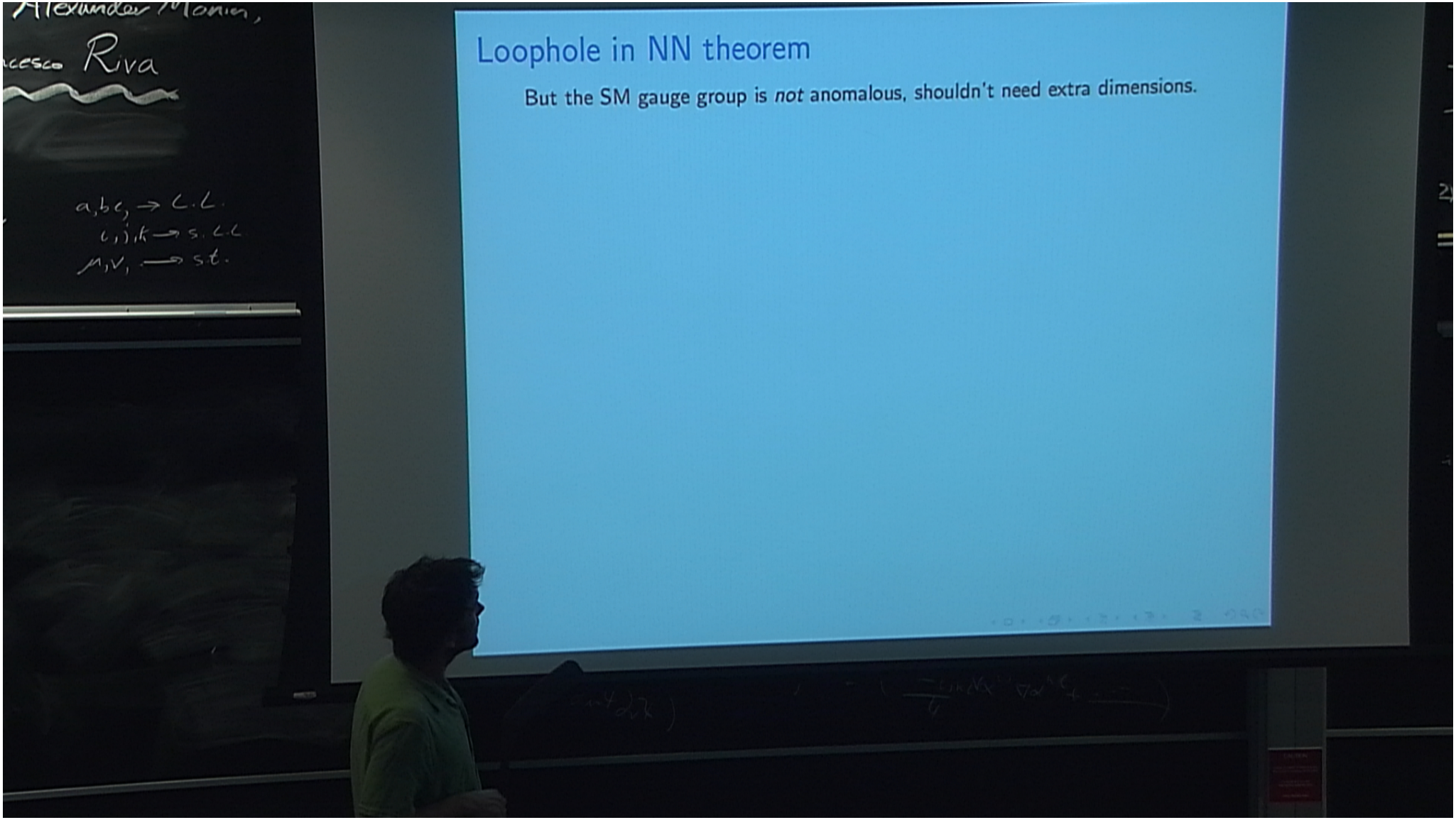
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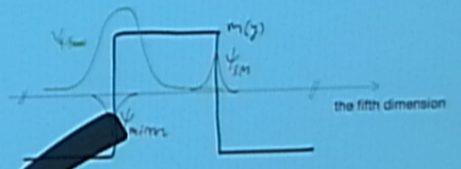

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New evidence for a special role of $n_F = 16 \cdot n$

Collapse of free-fermion classification:

Dimensional recursion strategy [Wang-Senthil, Qi, Ryu-Yao, Wen, You-BenTov-Xu]:

1. Consider neighboring phase where G is spontaneously broken $\langle \phi \rangle \neq 0$.
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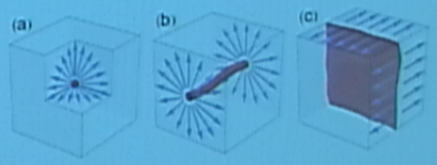

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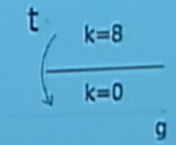

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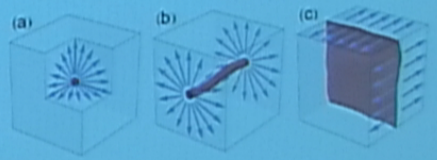

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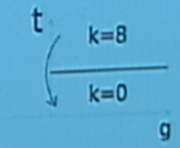

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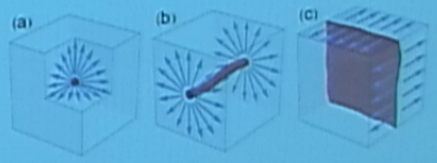

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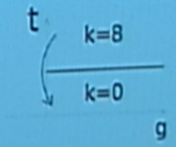
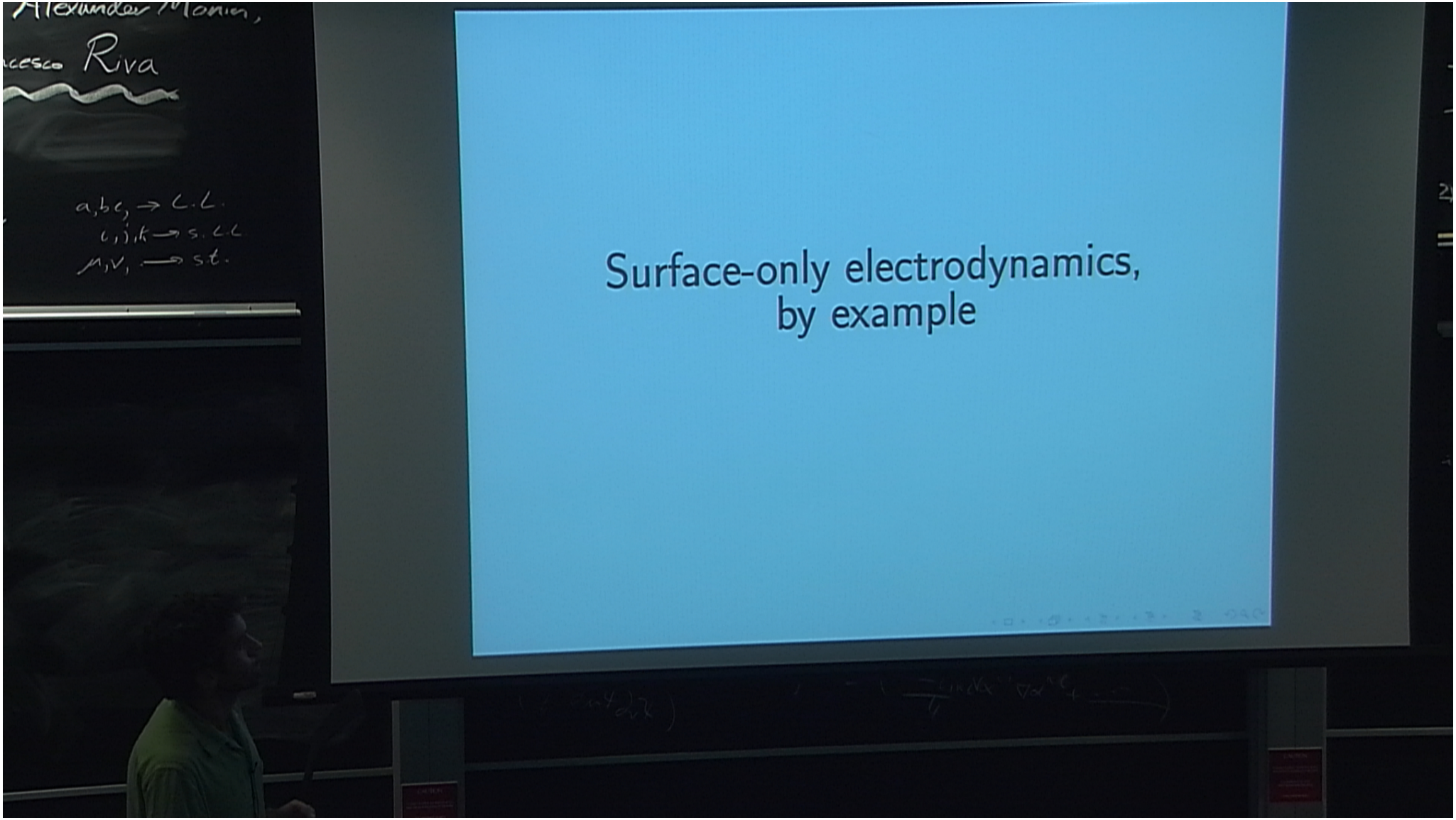


[You-BenTov-Xu]: In 4+1d, with many G, the collapse again happens at $k = 8 \simeq 0$ ($\rightarrow$ 16 Weyl fermions per domain wall.) awkward: $G \supset \mathbb{Z}_2^T$

Conclusion: This novel strategy for identifying obstructions to gapping the mirror fermions shows none when $n_F = 16n$.


Surface-only electrodynamics,
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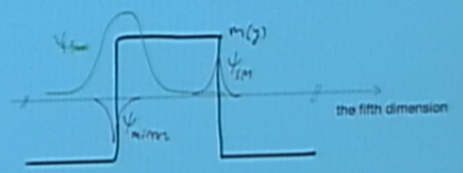

