Title: Einstein's equations from qubits

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Abstract: I will outline a path by which a semi-classical geometry obeying Einstein's equations emerges holographically from elementary quantum mechanical objects undergoing local dynamics. The key idea is that entanglement between the quantum degrees of freedom leads to the emergence of a dynamical geometry, that entanglement is the fabric of spacetime. Furthermore, although important technical challenges remain, I will argue that the conceptual ideas are in place. The core of the talk will be two new results that are crucial to this program, one establishing a new representation of entanglement in RG tensor networks and the other showing that the equivalence principle is encoded in the universality of entanglement.

Einstein's equations from qubits

Brian Swingle
with Mark van Raamsdonk

1405.2933 and 1406.????

SIMONS FOUNDATION

Three clues for quantum gravity

1. The Hamiltonian is measurable at infinity

$$F = \frac{G m_{\text{test}}}{r^2} \left(\frac{E}{c^2} \right)$$ [Newton]

2. Black holes have entropy proportional to their area

$$S_{BH} = \frac{c^3 A_{BH}}{4\hbar G_N}$$ [Bekenstein-Hawking]

3. Gravitational redshift

$$E_\infty = E_0 \sqrt{1 - \frac{2GM}{rc^2}}$$

[Einstein]

Begin in the middle

1. Hilbert space = (tensor product) space of the (regulated) CFT, similarly for Hamiltonian [Maldacena]
2. For some set of CFT states → semi-classical geometry and local bulk field theory

