Title: Effective theories of vortex lines

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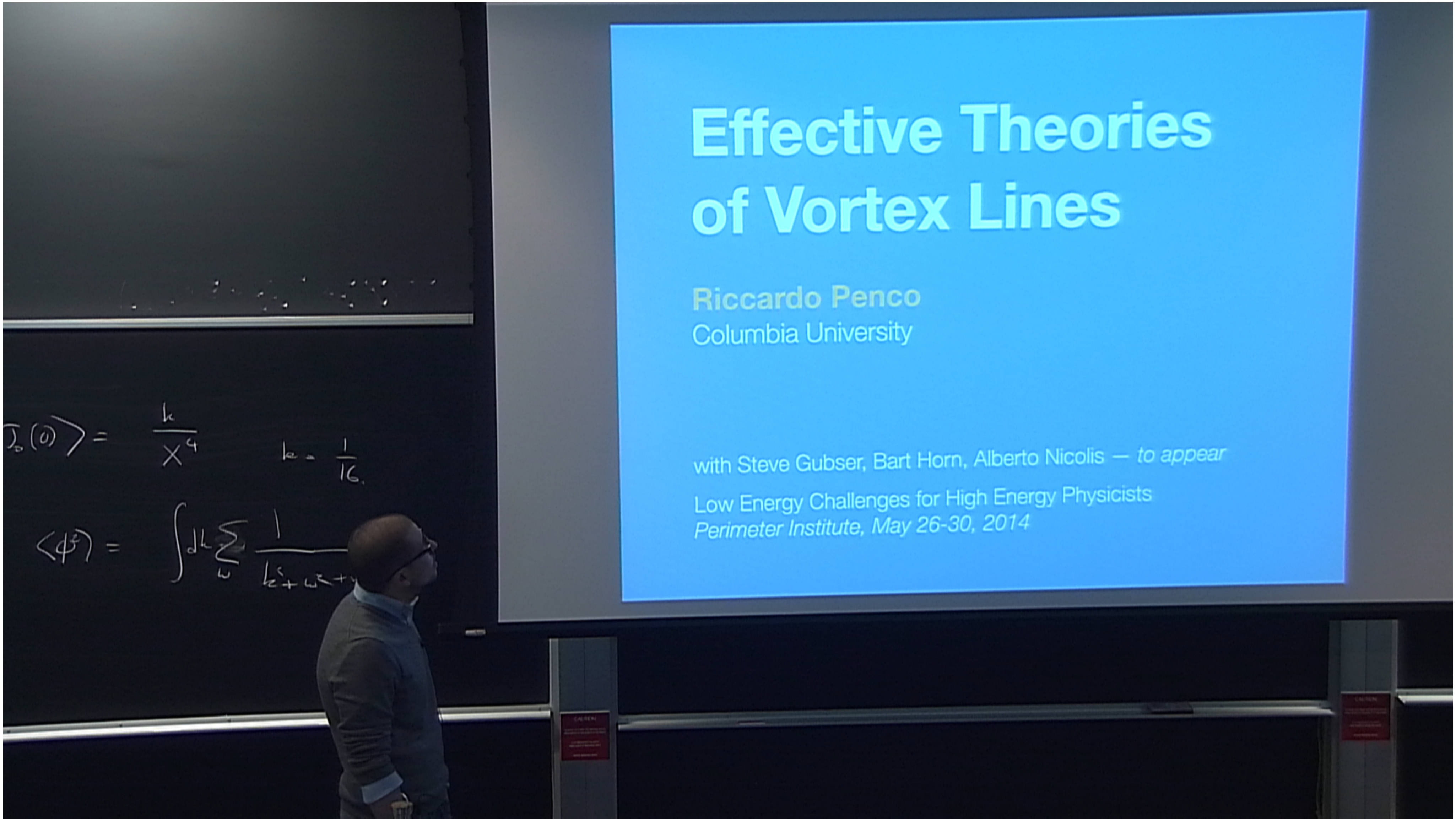
Abstract: Vortex lines are a distinctive feature of superfluids and are characterized by a very peculiar dynamics. In this talk, I will first discuss the behavior of vortex lines in a non-relativistic superfluids in the incompressible limit. I will then introduce an effective theory of vortex lines coupled to sound which applies to relativistic superfluids. I will conclude by briefly discussing the similarities between the effective theory for vortex lines and non-relativistic General Relativity.


Effective Theories of Vortex Lines
Riccardo Penco
Columbia University
with Steve Gubser, Bart Horn, Alberto Nicolis — to appear
Low Energy Challenges for High Energy Physicists
Perimeter Institute, May 26-30, 2014

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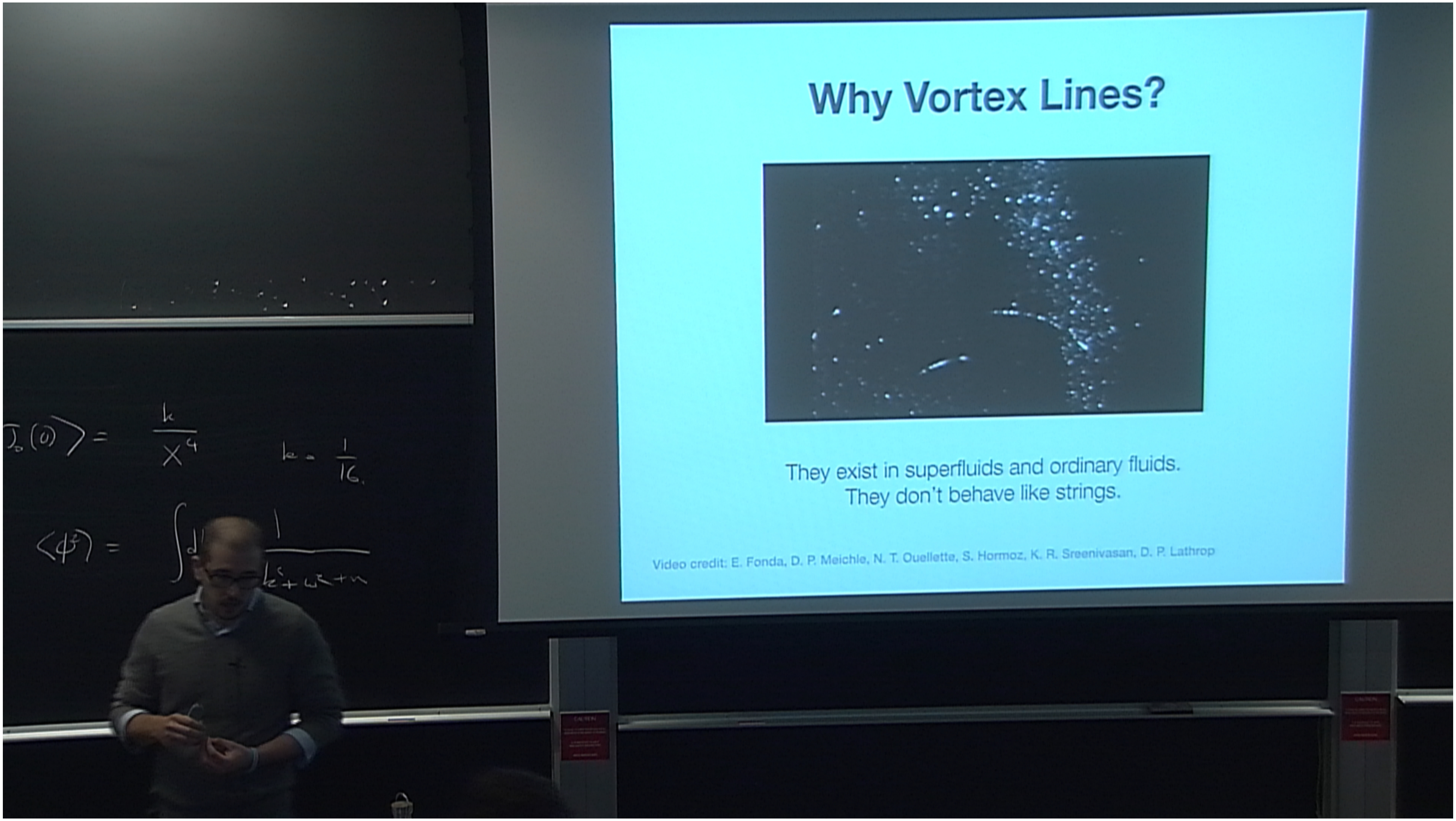
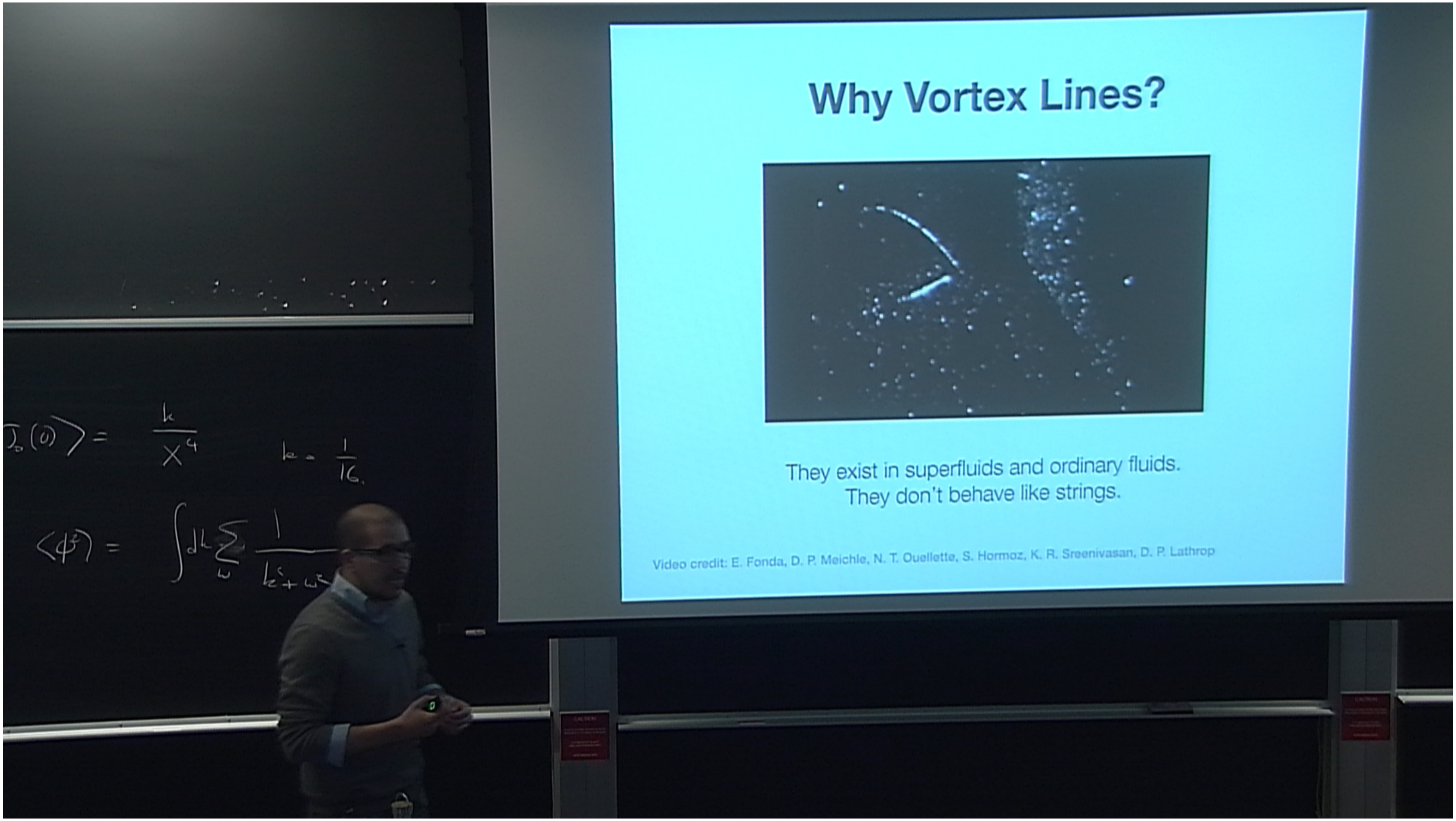


