Title: Wilsonian and Large N Approaches to Non-Fermi Liquids

Date: May 26, 2014 09:05 AM

URL: http://pirsa.org/14050095

Abstract: We study the problem of metals near a quantum critical point using a local Wilsonian effective field theory of Fermi surface fermions coupled to massless boson (i.e. order parameter) fields, in particular in a large N limit where the boson is matrix-valued. We focus on regions of parameter space where the boson dresses the fermions into a non-Fermi liquid while the bosons are approximately controlled by the Wilson-Fisher fixed point.

Wilsonian and Large N approaches to Non-Fermi Liquids

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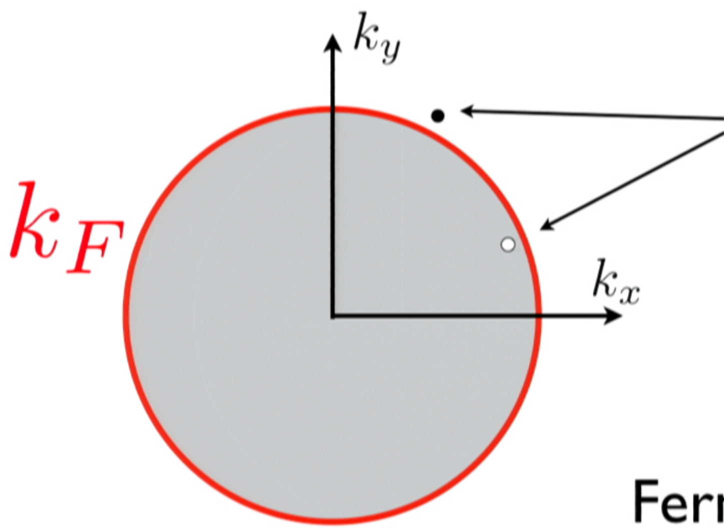
Introduction to Fermi Liquids

Fermions at finite density
have a Fermi surface



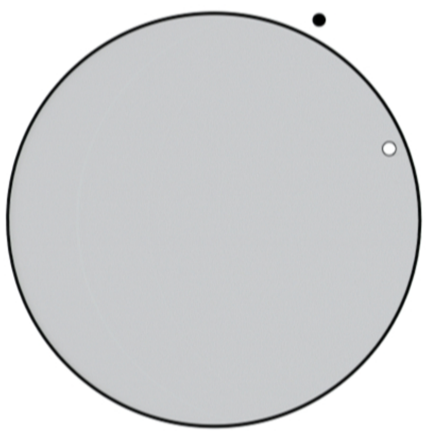
Fermi energy

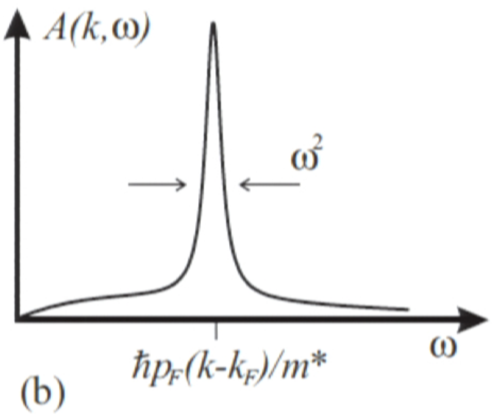
$$E = \frac{k^2}{2m} - \epsilon_F$$

Fermi momentum: $\frac{k_F^2}{2m} = \epsilon_F$

Landau Fermi Liquids

In simple metals, excitations are weakly coupled quasi-particles





$$\frac{1}{\tau} = \mathrm{Im}(\Sigma) \sim \frac{\omega^2}{k_F}$$

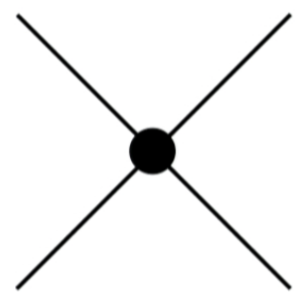
Landau Fermi Liquids

Why are emergent quasiparticles well-described by weak coupling?

Modern EFT description:
(almost) all interactions are irrelevant

Shankar
Polchinski

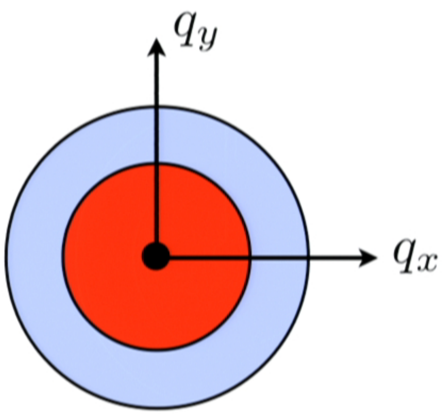
$$\frac{\psi^\dagger \psi \psi^\dagger \psi}{\Lambda}$$



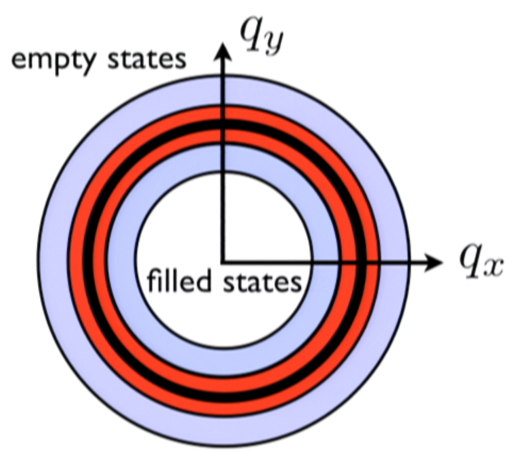
Landau Fermi Liquids

Scaling:

Standard:



Fermi Surface:



Fix angle and scale toward nearest point on Fermi surface:

$$\vec{q} = \hat{\theta}(k_F + \ell)$$

$$\omega \to e^{\lambda}\omega \quad \ell \to e^{\lambda}\ell$$

Landau Fermi Liquids

$$S_2 = \int dS^{d-1} \left[\int d\omega d\ell \psi^\dagger (\omega - v_F \ell) \psi \right]$$

$$\ell \equiv |k| - k_F$$

$$\omega \to e^\lambda \omega$$

$$\ell \to e^\lambda \ell$$

So we see that the fermions should scale as

$$\psi \to e^{-\frac{3}{2}\lambda} \psi$$

Landau Fermi Liquids

First interaction is four-fermion interaction

$$S_4 = \int d^{d-1}S_1 d\omega_1 d\ell_1 \ldots d^{d-1}S_4 d\omega_4 d\ell_4\, \delta(\omega_1 + \omega_2 + \omega_e + \omega_4)$$

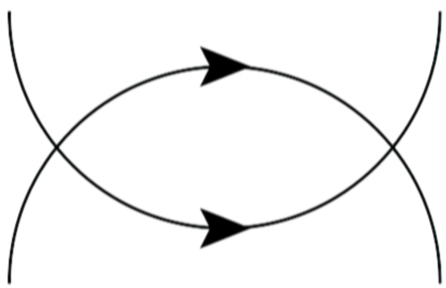
$$V(\theta_i)\psi_1^\dagger \psi_2^\dagger \psi_3 \psi_4 \quad \delta^d(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4)$$

