

Title: Quantum matter without quasiparticles

Date: May 29, 2014 10:30 AM

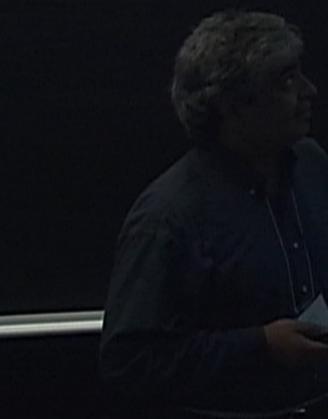
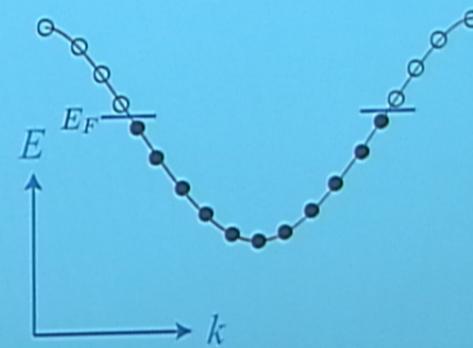
URL: <http://pirsa.org/14050109>

Abstract: <span>Modern materials abound in systems to which the quasiparticle picture does not apply, and developing their theoretical description remains an important challenge in condensed matter physics. I will describe recent progress in understanding the dynamics of two systems without quasiparticles: (i) ultracold atoms in optical lattices, and (ii) the nematic quantum critical point of metals with applications to the `strange metalâ€™ found in the high temperature superconductors. A combination of field-theoretic, holographic, and numerical methods will be used.</span><span></span>

Foundations of quantum many body theory:

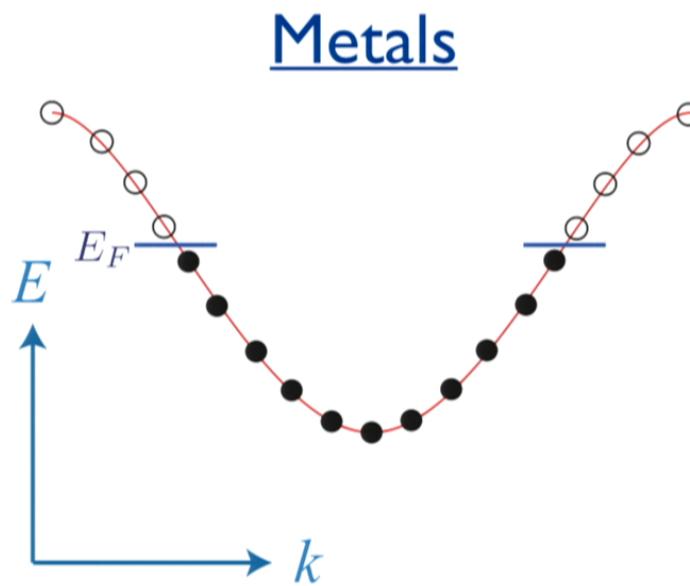
I. Ground states connected adiabatically to independent electron states

Metals



*Foundations of quantum many body theory:*

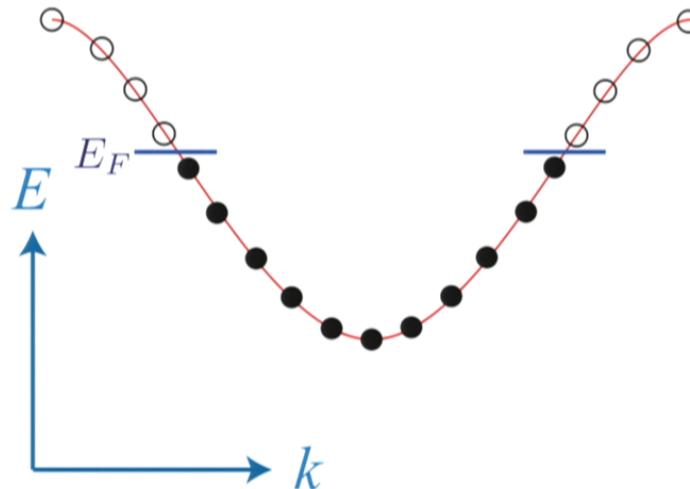
*I. Ground states connected adiabatically to independent electron states*



## *Foundations of quantum many body theory:*

### *I. Ground states connected adiabatically to independent electron states*

Metals



Modern phases of quantum matter:

1. *Ground states disconnected from independent electron states: many-particle entanglement*
2. *Boltzmann-Landau theory of quasiparticles*

Famous examples:

The fractional quantum Hall effect of electrons in two dimensions (e.g. in graphene) in the presence of a strong magnetic field. The ground state is described by Laughlin's wavefunction, and the excitations are *quasiparticles* which carry fractional charge.

Modern phases of quantum matter:

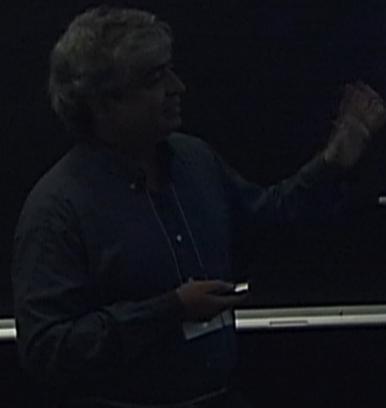
- 1. Ground states disconnected from independent electron states: many-particle entanglement*
- 2. No quasiparticles*

Modern phases of quantum matter:

1. Ground states disconnected from independent electron states: many-particle entanglement
2. No quasiparticles

Only 2 examples:

