Title: Effective field theory of two-dimensional nonrelativistic chiral superfluid

Date: May 27, 2014 10:30 AM

URL: http://pirsa.org/14050101

Abstract: Due to the current search of Majorana fermions, the physics of two-dimensional identical fermions with short-range p-wave interactions is of considerable interest. My talk will be about the effective theory of a chiral p+ip fermionic superfluid at zero temperature. This theory naturally incorporates the parity and time reversal violating effects such as the Hall viscosity and the edge current. I will present some applications of this theory such as the linear response to external electromagnetic and gravitational fields and the density profile of an isolated vortex. Finally, the dual gauge reformulation of this theory will be presented.

Effective field theory of planar chiral superfluids

Sergej Moroz
University of Washington

Outline

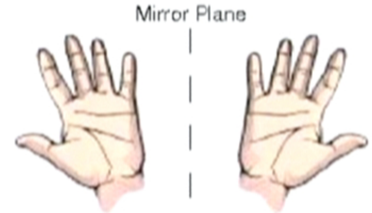
- Chiral superfluids in 2d
- Hall viscosity and edge currents
- Linear response and vortex solution
- Dual gauge theory

together with Carlos Hoyos and Dam Thanh Son

Superfluids

- T=0 state of a neutral many-body system
- No dissipation, quantum vortices, ...
- Old: ^4He and ^3He
- New: Bose and Fermi ultracold atoms

Chiral 2d superfluid



Spinless fermions in flatland:

- Chiral condensate $\Delta_{\mathbf{p}} = (p_x \pm i p_y)\hat{\Delta}$ preferred
- Topological phase transition at $\mu = 0$
- Chiral Majorana mode on boundaries
- Toy model for a film of ^{3}He
- Moore-Read $\nu = 5/2$ QH state *Volovik, Read, Green,...*

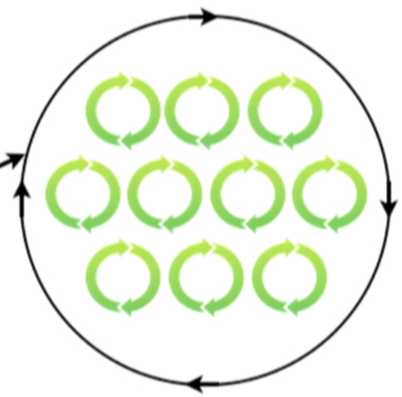
Symmetry breaking

- Chiral condensate $\Delta_{\mathbf{p}} = (p_x \pm ip_y)\hat{\Delta}$
- SSB pattern: $U(1)_N \times SO(2)_V \to U(1)_D$
- Single gapless Goldstone mode
- Breaks parity and time reversal!