Why Vortex Lines?
They exist in superfluids and ordinary fluids.
They don't behave like strings.
Video credit: E. Fonda, D. P. Meichle, N. T. Ouellette, S. Hormoz, K. R. Sreenivasan, D. P. Lathrop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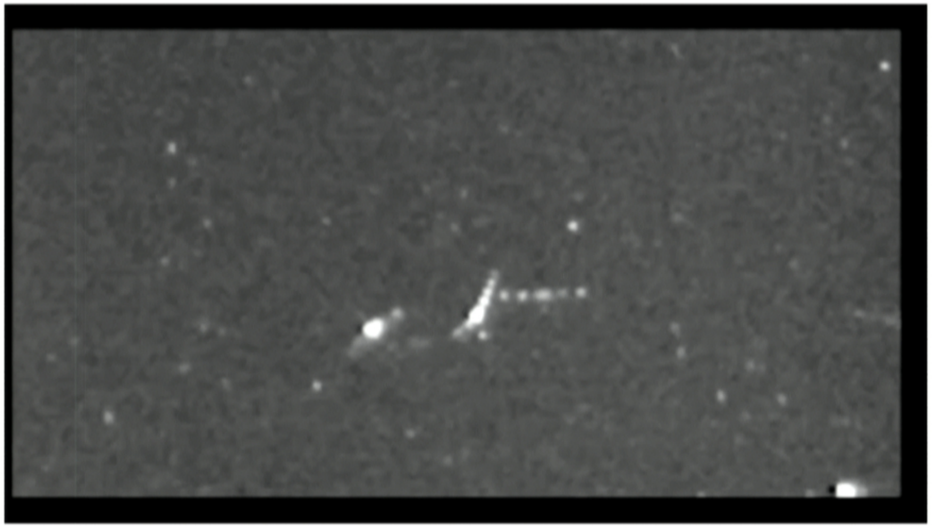

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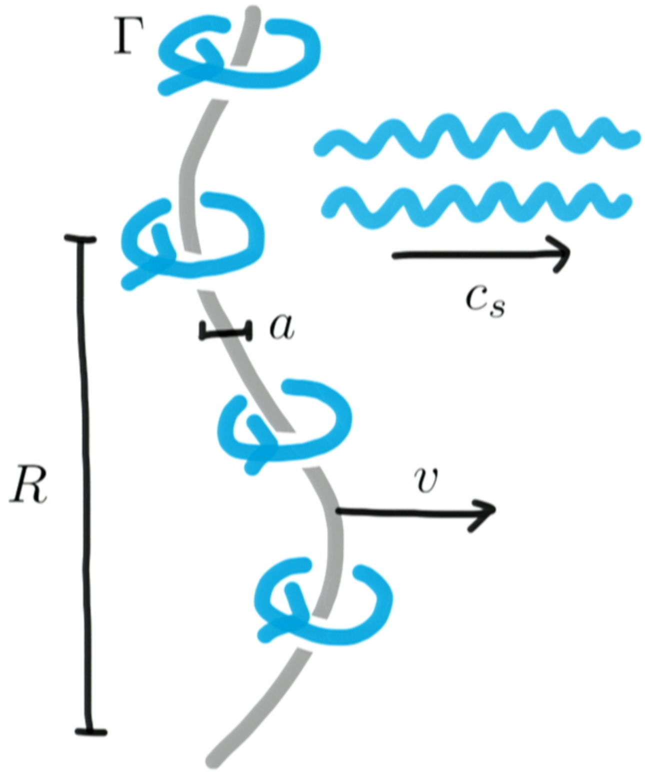
Basic Quantities



Vortex lines: $\nabla \times \vec{v} \neq 0$

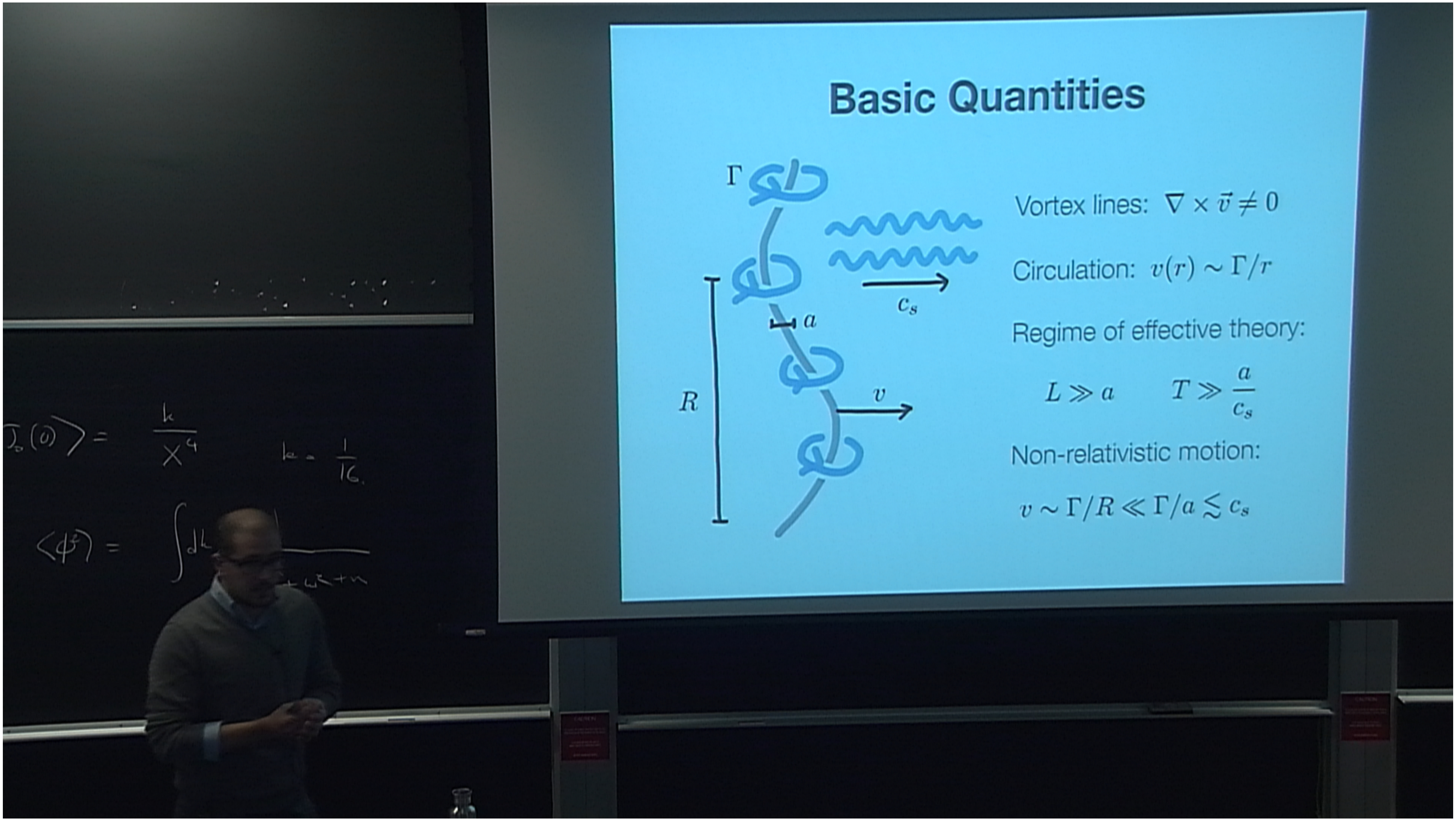
Circulation: $v(r) \sim \Gamma/r$

Regime of effective theory:

$$L \gg a \qquad T \gg \frac{a}{c_s}$$

Non-relativistic motion:

$$v \sim \Gamma/R \ll \Gamma/a \lesssim c_s$$



Basic Quantities
Γ
a
c_s
R
v
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Superfluids

Superfluid = spontaneously broken U(1) at finite density

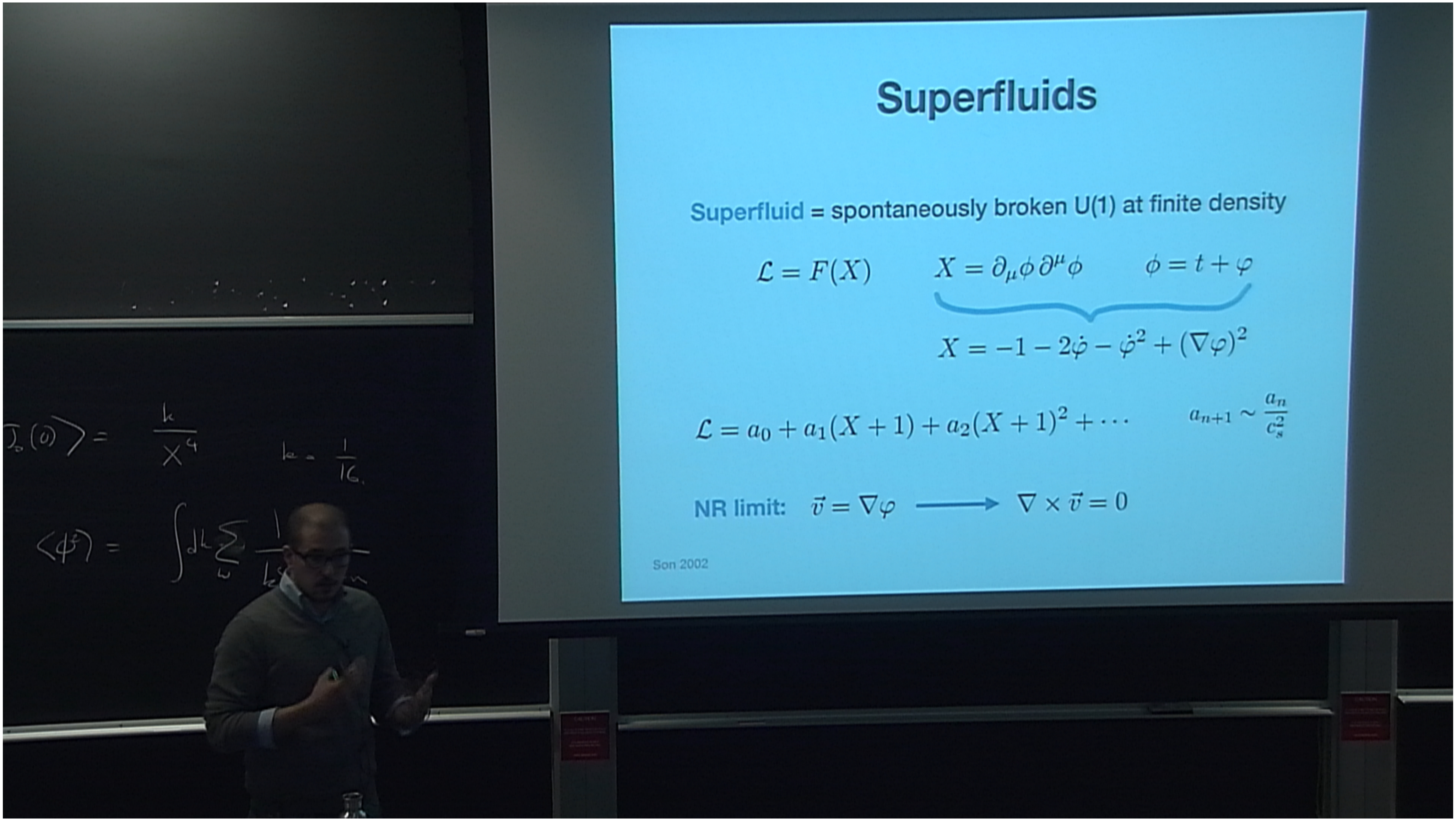
$$\mathcal{L} = F(X) \qquad X = \partial_\mu \phi \, \partial^\mu \phi \qquad \phi = t + \varphi$$