It naively scales like e^λ and is irrelevant

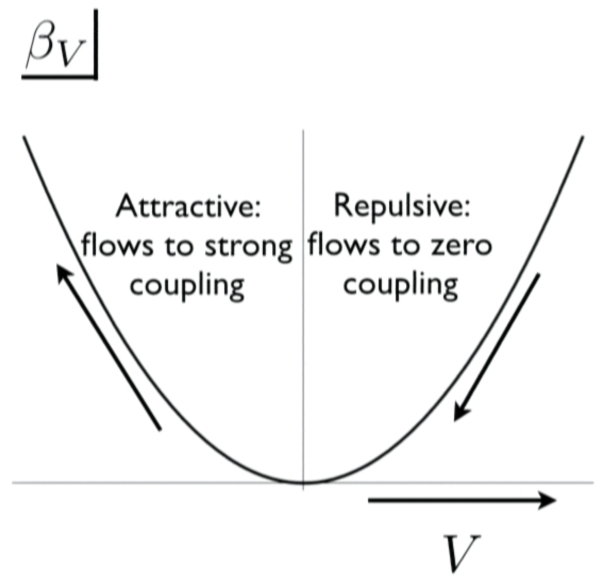
But for certain kinematic configurations, the delta function scales like $e^{-\lambda}$ and the interaction becomes *marginal*

Landau Fermi Liquids

BCS instability:
At one-loop, the interaction between antipodal points runs and becomes marginally relevant/irrelevant



$$\frac{dV}{d\log\mu} = V^2$$

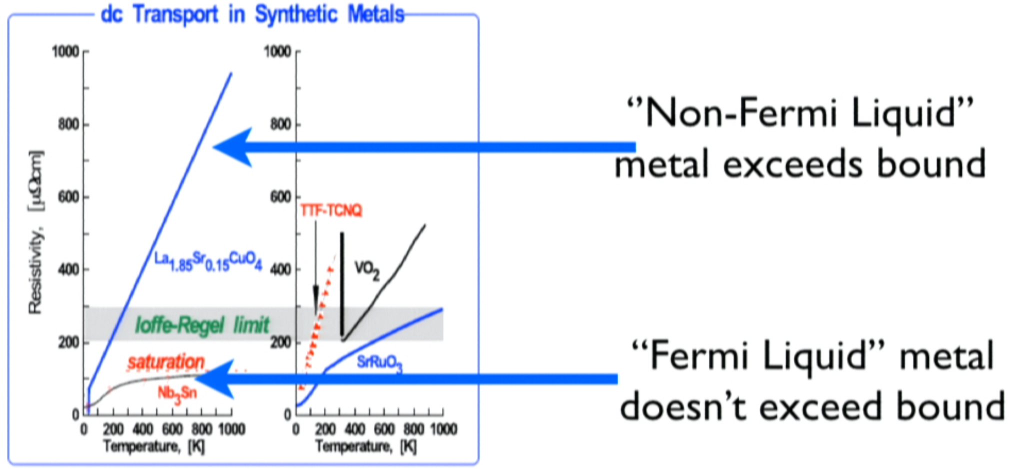


Fermi vs. Non-Fermi

Ioffe-Regel Resistivity Limit on Fermi Liquids

Quasi-particle transport:

resistivity is inversely proportional to mean free path



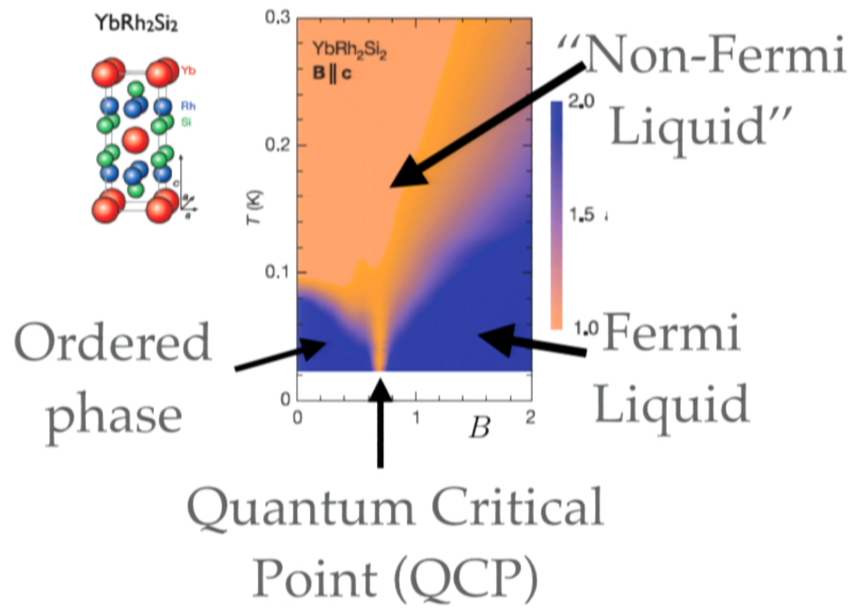
Courtesy of D. Basov (UCSD)

If resistivity is too large, then mean free path is shorter than lattice spacing, and quasi-particle description doesn't make sense

“Non-Fermi” Liquids

Many materials have fascinating new properties that make them fall outside of the Fermi Liquid description

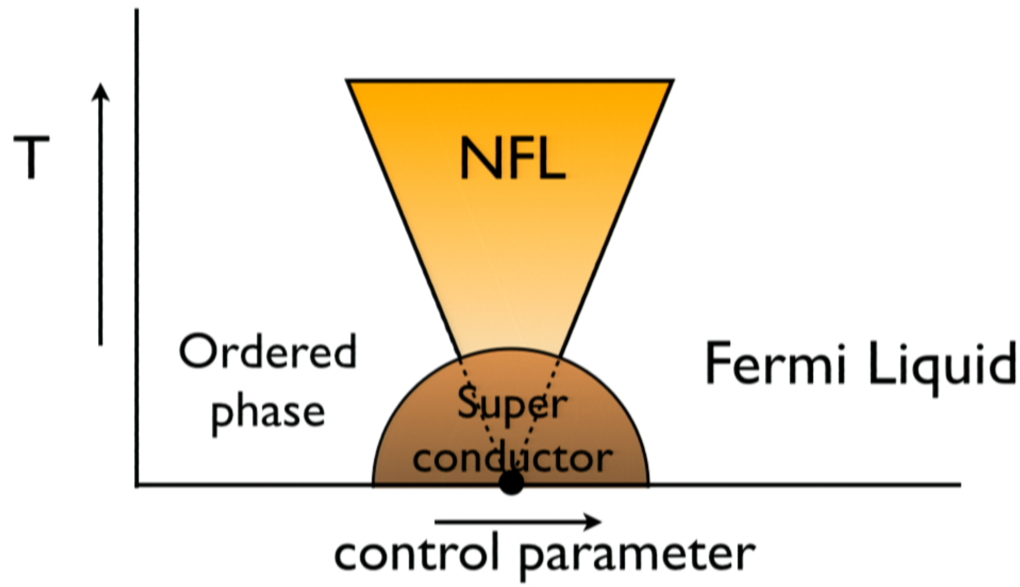
- Resistivity Linear in T
- Violate Ioffe-Regel bound
- Superconductivity often occurs at high temperature
- Often Located near Quantum Critical Points