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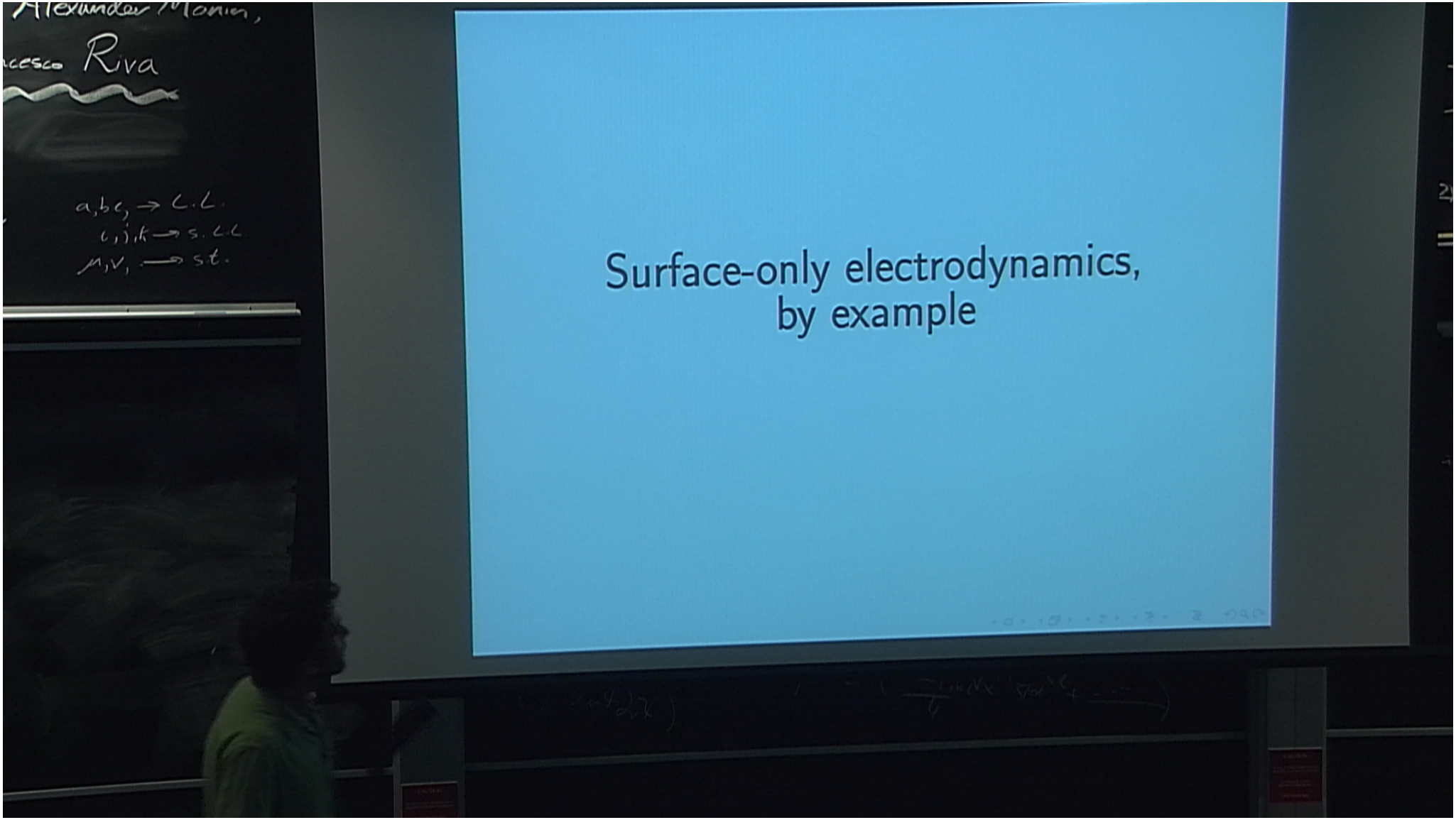
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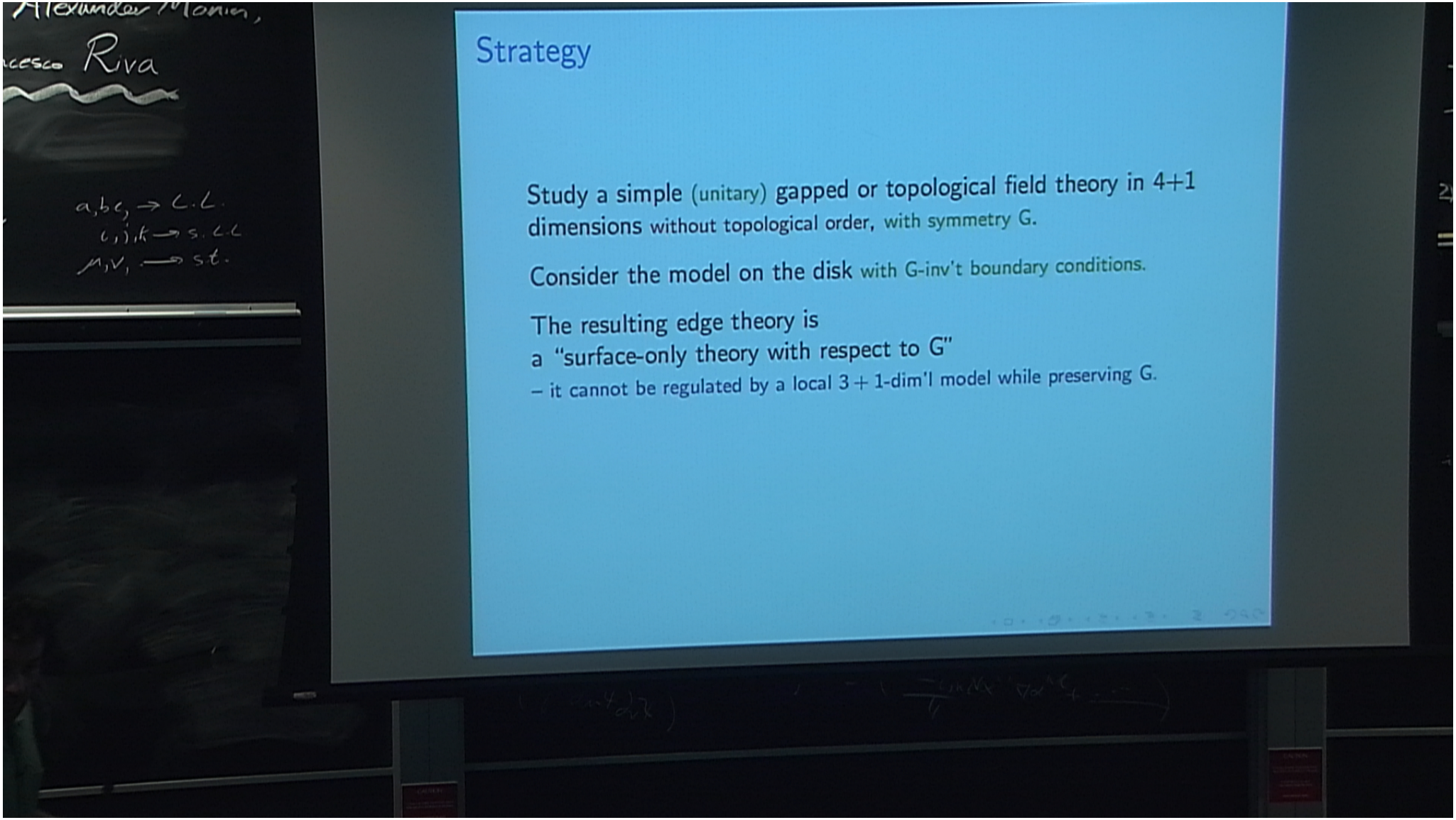
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Consider the model on the disk with G-inv't boundary conditions.
The resulting edge theory is
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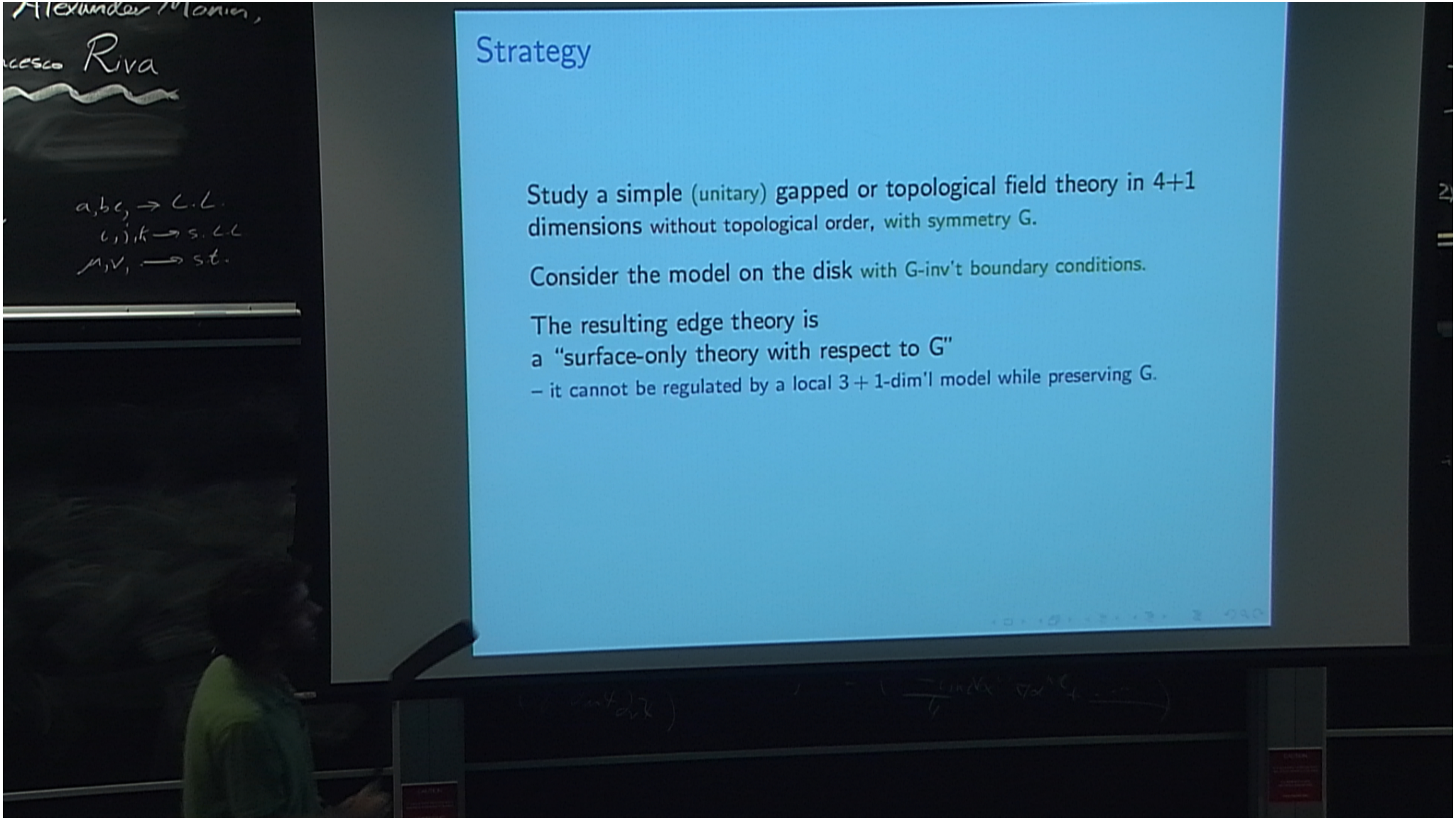
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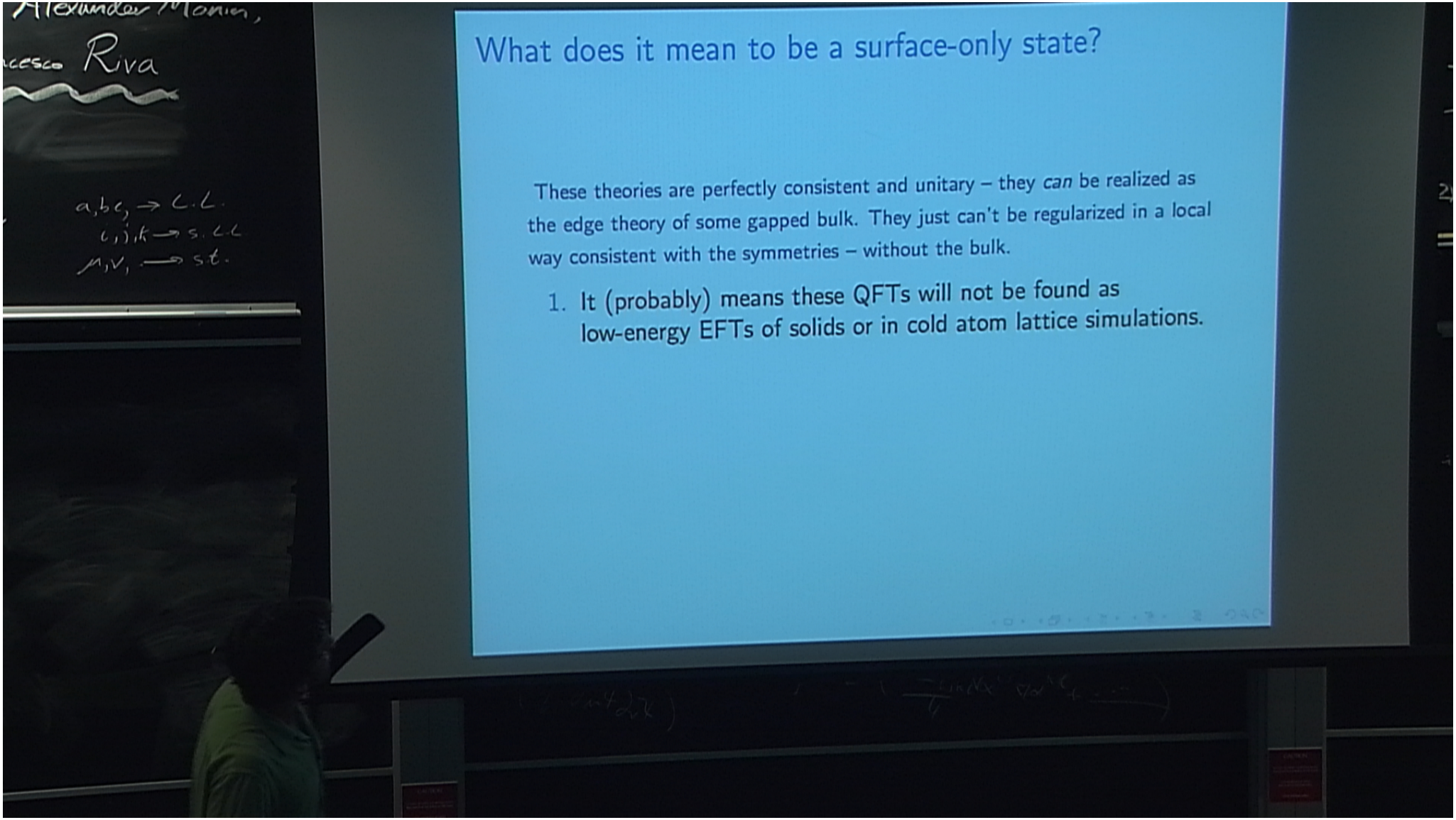
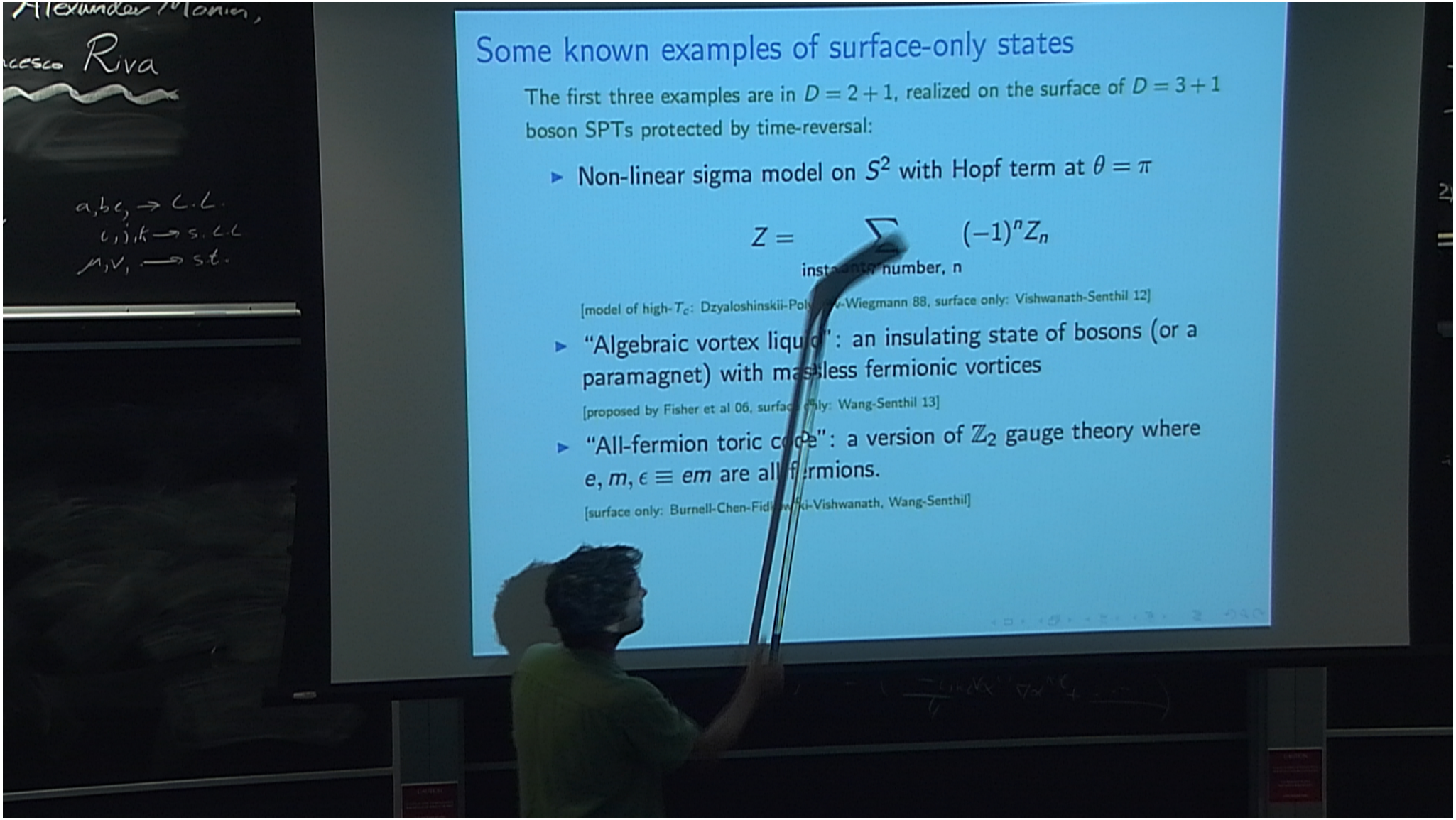

What does it mean to be a surface-only state?
These theories are perfectly consistent and unitary – they *can* be realized as the edge theory of some gapped bulk. They just can't be regularized in a local way consistent with the symmetries – without the bulk.
1. It (probably) means these QFTs will not be found as low-energy EFTs of solids or in cold atom lattice simulations.


Some known examples of surface-only states
The first three examples are in $D = 2+1$, realized on the surface of $D = 3+1$ boson SPTs protected by time-reversal:
Non-linear sigma model on S^2 with Hopf term at $\theta = \pi$
$Z = \sum_{\text{instanton number, n}} (-1)^n Z_n$
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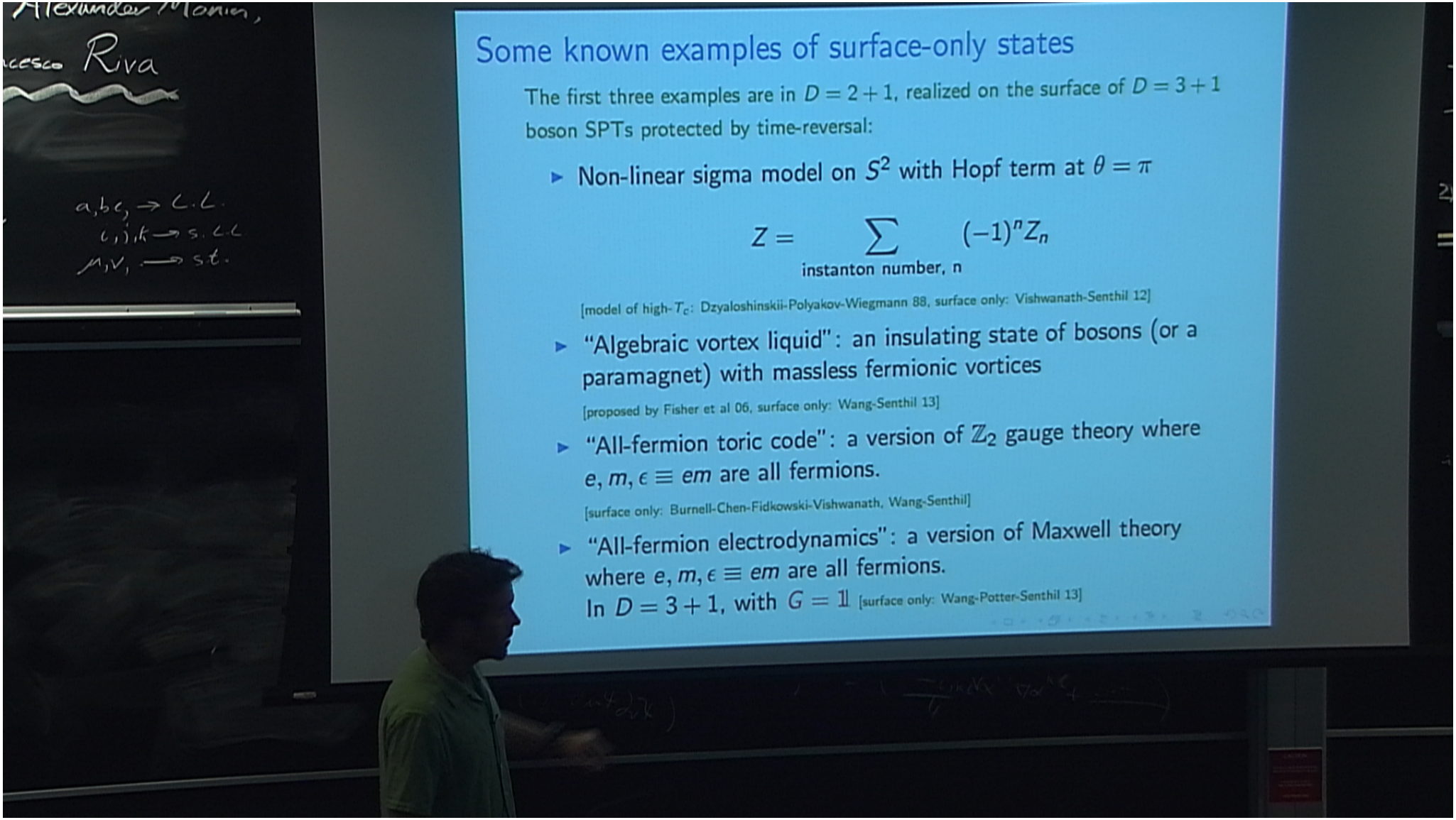
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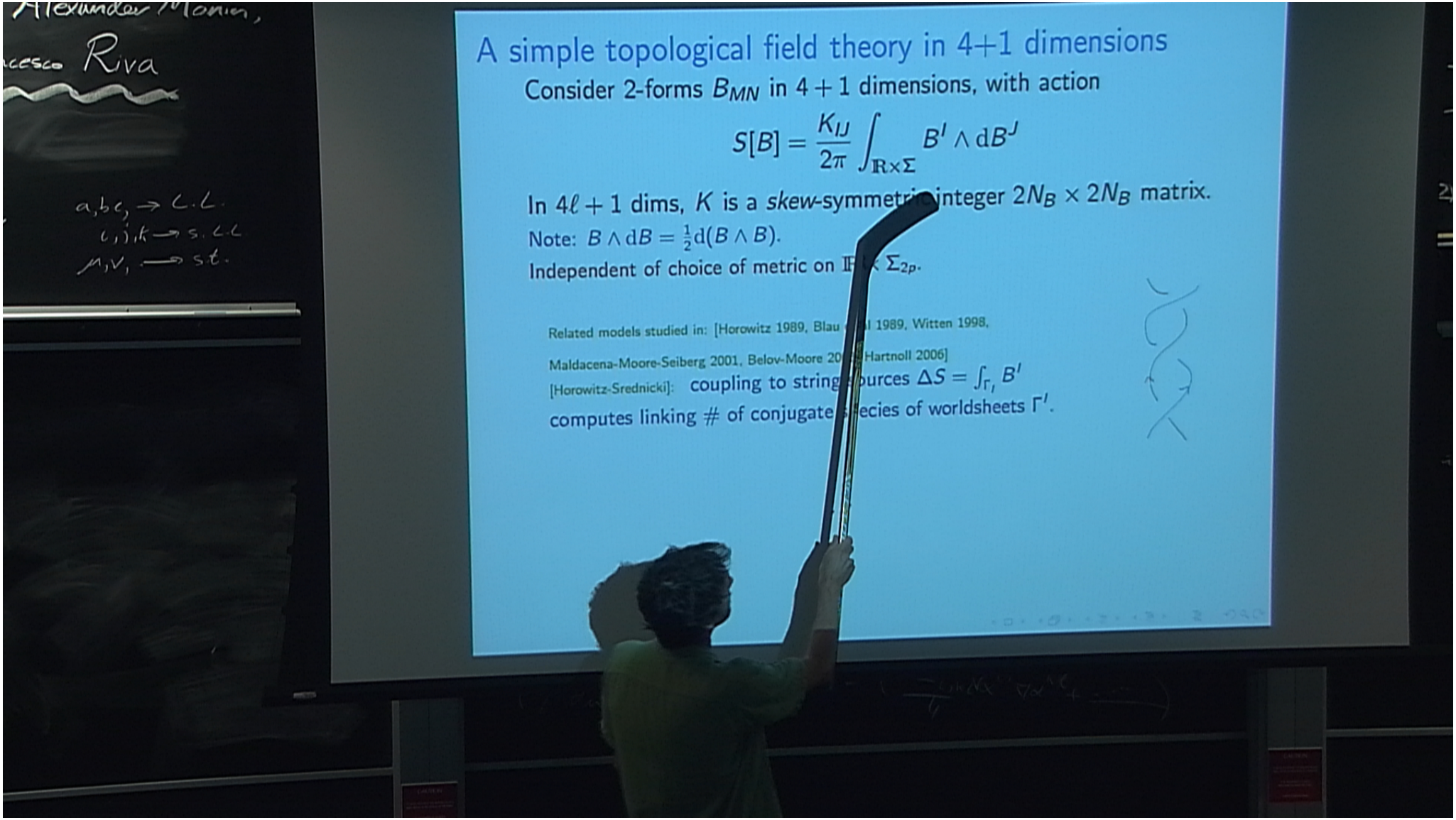
In $D = 3+1$, with $G = \mathbb{1}$ [surface only: Wang-Potter-Senthil 13]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$$S[B] = \frac{K_{IJ}}{2\pi} \int_{\mathbb{R}\times\Sigma} B^I \wedge \mathrm{d}B^J$$
In $4\ell+1$ dims, K is a *skew*-symmetric integer $2N_B \times 2N_B$ matrix.
Note: $B \wedge \mathrm{d}B = \frac{1}{2}\mathrm{d}(B \wedge B)$.
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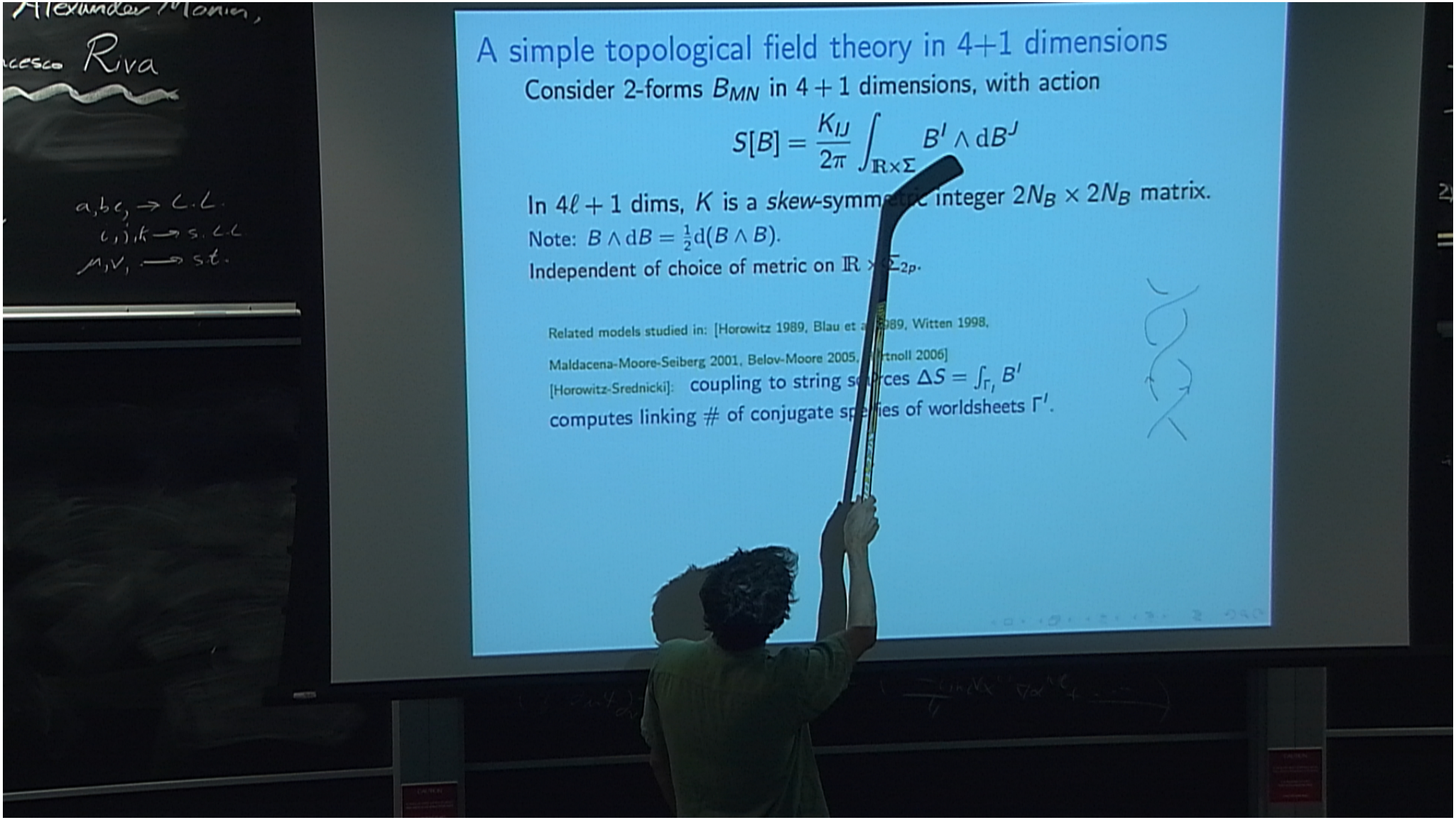
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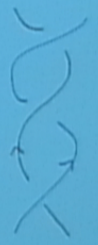