$$X = -1 - 2\dot{\varphi} - \dot{\varphi}^2 + (\nabla\varphi)^2$$

$$\mathcal{L} = a_0 + a_1(X+1) + a_2(X+1)^2 + \cdots \qquad a_{n+1} \sim \frac{a_n}{c_s^2}$$

NR limit: $\vec{v} = \nabla\varphi \longrightarrow \nabla \times \vec{v} = 0$

Son 2002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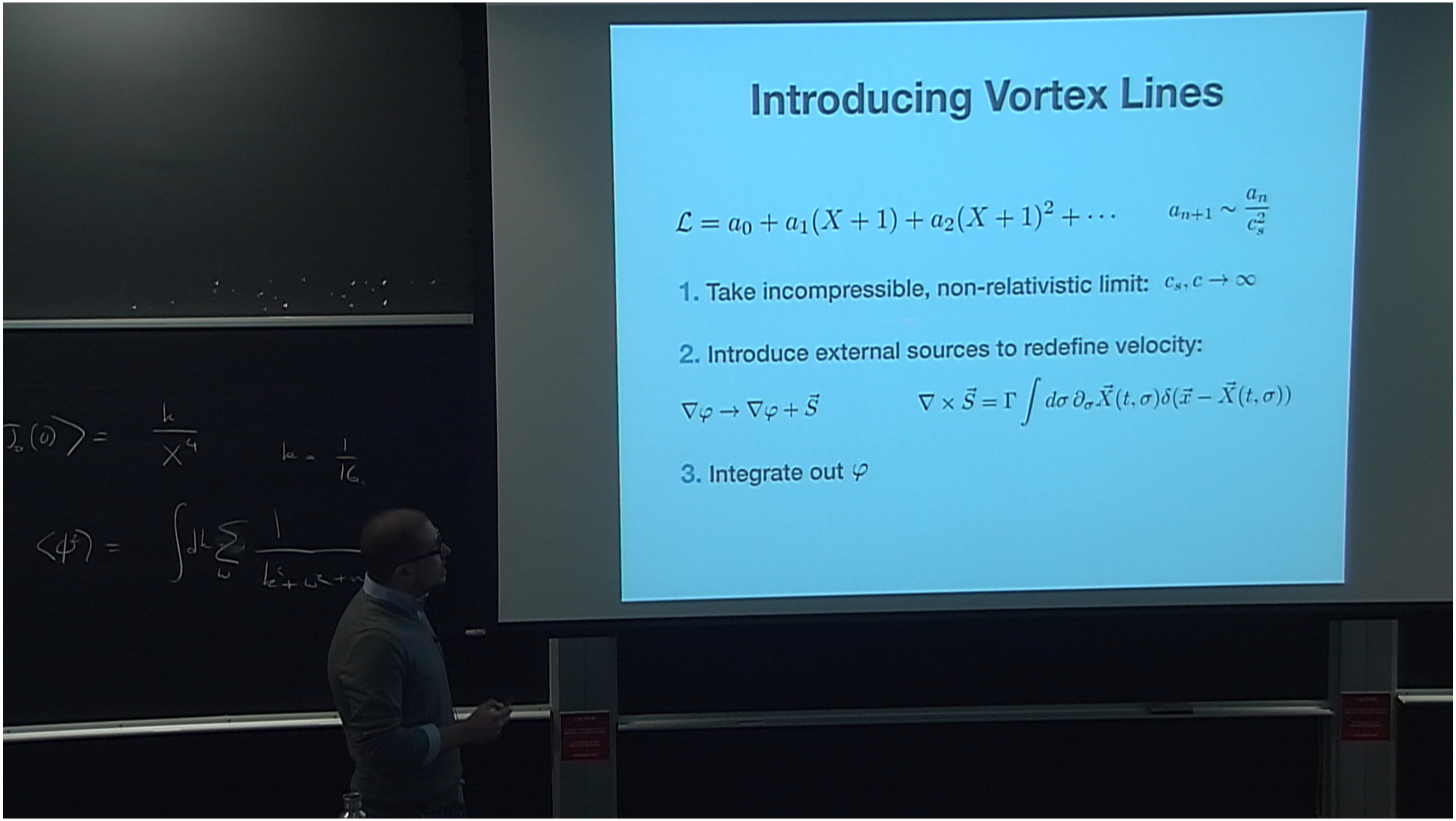
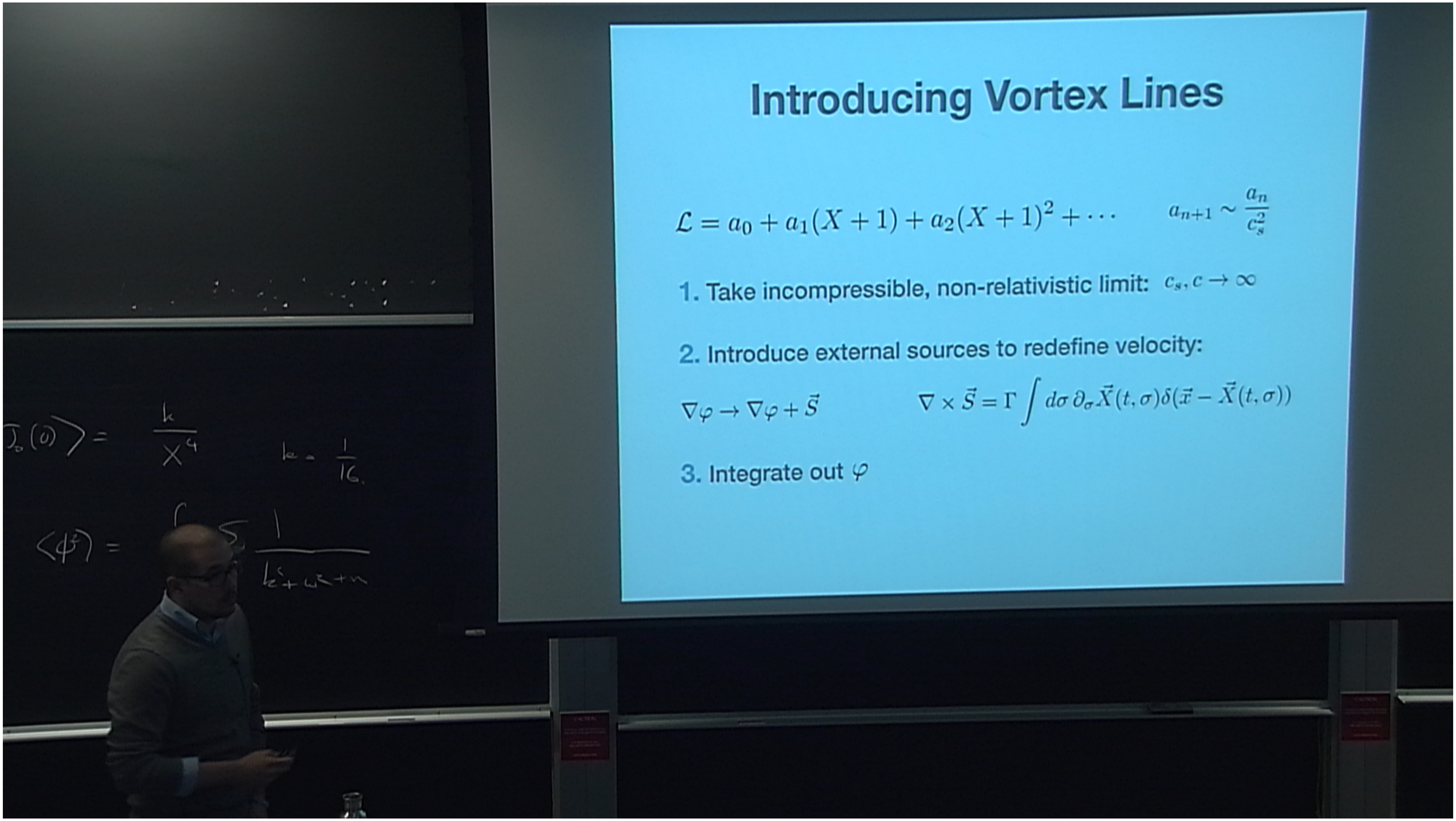

