Title: Universal incoherent metallic transport

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Abstract: In an incoherent metal, transport is controlled by the collective diffusion of energy and charge rather than by quasiparticle or momentum relaxation. We explore the possibility of a universal bound D \gtrsim \hbar v_F^2 /(k_B T) on the underlying diffusion constants in an incoherent metal. Such a bound is loosely motivated by results from holographic duality, the uncertainty principle and from measurements of diffusion in strongly interacting non-metallic systems. Metals close to saturating this bound are shown to have a linear in temperature resistivity with an underlying dissipative timescale matching that recently deduced from experimental data on a wide range of metals. The phenomenology of universal incoherent transport is found to reproduce various further observations in strongly correlated metals, and motivates direct probes of diffusive processes and measurements of charge susceptibilities. We suggest that this bound may be responsible for the ubiquitous appearance of high temperature regimes in metals with T-linear resistivity.

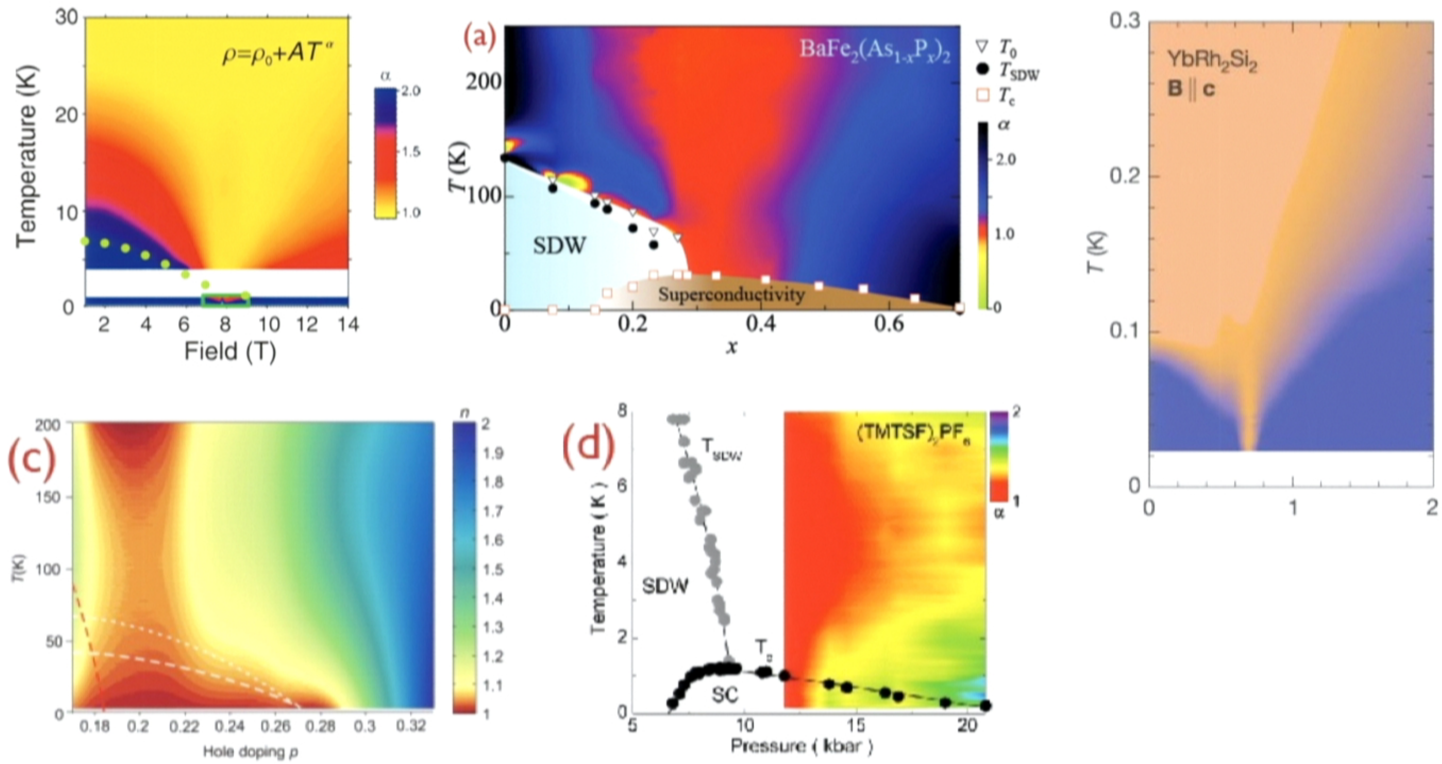
Universal incoherent metallic transport

Sean Hartnoll (Stanford)

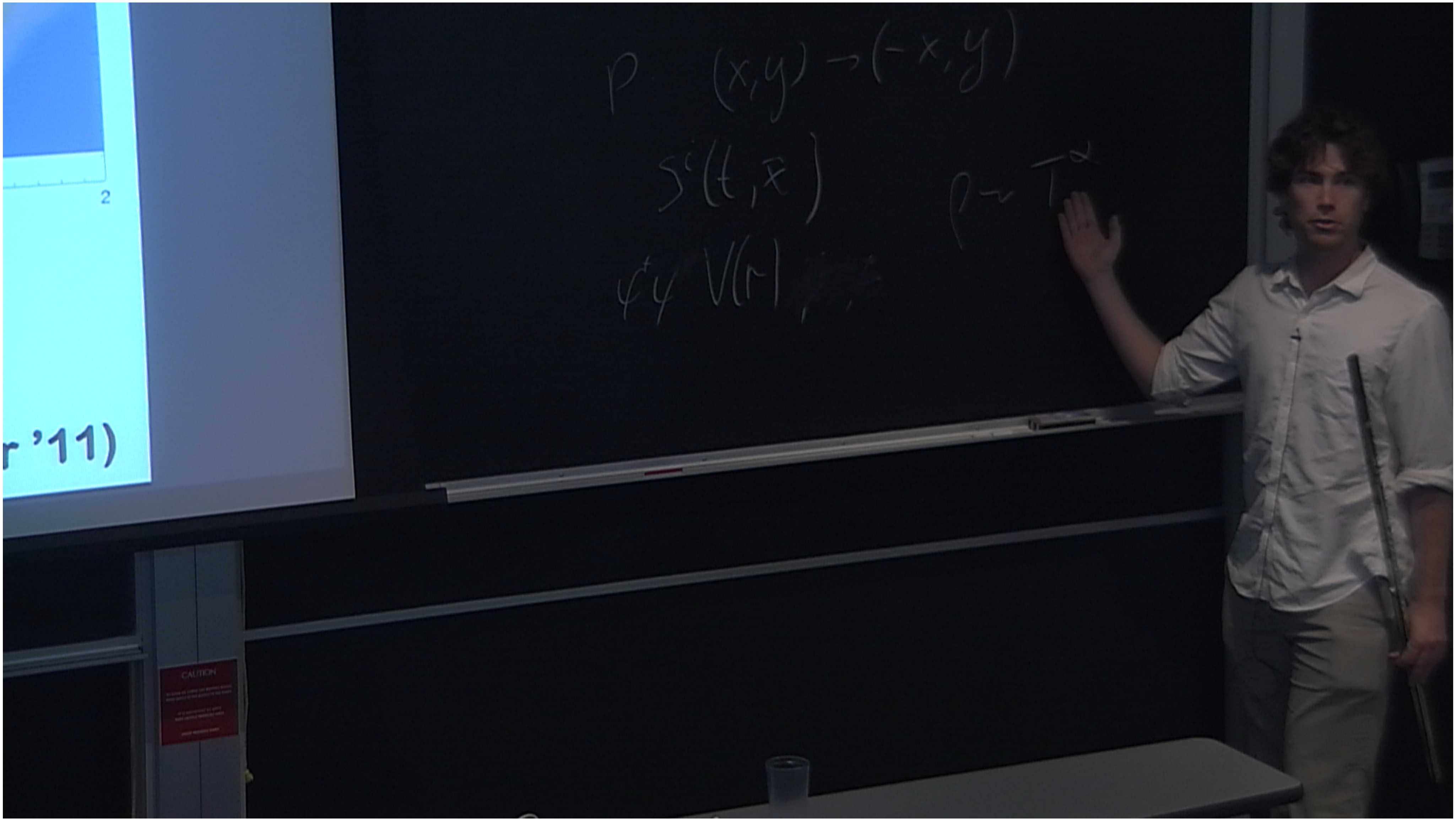
1405.3651

PI - May 2014

Ubiquity of T-linear resistivity

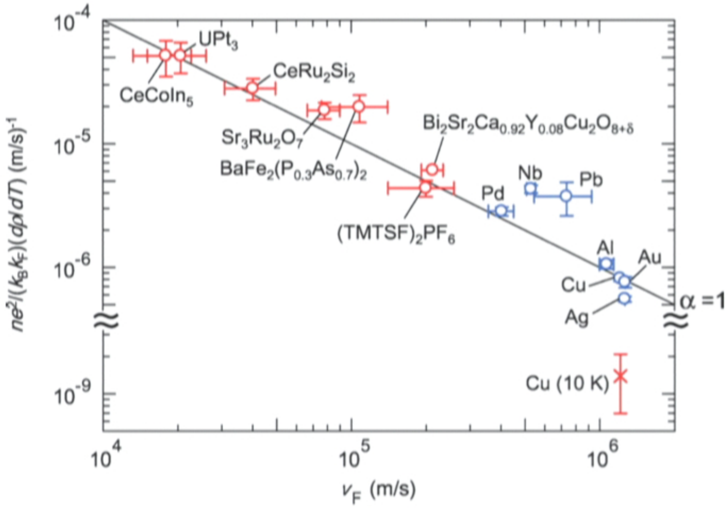


(after Sachdev-Keimer '11)


r ’11)
(x,y) → (-x,y)
CAUTION

Universal dynamics at work?

- **Write:** $\sigma = \dfrac{ne^2\tau}{m}$ $\left(n = \dfrac{k_F^2}{2\pi d}\right)$
- **Measure: σ (from resistivity), k_F,m (quantum oscillations).**
- **Extract τ. Find:** $\tau = \alpha \dfrac{\hbar}{k_B T}$ $(\alpha \approx 1 - 2)$



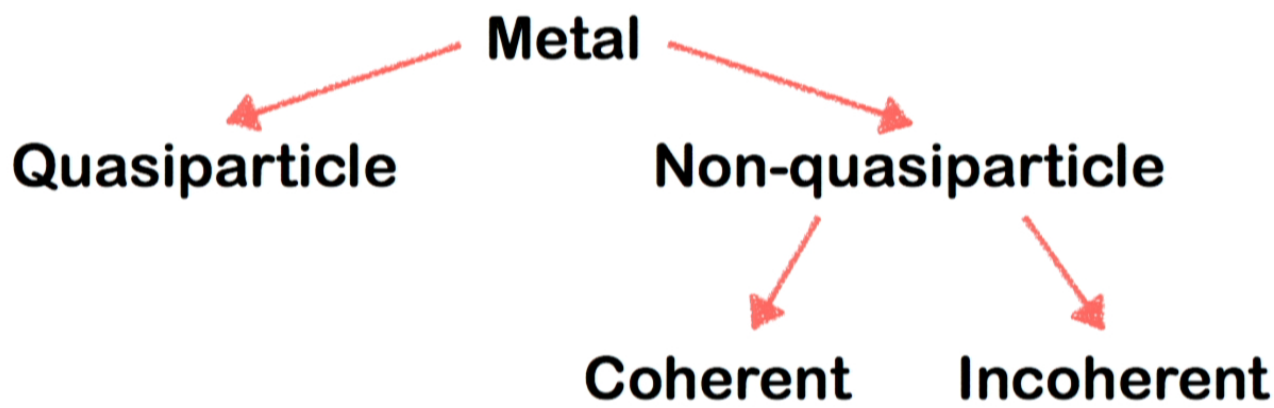
(Bruin et al. '13)

Mission statement

- Perhaps these systems show universal behavior because they are close to saturating a universal bound?
- ∃ longstanding desire to apply the logic of the KSS viscosity bound to metals (cf. Sachdev, Zaanen, Bruin et al.).
- Complicated at various levels, as I will review.
- This talk: quasi-concrete proposal for such a bound. Lots of nice phenomenology.