$$|\psi_{\rm CFT}\rangle \leftrightarrow (M, |\psi_{\rm bulk}\rangle)$$

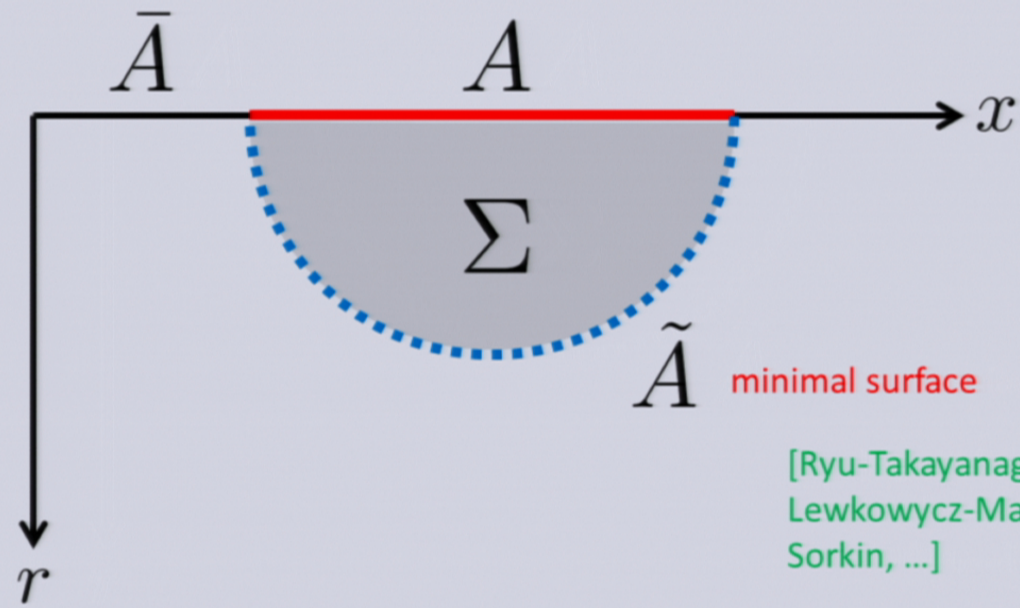
3. Example: ground state,

$$|0_{\rm CFT}\rangle \leftrightarrow (AdS, |0_{\rm bulk}\rangle)$$

$$M : ds^2 = \frac{R^2_{AdS}}{r^2}(-dt^2 + d\vec{x}^2 + dr^2)$$

fixed by symmetry

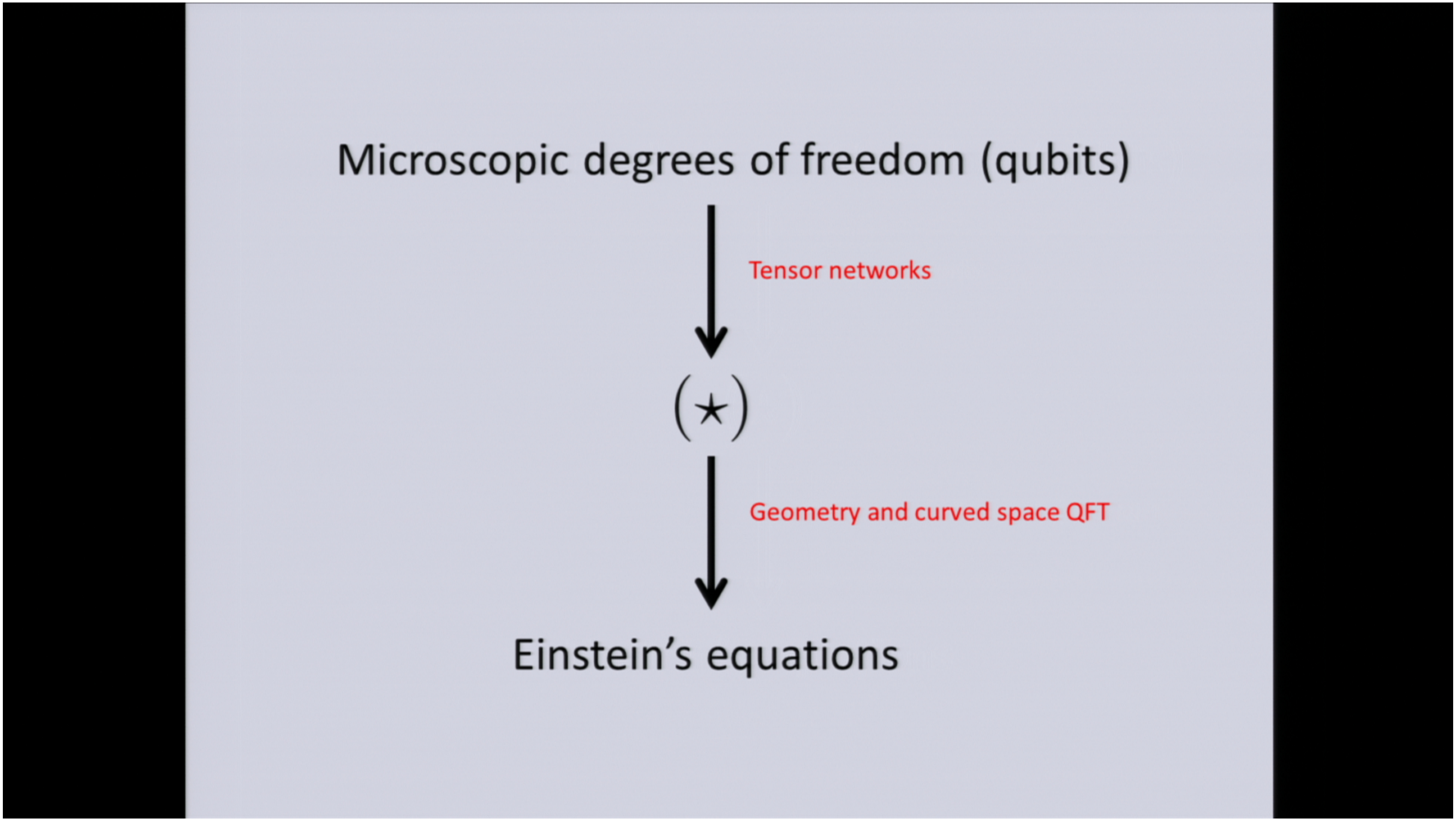
Assumption: entanglement and geometry



[Ryu-Takayanagi, Faulkner-Lewkowycz-Maldacena, Sorkin, ...]

$$(\star) \quad \boxed{S_A(|\psi_{\mathrm{CFT}}\rangle) = \frac{|\tilde{A}|_M}{4G_N} + S_\Sigma(|\psi_{\mathrm{bulk}}\rangle)}$$

[finite: Susskind-Uglum, Cooperman-Luty, Satz-Jacobson, Bianchi-Myers, ...]


Microscopic degrees of freedom (qubits)
Tensor networks
(⋆)
Geometry and curved space QFT
Einstein's equations

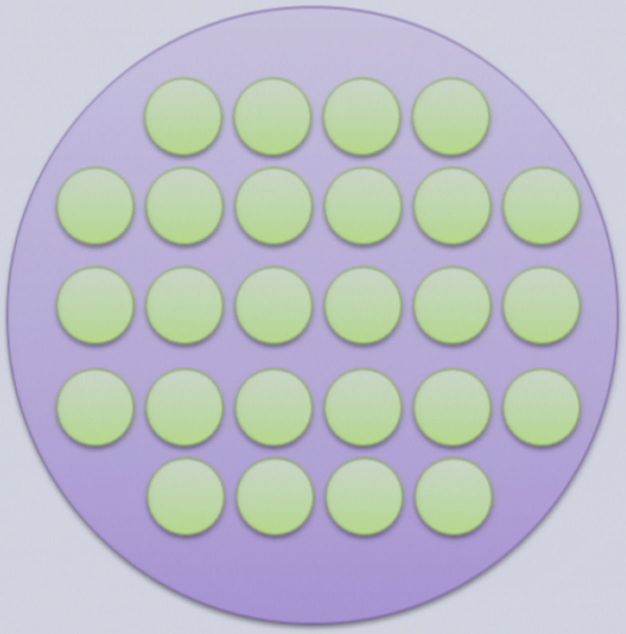
local DOF

$\dim(\bigcirc) = 2^N$

qubit

$\dim(\bigcirc) = 2$

Local DOFs composed of many interacting qubits (e.g. large N gauge theory)



Tensor product Hilbert space: $\mathcal{V}_{\rm sys} = \bigotimes_x \mathcal{V}_x(\bigcirc)$

Local Hamiltonian: $H = \sum_{x=1}^{L} H_{x,x+1}$

(assumed to have DOF at all energy scales, strongly interacting)