Introducing Vortex Lines
$\mathcal{L} = a_0 + a_1(X+1) + a_2(X+1)^2 + \cdots$
$a_{n+1} \sim \frac{a_n}{c_s^2}$
1. Take incompressible, non-relativistic limit: $c_s, c \to \infty$
2. Introduce external sources to redefine velocity:
$\nabla\varphi \to \nabla\varphi + \vec{S}$
$\nabla \times \vec{S} = \Gamma \int d\sigma\, \partial_\sigma \vec{X}(t,\sigma)\delta(\vec{x} - \vec{X}(t,\sigma))$
3. Integrate out φ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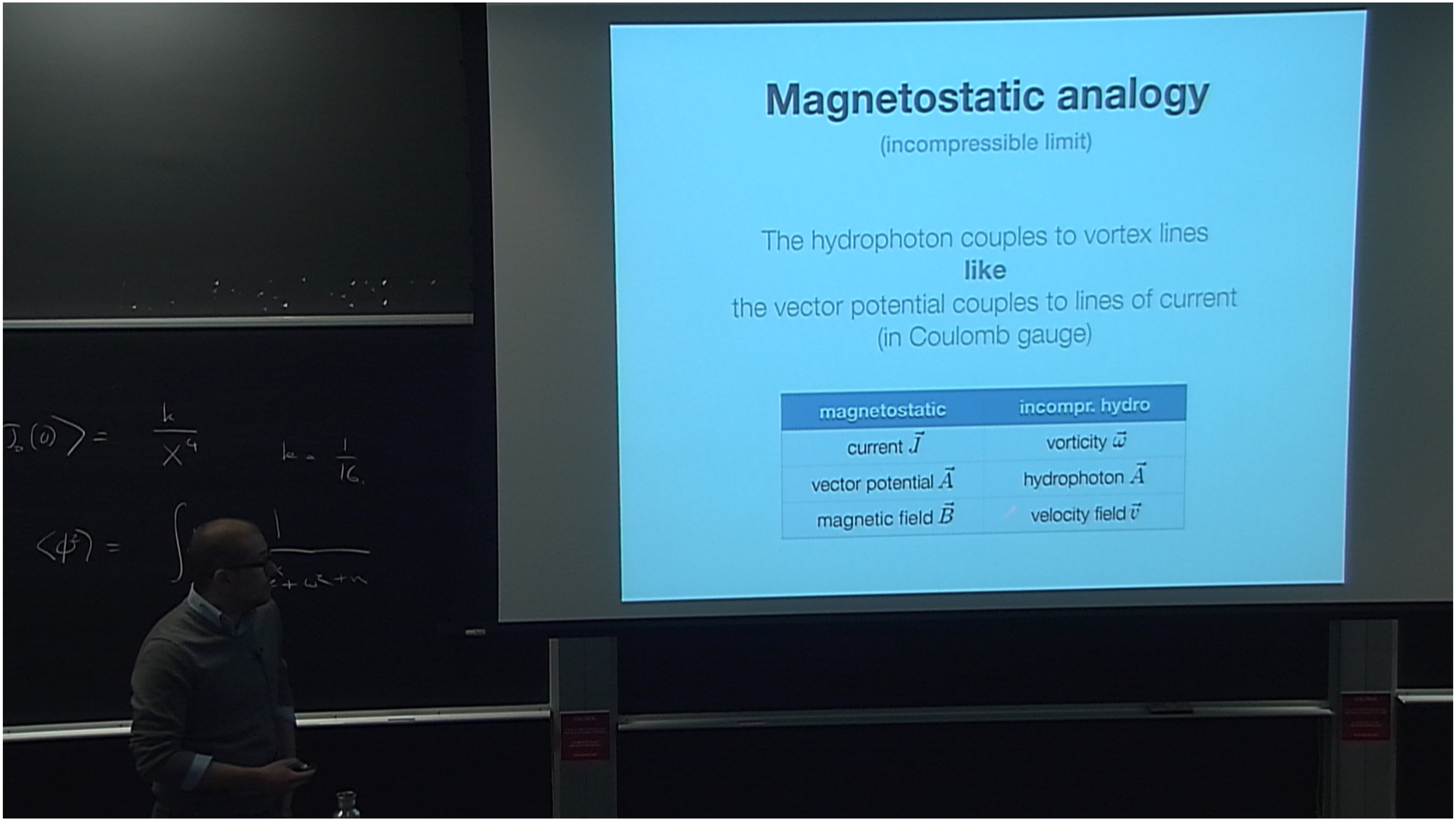
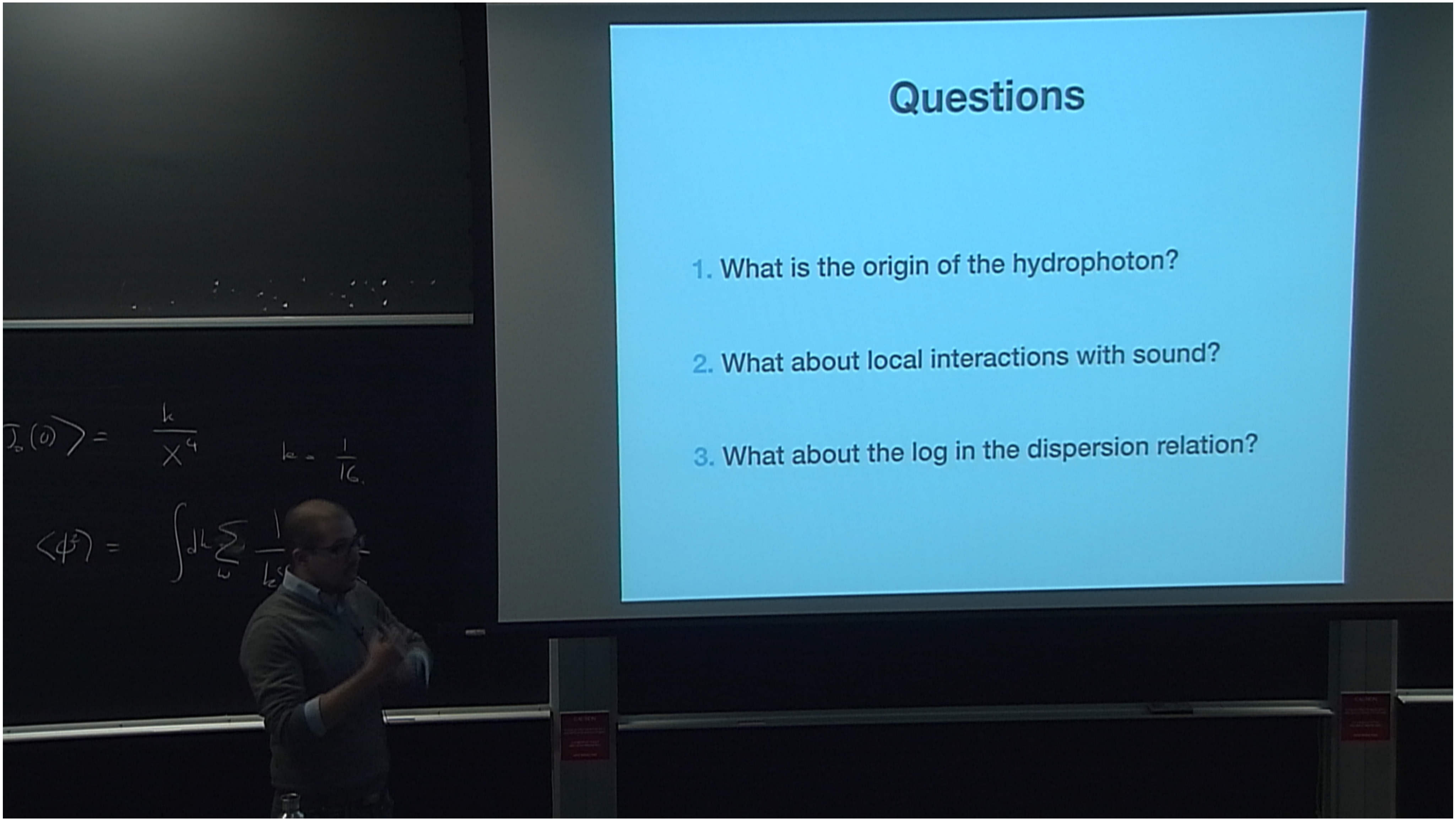
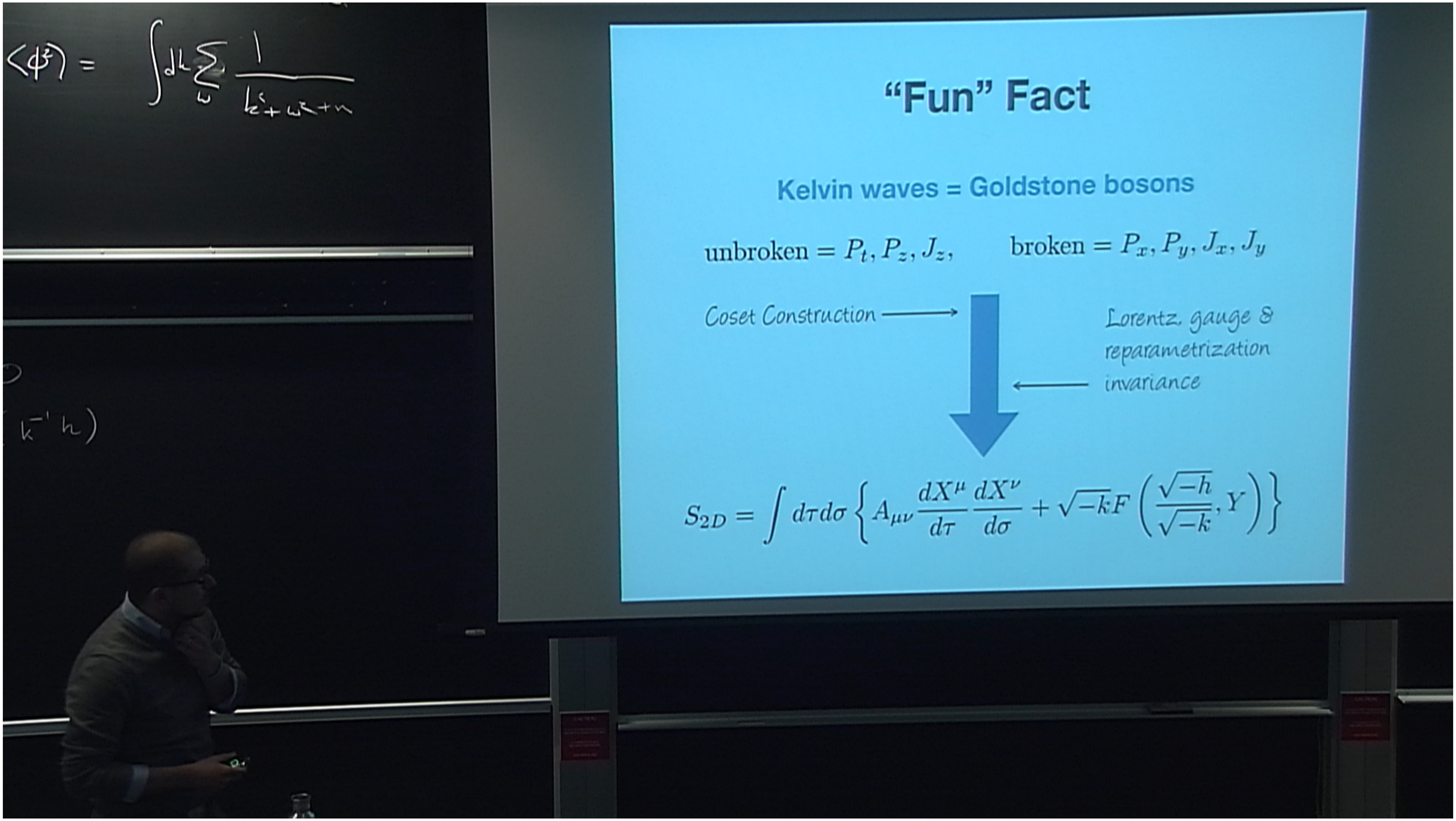
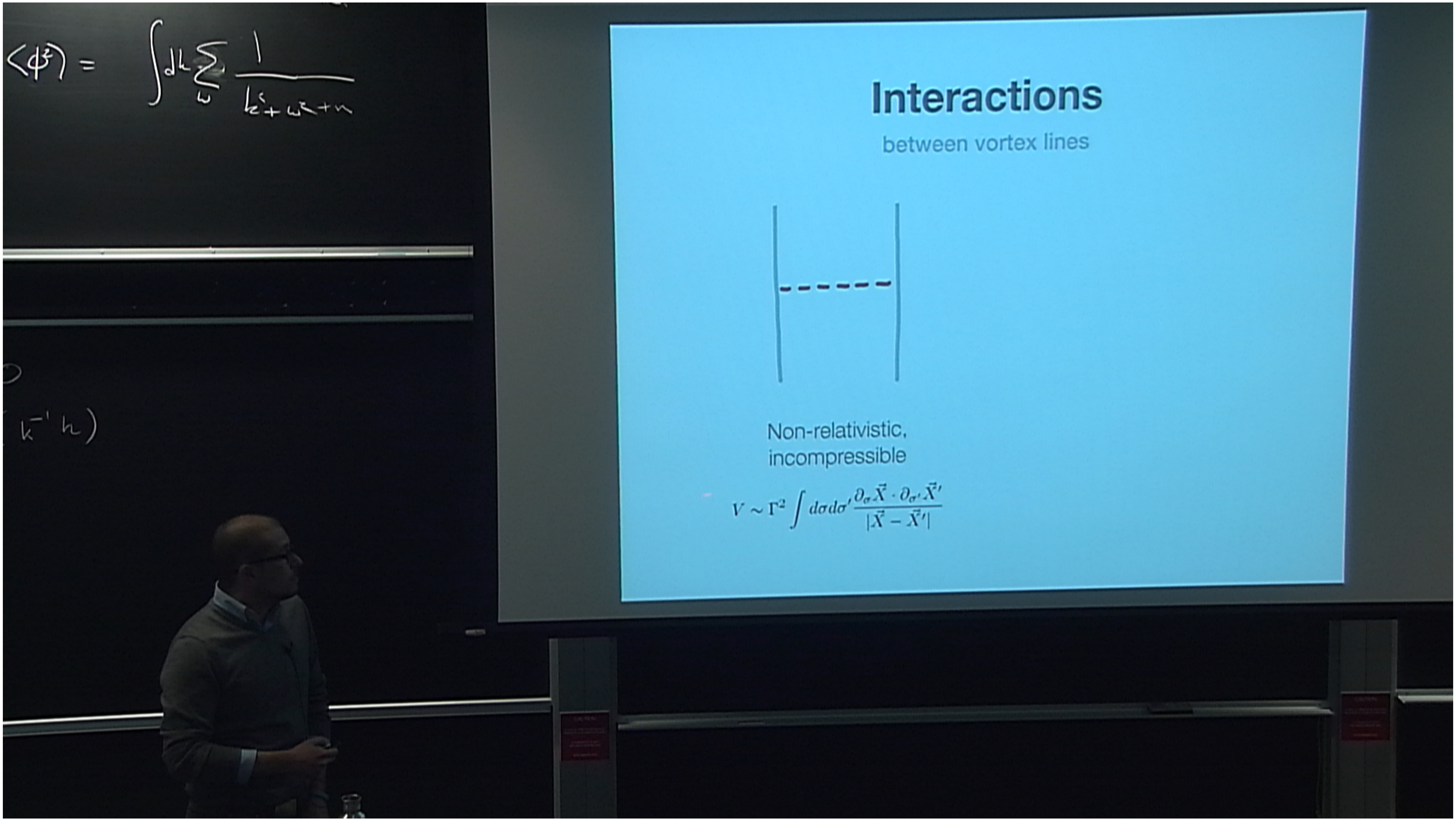