Classification of metals by transport

- The resistivity of a metal is determined by the longest lived excitations that carry charge (or heat).



Quasiparticle transport

- Longest lived excitations: δn_k.
- Study with Boltzmann equation.

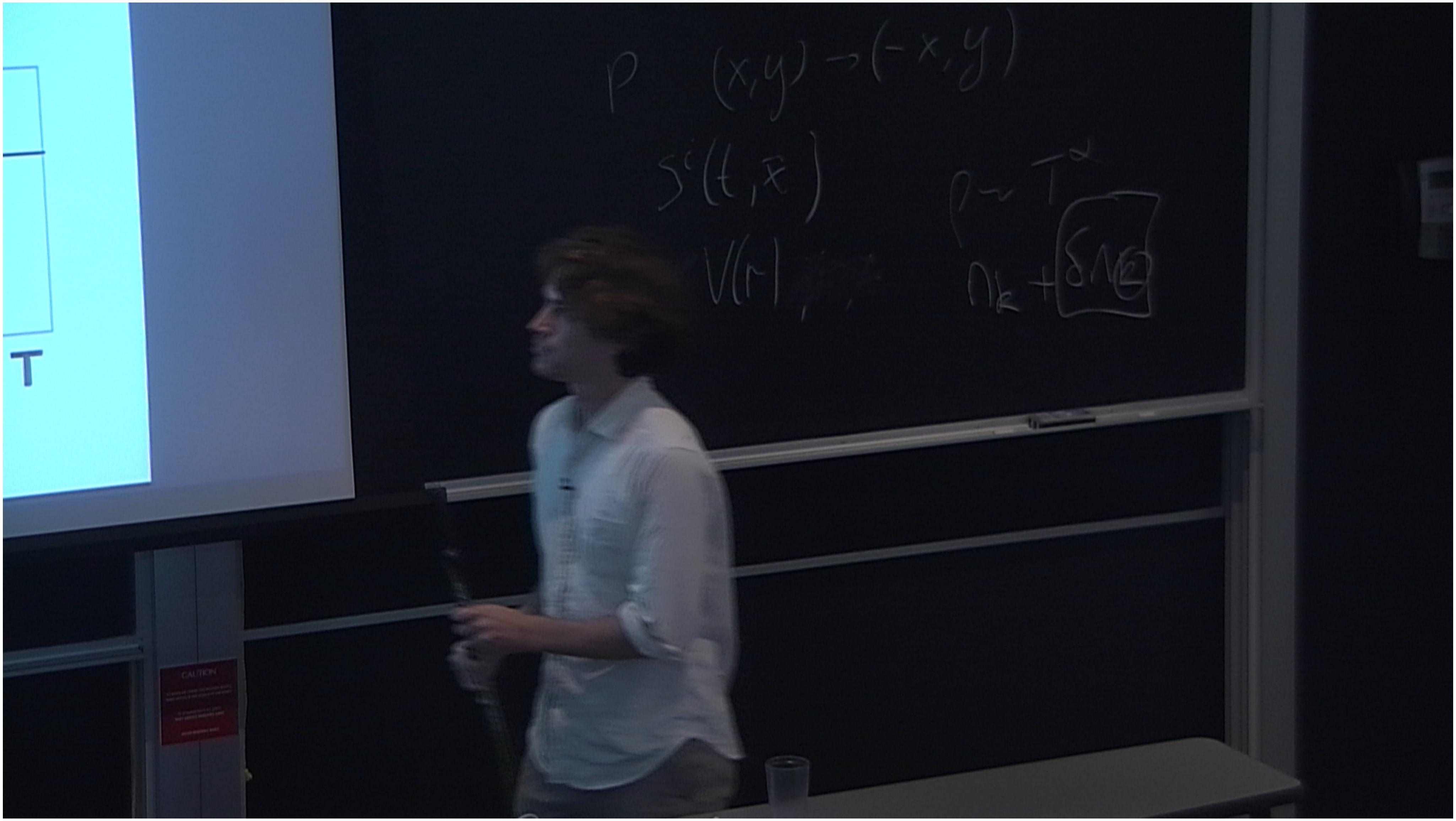
$$\sigma = \frac{ne^2\tau}{m}$$

$$\sim (k_F \ell)\, k_F^{d-2} \frac{e^2}{\hbar}$$

$$\gtrsim k_F^{d-2} \frac{e^2}{\hbar}$$

mean free path

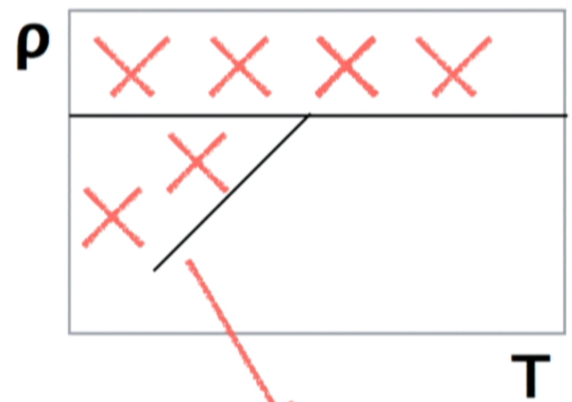
MIR bound



Quasiparticle transport

- **Lifetime instead of mean free path:**

$$\sigma = \frac{ne^2\tau}{m}$$
$$\sim \frac{\tau E_F}{\hbar} k_F^{d-2} \frac{e^2}{\hbar}$$
$$\gtrsim \frac{E_F}{k_B T} k_F^{d-2} \frac{e^2}{\hbar}$$



if qp have energy ~ k_BT.
uncertainty principle: $k_B T \tau \gtrsim \hbar$

Saturated by e-ph scattering above Debye scale.

Quasiparticle transport

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mean free path

MIR bound

Coherent metals

- ∃ long-lived quantity P s.t.

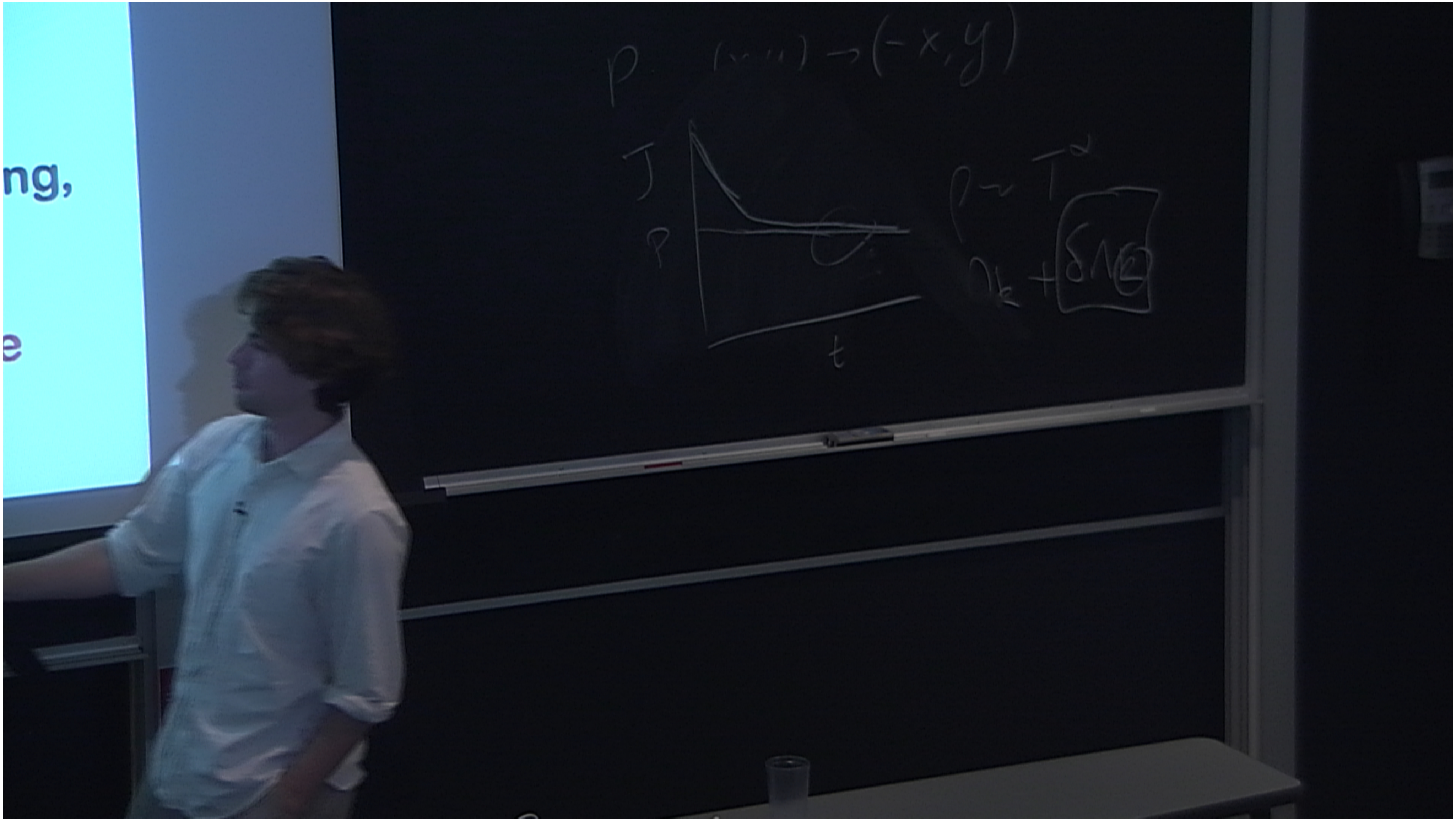
$$\chi_{JP} = i \int_0^\infty \langle [J(t), P(0)] \rangle \neq 0$$

i.e. $\langle P(t) \rangle \sim e^{-\Gamma t}, \qquad \Gamma \ll k_B T$

- E.g. P is the momentum. (momentum-conserving interactions strong, umklapp+disorder weak)
- Then:

$$\sigma = \frac{\chi_{JP}^2}{\chi_{PP}} \frac{1}{-i\omega + \Gamma}$$

Narrow Drude peak



Incoherent metals

- Nothing is long-lived that overlaps with the current J.
- Longest lived quantities are total energy and charge. Relate to currents:

