Title: Quantum Turbulence

Date: May 28, 2014 10:30 AM

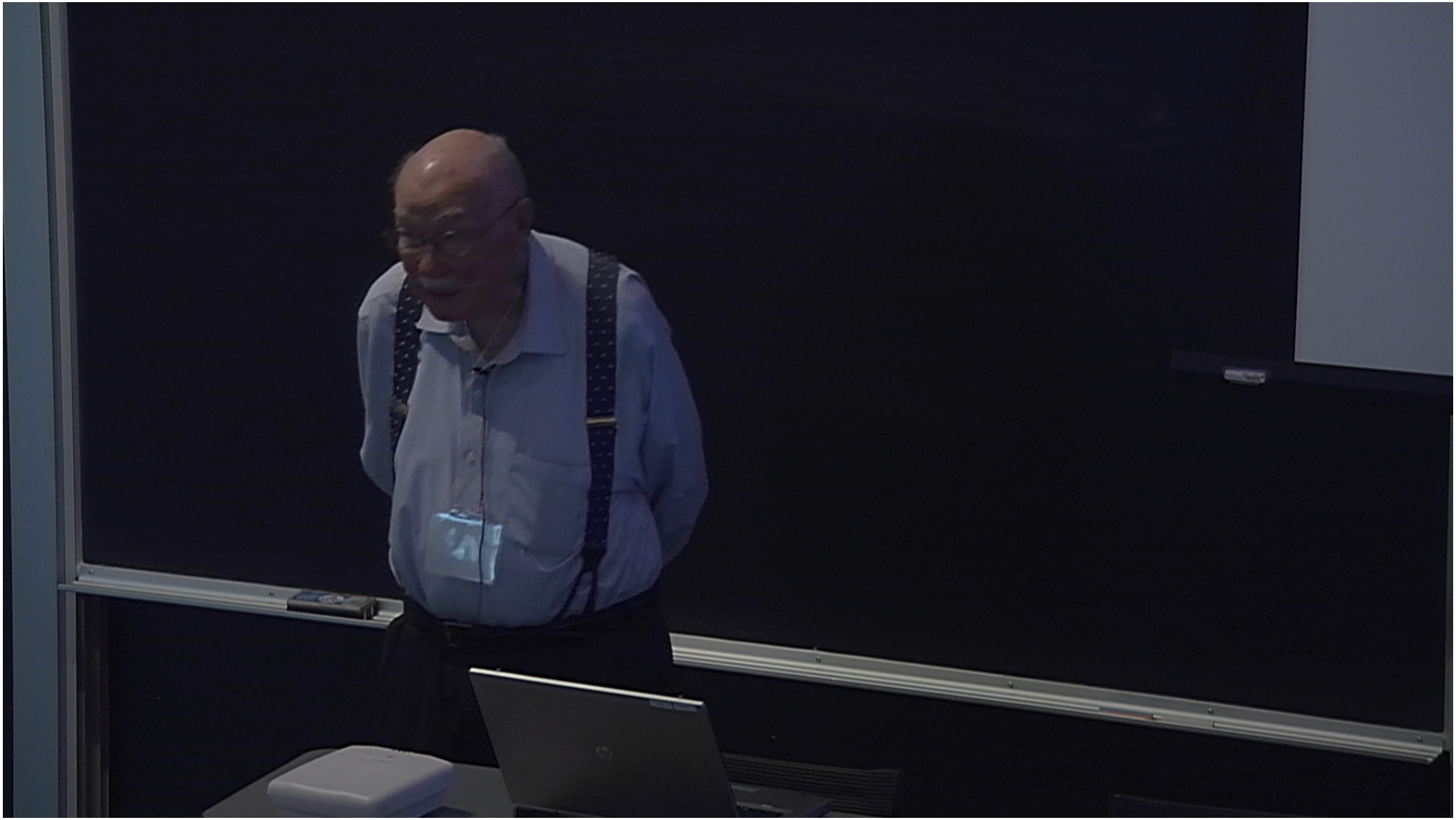
URL: http://pirsa.org/14050104

Abstract: This hour will be devoted to a description of quantum turbulence,that is turbulence in superfluids. The first talk (~20 minutes) will be given by Russell Donnelly. He will describe briefly the problem of classical turbulence and how turbulence in superfluids is different. The second talk will be given by Carlo Barenghi who will discuss progress in the simulation of quantum turbulence which is capable of suggesting insights so far inaccessible to experiment.

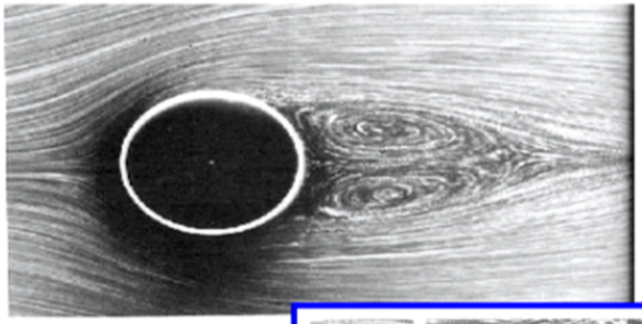
Quantum Turbulence

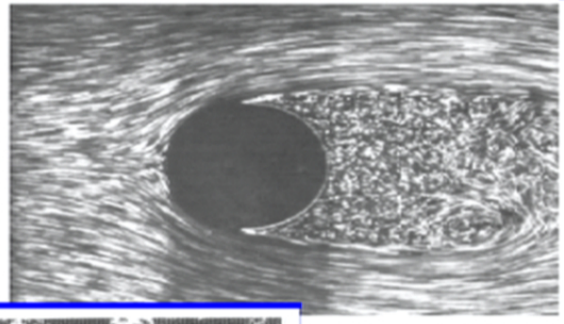
Quantum Turbulence

Russell J. Donnelly
Department of Physics
University of Oregon



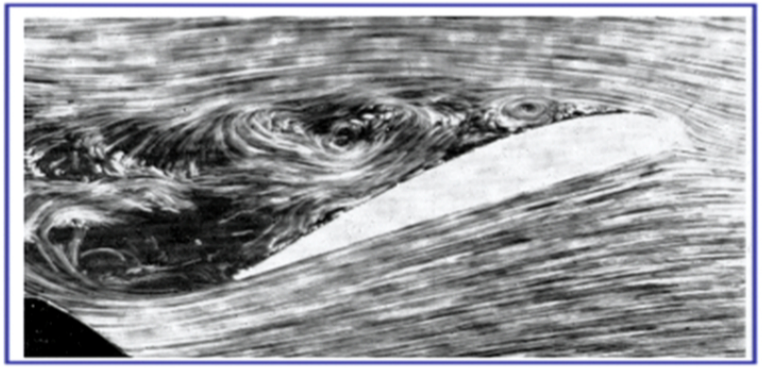
Classical and Quantum Turbulence



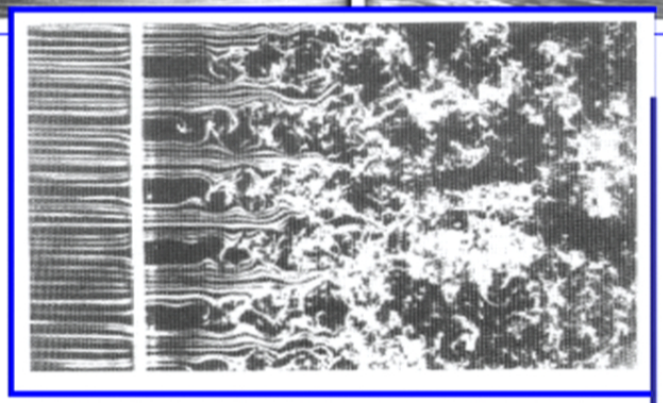


Water flowing past a cylinder → turbulent wake

Flow past an aerofoil



Water flowing through a grid → homogeneous turbulence



- The flow of **water** is governed by **classical mechanics**.
- **How is turbulence modified if the water in replaced by a superfluid?**

  (Remember flow of a **superfluid** is strongly influenced by **quantum effects**).
- How does quantum turbulence differ from classical turbulence?

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Turbulence is the chaotic motion of a fluid that occurs at sufficiently high flow rates. A common example is the flow of water from a tap: turn the tap on a little and the water runs out in a regular fashion called laminar flow. Turn the tap on fully and the flow becomes an irregular torrent. Such turbulent flow is characterized by regions of circulating fluid called eddies or vortices. If these features are ordered, they can lead to large scale structures such as whirlpools or tornados. Usually the eddies are highly irregular and interact in a complex and unpredictable manner.

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Classical turbulence consists of eddies (Leonardo 1452-1519)

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Classical Turbulence

- Based on the Navier-Stokes equation.

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{1}{\rho}\nabla p + \nu \nabla^2 v$$

Non-linear inertial term

Dissipative term

- In turbulent flows the non-linear (inertial) term of this equation becomes important relative to the dissipative term.
- The ratio $\frac{\text{non-linear inertial term}}{\text{viscous term}}$ is the Reynolds number, Re.

$$\mathrm{Re} = \frac{LU}{\nu}$$

At high Re dissipation can be neglected.

- Behaviour of turbulence at very high Re when fully developed: Richardson cascade and Kolmogorov energy spectrum).

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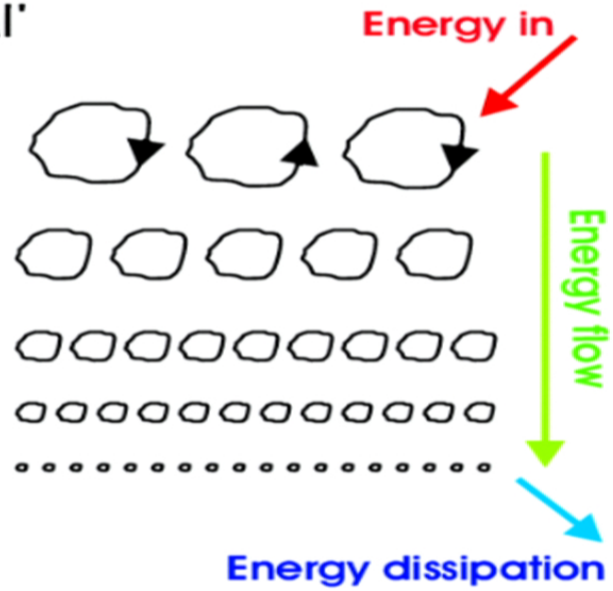
Richardson cascade and the Kolmogorov spectrum

- Non-linearities cause coupling of eddies of different sizes.
- This coupling causes turbulent energy to flow (at a rate ε per unit mass) from large eddies to smal' eddies in a cascade.

$$u'l/\nu >> 1$$

ε



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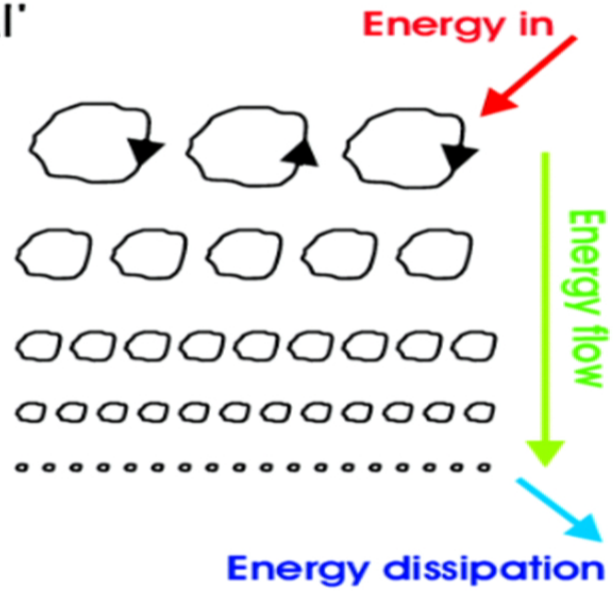

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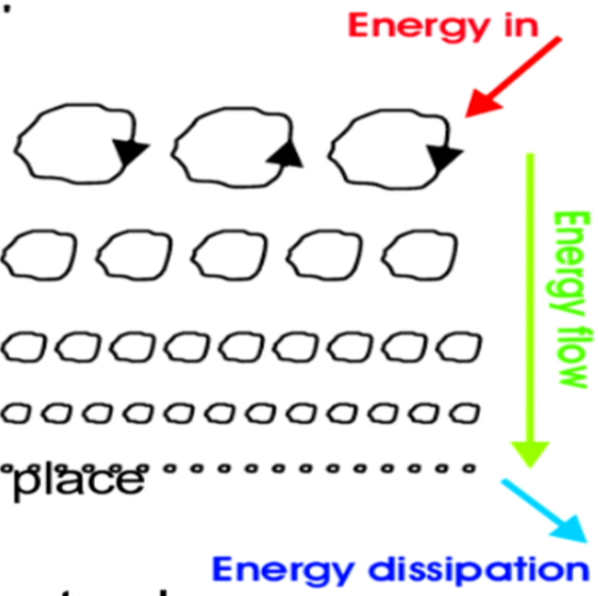
Richardson cascade and the Kolmogorov spectrum

- Non-linearities cause coupling of eddies of different sizes.
- This coupling causes turbulent energy to flow (at a rate ε per unit mass) from large eddies to smal' eddies in a cascade.
- Let u' be the characteristic velocity in an eddy size *l*.
- If $u'l/\nu >> 1$ the flow of energy in the casc is unaffected by viscosity and conserves energy. We call this an inertial flow of energy.
- When $u'l/\nu \sim 1$ **viscous dissipation** takes place and the cascade is terminated.
- How is energy distributed over eddy size in the steady state?
- According to Kolmogorov, for the inertial range: $u_r^2 = \varepsilon^{2/3} l^{2/3}$
- Cut-off due to viscosity is at $\eta = \left(\nu^3/\varepsilon\right)^{1/4}$



Kolmogorov spectrum

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Homogeneous Isotropic Turbulence

The energy spectrum E is independent of energy production processes for all wavenumbers large compared to those at which the production occurs. Then E depends only on wavenumber, dissipation and viscosity

$$E=E(k, \varepsilon, \nu).$$

If the cascade is long enough, there may be an intermediate range (the inertial sub-range) in which the action of viscosity has not yet come in, that is $E=E(k,\varepsilon)$. Then dimensional analysis gives

$$E=A\varepsilon^{2/3} k^{5/3}$$

which is the famous Kolmogorov result. Here A is a constant.