1. Conformal field theories in spatial dimension  $d > 1$
2. Quantum critical metals in dimension  $d = 2$



# Outline

## 1. Conformal field theories in 2+1 dimensions

*Superfluid-insulator transition*

*of ultracold bosonic atoms in an optical lattice*

## 2. Theory of a non-Fermi liquid

*Non-quasiparticle transport at the*

*Ising-nematic quantum critical point*

## The dynamics of quantum criticality revealed by quantum Monte Carlo and holography

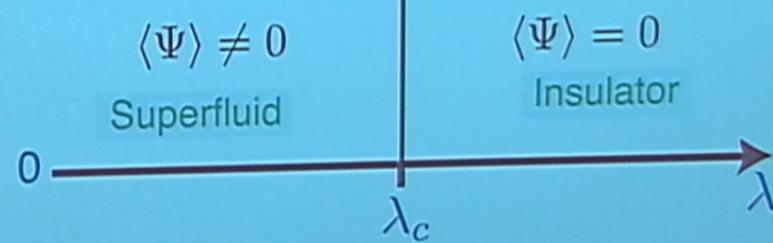


William Witczak-Krempa  
Perimeter

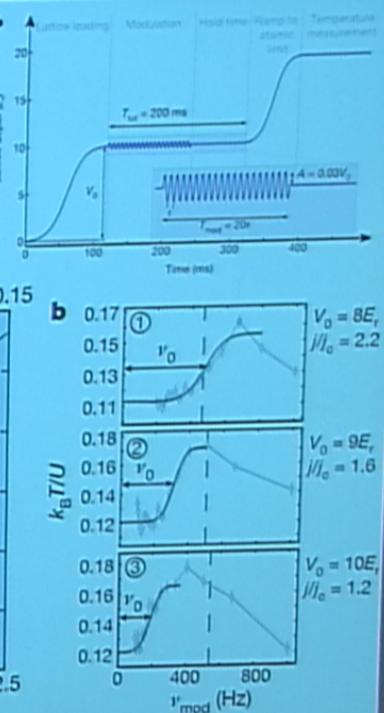
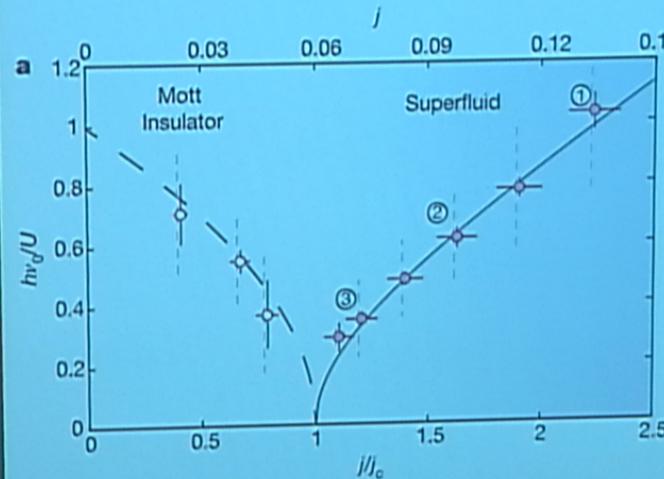


Erik Sorensen  
McMaster

$$\begin{aligned} S &= \int d^2 r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)] \\ V(\Psi) &= (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2 \end{aligned}$$



## Excitations across the superfluid-insulator transition

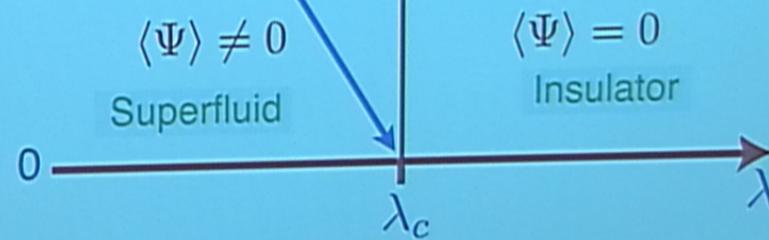


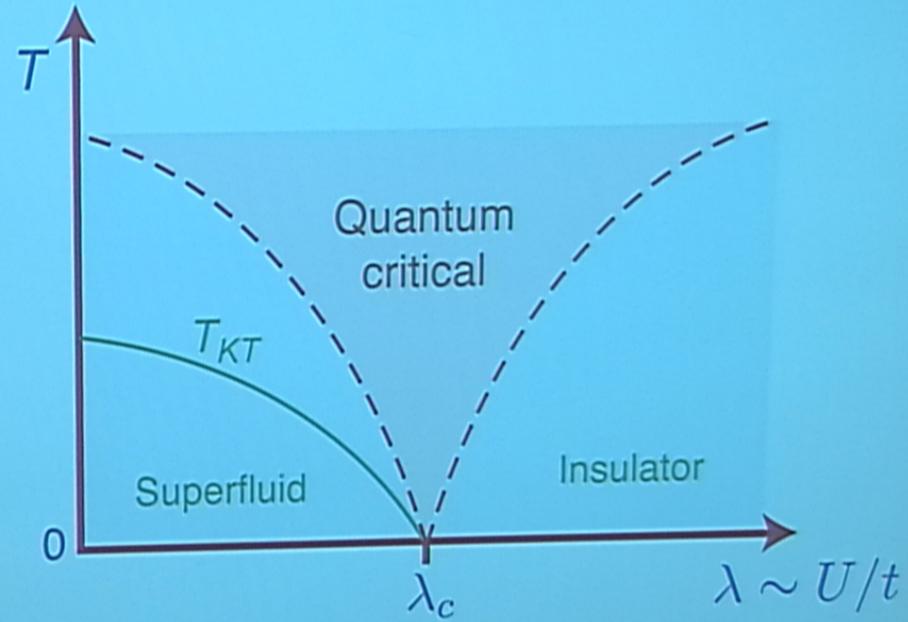
Manuel Endres, Takeshi Fukuhara, David Pekker, Marc Cheneau, Peter Schaub, Christian Gross, Eugene Demler, Stefan Kuhr, and Immanuel Bloch, *Nature* 487, 454 (2012).