Edge current

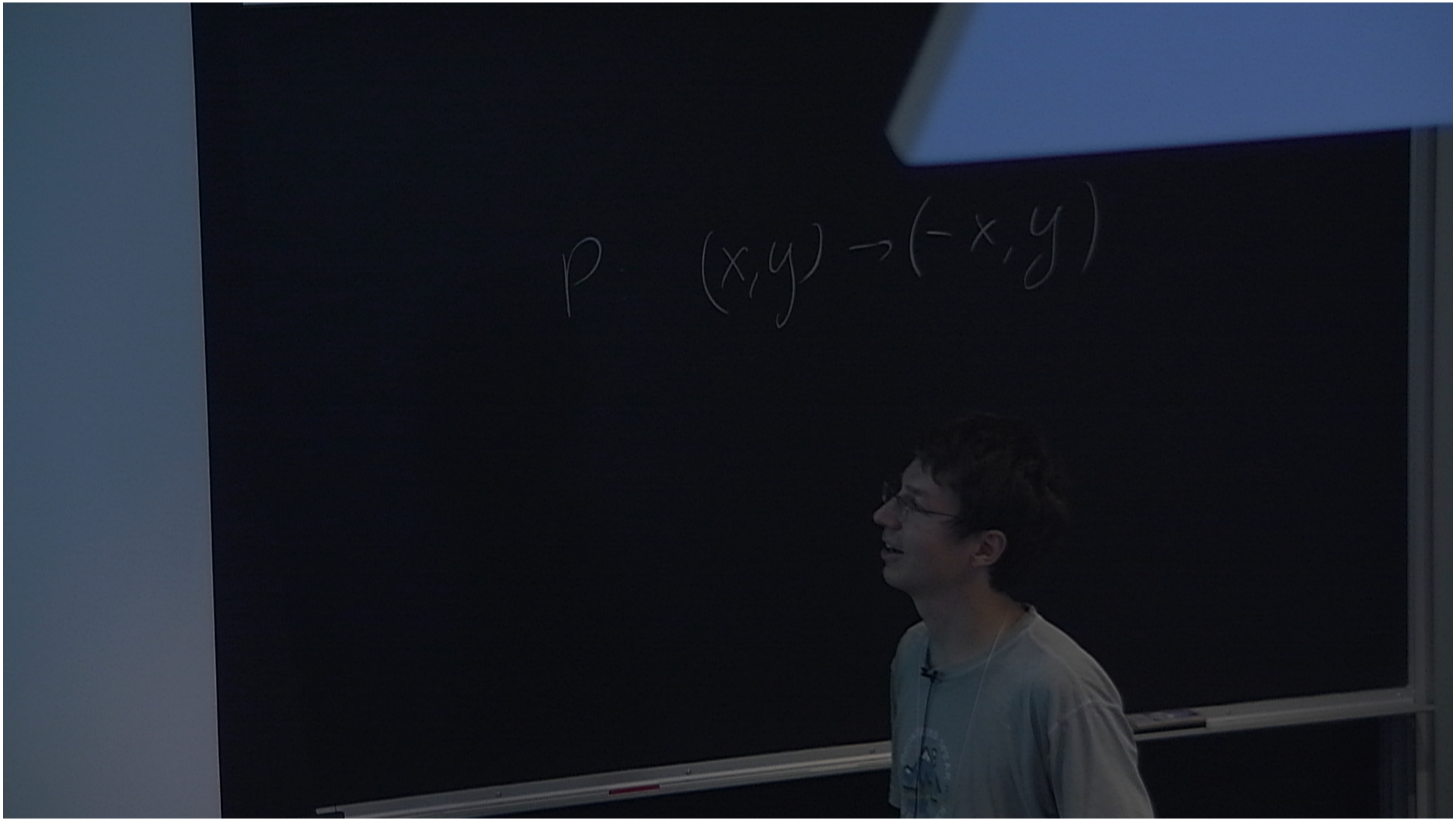
- Chiral ground state rotates

edge particle current

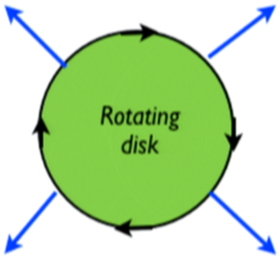


- Angular momentum of p+ip superfluid

$$L_{\text{GS}} = \int d^2x \epsilon_{kl} x^k J^l = 1/2 \underbrace{\int d^2x \rho}_{N}$$


P (x,y) → (-x,y)

Hall viscosity



- Specific to 2d with broken P and T *Avron, Seiler, Zograf*
- Non-dissipative effect

$$T^{ij}_{\mathsf{Hall}} = -\eta_{\mathsf{H}}(\varepsilon^{ik} g^{jl} + \varepsilon^{jk} g^{il}) V_{kl}$$

$$f^{i}_{\mathsf{Hall}} = \eta_{\mathsf{H}} \epsilon^{ij} \Delta v_j$$

- Counts internal angular momentum density

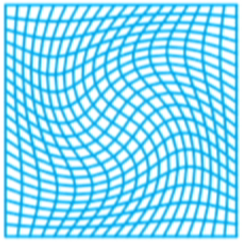
$$\eta_{\mathsf{H}} \sim \hbar / l^2$$

Read

Galilean-invariant examples:
IQHE, FQHE, p+ip SF

General coordinate invariance

Son, Wingate



- Put superfluid into curved space and turn on electromagnetic source
- Under diffeomorphism

Goldstone field
$\theta = \mu t - \varphi$

$$\delta\theta = -\xi^k \partial_k \theta$$

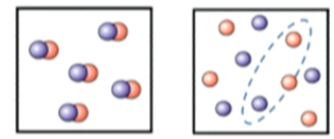
$$\delta A_t = -\xi^k \partial_k A_t - A_k \dot{\xi}^k$$

$$\delta A_i = -\xi^k \partial_k A_i - A_k \partial_i \xi^k + g_{ik} \dot{\xi}^k$$

$$\delta g_{ij} = -\xi^k \partial_k g_{ij} - g_{ik} \partial_j \xi^k - g_{kj} \partial_i \xi^k$$

