Title: Can Eigenstate Thermalization Breakdown without Disorder?

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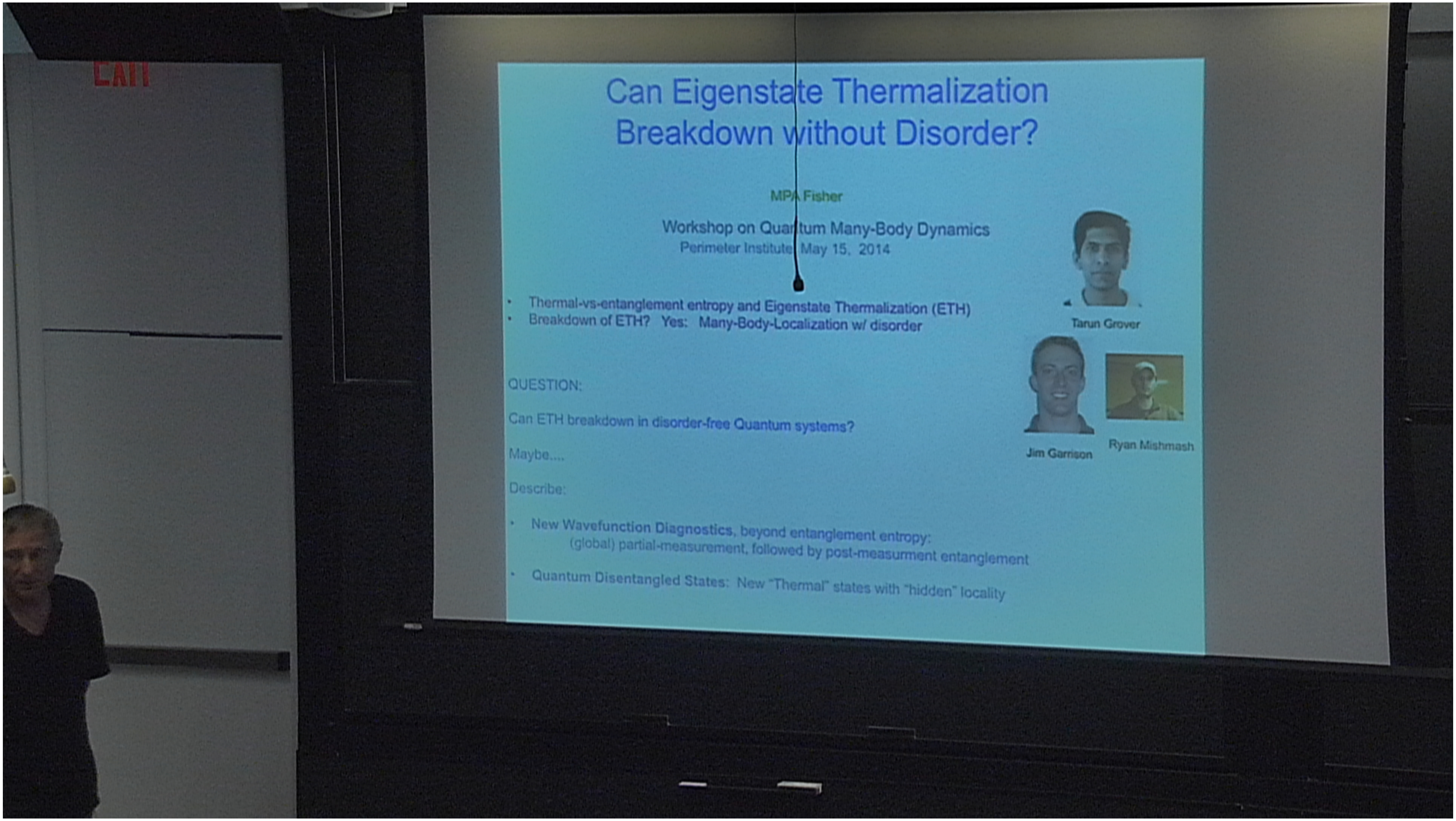
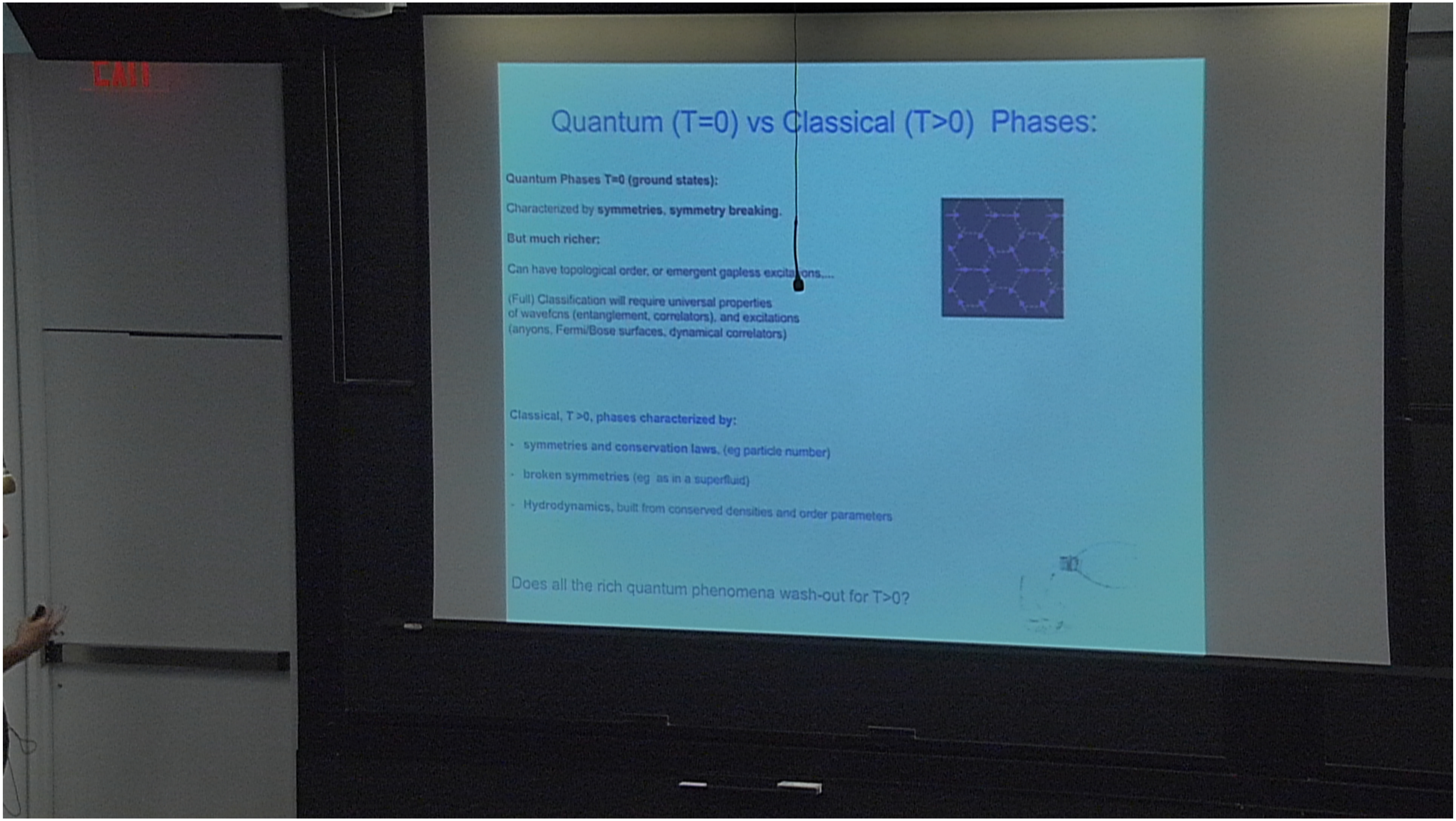
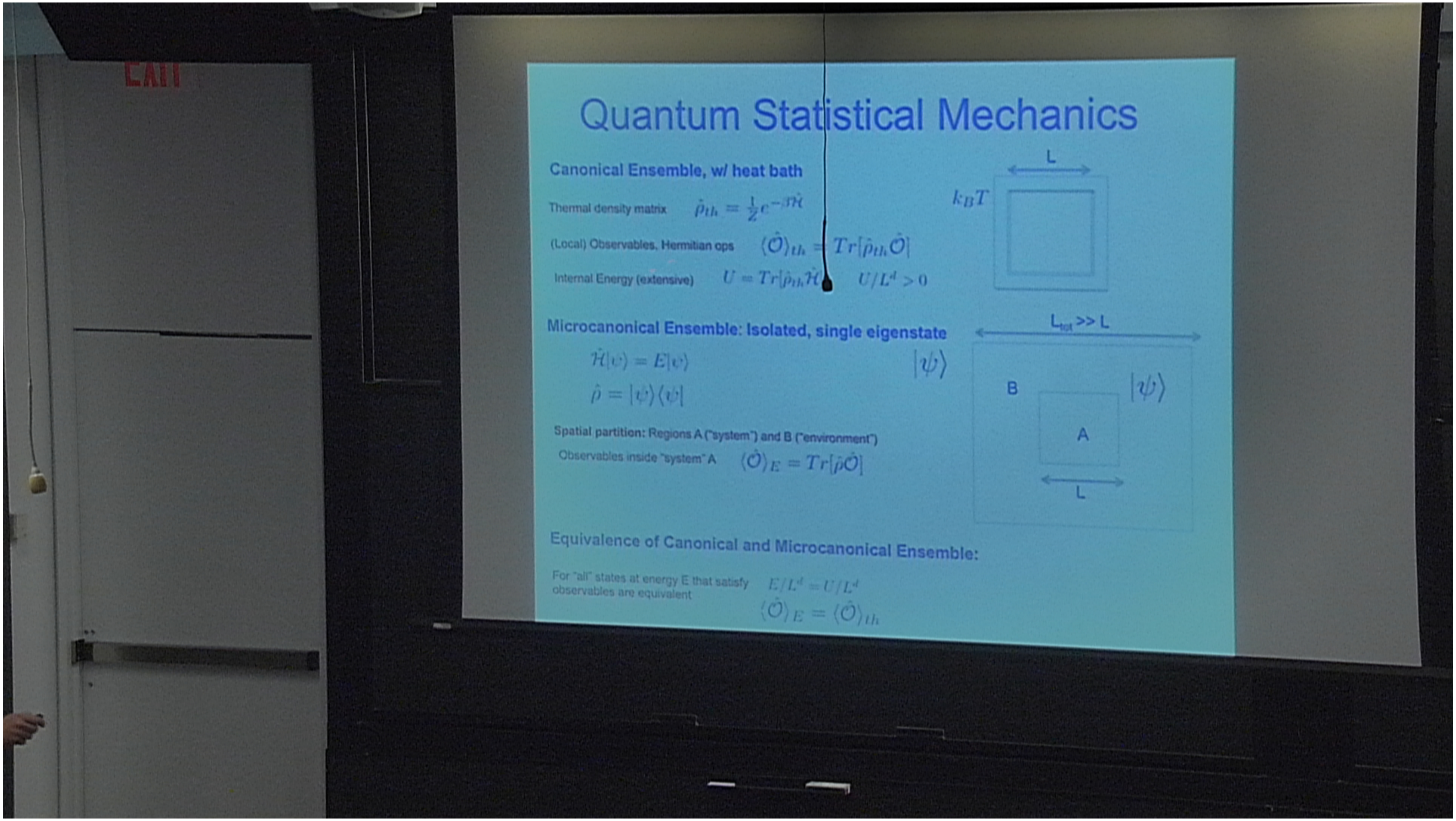
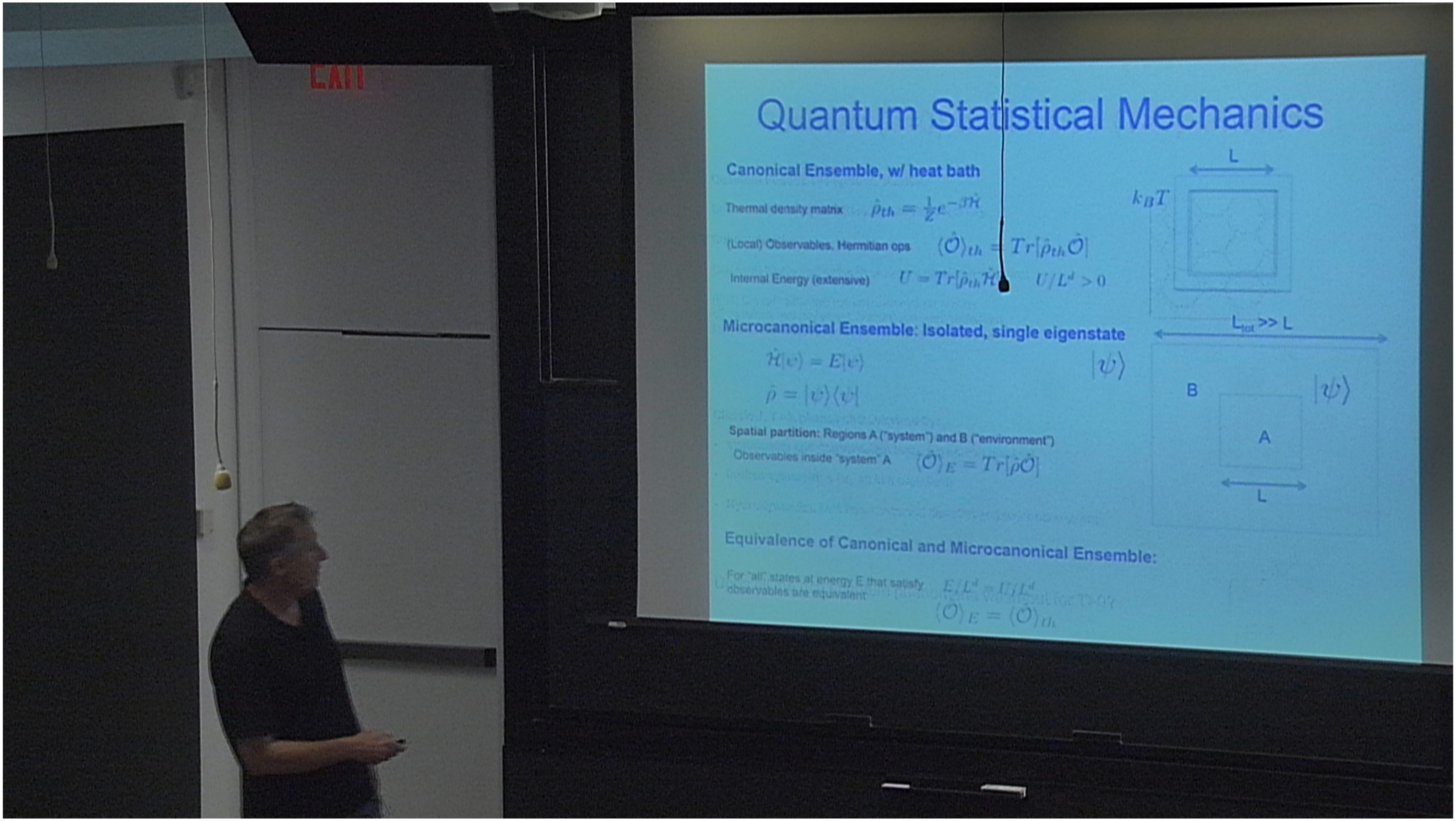
Abstract: We describe a new diagnostic for many-body wavefunctions which generalizes the spatial bipartite entanglement entropy. By was of illustration, for a two-component wavefunction of heavy and light particles, a partial (projective) measurement of the coordinates of the heavy (but not light) particles is first performed, and then the entanglement entropy of the projected wavefunction for the light particles is computed. If the two-component wavefunction has a volume law entanglement entropy, yet the post measurement wavefunction of the light particles is disentangled with an area law entanglement, we refer to the original wavefunction as a â€œQuantum Disentangled Stateâ€•. This diagnostic can be generalized to include other partial measurements, such as measuring the charge, but not spin, for finite-energy density eigenstates of Fermion Hubbard-type model. Quantum disentanglement results if the post measurement spin-wavefunction has an area law entanglement entropy. Recent numerics searching for such Quantum Disentangled States in 1d Hubbard-type models will be discussed in detail.