$$S = \int d^2 r dt [|\partial_t \Psi|^2 - c^2 |\nabla_r \Psi|^2 - V(\Psi)]$$

$$V(\Psi) = (\lambda - \lambda_c) |\Psi|^2 + u (|\Psi|^2)^2$$

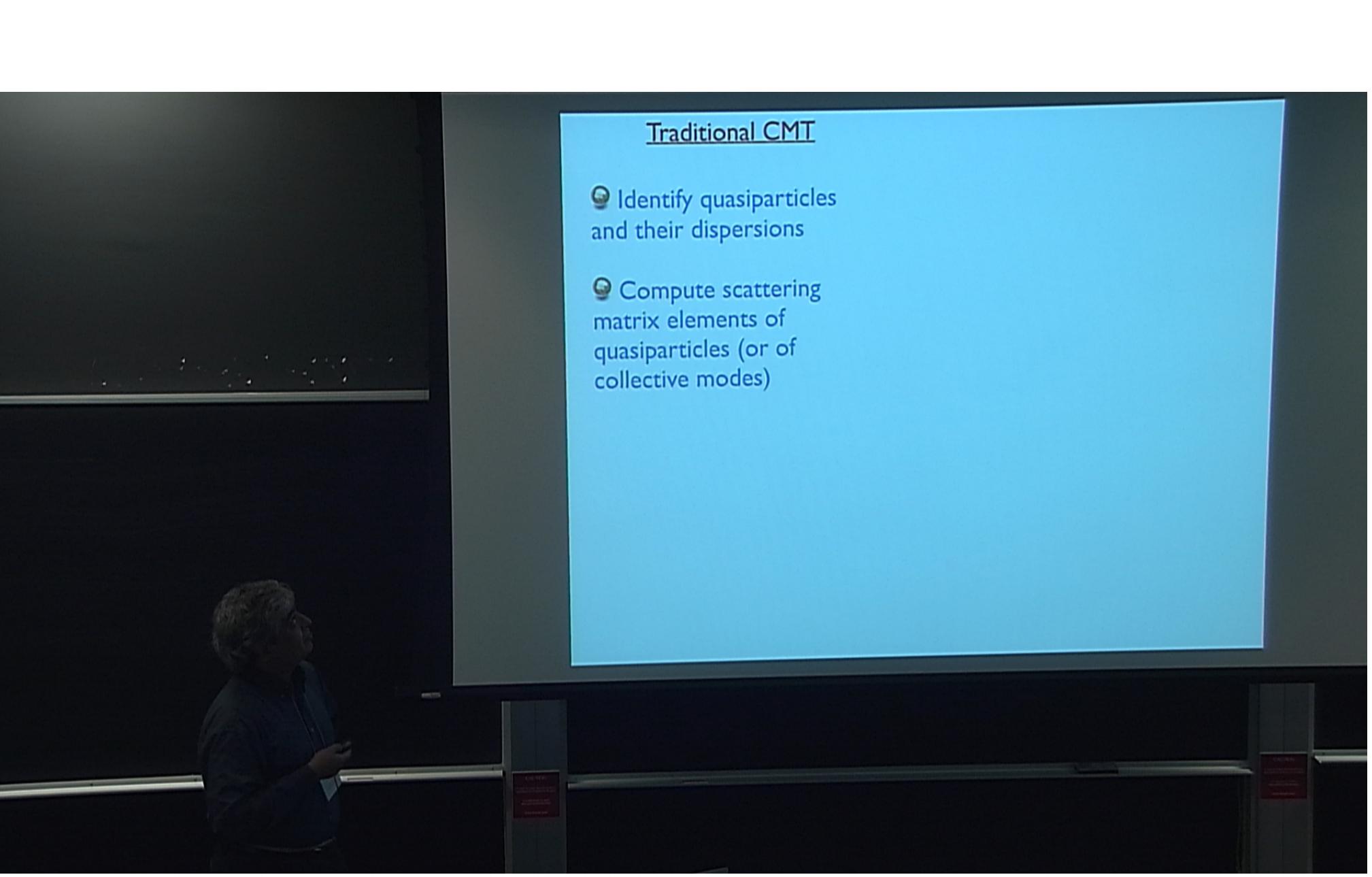
Quantum state with  
“long-range” quantum entanglement  
**and no quasiparticles.**  
A 2+1 dim. conformal field theory (CFT3)



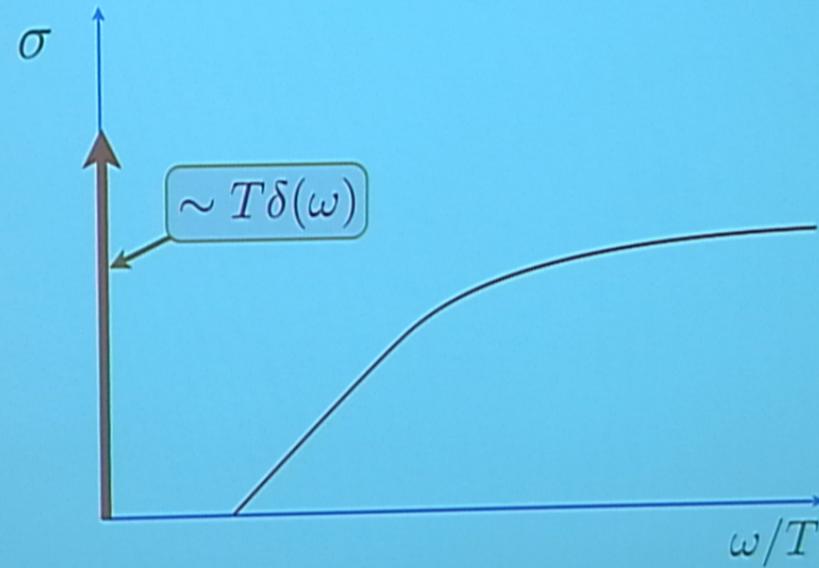


### Traditional CMT

- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)



Quasiparticle view of quantum criticality (Boltzmann equation):  
Electrical transport for a free CFT3



Quasiparticle view of quantum criticality (Boltzmann equation):  
Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left( \frac{\hbar\omega}{k_B T} \right)$$

Re[ $\sigma(\omega)$ ]