- Generalizes Galilean transformation

s-wave superfluid



Popov; Greiter&Wilczek&Witten; Son&Wingate

$$S[\theta] = \int dt d\mathbf{x} \sqrt{g} P\left(X\right)$$

pressure

$D_\nu \theta \equiv \partial_\nu \theta - A_\nu$

GCT invariant:

$$X = D_t\theta - \frac{g^{ij}}{2} D_i\theta D_j\theta$$

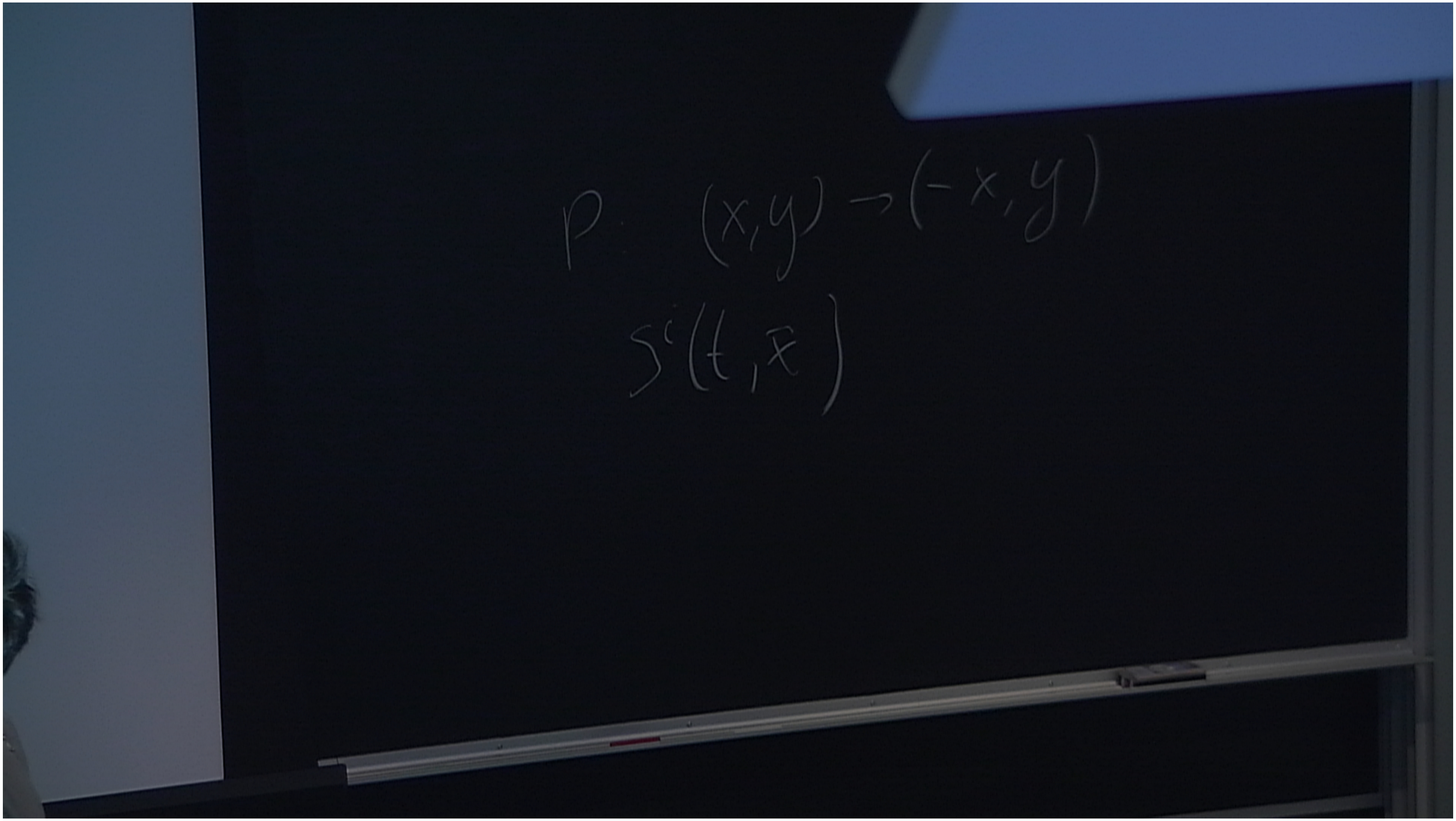
- Ideal superfluid hydrodynamics with

$$\rho \equiv dP/dX \qquad v_j \equiv -D_j\theta$$

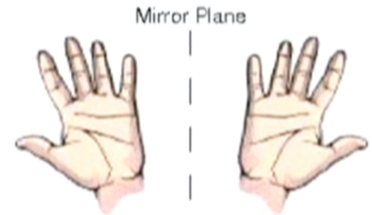
- Leading order in power-counting

$$\partial_\nu \theta \sim A_\nu \sim g_{ij} \sim O(1) \qquad [\partial_\nu O] = 1 + [O]$$