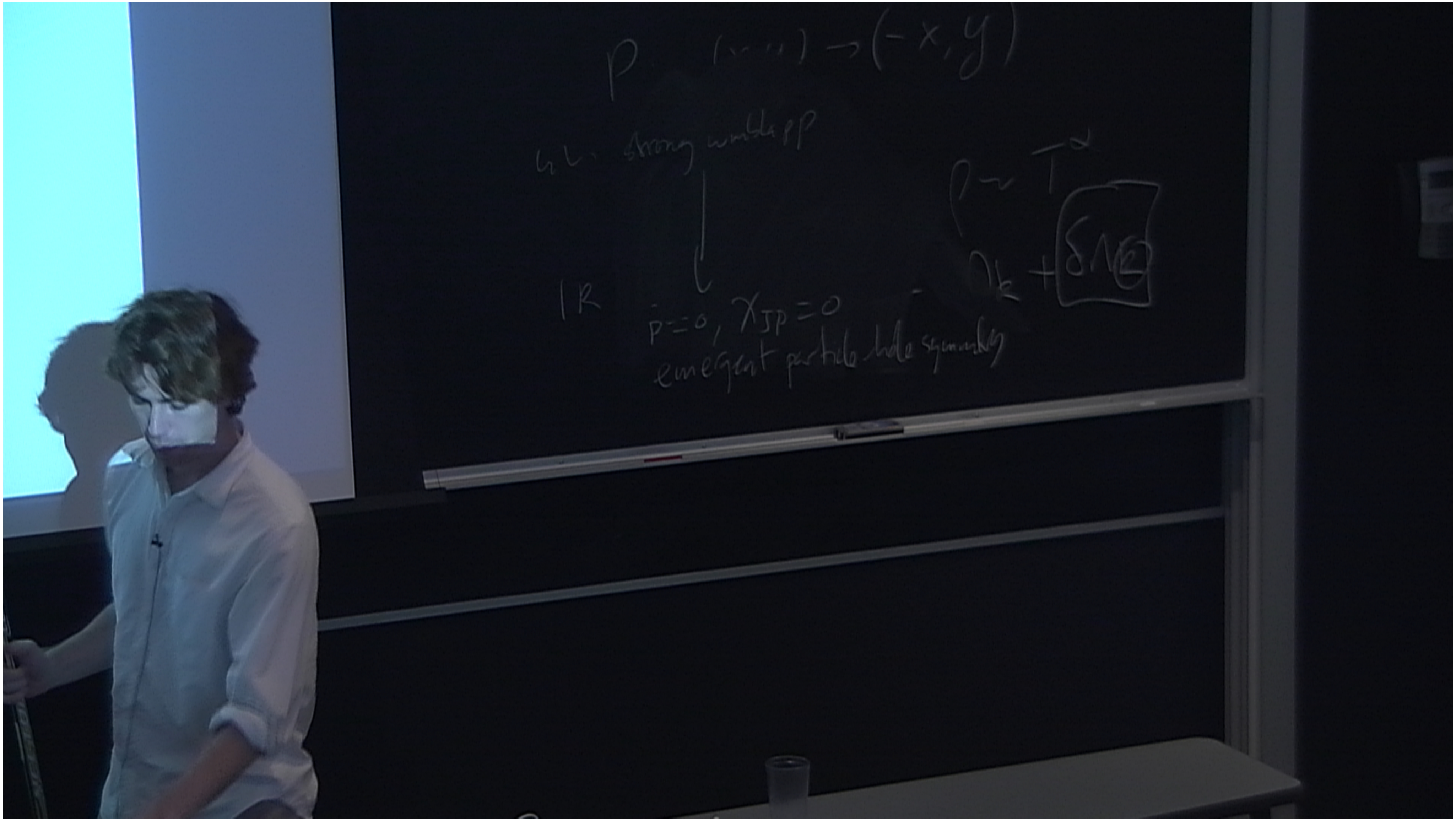
$$\frac{\partial n_A}{\partial t} + \nabla \cdot j_A = 0\,, \qquad (n_A = \{\epsilon, \rho\})$$

- Conductivities:

$$j_A = -\sigma_{AB} \nabla \mu_B\,, \qquad (\mu_A = \{T, \mu\})$$

- Susceptibilities:

$$\nabla n_A = \chi_{AB} \nabla \mu_B$$



emergent particle hole symmetry

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Incoherent metals

- **Implies diffusion:**

$$\frac{\partial n_A}{\partial t} = D_{AB} \nabla^2 n_B$$

- **With the Einstein relations:**

$$\sigma_{AB} = D_{AC} \chi_{CB}$$

- **In particular, diffusion rates:**

$$D_+ D_- = \frac{\sigma}{\chi} \frac{\kappa}{c},$$

$$D_+ + D_- = \frac{\sigma}{\chi} + \frac{\kappa}{c} + \frac{T(\xi\sigma - \chi\alpha)^2}{c\chi^2\sigma}.$$

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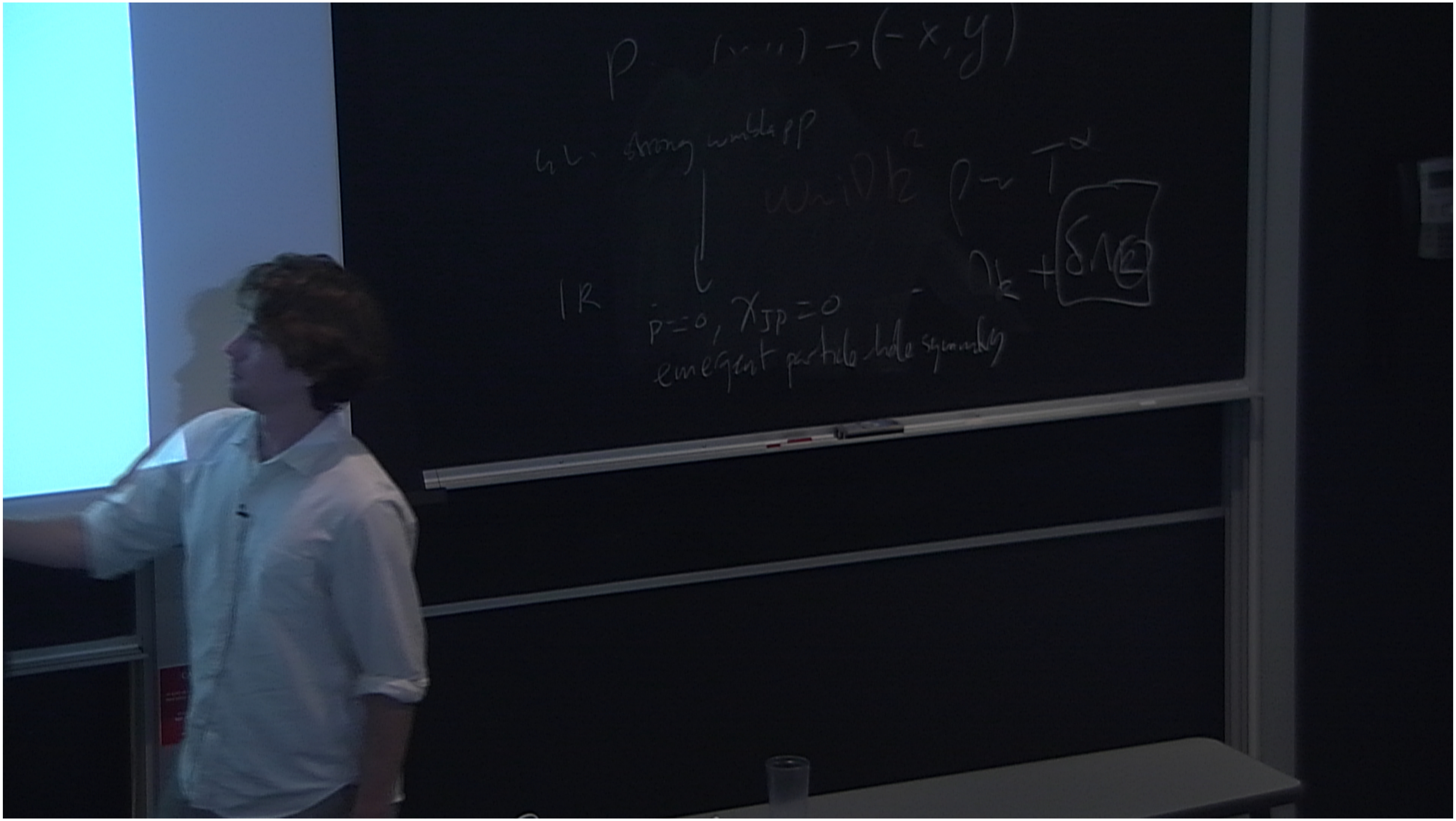
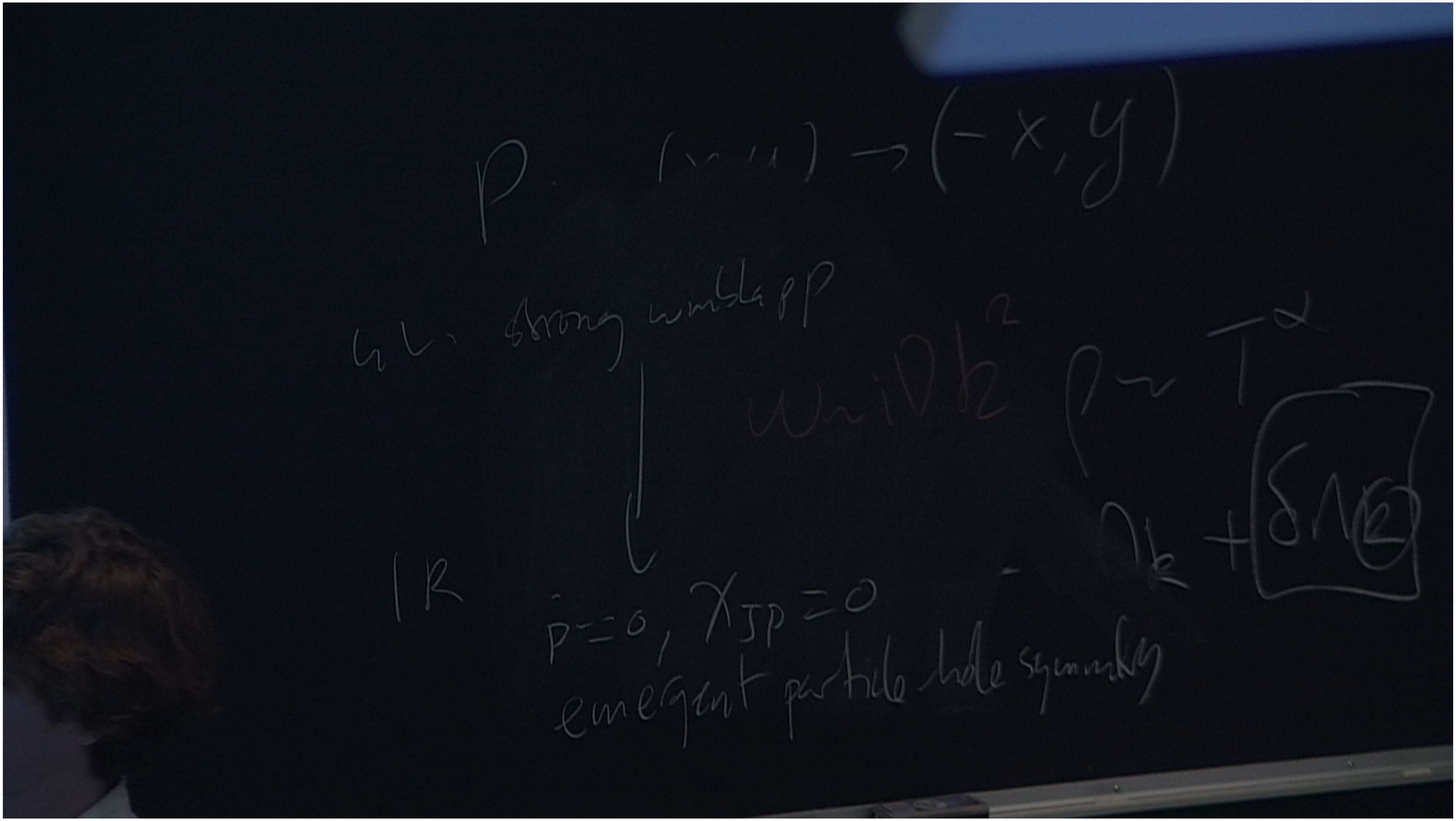
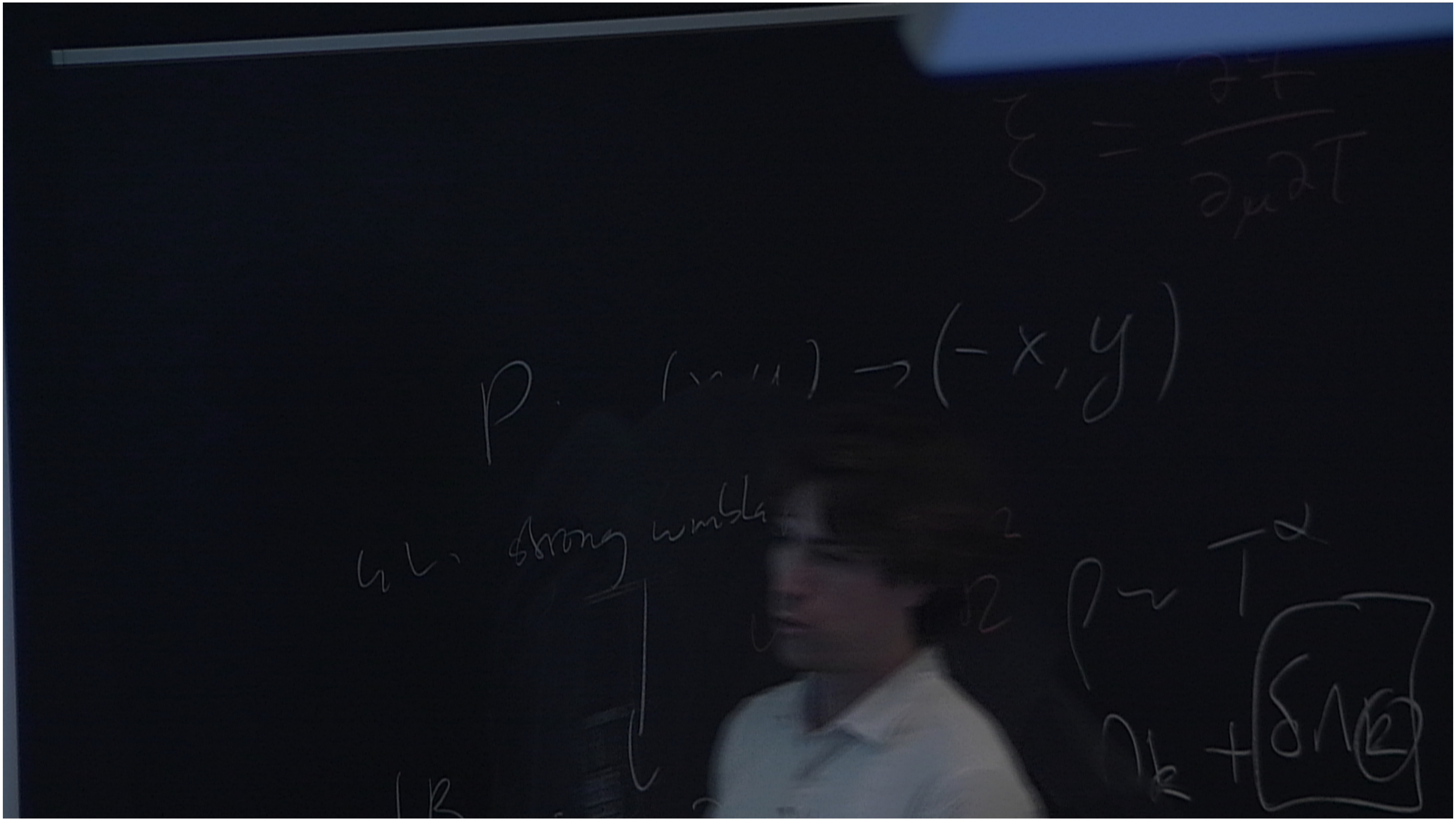
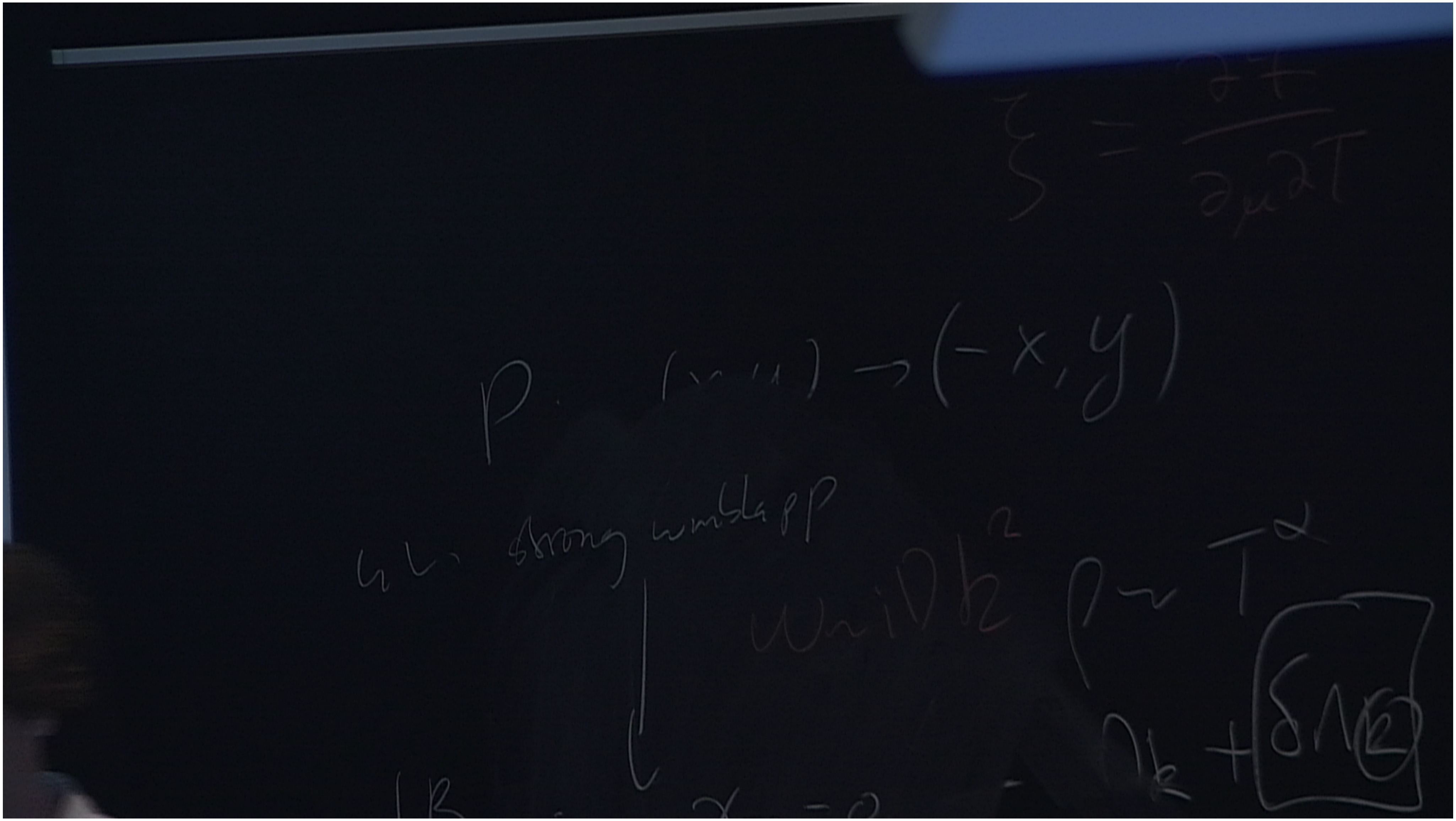