Basic idea

Consider the ground state of H: $|0_{\rm CFT}\rangle$

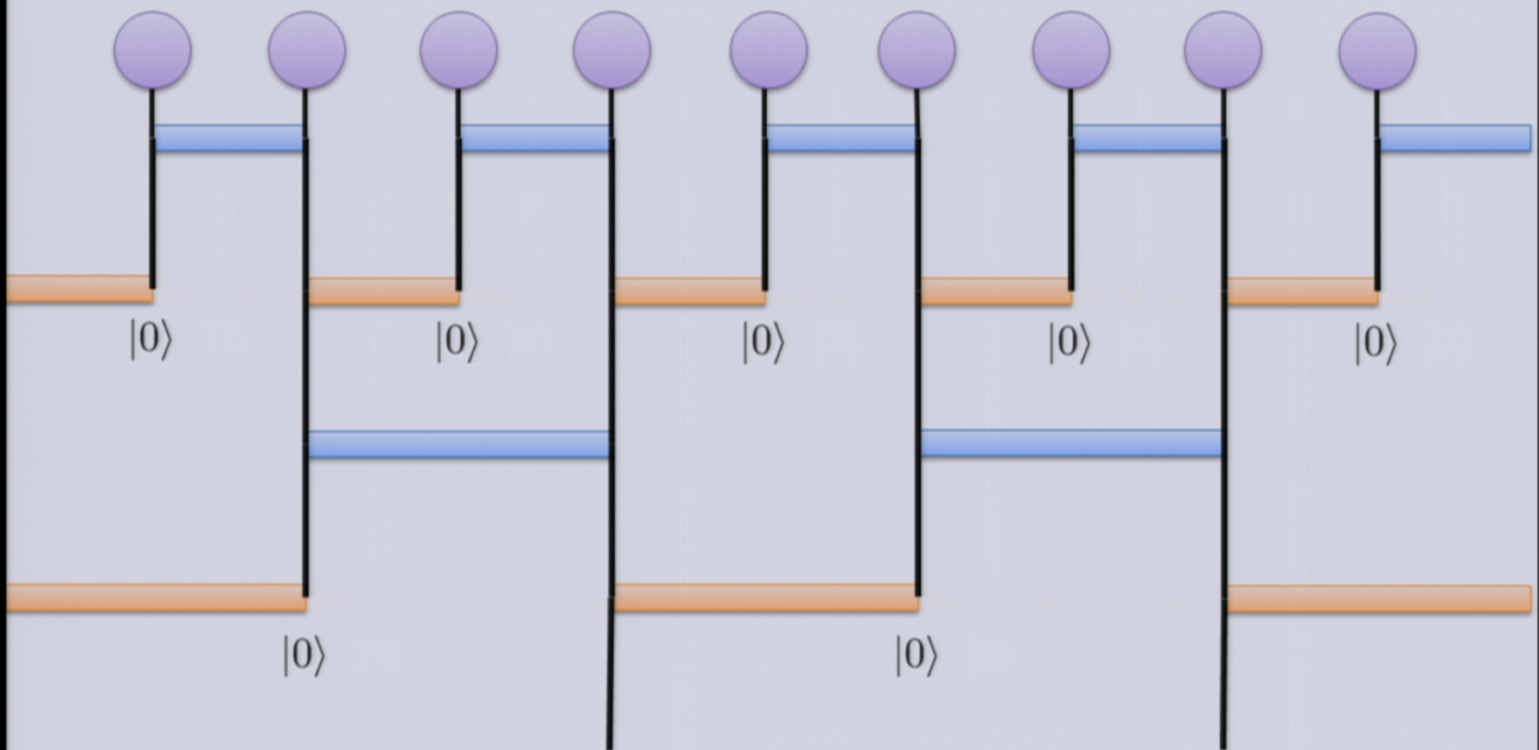
We would like a way to represent the ground state (and other states) which