Can Eigenstate Thermalization
Breakdown without Disorder?
MPA Fisher
Workshop on Quantum Many-Body Dynamics
Perimeter Institute May 15, 2014
Thermal-vs-entanglement entropy and Eigenstate Thermalization (ETH)
Breakdown of ETH? Yes: Many-Body-Localization w/ disorder
Tarun Grover
QUESTION:
Can ETH breakdown in disorder-free Quantum systems?
Maybe....
Jim Garrison
Ryan Mishmash
Describe:
New Wavefunction Diagnostics, beyond entanglement entropy:
(global) partial-measurement, followed by post-measurment entanglement
Quantum Disentangled States: New "Thermal" states with "hidden" locality


Quantum (T=0) vs Classical (T>0) Phases:
Quantum Phases T=0 (ground states):
Characterized by symmetries, symmetry breaking.
But much richer:
Can have topological order, or emergent gapless excitations,...
(Full) Classification will require universal properties of wavefcns (entanglement, correlators), and excitations (anyons, Fermi/Bose surfaces, dynamical correlators)
Classical, T >0, phases characterized by:
symmetries and conservation laws. (eg particle number)
broken symmetries (eg as in a superfluid)
Hydrodynamics, built from conserved densities and order parameters
Does all the rich quantum phenomena wash-out for T>0?


Quantum Statistical Mechanics
Canonical Ensemble, w/ heat bath
Thermal density matrix $\hat{\rho}_{th} = \frac{1}{Z} e^{-\beta \hat{\mathcal{H}}}$
(Local) Observables, Hermitian ops $\langle \hat{\mathcal{O}} \rangle_{th} = Tr[\hat{\rho}_{th} \hat{\mathcal{O}}]$
Internal Energy (extensive) $U = Tr[\hat{\rho}_{th} \hat{\mathcal{H}}]$ $U/L^d > 0$
L
$k_B T$
Microcanonical Ensemble: Isolated, single eigenstate
$\hat{\mathcal{H}}|\psi\rangle = E|\psi\rangle$
$\hat{\rho} = |\psi\rangle\langle\psi|$
$|\psi\rangle$
L_{tot} >> L
B
A
L
Spatial partition: Regions A ("system") and B ("environment")
Observables inside "system" A $\langle \hat{\mathcal{O}} \rangle_E = Tr[\hat{\rho} \hat{\mathcal{O}}]$
Equivalence of Canonical and Microcanonical Ensemble:
For "all" states at energy E that satisfy $E/L^d = U/L^d$
observables are equivalent
$\langle \hat{\mathcal{O}} \rangle_E = \langle \hat{\mathcal{O}} \rangle_{th}$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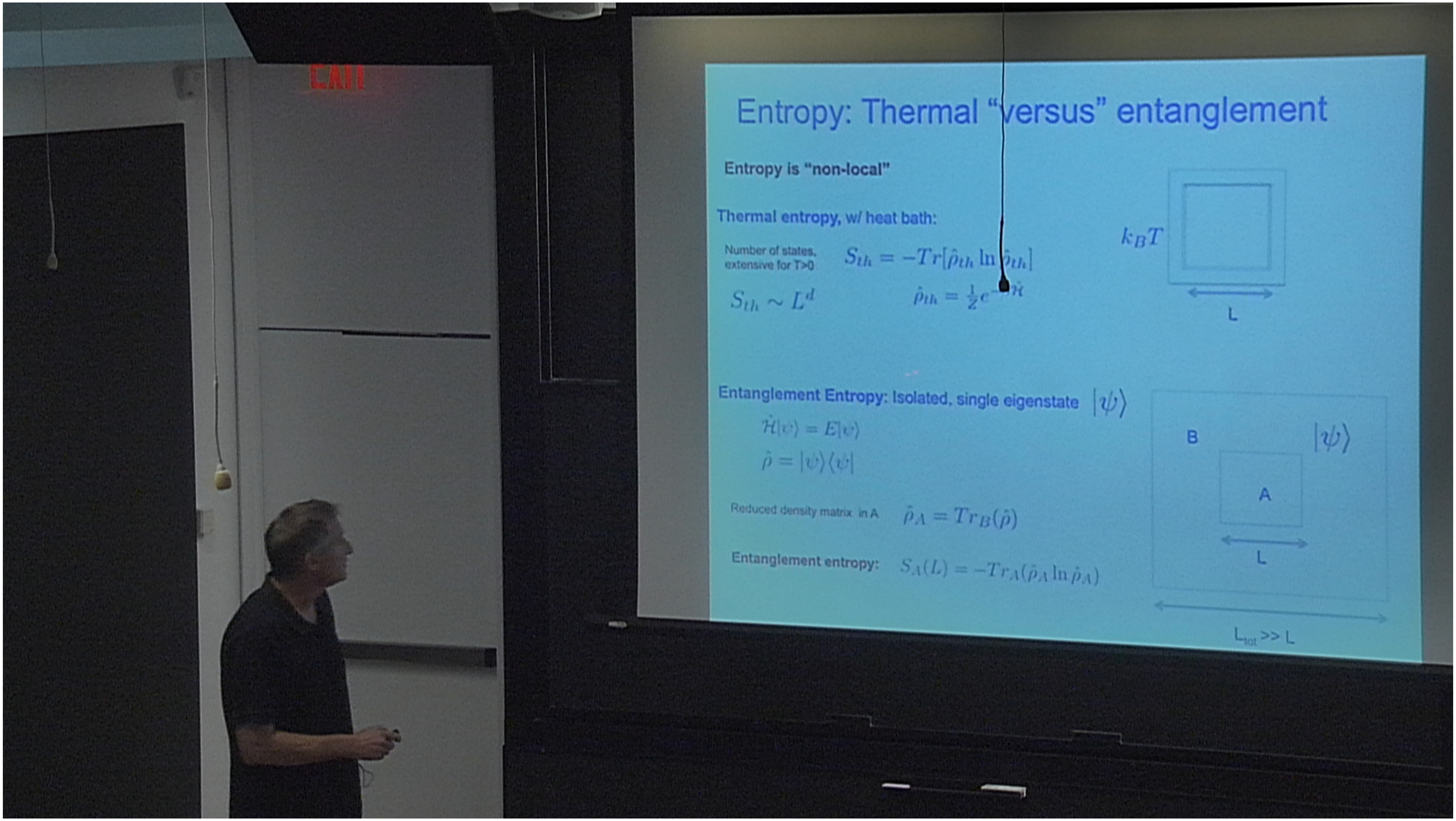

Entropy: Thermal "versus" entanglement
Entropy is "non-local"
Thermal entropy, w/ heat bath:
Number of states, extensive for T>0
$S_{th} = -Tr[\hat{\rho}_{th} \ln \hat{\rho}_{th}]$
$S_{th} \sim L^d$
$\hat{\rho}_{th} = \frac{1}{Z}e^{-\beta\hat{\mathcal{H}}}$
k_BT
L
Entanglement Entropy: Isolated, single eigenstate $|\psi\rangle$
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Reduced density matrix in A $\hat{\rho}_A = Tr_B(\hat{\rho})$
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B
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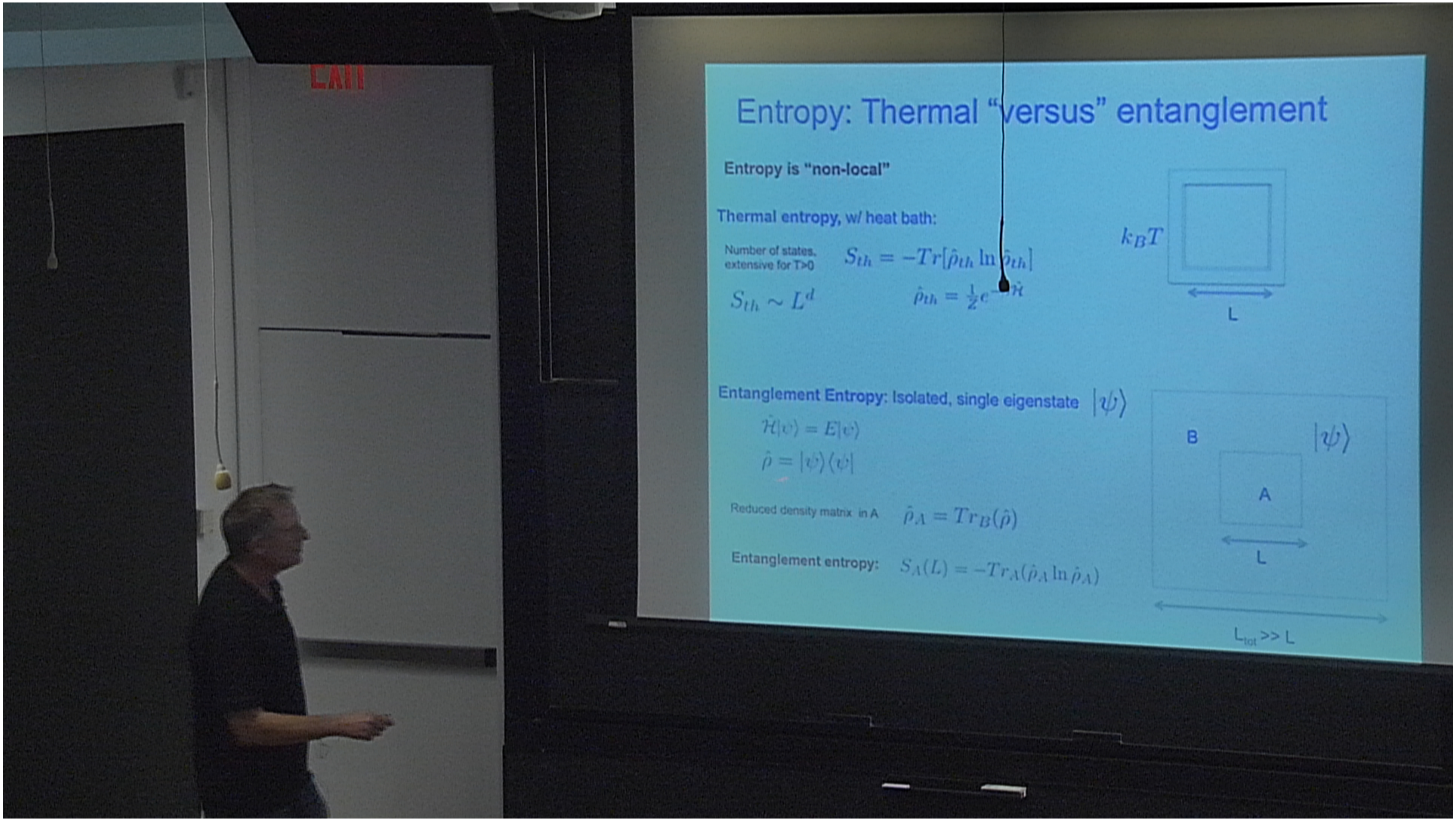