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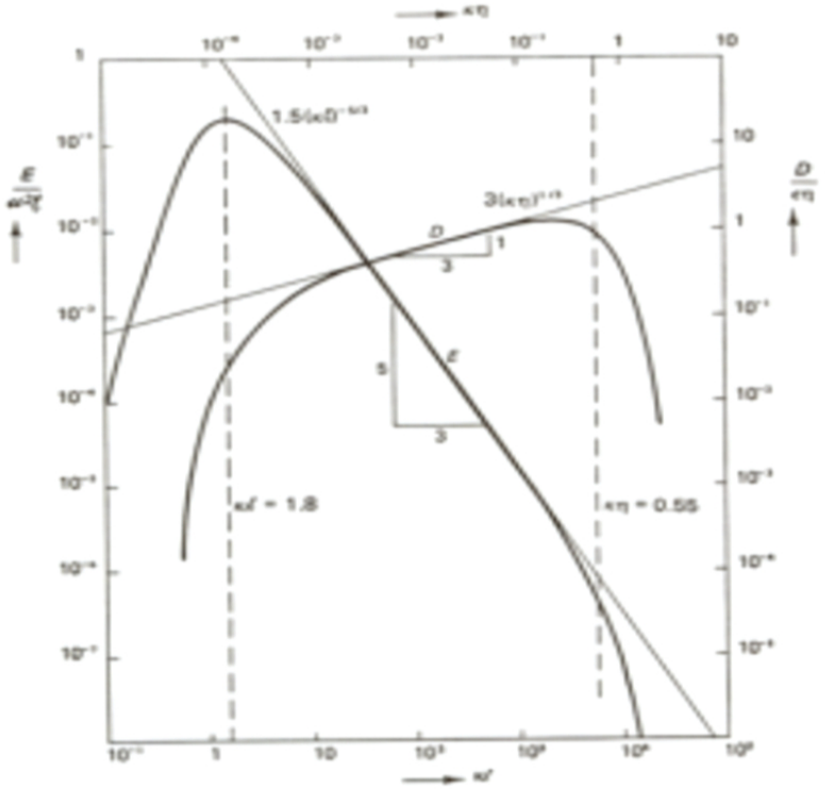
Energy spectrum on Kolmogorov ideas



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Search for the 5/3 spectrum
The difference in tidal phase drives a steady current through the tidal channel at about Re~4x10 ^{7}

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Turbulence probe

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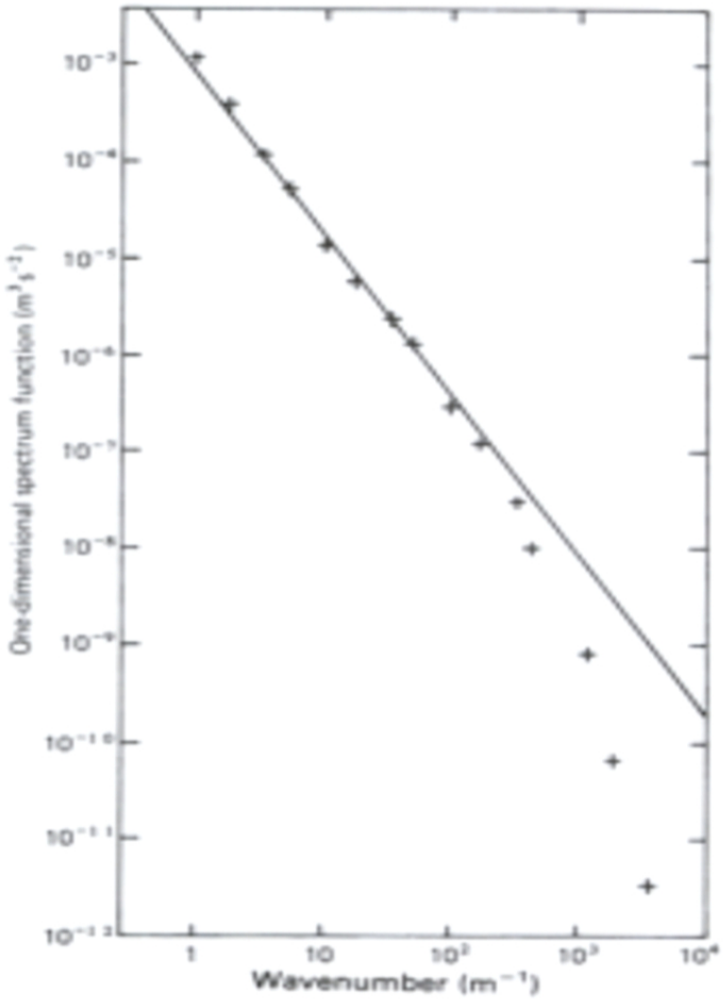

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What is Quantum Turbulence and Why Study It?

Helium II and ^{3}He-B are regarded as a mixture of a normal viscous fluid and a totally inviscid superfluid. In principle either or both fluids could become turbulent

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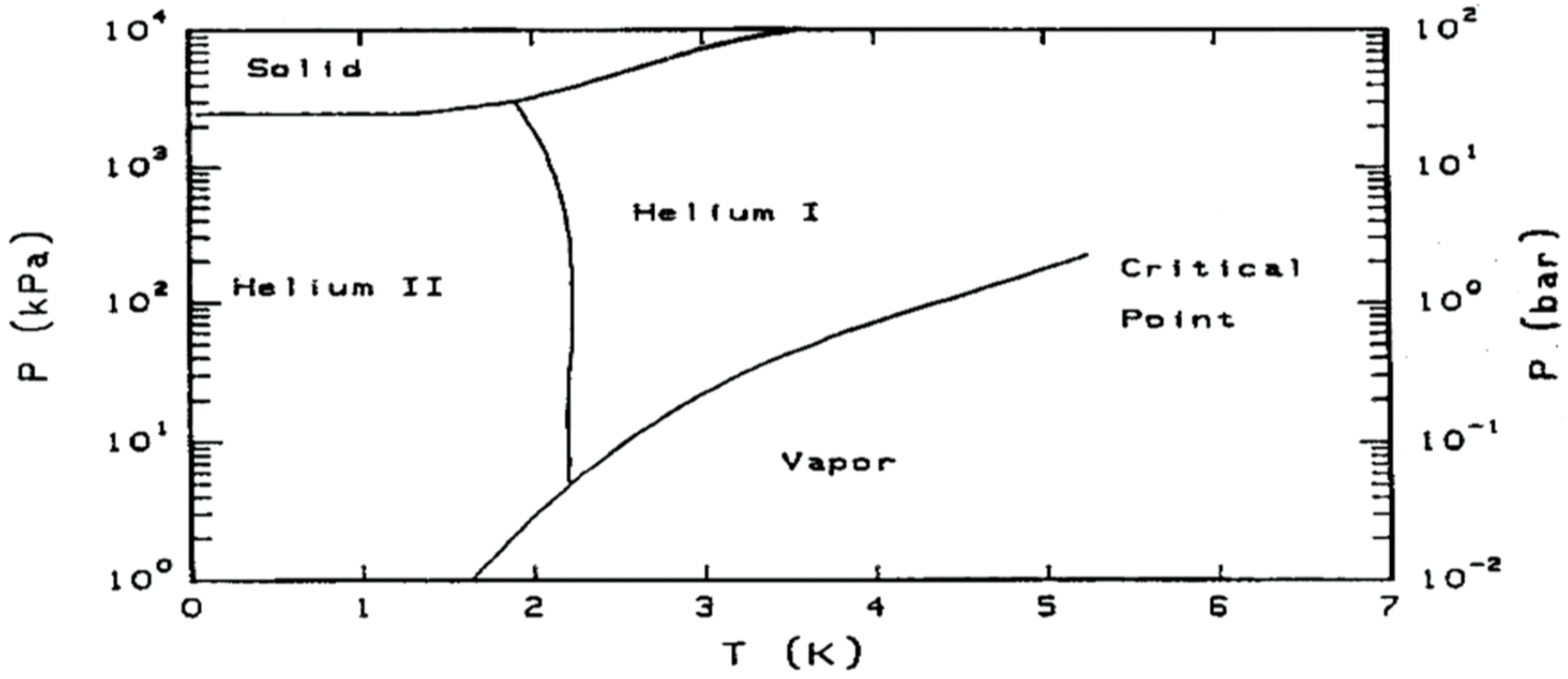
Phase diagram for liquid ^{4}He



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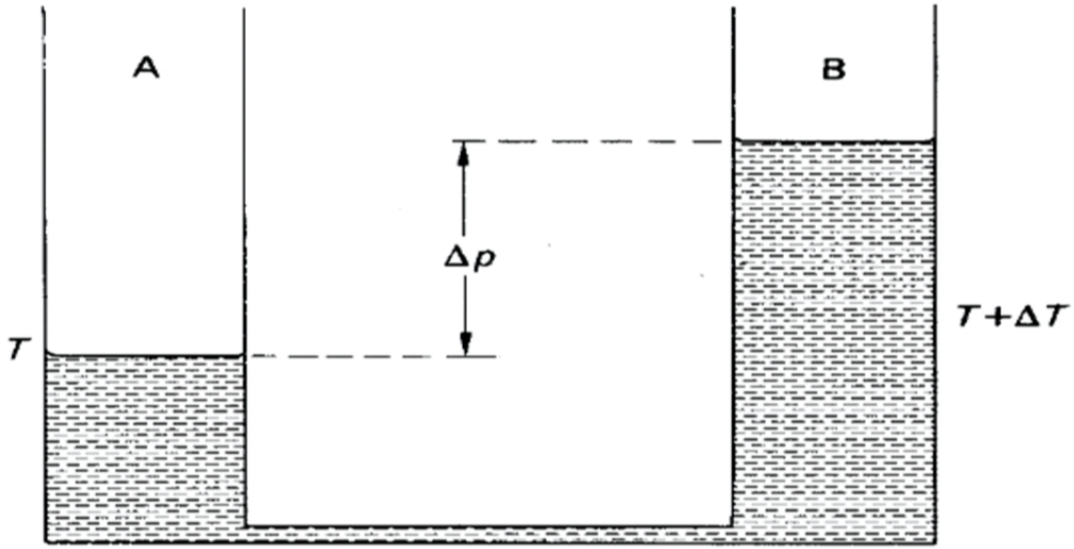
Fountain Pressure



$$\Delta P / \Delta T = \rho S$$

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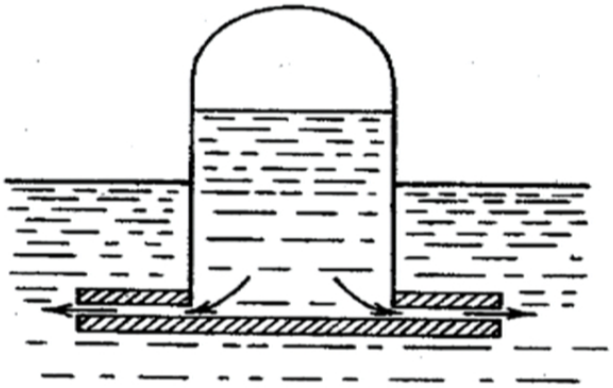
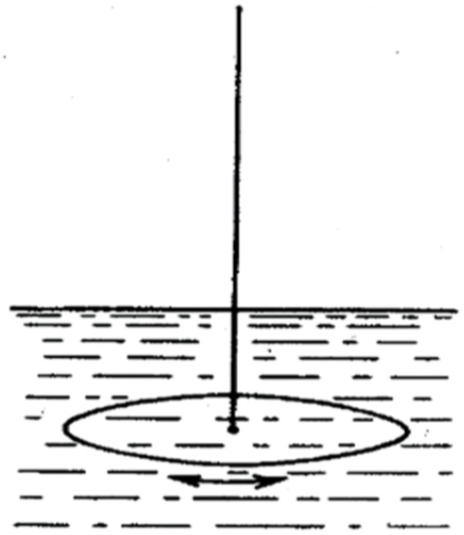
Viscosity Paradox in He II