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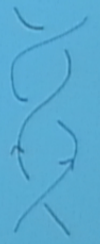

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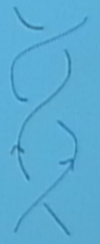

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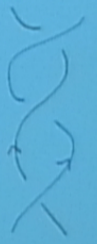

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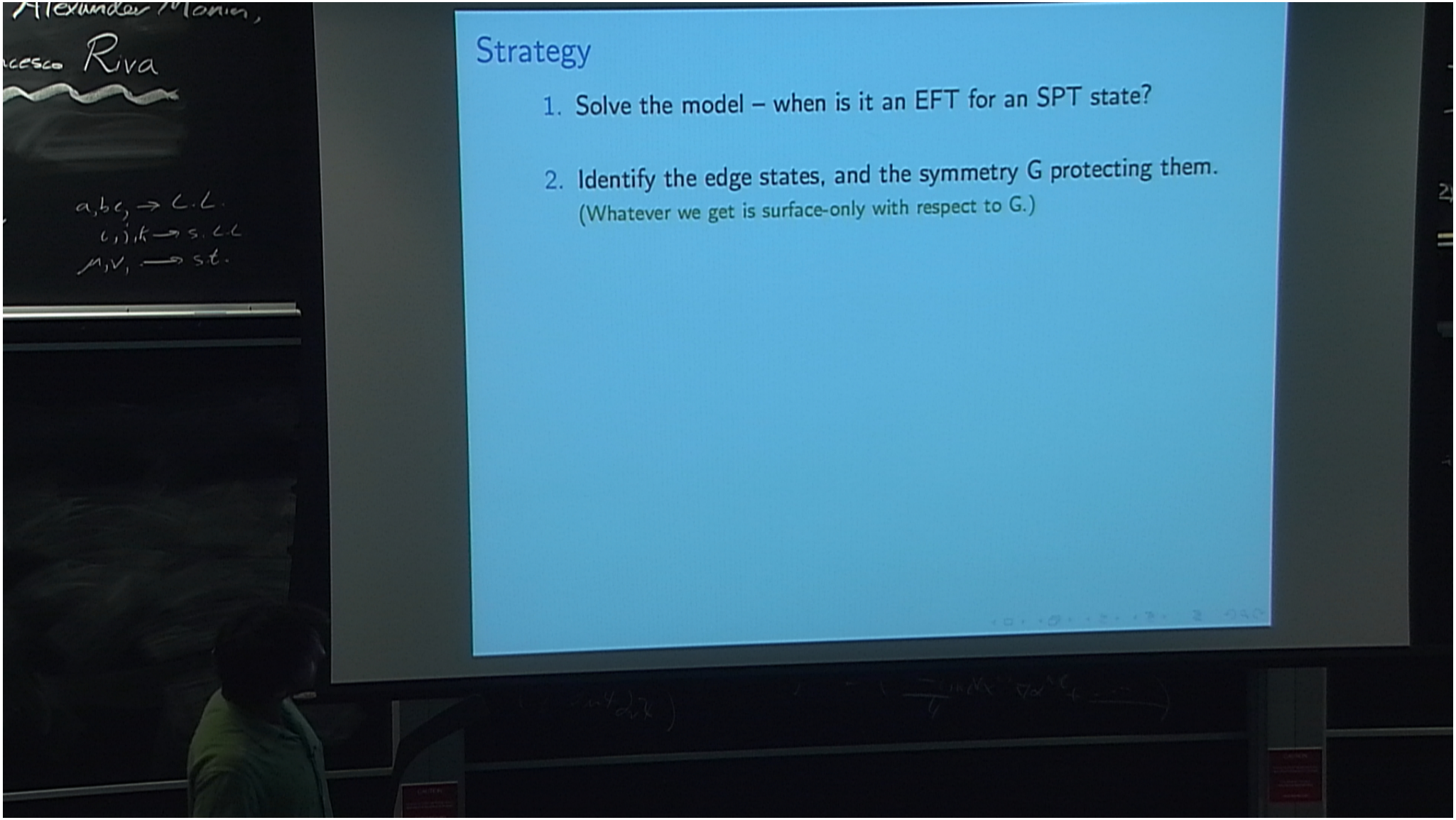
Simplest case ($N_B = 1$) is realized in IIB strings on $AdS_5 \times S^5$, $B \equiv B_{NSNS}$, $C \equiv C_{RR}$:

$$S = \frac{1}{2\pi} \int_{AdS_5\times S^5} F^{(5)}_{RR} \wedge B \wedge \mathrm{d}C = \frac{N}{2\pi} \int_{\mathbb{R}\times\Sigma} B \wedge \mathrm{d}C$$

Crucial: IIB S-duality acts by B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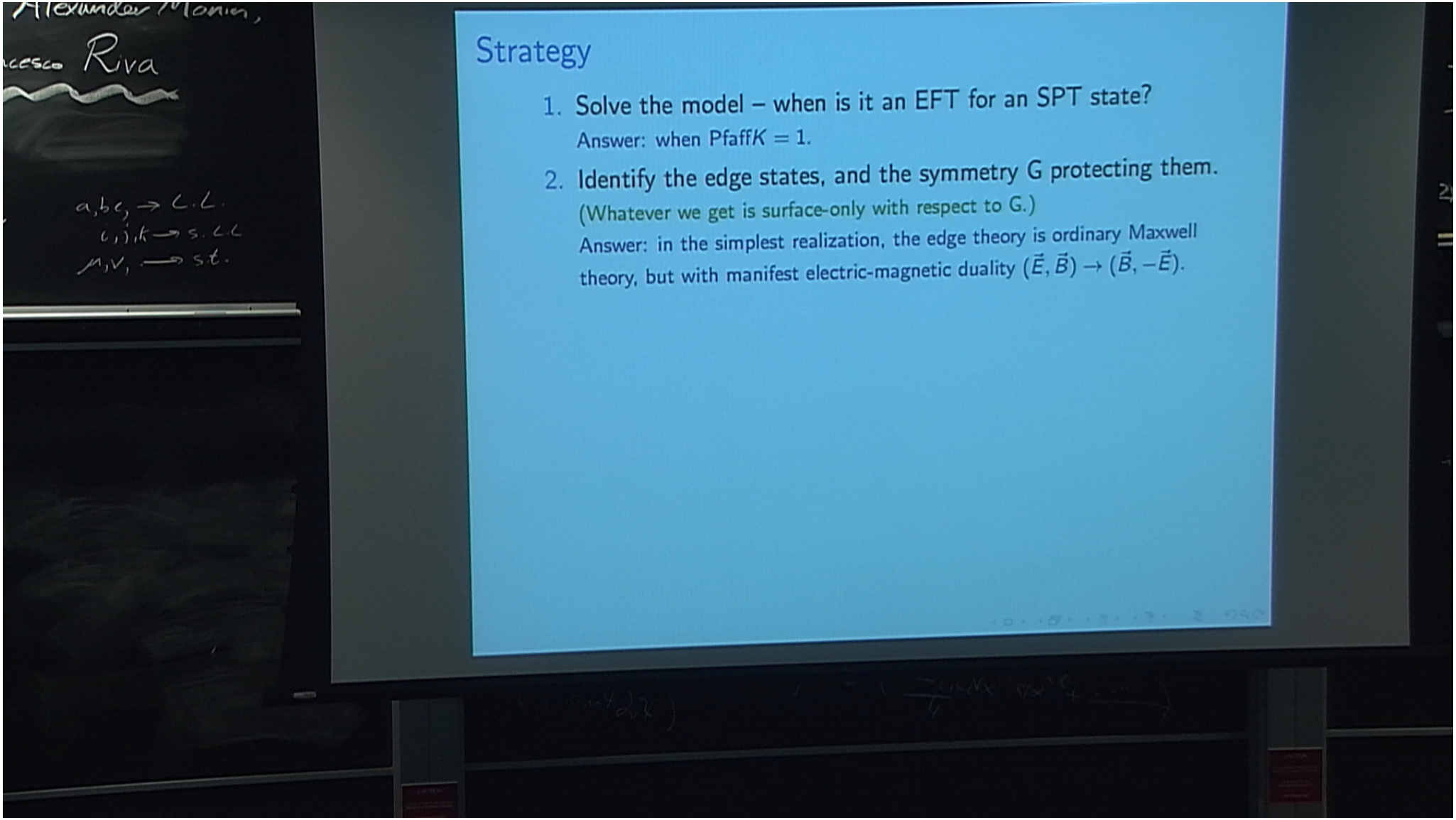
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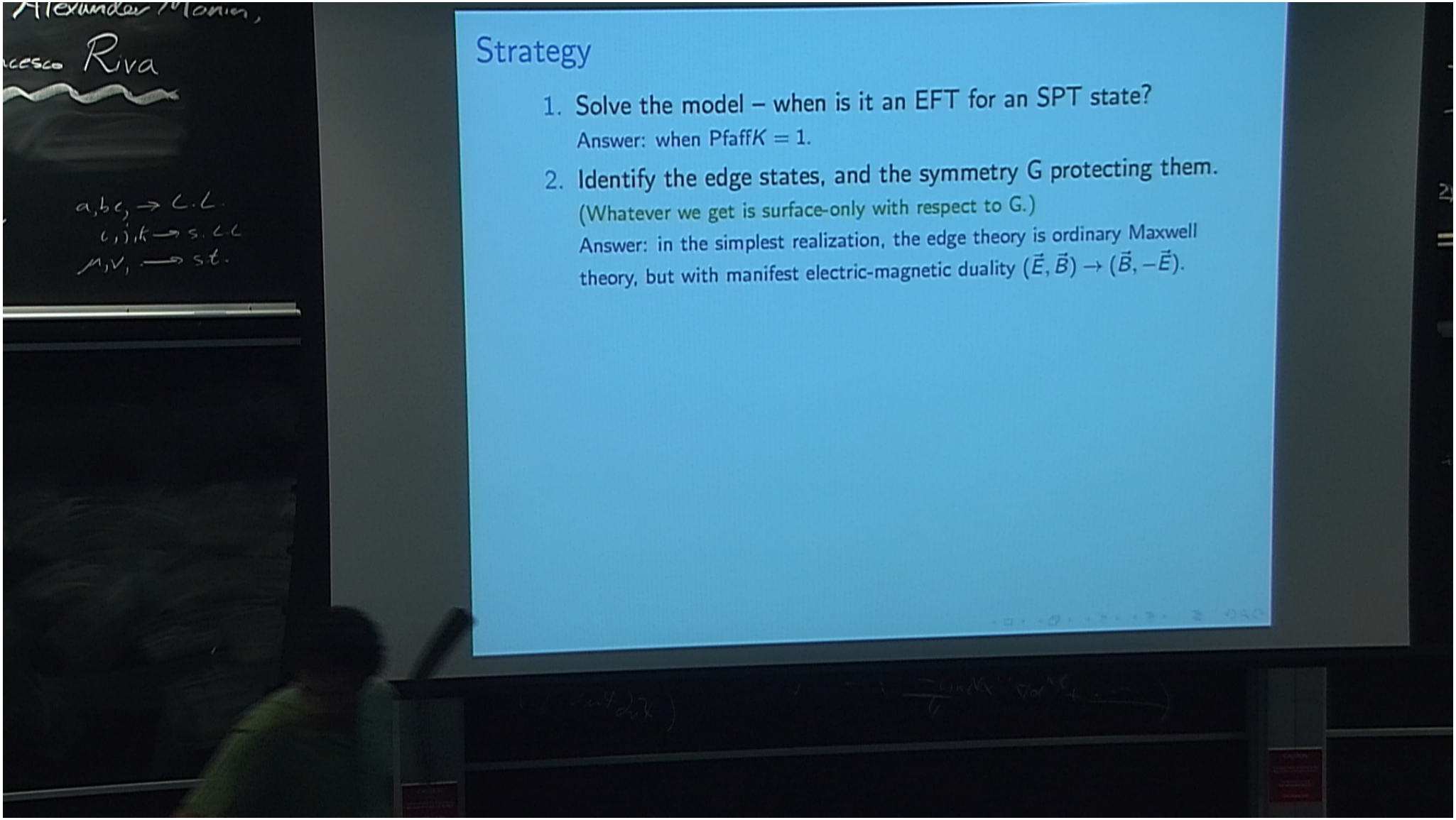
Answer: in the simplest realization, the edge theory is ordinary Maxwell theory, but with manifest electric-magnetic duality $(\vec{E}, \vec{B}) \to (\vec{B}, -\vec{E})$.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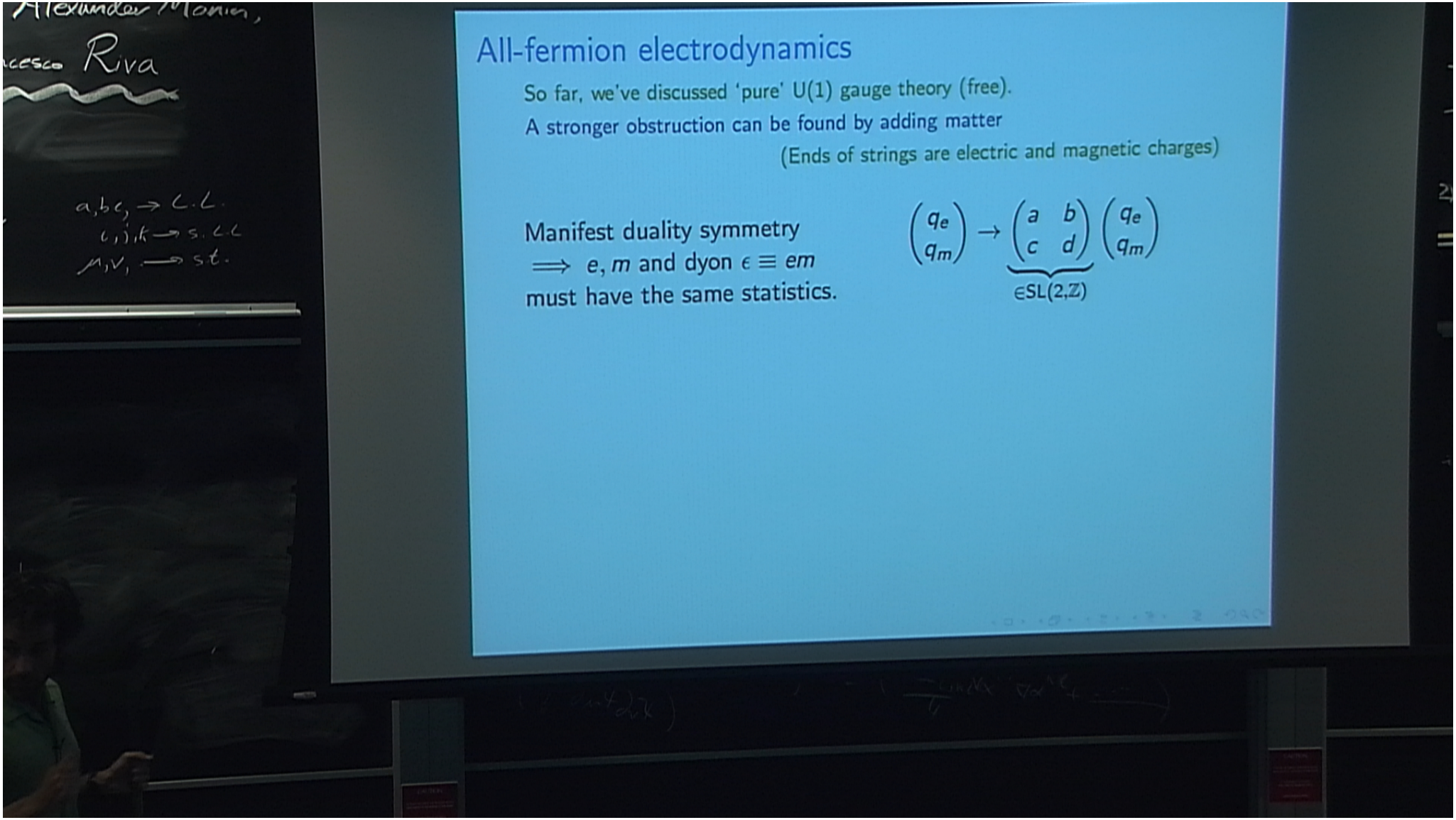
Answer: in the simplest realization, the edge theory is ordinary Maxwell theory, but with manifest electric-magnetic duality $(\vec{E}, \vec{B}) \to (\vec{B}, -\vec{E})$.