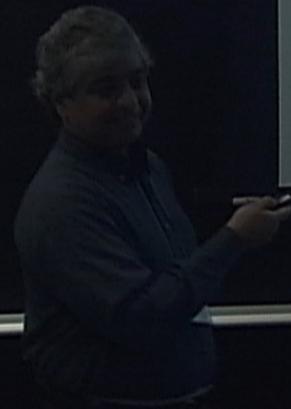
$$\Sigma(\infty) = 2\pi \times (1/16) + \mathcal{O}(1/N)$$

in a vector large  $N$  limit

1

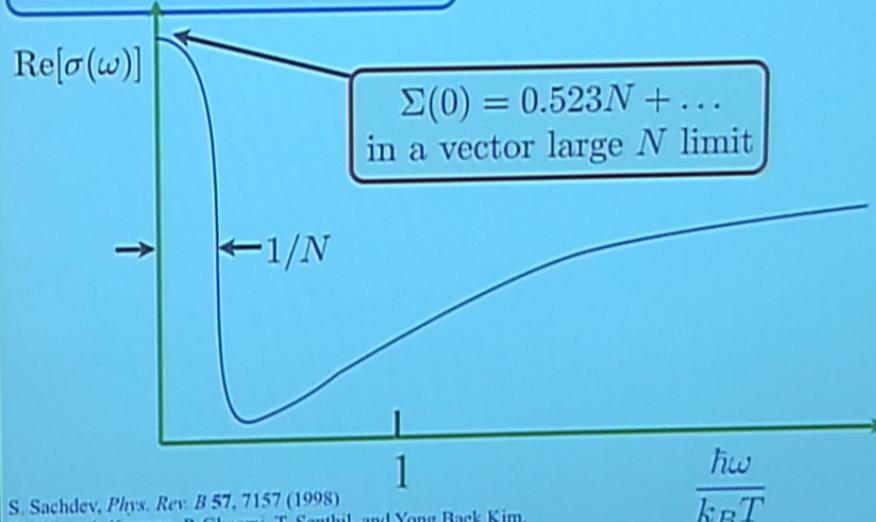
$\frac{\hbar\omega}{k_B T}$

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

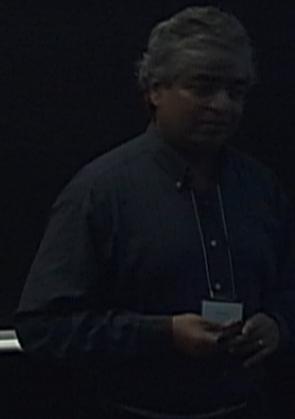


Quasiparticle view of quantum criticality (Boltzmann equation):  
Electrical transport for a (weakly) interacting CFT3

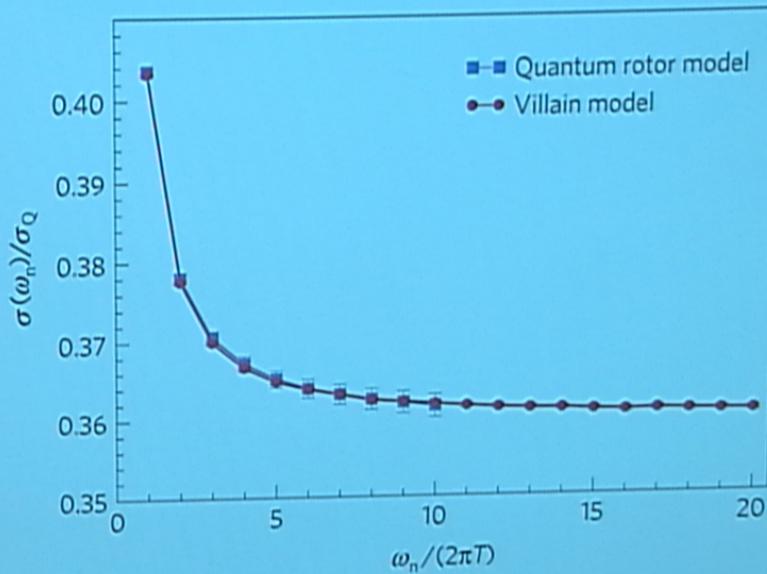
$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left( \frac{\hbar\omega}{k_B T} \right)$$



S. Sachdev, *Phys. Rev. B* **57**, 7157 (1998)  
W. Witczak-Krempa, P. Ghaemi, T. Senthil, and Yong Baek Kim,  
*Phys. Rev. B* **86**, 24102 (2012)

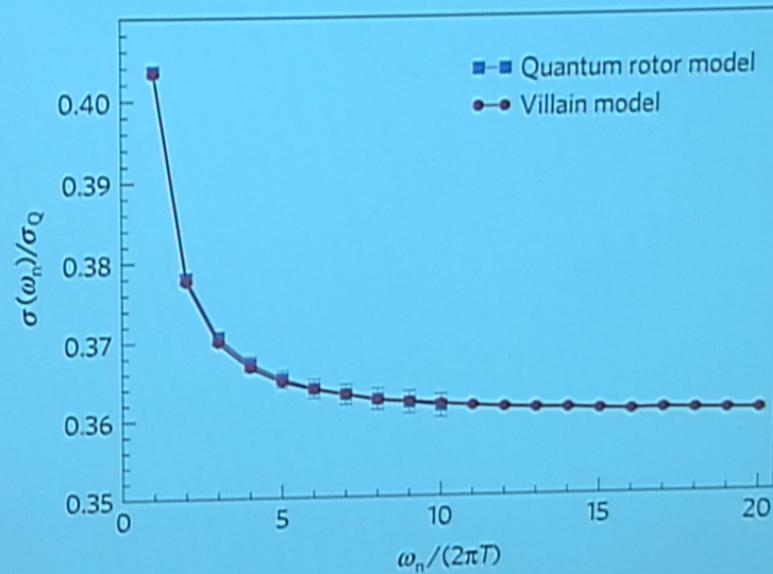


## Quantum Monte Carlo for lattice bosons



W. Witczak-Krempa, E. Sorensen, and S. Sachdev, Nature Physics **10**, 361 (2014)  
See also K. Chen, L. Liu, Y. Deng, L. Pollet, and N. Prokof'ev, Phys. Rev. Lett. **112**, 030402 (2013)