$\eta < 10^{-11}$ poise

(*a*)



$\eta \sim 10^{-5}$ poise

(*b*)

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These experiments suggest there are two different fluid present: a "normal fluid" which damps the oscillating disc, and a "superfluid" which can flow through the smallest channels.

$$\rho = \rho_n + \rho_s$$

The two fluids can also have their own velocity fields

$$\mathbf{v}_n, \mathbf{v}_s$$

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Tisza Two Fluid Equations

$$\rho_s \frac{D\mathbf{v}_s}{Dt} = -\frac{\rho_s}{\rho}\nabla P + \rho_s S \nabla T$$

$$\rho_n \frac{D\mathbf{v}_n}{Dt} = -\frac{\rho_n}{\rho}\nabla P - \rho_s S \nabla T + \eta \nabla^2 \mathbf{v}_n$$

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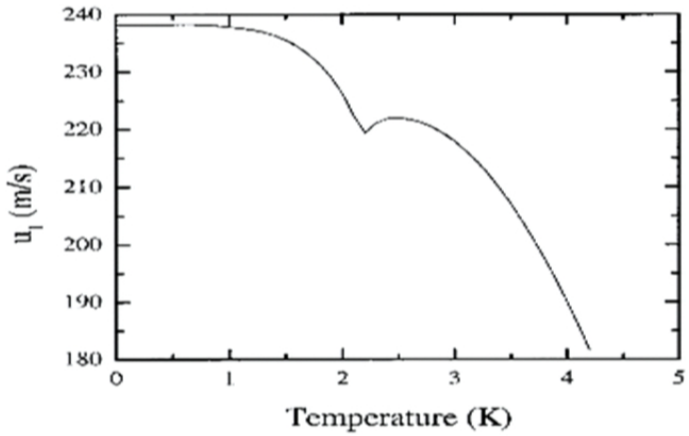
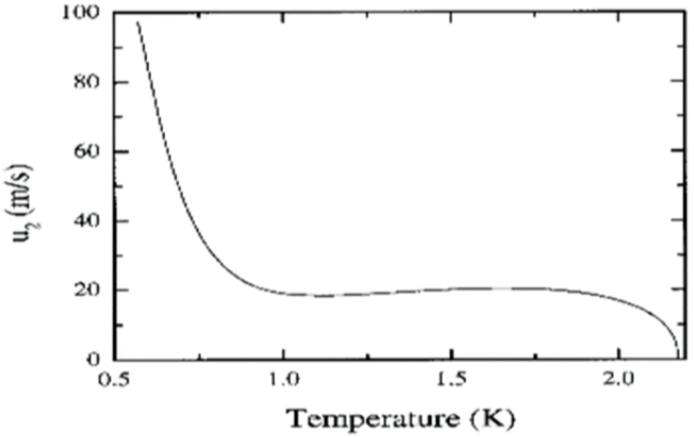
These equations result in two forms of sound, called “first” and “second” sound

$$u_1^2 \approx dp/d\rho, \quad u_2^2 \approx TS^2\rho_s/C\rho_n$$



First sound



Second sound

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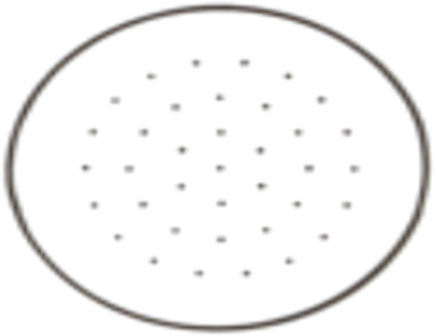
Side view



Top view

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Shortly thereafter Schwarz and Donnelly built an apparatus the could fire vortex rings at an array of quantized vortex lines such as is shown in Figure 5 [9]. The results show that the cross section for scattering of rings from lines is the order of the ring diameter, which indicates that the line and ring need to be very nearly in contact to have a strong interaction. This result is in good agreement with Feynman's speculations as quoted in the discussions by Feynman of Figure 4. Schwarz then began his careful numerical simulations of vortex line turbulence continuing over a period of years. For an interesting example of Schwarz's work see Figure 7

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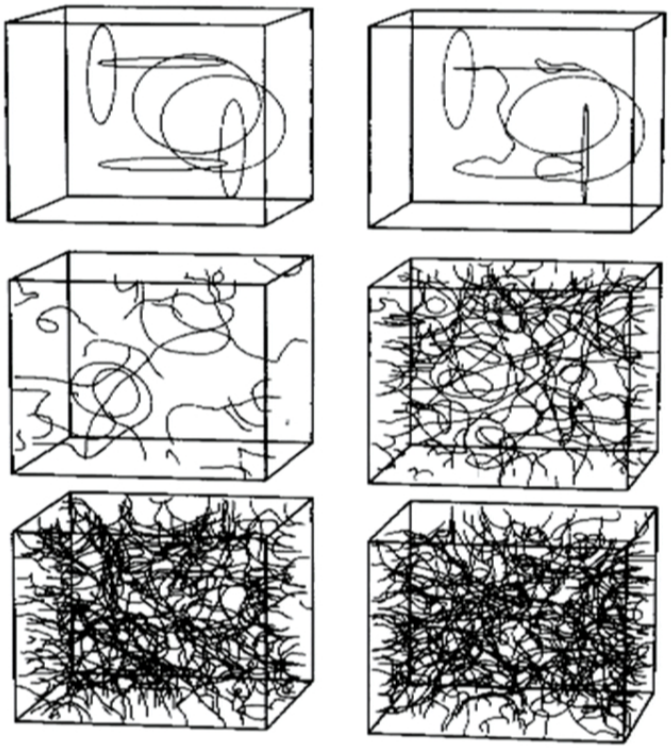
In turbulent counterflows the vortex lines are considered to be a tangle of quantized vortex lines (a suggestion of Feynman).



Much can be learned about quantized vortex lines by computer simulations pioneered by Schwarz and continued by Barenghi, Tsubota and others.

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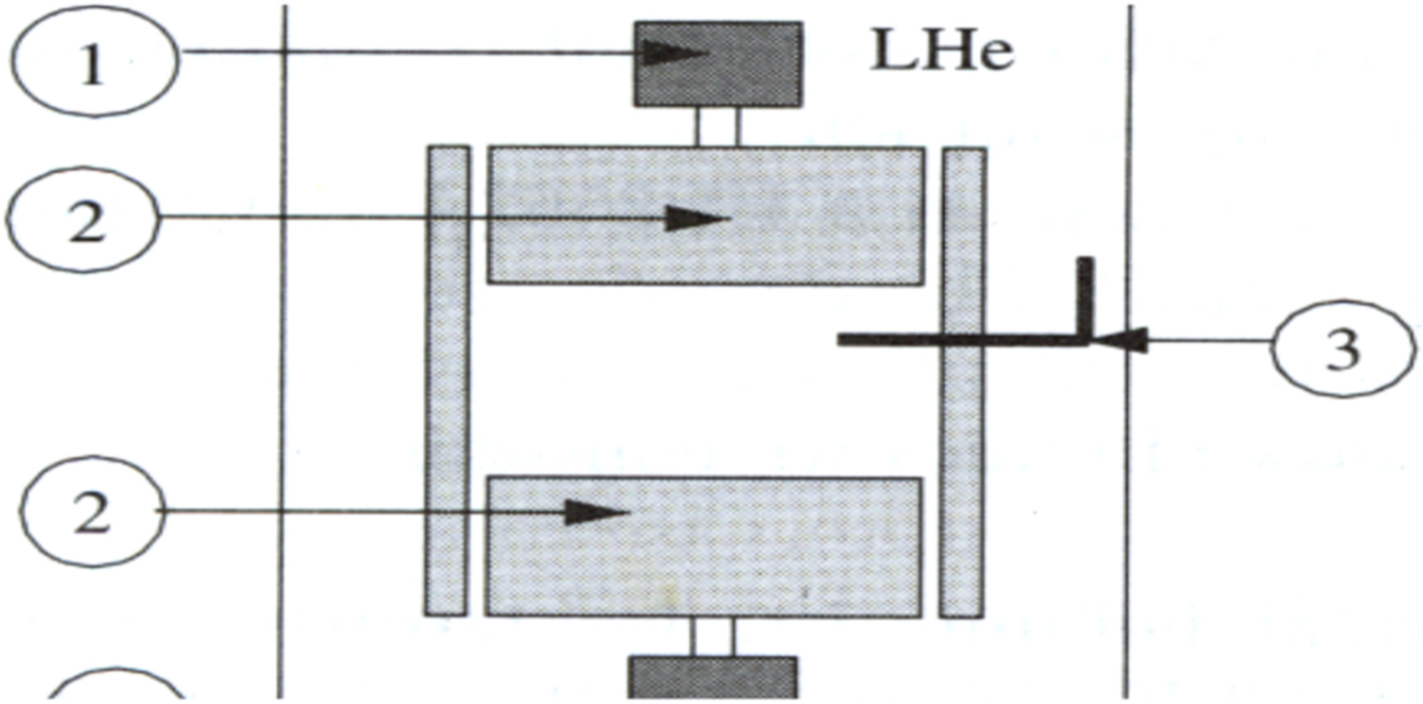

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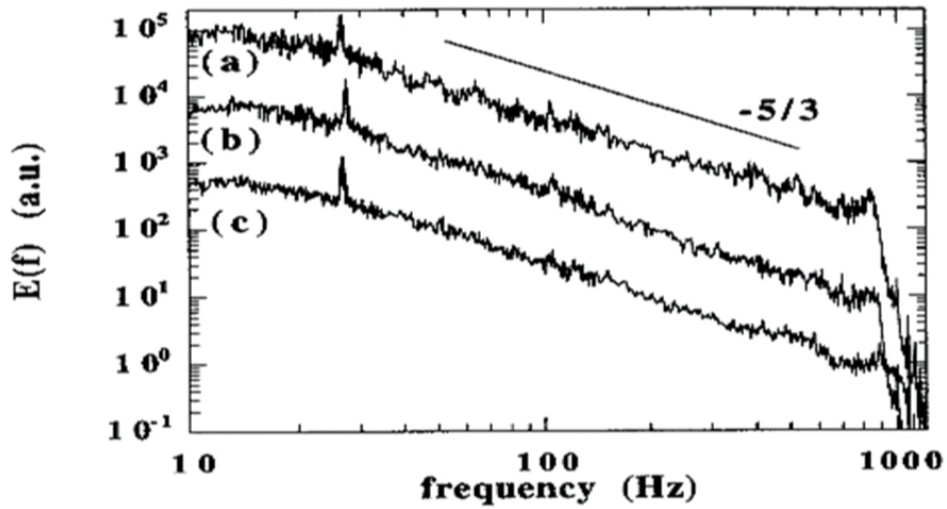
Experimental results on superfluid ^{4}He

- **Pressure fluctuations**: experiment of Maurer and Tabeling on swirling flow generated by two counter-rotating blades.