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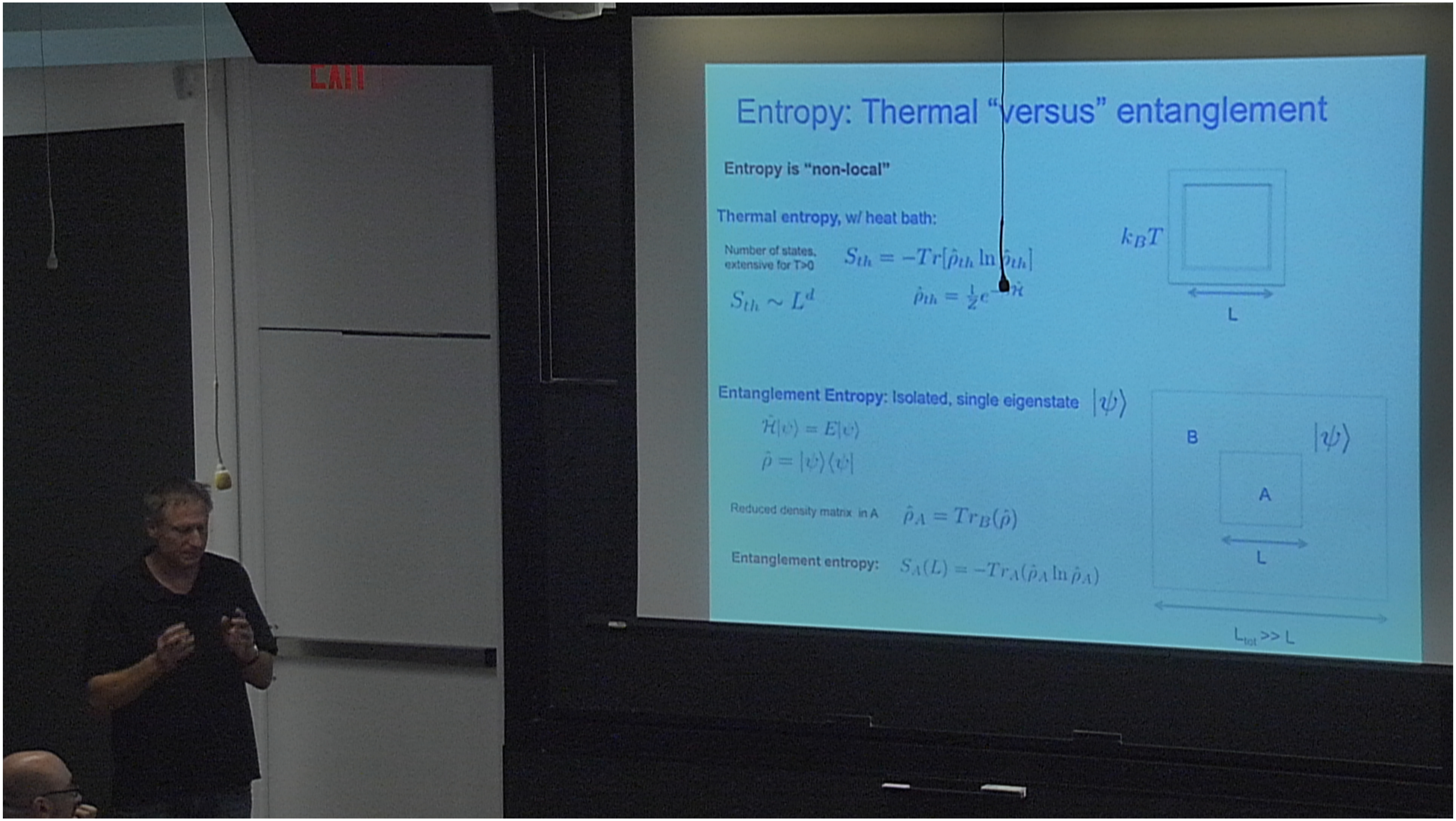
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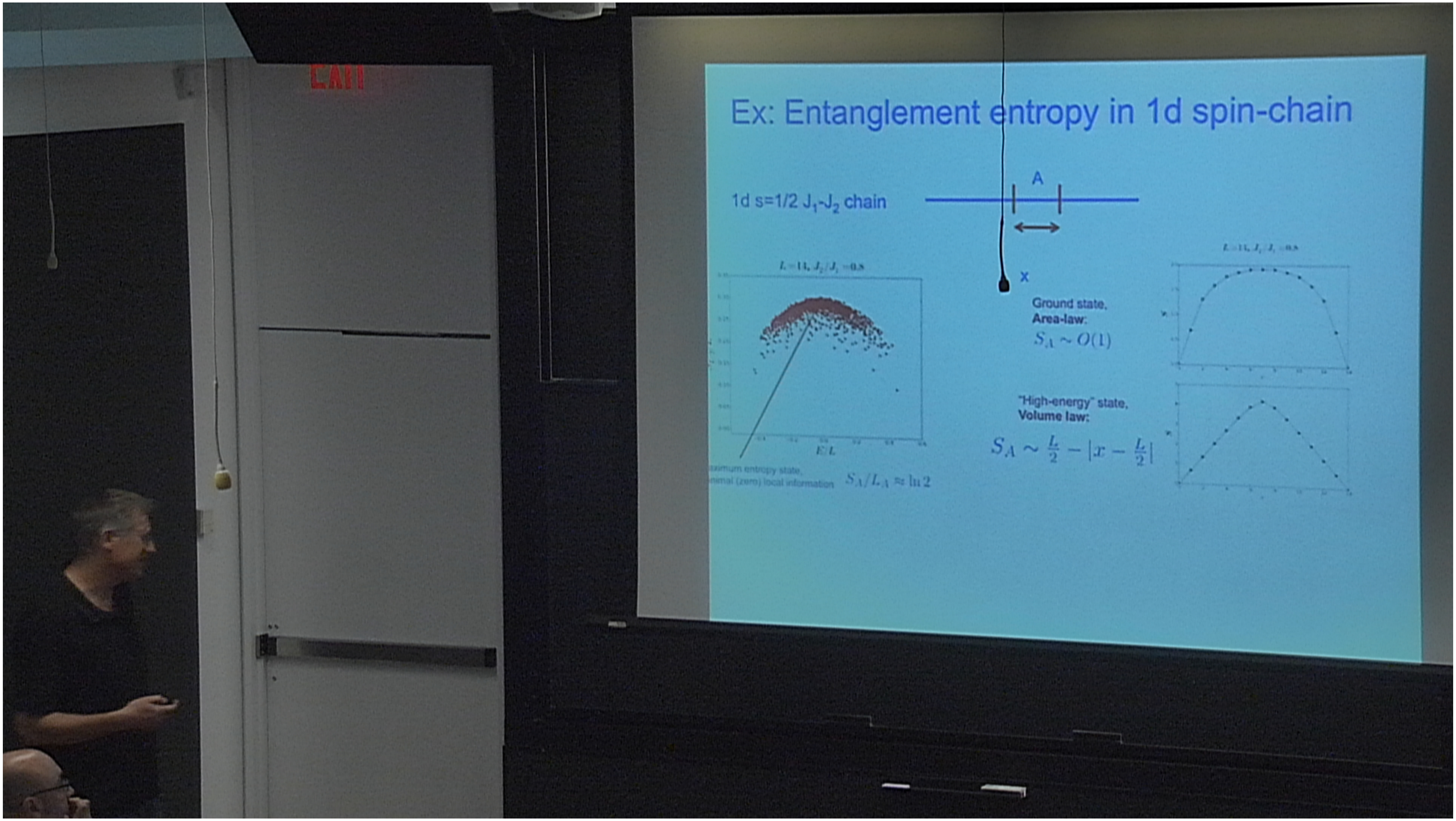
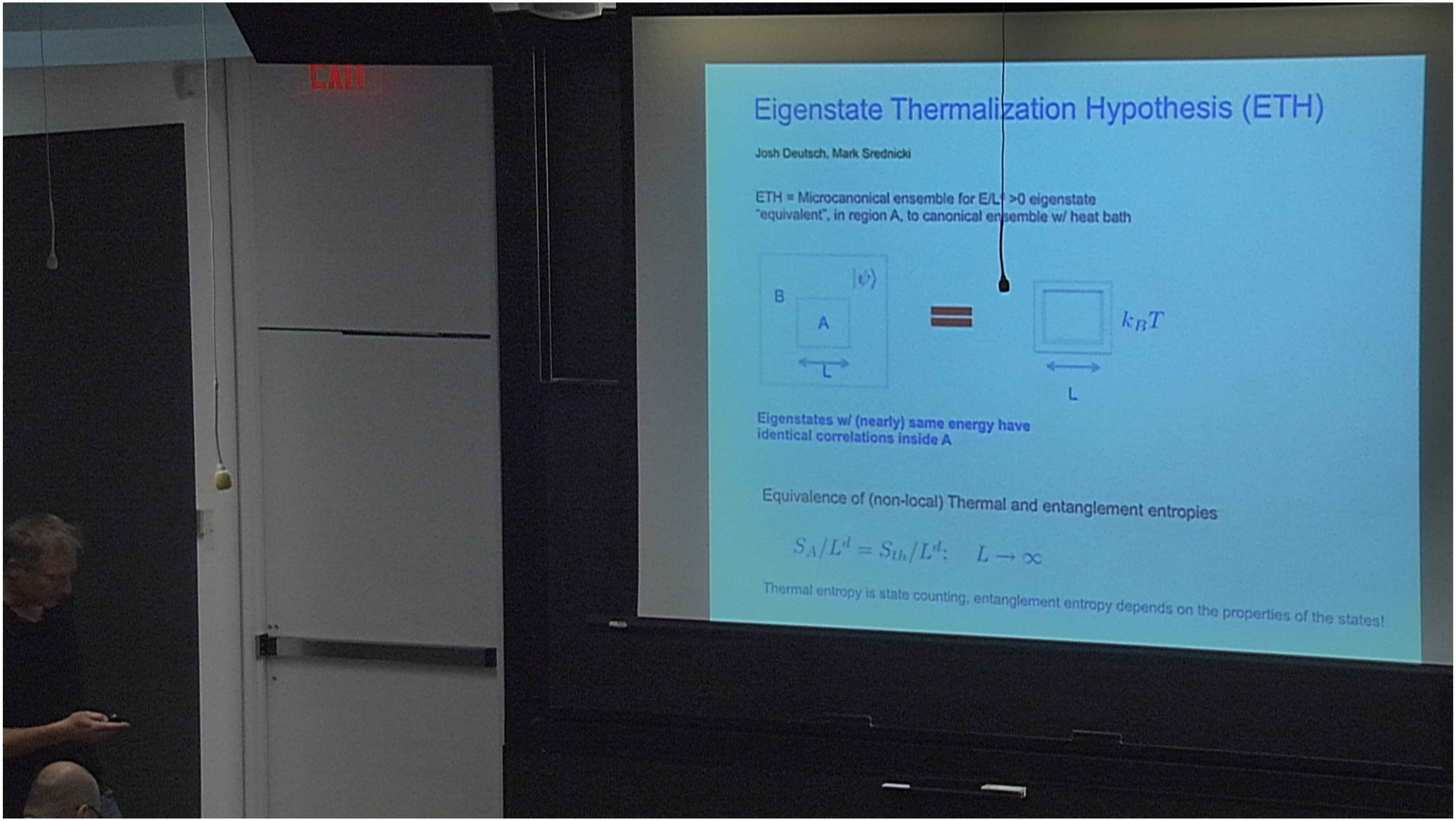


Ex: Entanglement entropy in 1d spin-chain
1d s=1/2 J1-J2 chain
A
x
Ground state,
Area-law:
$S_A \sim O(1)$
"High-energy" state,
Volume law:
$S_A \sim \frac{L}{2} - |x - \frac{L}{2}|$
$S_A/L_A \approx \ln 2$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Josh Deutsch, Mark Srednicki
ETH = Microcanonical ensemble for E/L >0 eigenstate
"equivalent", in region A, to canonical ensemble w/ heat bath
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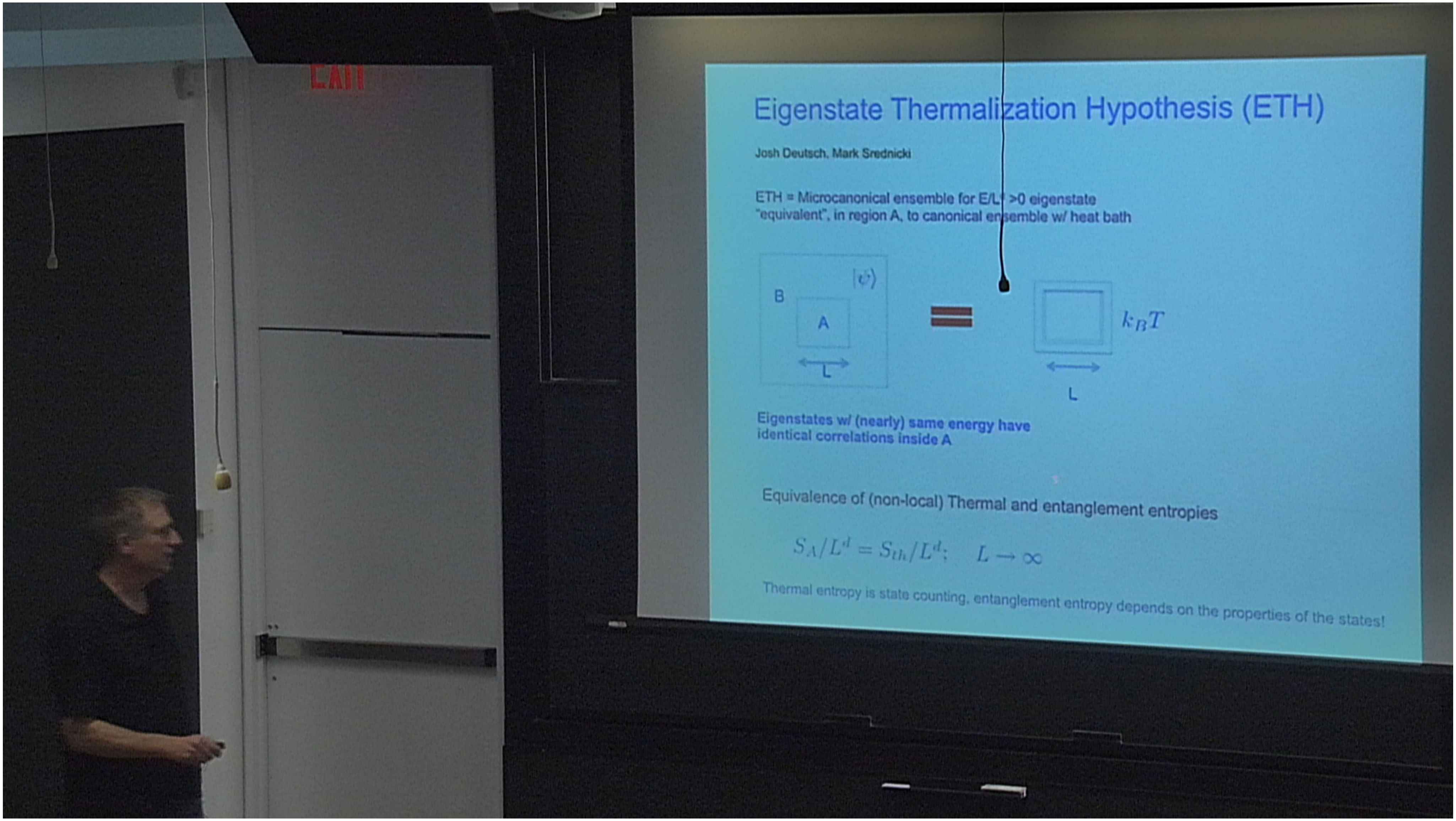
Eigenstates w/ (nearly) same energy have
identical correlations inside A
Equivalence of (non-local) Thermal and entanglement entropies
$S_A/L^d = S_{th}/L^d; \quad L \to \infty$
Thermal entropy is state counting, entanglement entropy depends on the properties of the states!