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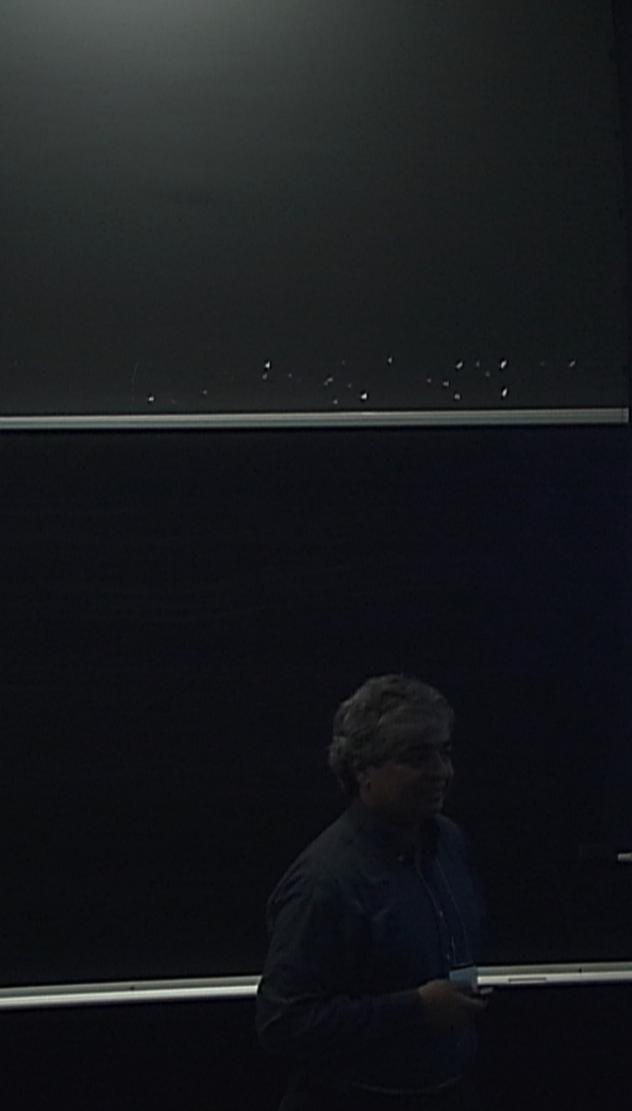
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### Traditional CMT

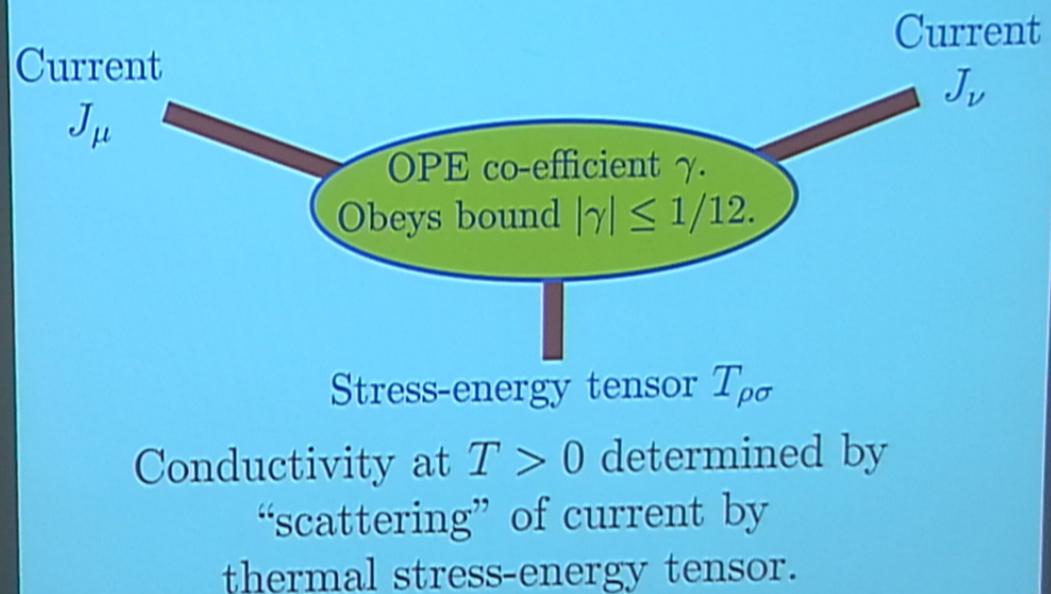
- Identify quasiparticles and their dispersions
- Compute scattering matrix elements of quasiparticles (or of collective modes)
- These parameters are input into a quantum Boltzmann equation
- Deduce dissipative and dynamic properties at non-zero temperatures

### Dynamics without quasiparticles

- Start with strongly interacting CFT without particle- or wave-like excitations
- Compute scaling dimensions and OPE co-efficients of operators of the CFT



## Physical picture of electrical transport in a CFT3



R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)  
D. Chowdhury, S. Raju, S. Sachdev, A. Singh, and P. Strack, *Physical Review B* **87**, 085138 (2013).  
D. M. Hofman and J. Maldacena, *JHEP* **0805** (2008) 012.

## AdS<sub>4</sub> theory of quantum criticality

Most general effective holographic theory for linear charge transport with 4 spatial derivatives:

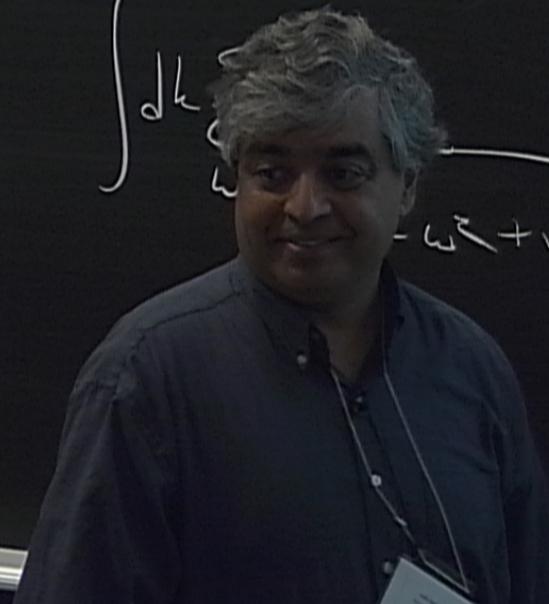
$$\begin{aligned} S_{\text{bulk}} = & \frac{1}{g_M^2} \int d^4x \sqrt{g} \left[ \frac{1}{4} F_{ab} F^{ab} + \gamma L^2 C_{abcd} F^{ab} F^{cd} \right] \\ & + \int d^4x \sqrt{g} \left[ -\frac{1}{2\kappa^2} \left( R + \frac{6}{L^2} \right) \right], \end{aligned}$$