(a) 2.3 K
(b) 2.08K
(c) 1.4 K



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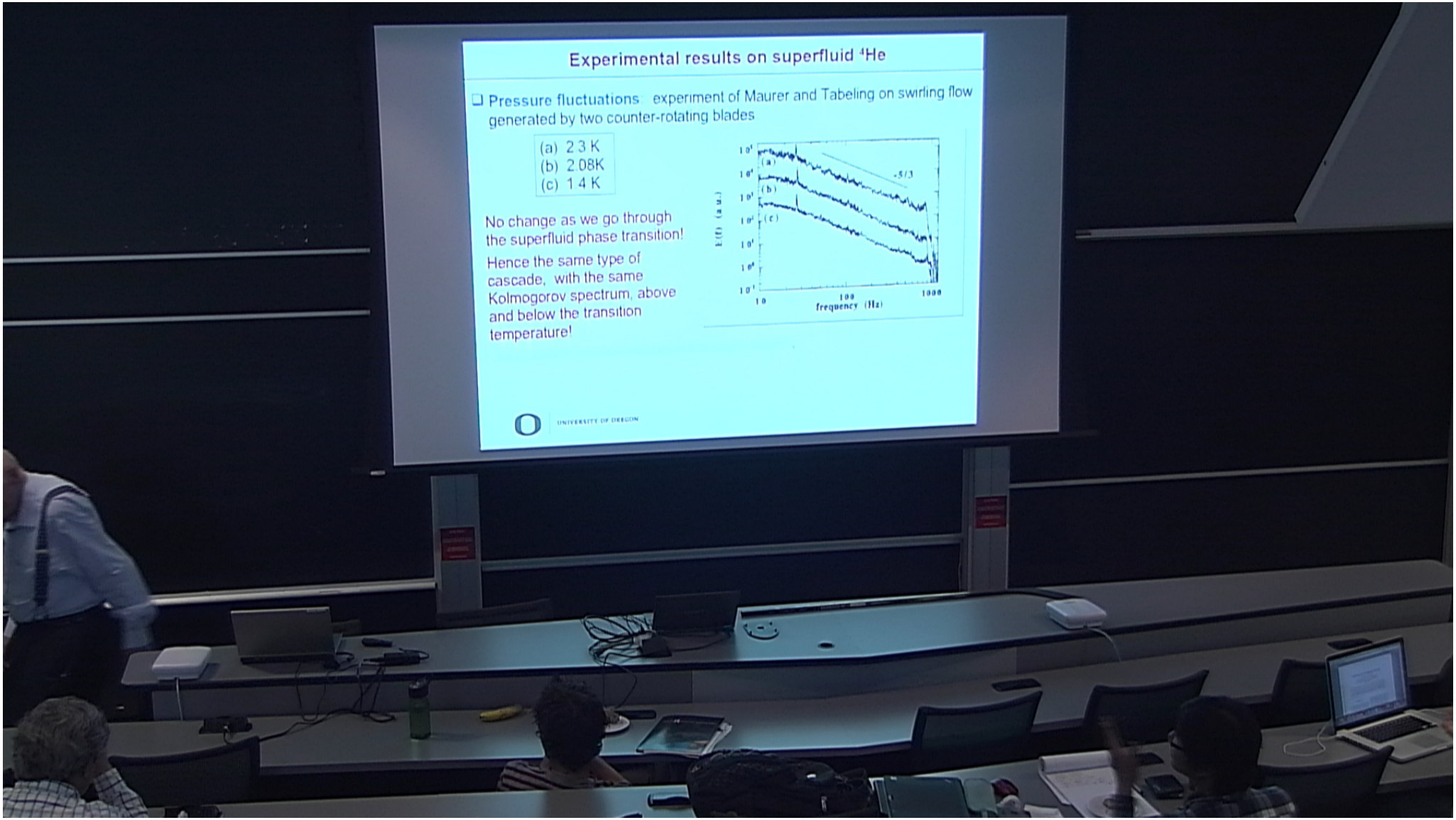
Experimental results on superfluid ⁴He
Pressure fluctuations: experiment of Maurer and Tabeling on swirling flow generated by two counter-rotating blades
(a) 2.3 K
(b) 2.08K
(c) 1.4 K
No change as we go through the superfluid phase transition!
Hence the same type of cascade, with the same Kolmogorov spectrum, above and below the transition temperature!
-5/3
(a)
(b)
(c)
E(f) (a.u.)
10
100
1000
frequency (Hz)
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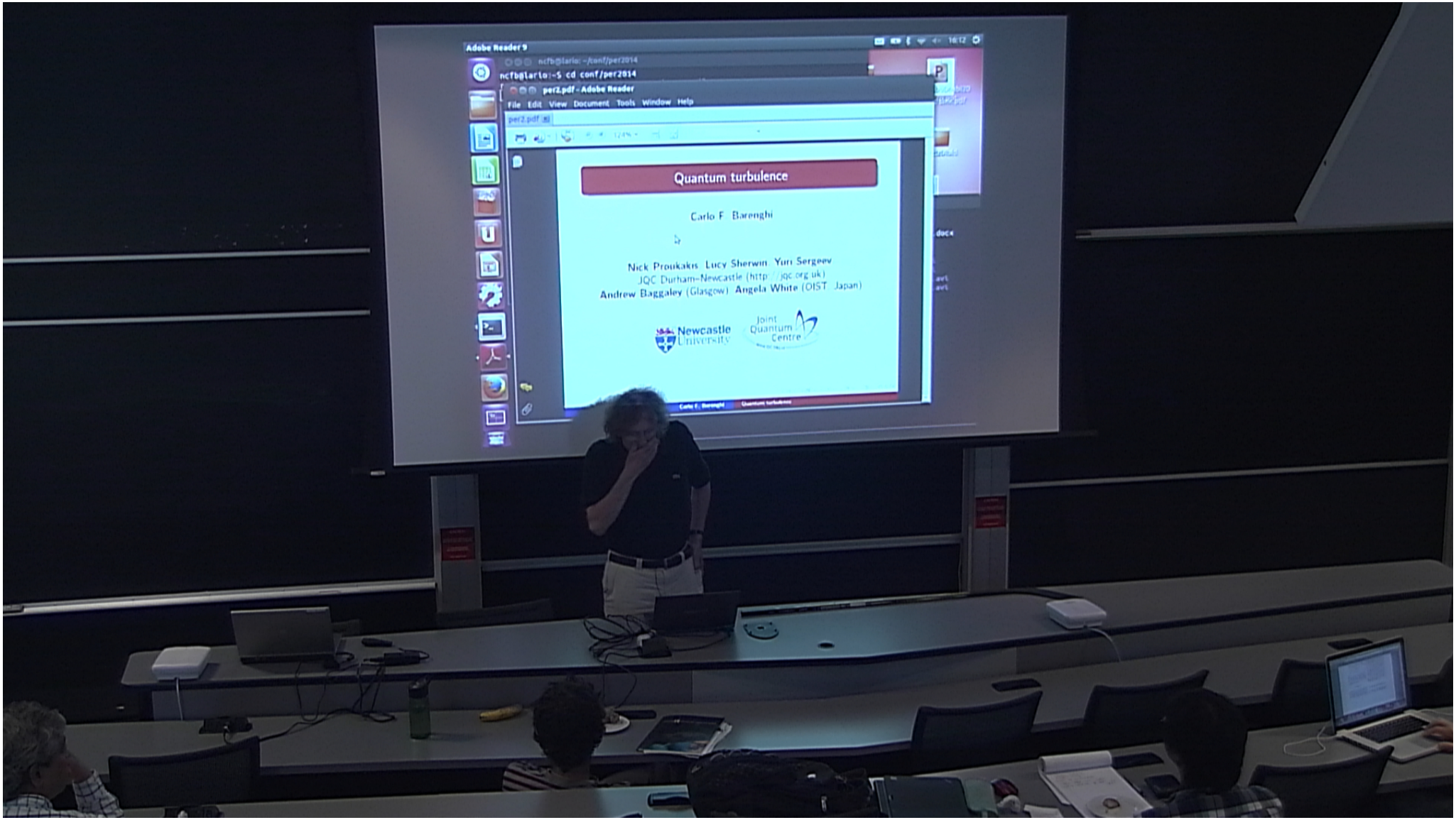
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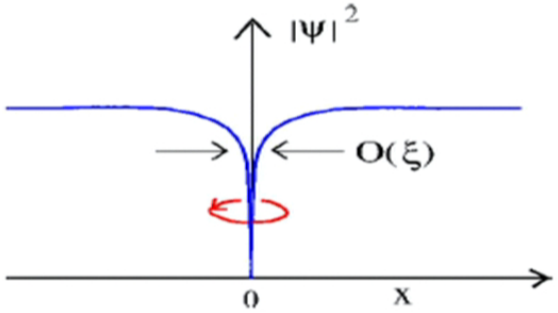
Quantum turbulence
Carlo F. Barenghi
Nick Proukakis Lucy Sherwin Yuri Sergeev
JQC Durham-Newcastle (http://jqc.org.uk)
Andrew Baggaley (Glasgow) Angela White (OIST Japan)
Newcastle University
Joint Quantum Centre

Vortex lines

$$\Psi = |\Psi| e^{i\phi} \quad \Rightarrow \text{Velocity } \mathbf{v} = (\hbar/m)\nabla\phi$$

$$\oint_C \mathbf{v} \cdot \mathbf{dr} = \frac{h}{m} = \kappa \quad \text{if C encloses a vortex line}$$

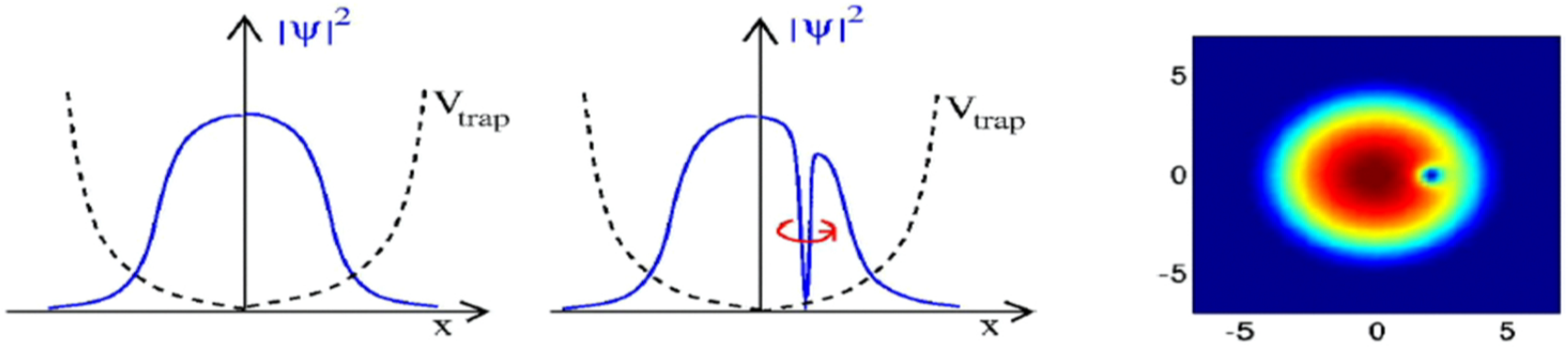
A vortex is a hole (radius $\xi \approx 10^{-8}\mathrm{cm}$ in ^{4}He) around which the phase changes by 2π, and $v = \kappa/(2\pi r)$



Vortex in a homogeneous condensate (e.g. liquid helium)

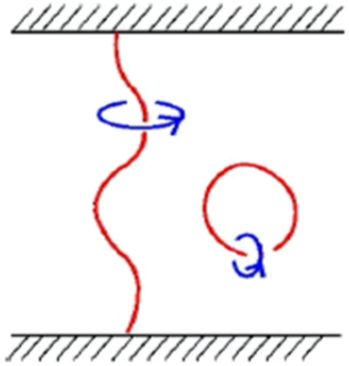
Phase

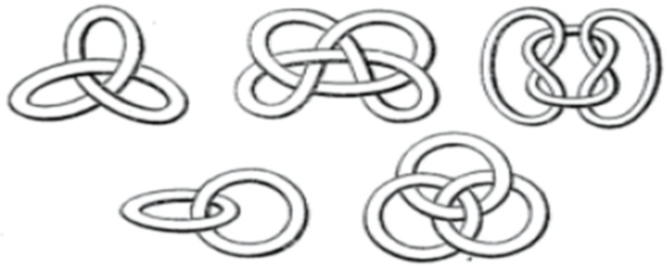
Vortex lines



Vortex in an inhomogenous condensate (e.g. trapped atomic gas)

Vortex lines



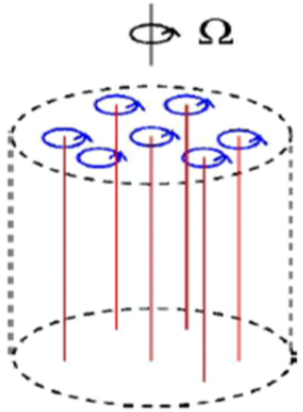


- Vortex rings, Kelvin waves, vortex knots:
  studied since the times of Helmholtz, Lord Kelvin and Tait

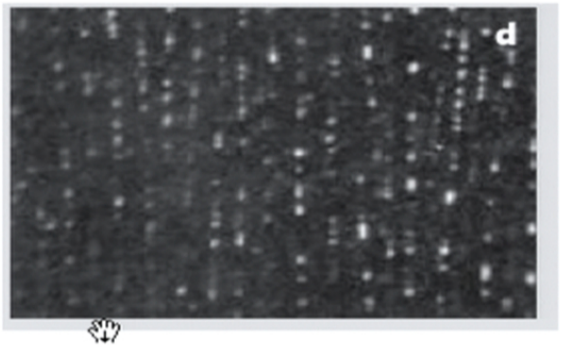
- Vortex lines = stable topological defects in a perfect fluid =
  = attractive models of elementary particles

W. Rankine *Molecular vortices* 1849
W. Thomson (Lord Kelvin) *On vortex atoms* 1867
J.J. Thomson *A treatise on the motion of vortex rings* 1883

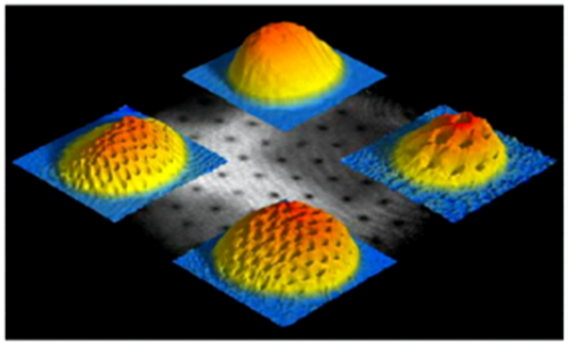
Visualization of individual vortex lines



Vortex lattice in
a rotating superfluid



(Maryland)