Magnetostatic analogy
(incompressible limit)
The hydrophoton couples to vortex lines
like
the vector potential couples to lines of current
(in Coulomb gauge)
magnetostatic
incompr. hydro
current $\vec{J}$
vorticity $\vec{\omega}$
vector potential $\vec{A}$
hydrophoton $\vec{A}$
magnetic field $\vec{B}$
velocity field $\vec{v}$


Questions
1. What is the origin of the hydrophoton?
2. What about local interactions with sound?
3. What about the log in the dispersion relation?


"Fun" Fact
Kelvin waves = Goldstone bosons
unbroken = $P_t, P_z, J_z,$ broken = P_x, P_y, J_x, J_y
Coset Construction
Lorentz, gauge & reparametrization invariance
$S_{2D} = \int d\tau d\sigma \left\{ A_{\mu\nu} \frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\sigma} + \sqrt{-k} F \left(\frac{\sqrt{-h}}{\sqrt{-k}}, Y \right) \right\}$



Interactions
between vortex lines
Non-relativistic,
incompressible
$V \sim \Gamma^2 \int d\sigma d\sigma' \frac{\partial_\sigma \vec{X} \cdot \partial_{\sigma'} \vec{X}'}{|\vec{X} - \vec{X}'|}$

Classical RG Running

The tension T can be fixed by demanding that the self-energy of a static vortex line is finite.

$$\frac{dE}{d\sigma} = \underbrace{\Gamma^2 \int d\sigma' \frac{\partial_\sigma \vec{X} \cdot \partial_{\sigma'} \vec{X}'}{|\vec{X} - \vec{X}'|}}_{\sim \Gamma^2 \log(L/a)} + T|\partial_\sigma \vec{X}|$$

$$\longrightarrow \quad T(\mu) \sim \Gamma^2 \log(\mu a) + \text{finite}$$

Lund & Regge, 1976

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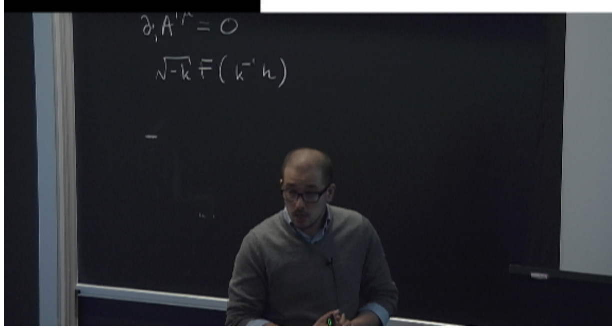

Now, $\omega = \pm \frac{\Gamma}{4\pi} k^2 \log(k/\mu)$

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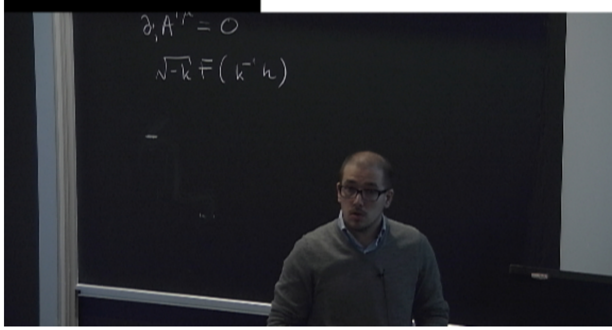

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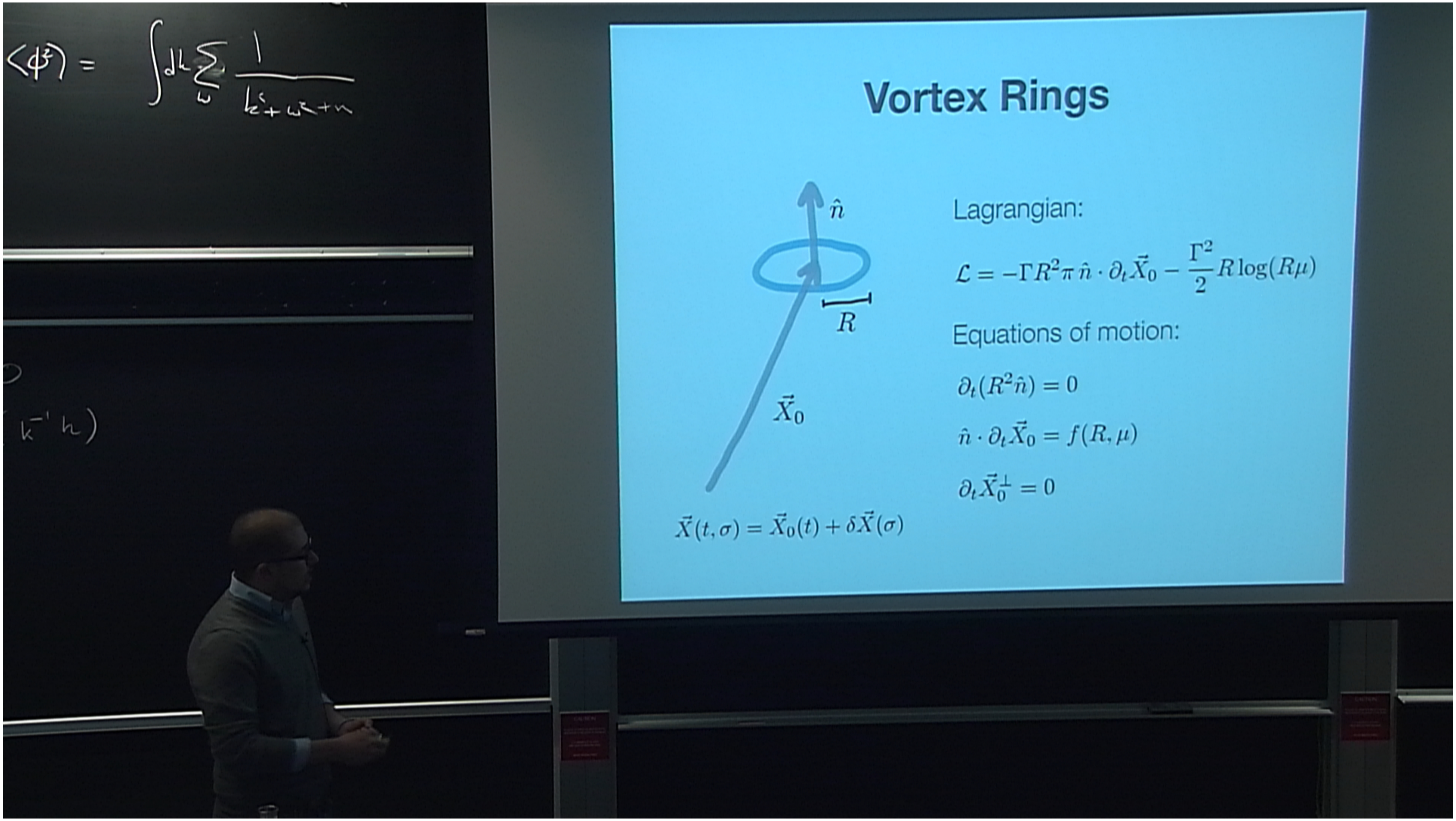
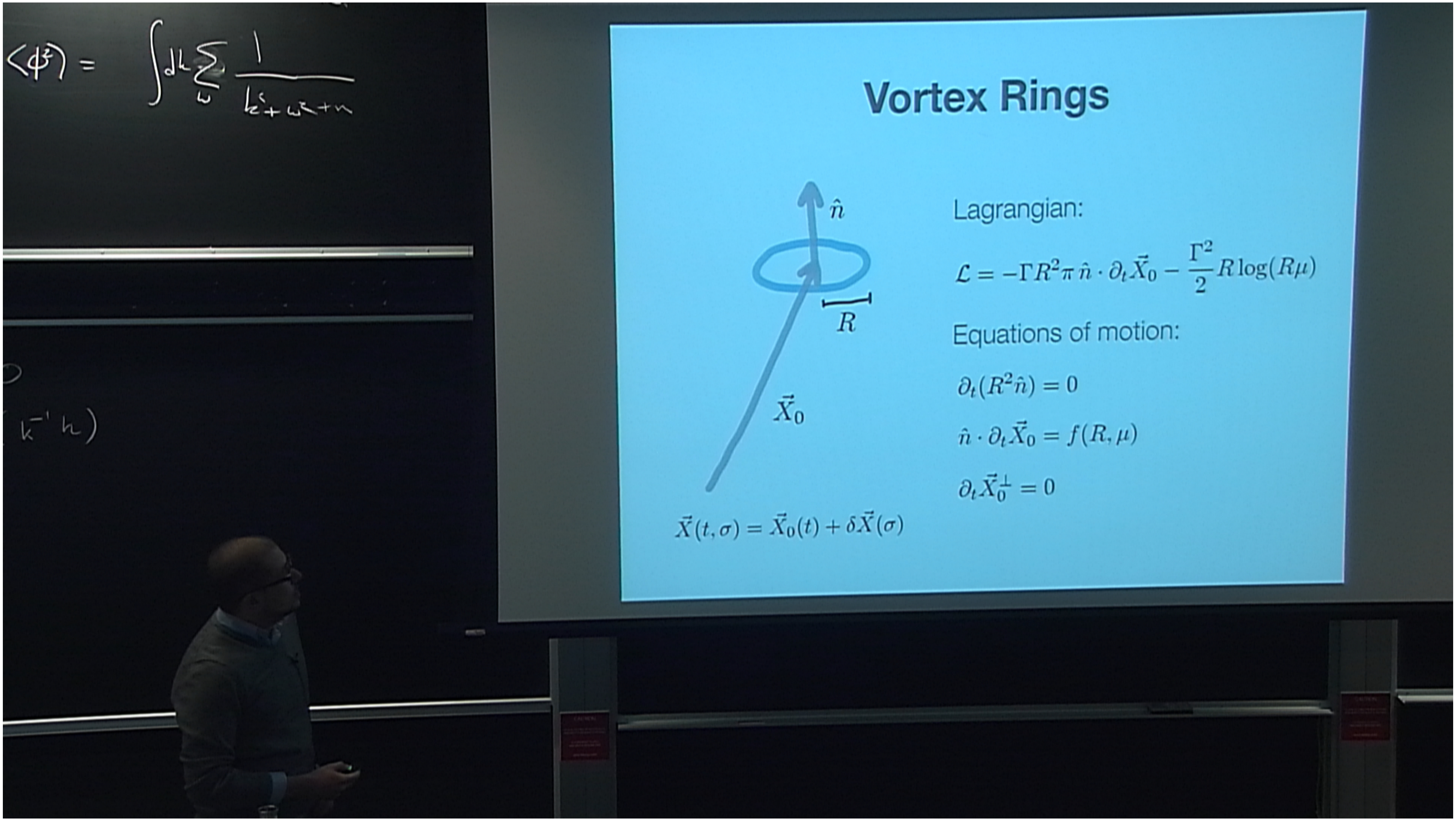