This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current  $J_\mu$  and the stress energy tensor  $T_{\mu\nu}$ , and a 3-point  $T, J, J$  correlator. Constraints from both the CFT and the gravitational theory bound  $|\gamma| \leq 1/12 = 0.0833..$

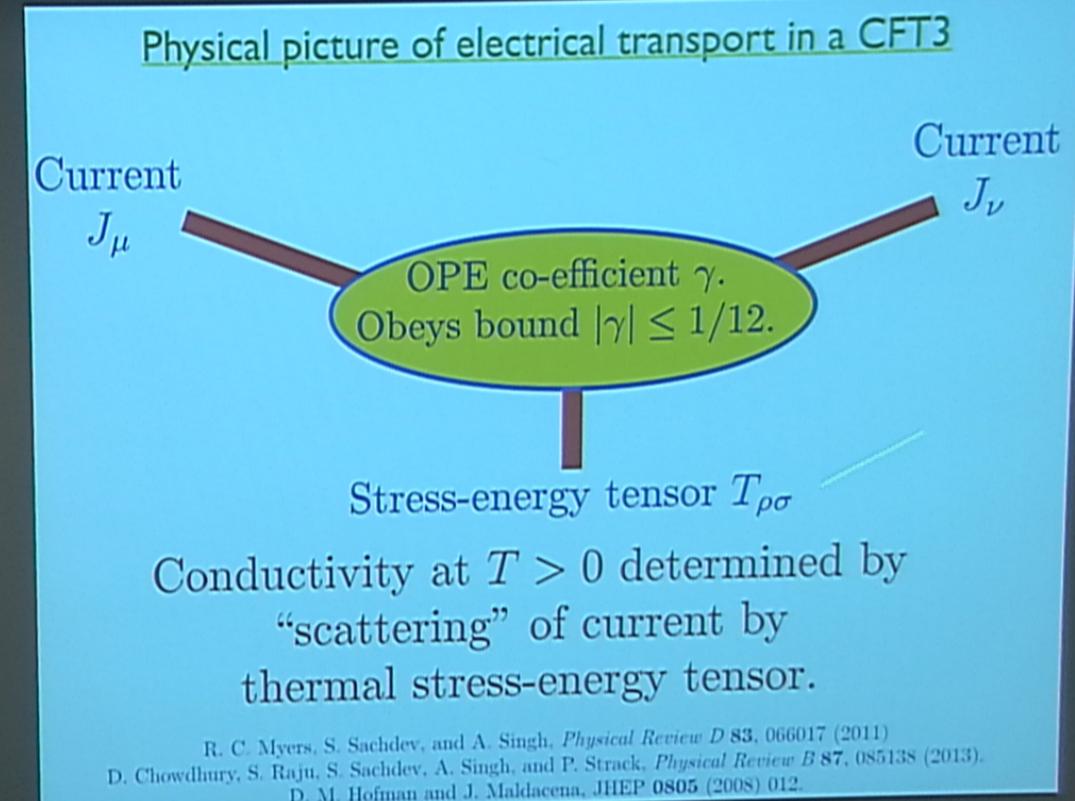
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1

$$\langle \phi^2 \rangle = \int dk \zeta \zeta - \omega \zeta + n$$



$$\langle \phi \rangle = \int dL \sum \dots$$



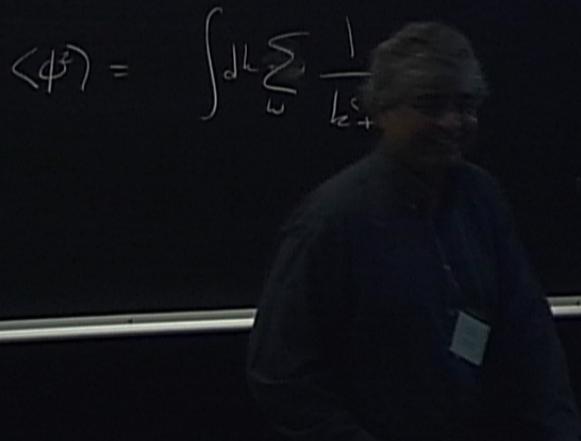
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## AdS<sub>4</sub> theory of quantum criticality

- The AdS<sub>4</sub> solutions satisfy two sum rules which are expected to be satisfied by all CFT3s:

$$\int_0^\infty d\omega [\Sigma(\omega) - \Sigma(\infty)] = 0$$
$$\int_0^\infty d\omega \left[ \frac{1}{\Sigma(\omega)} - \frac{1}{\Sigma(\infty)} \right] = 0$$

The second sum rule relies on the existence of a S-dual CFT3.