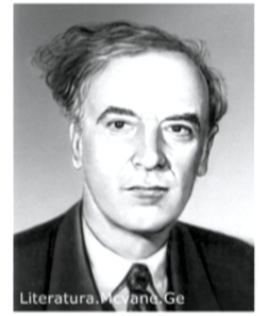
Quantum Critical Points

A Recurring theme: NFLs arise near Quantum Phase Transitions

(Phase transition at zero temp)



Landau-Ginzburg-Wilson

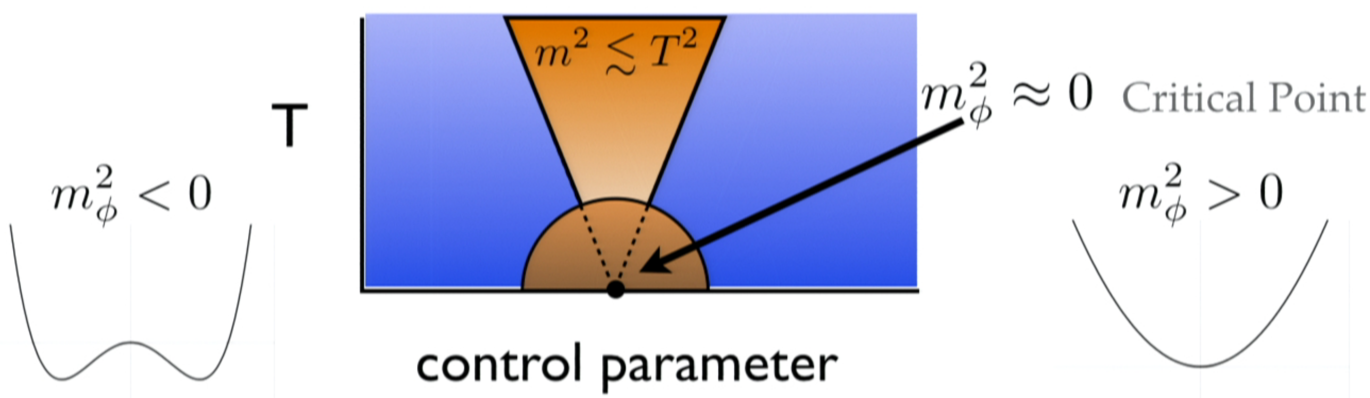


Write down Lagrangian for the order parameter of the phase transition

$$\mathcal{L} \sim \dot{\phi}^2 - (\nabla\phi)^2 - m^2\phi^2 - \lambda\phi^4 + \ldots$$

Example: Ferromagnetic transition $\phi = M_z$

Near critical point: ϕ is a nearly massless fluctuating boson



EFTs of Non-Fermi Liquids

As a high energy physicist, I will take some lessons from the study of QCD:

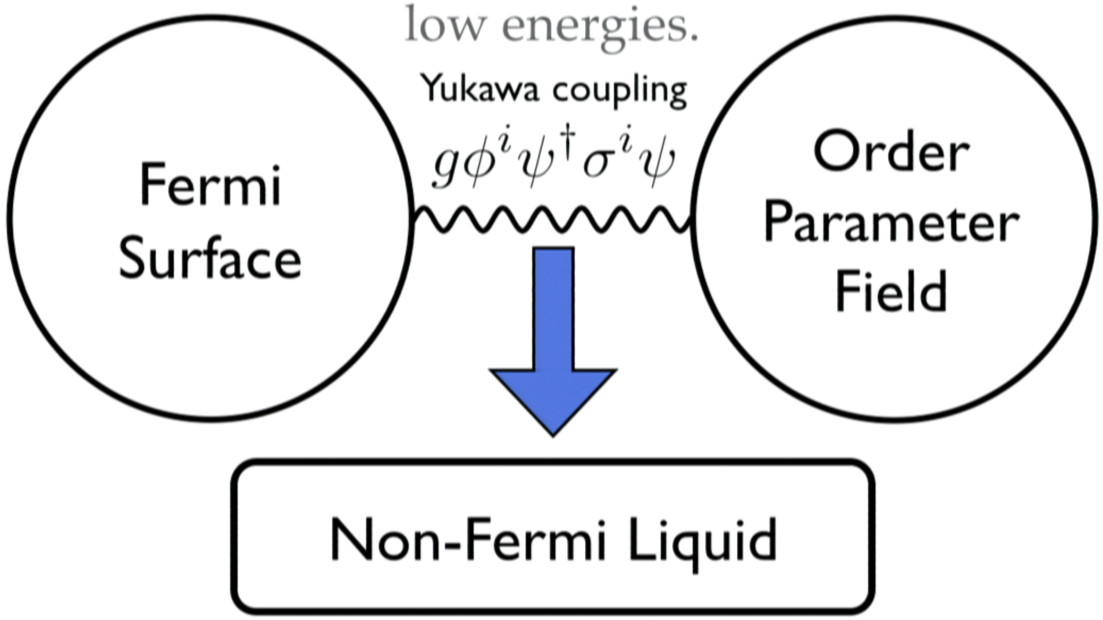
1) It was hard to see *a priori* what QFTs (if any!) could explain deep inelastic scattering

The classification and study of local QFTs was wildly successful

2) Confinement especially was hard to tackle directly, and simplifying special cases (2d, large N, SUSY) played a crucial role in our qualitative understanding

EFTs and Non-Fermi Liquids

Treat this as a Wilsonian EFT problem:
Choose our light degrees of freedom and add interactions.
Integrate out high energy modes and see what happens at low energies.



EFTs of Non-Fermi Liquids

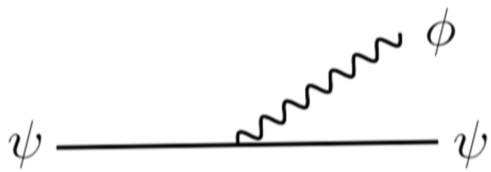
Wilsonian approach: start with *local* action in UV and integrate out high energy modes

We will not add *by hand* any terms like

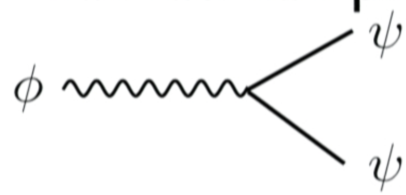
$$\phi \frac{1}{\nabla^2} \phi$$

Marriage of Landau's Two Great Frameworks

Fermions can decay:
Non-Fermi Liquid



Bosons can decay to particle/hole pairs:
"Landau damping"

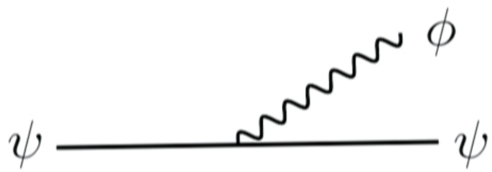


Titanic Struggle

Fermions renormalize bosons and vice versa

Who wins?