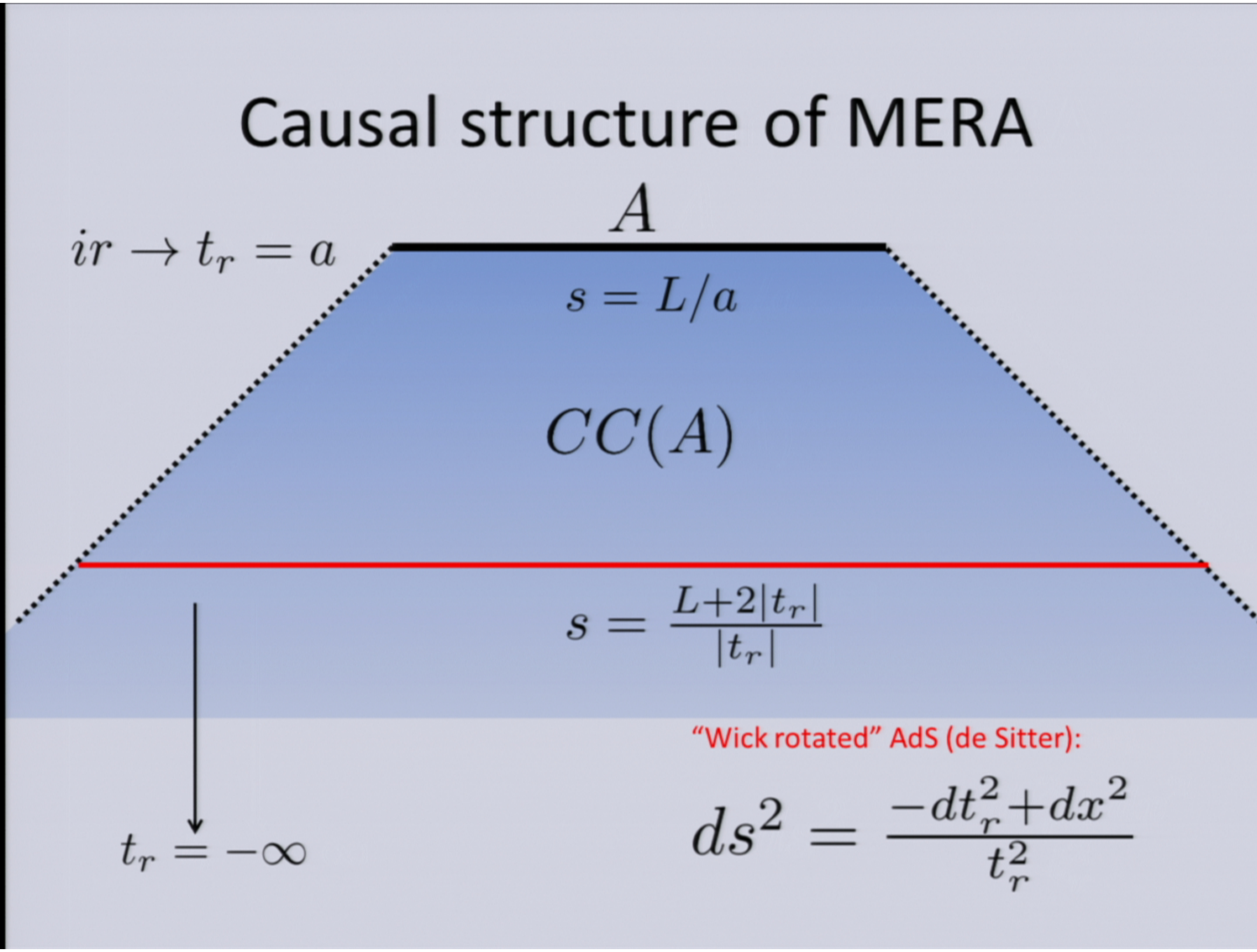
1. makes clear the physics at different length scales,
2. explicitly takes entanglement into account, and
3. represents entanglement geometrically.

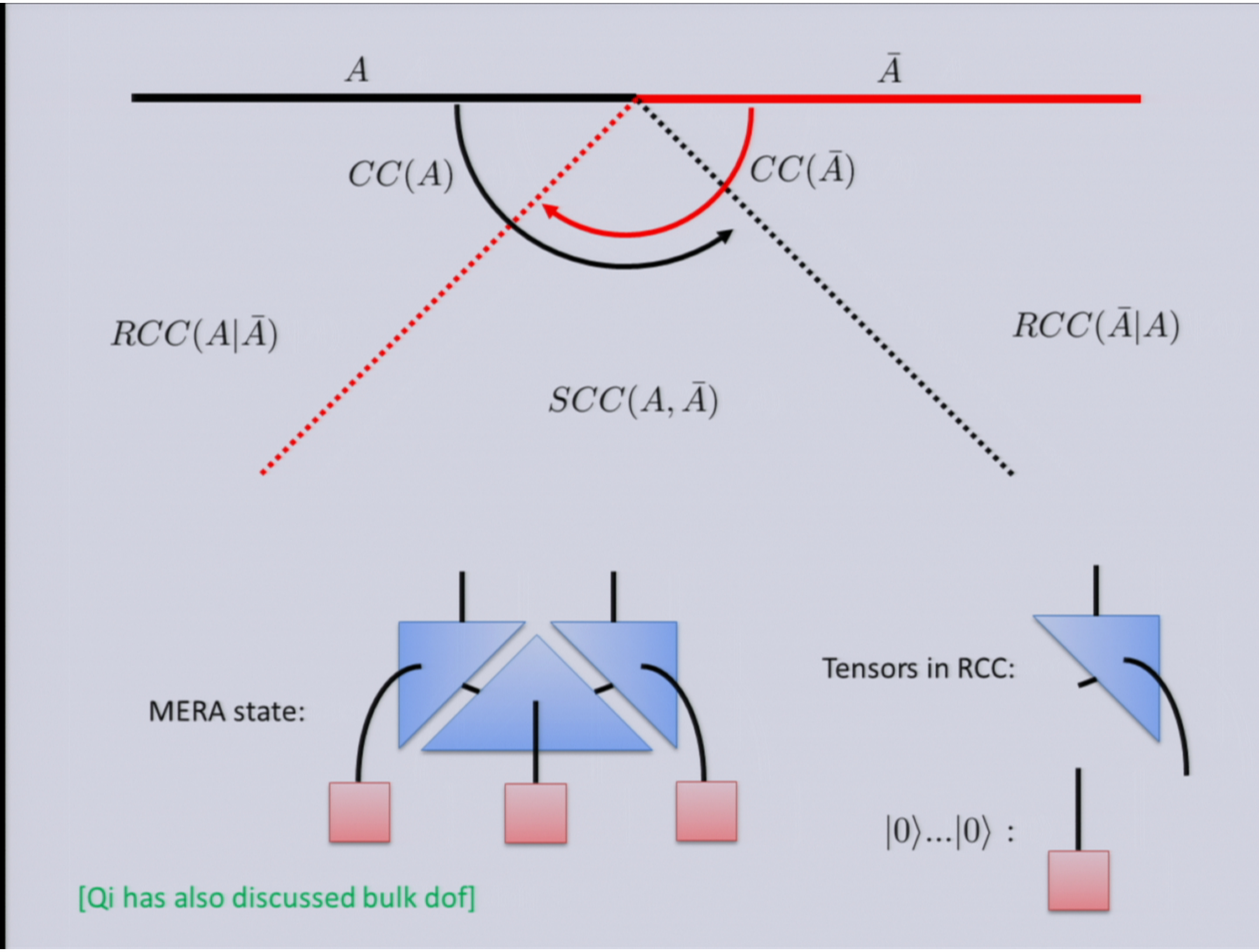
$|0_{\rm CFT}\rangle$ → entanglement network [Vidal, BGS]