All-fermion electrodynamics
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A stronger obstruction can be found by adding matter
(Ends of strings are electric and magnetic charges)
Manifest duality symmetry
$\implies$ e, m and dyon $\epsilon \equiv em$
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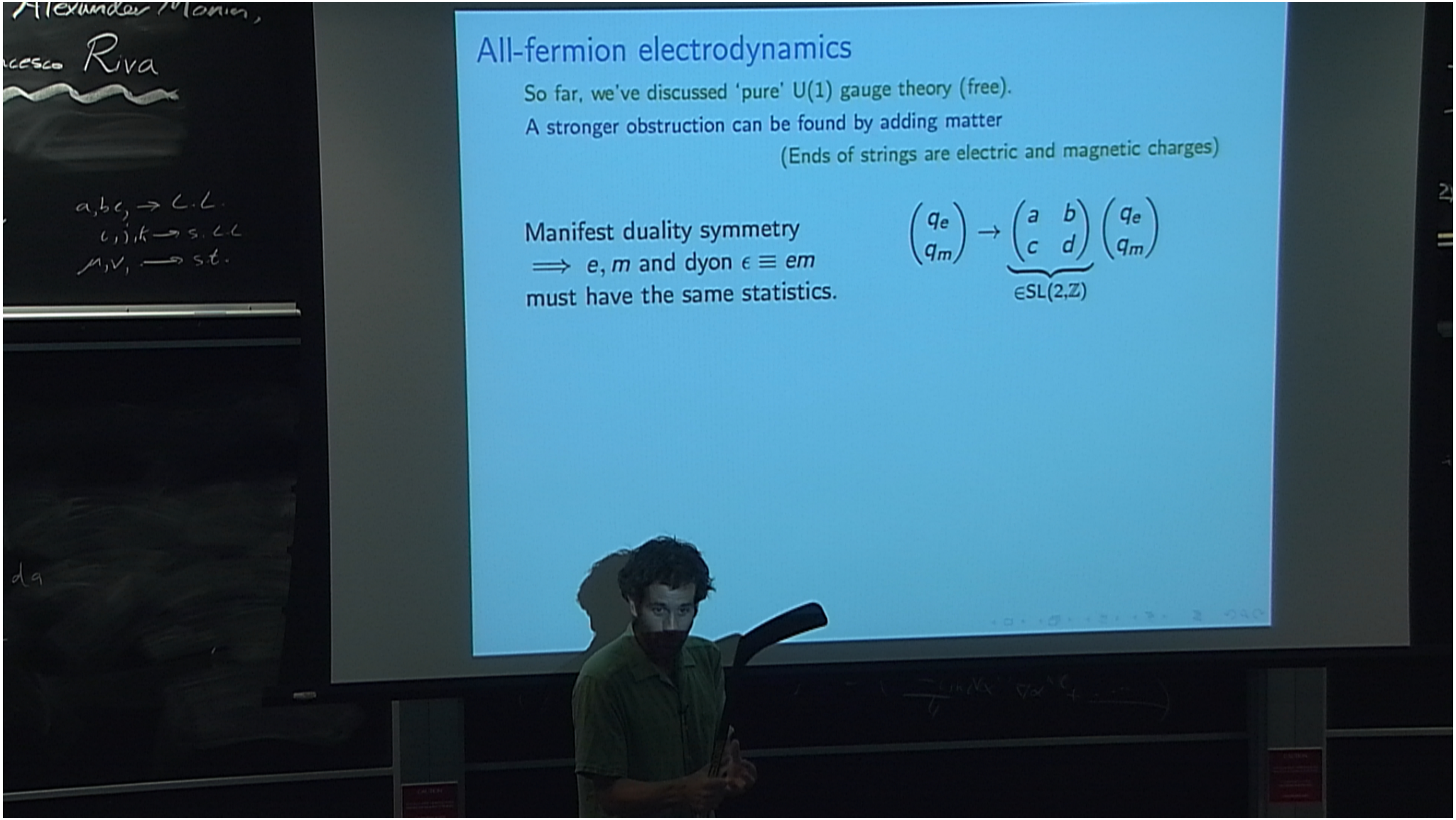
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$$\psi(x_1, x_2) = e^{i\varphi}\psi(x_2, x_1)$$

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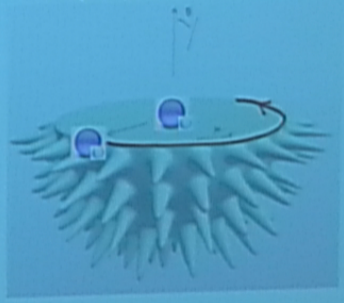

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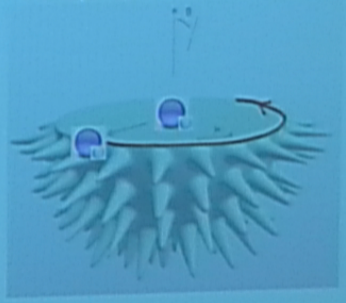

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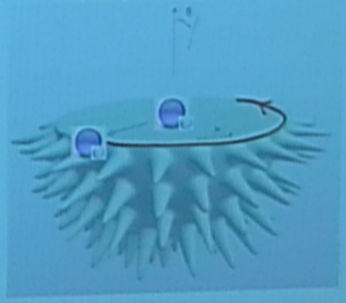

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This is evidence for a 4+1d SPT
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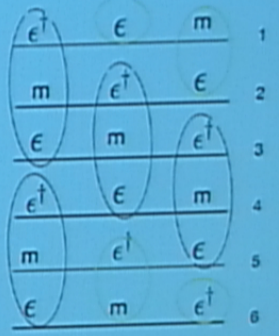

Idea: 'dyon string condensation' (like [Metlitski-Kane-Fisher])

- Each layer is ordinary electrodyamics with bosonic charges.
- $b_i \equiv \epsilon_i^\dagger m_{i+1} \epsilon_{i+2}$ are mutually-local bosons.
- Condensing b_i (obliquely) *confines* a_{i+1}, $i+1 = 2 \ldots N-1$.
- At top layer: $m_1\epsilon_2, \epsilon_1^\dagger m_1 \epsilon_2, \epsilon_1^\dagger$ survive, are fermions,

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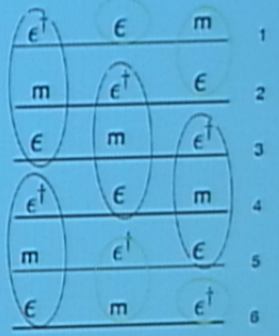

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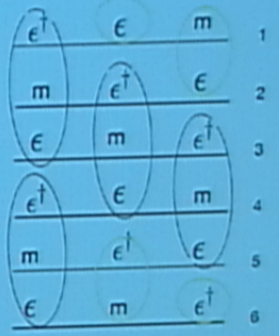

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In the bulk, continuum: this is the [illegible] TQFT theory with gapped string matter.

Dyon string condensation

If we did this: $\longrightarrow$

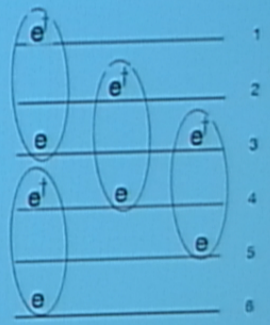
$\Delta H = V\sum_i(|\tilde{b}_i|^2 - v)^2$, $\tilde{b}_i \equiv e_i^\dagger e_{i+2} = ve^{\mathrm{i}a_{i,i+2}}$

would higgs $\prod_i \mathrm{U}(1)_i \to \mathrm{U}(1)_{\mathrm{even}} \times \mathrm{U}(1)_{\mathrm{odd}}$.

$4+1d$ Maxwell theory with $\mathrm{G} = \mathrm{U}(1)_{\mathrm{even}} \times \mathrm{U}(1)_{\mathrm{odd}}$, bulk photons.

Can dualize to 2-form potentials: $f^{o/e} = da^{o/e} = \star dC^{o/e}$

$$S = \sum_{\alpha=o,e} \int_{5d} \left(\frac{1}{g_\alpha^2} dC^\alpha \wedge \star dC^\alpha + C^\alpha \wedge \star j_m^\alpha \right).$$



Magnetic flux tubes of broken U(1)s collimate monopoles into *monopole strings.*

If instead we do *this* : $\longrightarrow$

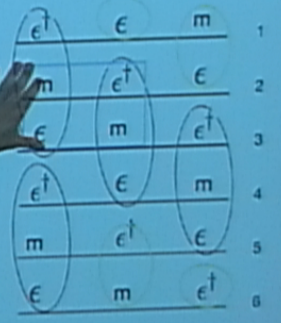
$b_i \equiv \epsilon_i^\dagger m_{i+1}\epsilon_{i+2}$ are mutually-local bosons.

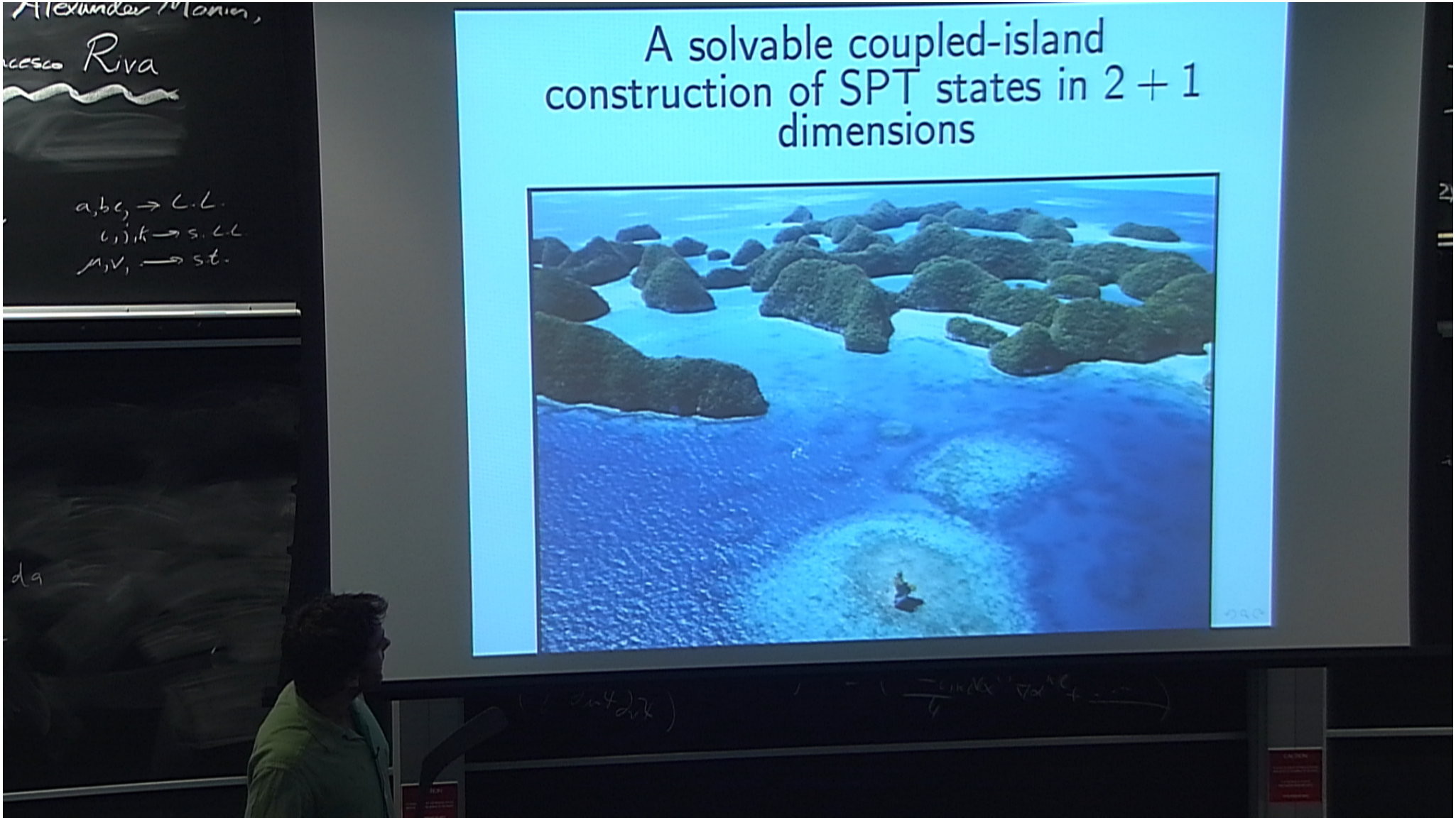
$\Delta H = V\sum_i(|b_i|^2 - v)^2$. Condensing b_i (obliquely) *confines* $a_{i+1}, i+1 = 2 \ldots N-1$.

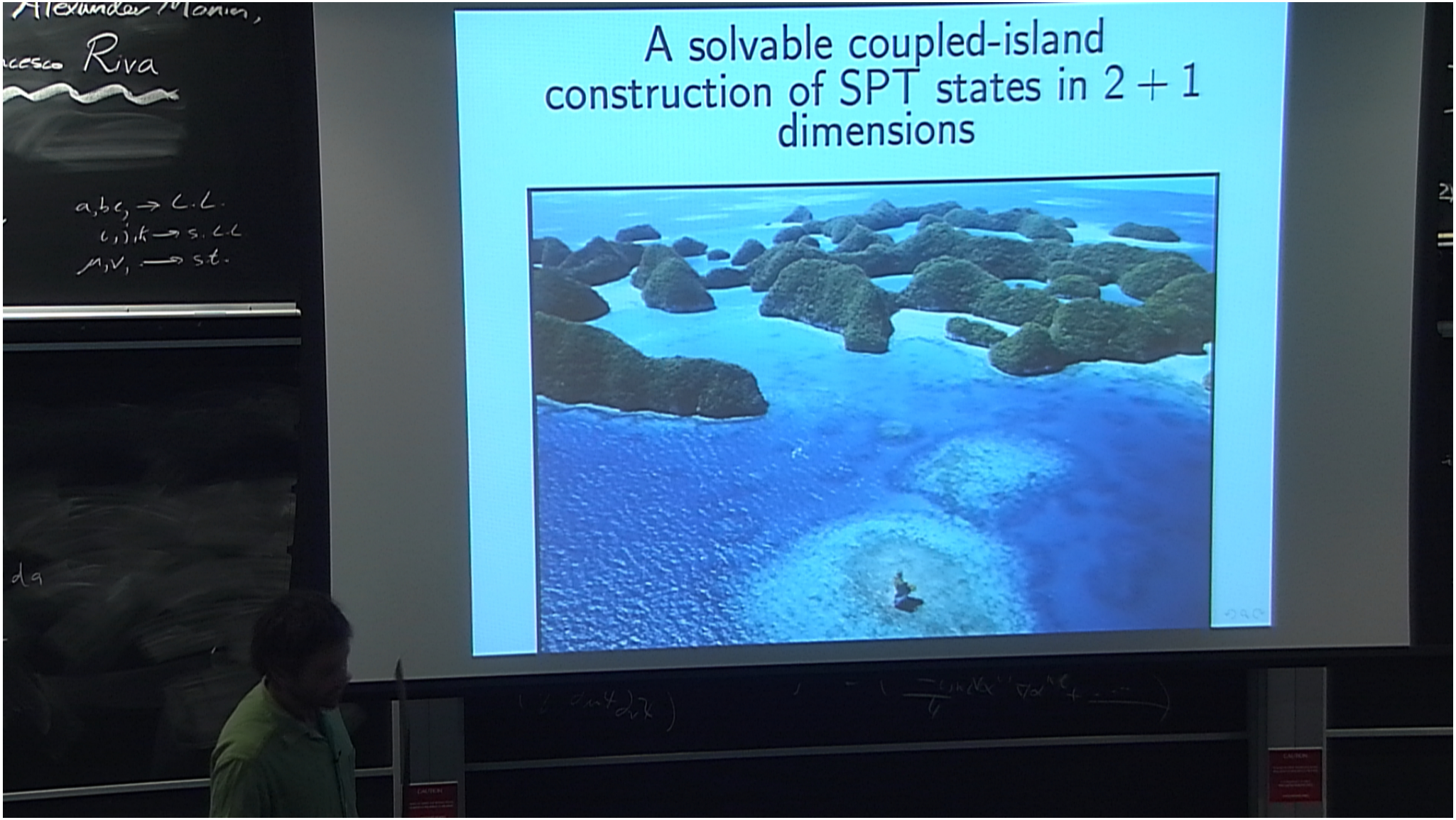
Binds monopole strings of $a^{e/o}$ to electric flux lines of $a^{o/e}$.