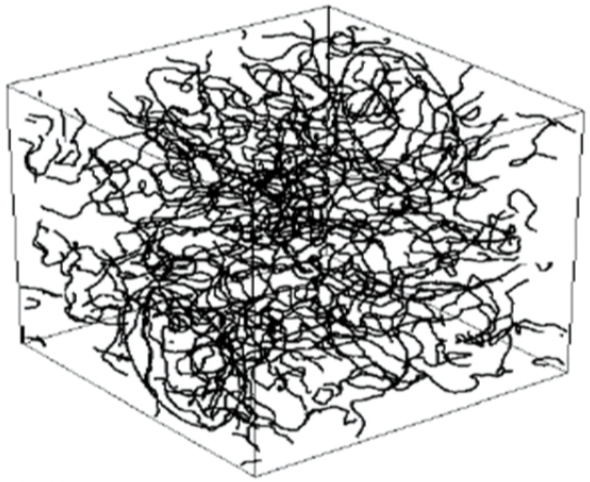
(MIT)

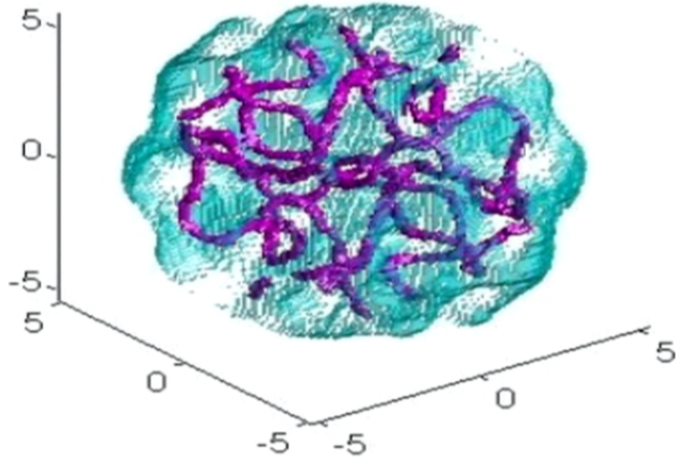
(Berkeley)

Vortex tangle

Quantum turbulence is a **tangle of vortex lines**, generated

- with propellers, grids, forks, wires, heat flows, etc in liquid helium
- with laser stirrers, shaking the trap, etc in atomic condensates





Within a turbulent tangle, we recognize processes such as
Kelvin waves, links and knots, reconnections

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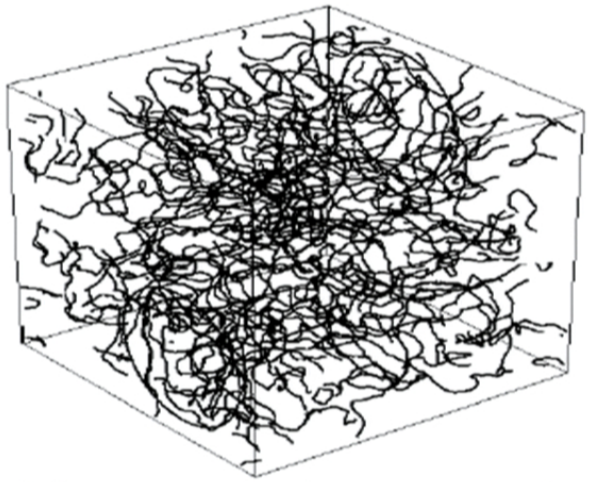

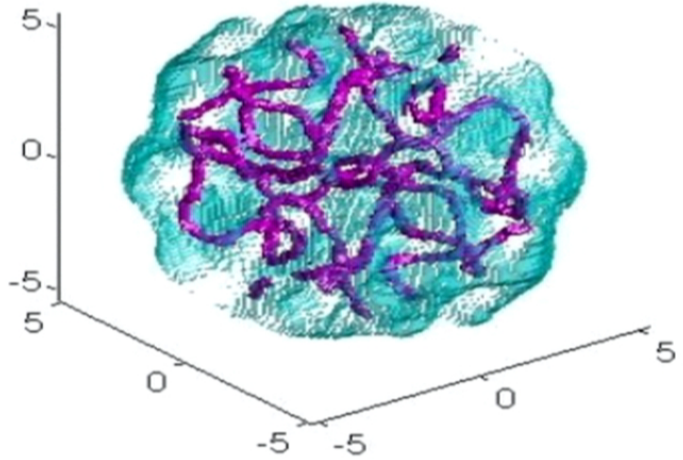

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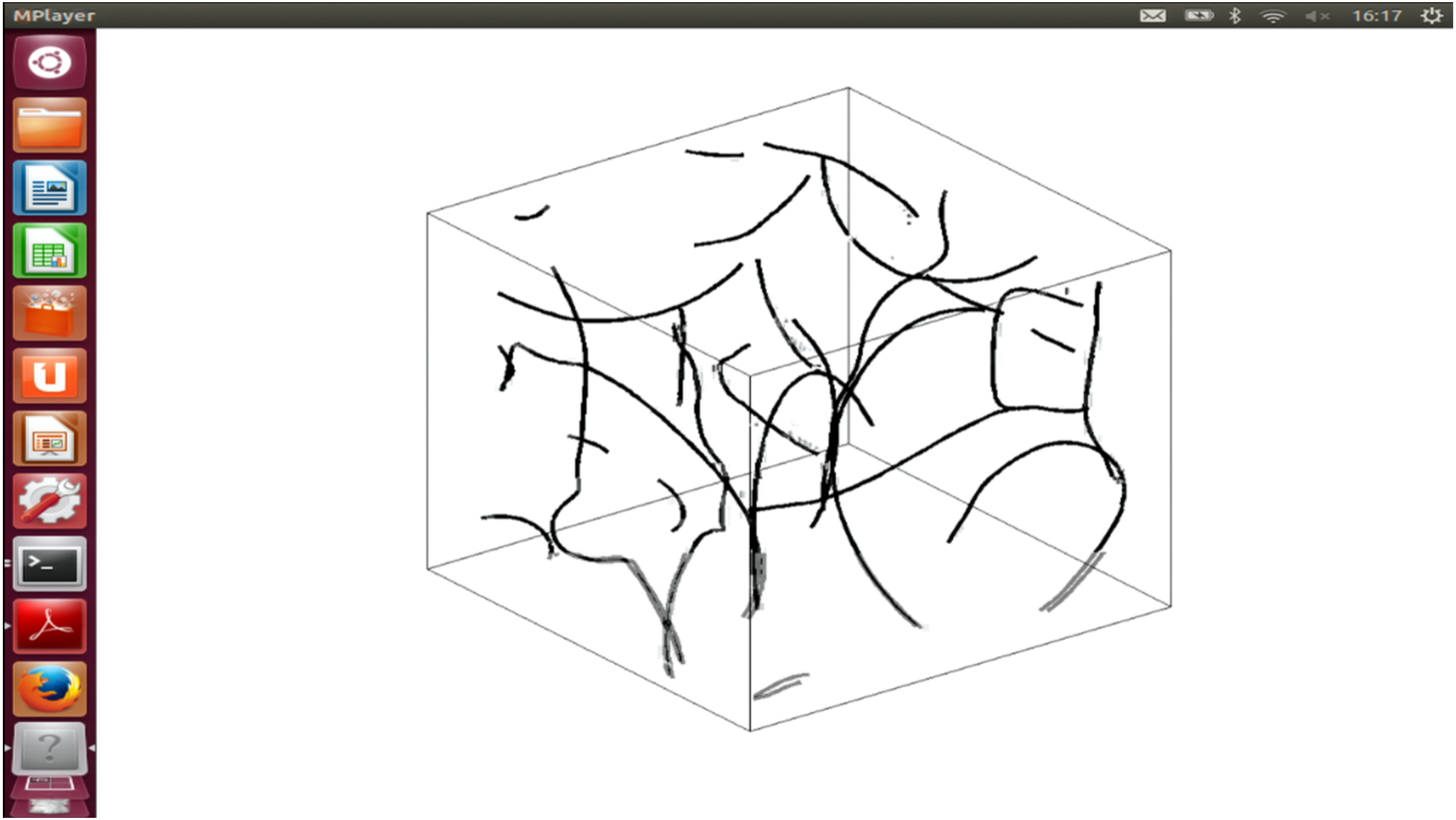
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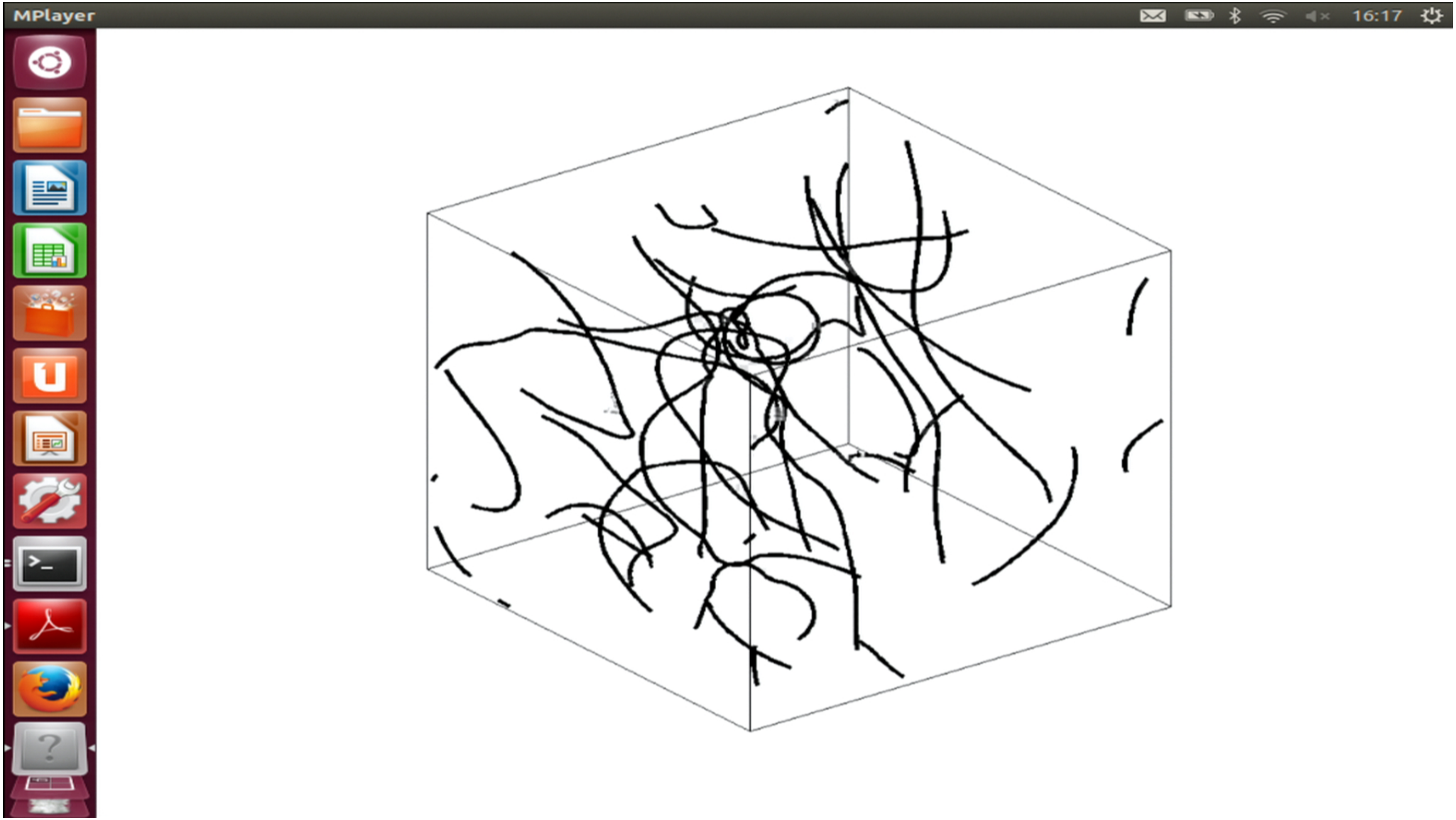
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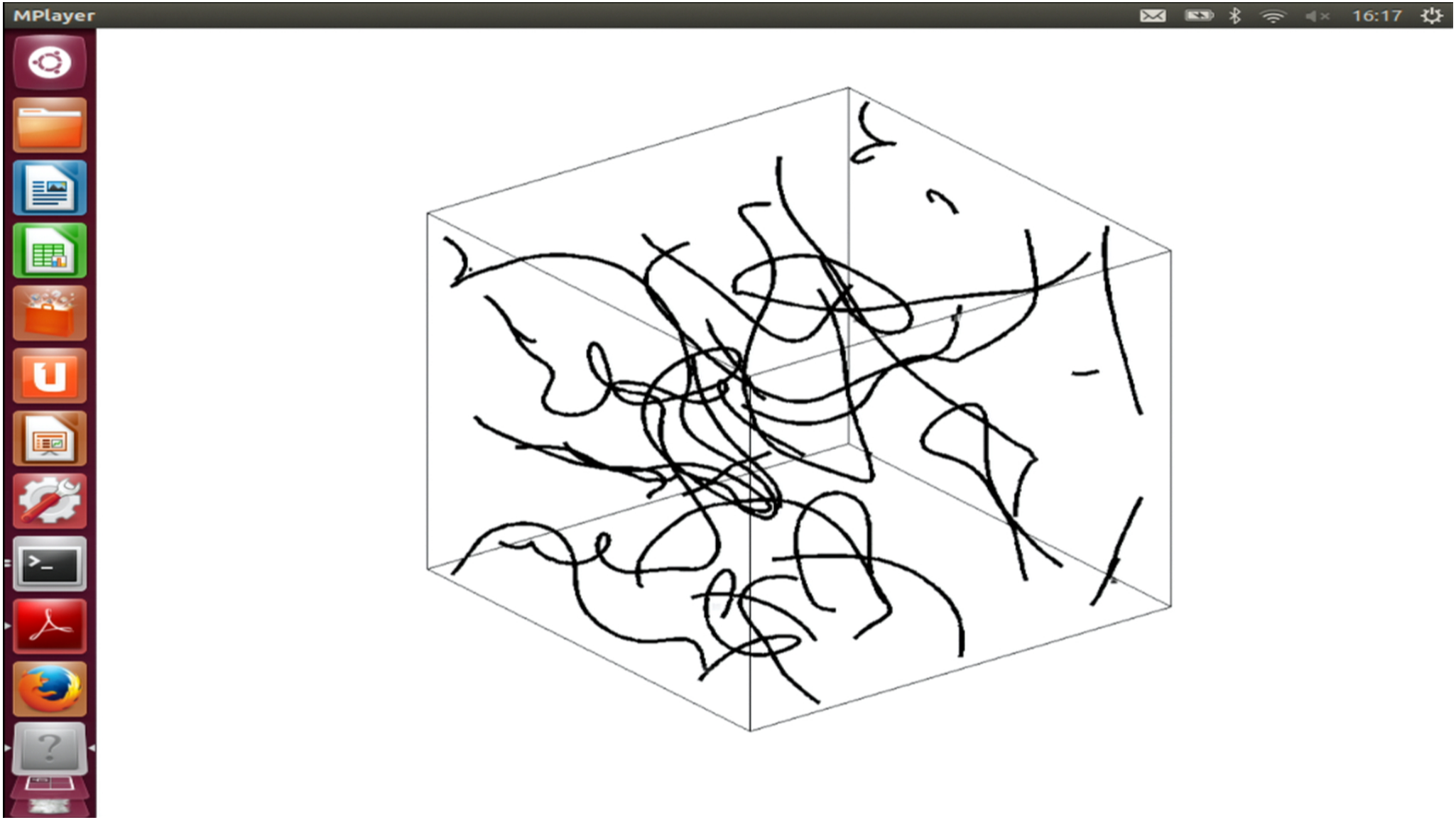
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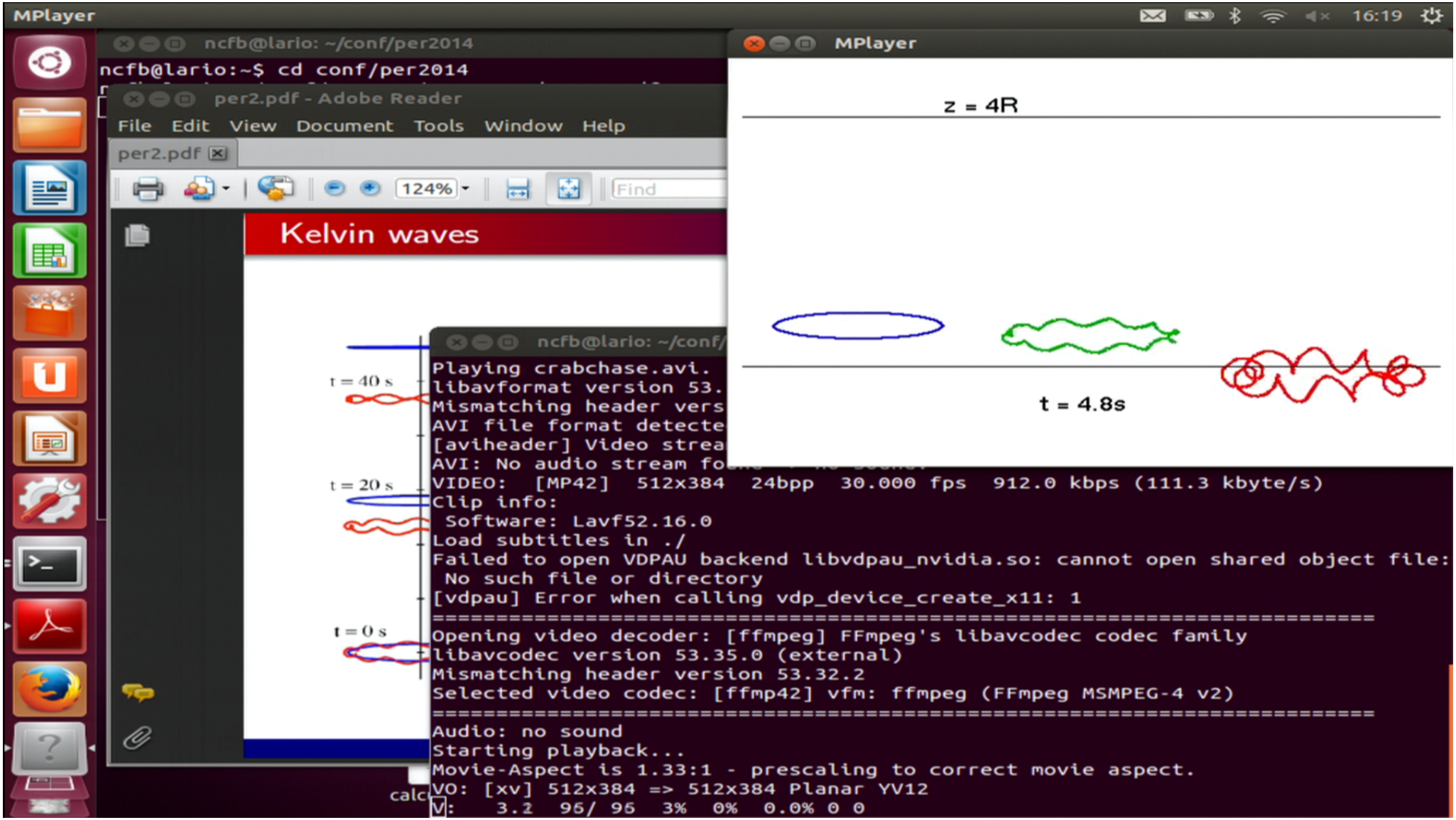
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124%
Find
Kelvin waves
t = 40 s
t = 20 s
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MPlayer
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Playing crabchase.avi.
libavformat version 53.
Mismatching header vers
AVI file format detecte
[aviheader] Video strea
AVI: No audio stream fo
VIDEO: [MP42] 512x384 24bpp 30.000 fps 912.0 kbps (111.3 kbyte/s)
Clip info:
Software: Lavf52.16.0
Load subtitles in ./
Failed to open VDPAU backend libvdpau_nvidia.so: cannot open shared object file:
No such file or directory
[vdpau] Error when calling vdp_device_create_x11: 1
Opening video decoder: [ffmpeg] FFmpeg's libavcodec codec family
libavcodec version 53.35.0 (external)
Mismatching header version 53.32.2
Selected video codec: [ffmp42] vfm: ffmpeg (FFmpeg MSMPEG-4 v2)
Audio: no sound
Starting playback...
Movie-Aspect is 1.33:1 - prescaling to correct movie aspect.
VO: [xv] 512x384 => 512x384 Planar YV12
V: 3.2 95/ 95 3% 0% 0.0% 0 0