Fermions can decay:
Non-Fermi Liquid



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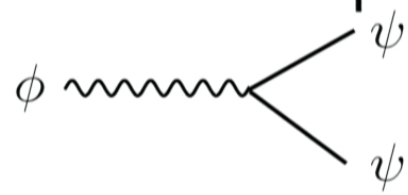

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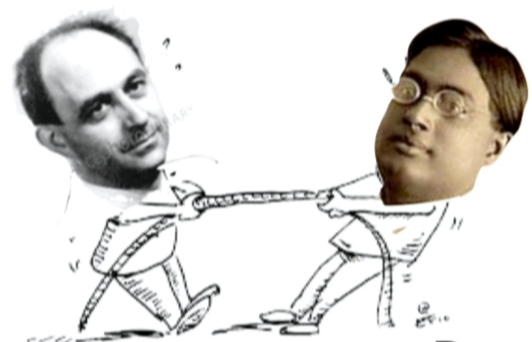
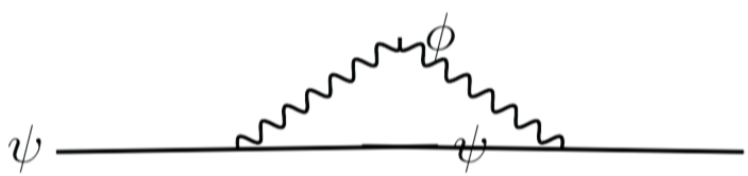


Fermions can decay:
Non-Fermi Liquid



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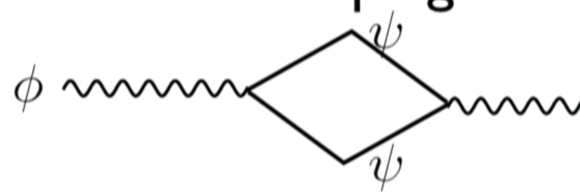
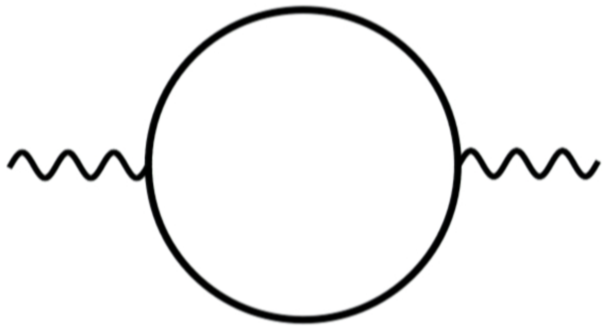
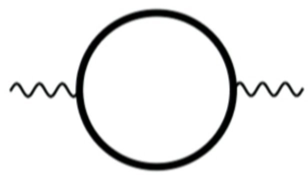


Landau Damping

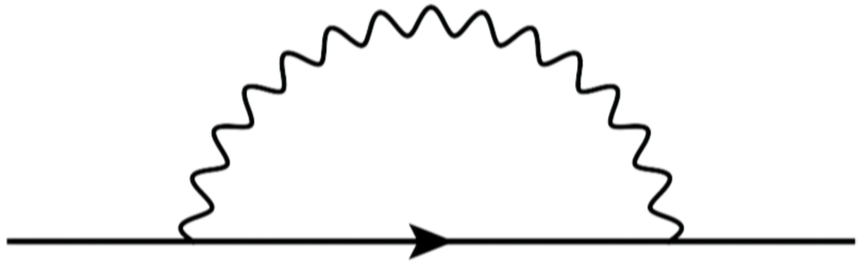


One-loop boson self-energy has non-analytic term  $\sim M^2 F(\omega/p)$

One loop vs tree-level

Strong coupling at IR scale:  > 

Anomalous Dimension



Wavefunction
renormalization

This is a more familiar effect from a particle physicist's point of view:

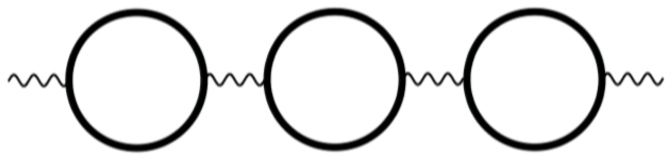
The log divergent piece changes the scaling dimension of the fermion field

Landau Damping

Mainstream philosophy
Hertz (1976):

"Fermions Win"

"Keep 1PI diagrams but drop all others, resum to get new kinetic term"



"Then feed this back into corrections to fermion"

Landau Damping

Mainstream philosophy
Hertz (1976):

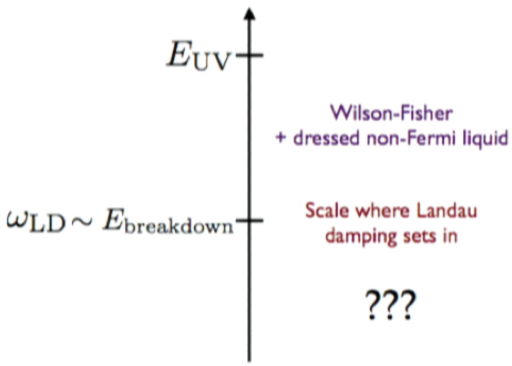
"Fermions Win"

Long line of work in this direction:
Millis '93, Polchinski '94,
Nakak, Wilczek '94,
Oganesyan, Kivelson, Fradkin '01,
Chubukov et al. '06, S.S. Lee '09,
Metlitski, Sachdev '10,
Mross, McGreevy, Liu, Senthil '10

We will go a different direction

Wilsonian Perturbation Theory

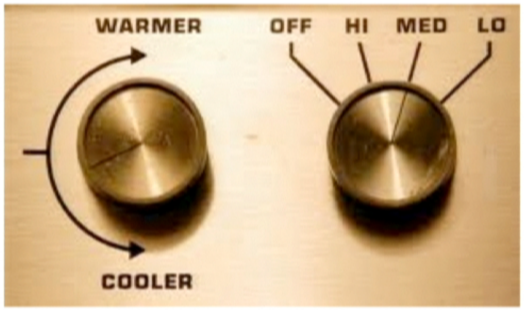
We will start by perturbing around the free UV theory and look for limits where we can do perturbative calculations in a small parameter



- Look for intermediate regimes governed by approximate fixed points.
- Estimate the scales where corrections become large and our description breaks down.

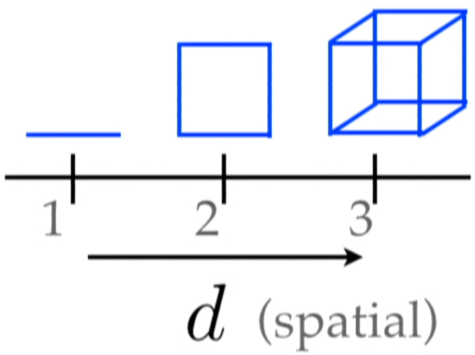
In a theory with superconducting instabilities, we are mainly interested in pushing the breakdown scale below the superconducting dome.