This is the effect of the additional term

$$\Delta S = \int \frac{1}{4\pi} C^e \wedge$$




A solvable coupled-island construction of SPT states in 2 + 1 dimensions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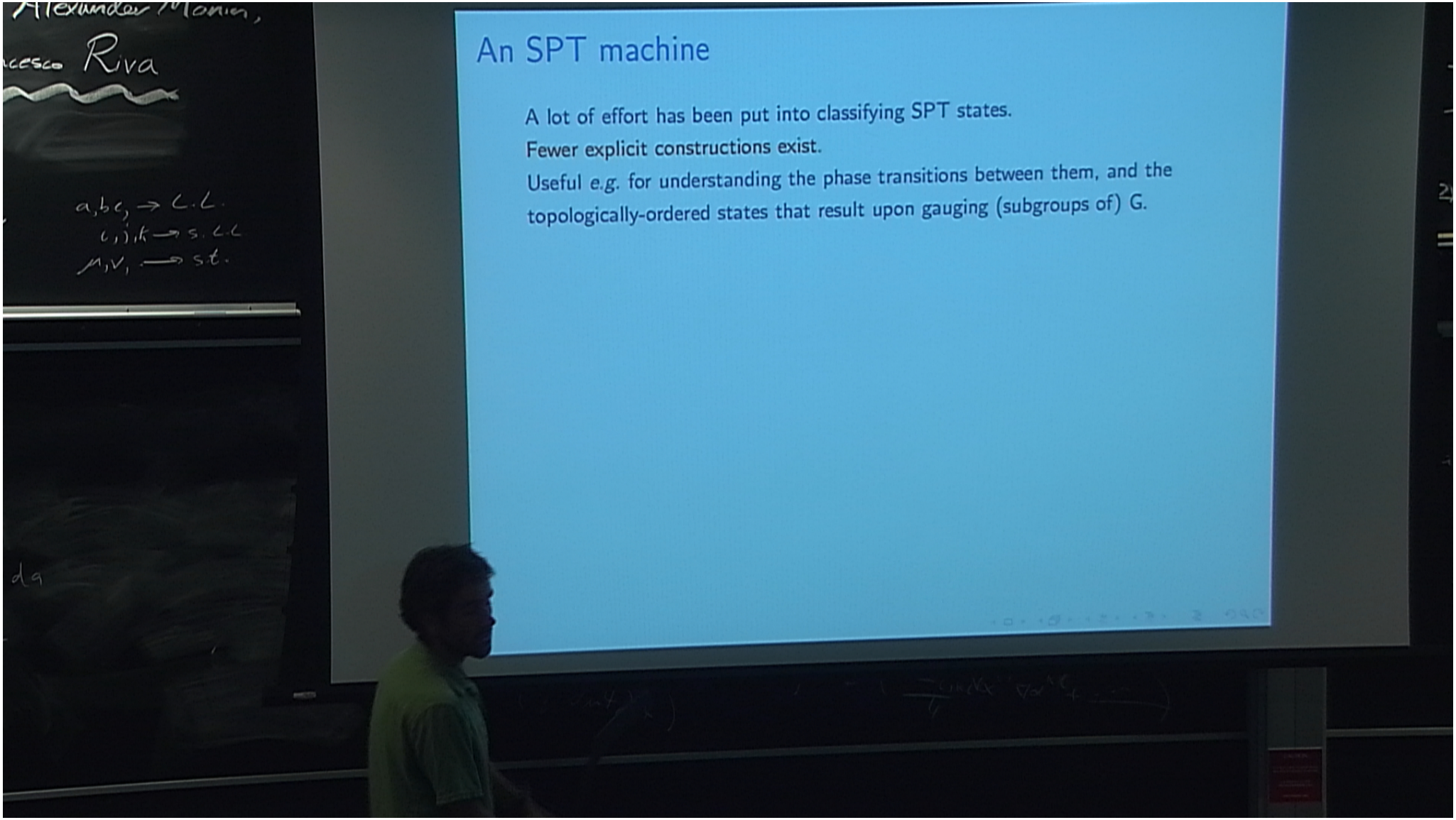
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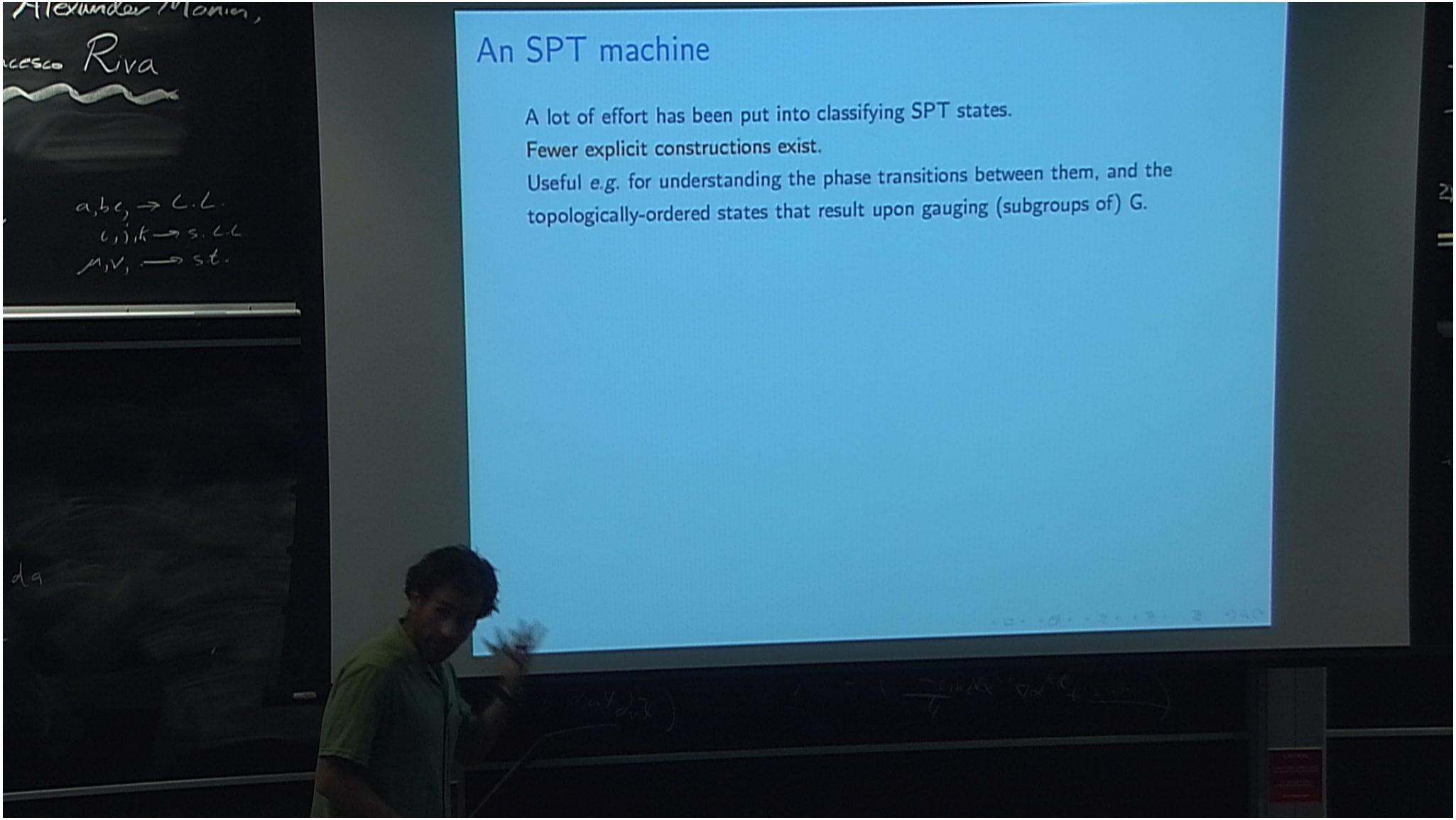
Fewer explicit constructions exist.
Useful e.g. for understanding the phase transitions between them, and the topologically-ordered states that result upon gauging (subgroups of) G.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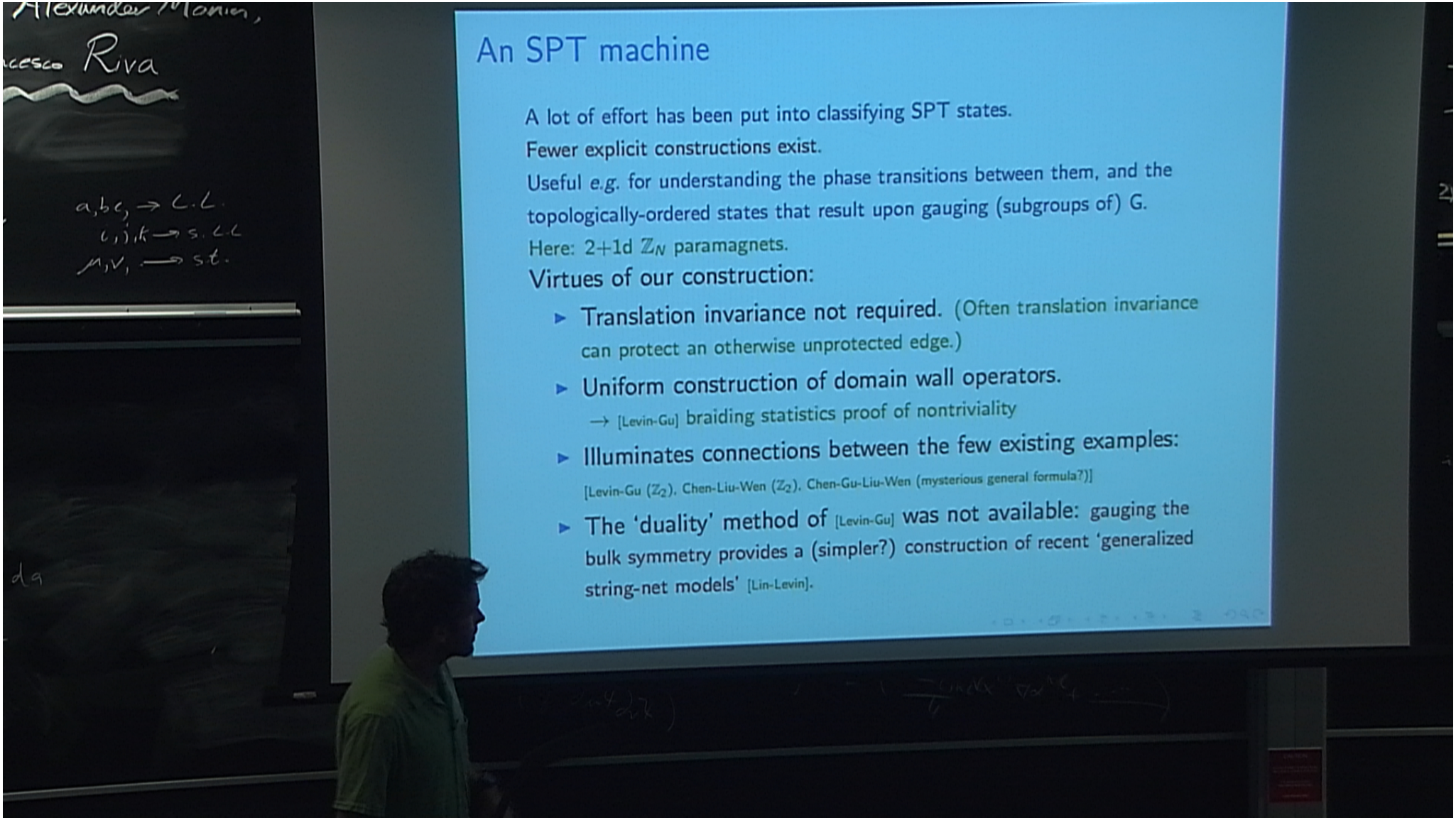
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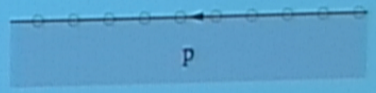
Here: 2+1d $\mathbb{Z}_N$ paramagnets.
Virtues of our construction:
Translation invariance not required. (Often translation invariance can protect an otherwise unprotected edge.)
Uniform construction of domain wall operators.
→ [Levin-Gu] braiding statistics proof of nontriviality
Illuminates connections between the few existing examples:
[Levin-Gu ($\mathbb{Z}_2$), Chen-Liu-Wen ($\mathbb{Z}_2$), Chen-Gu-Liu-Wen (mysterious general formula?)]
The 'duality' method of [Levin-Gu] was not available: gauging the bulk symmetry provides a (simpler?) construction of recent 'generalized string-net models' [Lin-Levin].

Non-onsite symmetry

An anomalous symmetry can be realized in a lattice model if it is *not on-site*: its action on one site depends on others.

This means you can't gauge it just by coupling to link variables (without coarse-graining first).

(Like chiral symmetry with staggered fermions.)



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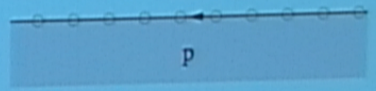

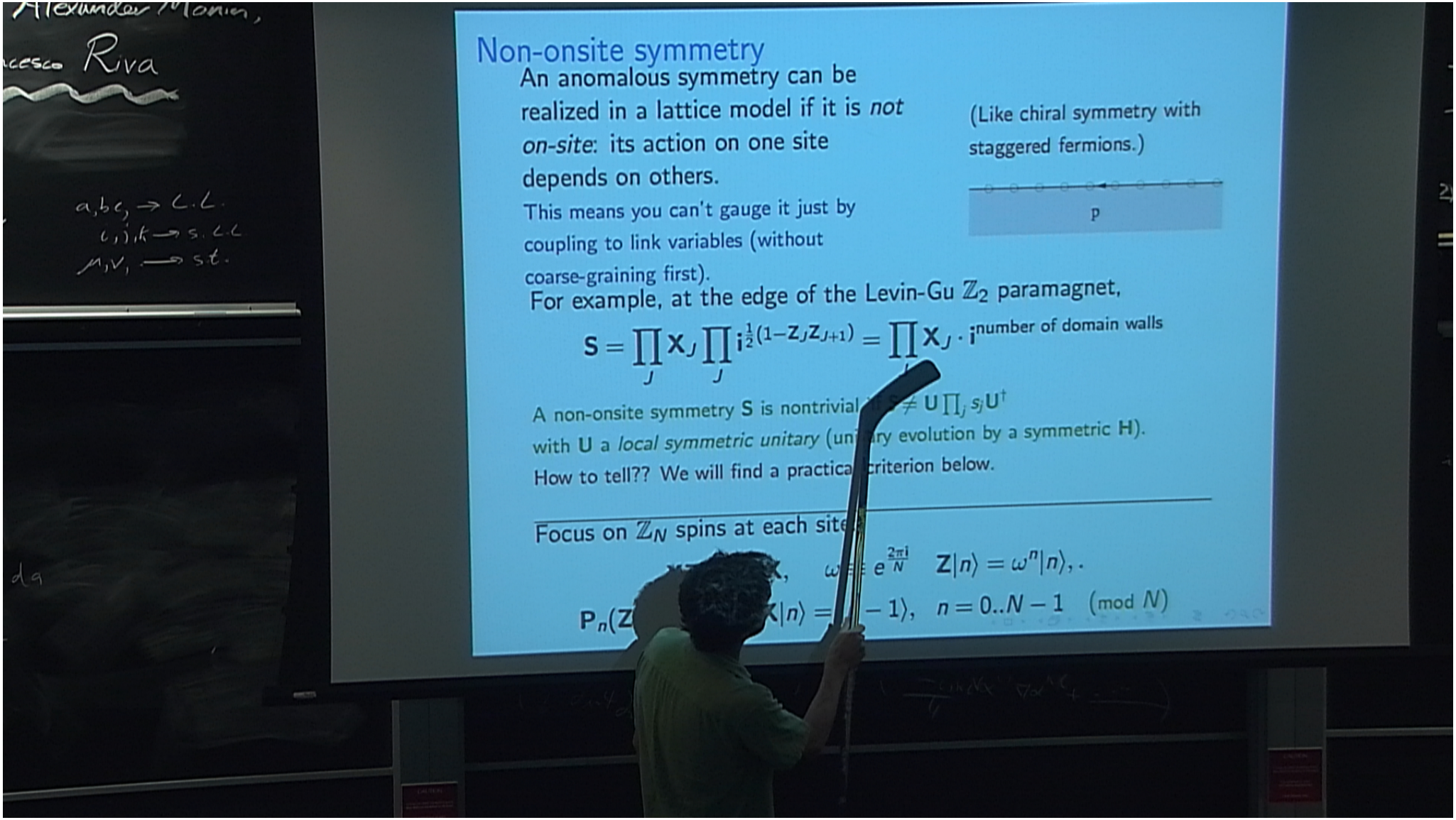
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For example, at the edge of the Levin-Gu $\mathbb{Z}_2$ paramagnet,
$\mathbf{S} = \prod_j \mathbf{X}_j \prod_j \mathbf{i}^{\frac{1}{2}(1-\mathbf{Z}_j\mathbf{Z}_{j+1})} = \prod_j \mathbf{X}_j \cdot \mathbf{i}^{\text{number of domain walls}}$
A non-onsite symmetry S is nontrivial if $\mathbf{S} \neq \mathbf{U}\prod_j s_j \mathbf{U}^\dagger$
with U a *local symmetric unitary* (unitary evolution by a symmetric H).
How to tell?? We will find a practical criterion below.
Focus on $\mathbb{Z}_N$ spins at each site
$\omega = e^{\frac{2\pi i}{N}}$ $\mathbf{Z}|n\rangle = \omega^n|n\rangle,$.
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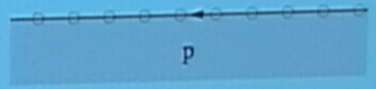