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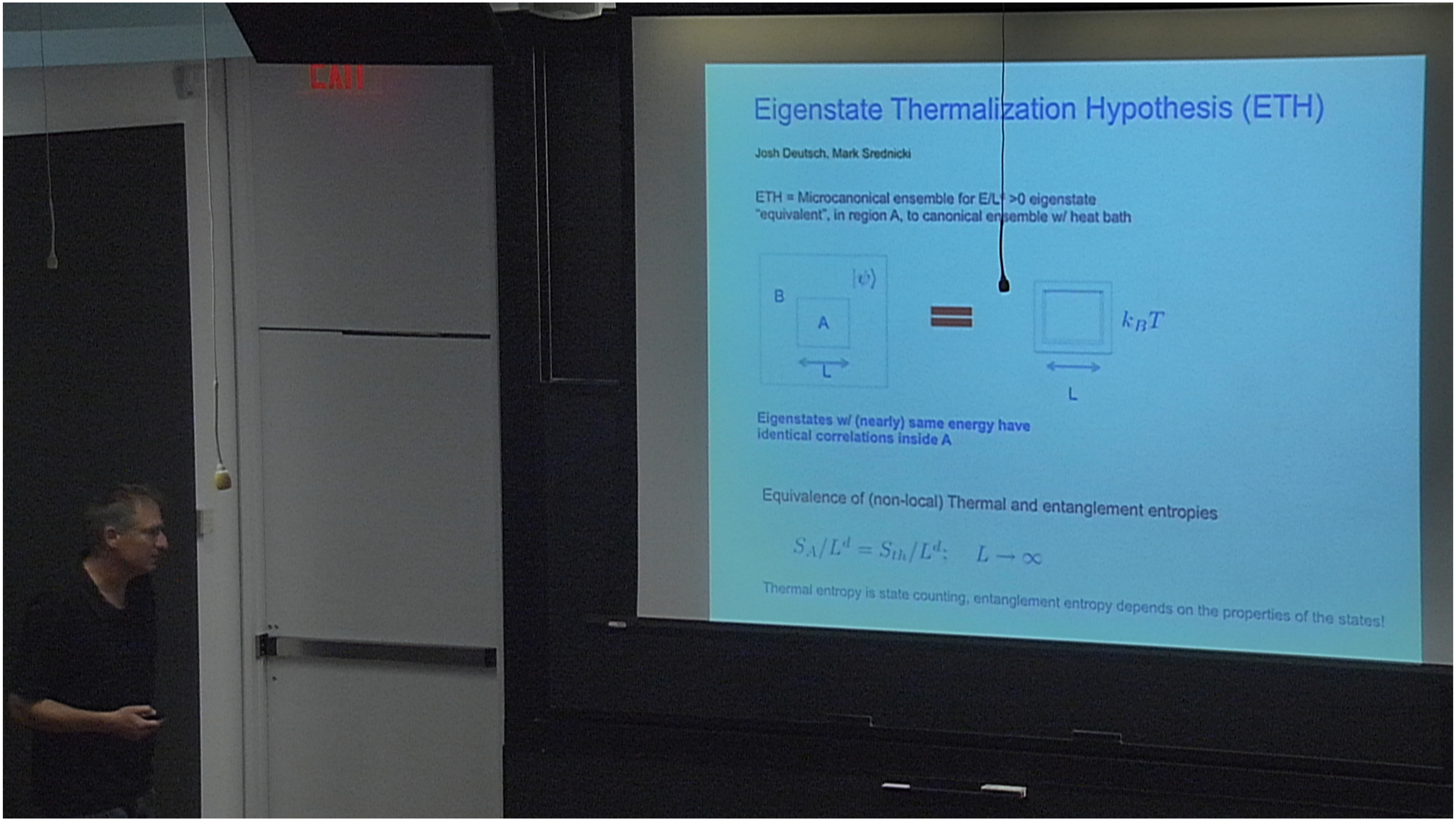
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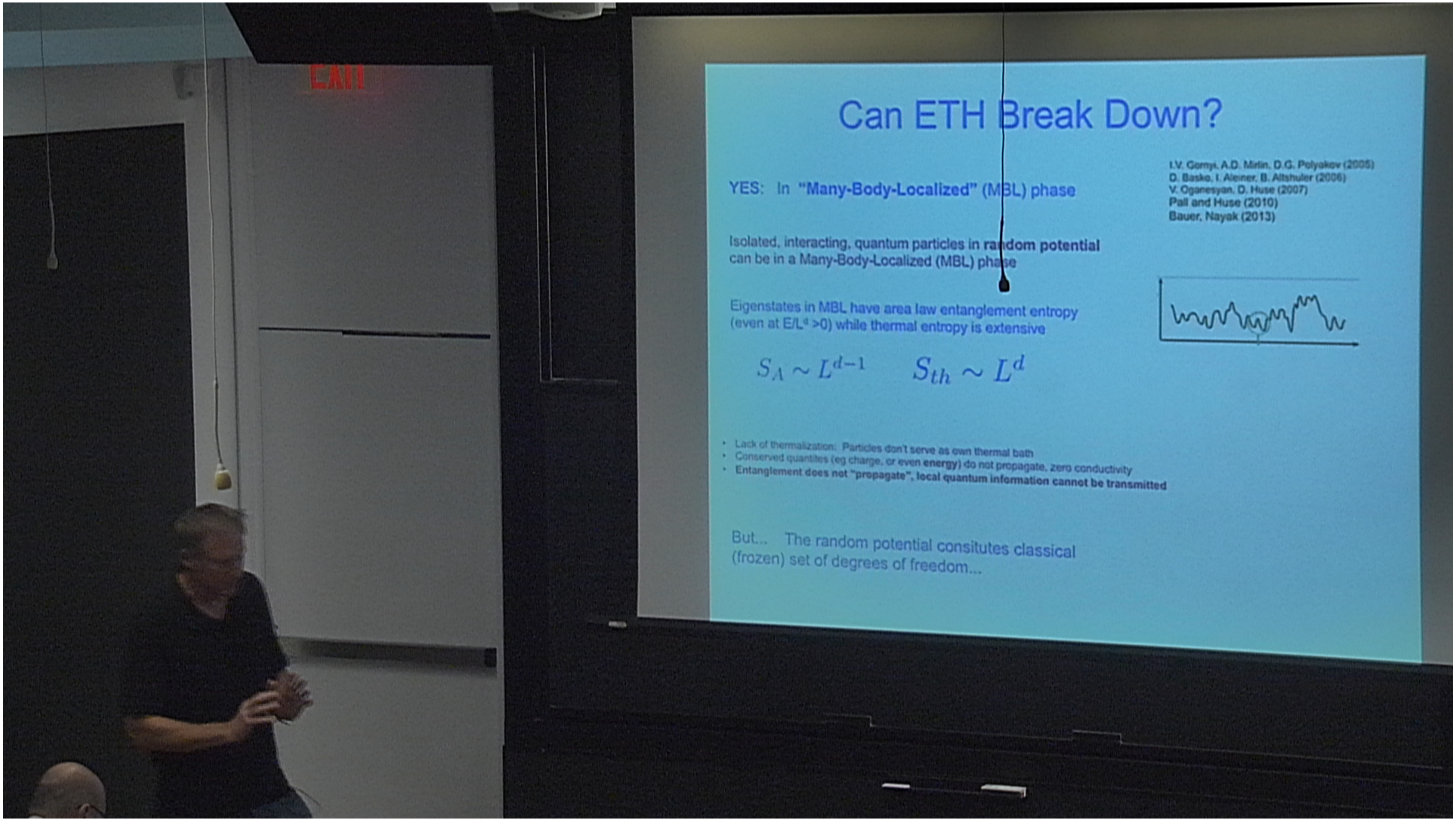
Can ETH Break Down?
YES: In "Many-Body-Localized" (MBL) phase
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D. Basko, I. Aleiner, B. Altshuler (2006)
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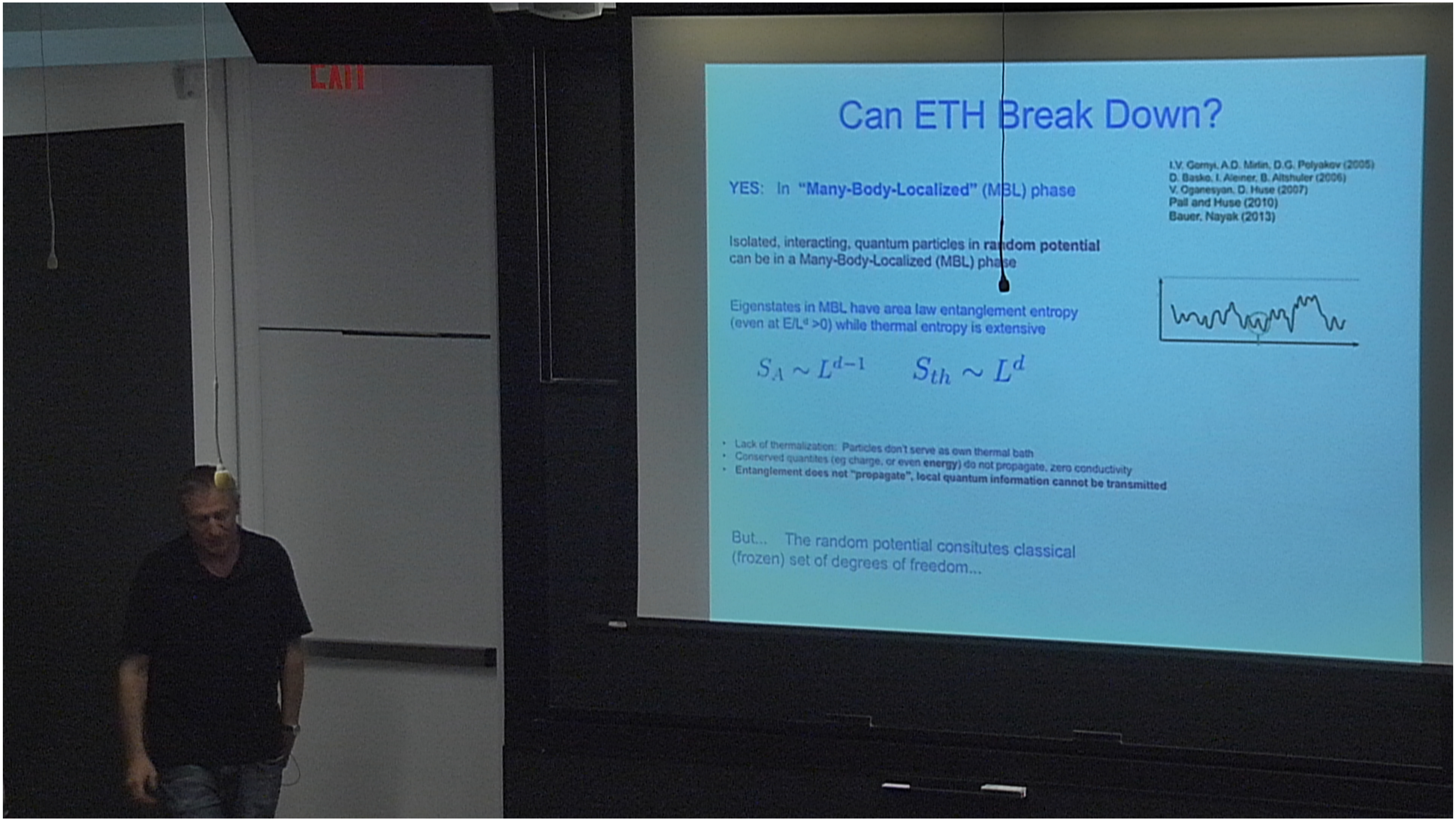
Pall and Huse (2010)
Bauer, Nayak (2013)
Isolated, interacting, quantum particles in random potential can be in a Many-Body-Localized (MBL) phase
Eigenstates in MBL have area law entanglement entropy (even at E/L^d >0) while thermal entropy is extensive
$S_A \sim L^{d-1}$ $S_{th} \sim L^d$
• Lack of thermalization: Particles don't serve as own thermal bath
• Conserved quantites (eg charge, or even energy) do not propagate, zero conductivity
• Entanglement does not "propagate", local quantum information cannot be transmitted
But... The random potential consitutes classical (frozen) set of degrees of freedom...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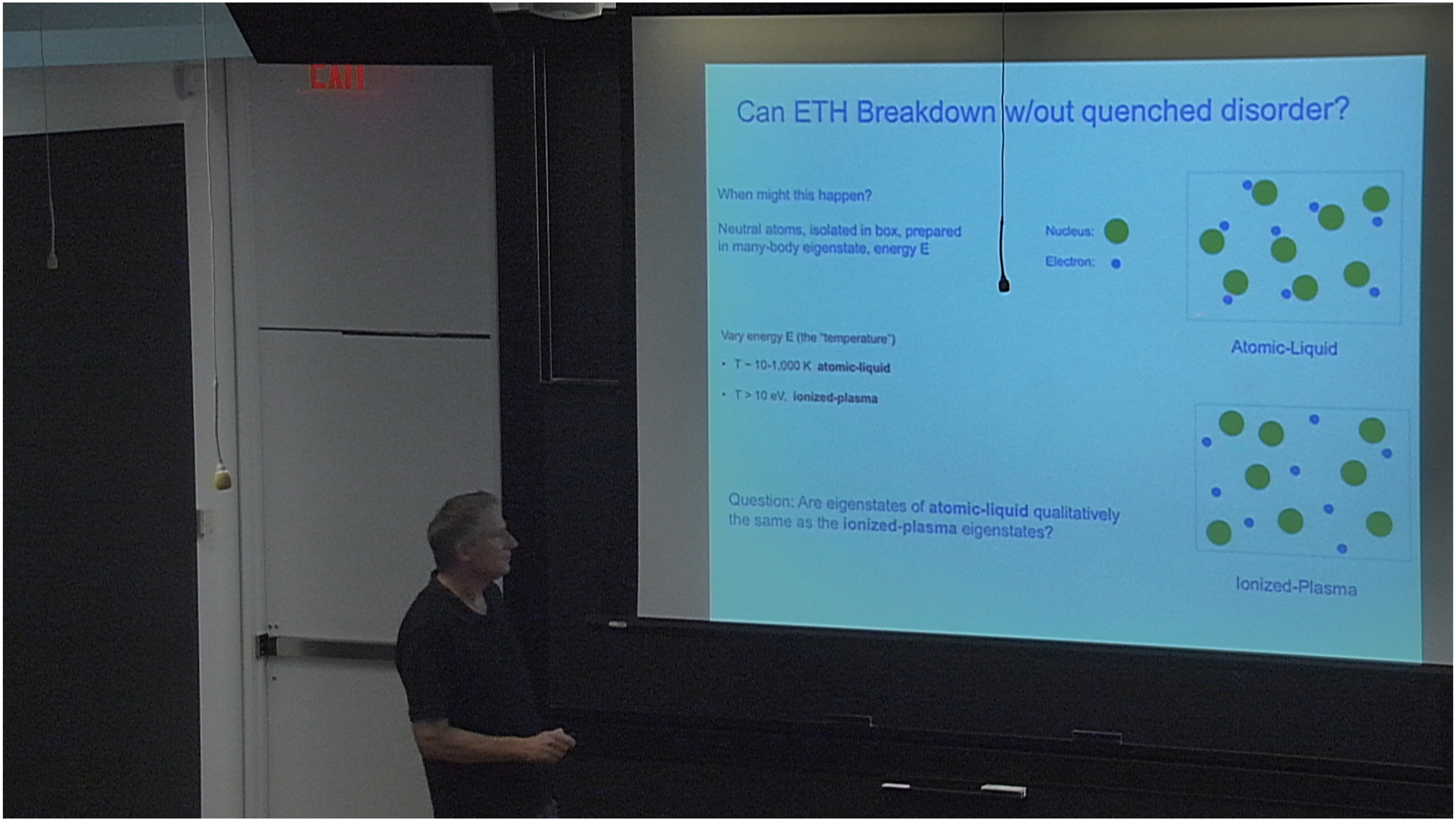
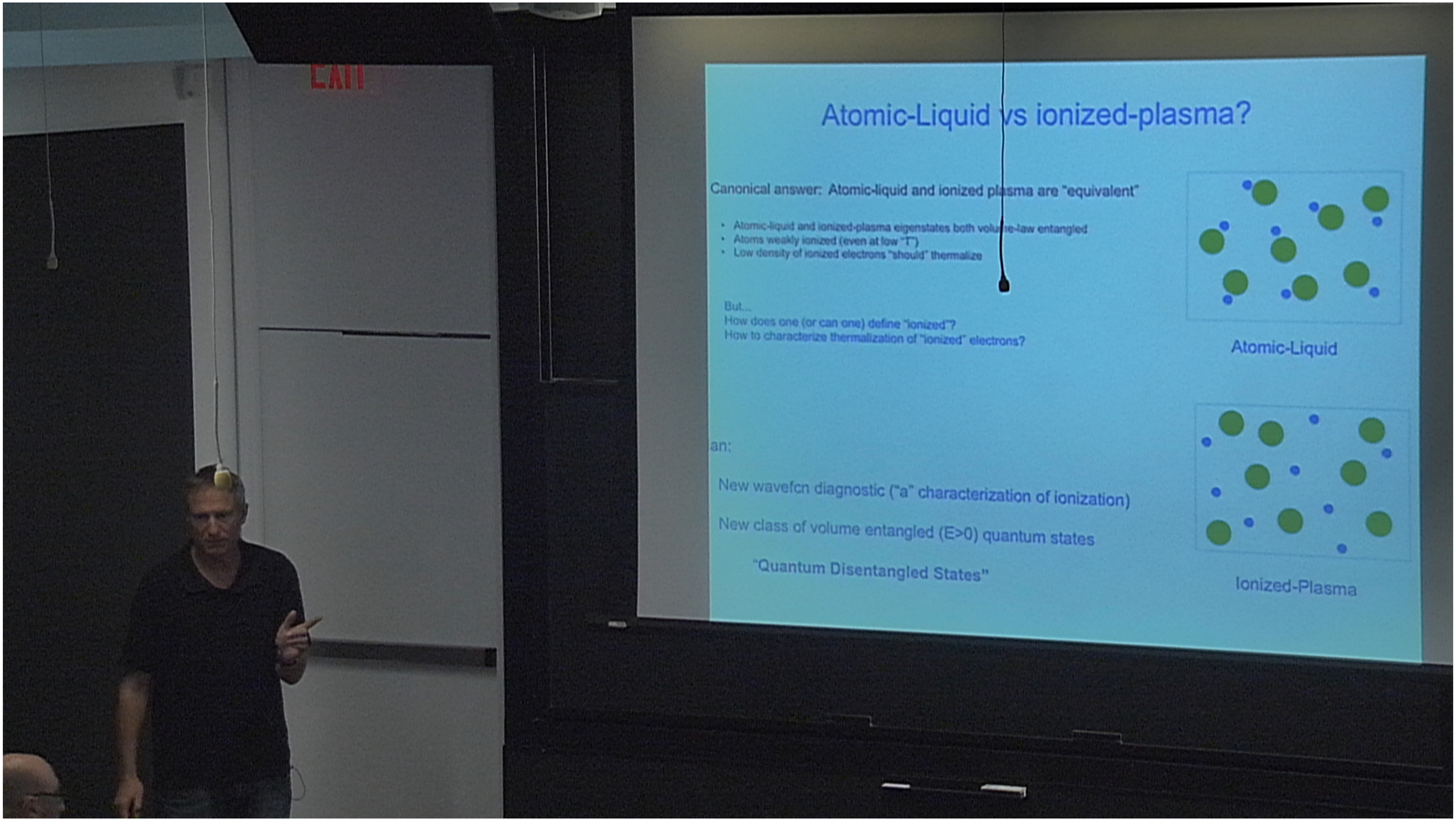