Adobe Reader 9

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MPlayer

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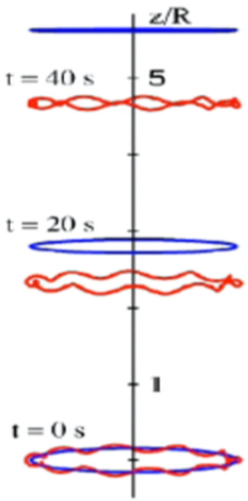
Kelvin waves



Dispersion relation of Kelvin waves
was determined in 1880,
but it took more than a century
to notice that a wavy vortex ring
moves slower

Kiknadze & Mamaladze, JLTP 2002
Hänninen, Tsubota & CFB, PRE 2006
Helm, CFB & al, PRA 2011

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```
V:   1.3   40/ 40   4%   0%   0.0% 0 0
=====  PAUSE  =====
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MPlayer

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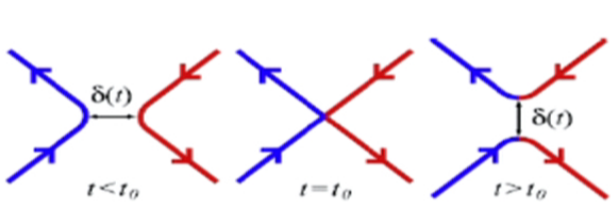
Find

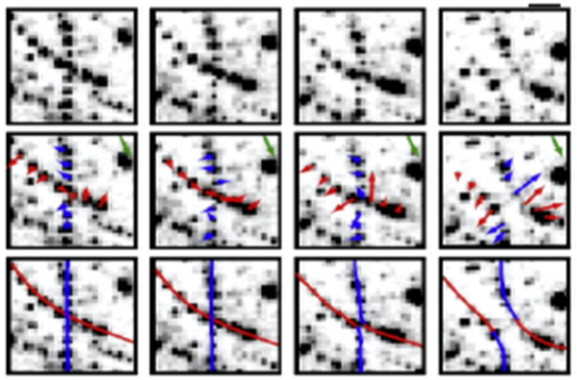
Reconnections

- Bewley, Paoletti, Sreenivasan, & Lathrop 2008

Direct observation of vortex reconnections in superfluid helium
(vortex lines visualised by trapped hydrogen particles)



$\delta(t) \sim (t_0 - t)^{1/2}$ before
$\delta(t) \sim (t - t_0)^{1/2}$ after



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VO: [XV] 512x384 => 512x384 Planar YV12
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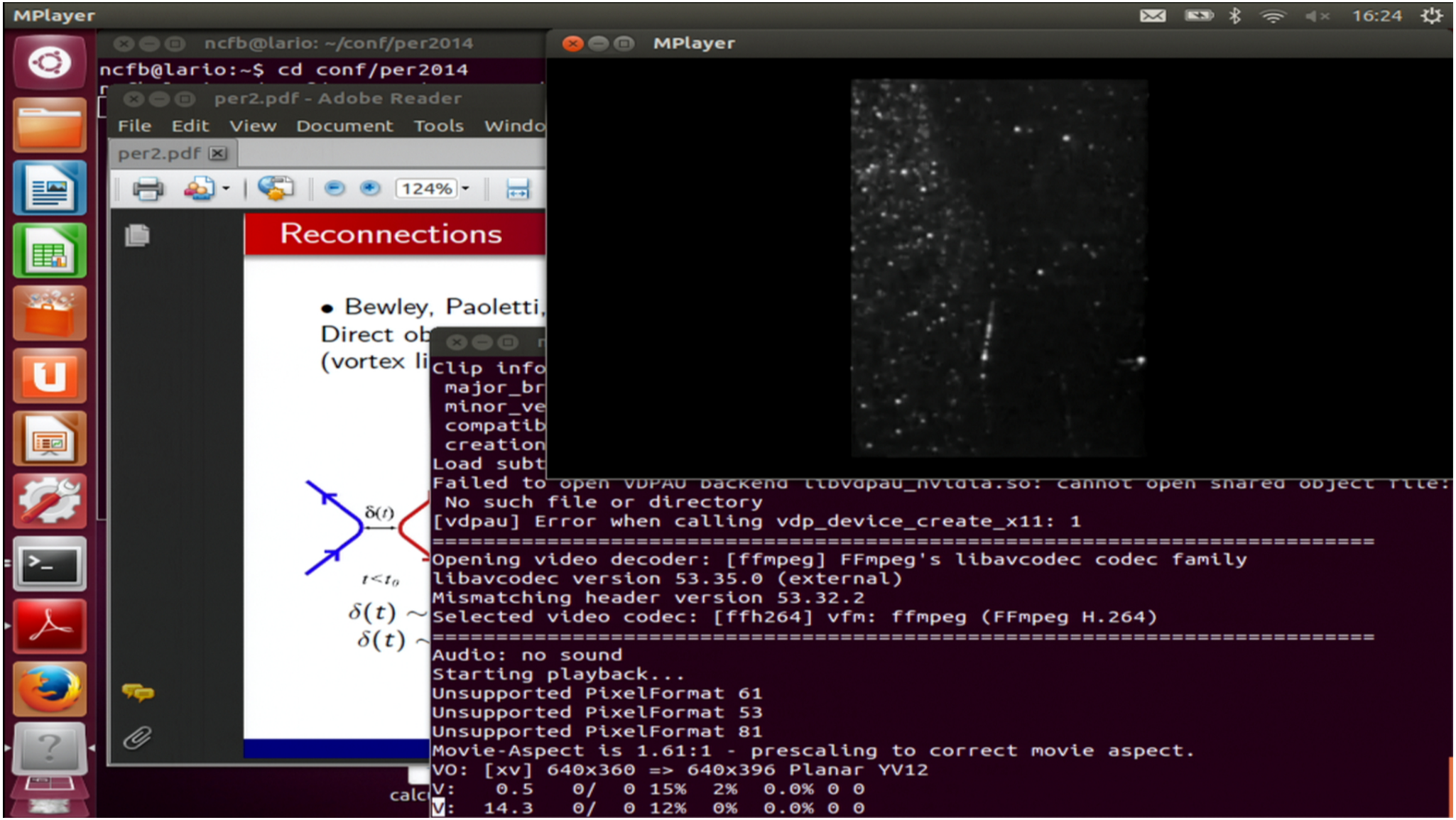
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libavcodec version 53.35.0 (external)
Mismatching header version 53.32.2
Selected video codec: [ffh264] vfm: ffmpeg (FFmpeg H.264)
Audio: no sound
Starting playback...
Unsupported PixelFormat 61
Unsupported PixelFormat 53
Unsupported PixelFormat 81
Movie-Aspect is 1.61:1 - prescaling to correct movie aspect.
VO: [xv] 640x360 => 640x396 Planar YV12
V: 0.5 0/ 0 15% 2% 0.0% 0 0
V: 14.3 0/ 0 12% 0% 0.0% 0 0