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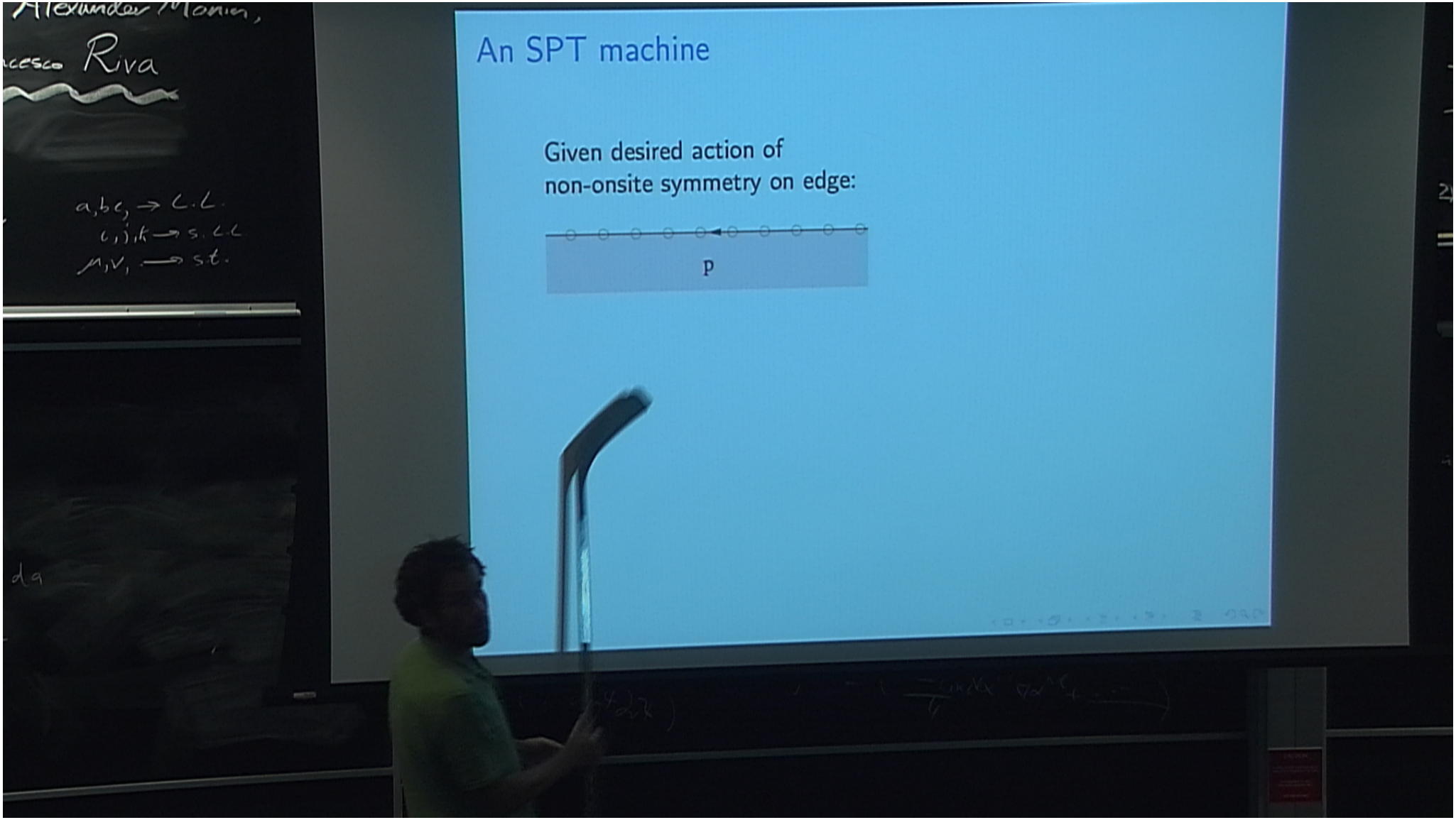
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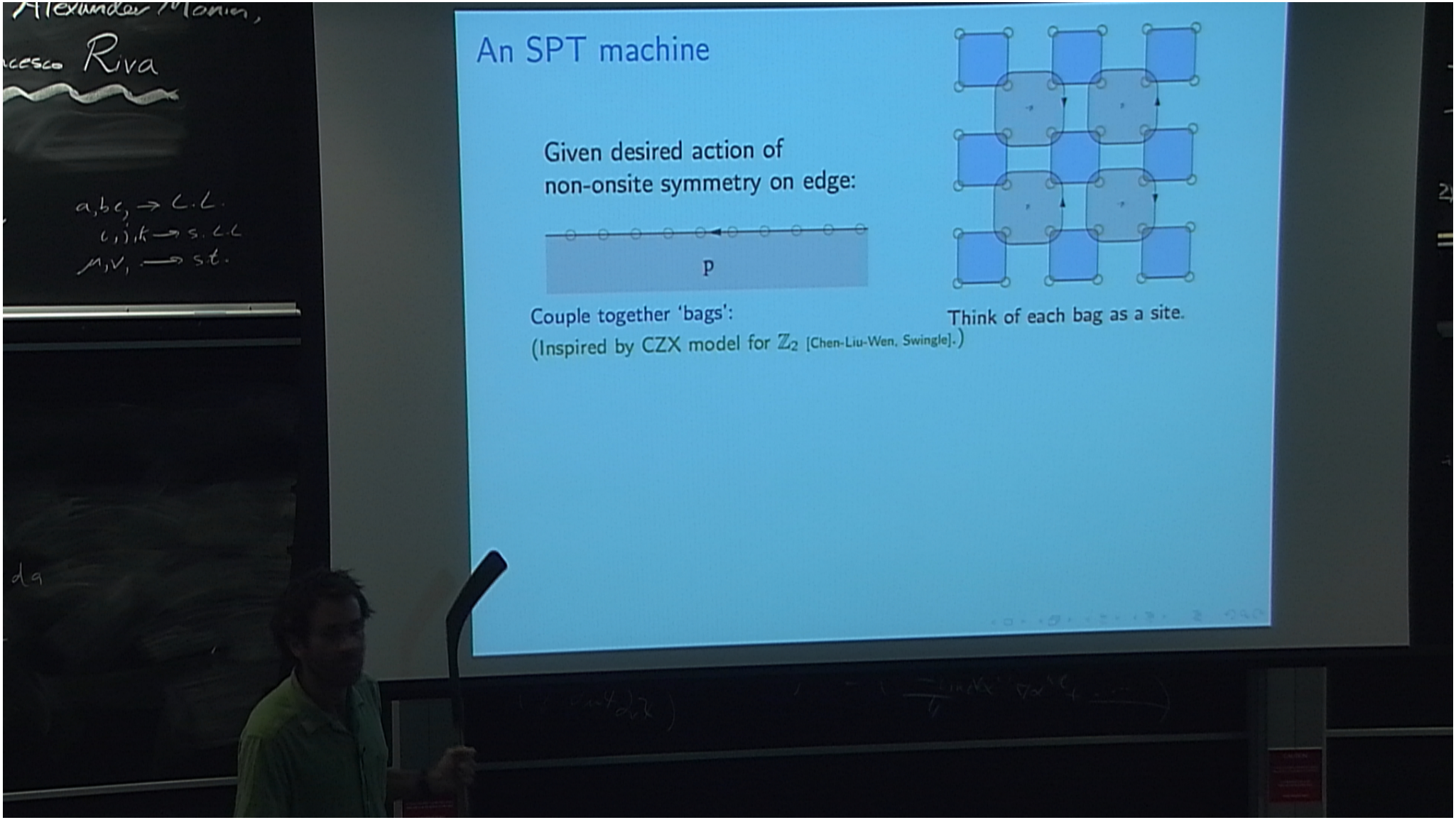
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(Inspired by CZX model for $\mathbb{Z}_2$ [Chen-Liu-Wen, Swingle].)
Think of each bag as a site.

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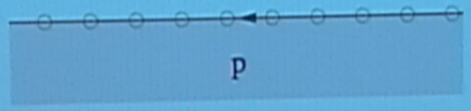

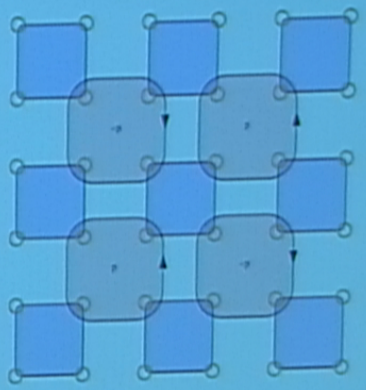

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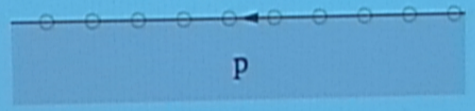

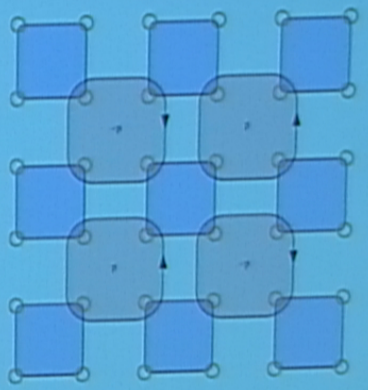

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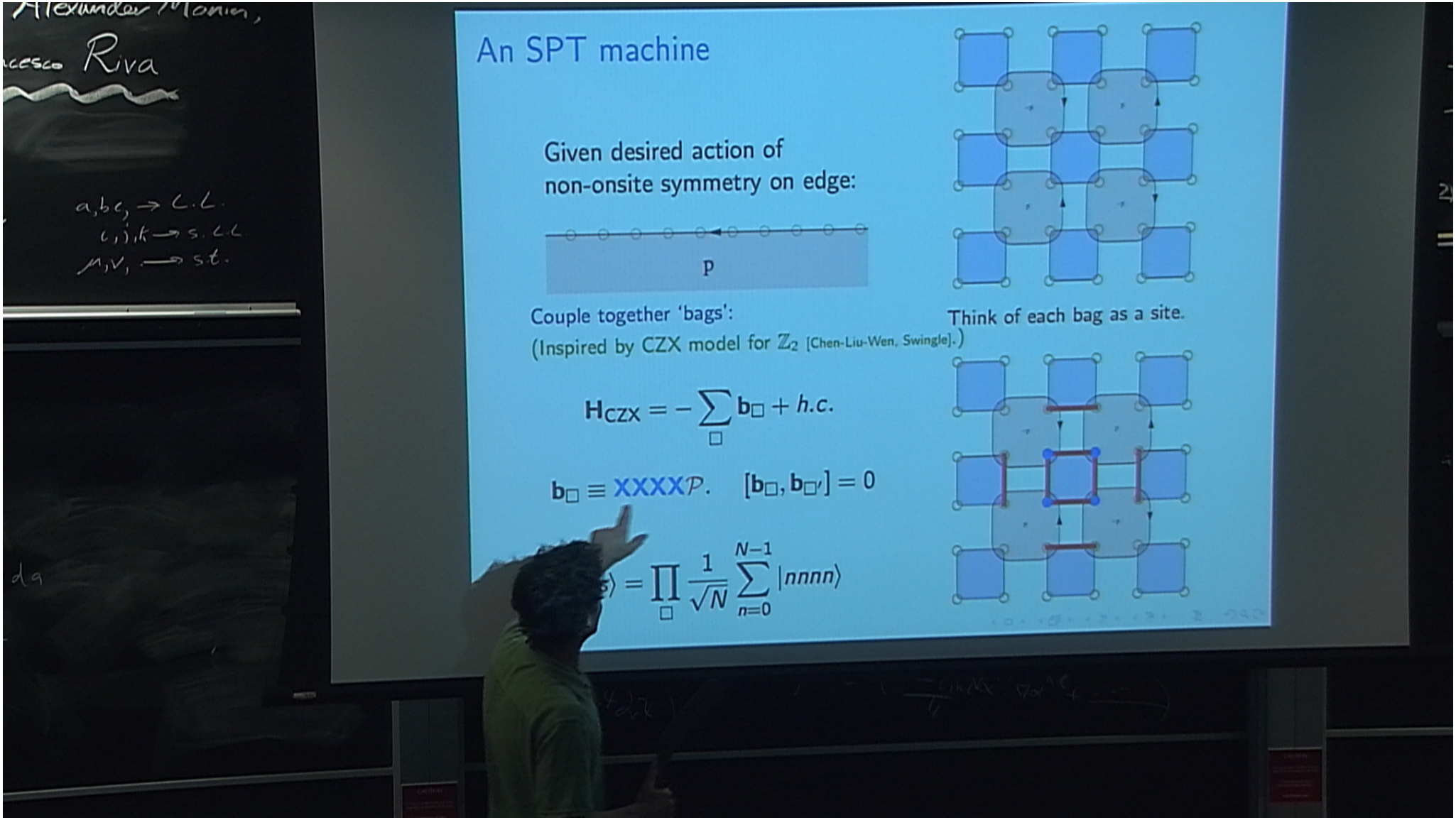
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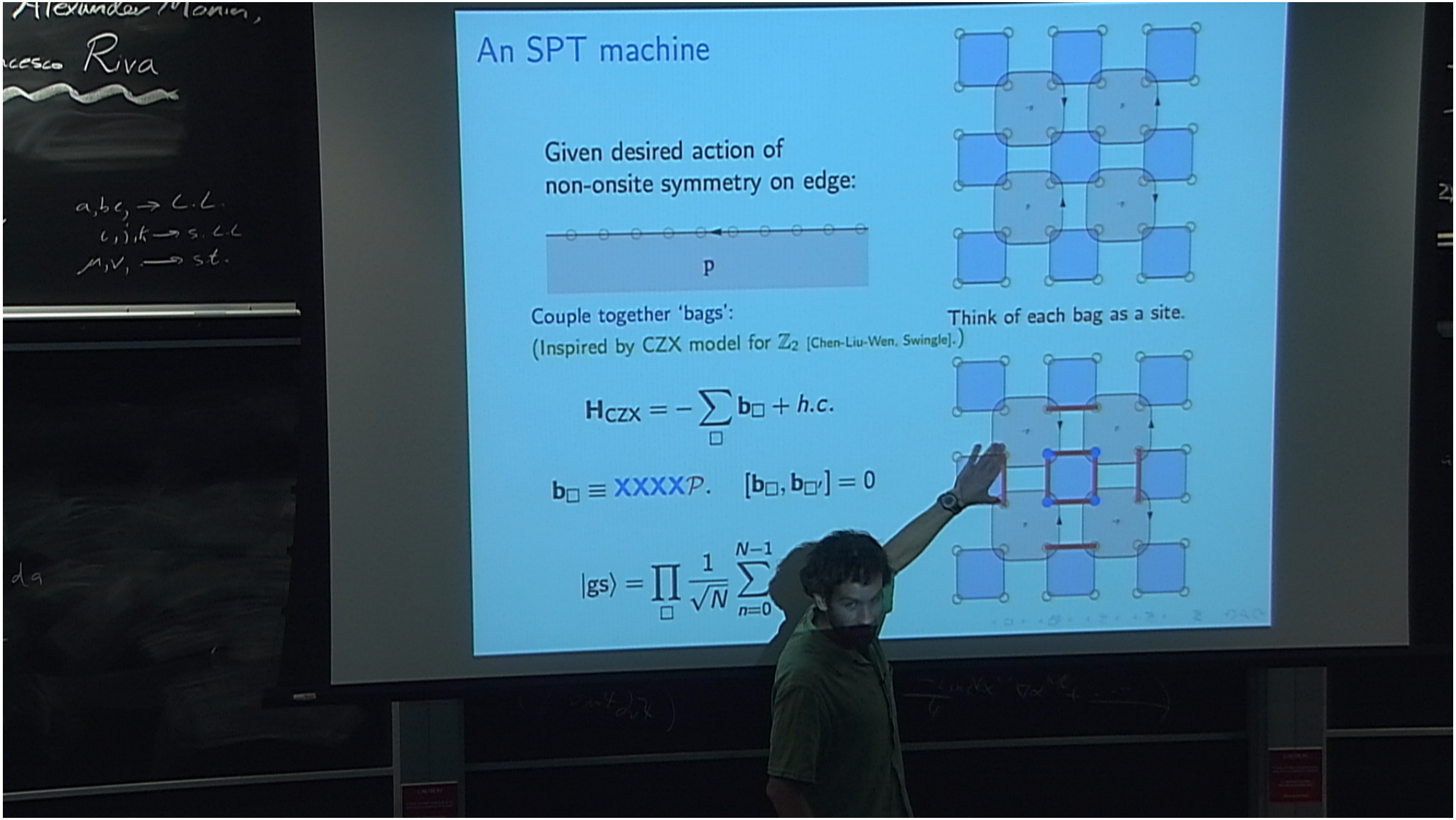
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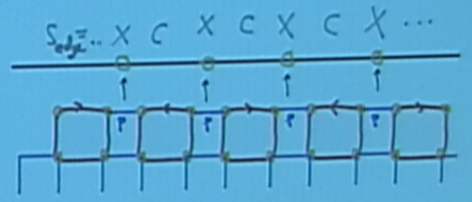
Symmetry-protected edge states

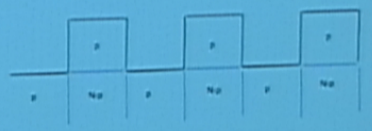
Note: **H** doesn't know about p; **S** does: ($[\mathbf{S}, \mathbf{H}_{\mathrm{CZX}}] = 0$)

$$\mathbf{S} = \prod_j \mathbf{X}_j \left(\prod_{\text{bags}} \prod_j C_p(\mathbf{Z}_j, \mathbf{Z}_{j+1}) \right)$$



Not onsite on edge: $\longrightarrow$

Rough edge realizes the desired **S** on edge modes:



Symmetry-protected edge states

Note: **H** doesn't know about p; **S** does: ($[\mathbf{S}, \mathbf{H}_{\mathrm{CZX}}] = 0$)

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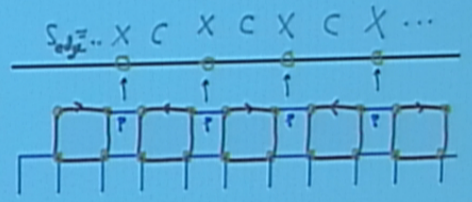
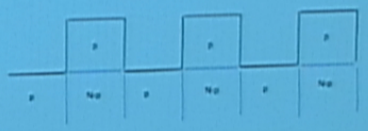