$$\langle \phi^2 \rangle = \int d\omega \sum \dots$$

W. Witczak-Krempa and S. Sachdev, *Phys. Rev. B* 86, 235115 (2012)

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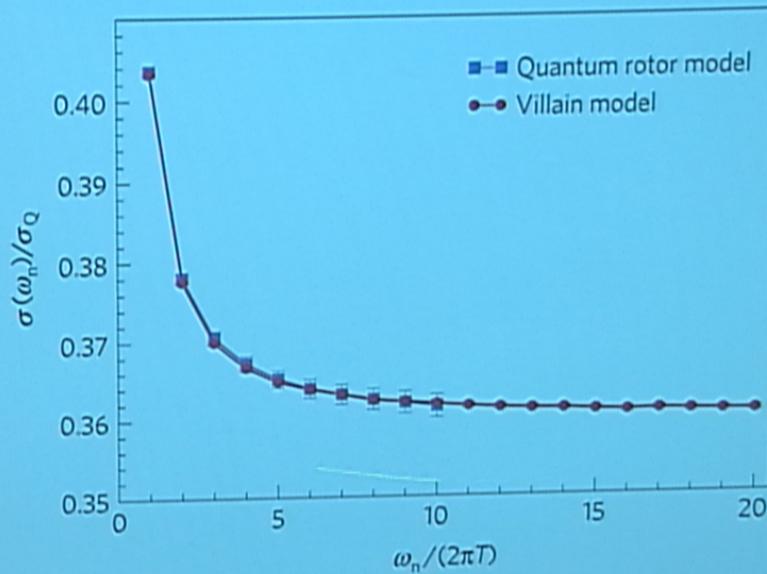
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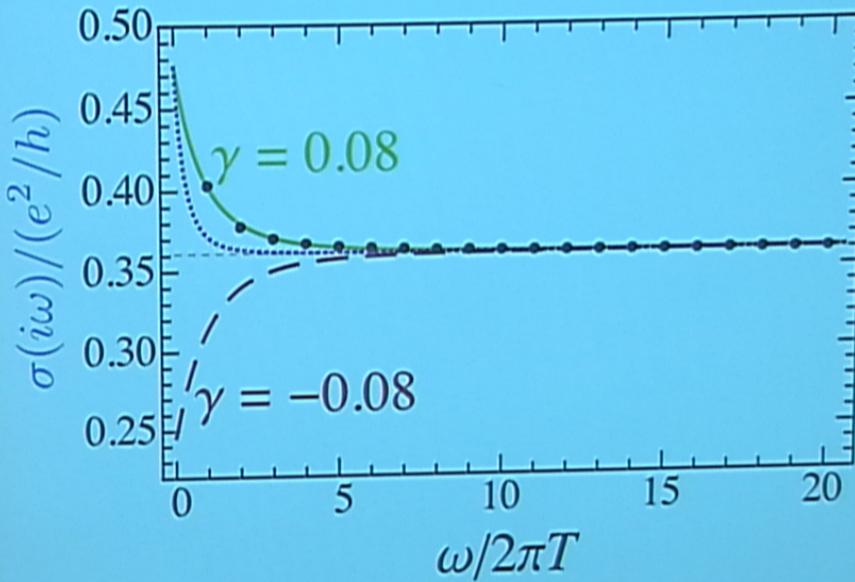


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$$\langle \phi^2 \rangle = \int dL \phi^2$$

$$\langle \phi^2 \rangle = \frac{1}{L^2 + \omega^2 + m}$$

### Transport revealed by QMC and holography

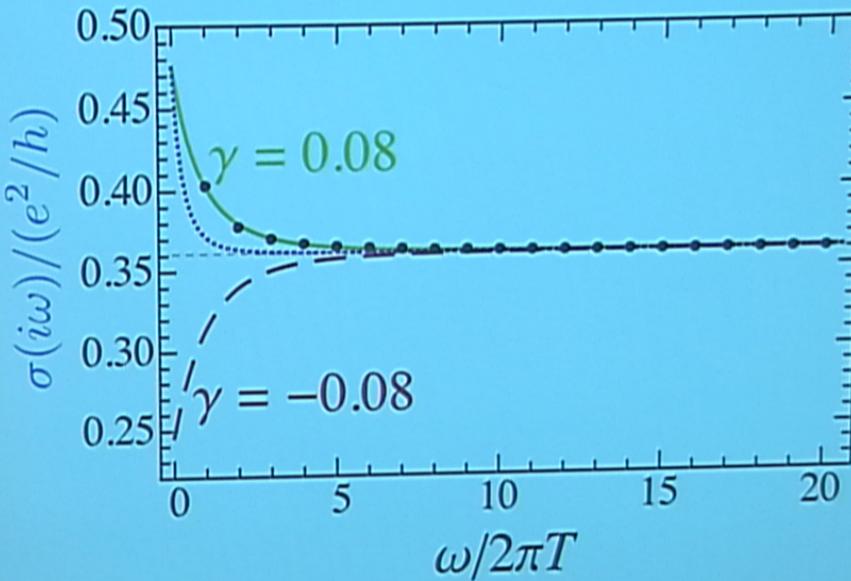


Fit of holographic theory to Monte Carlo,  
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$$\langle \phi^2 \rangle = \int dL \sum_c \frac{1}{L c_+}$$

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This action is characterized by 3 dimensionless parameters, which can be linked to data of the CFT (OPE coefficients): 2-point correlators of the conserved current  $J_\mu$  and the stress energy tensor  $T_{\mu\nu}$ , and a 3-point  $T, J, J$  correlator. Constraints from both the CFT and the gravitational theory bound  $|\gamma| \leq 1/12 = 0.0833..$

$$\langle \hat{\phi} \rangle = \int dk \sum_{\omega} \frac{1}{k^2 + \omega^2 + m^2}$$

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$$\langle J_m(x) J_o(0) \rangle = \frac{k}{x^4} \quad k = \frac{1}{16}$$

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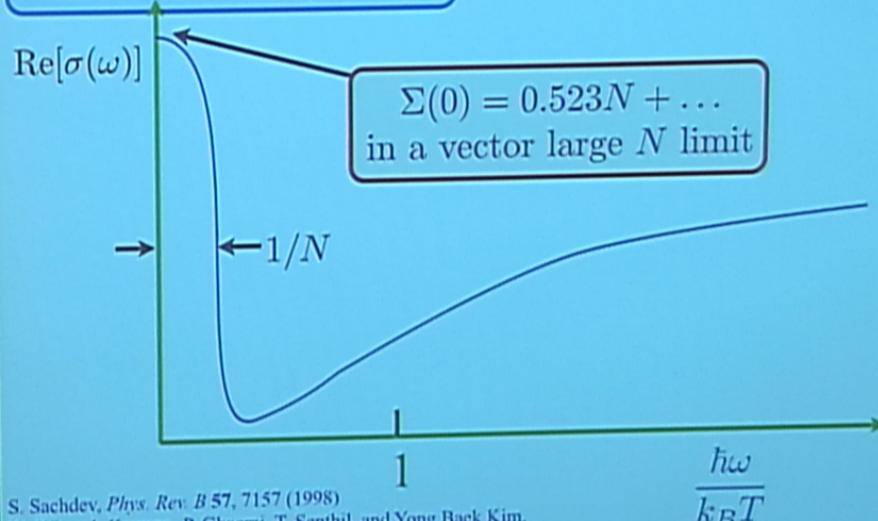
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$$\langle J_\mu(\omega) \rangle = \frac{k}{X^4} \quad k = \frac{1}{16},$$
$$\langle J_\mu(\omega) J_\nu(\omega') \rangle = \int dL \sum_\omega \frac{1}{k^2 + \omega^2 + m^2}$$