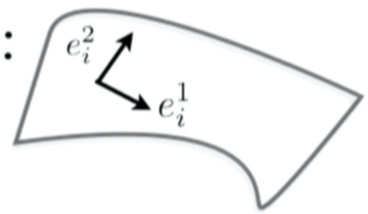
- Nonlinear in Goldstones $\quad [(\partial\theta)^n] = n[\partial\theta] = 0$


$P: (x,y) \to (-x,y)$
$S^i(t,\bar{x})$

Chiral superfluid



- New gauge field needed $U(1)_N \times SO(2)_V \to U(1)_V$
- Orthonormal spatial vielbein:



- Spin connection:

$$\omega_t \equiv \frac{1}{2}\left(\epsilon^{ab} e^{aj} \partial_t e_j^b + B\right)$$

$$\omega_i \equiv \frac{1}{2}\epsilon^{ab} e^{aj} \nabla_i e_j^b = \frac{1}{2}\left(\epsilon^{ab} e^{aj} \partial_i e_j^b - \varepsilon^{jk} \partial_j g_{ik}\right)$$

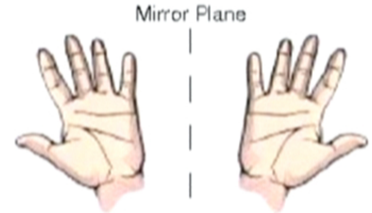
$SO(2)_V$: gauge field
diffeo: one-form

- Just introduce new covariant derivative

$$\boxed{D_\nu \theta \equiv \partial_\nu \theta - A_\nu - s\omega_\nu}$$

for p+ip SF
$s = 1/2$

Chiral superfluid



- U(1) current:

$$J^i \equiv -\frac{1}{\sqrt{g}}\frac{\delta S}{\delta A_i} = \underbrace{\rho g^{ij} v_j}_{\text{convective}} + \underbrace{\frac{s}{2}\varepsilon^{ij}\partial_j \rho}_{\text{edge}}$$

- Stress tensor:

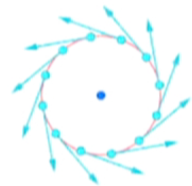
$$\Delta T^{ij}_{\mathsf{ch}} \equiv \frac{2}{\sqrt{g}}\frac{\delta S_{\mathsf{ch}}}{\delta g_{ij}}$$

$$\eta_H = -\frac{s}{2}\rho^{\mathsf{GS}}$$

$$= (v^i J^j_{\mathsf{edge}} + v^j J^i_{\mathsf{edge}}) + T^{ij}_{\mathsf{Hall}} - \frac{s^2}{4}\rho R g^{ij}$$

<u>Superfluid parity-violating hydrodynamics</u>

Vorticity



$$\omega = \frac{1}{2}\epsilon^{ij}\partial_i v_j = \frac{\sqrt{g}}{2}\left(B + \frac{s}{2}R\right)$$

- Vorticity is sourced by magnetic field and curvature
- p-wave superfluid on a sphere without B

$$\int_{S^2} \omega = \pi$$

two quantum vortices

Linear response

- Electromagnetic $J^i = \sigma_H(\omega, \mathbf{p})\epsilon^{ij}E_j + \ldots$

$$\sigma_H(\omega, \mathbf{p}) = \frac{s\rho^{\mathsf{GS}}}{2}\frac{-\mathbf{p}^2}{\omega^2 - c_s^2\mathbf{p}^2}$$