IR
emergent particle hole symmetry


(-x, y)
strong
IR
emergent particle hole symmetry





Incoherent metals

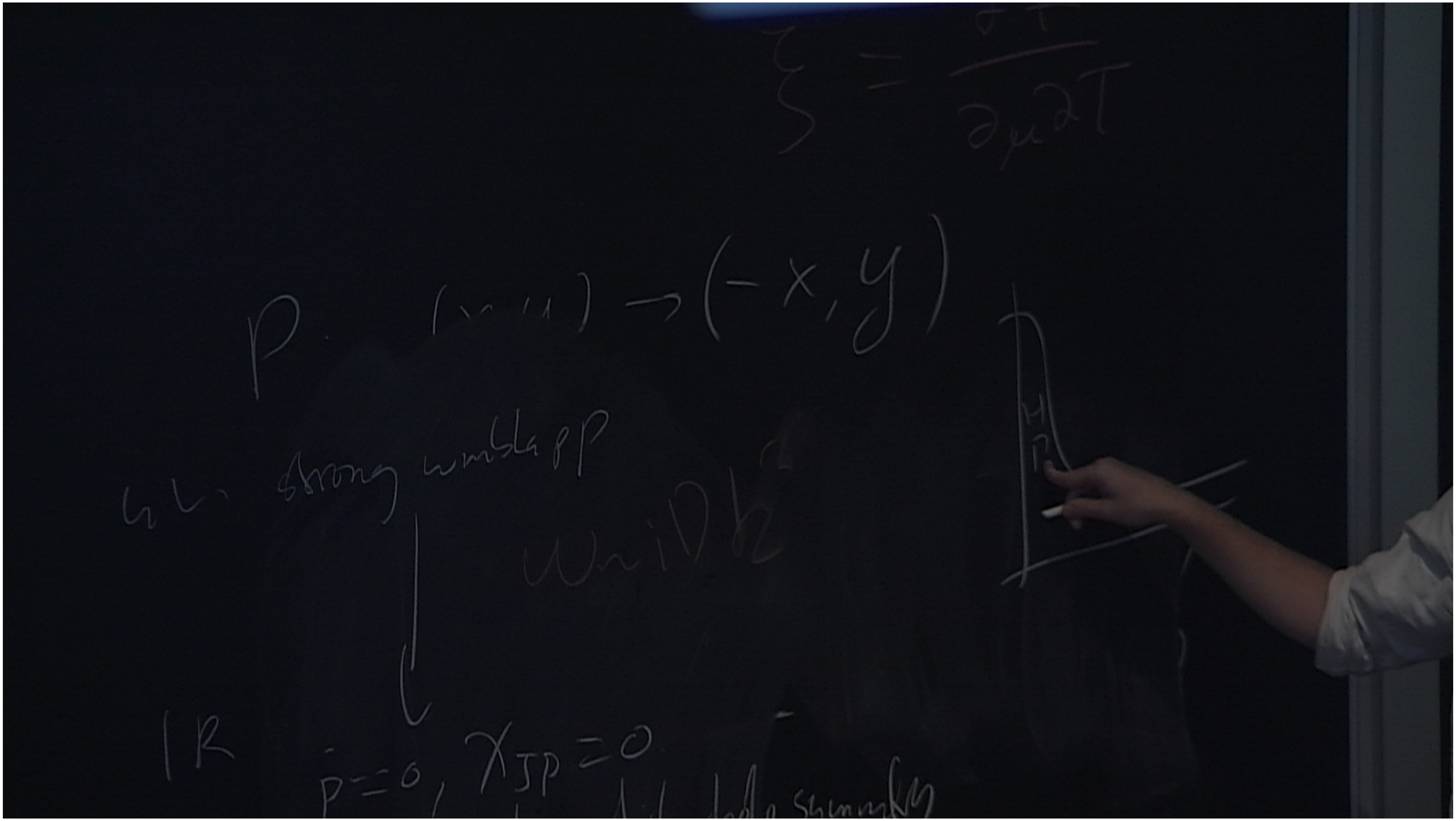
- **Dropping the 'thermoelectric' terms:**

$$\sigma = \chi D_+ \,,$$
$$\kappa = c D_- \,.$$

- **Unlike momentum relaxation, diffusion is a process that is intrinsic to the system.**

- **Might the D's be fundamentally bounded? e.g. with quasiparticles:**

$$D \sim v_F^2 \tau \gtrsim \frac{v_F^2 \hbar}{k_B T}$$



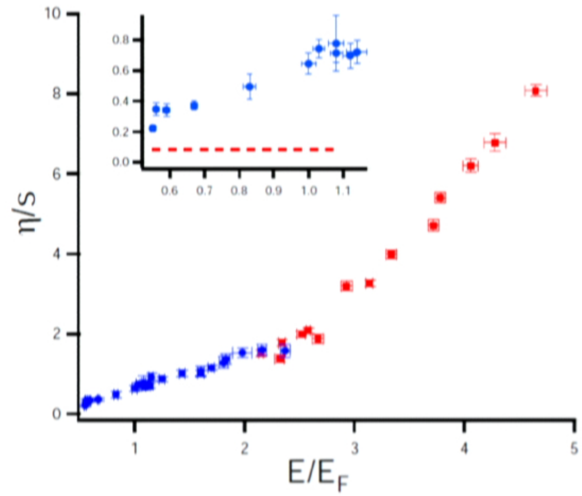
Universal bounds?