Can ETH Breakdown w/out quenched disorder?
When might this happen?
Neutral atoms, isolated in box, prepared in many-body eigenstate, energy E
Nucleus:
Electron:
Atomic-Liquid
Vary energy E (the "temperature")
• T ~ 10-1,000 K atomic-liquid
• T > 10 eV, ionized-plasma
Question: Are eigenstates of atomic-liquid qualitatively the same as the ionized-plasma eigenstates?
Ionized-Plasma


Atomic-Liquid vs ionized-plasma?
Canonical answer: Atomic-liquid and ionized plasma are "equivalent"
Atomic-liquid and ionized-plasma eigenstates both volume-law entangled
Atoms weakly ionized (even at low "T")
Low density of ionized electrons "should" thermalize
But...
How does one (or can one) define "ionized"?
How to characterize thermalization of "ionized" electrons?
Atomic-Liquid
an:
New wavefcn diagnostic ("a" characterization of ionization)
New class of volume entangled (E>0) quantum states
"Quantum Disentangled States"
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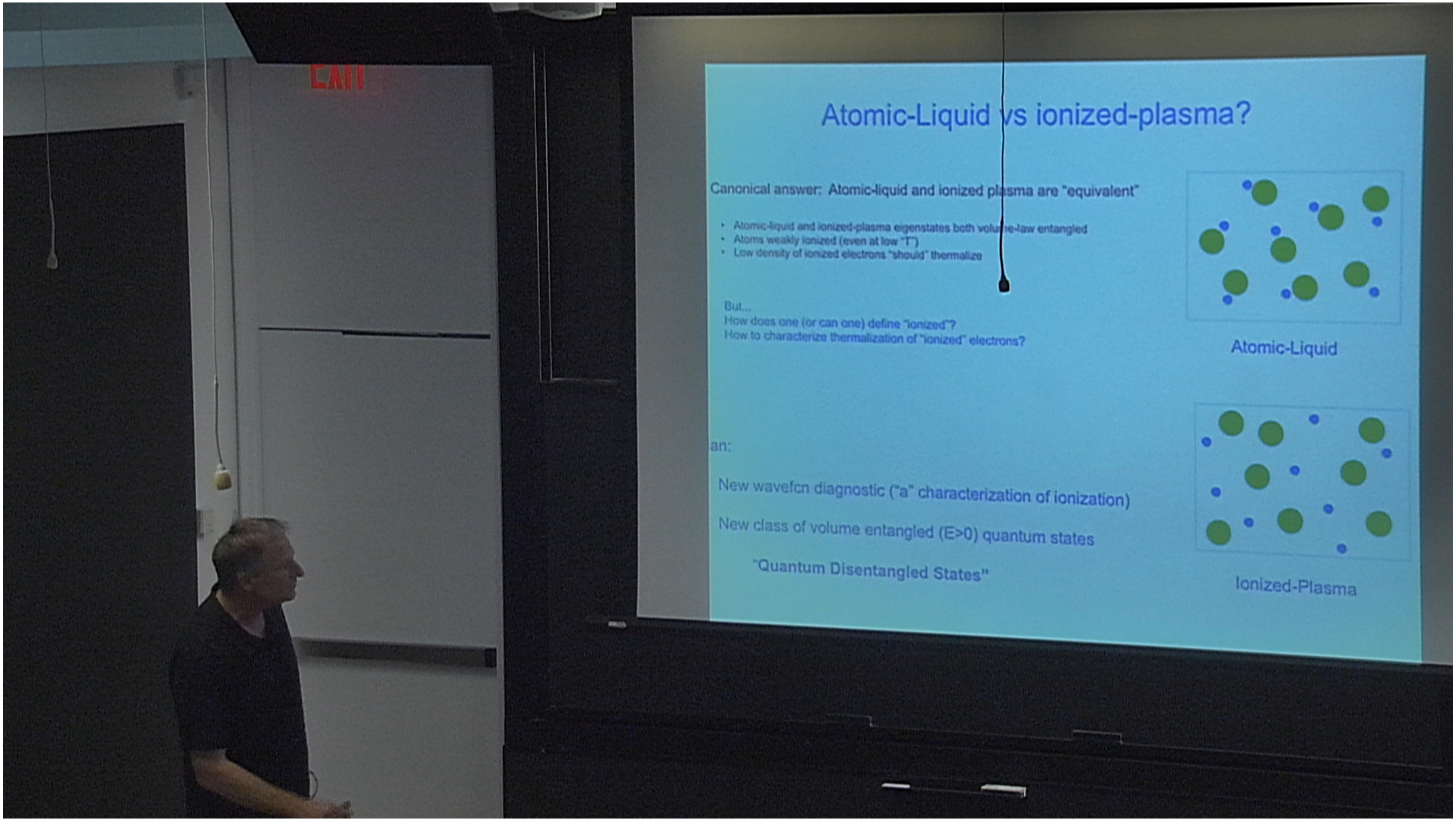
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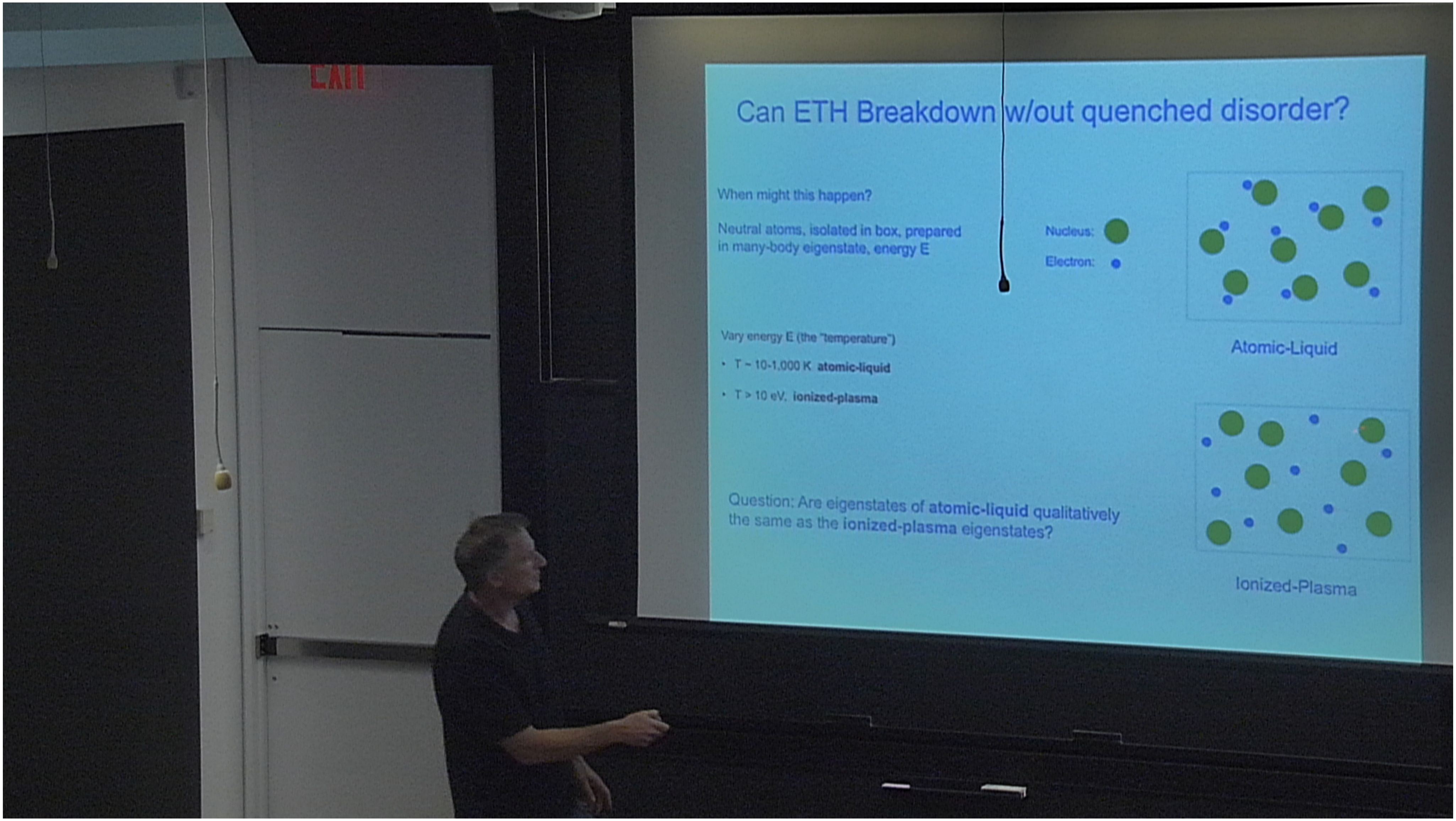
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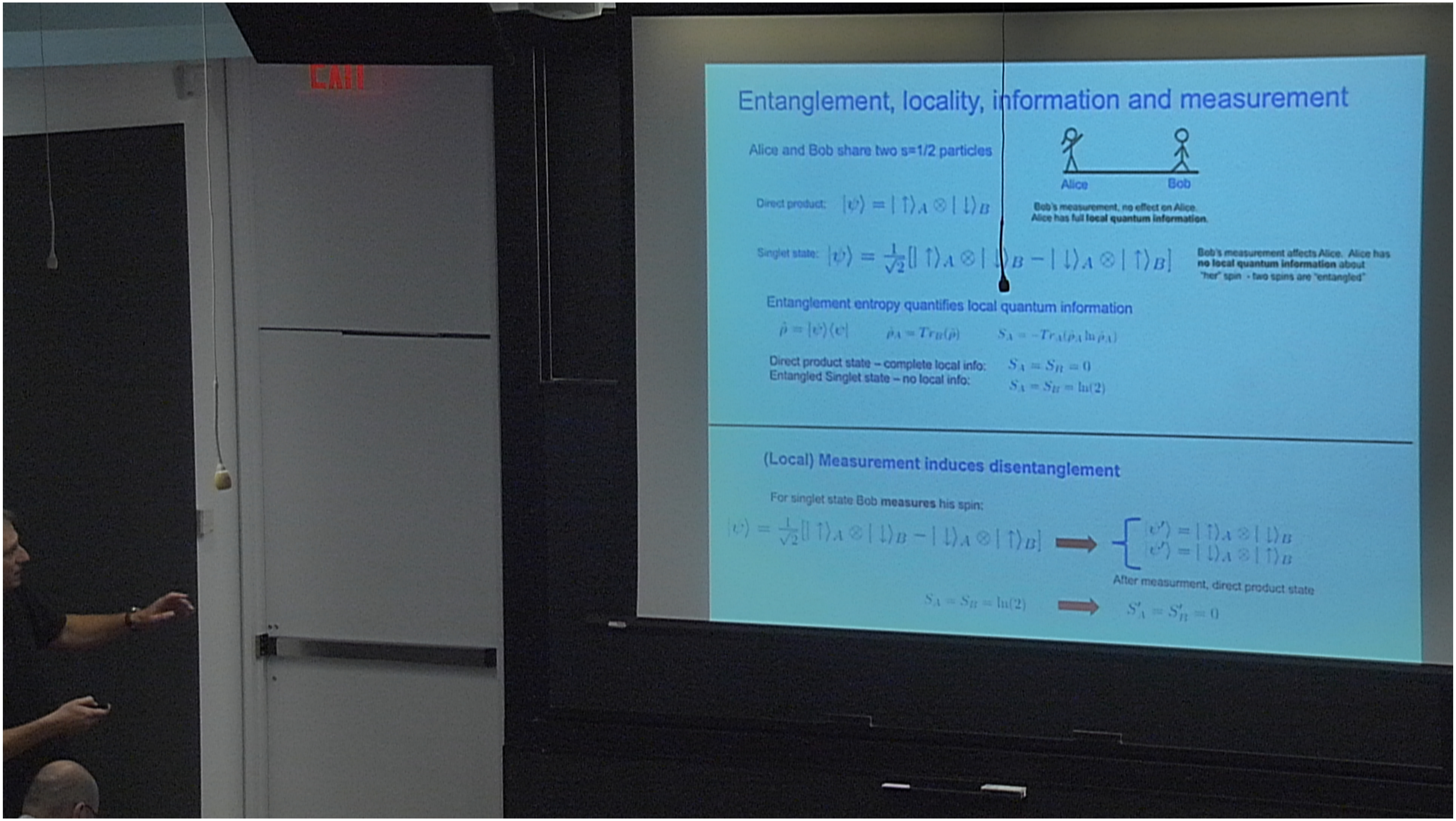
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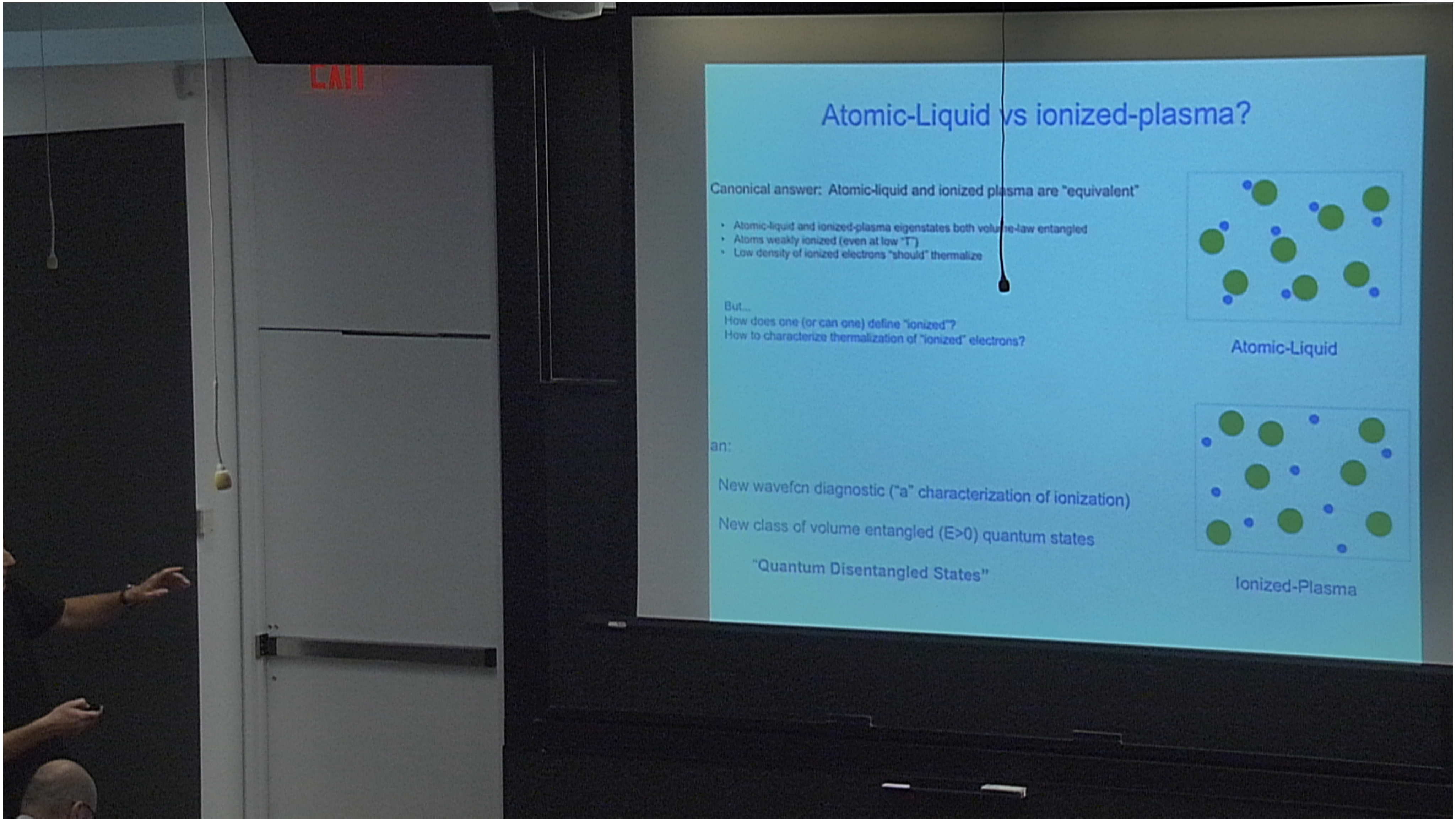