Vortex Rings
$\hat{n}$
R
$\vec{X}_0$
$\vec{X}(t,\sigma) = \vec{X}_0(t) + \delta\vec{X}(\sigma)$
Lagrangian:
$\mathcal{L} = -\Gamma R^2 \pi \, \hat{n} \cdot \partial_t \vec{X}_0 - \frac{\Gamma^2}{2} R \log(R\mu)$
Equations of motion:
$\partial_t (R^2 \hat{n}) = 0$
$\hat{n} \cdot \partial_t \vec{X}_0 = f(R, \mu)$
$\partial_t \vec{X}_0^{\perp} = 0$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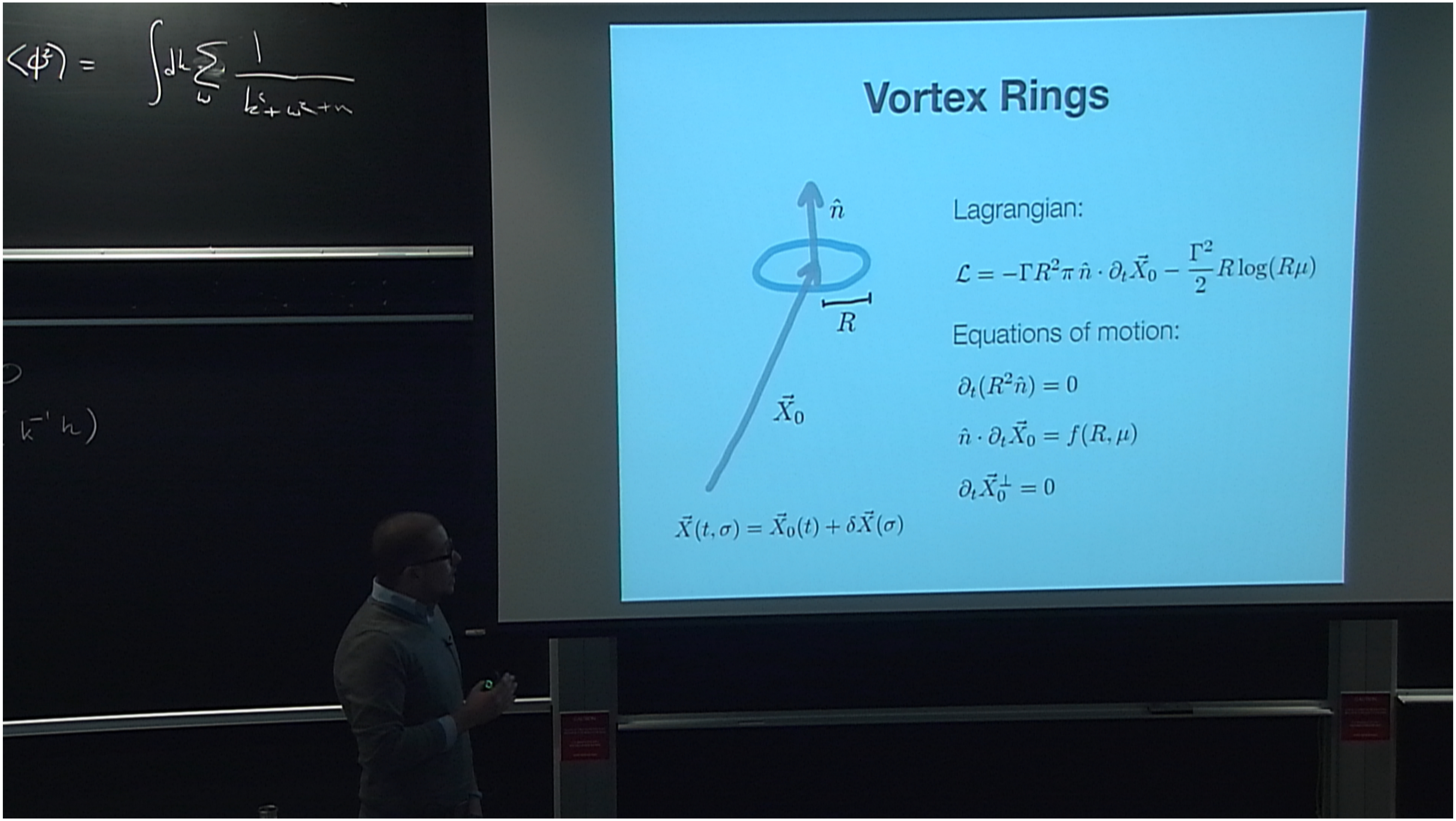
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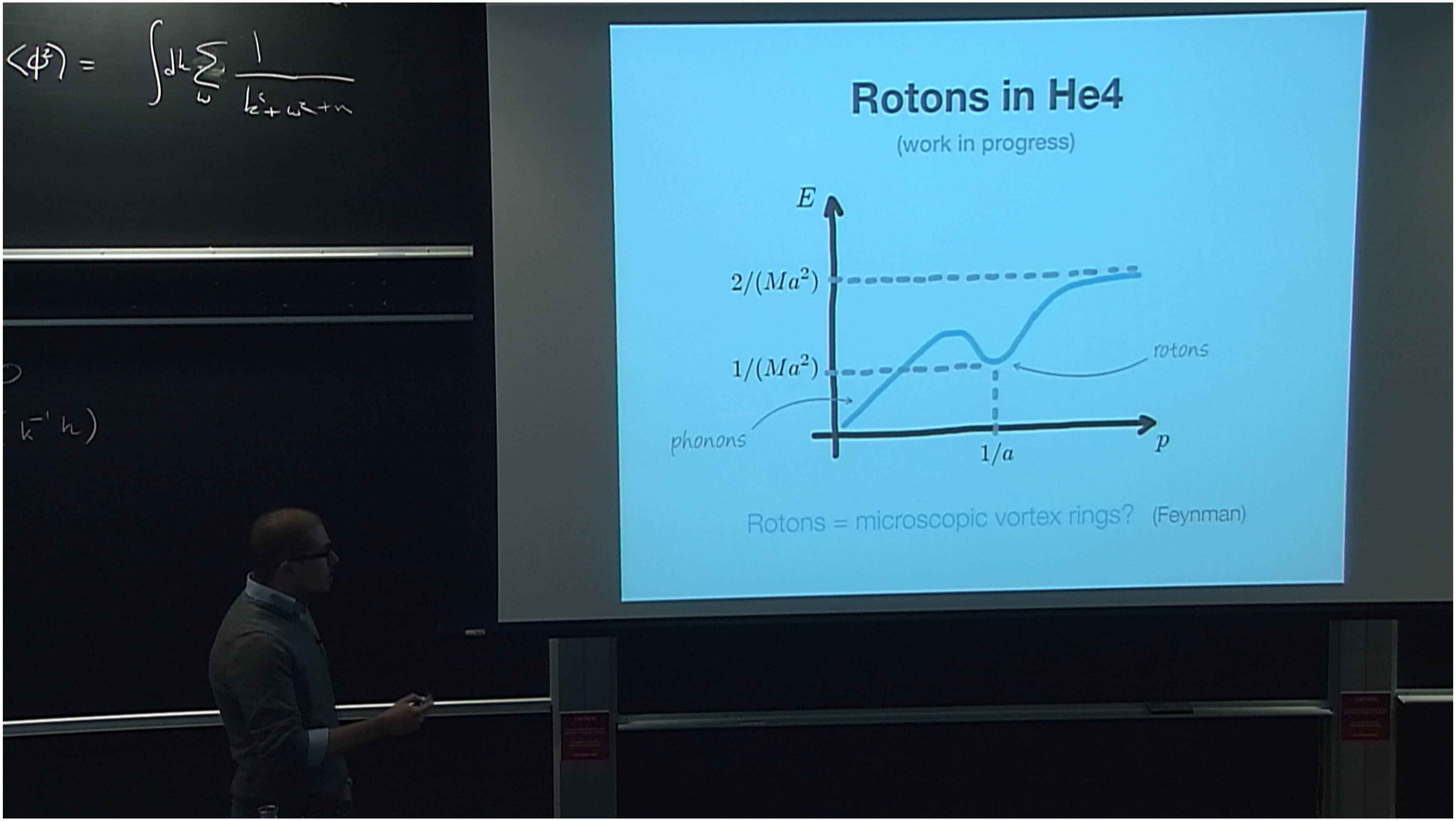
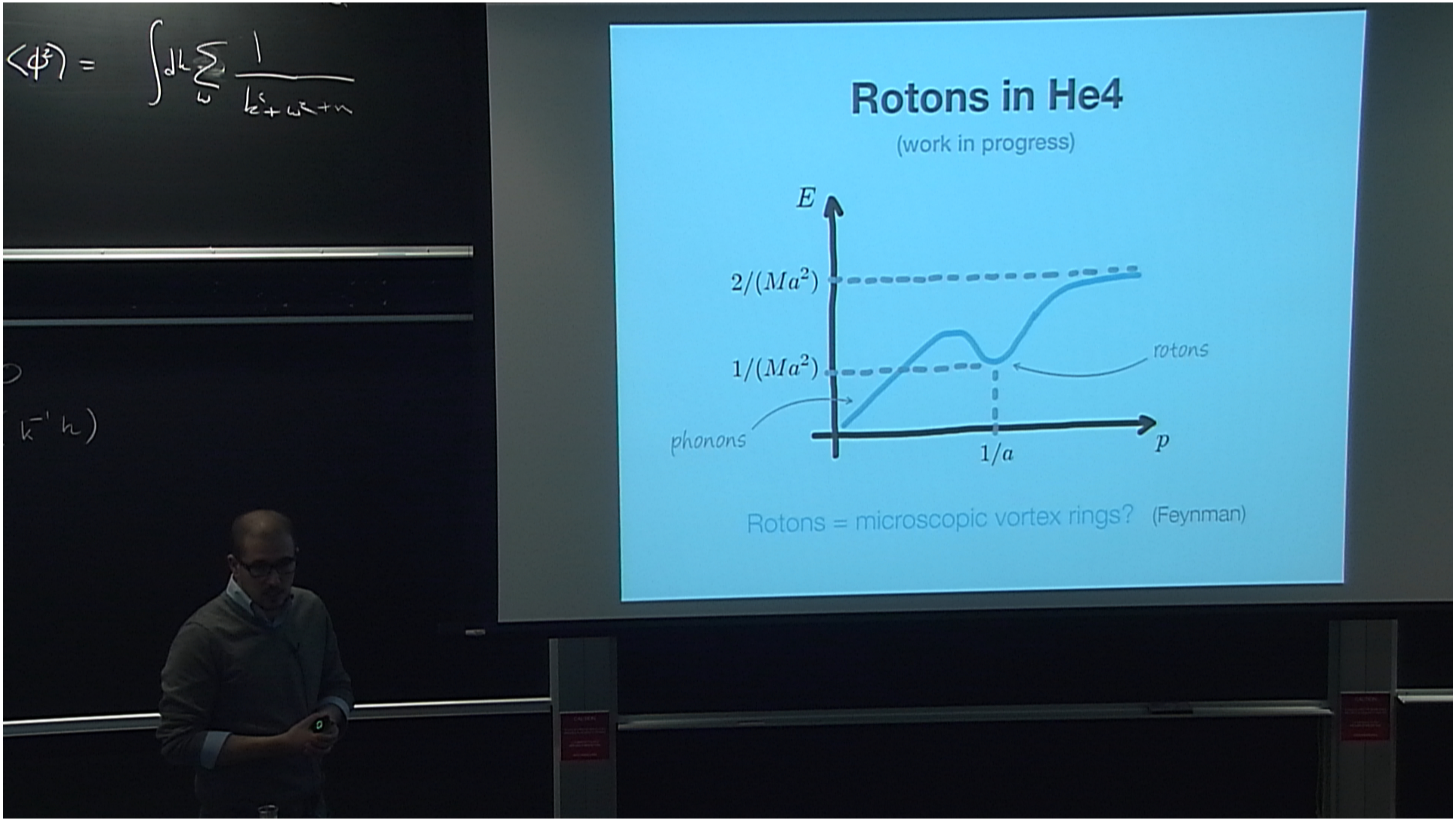