Looking for Parameters to Dial

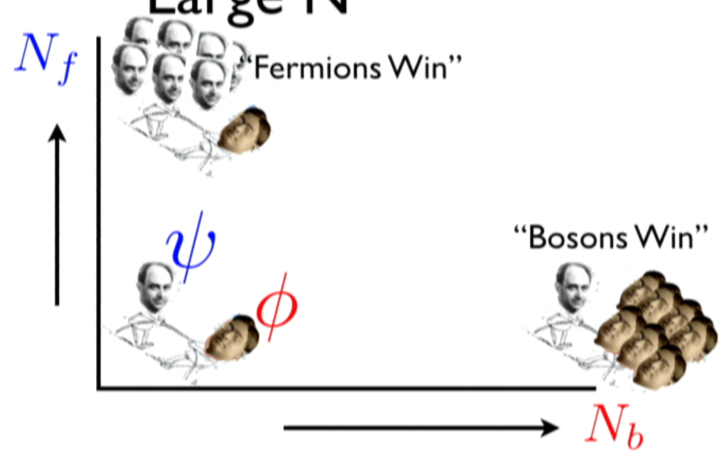


We want to find parameters of the theory where a drastic simplification occurs (e.g. Landau damping is highly suppressed)

Dimension: small $\epsilon = 3 - d$



Large N



Epsilon Expansion

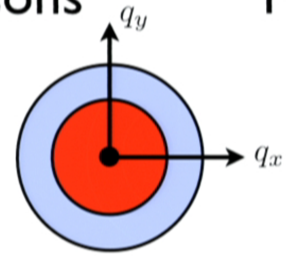
Work near upper critical dimension to find a scale-invariant fixed point at weak coupling

All three couplings are classically marginal in $d = 3$

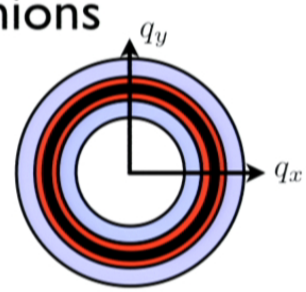
$$g\phi\psi^\dagger\psi \qquad \lambda_\phi\phi^4$$

$$\lambda_\psi(\psi^\dagger\psi)^2$$

Bosons



Fermions



$$g \to e^{\frac{3-d}{2}\lambda} g$$

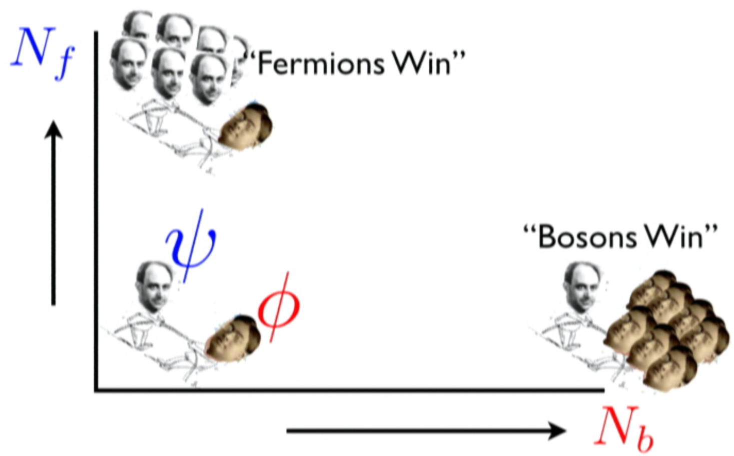
$$\lambda_\phi \to e^{(3-d)\lambda} \lambda_\phi$$

$$\lambda_\psi \to \lambda_\psi$$

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

Furthermore, consider simplifications in large N limits



Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
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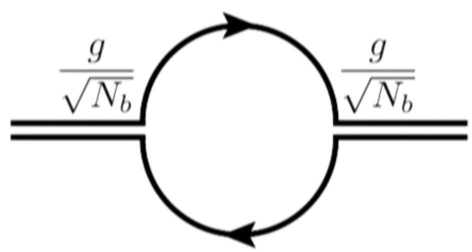
At $N_b \to \infty$ N_f fixed

"Bosons Win"



Landau Damping is a non-planar diagram and has no effect at infinite N_b

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed

$$\frac{\lambda_\phi^{(2)}}{8N_b^2}(\mathrm{tr}[\phi^2])^2 \qquad \frac{\lambda_\phi^{(1)}}{8N_b}\mathrm{tr}[\phi^4]$$

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_b \to \infty$ N_f fixed

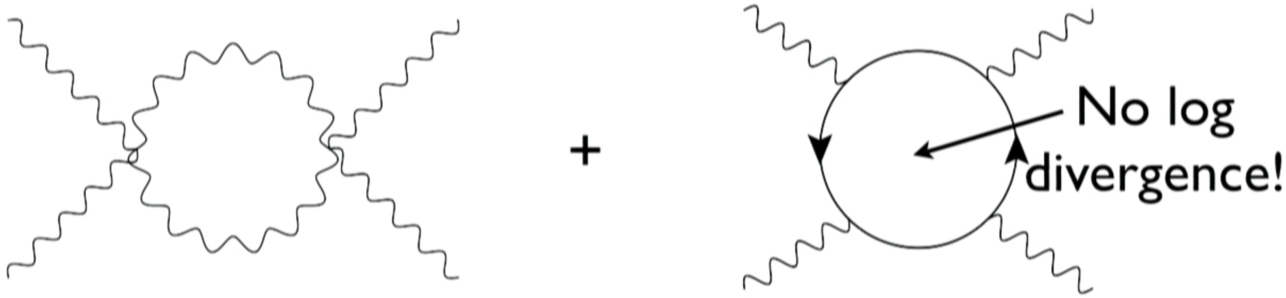
$\frac{\lambda_\phi^{(2)}}{8N_b^2}(\mathrm{tr}[\phi^2])^2$ $\frac{\lambda_\phi^{(1)}}{8N_b}\mathrm{tr}[\phi^4]$

One can set $\lambda_\phi^{(1)} = 0$ naturally (in the 't Hooft sense)

Then the ϕ sector is isomorphic to the SO(N_b^2) Wilson-Fisher fixed point

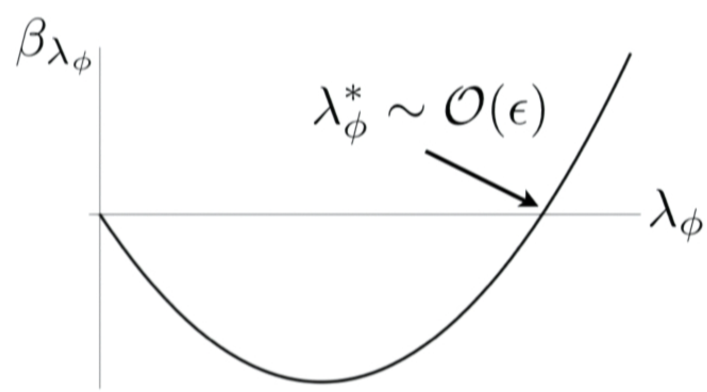
Epsilon Expansion

$$d = 3 - \epsilon$$



Scalar quartic running is the same as in Wilson Fisher

$$\frac{d}{d \log \mu} \lambda_\phi = -\epsilon \lambda_\phi + a_{\lambda_\phi} \lambda_\phi^2$$