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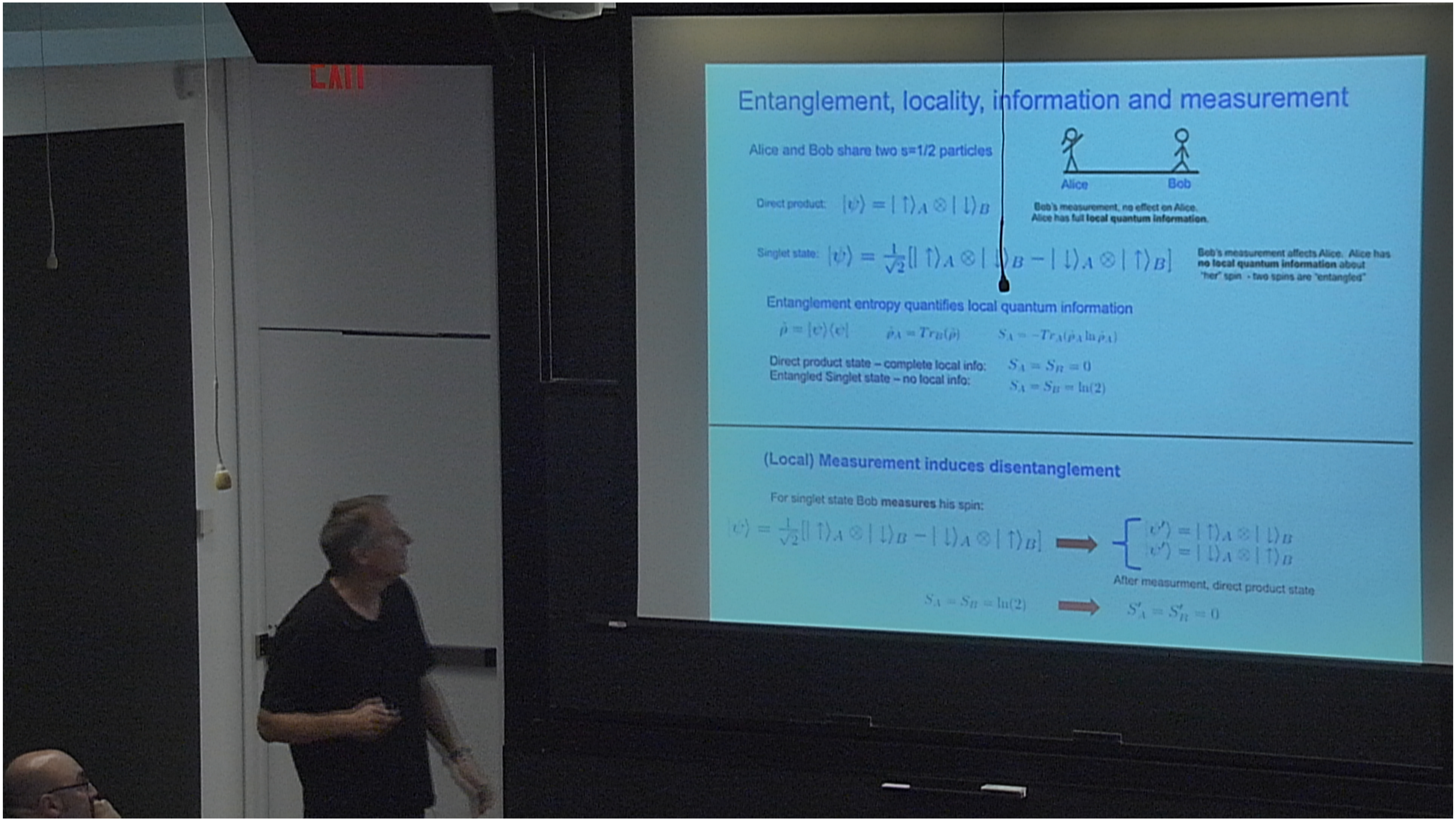

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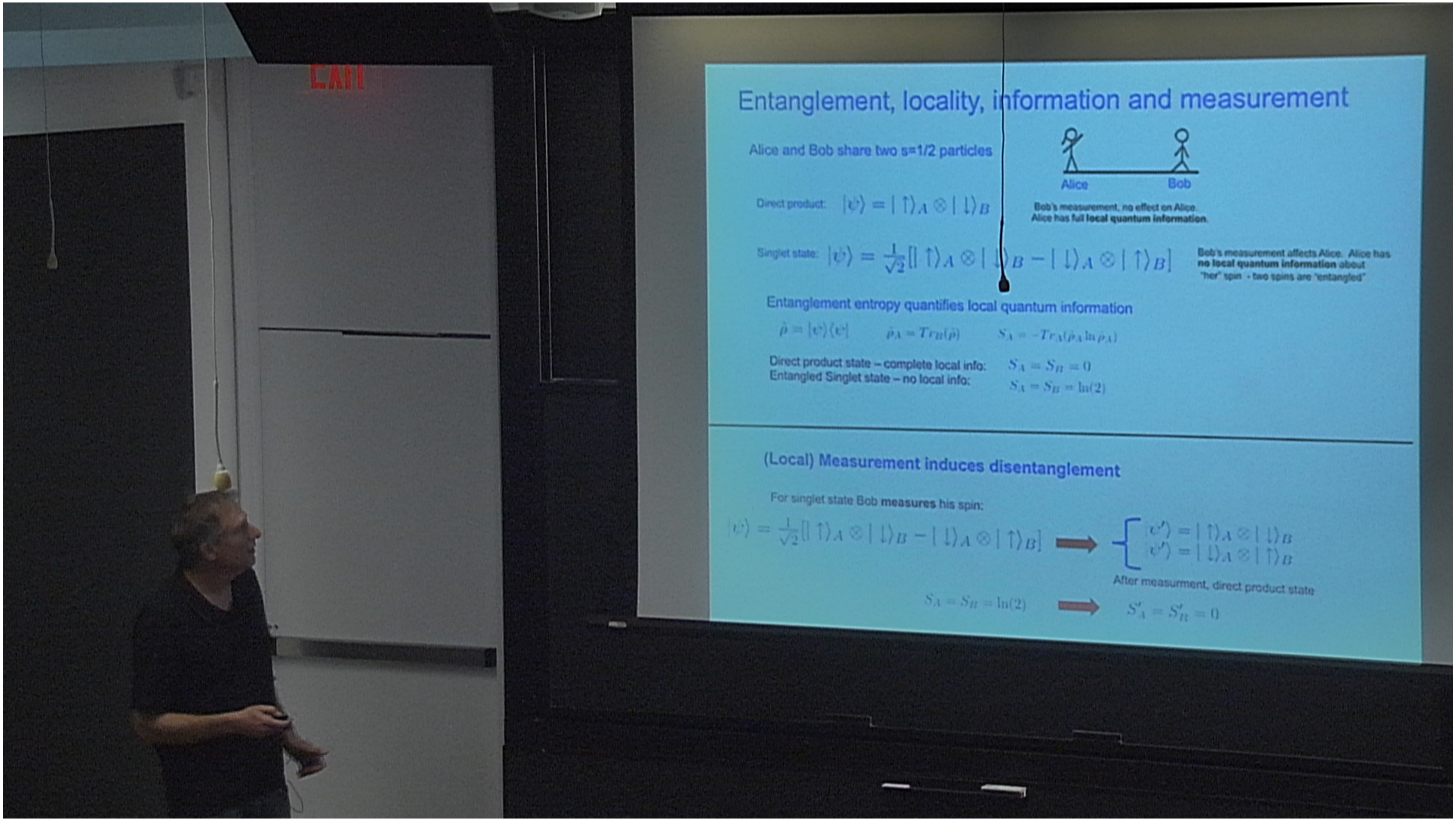

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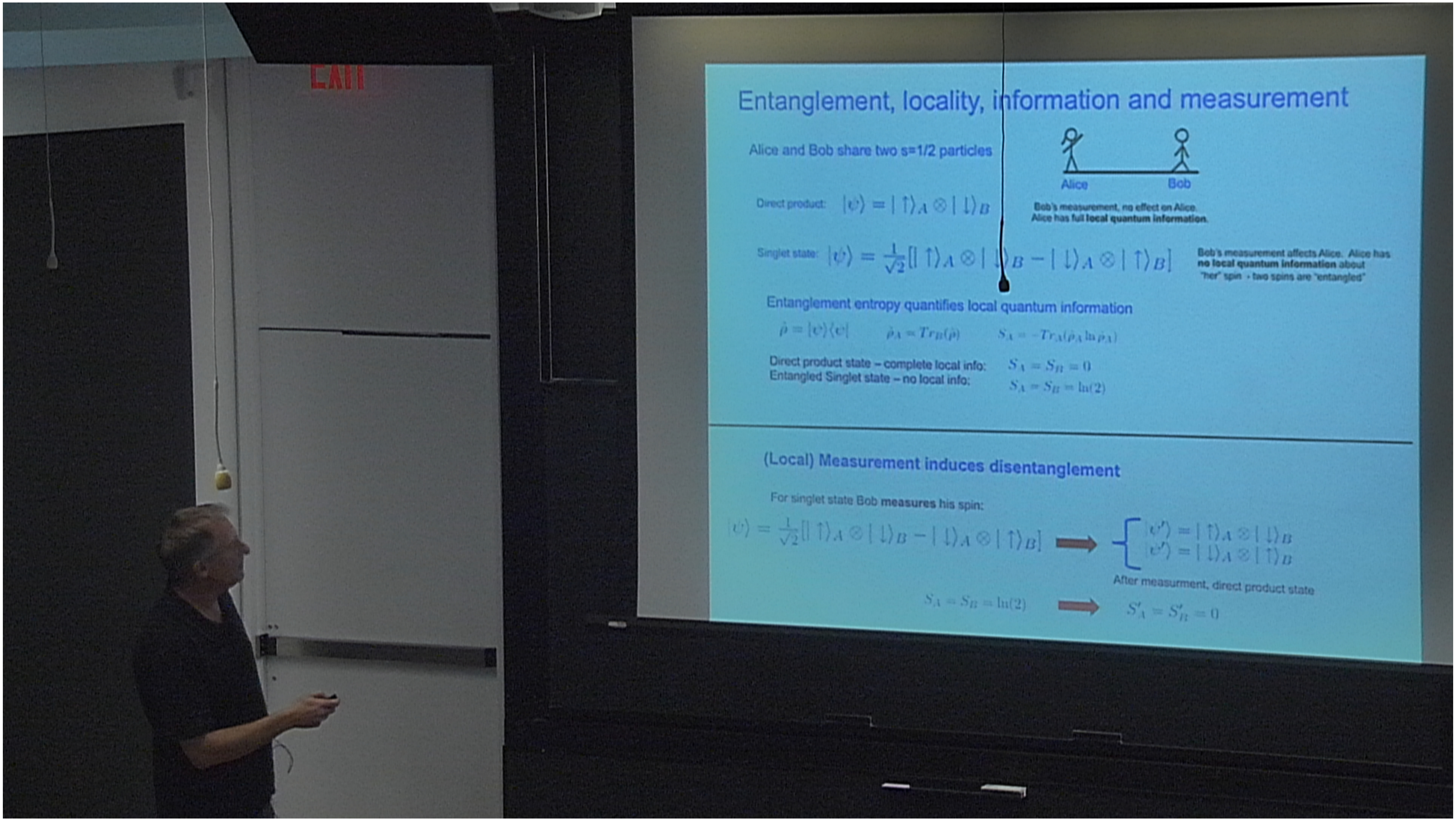
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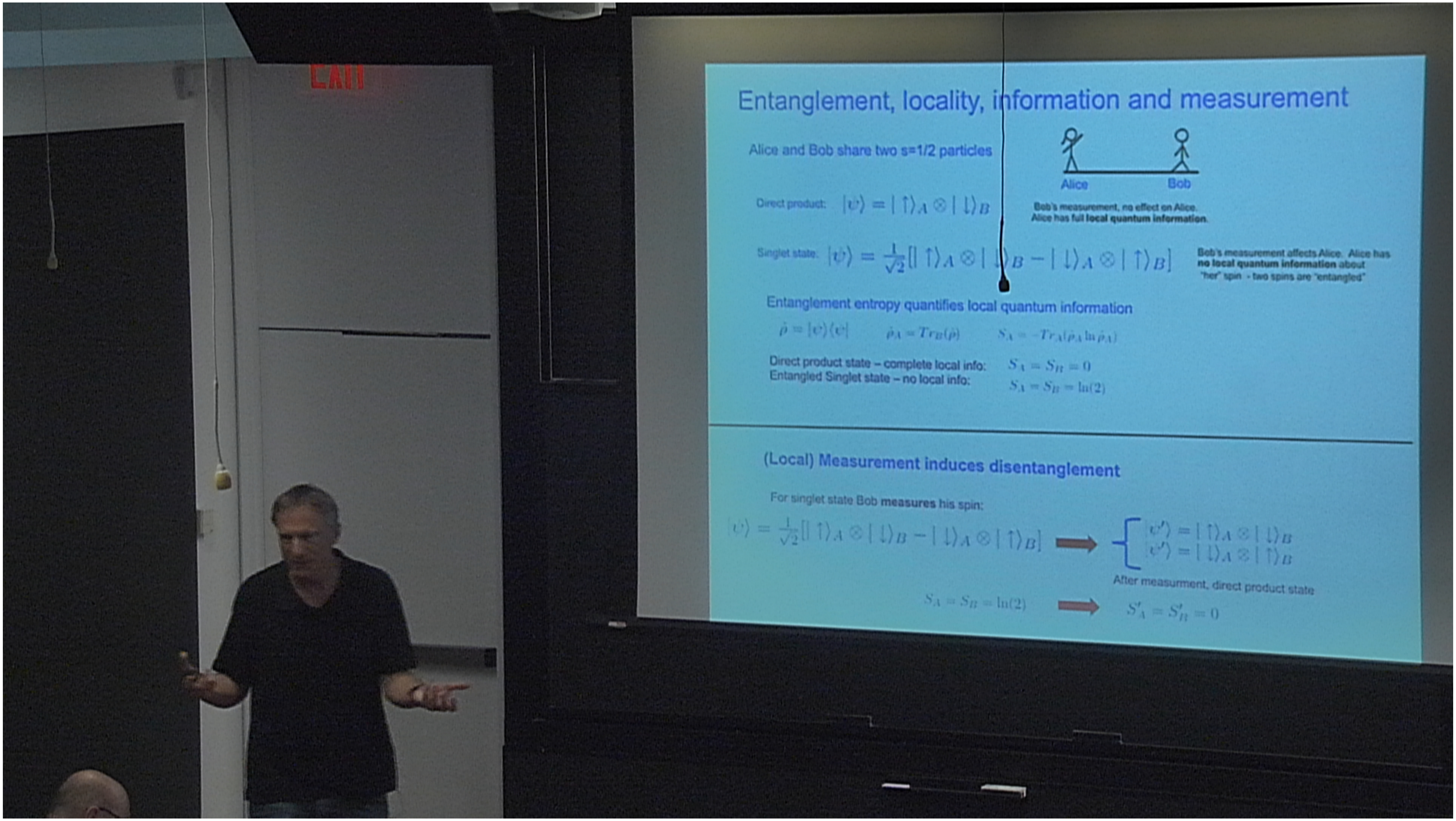