Entanglement Renormalization (MERA)

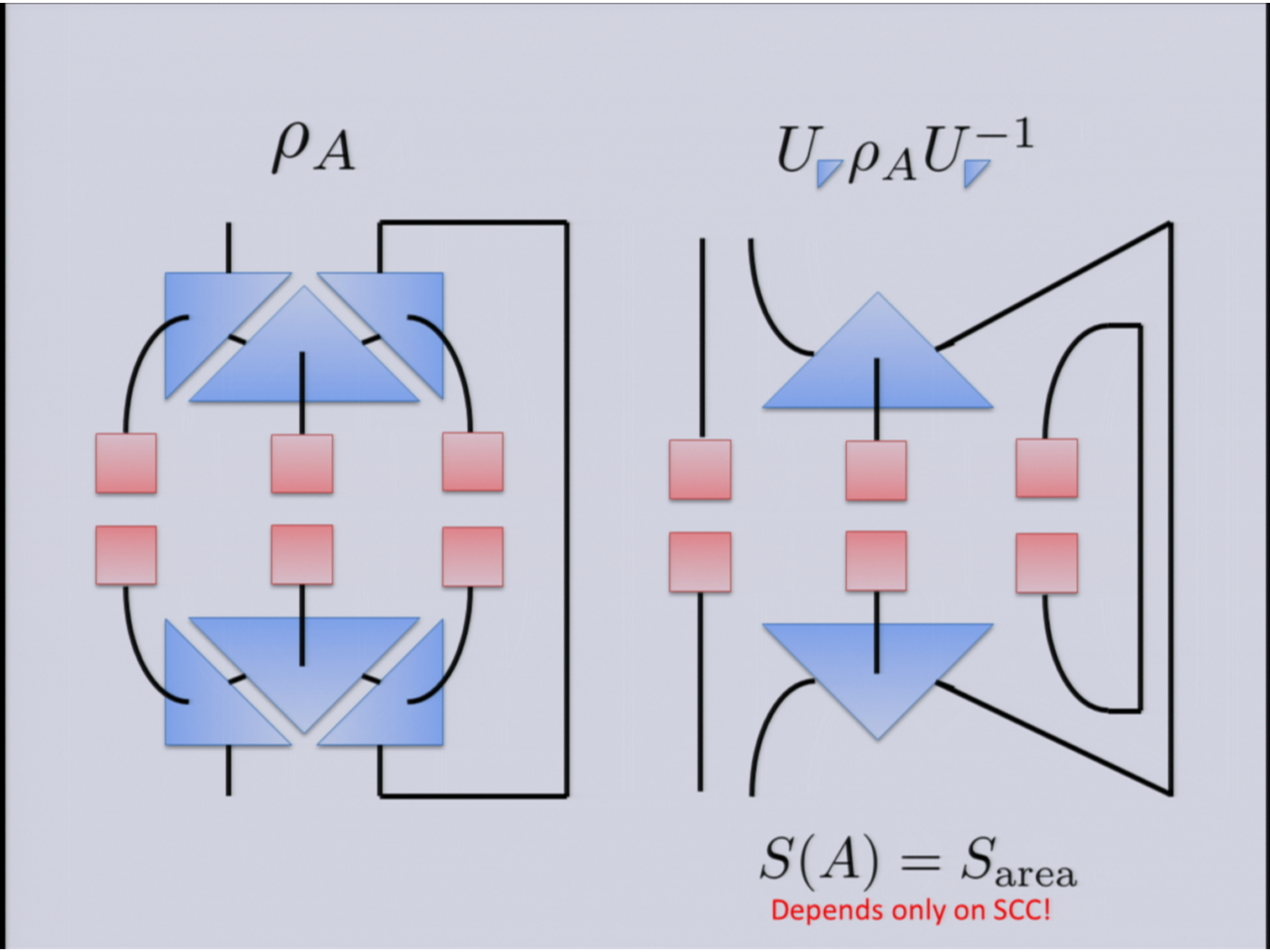
[Vidal]