- Kovtun-Son-Starinets:

$$\frac{\eta}{s} \sim \frac{\epsilon\tau}{k_B n} \gtrsim \frac{\hbar}{k_B}$$

- They proposed that this bound continued to hold in the absence of quasiparticles.
- Evidence: (i) holographic models without quasiparticles and (ii) data on strongly interacting systems are consistent with a bound.

- **'Cold' Fermi gas at unitarity:**



- **Quark-gluon plasma close to Tc:**

$$0.1 \lesssim \frac{\eta}{s} \lesssim 0.4$$

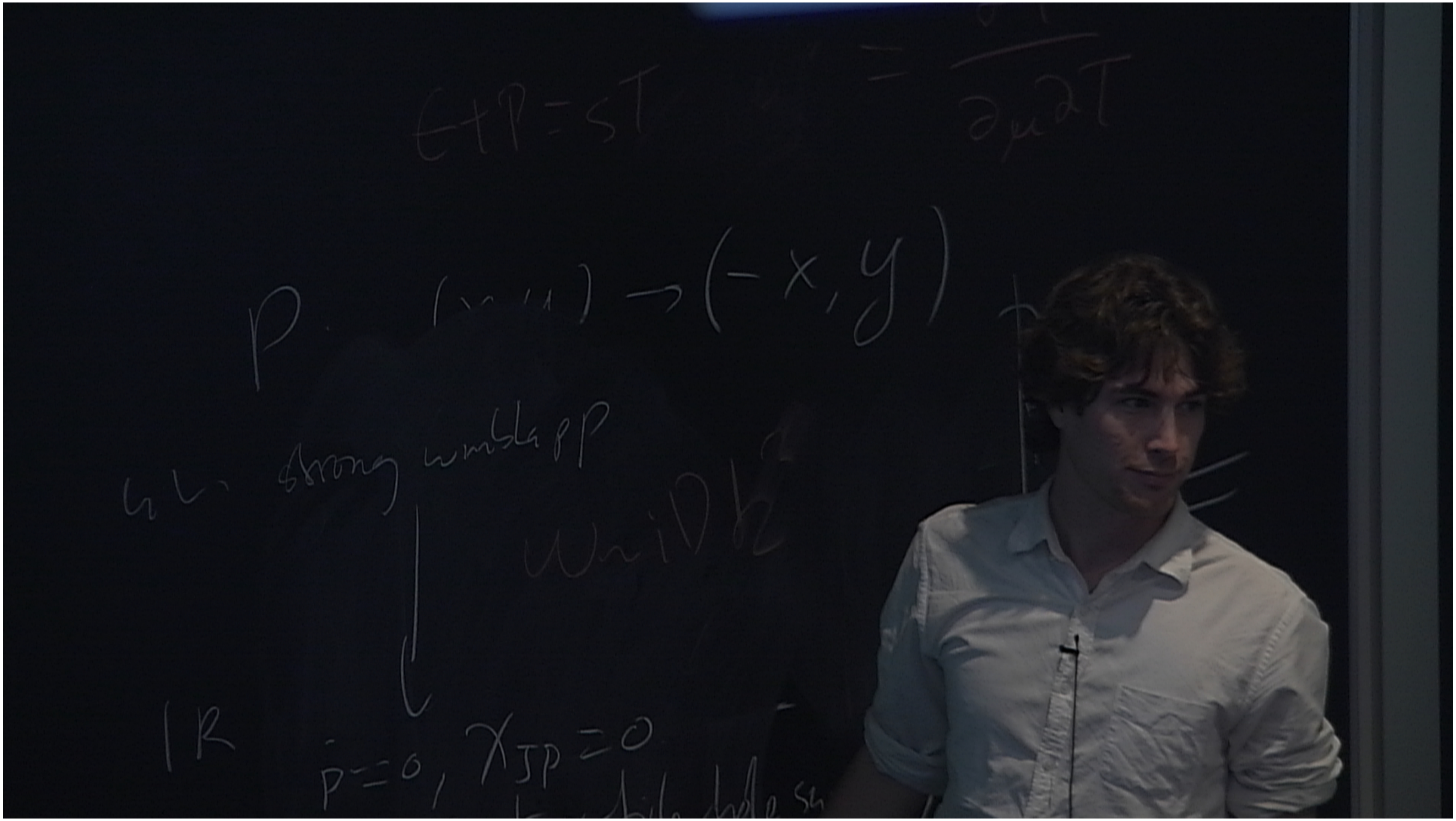
Universal bounds?

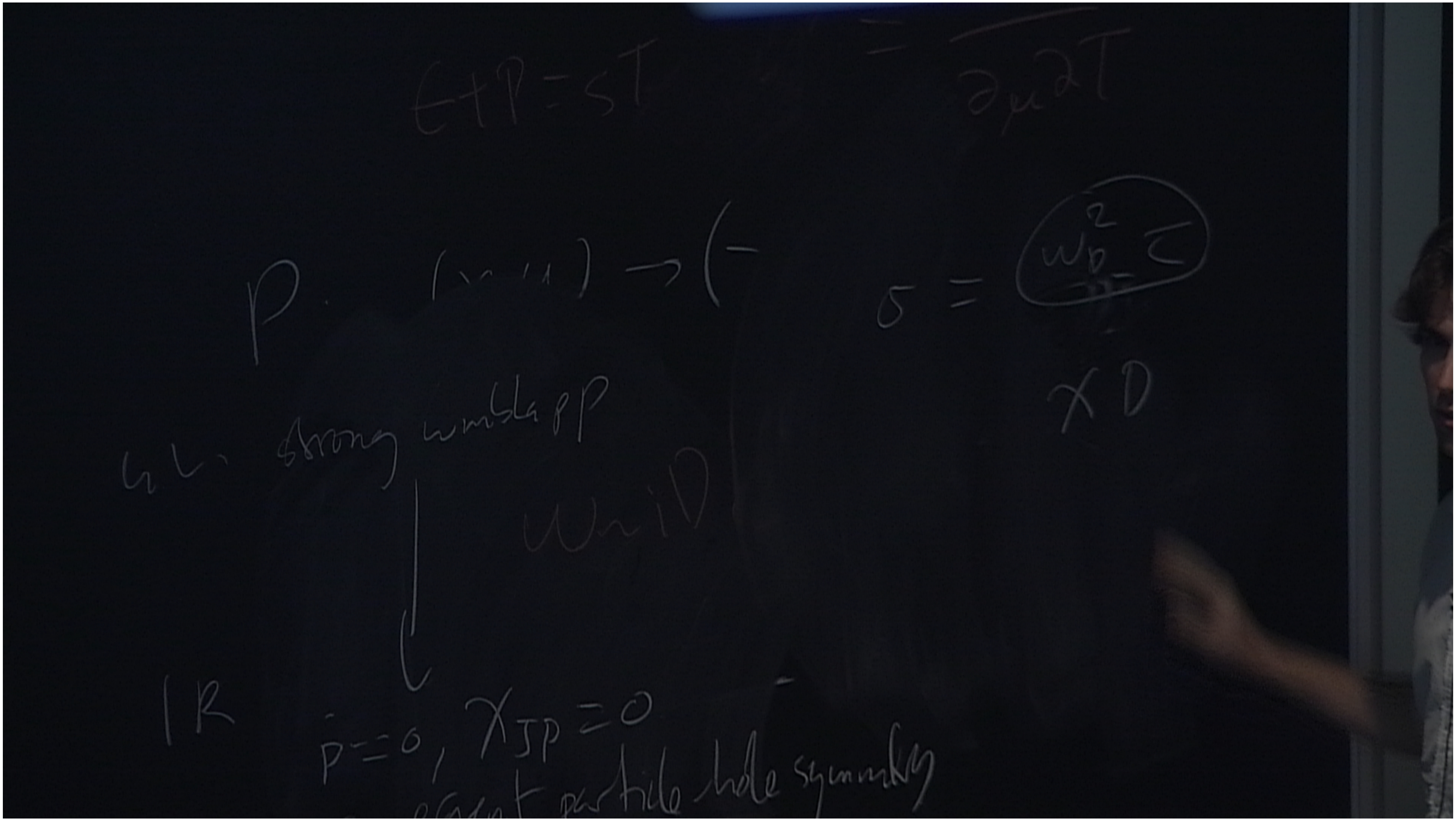
- The shear viscosity quantifies diffusion of momentum.
- In a zero density, relativistic plasma:

$$\frac{\eta}{s} = \frac{DT}{c^2} \quad \Rightarrow \quad D \gtrsim \frac{\hbar c^2}{k_B T}$$

- Could there be a similar bound for charge and energy diffusion in metals?
- The qp bound in metals suggests: $c \to v_F$

$$D_{\pm} \gtrsim \frac{\hbar v_F^2}{k_B T}$$


→ (−x, y)
IR


ε+P = sT
σ =
χ D
strong
IR
particle hole symmetry

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$$\boxed{D_{\pm} \gtrsim \frac{\hbar v_F^2}{k_B T}}$$

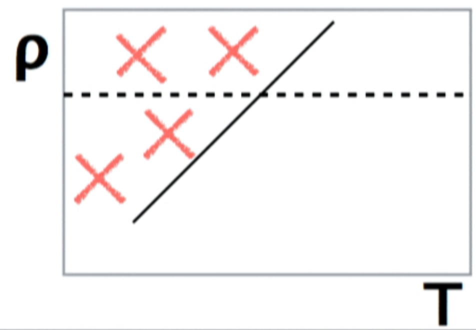
Phenomenology (i)

- A system approximately saturating the bound will have:

$$\rho \sim \frac{1}{\chi D} \sim \frac{\hbar}{k_F^{d-2} E_F} \frac{k_B T}{e^2} \,, \qquad (\text{with } \chi \sim e^2 k_F^d / E_F)$$

- Linear resistivity. If analyzed à la Bruin et al. would give the measured: $\tau \sim \hbar/(k_B T)$

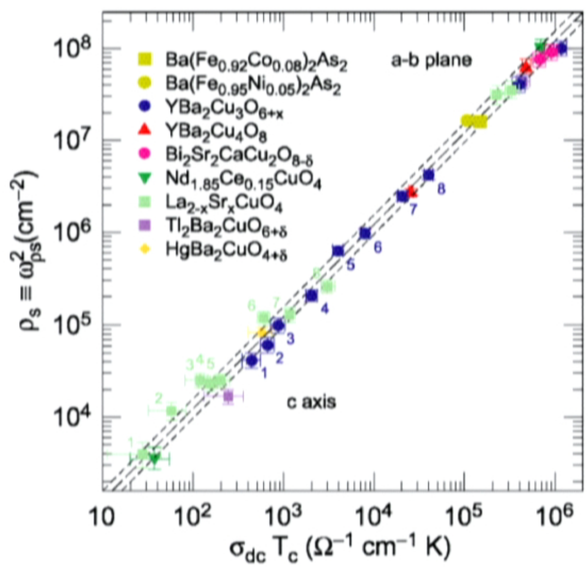
- Can cross MIR bound:



Phenomenology (ii)

- **Slope of linear resistivity matches data:**

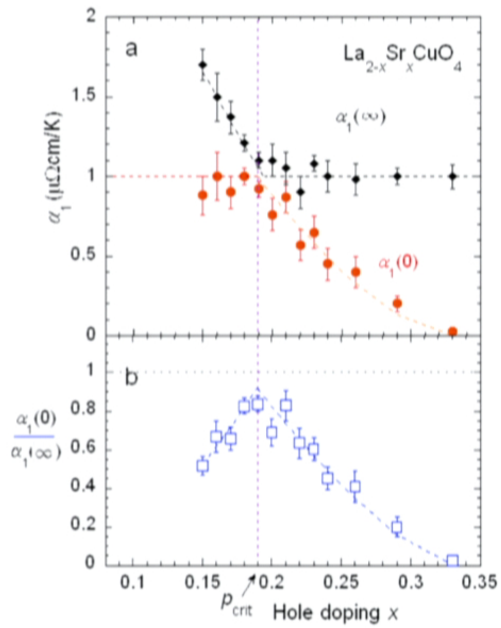
$$\rho_s = \frac{e^2 n_s}{m} \sim k_F^{d-2} E_F \frac{e^2}{\hbar^2} \sim \sigma(T_c) T_c \frac{k_B}{\hbar}$$



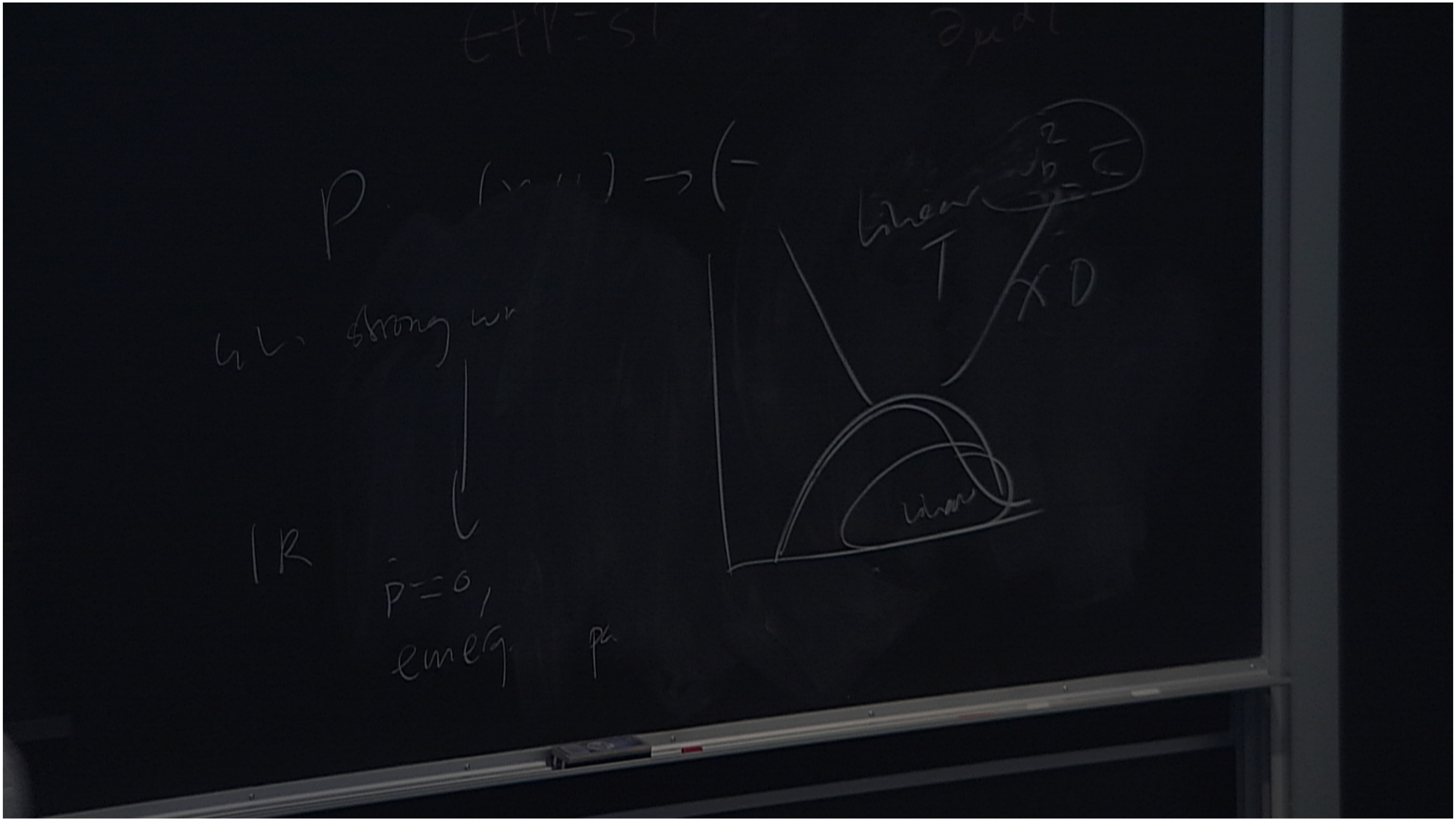
- **Homes et al. '04**
- **Wu et al. '10**
- **Zaanen '04**

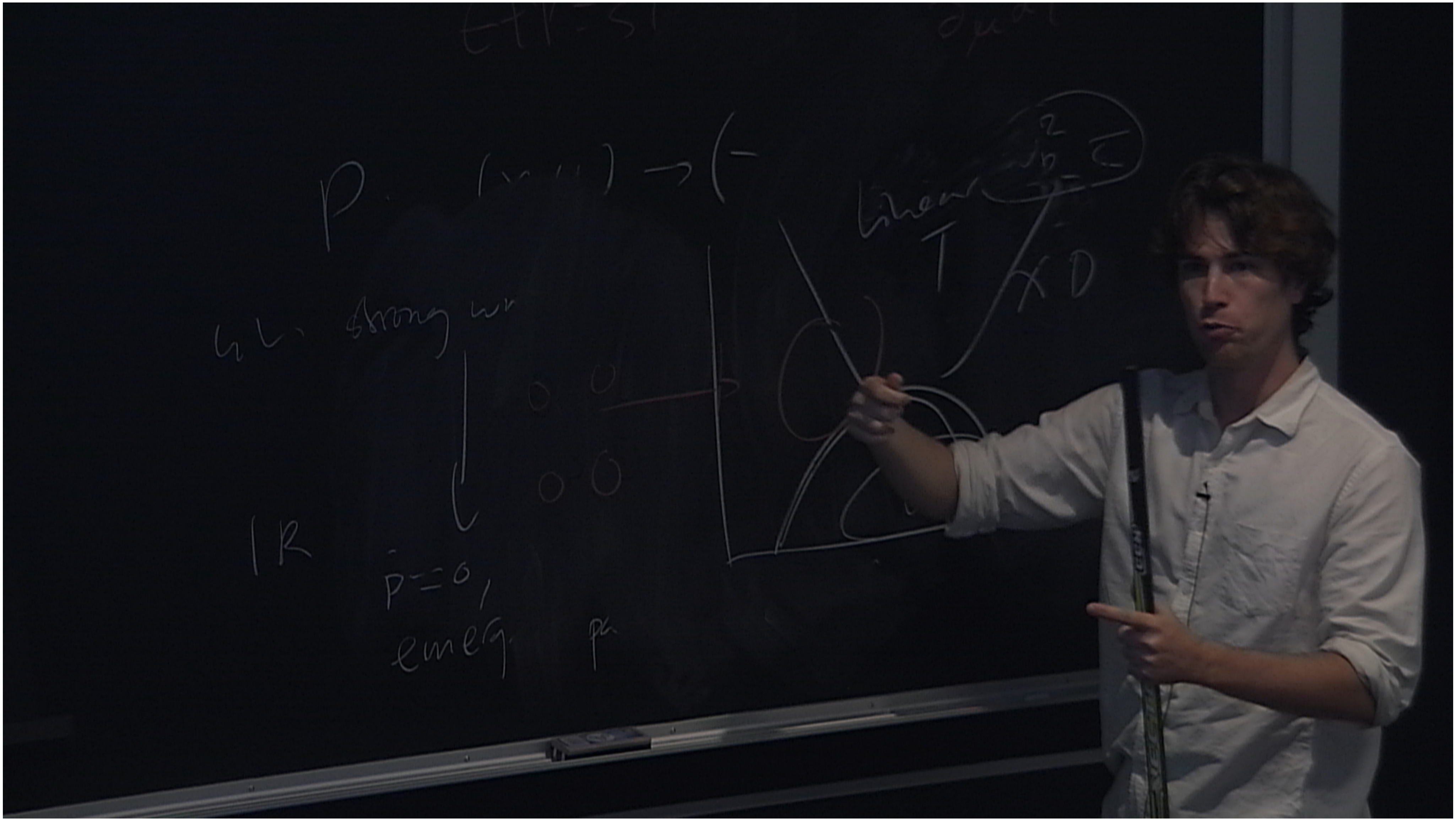
Phenomenology (iii)

- **Slope** of linear resistivity ~ $1/E_F$ agrees with trend in underdoped cuprates as a function of doping:



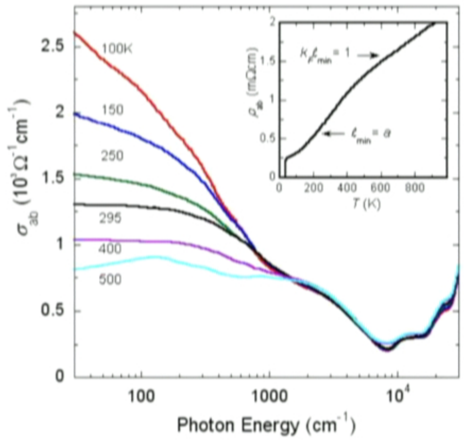
- Hussey et al. '09


strong
linear
XD
IR

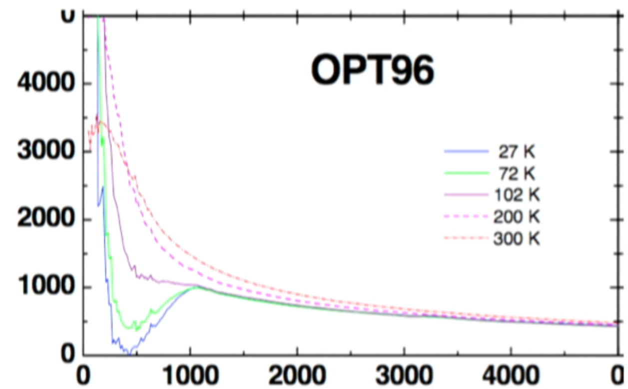


Phenomenology (iv)

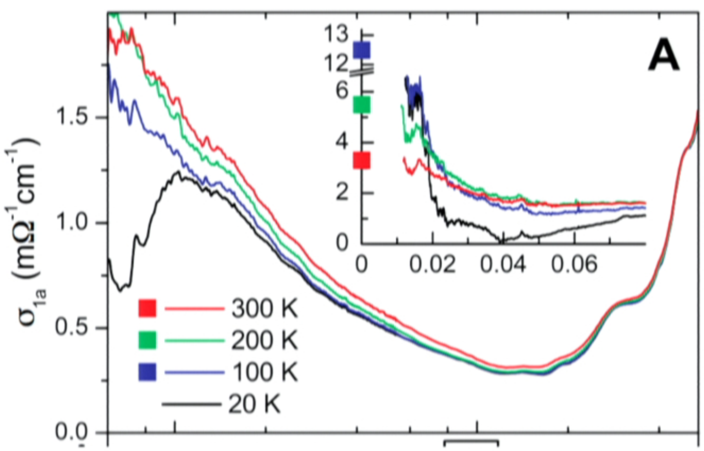
- Optical conductivities give a direct probe of the Drude peak and hence of (in)coherence.
- All materials in which I am aware of measurements show the onset of incoherence at increasing temperature in T-linear resistivity regimes…