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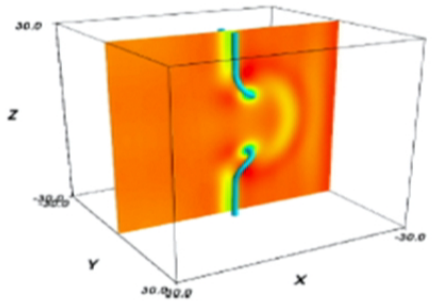
Find

Reconnections

- In classical fluids reconnections are associated with **losses of kinetic energy** arising from **viscous** or **resistive** effects
- In quantum fluids near absolute zero such losses are **acoustic** (sound = sink of kinetic energy)

Sound waves emitted by:

- Kelvin waves
  Vinen, Kozik & Svistunov, Sonin, L'vov & Nazarenko, Krstulovic, Baggaley & Hänninen, etc
- Reconnections
  Zuccher, CFB & al, PoF 2012
  Leadbeater, CFB & al, PRL 2001



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```
Exiting... (End of file)
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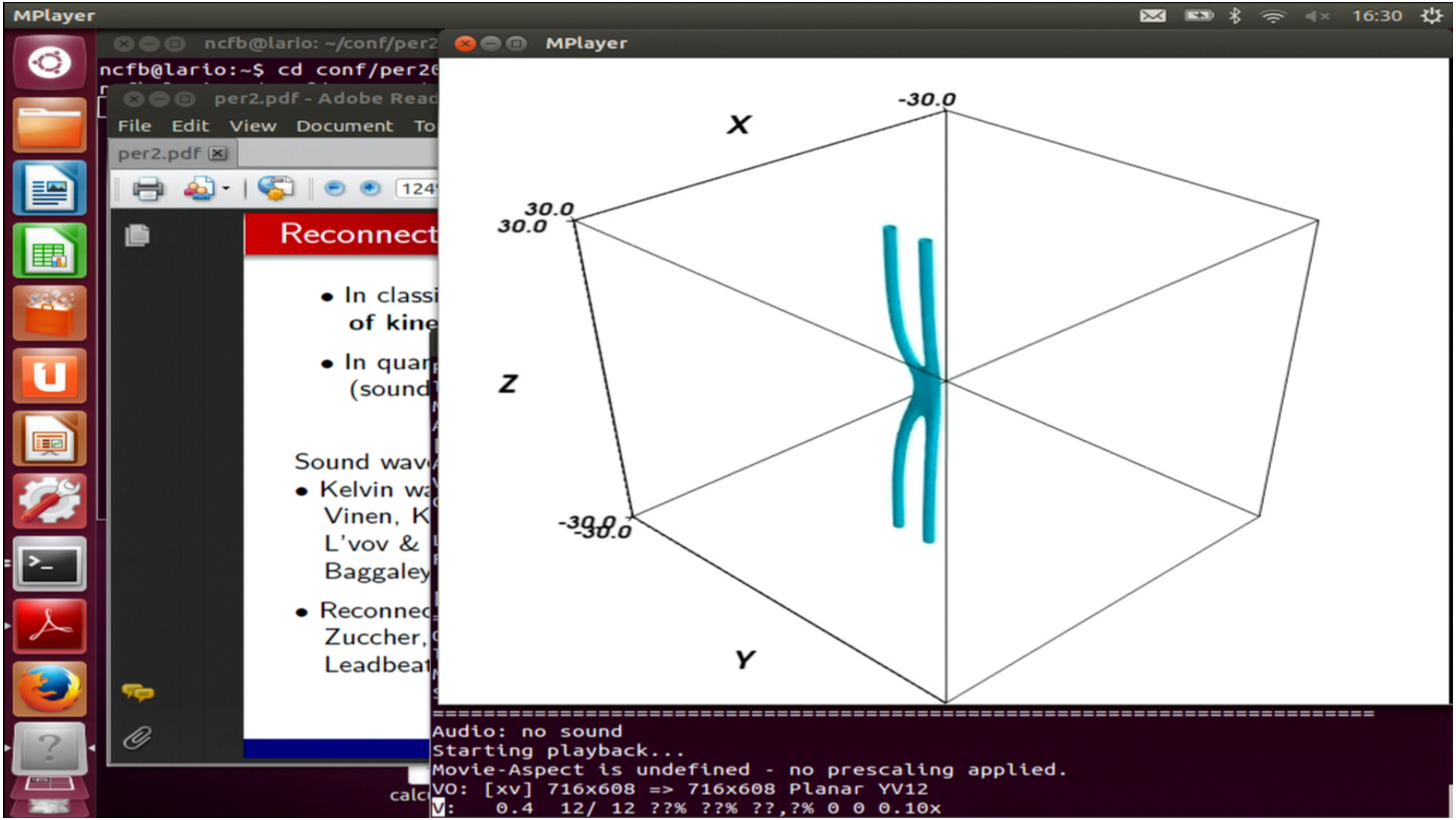
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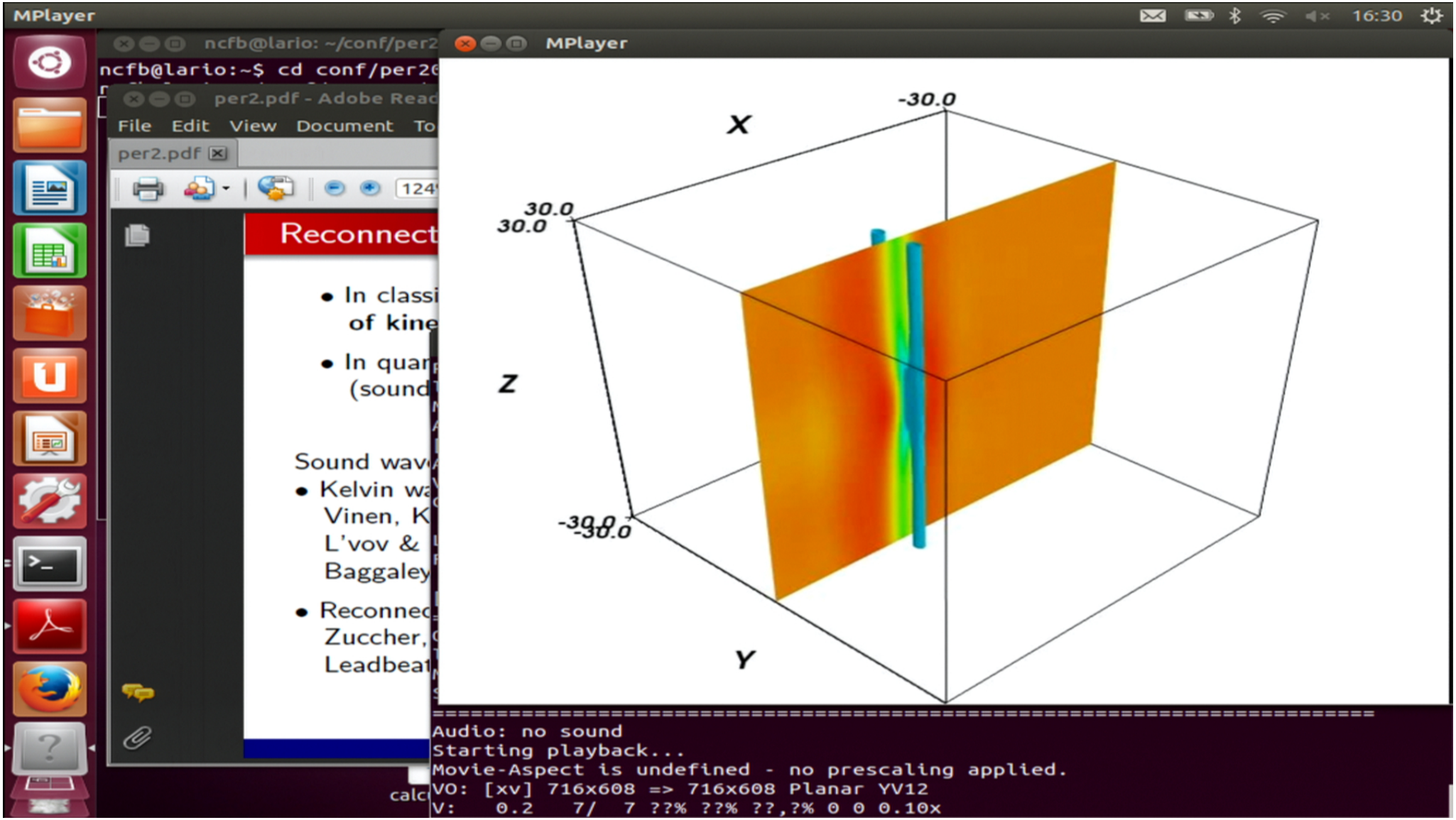
VO: [xv] 716x608 => 716x608 Planar YV12
V: 0.4 12/ 12 ??% ??% ??,?% 0 0 0.10x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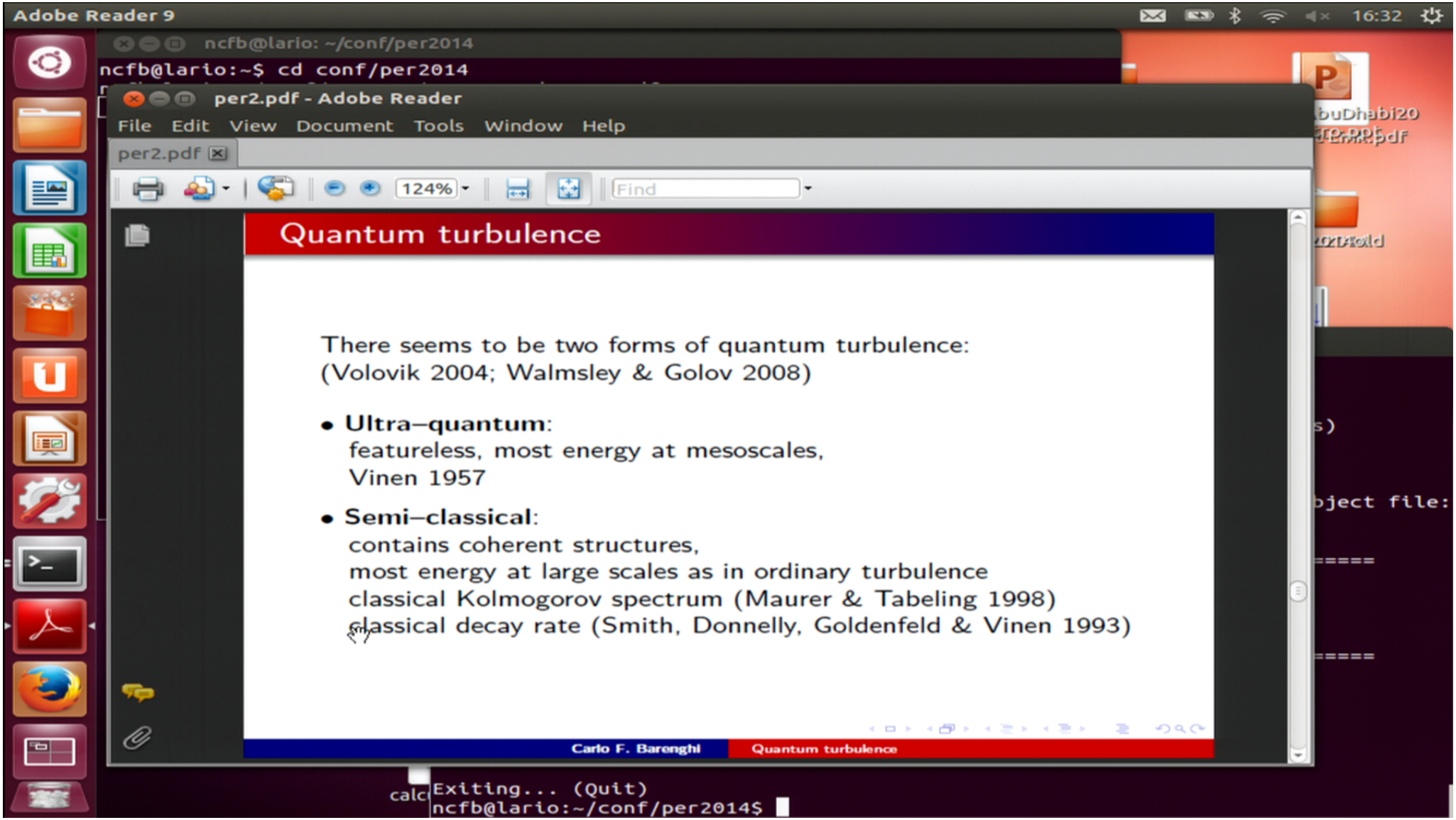
V: 0.2 7/ 7 ??% ??% ??,?% 0 0 0.10x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124%
Find
Quantum turbulence
There seems to be two forms of quantum turbulence:
(Volovik 2004; Walmsley & Golov 2008)
• Ultra–quantum:
featureless, most energy at mesoscales,
Vinen 1957
• Semi–classical:
contains coherent structures,
most energy at large scales as in ordinary turbulence
classical Kolmogorov spectrum (Maurer & Tabeling 1998)
classical decay rate (Smith, Donnelly, Goldenfeld & Vinen 1993)
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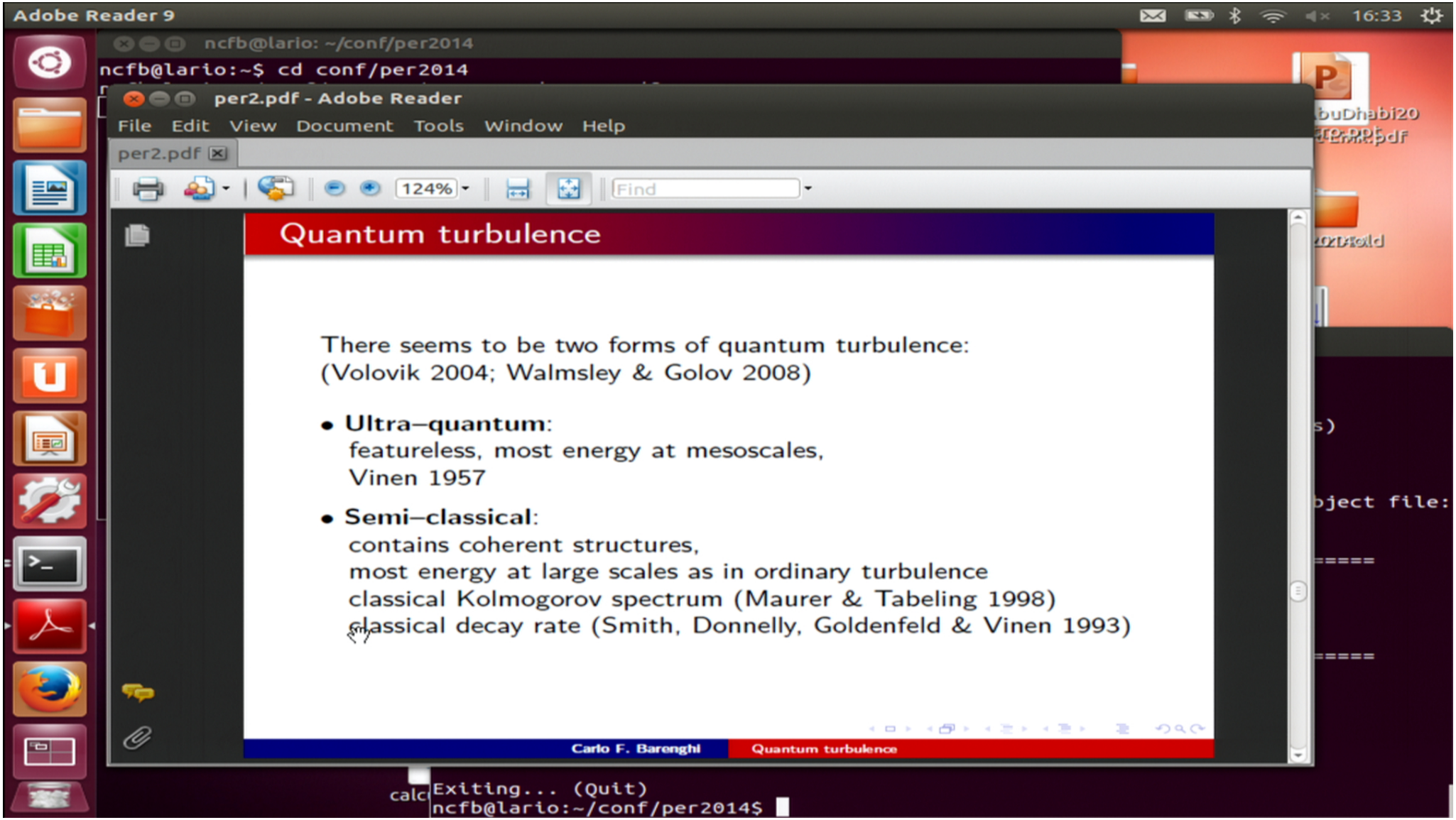
Quantum turbulence
Exiting... (Quit)
ncfb@lario:~/conf/per2014$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Find
Quantum turbulence
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(Volovik 2004; Walmsley & Golov 2008)
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Quantum turbulence
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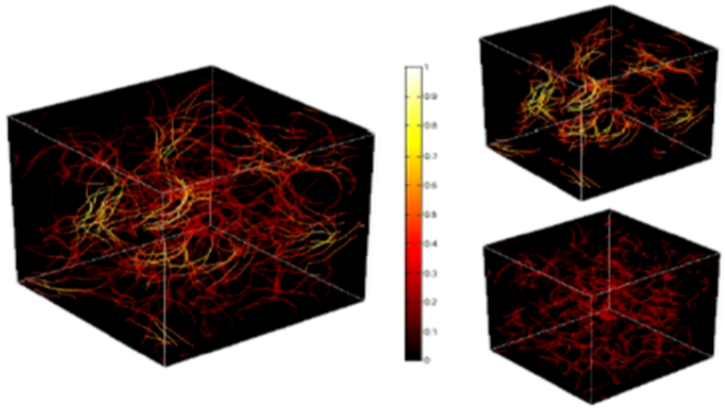
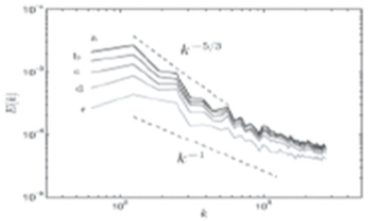
Find