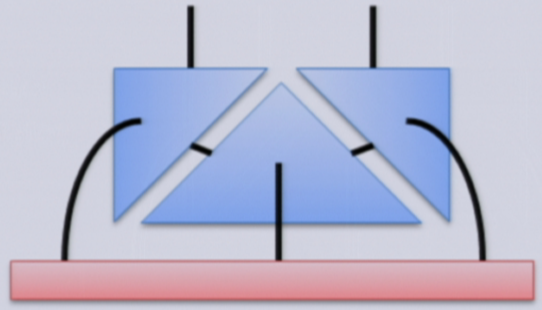

Causal structure of MERA
A
$ir \to t_r = a$
$s = L/a$
$CC(A)$
$s = \frac{L+2|t_r|}{|t_r|}$
"Wick rotated" AdS (de Sitter):
$ds^2 = \frac{-dt_r^2+dx^2}{t_r^2}$
$t_r = -\infty$



A
$\bar{A}$
$CC(A)$
$CC(\bar{A})$
$RCC(A|\bar{A})$
$RCC(\bar{A}|A)$
$SCC(A,\bar{A})$
MERA state:
Tensors in RCC:
$|0\rangle...|0\rangle$:
[Qi has also discussed bulk dof]


ρ_A
$U \rho_A U^{-1}$
$S(A) = S_{\text{area}}$
Depends only on SCC!

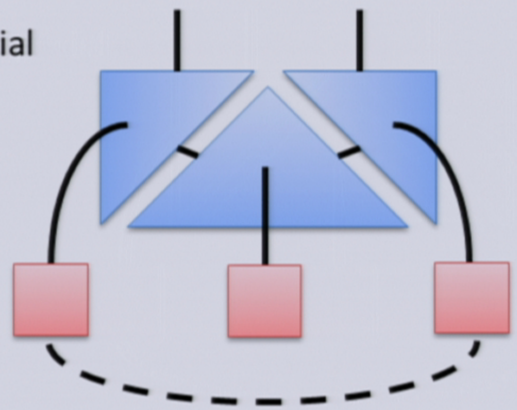
MERA with more general bulk state:



$$S(A) = ?$$

Can't say much in complete generality ...

MERA with special bulk state:



$$\frac{1}{\sqrt{M}}\sum_{i=1}^{M}|ii\rangle$$

$$S(A) = S_{\text{area}} + S_{\text{bulk}}$$

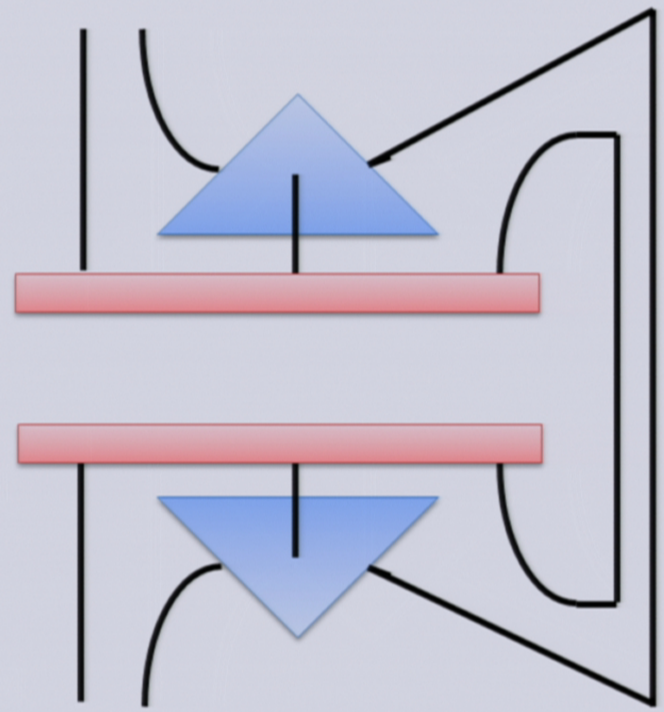
$$S_{\text{bulk}} = \log(M)$$

S(A) “only” differs from entanglement of bulk dof due to bulk “local” unitaries acting in SCC

Long-range parts are preserved, e.g. visible Fermi surface, bulk long-range entanglement, etc.

[BGS-Huijse-Sachdev]

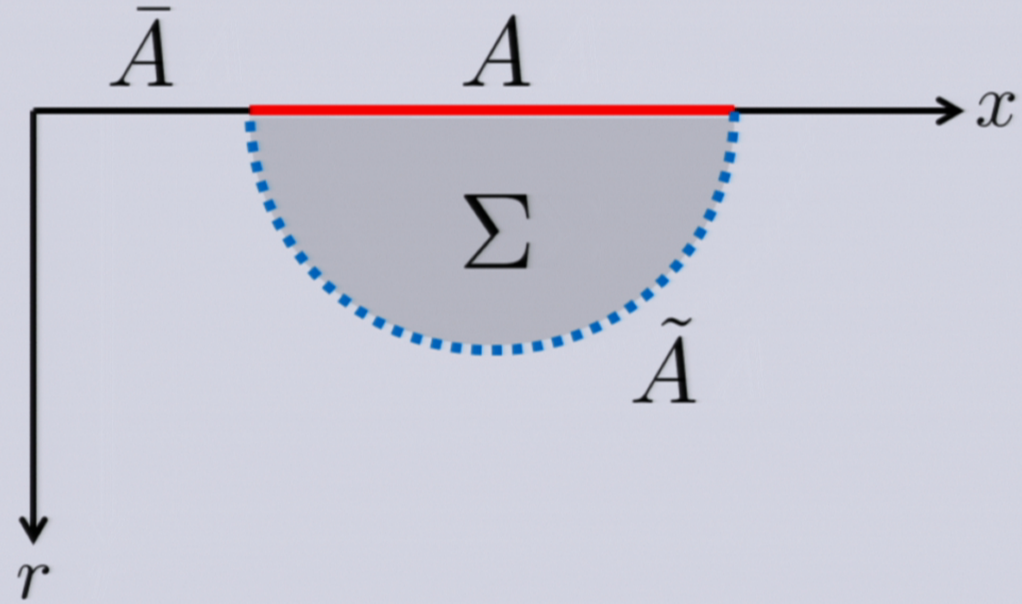
Some further statements are possible in a crude large N model, but too little time …