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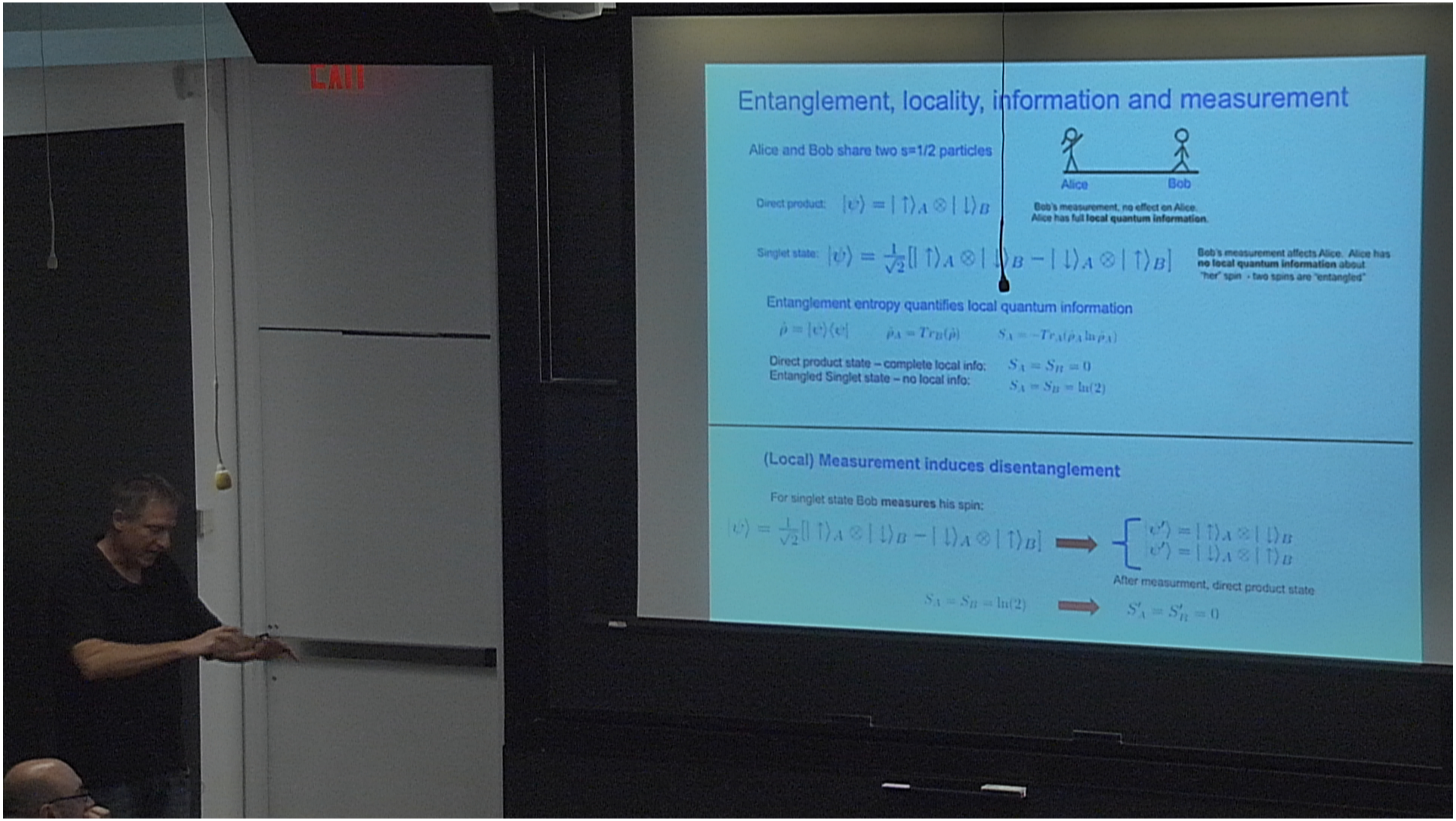
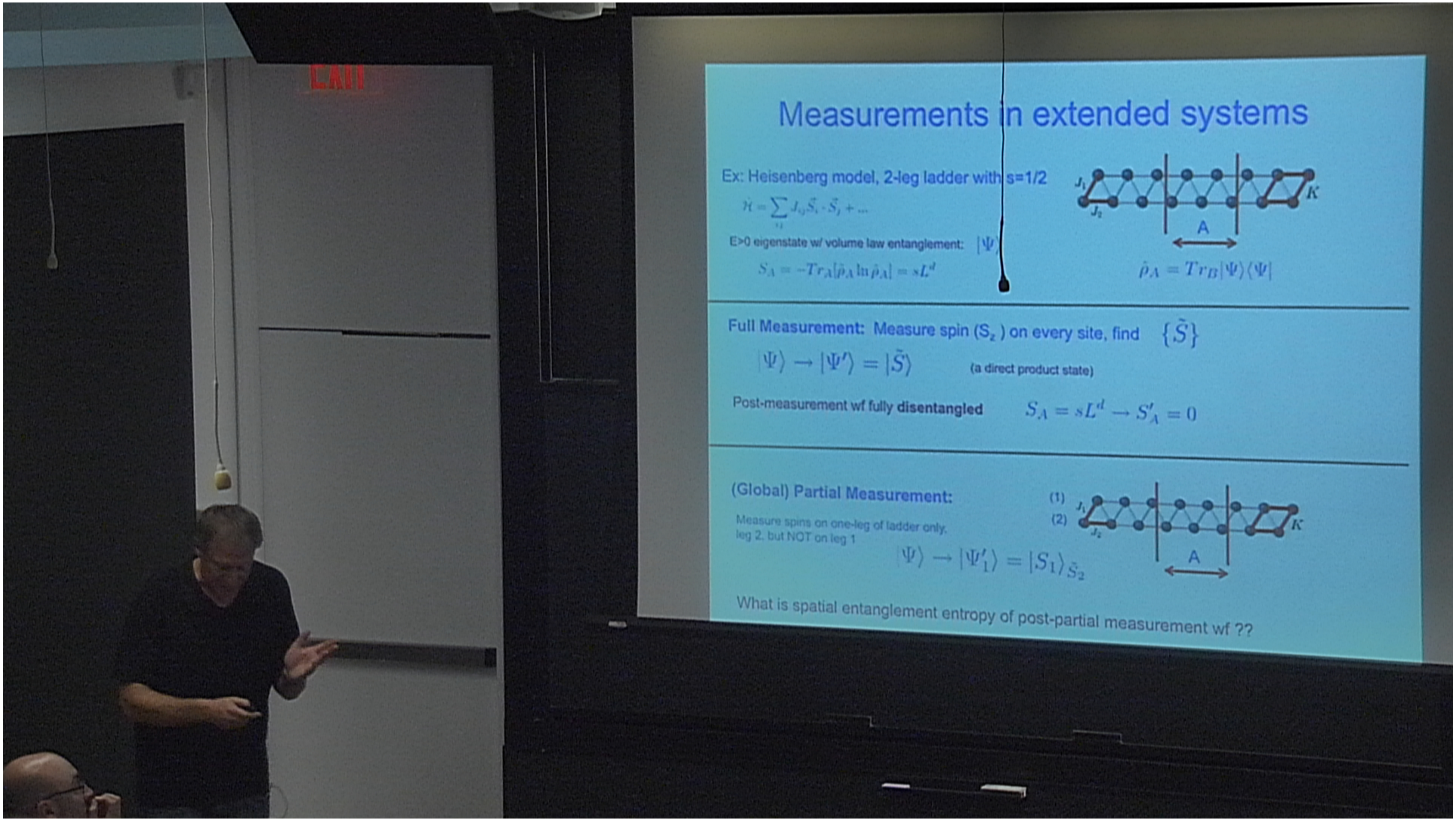
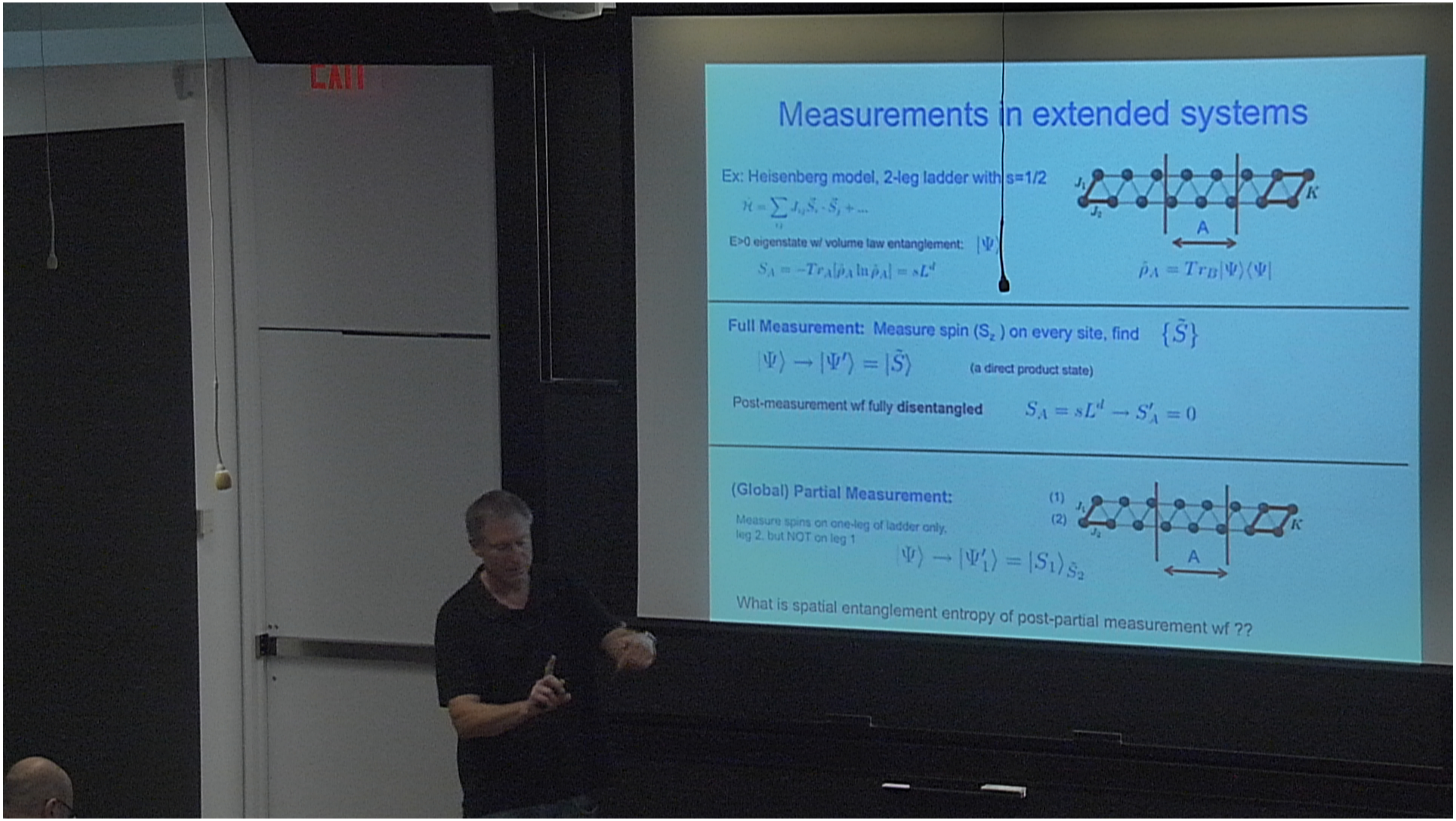
After measurment, direct product state
$S_A = S_B = \ln(2)$
$S'_A = S'_B = 0$


Entanglement, locality, information and measurement
Alice and Bob share two s=1/2 particles
Alice
Bob
Direct product: $|\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B$
Bob's measurement, no effect on Alice.
Alice has full local quantum information.
Singlet state: $|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B]$
Bob's measurement affects Alice. Alice has no local quantum information about "her" spin - two spins are "entangled"
Entanglement entropy quantifies local quantum information
$\hat{\rho} = |\psi\rangle\langle\psi|$ $\hat{\rho}_A = Tr_B(\hat{\rho})$ $S_A = -Tr_A(\hat{\rho}_A \ln \hat{\rho}_A)$
Direct product state – complete local info: $S_A = S_B = 0$
Entangled Singlet state – no local info: $S_A = S_B = \ln(2)$
(Local) Measurement induces disentanglement
For singlet state Bob measures his spin:
$|\psi\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B]$ → $|\psi'\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B$; $|\psi'\rangle = |\downarrow\rangle_A \otimes |\uparrow\rangle_B$
After measurment, direct product state
$S_A = S_B = \ln(2)$ → $S'_A = S'_B = 0$


Measurements in extended systems
Ex: Heisenberg model, 2-leg ladder with s=1/2
E>0 eigenstate w/ volume law entanglement: $|\Psi\rangle$
$S_A = -Tr_A[\hat{\rho}_A \ln \hat{\rho}_A] = sL^d$
$\hat{\rho}_A = Tr_B|\Psi\rangle\langle\Psi|$
A
Full Measurement: Measure spin (S_z) on every site, find $\{\tilde{S}\}$
$|\Psi\rangle \to |\Psi'\rangle = |\tilde{S}\rangle$
(a direct product state)
Post-measurement wf fully disentangled
$S_A = sL^d \to S'_A = 0$
(Global) Partial Measurement:
Measure spins on one-leg of ladder only, leg 2, but NOT on leg 1
$|\Psi\rangle \to |\Psi'_1\rangle = |S_1\rangle_{\tilde{S}_2}$
(1)
(2)
A
What is spatial entanglement entropy of post-partial measurement wf ??