Not onsite on edge: $\longrightarrow$



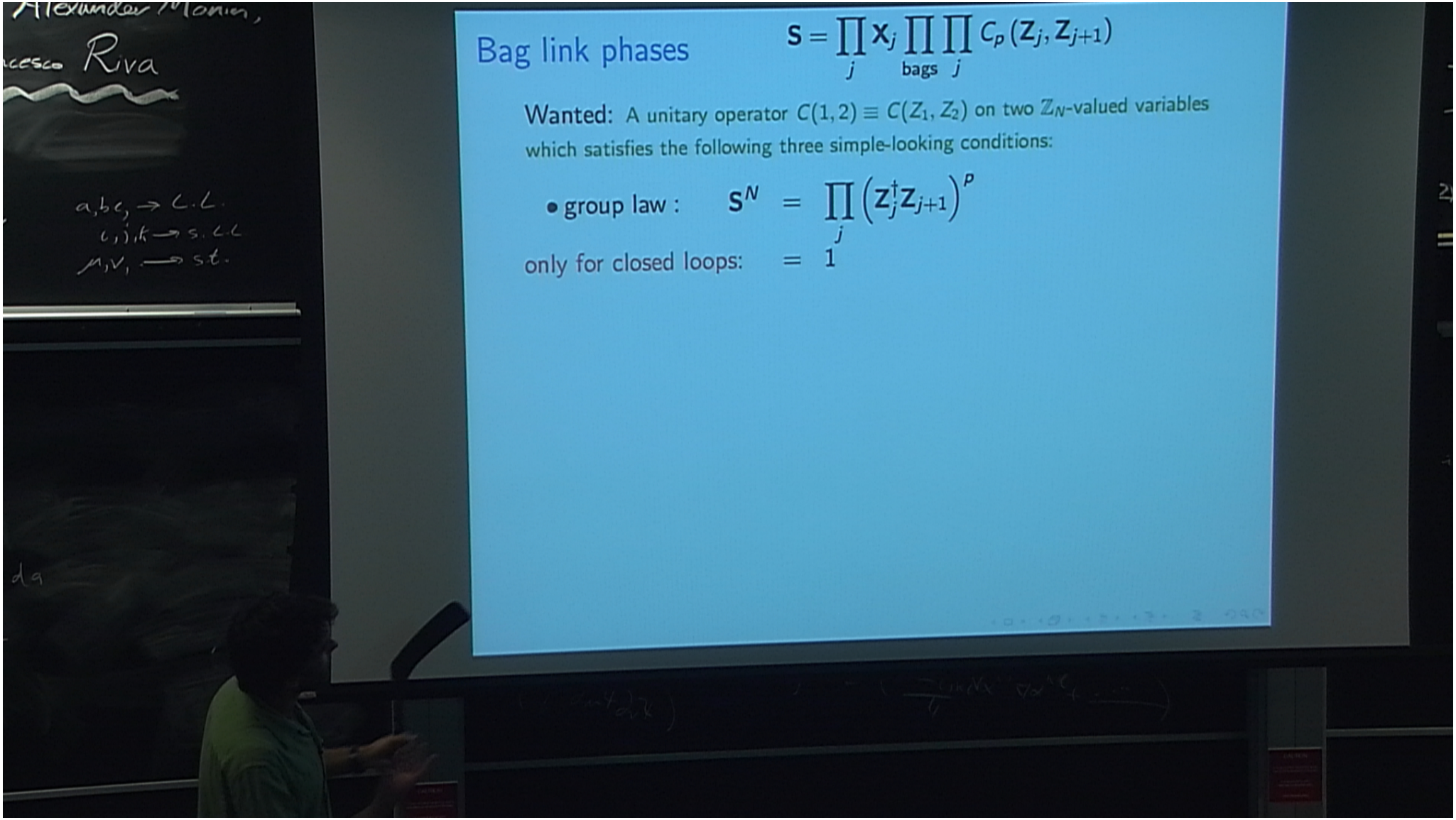
Rough edge realizes the desired **S** on edge modes:

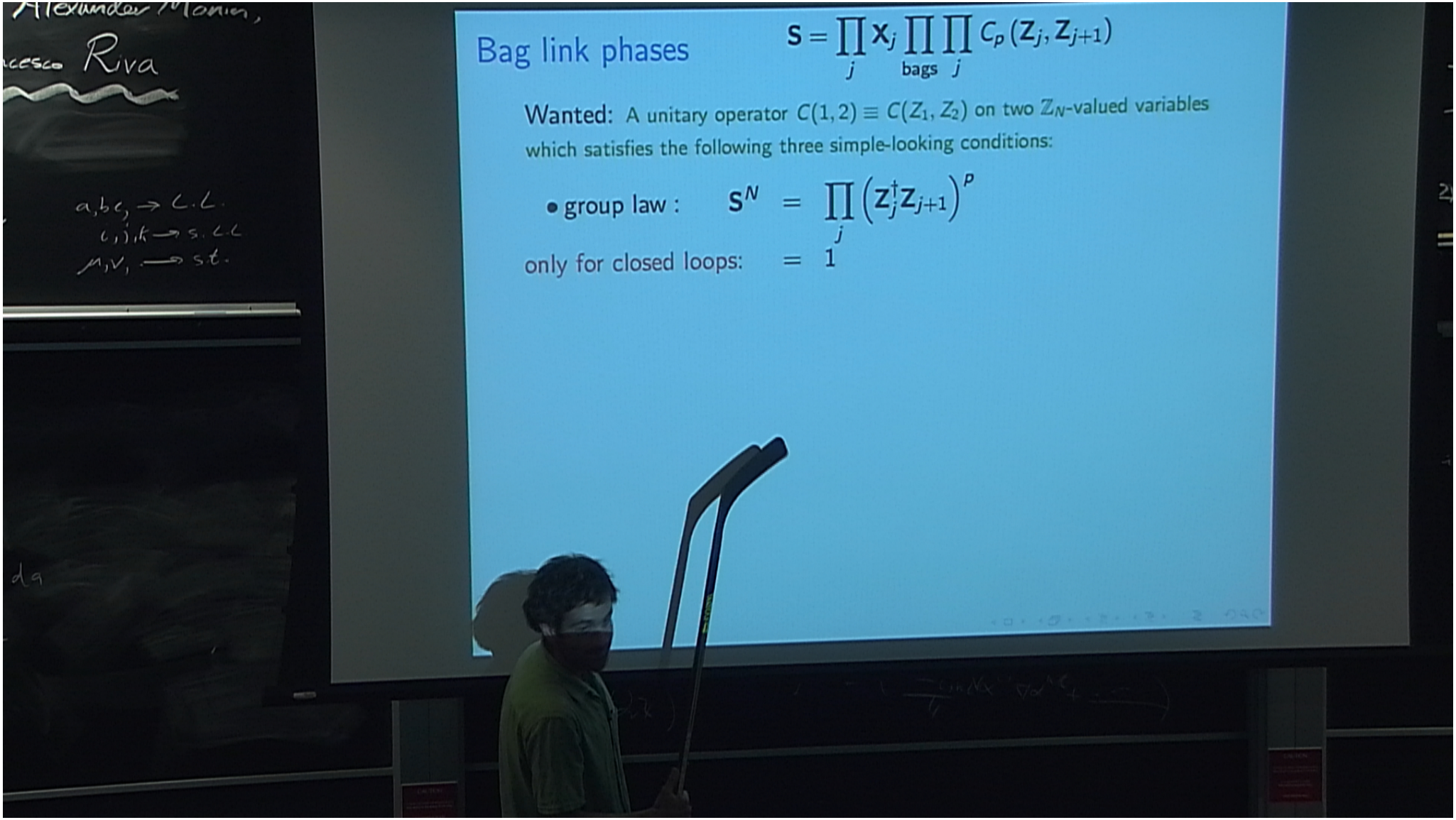
Claim of robustness: perturbing $\mathbf{H}_{\mathrm{CZX}}$ by terms respecting **S**, you cannot remove this edge stuff.

i.e. no local, symmetric uni… can make $|gs\rangle_{\text{edge}}$ a product state. (Gapless or …try-breaking degen…acy.)

…ming (?): require… bipartite graph of connections between bags.

No… draw lattices fo… mplicity of drawing, but translation invariance is not …necessary.


Bag link phases
$\mathbf{S} = \prod_j \mathbf{X}_j \prod_{\text{bags}} \prod_j C_p(\mathbf{Z}_j, \mathbf{Z}_{j+1})$
Wanted: A unitary operator $C(1,2) \equiv C(Z_1, Z_2)$ on two $\mathbb{Z}_N$-valued variables which satisfies the following three simple-looking conditions:
• group law : $\mathbf{S}^N = \prod_j \left(\mathbf{Z}_j^\dagger \mathbf{Z}_{j+1}\right)^p$
only for closed loops: $= 1$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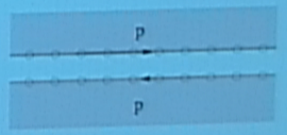
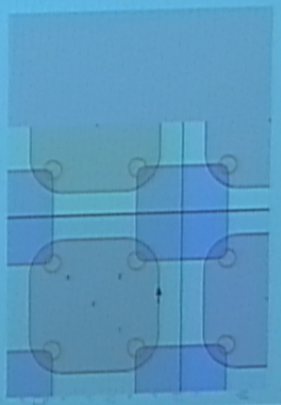


$=$

- gappability : $C_p(1,2)C_{-p}(1,2) = 1$

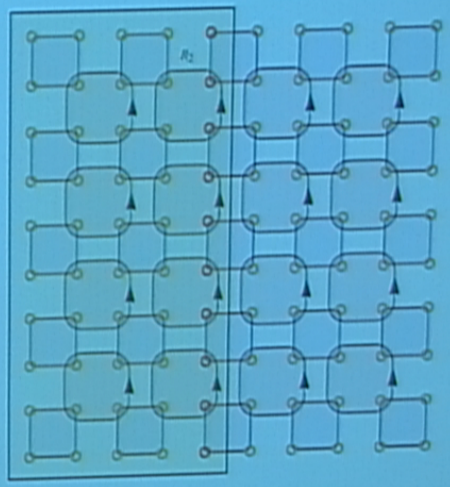


- non-triviality (flux braiding) :

$$\frac{X_1^{-k_2} X_3^{k_1} C_{-p}^{k_2}(23) X_3^{-k_1}}{C_{-p}^{k_2}(23)} \stackrel{?}{=} e^{2\pi i \frac{2p}{N} k_1 k_2}.$$

A uniform construction of domain-wall operators

$$W(R) \equiv \prod_{j \in R \setminus \text{last col}} \mathbf{X}_j \prod_{\text{bags in } R} \mathcal{P} \cdot \prod_{j \in \text{bag}} C_p(\mathbf{Z}_j, \mathbf{Z}_{j+1})$$



Acts like **S** in the interior of R.
Threads $2\pi/N$-flux along its boundary.
Becomes the string operator in the topologically-ordered model with G gauged and deconfined.

Braiding of flux insertions

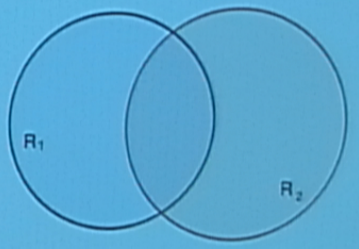
[Levin-Gu]

$\mathbf{W}_R$ generate symmetries:

$$[\mathbf{H}, \mathbf{W}_R] = 0 \ .$$

If domain walls intersect *once*:

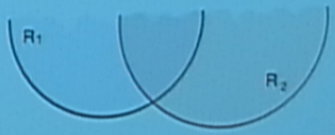
$$\mathbf{W}_{R_1}^{k_1} \mathbf{W}_{R_2} = e^{2\pi i k_1 k_2 \frac{2p}{N^2}} \mathbf{W}_{R_2}^{k_2} \mathbf{W}_{R_1}^{k_1}$$



(annoying fine print:
this formula works for $k_1 k_2 \propto N$)

This must be represented on groundstates
(in fact, the whole spectrum)

$\Longrightarrow$ nontrivial edge spectrum.



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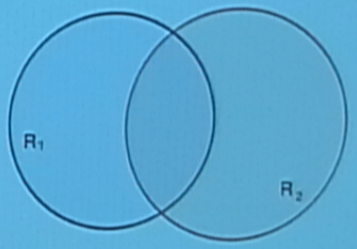

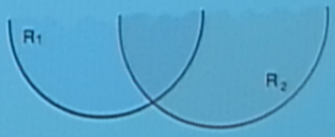

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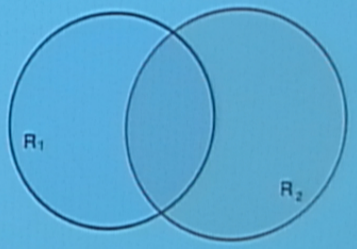

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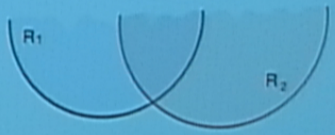