Quantum turbulence





Kolmogorov energy spectrum

Total vorticity (left) decomposed into
- coherent (right, top) and
- incoherent part (right, bottom)

Baggaley, Laurie & CFB, PRL 2012

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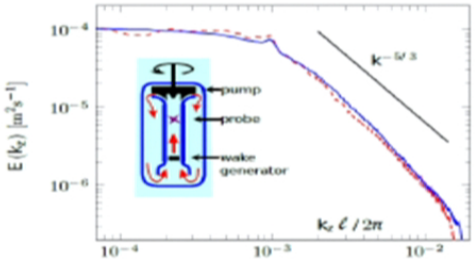

Energy spectrum,
Roche & al, Grenoble

Double cascade

Possibility of **bottleneck** accumulation of energy between the semi–classical (3D) $k^{-5/3}$ **Kolmogorov cascade** for $k \ll 2\pi/\ell$ and the (1D) **Kelvin wave cascade** for $k \gg 2\pi/\ell$, where ℓ is the average distance between vortex lines

CFB, L'vov and Roche, PNAS 2014

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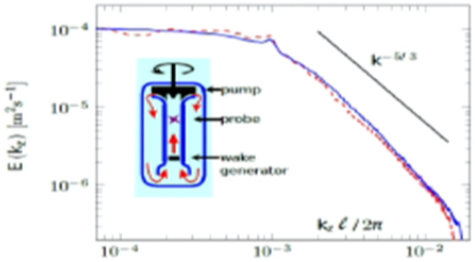
Quantum turbulence



Energy spectrum,
Roche & al, Grenoble

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CFB, L'vov and Roche, PNAS 2014

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Exiting... (Quit)
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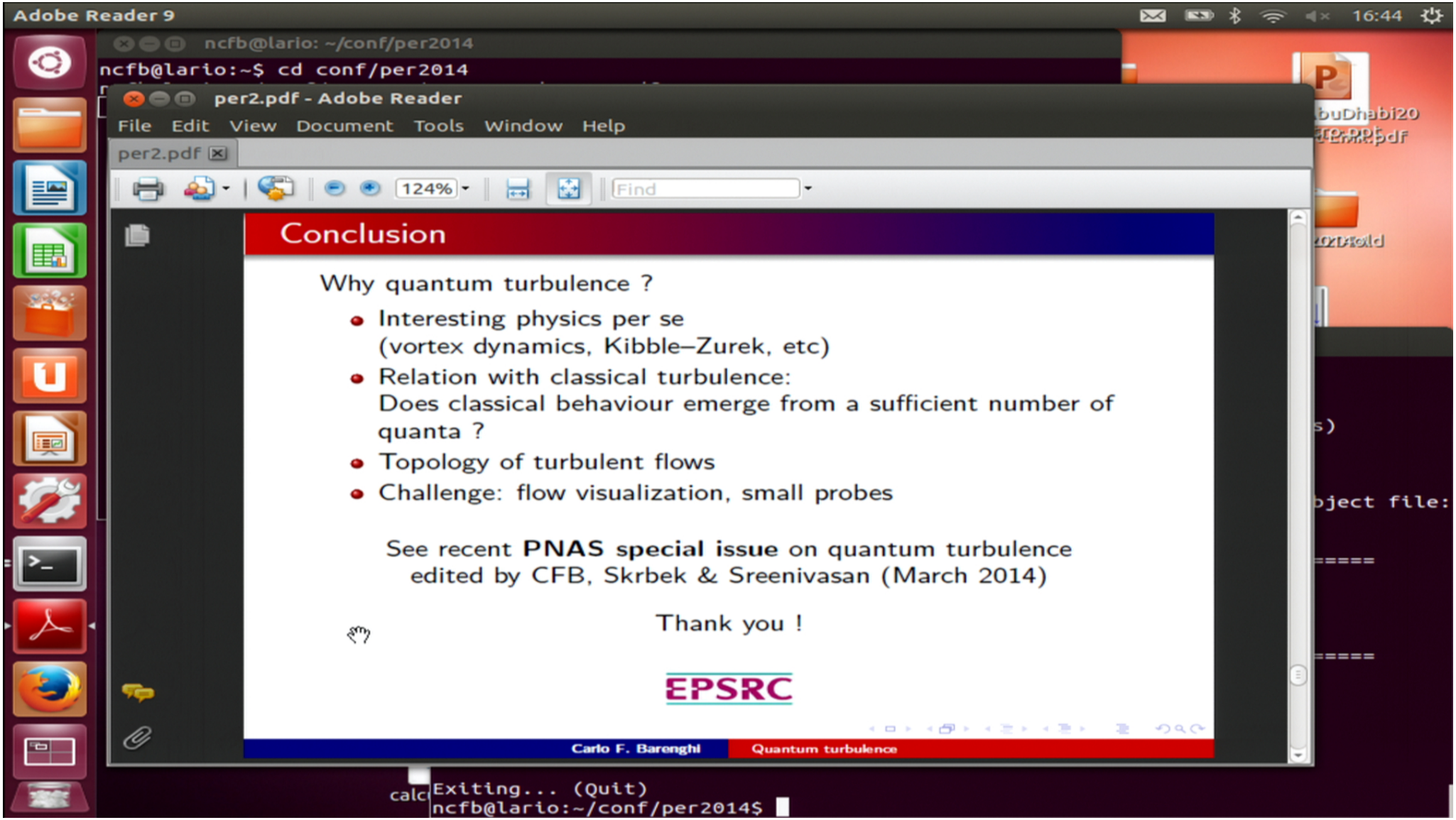
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Conclusion
Why quantum turbulence ?
Interesting physics per se
(vortex dynamics, Kibble–Zurek, etc)
Relation with classical turbulence:
Does classical behaviour emerge from a sufficient number of quanta ?
Topology of turbulent flows
Challenge: flow visualization, small probes
See recent PNAS special issue on quantum turbulence
edited by CFB, Skrbek & Sreenivasan (March 2014)
Thank you !
EPSRC
Carlo F. Barenghi
Quantum turbulence
Exiting... (Quit)
ncfb@lario:~/conf/per2014$