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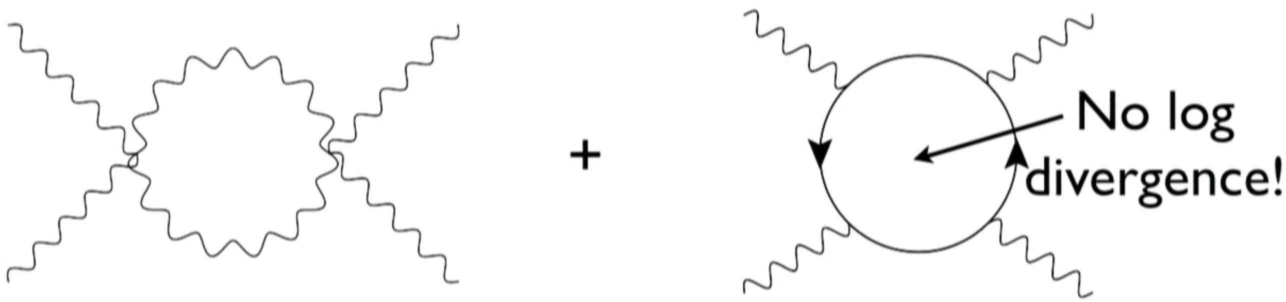

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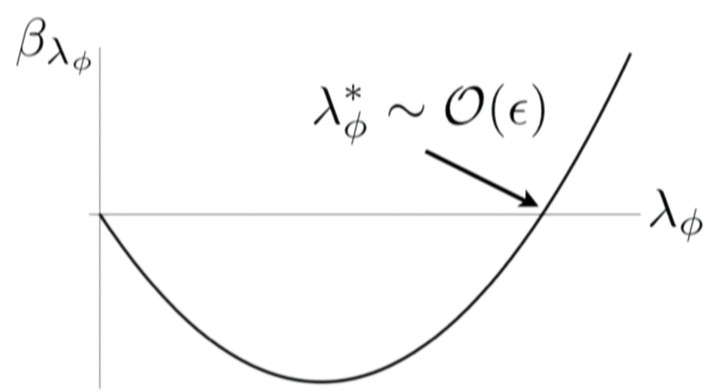


Running Velocity

Look at renormalization of quadratic terms

$$\mathcal{L} \supset \psi^\dagger(\omega - \epsilon(\ell))\psi$$

$$\ell = |k| - k_F$$

$$\epsilon(\ell) = v\ell + w\ell^2 + \ldots \qquad = k_x + \frac{k_y^2}{k_F} + \mathcal{O}(k_x^2, k_y^4)$$

Focus on limit where higher-derivative term is small

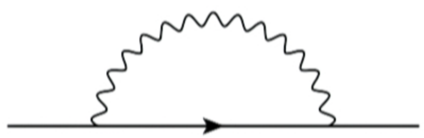
$$w\ell \ll v$$

Running Velocity

Look at renormalization of quadratic terms

$$\epsilon(\ell) = v\ell + w\ell^2 + \dots$$

When $w\ell \ll v$ one finds



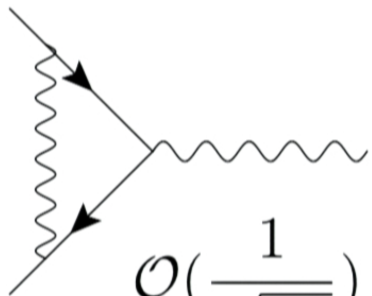
$$\sim \quad (Z\omega + Z_\ell \ell)\log(\Lambda)$$

$$\beta_v = \frac{g^2}{(2\pi)^2 c^2} \quad \Rightarrow \quad v \sim v_0 - \frac{g^2}{(2\pi)^2 c^2}\log\frac{\Lambda}{\mu}$$

Velocity flows toward zero faster than any power of RG scale μ

Epsilon Expansion

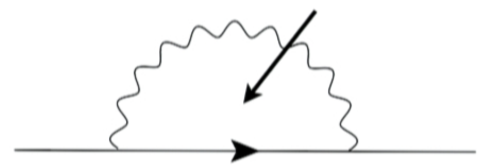
$$d = 3 - \epsilon$$



$\mathcal{O}(\frac{1}{\sqrt{N_b}})$

+

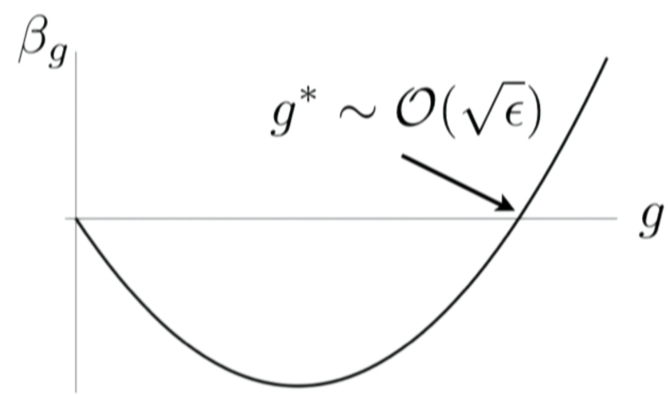
γ_ψ from Wavefunction renormalization



$\mathcal{O}(N_b^0)$

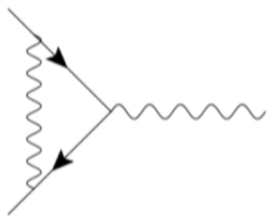
Yukawa runs to IR fixed point

$$\frac{d}{d\log\mu} g = -g\left(\frac{\epsilon}{2} - a_g g^2\right) + \ldots$$

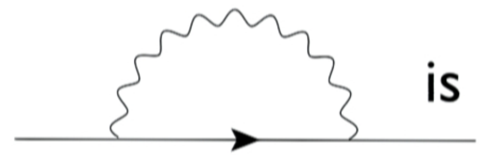


Large N Dials

At $N_b \to \infty \quad N_f$ fixed



is $\mathcal{O}(\frac{1}{\sqrt{N_b}})$



is $\mathcal{O}(N_b^0)$

So all running of g is through wavefunction renormalization: $$\frac{d}{d\log\mu}g = -g\left(\frac{\epsilon}{2} - 2\gamma_\psi(g)\right)$$

Scale-invariant fixed point even for $\epsilon \sim \mathcal{O}(1)$ $$2\gamma_\psi = \frac{\epsilon}{2}$$