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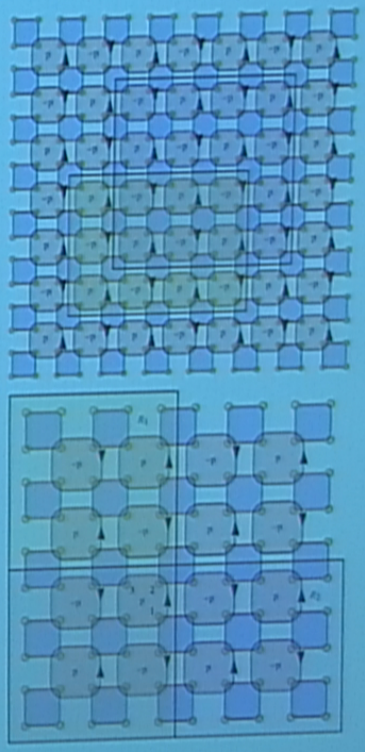
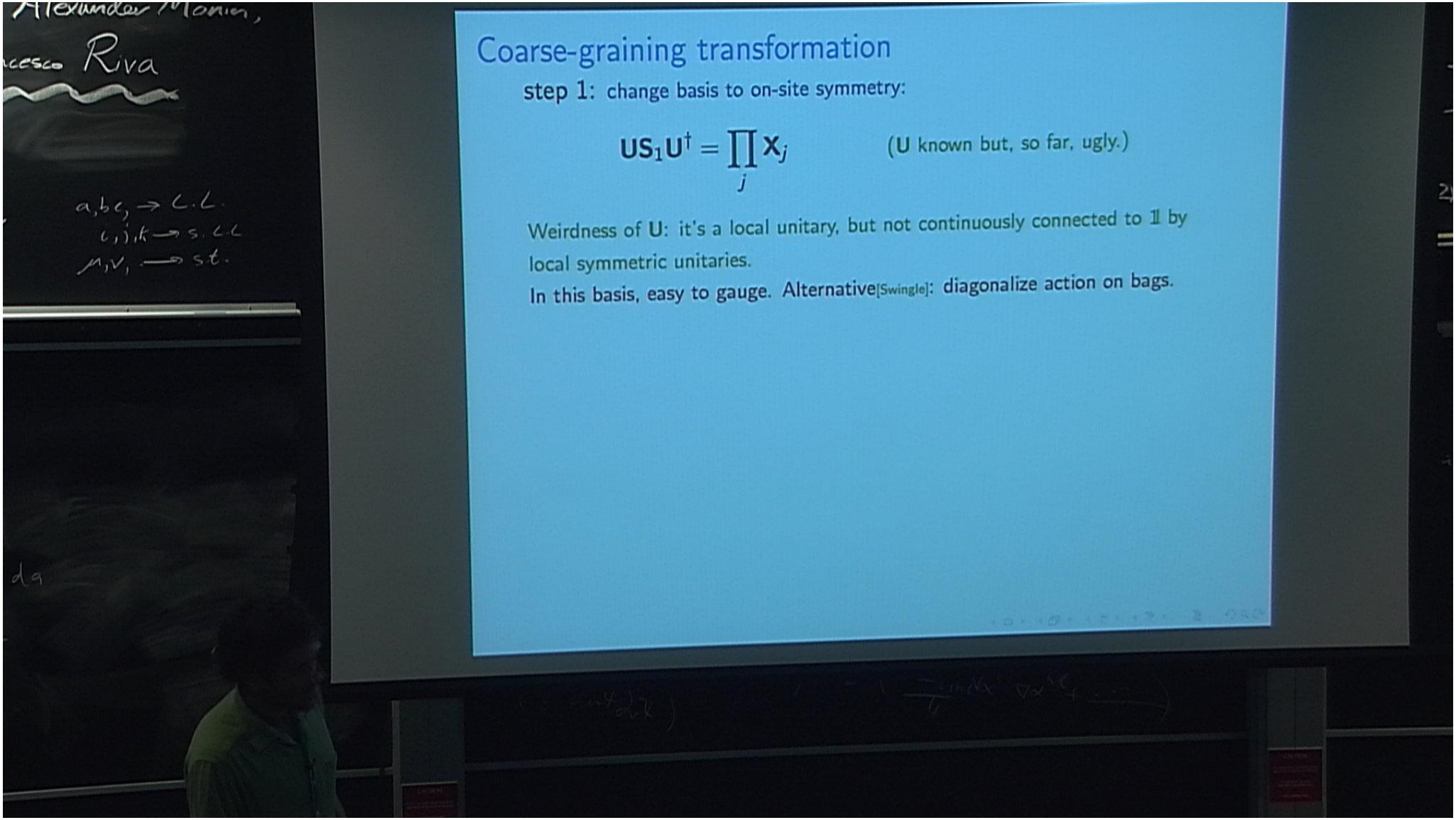
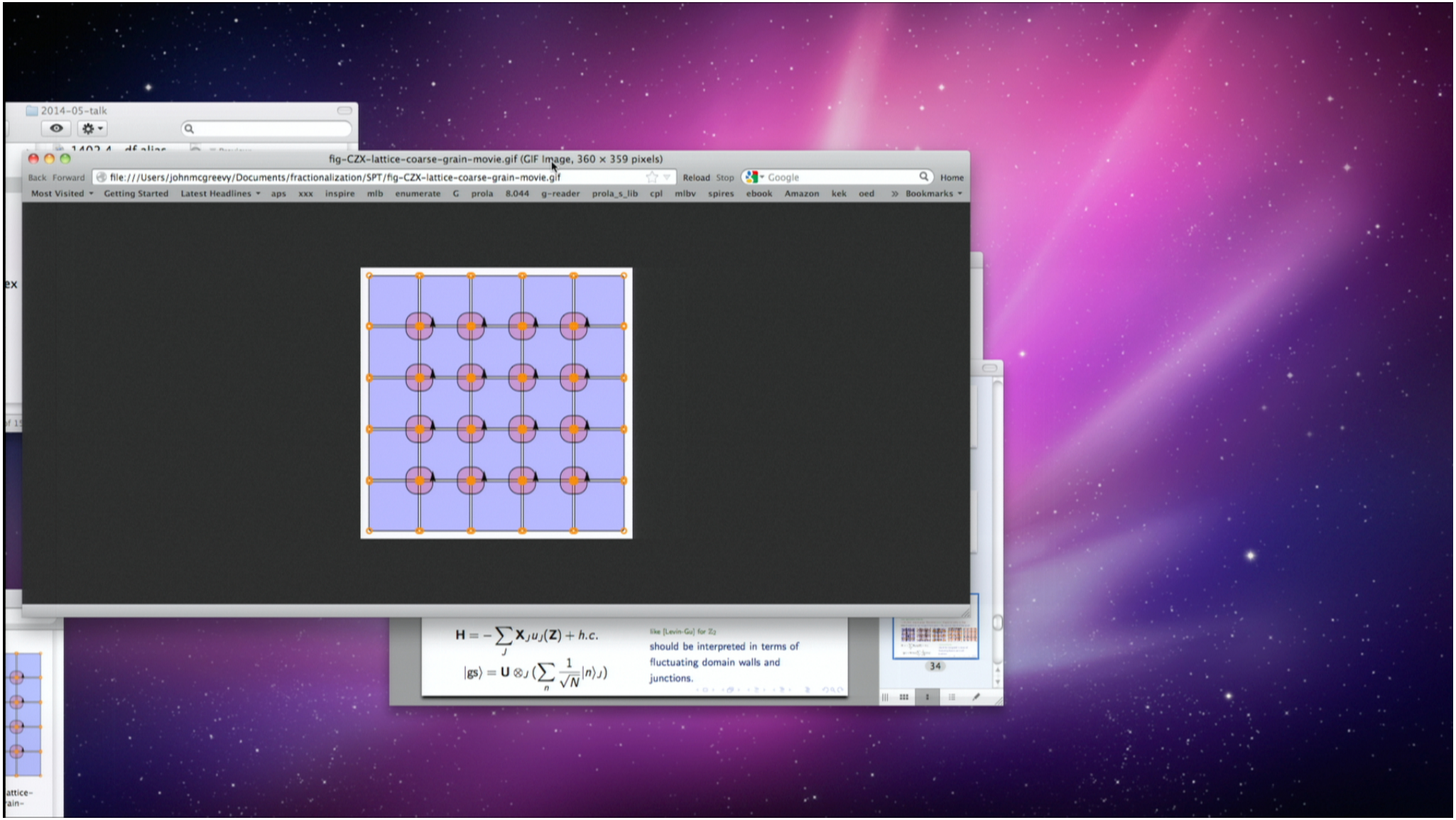
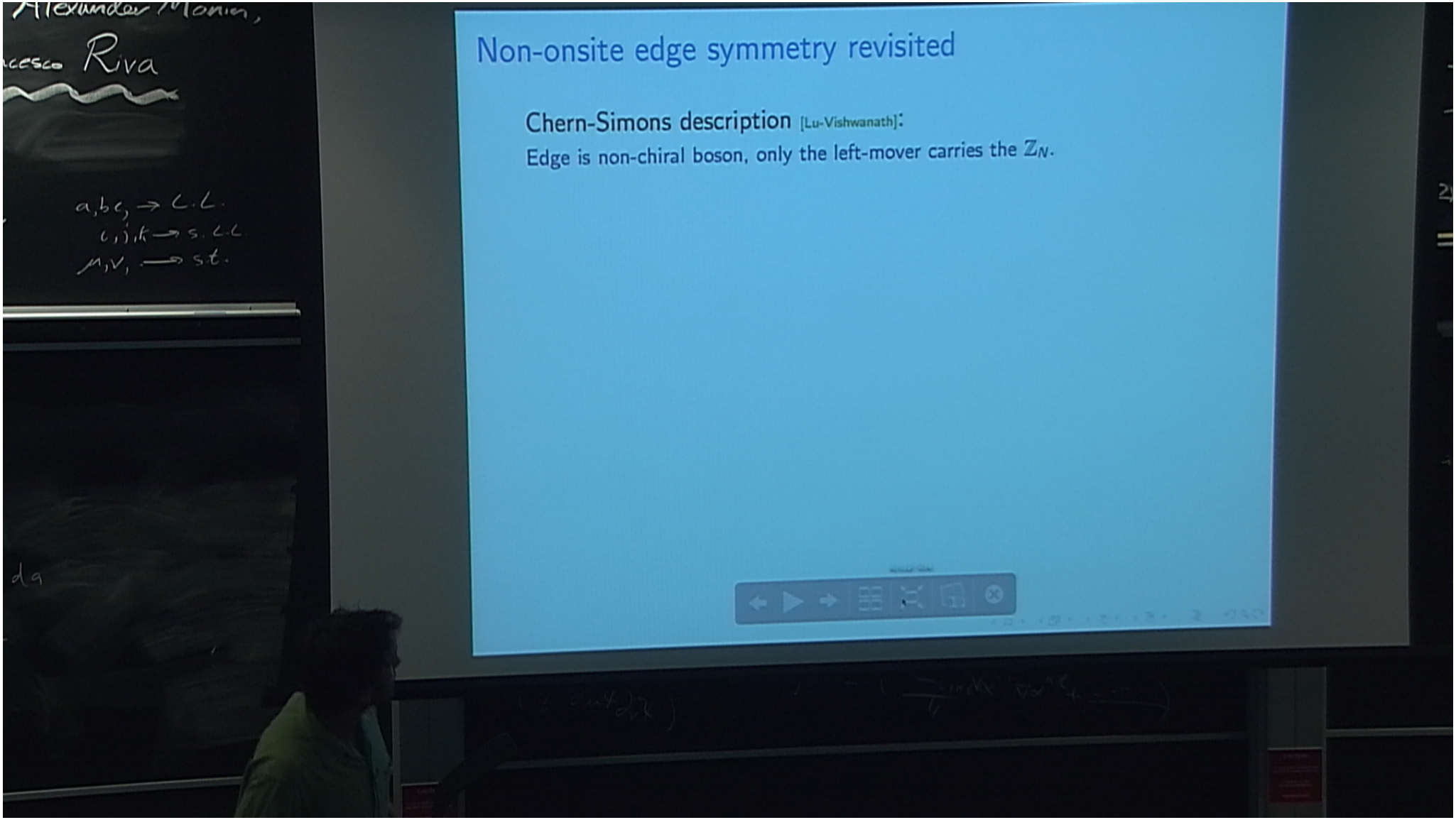
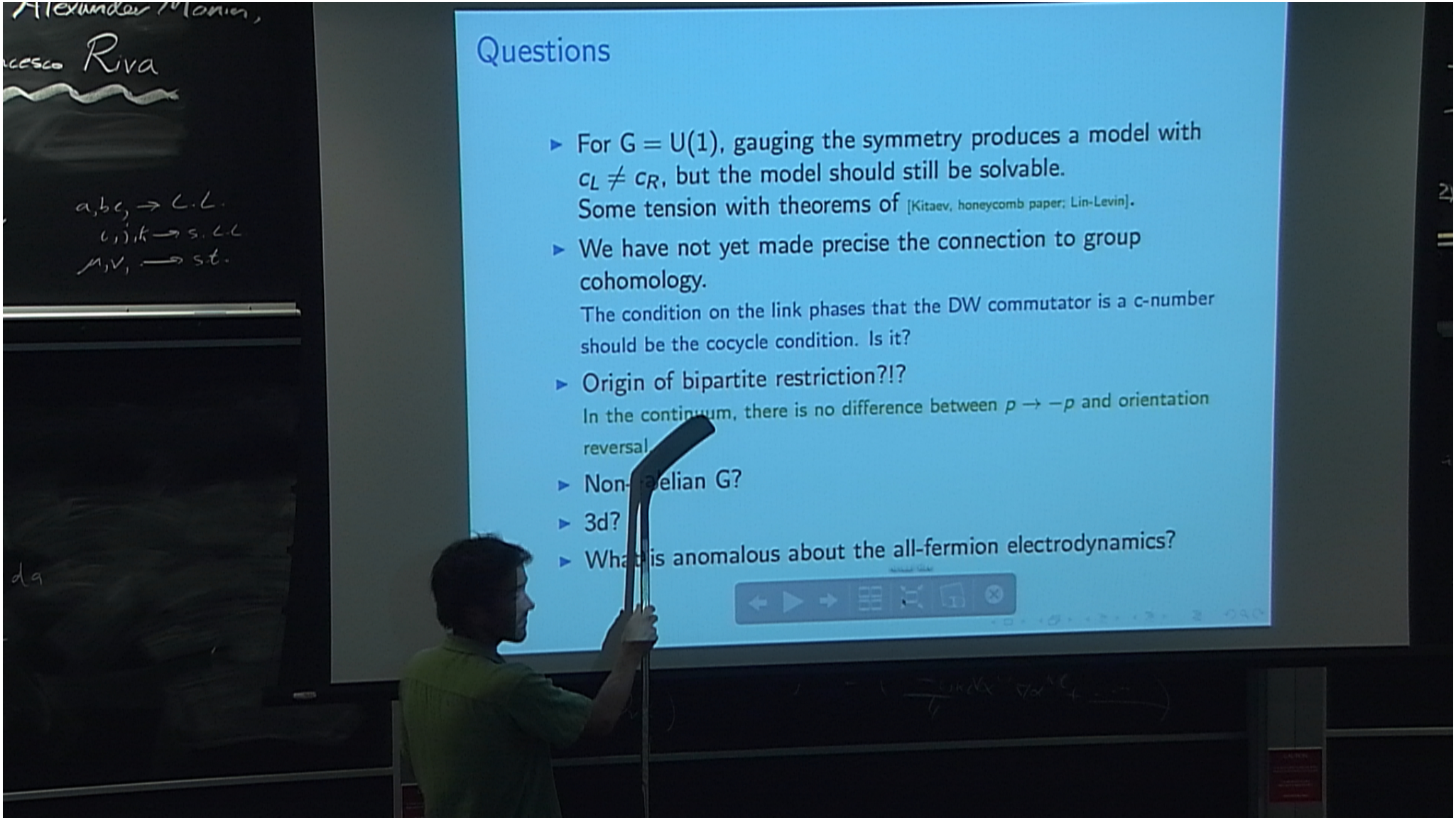
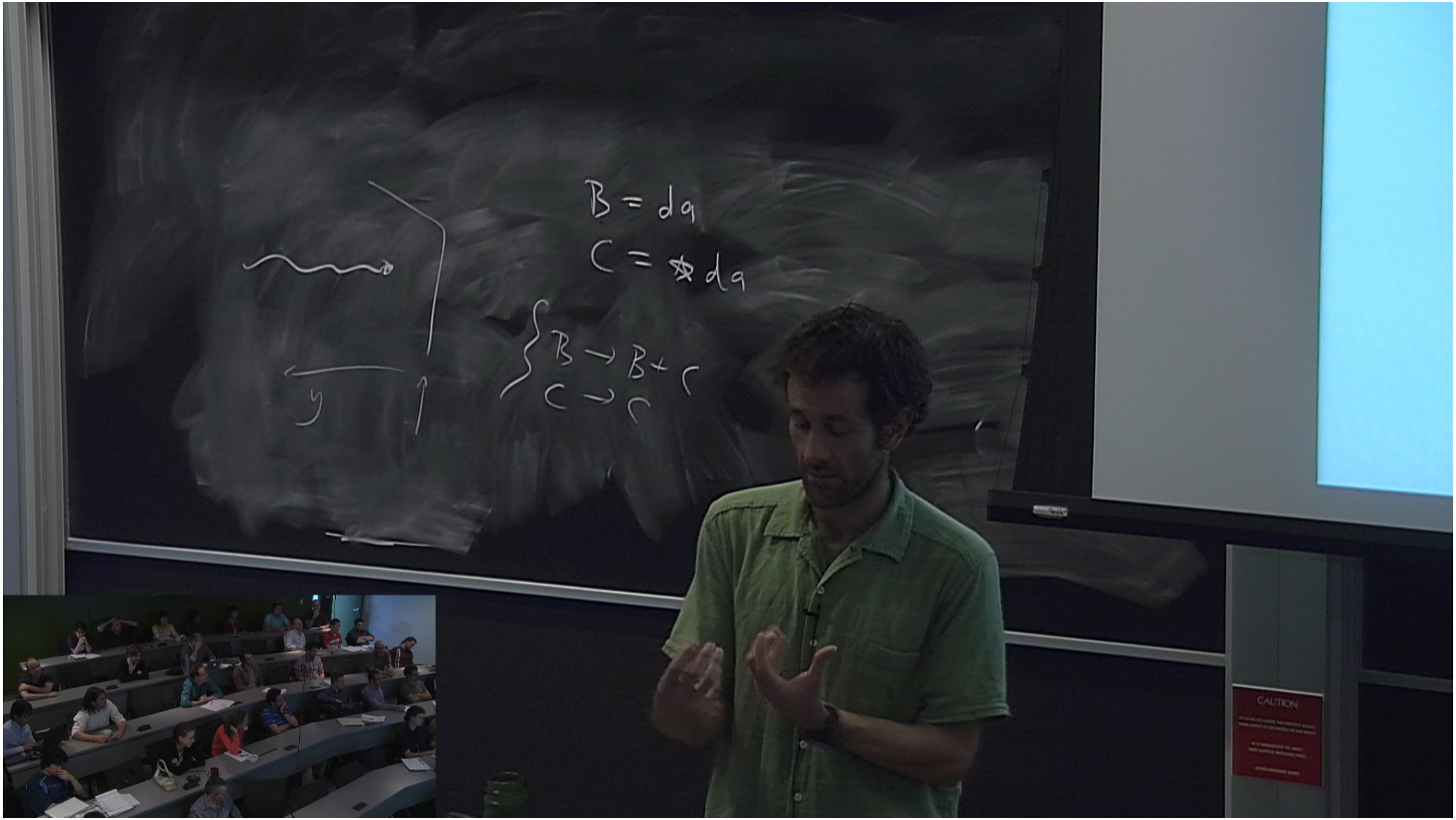


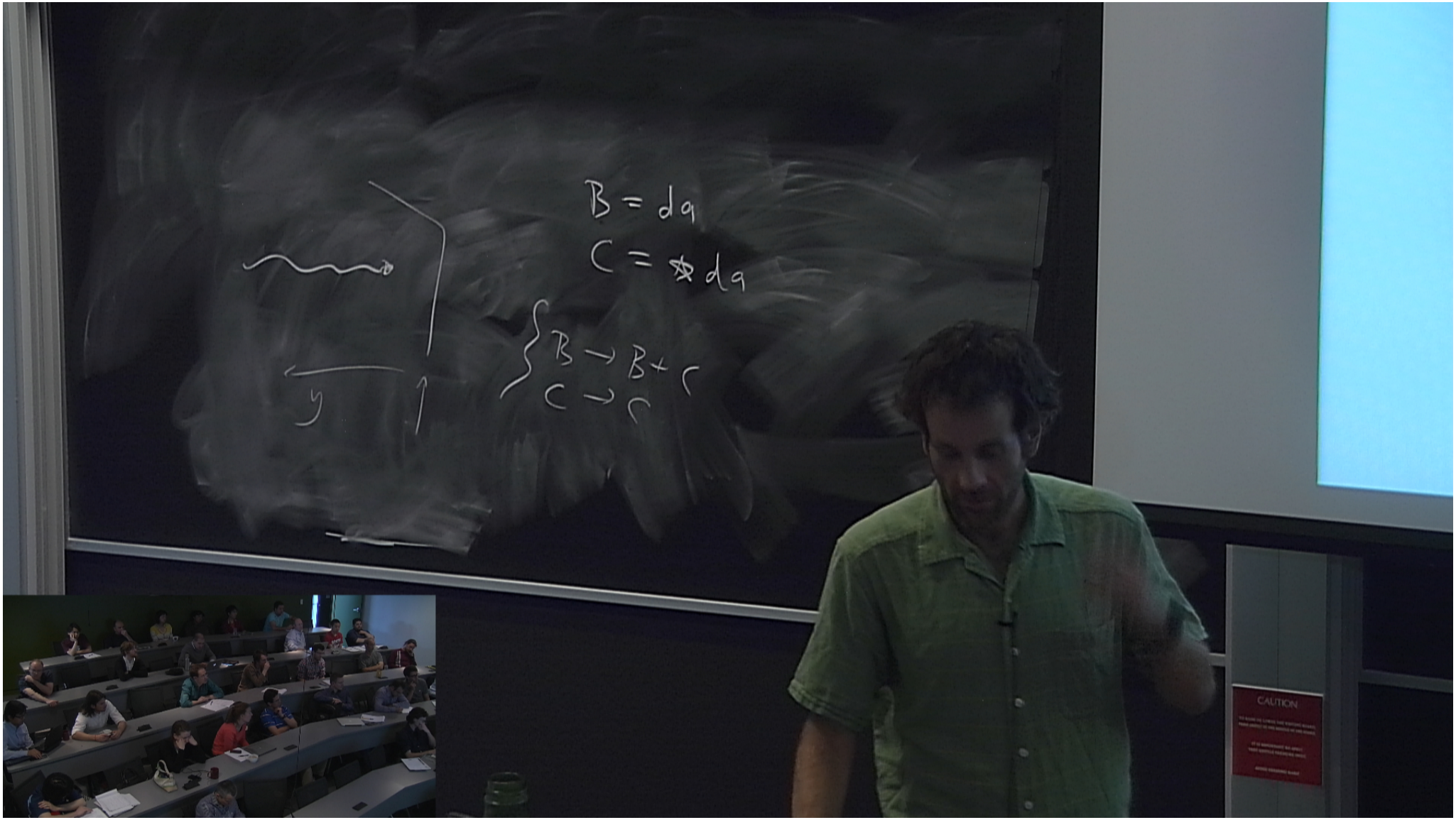

Coarse-graining transformation
step 1: change basis to on-site symmetry:
$\mathbf{U}S_1\mathbf{U}^\dagger = \prod_j \mathbf{X}_j$ (U known but, so far, ugly.)
Weirdness of U: it's a local unitary, but not continuously connected to $\mathbb{1}$ by local symmetric unitaries.
In this basis, easy to gauge. Alternative[Swingle]: diagonalize action on bags.


2014-05-talk
fig-CZX-lattice-coarse-grain-movie.gif (GIF Image, 360 × 359 pixels)
Back Forward
file:///Users/johnmcgreevy/Documents/fractionalization/SPT/fig-CZX-lattice-coarse-grain-movie.gif
Reload Stop
Google
Home
Most Visited
Getting Started
Latest Headlines
aps
xxx
inspire
mlb
enumerate
G
prola
8.044
g-reader
prola_s_lib
cpl
mlbv
spires
ebook
Amazon
kek
oed
Bookmarks
$H = -\sum_J X_J u_J(Z) + h.c.$
$|gs\rangle = U \otimes_J (\sum_n \frac{1}{\sqrt{N}} |n\rangle_J)$
like [Levin-Gu] for Z_2
should be interpreted in terms of fluctuating domain walls and junctions.
34


Non-onsite edge symmetry revisited
Chern-Simons description [Lu-Vishwanath]:
Edge is non-chiral boson, only the left-mover carries the $\mathbb{Z}_N$.


Questions
For G = U(1), gauging the symmetry produces a model with $c_L \neq c_R$, but the model should still be solvable.
Some tension with theorems of [Kitaev, honeycomb paper; Lin-Levin].
We have not yet made precise the connection to group cohomology.
The condition on the link phases that the DW commutator is a c-number should be the cocycle condition. Is it?
Origin of bipartite restriction?!?
In the continuum, there is no difference between $p \to -p$ and orientation reversal.
Non-Abelian G?
3d?
What is anomalous about the all-fermion electrodynamics?


B = dq
C = *dq
B → B + C
C → C
y
CAUTION


B = dq
C = ⋆ dq
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C → C
y
CAUTION