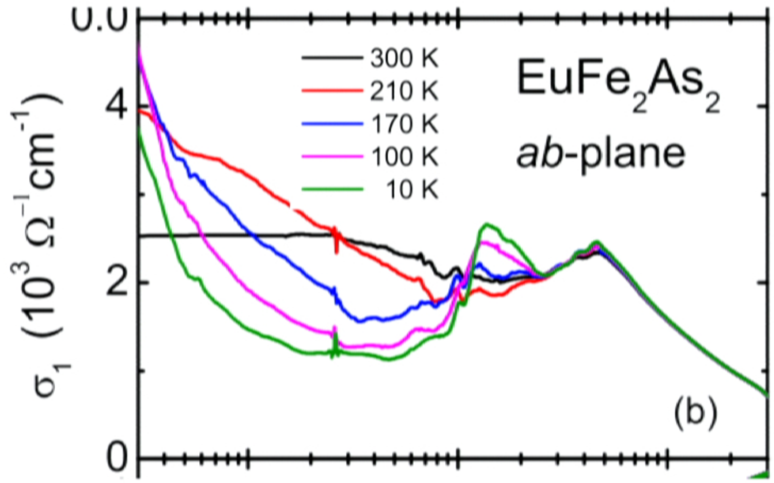
- LSCO, Takenaka et al. ‘03



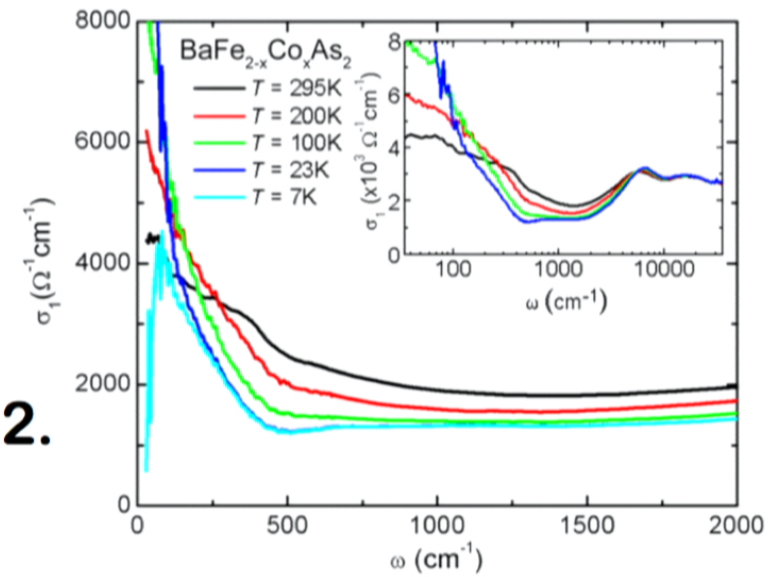
- BSCCO, Hwang et al. ‘07



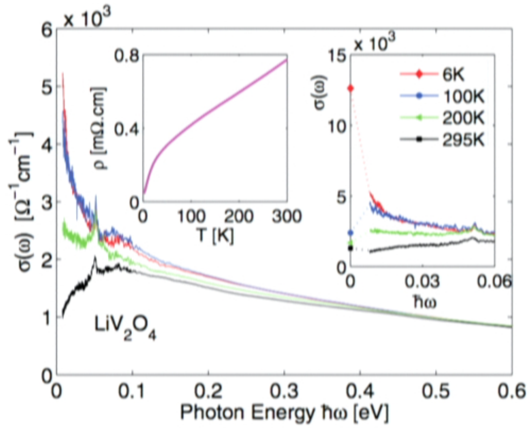
- YBCO, Boris et al. ‘04



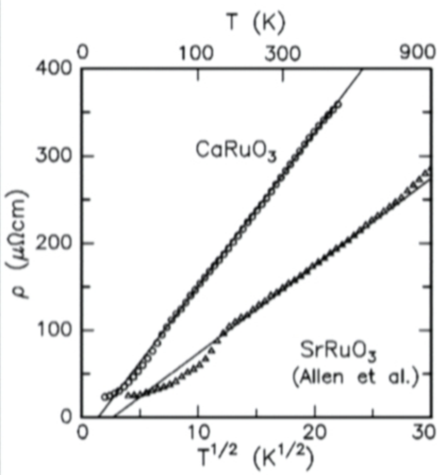
- **Wu et al. '09.**

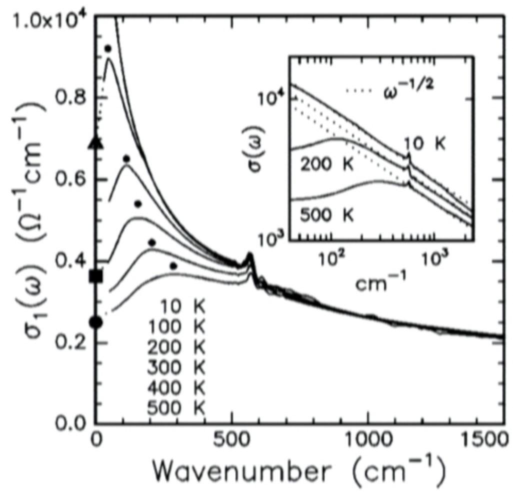


- **Schafgans et al. '12.**

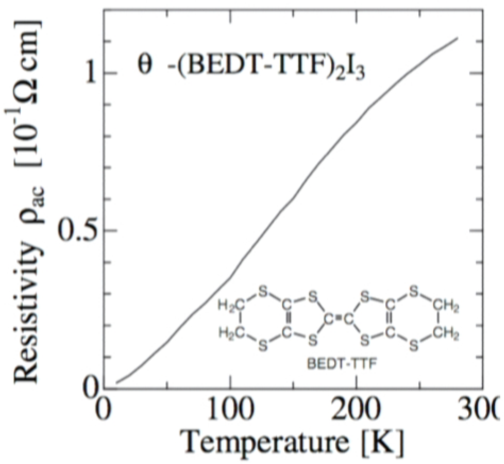


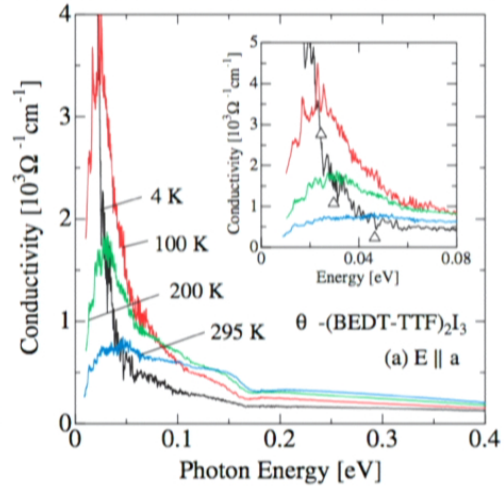
- **Jonsson et al. '07.**



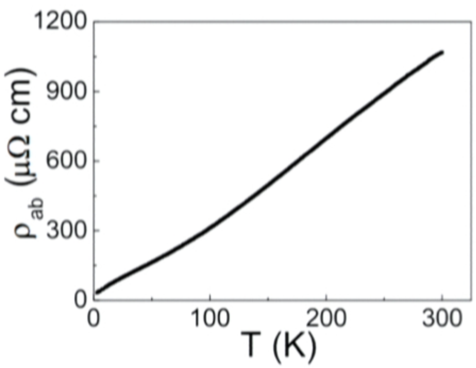


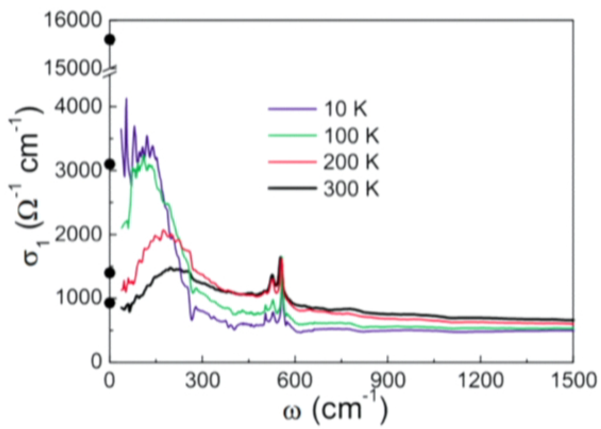
- **Lee et al. '02.**





- **Takenaka et al. '05.**





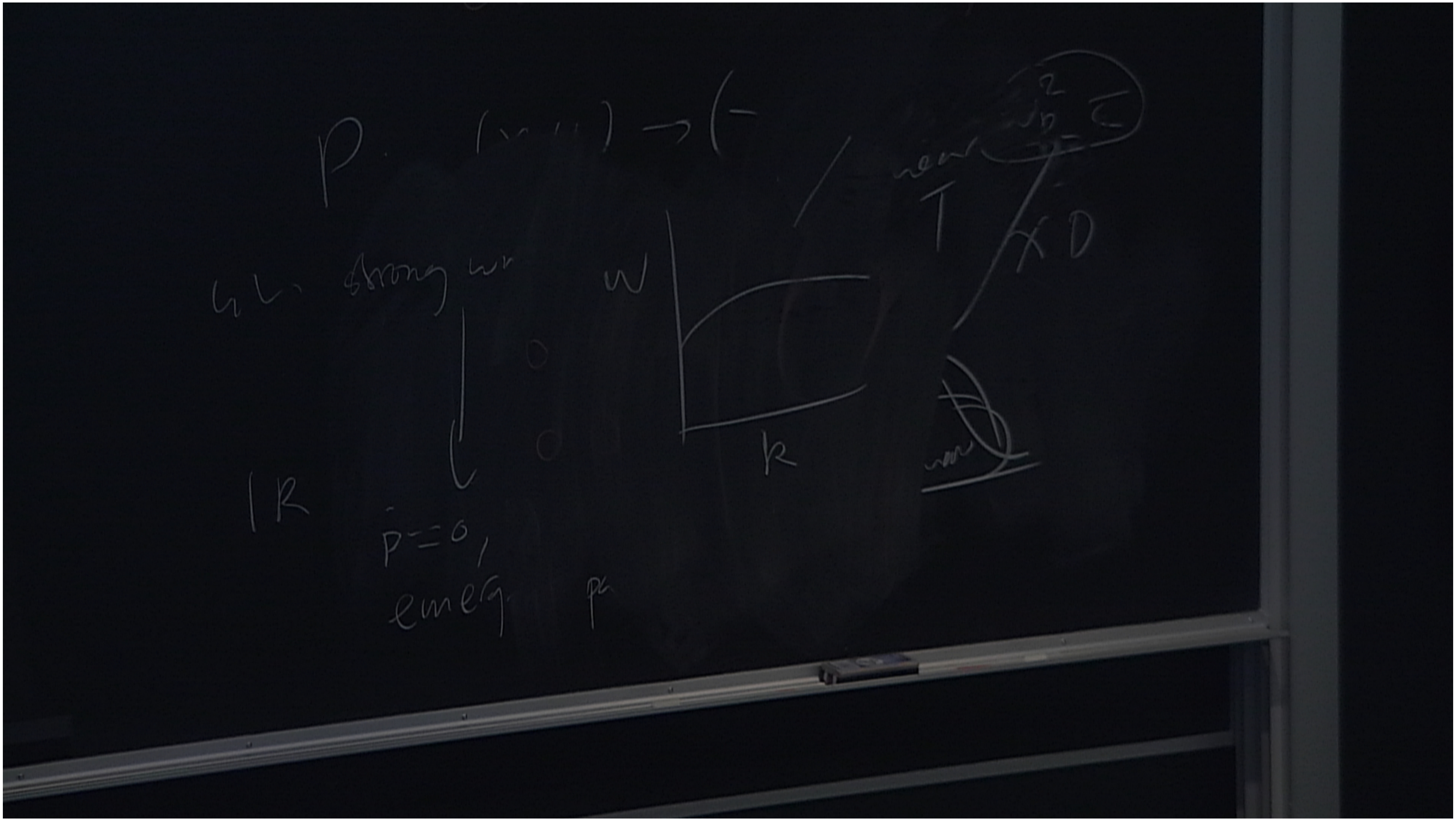
- **Wang et al. '04.**

$Na_{0.7}CoO_2$

Incoherence vs. phonons

- Electron-phonon-type scattering above a 'Debye' scale mimics many features of incoherent transport.
- However:
  (i) e-ph scattering cannot cross MIR bound.
  (ii) Above Debye scale, elastic scattering, and hence the Wiedemann-Franz law:

$$\frac{\kappa}{\sigma T} = \frac{\pi^2}{3} \frac{k_B^2}{e^2}$$


strong
IR
k

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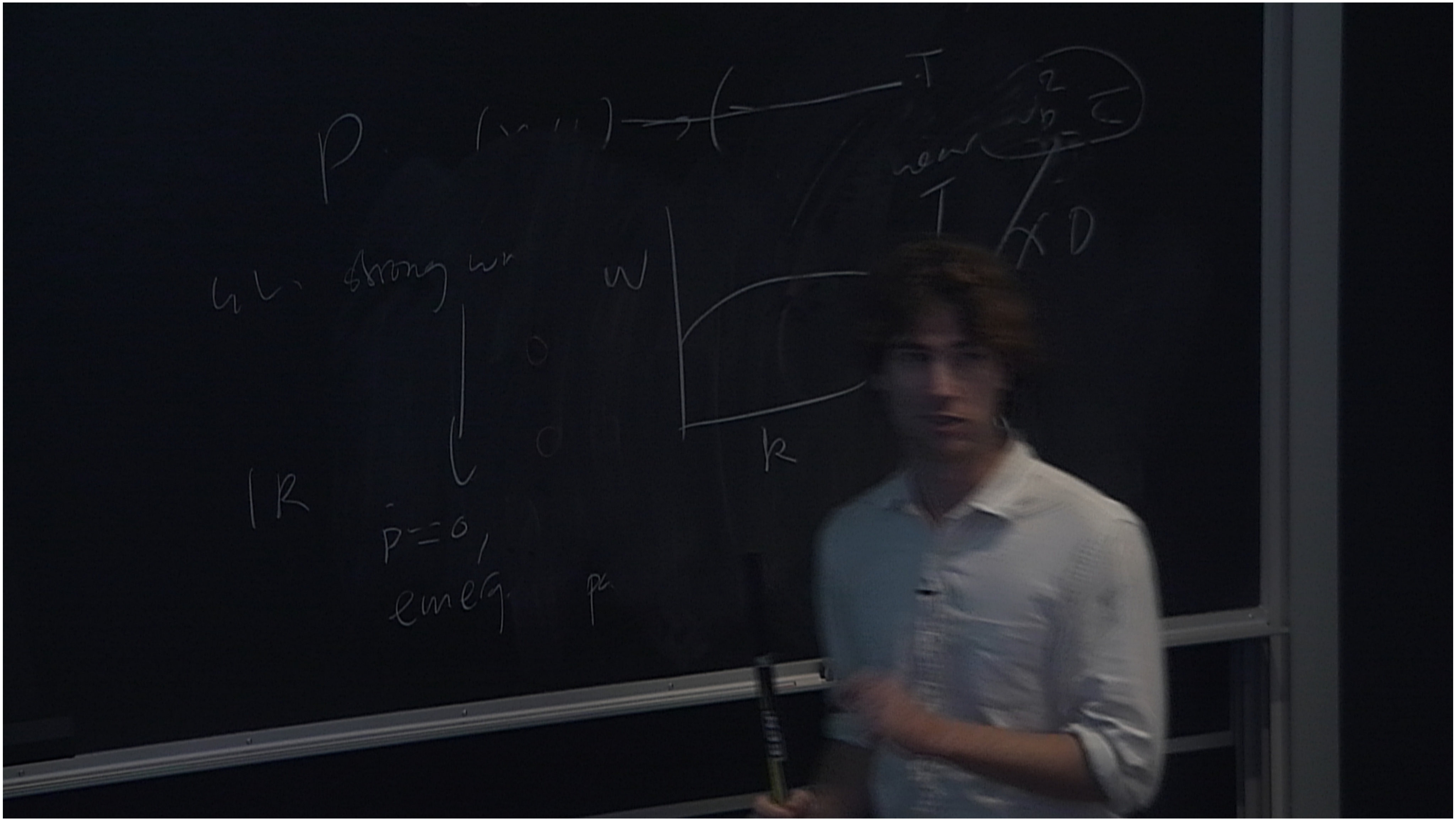
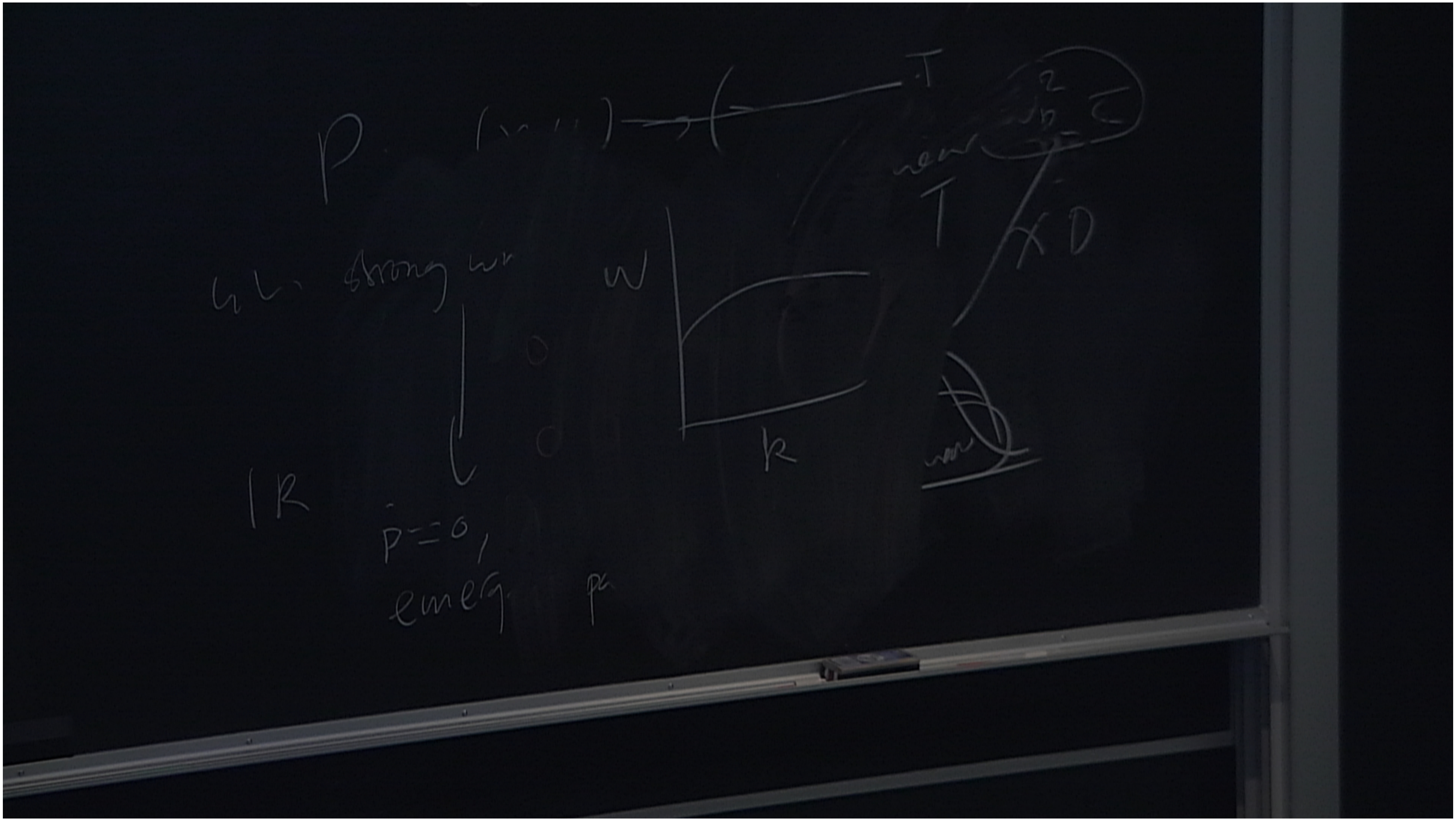


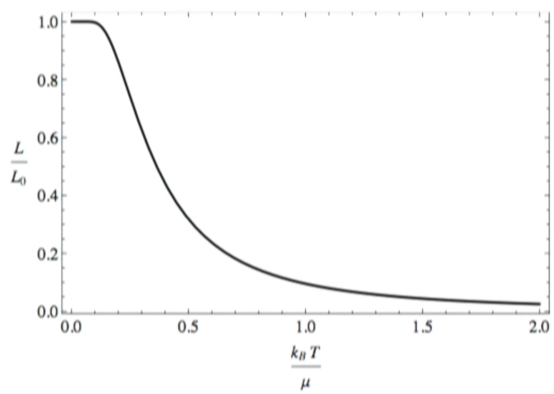


Incoherence vs. phonons

- **In an incoherent metal:** $\frac{\kappa}{\sigma T} = \frac{cD_+}{T\chi D_-} \sim \frac{c}{T\chi}$

- **Rough estimate using FD distribution:**

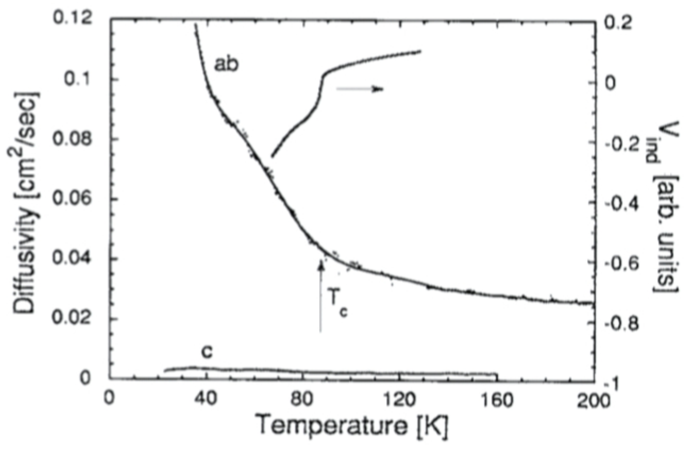
$$f = g_o T \int_0^{E_B} d\epsilon \log\left(1 + \epsilon^{-(\epsilon-\mu)/T}\right)$$



- **Measurements in YBCO and heavy fermions indeed show $L/L_0 < 1$.**

Looking forward (i)

- Direct measurements of the diffusion constants can distinguish different scenarios and potential falsify the bound.
- An old measurement of thermal diffusivity in BSCCO exists:



- Wu et al. '93. BSCCO
- Compatible with bounds once phonons subtracted.

Looking forward (ii)

- Can theoretical or experimental counterexamples be found?
- Can the role of v_F be clarified?
- The effects of 'thermoelectric' conductivities should be included more carefully.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ounded?
CAUTION