Summary of Part 1

1. Entanglement network for states, emergent dimension associated with length scale
2. Network encodes entanglement quasi-geometrically
3. Network is dynamical
4. New physics of bulk entanglement
5. In special cases, e.g. large N, some additional (holographic) features are crudely captured

A slight leap



$$(\star)\quad S_A(|\psi_{\rm CFT}\rangle) = \frac{|\tilde{A}|_M}{4G_N} + S_\Sigma(|\psi_{\rm bulk}\rangle)$$

Dynamics

$$\delta|\psi_{\rm CFT}\rangle \leftrightarrow (\delta M, \delta|\psi_{\rm bulk}\rangle)$$

CFT side:

$$|\psi_{\rm CFT}\rangle = |0_{\rm CFT}\rangle + \epsilon\delta|\psi_{\rm CFT}\rangle$$

Gravity side:

$$ds^2 = \frac{R^2_{AdS}}{r^2}(-dt^2 + d\vec{x}^2 + dr^2 + h_{\mu\nu}dx^\mu dx^\nu)$$

$$|\psi_{\rm bulk}\rangle = |0_{\rm bulk}\rangle + \epsilon\delta|\psi_{\rm bulk}\rangle$$

CFT First Law

Symmetry: $A = B(R, x_0)$ ball of radius R

$$\Sigma_B = \text{hemi-sphere}$$

$$\boxed{\delta S_B = \delta E_B}$$

[Hislop-Longo, Casini-Myers-Huerta]

$$\delta S_B = -\mathrm{tr}(\delta\rho_B \log(\rho_B))$$

$$\delta E_B = 2\pi \int_B \frac{R^2 - |\vec{x}|^2}{2R} \mathrm{tr}(\delta\rho_B T_{00}^{\mathrm{CFT}})$$

Aside: why is B special?

Answer: B supports a boundary conformal Killing field

$$\zeta = -\frac{2\pi}{R}(t-t_0)(x^i - x_0^i)\partial_i + \frac{2\pi}{2R}(R^2 - (x-x_0)^2 - (t-t_0)^2)\partial_t$$

Answer: Σ_B supports a bulk Killing field

$$\xi = -\frac{2\pi}{R}(t-t_0)(r\partial_r + (x^i - x_0^i)\partial_i) + \frac{2\pi}{2R}(R^2 - (x-x_0)^2 - r^2 - (t-t_0)^2)\partial_t$$

Answer: $\tilde{B}$ is the bifurcation surface of a Killing horizon

CFT first law in gravitational variables

$\exists$ a form $\chi(h)$:

[Lashkari et al., Faulkner et al., Iyer-Wald, Jacobson]

Area variation: $\delta \frac{|\tilde{B}|_M}{4G_N} = \int_{\tilde{B}} \chi$

variation wrt surface = 0

Energy variation: $\delta E_B = \int_B \chi$

CFT T given by asymptote of h

Differential: $d\chi = -2\xi^a \delta E^g_{ab} \epsilon^b$

$$E^g_{ab} = \frac{1}{\sqrt{-g}} \frac{\delta W_{\text{grav}}}{\delta g^{ab}} \qquad W_{\text{grav}} = \frac{1}{16\pi G_N} \int \sqrt{-g}(R - 2\Lambda)$$

Including the matter term

$$\delta S_\Sigma = \mathrm{tr}(\delta\rho_\Sigma \log(\rho_\Sigma))$$ variation wrt surface = 0

$$-\log(\rho_{\Sigma_B}) = \int_{\Sigma_B} \xi^a T_{ab}^{\mathrm{bulk}} \epsilon^b$$ bulk region is rindler wedge

$$\boxed{\delta S_{\Sigma_B} = \int_{\Sigma_B} \xi^a \delta \langle T_{ab}^{\mathrm{bulk}} \rangle \epsilon^b}$$

Main argument

CFT first law $$0 = \delta S_B - \delta E_B$$

Holographic variables $$0 = \delta S_B^{\text{grav}} - \delta E_B^{\text{grav}}$$

Use calculated variations $$0 = \int_{\tilde{B}} \chi + \int_{\Sigma_B} \xi^a \delta \langle T_{ab}^{\text{bulk}} \rangle \epsilon^b - \int_B \chi$$

Stokes theorem $$0 = \int_{\Sigma_B} \xi^a \left(-2\delta E^g_{ab} + \delta \langle T_{ab}^{\text{bulk}} \rangle \right) \epsilon^b$$

$$0 = \int_{\Sigma_B} \xi^a \left(-2\delta E^g_{ab} + \delta\langle T^{\text{bulk}}_{ab}\rangle\right) \epsilon^b$$