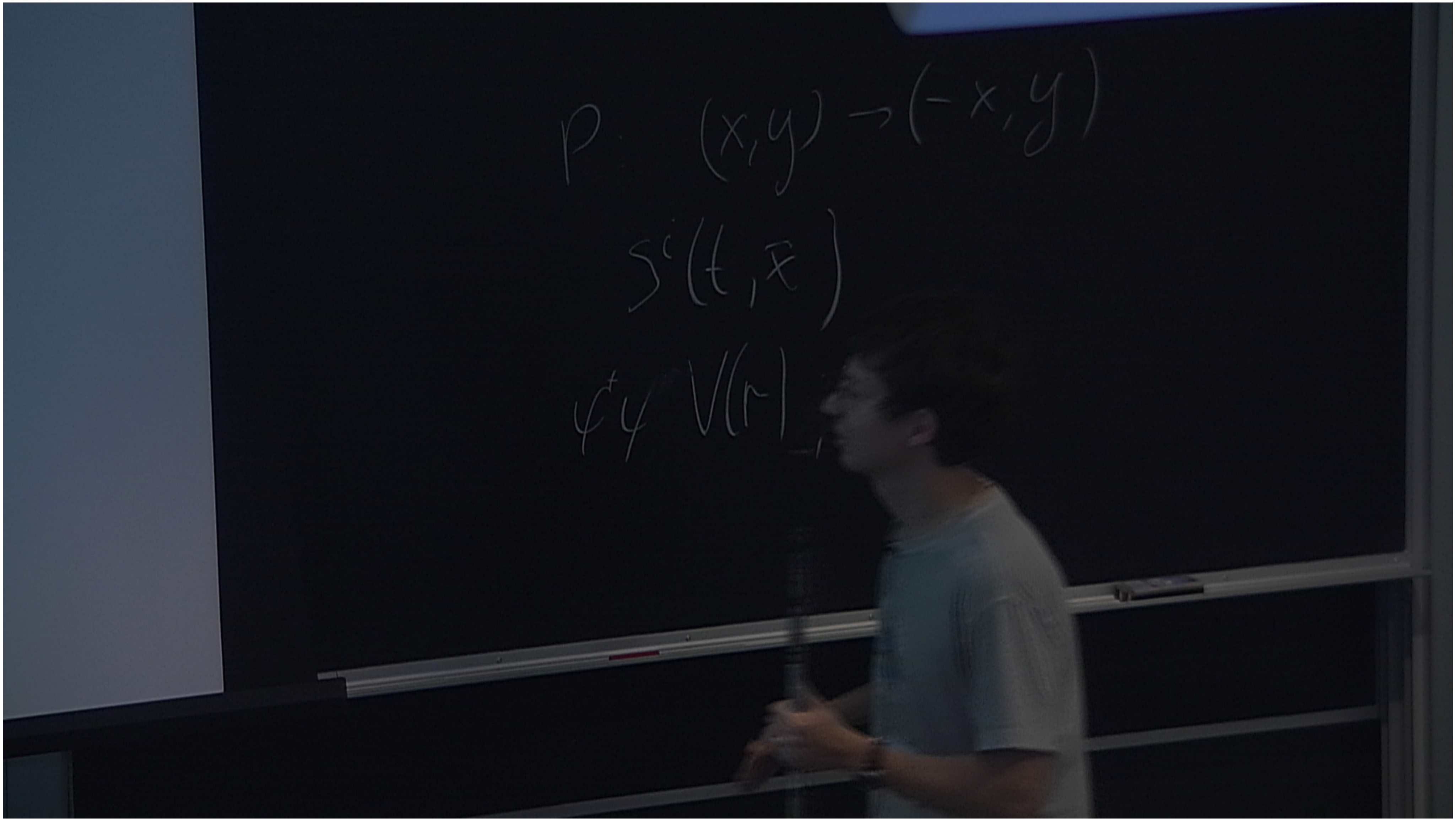
- Gravitational $\delta T^{xy} = -i\omega\frac{\eta_H(\omega)}{2}(h_{xx} - h_{yy}) + \ldots$

$$\eta_H(\omega) = -\frac{s}{2}\rho^{\mathsf{GS}}$$

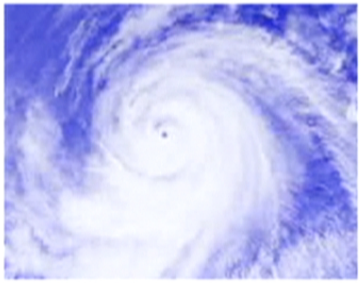
- Universal relation:

Hoyos, Son; Bradlyn et al

$$\boxed{\eta_H(\omega) = \frac{\omega^2}{2}\frac{\partial^2}{\partial p_x^2}\sigma_H(\omega, \mathbf{p})\Big|_{\mathbf{p}=0}}$$


P: (x,y) → (−x,y)
S^i(t, x̄)
ψ†ψ V(r)