Rotons in He4
(work in progress)
E
2/(Ma²)
1/(Ma²)
rotons
phonons
1/a
p
Rotons = microscopic vortex rings? (Feynman)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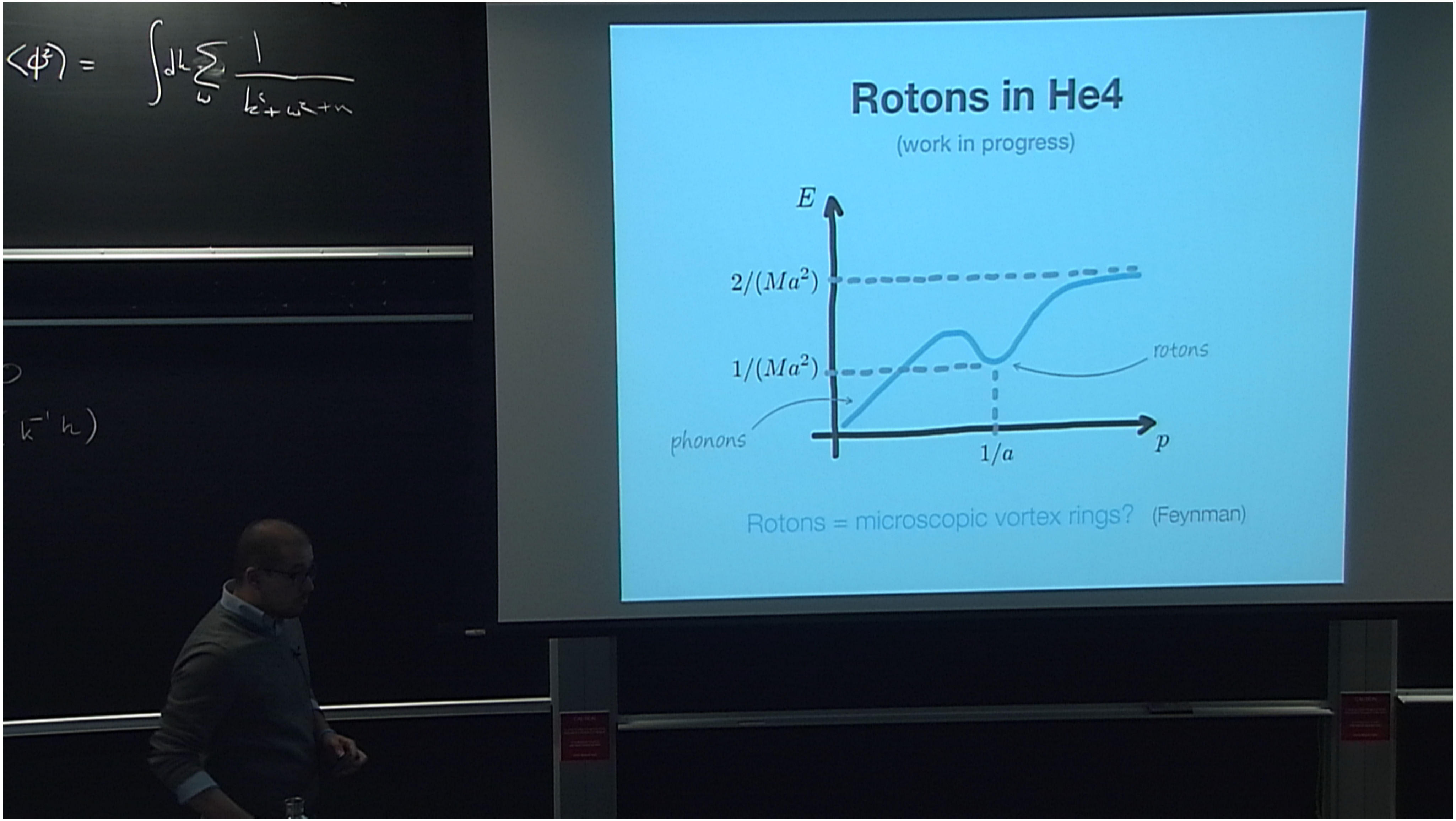

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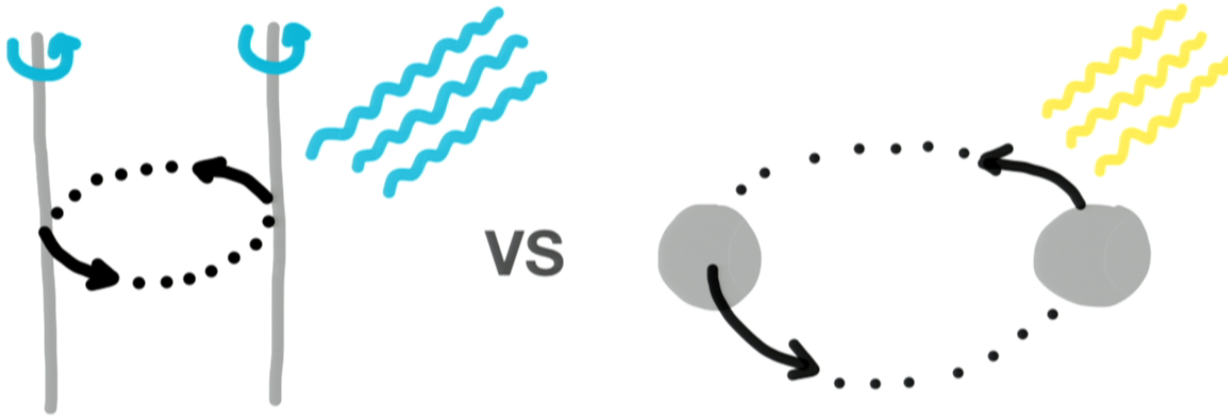
Vortex Lines vs Binaries

(outlook)