Quasiparticle view of quantum criticality (Boltzmann equation):  
Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h} \Sigma \left( \frac{\hbar\omega}{k_B T} \right)$$



S. Sachdev, *Phys. Rev. B* **57**, 7157 (1998)  
W. Witczak-Krempa, P. Ghaemi, T. Senthil, and Yong Baek Kim,  
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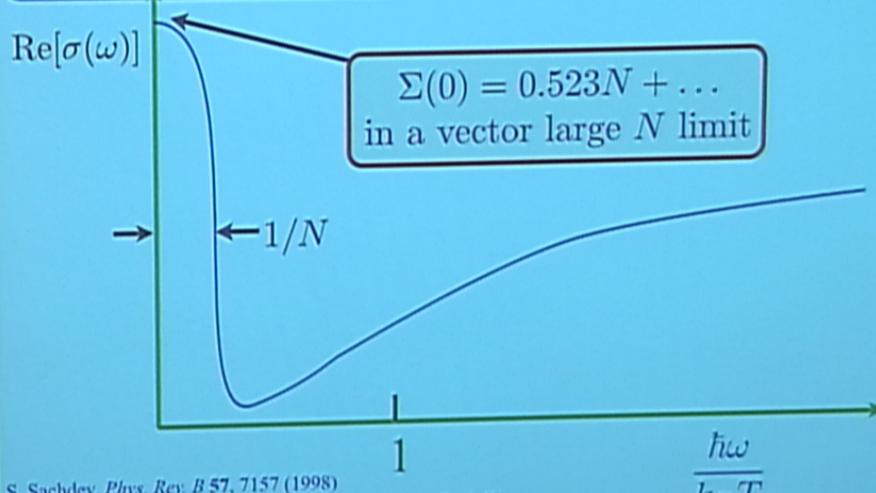
$$\langle \hat{J}_x(\vec{r}) \rangle = \frac{k}{X^4} \quad \hbar\omega = \frac{1}{4C_1}$$
$$\langle \hat{\phi}^2 \rangle = \int \frac{1}{\zeta + \omega^2 + m^2}$$

$$\langle J_\phi(\omega) \rangle = \frac{k}{X^4} \quad \text{where } k = \frac{1}{4\pi}$$

$$\langle \phi^2 \rangle = \int d\omega \sum_{\omega} \frac{1}{L^2}$$

Quasiparticle view of quantum criticality (Boltzmann equation):  
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## Outline

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## *Superfluid-insulator transition*

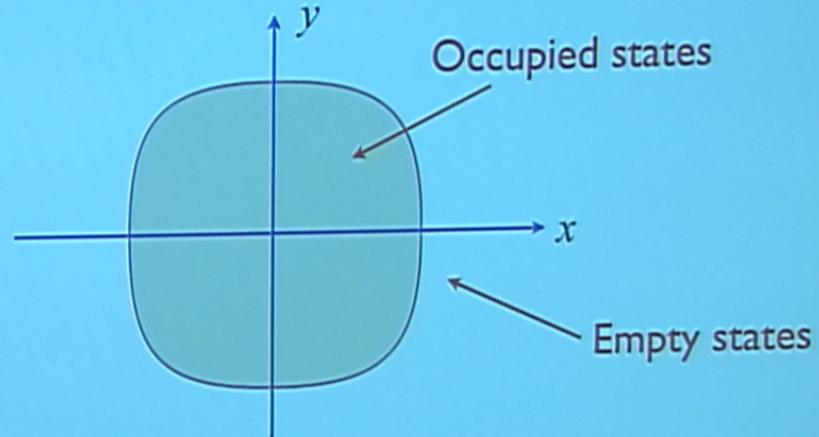
of ultracold bosonic atoms in an optical lattice

## 2. Theory of a non-Fermi liquid

## Non-quasiparticle transport at the Ising-nematic quantum critical point

$$\mathcal{D}(0) = \frac{k}{X^4} \quad k = \frac{1}{16}$$

## Quantum criticality of Ising-nematic ordering in a metal

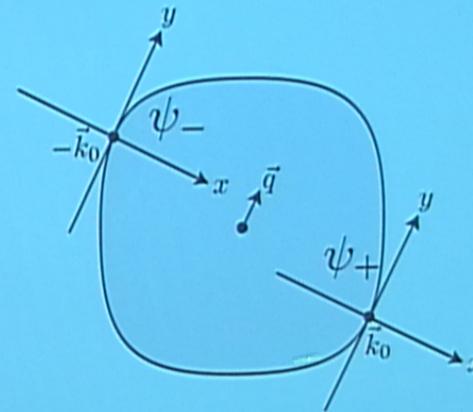


A metal with a Fermi surface  
with full square lattice symmetry

$$\langle \hat{\rho}(0) \rangle = \frac{k}{X^4} \quad k = \frac{1}{16}$$

$$\langle \phi^2 \rangle = \sum_{k,n} \frac{1}{k^2 + \omega_n^2 + m}$$

## Quantum criticality of Ising-nematic ordering in a metal



- $\phi$  fluctuation at wavevector  $\vec{q}$  couples most efficiently to fermions near  $\pm\vec{k}_0$ .

$$\langle \phi(\omega) \rangle = \frac{1}{X^4} \quad \omega = \frac{1}{TC}$$

$$\langle \phi^2 \rangle = \int dk \sum_{\omega} \frac{1}{E_{\pm}}$$