Vortex solution



- Quantum vortex

$$v_r = 0 \qquad v_\phi = \frac{n}{2r} \qquad n \in \mathbb{Z}$$

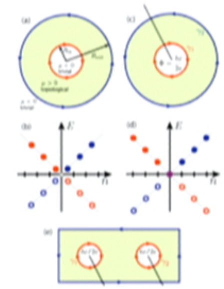
- Euler equation and its solution

$$\rho \frac{n^2}{4r^3} = \left[c_s^2 + \frac{sn}{2r^2}\right]\partial_r \rho \to \frac{\rho_\infty - \rho}{\rho_\infty} = \frac{n^2}{8c_{s\infty}^2 r^2} + O(r^{-4})$$

- Vortex and anti-vortex are different

$$\frac{\Delta\rho}{\rho_\infty} = \frac{s}{16c_{s\infty}^4 r^4} + O(r^{-6})$$

Gapless fermi modes



- Topological SF - gapless edge mode in BCS
- No explicit fermi modes in our EFT
- Non-analyticity of EoS at critical point
- This can appear only from integration of gapless modes

Edge modes are integrated out!

Alternative formulation

- Improved gauge potentials $\delta v^i = -\xi^k \partial_k v^i + v^k \partial_k \xi^i + \dot{\xi}^i$

one-form →

$$\tilde{A}_t \equiv A_0 + \frac{1}{2} g_{ij} v^i v^j$$
$$\tilde{A}_i \equiv A_i - g_{ij} v^j$$

← local Galilean boost

$$S[\theta, v^i] = \int dt d\mathbf{x} \sqrt{g} \left[\rho v^\mu D_\mu \theta - \epsilon(\rho) \right]$$

$$D_\mu \theta = \partial_\mu \theta - \tilde{A}_\mu - s\omega_\mu$$

- Velocity dof is massive and can be integrated

$$S[\theta] = \int dt d\mathbf{x} \sqrt{g} P(X)$$

Do we need vielbeins?

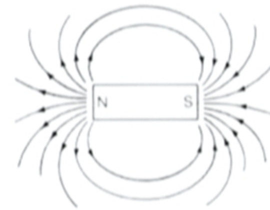
Reminder: $$D_\nu \theta \equiv \partial_\nu \theta - A_\nu - s\omega_\nu$$

$$\omega_t \equiv \frac{1}{2}\Big(\epsilon^{ab} e^{aj} \partial_t e^b_j + B\Big)$$
$$\omega_i \equiv \frac{1}{2}\epsilon^{ab} e^{aj} \nabla_i e^b_j = \frac{1}{2}\Big(\epsilon^{ab} e^{aj} \partial_i e^b_j - \varepsilon^{jk} \partial_j g_{ik}\Big)$$

We construct EFT of **bosons**, there are no fermions!

Can we write everything of terms of spatial metric?

Gauge description



- Duality relation $J^{\mu} \equiv \varepsilon^{\mu\nu\rho}\partial_{\nu}a_{\rho}$
- Conservation of current: Bianchi identity
- Nonlinear dual electrodynamics in 2+1

$$\boxed{\mathcal{L}_{sf} = \frac{g^{ij}e_i e_j}{2b} - \epsilon(b) - \varepsilon^{\mu\nu\rho}A_{\mu}\partial_{\nu}a_{\rho}}$$

$$b \equiv \varepsilon^{ij}\partial_i a_j \qquad e_j \equiv \partial_t a_j - \partial_j a_t$$

Dual description of s-wave superfluid

Wen-Zee term

- Dual description of chiral superfluid

$$\mathcal{L}_{WZ} = -s\varepsilon^{\mu\nu\rho}\omega_\mu \partial_\nu a_\rho$$

- Encodes Hall viscosity and edge current
- Up to surface term: $\mathcal{L}_{WZ} = -s\left(a_t B_\omega - \varepsilon^{ij} a_i E_{\omega j}\right)$

$$B_\omega = \frac{1}{2}R, \quad E_{\omega i} = \frac{1}{2}\left[-\partial_t(\Gamma^k_{ij})\varepsilon^{jl}g_{kl} - \partial_i B\right]$$

Vielbein eliminated in dual electrodynamics

Conclusion

- Effective hydro theory for Galilean parity-violating superfluid
- Hall viscosity and edge current
- Better understanding of Majorana modes?
- Critical theory of topological phase transition?
- Applications to films of He3A