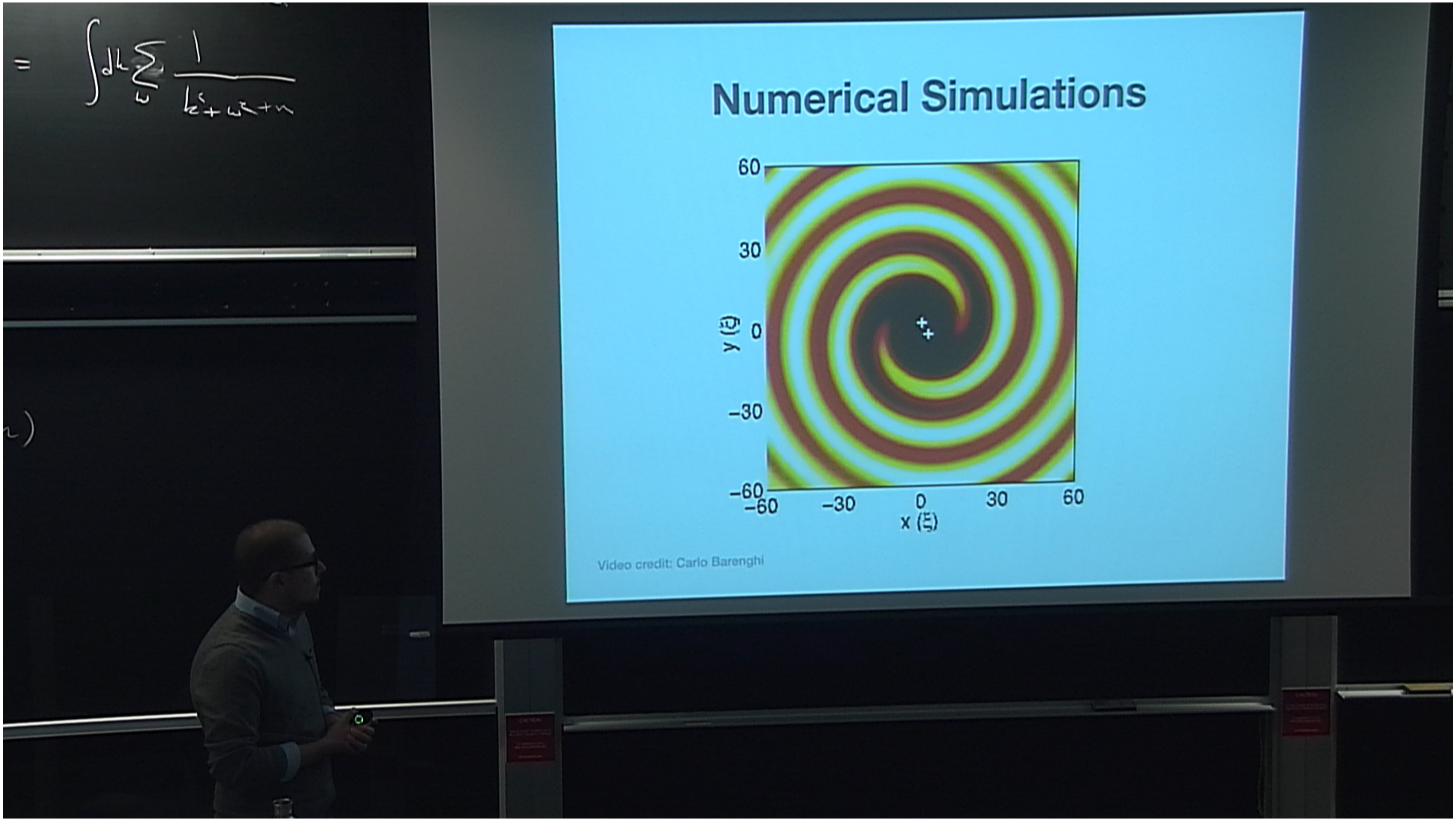


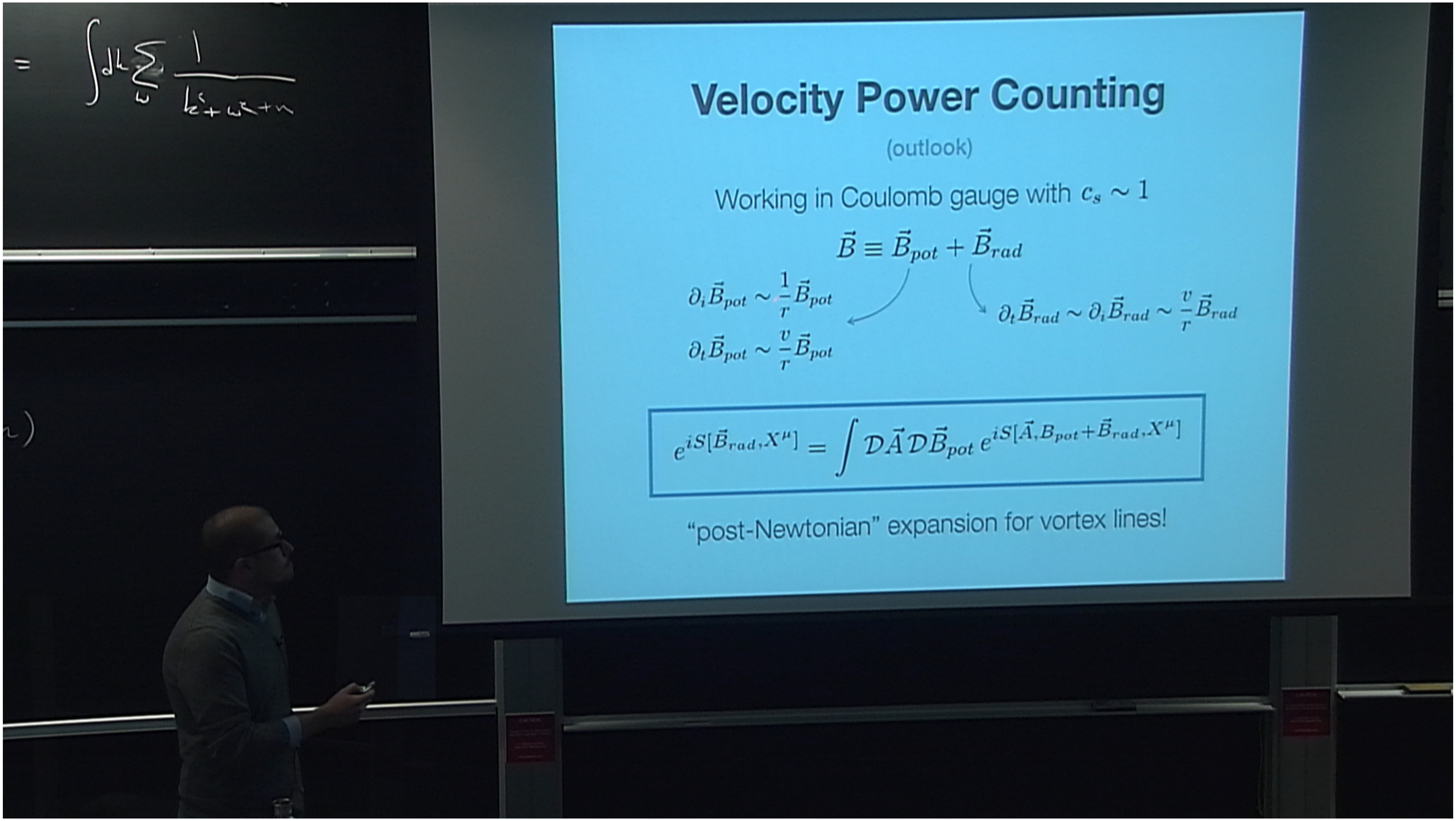
$$A_{\mu\nu} \longleftrightarrow g_{\mu\nu}$$

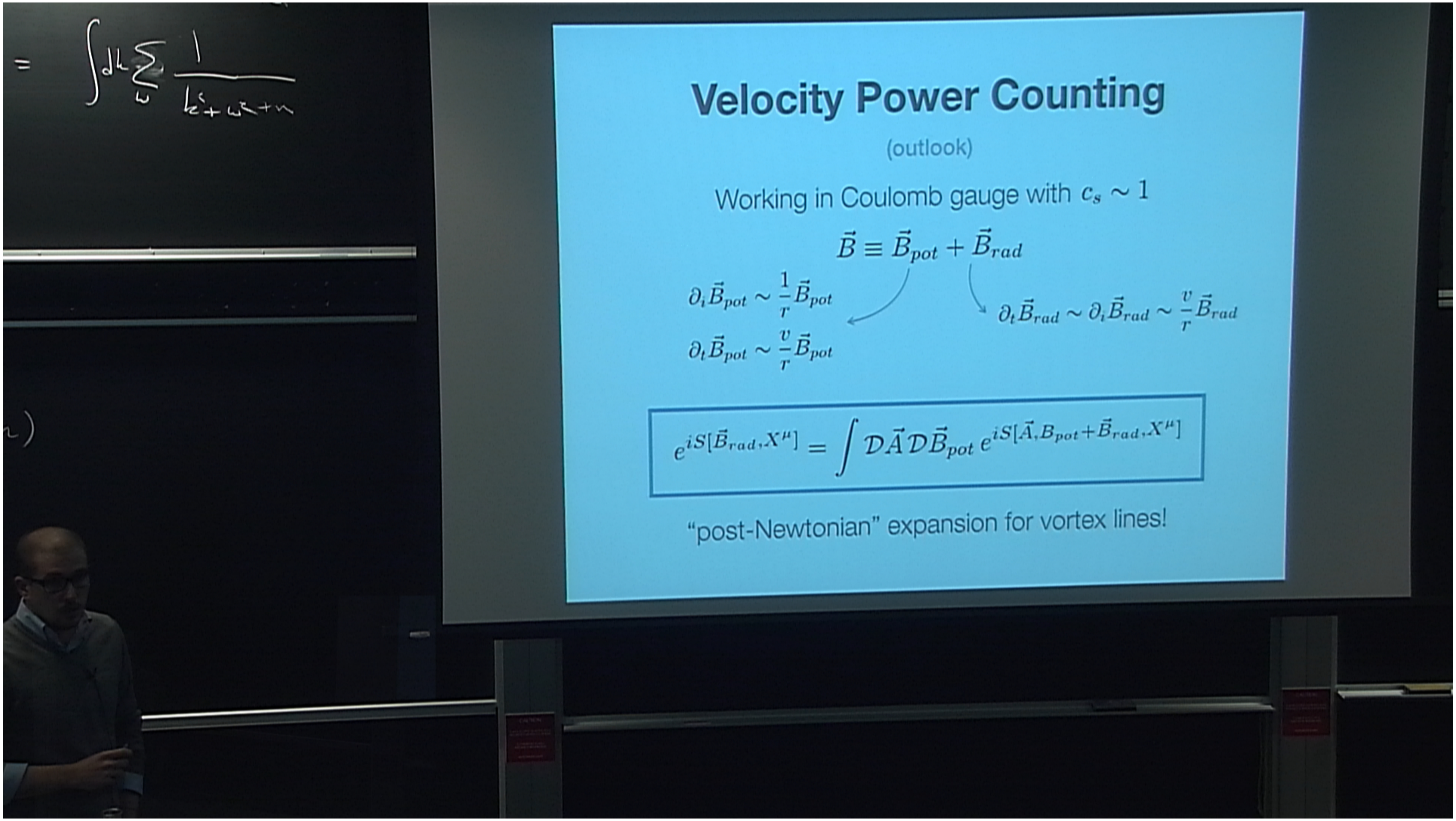
$$x^{\mu}(\tau, \sigma) \longleftrightarrow x^{\mu}(\tau)$$

Endlich & Nicolis, 2013

Golberger & Rothstein, 2006


Numerical Simulations
60
30
0
−30
−60
y (ξ)
−60
−30
0
30
60
x (ξ)
Video credit: Carlo Barenghi


Velocity Power Counting
(outlook)
Working in Coulomb gauge with $c_s \sim 1$
$\vec{B} \equiv \vec{B}_{pot} + \vec{B}_{rad}$
$\partial_i \vec{B}_{pot} \sim \frac{1}{r} \vec{B}_{pot}$
$\partial_t \vec{B}_{pot} \sim \frac{v}{r} \vec{B}_{pot}$
$\partial_t \vec{B}_{rad} \sim \partial_i \vec{B}_{rad} \sim \frac{v}{r} \vec{B}_{rad}$
$e^{iS[\vec{B}_{rad}, X^\mu]} = \int \mathcal{D}\vec{A}\,\mathcal{D}\vec{B}_{pot}\, e^{iS[\vec{A}, B_{pot} + \vec{B}_{rad}, X^\mu]}$
"post-Newtonian" expansion for vortex lines!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“post-Newtonian” expansion for vortex lines!