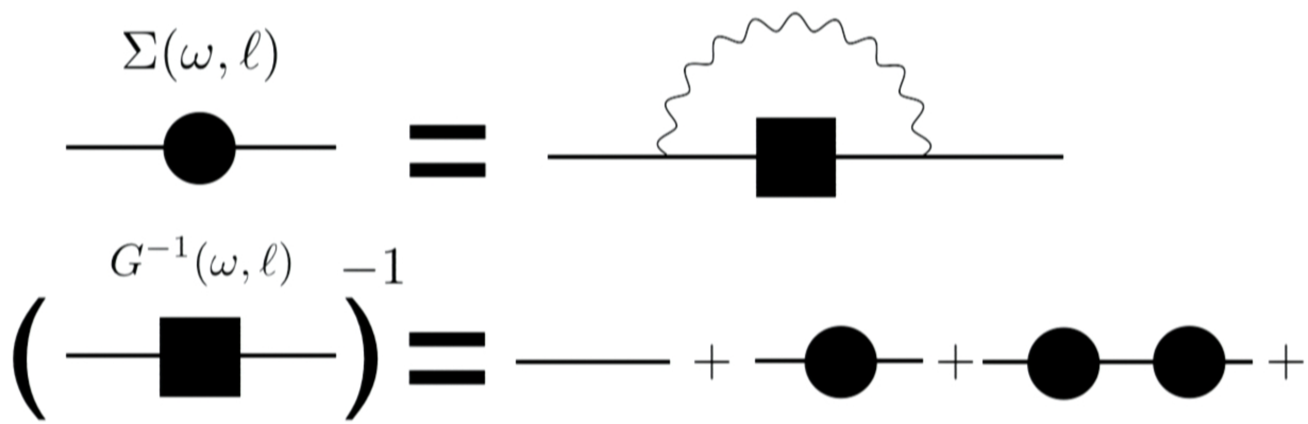
The fermion Green's function therefore takes the form $$G(\omega, p) = \frac{1}{\omega^{1-2\gamma_\psi}} f\left(\frac{\omega}{p}\right)$$

Large N Dials

At $N_b \to \infty$ N_f fixed

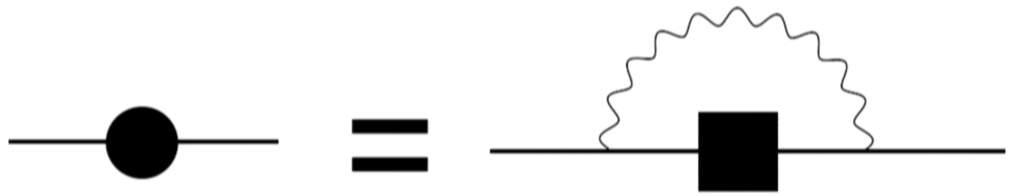
Actually, we can even calculate the scaling function $f\left(\frac{\omega}{p}\right)$

Gap equation for fermion Green's function



Large N Dials

At $N_b \to \infty$ N_f fixed



Solution: $G(\omega, p) = \frac{1}{\omega^{1-\frac{\epsilon}{2}}}$ $f\left(\frac{\omega}{p}\right) = 1$

Large N Dials

	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
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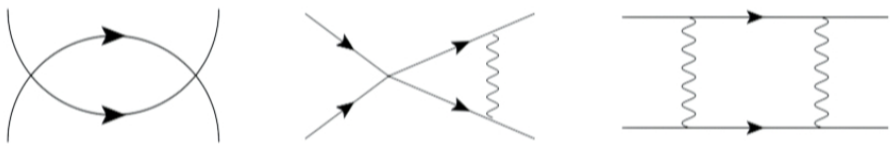
At $N_b \to \infty$ N_f fixed

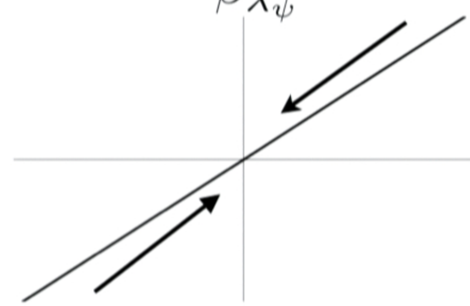


are all $\mathcal{O}(\frac{1}{N_b})$

The only contribution to four-fermi running is wavefunction renormalization

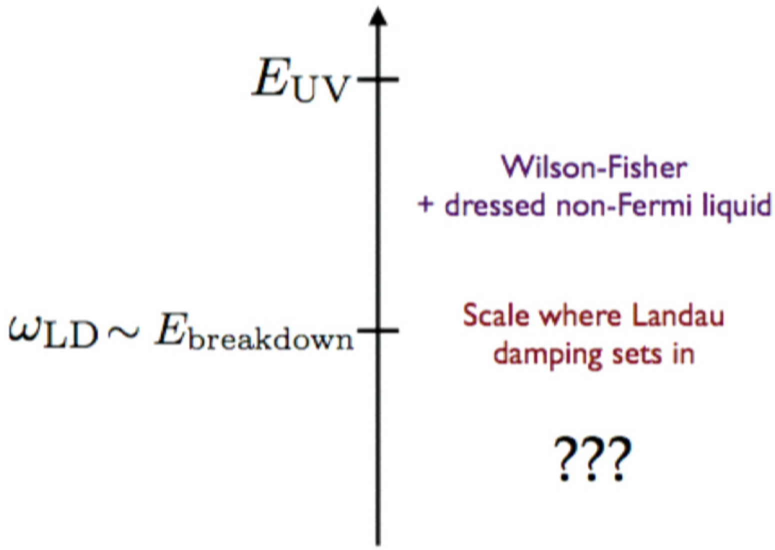
$$\frac{d\lambda_\psi}{d\log\mu} = 4\gamma_\psi\lambda_\psi$$

Stable against superconductivity near origin



Epsilon Expansion

Landau damping pushed to low scale

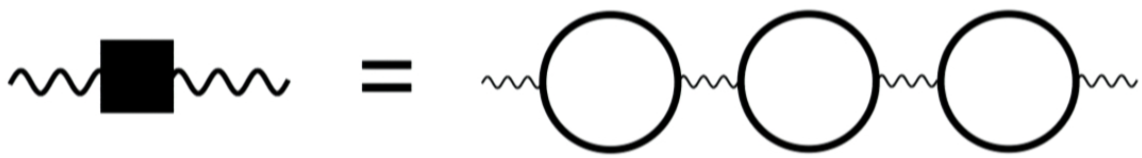


Large N Dials

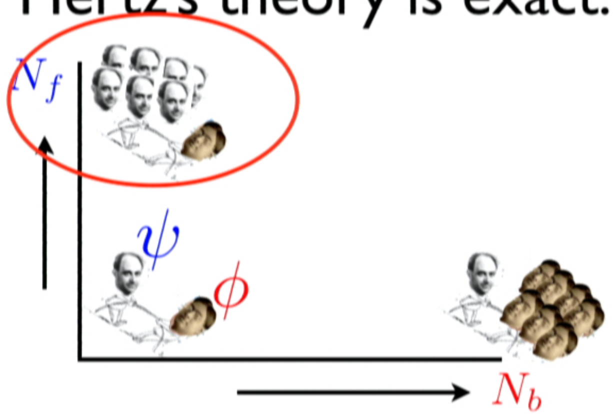
	$SU(N_b)$	$SU(N_f)$
ϕ_i^j	Adj	1
ψ_i^A	$\square$	$\bar{\square}$

At $N_f \to \infty \quad N_b$ fixed

"Fermions Win"



Hertz's theory is exact: $G_\phi(\omega, p) = \dfrac{1}{\omega^2 + c_s^2 p^2 + \Pi(\omega, p)}$