1. Valid for all B → integrand vanishes
2. Boosted frames give all field theory directions
3. Other directions are constraints:
   $$r\mu, rr \leftrightarrow \partial_\nu T^{\mu\nu} = 0, T^\mu_\mu = 0$$

$$\boxed{\delta E^g_{ab} = \frac{1}{2}\delta\langle T^{\text{bulk}}_{ab}\rangle}$$

Einstein's equations linearized about AdS, equivalence principle, Newtonian gravity

Some details and interpretations

$$\delta E^g_{ab} = \frac{1}{2}\delta\langle T^{\rm bulk}_{ab}\rangle$$

RHS includes graviton

Suppose a linearized operator equation: $K\hat{h} = \hat{O}$

We have matrix elements of $K\hat{h} = \hat{O}$ between vac and any state

Results fix K and O: $K = K_{\rm Einstein}$

$$\hat{O} = \hat{T}^{\rm bulk} + \hat{O}' =_{\rm local} \hat{T}^{\rm bulk}$$

$$\forall\psi\ \langle\psi|\hat{O}'|0\rangle = \langle 0|\hat{O}'|\psi\rangle = 0$$

Non-linear extension

$$W_{\text{grav}} = \frac{1}{16\pi G_N} \int \sqrt{-g}(R - 2\Lambda)$$

The only local* non-linear action consistent with

1. our linearized result and
2. our entanglement assumption

is Einstein's action universally coupled to matter.

* We only require that the action be local, not the physics. Also, any attempt to introduce real non-locality must contend with, e.g. causality constraints from the local graviton, etc.

Summary of Part 2

1. Assume the existence of a geometry and a certain representation of entanglement
2. Purely CFT constraints give linearized Einstein equations, equivalence principle
3. Universality of gravity comes from universality of entanglement
4. Only non-linear local action consistent with 1. and 2. is Einstein's action
5. Can be extended to higher curvature theories

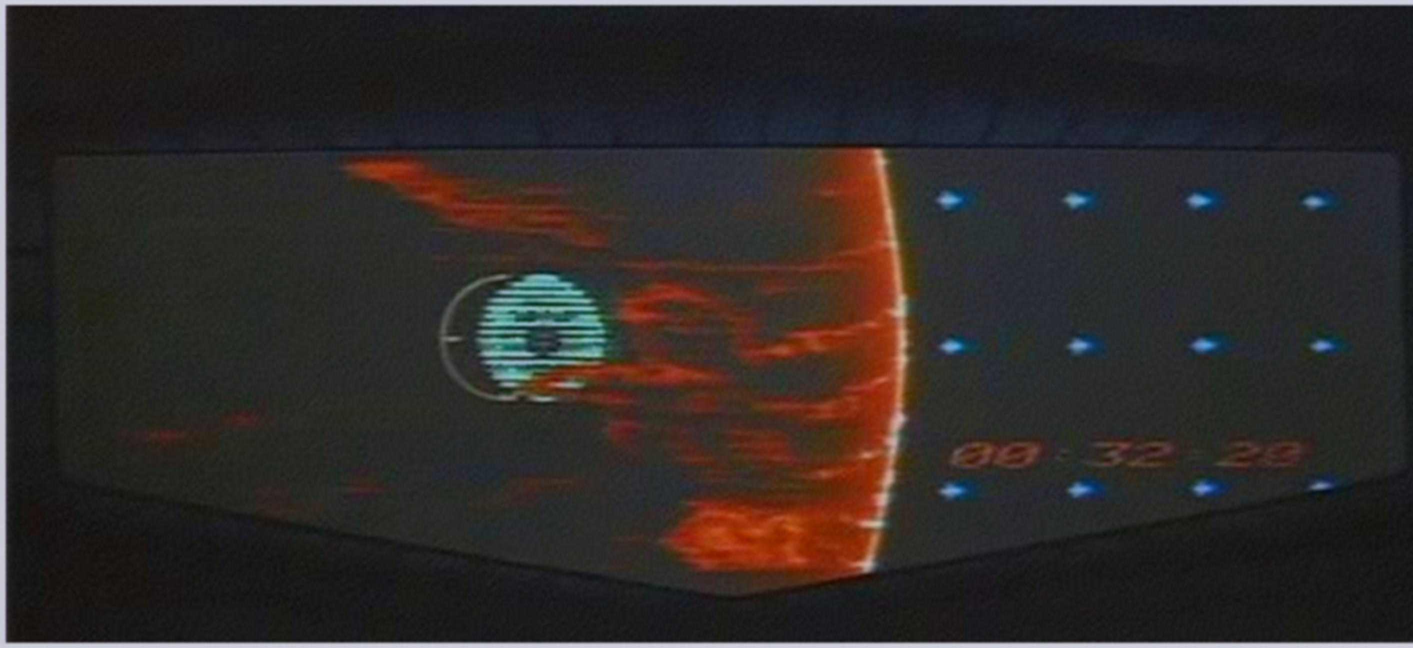
[Faulkner et al.]

I argued we have a conceptual path from qubits to Einstein and many key results, but many technical challenges remain.

Ideas/issues:

1. Tensor networks and CFT bootstrap
2. Explicit network dynamics
3. Short distance bulk locality
4. Probe approximation for networks

We’re almost in range …



THANKS!