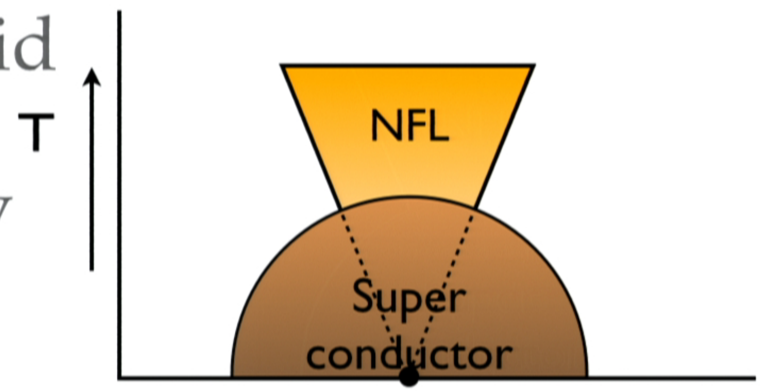
Ordering of Scales

We have three major scales in the problem:

1) Fermi Liquid
→ Non-Fermi Liquid

2) Superconductivity

3) Landau Damping



see also:
Metlitski, Mross, Sachdev, Senthil, '14

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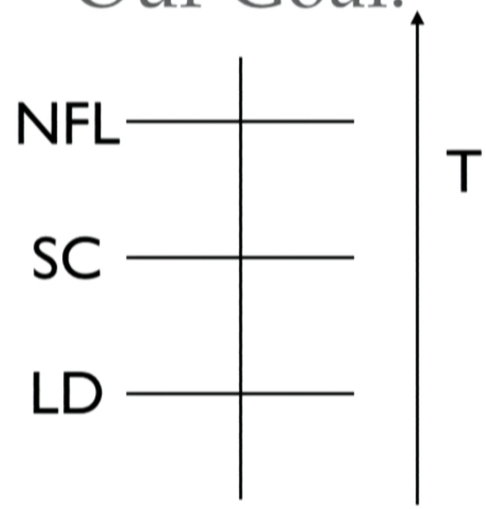


see also:
Metlitski, Mross, Sachdev, Senthil, '14

Ordering of Scales

In the large N theory so far, near d=3 we have the following ordering:

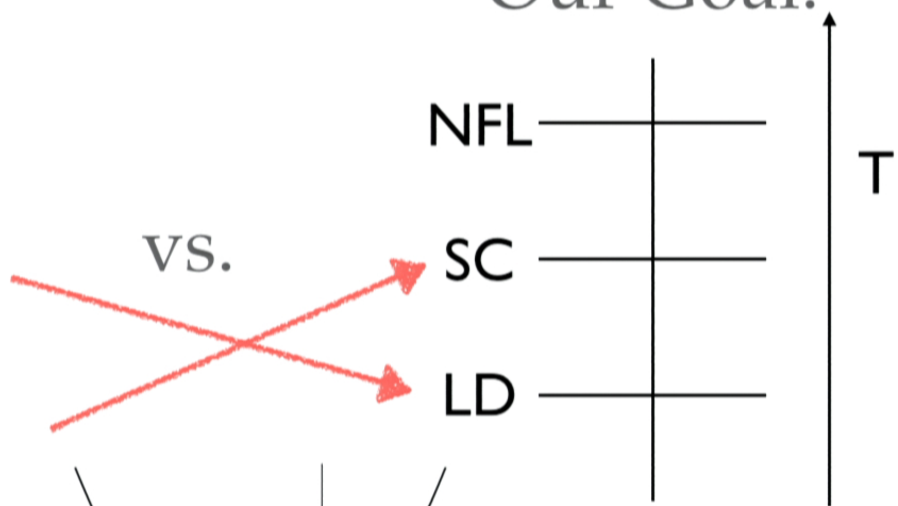
NFL: $\sim k_F e^{-1/g^2}$

LD: $\sim k_F \sqrt{\frac{g}{N}}$

SC: $\sim k_F e^{-N/g}$

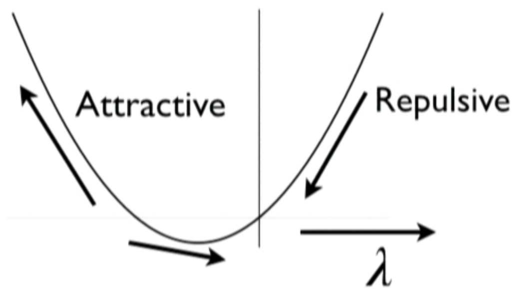
vs.

Our Goal:



$N = \infty$:

$\beta_\lambda \sim 4\gamma_\psi \lambda + \lambda^2$



Ordering of Scales

Cheap ad hoc solution:
add an attractive BCS interaction λ
in the UV by hand.

Now: SC $\sim k_F e^{-1/|\lambda|}$

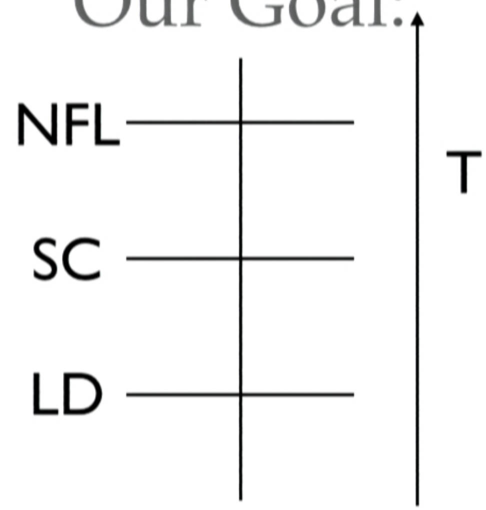
NFL: $\sim k_F e^{-1/g^2}$

LD: $\sim k_F \sqrt{\frac{g}{N}}$

SC: $\sim k_F e^{-1/\lambda}$

vs.

Our Goal:

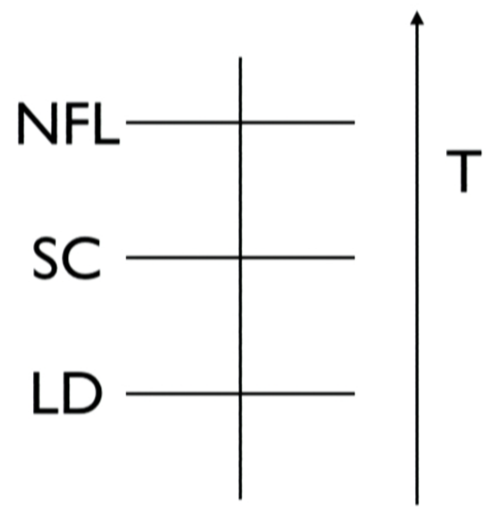


Ordering of Scales

But we want an example where the enhanced pairing scale is directly caused by the quantum critical point.

There are various parameters and modifications of the theory that can be explored. Hopefully, one of them has the desired ordering of scales. Experiments suggest such a limit should exist!

Our Goal:



Conclusion

Non-Fermi liquids have new dynamics in need of a theoretical description

We are looking for local EFTs of the Fermi surface (plus light states) that exhibit similar dynamics

A rich structure of such theories exists depending on various parameters of the theory

In some limits (large N, small ϵ) the theory can be analyzed perturbatively and leads to new approximate fixed points