Measurements in extended systems
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$\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots$
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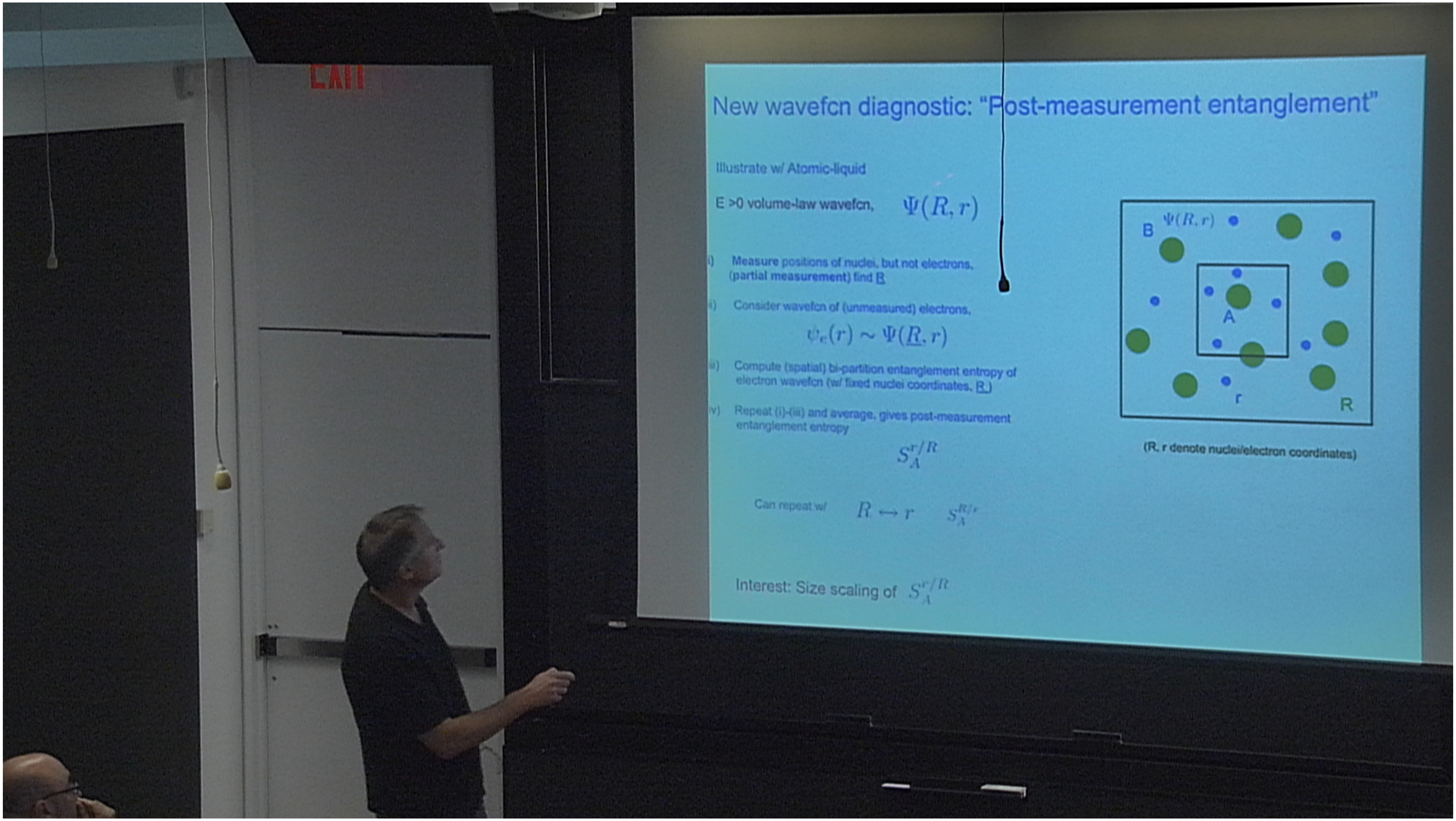
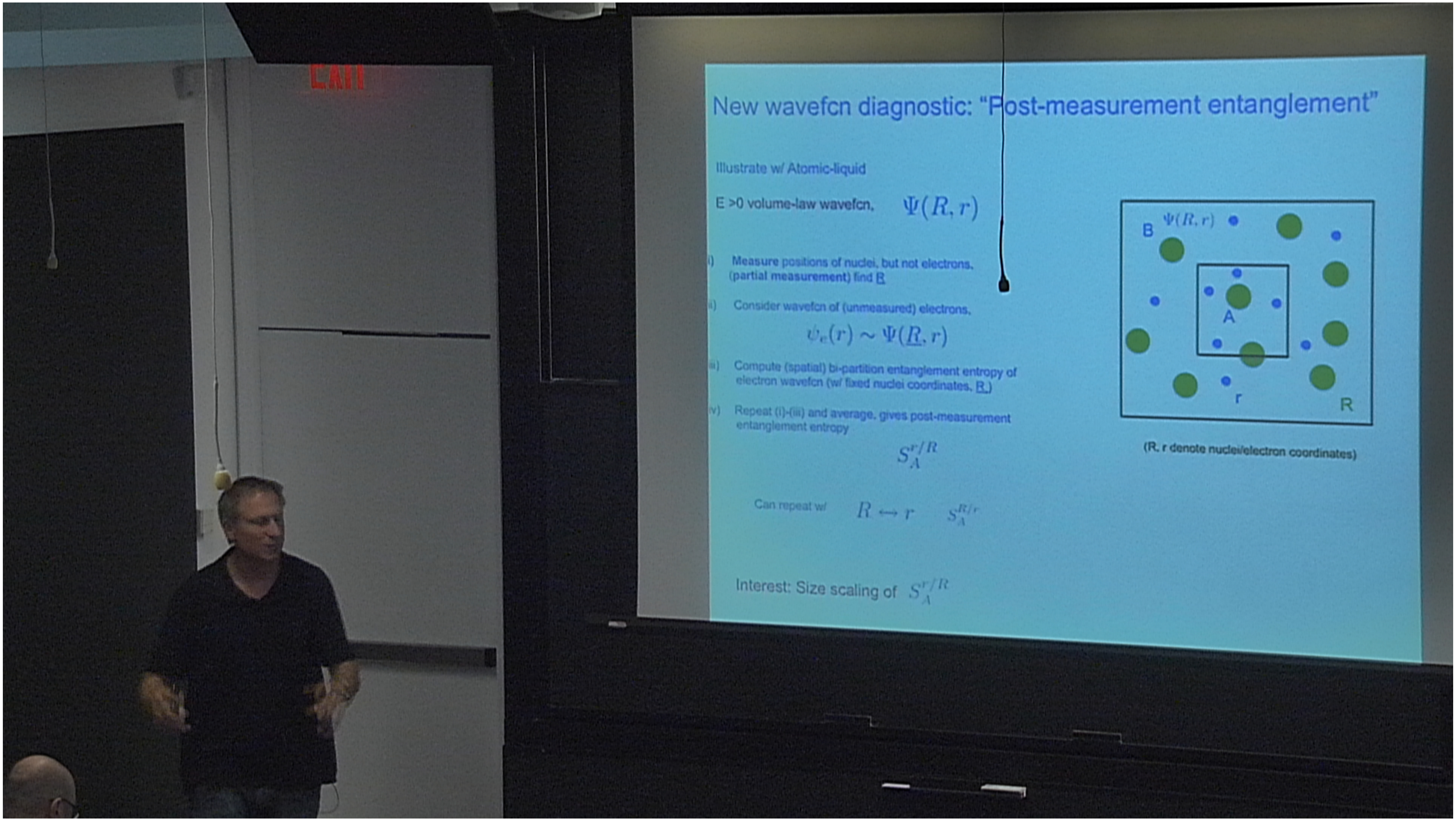

New wavefcn diagnostic: "Post-measurement entanglement"
Illustrate w/ Atomic-liquid
E >0 volume-law wavefcn, $\Psi(R,r)$
i) Measure positions of nuclei, but not electrons, (partial measurement) find R
ii) Consider wavefcn of (unmeasured) electrons, $\psi_e(r) \sim \Psi(\underline{R},r)$
iii) Compute (spatial) bi-partition entanglement entropy of electron wavefcn (w/ fixed nuclei coordinates, R)
iv) Repeat (i)-(iii) and average, gives post-measurement entanglement entropy $S_A^{r/R}$
Can repeat w/ $R \leftrightarrow r$ $S_A^{R/r}$
Interest: Size scaling of $S_A^{r/R}$
B
$\Psi(R,r)$
A
r
R
(R, r denote nuclei/electron coordinates)



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Repeat (i)-(iii) and average
entanglement entropy
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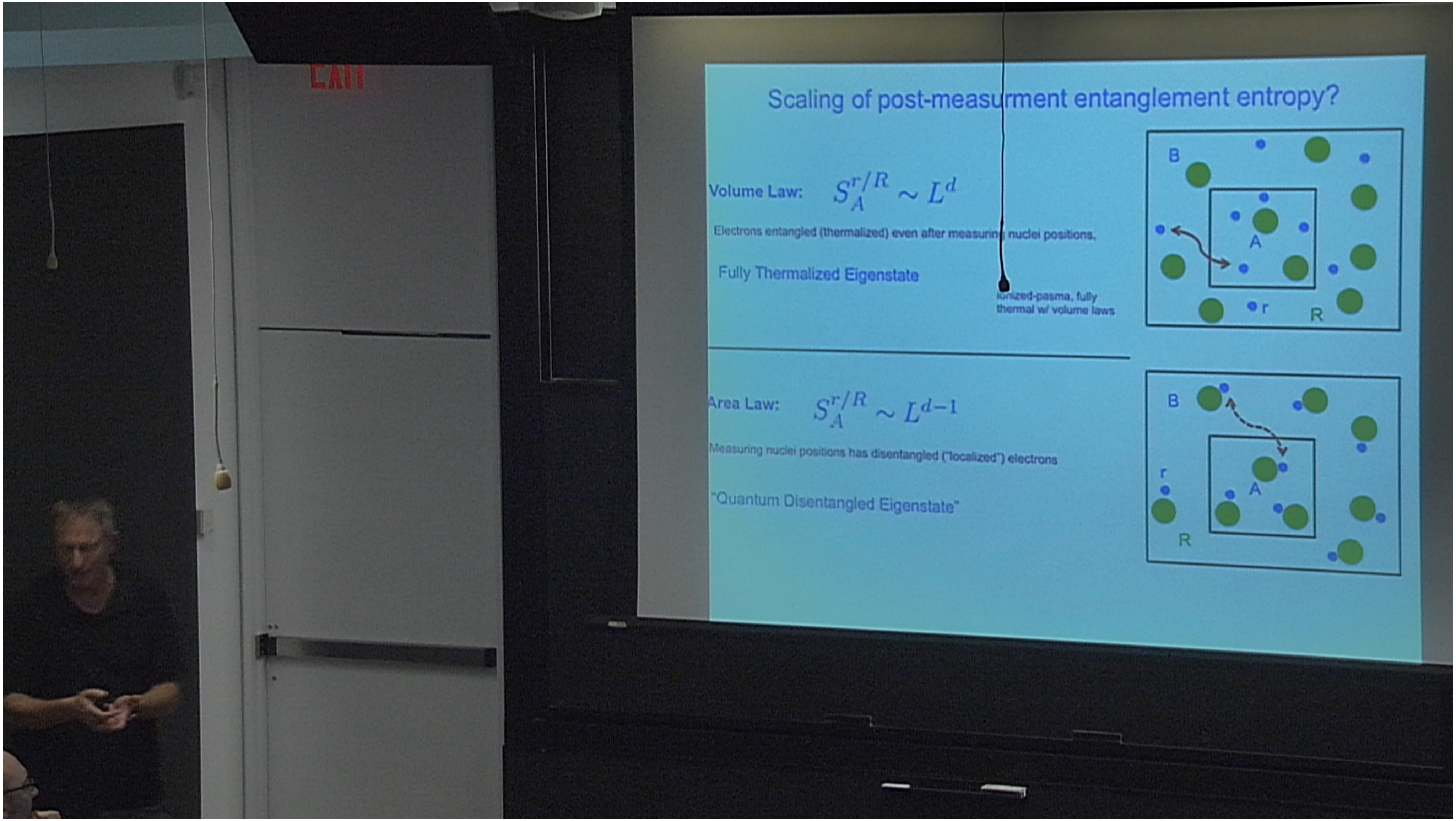
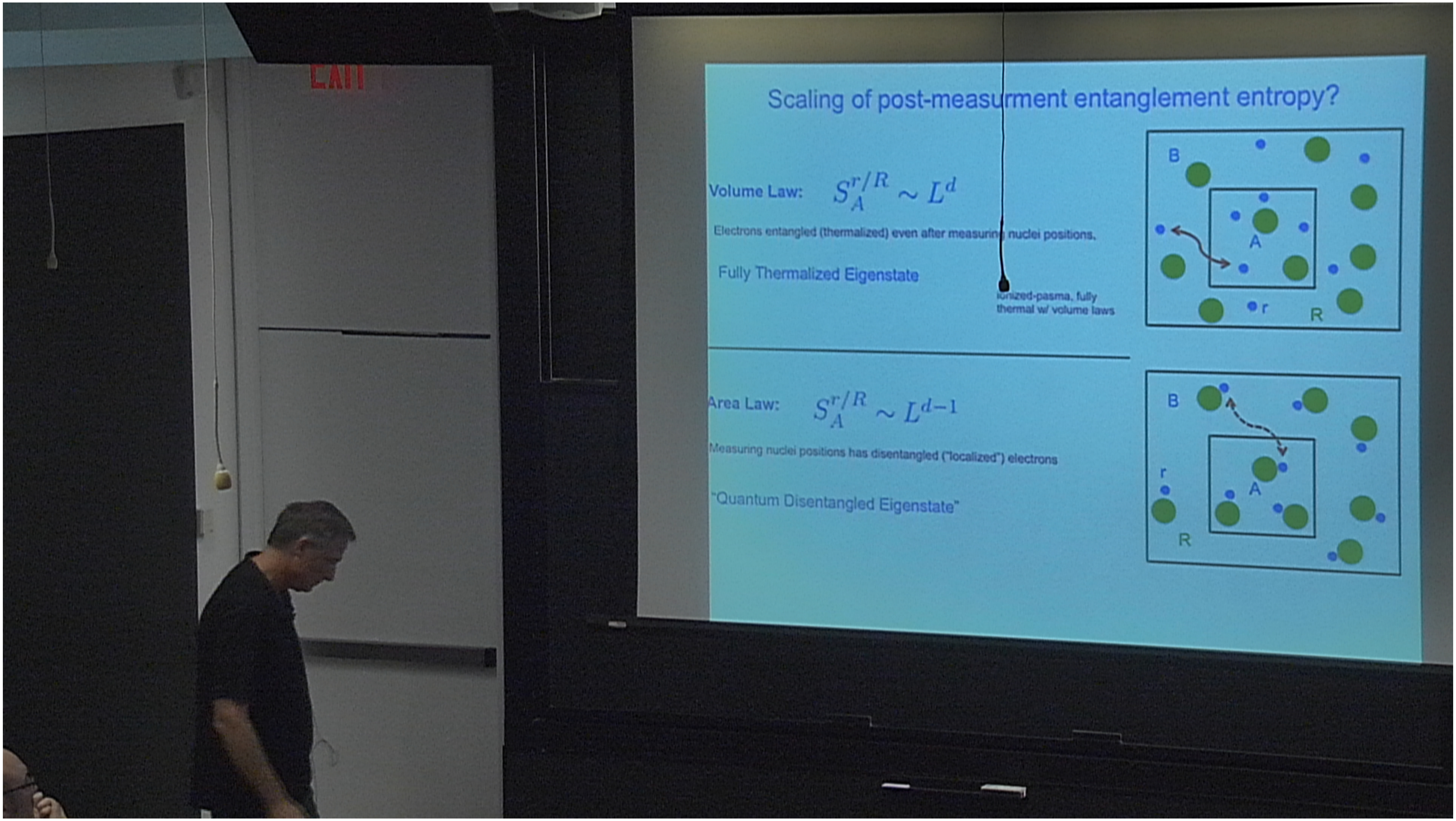

Scaling of post-measurment entanglement entropy?
Volume Law: $S_A^{r/R} \sim L^d$
Electrons entangled (thermalized) even after measuring nuclei positions.
Fully Thermalized Eigenstate
ionized-pasma, fully thermal w/ volume laws
B
A
r
R
Area Law: $S_A^{r/R} \sim L^{d-1}$
Measuring nuclei positions has disentangled ("localized") electrons
"Quantum Disentangled Eigenstate"
B
r
A
R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B
A
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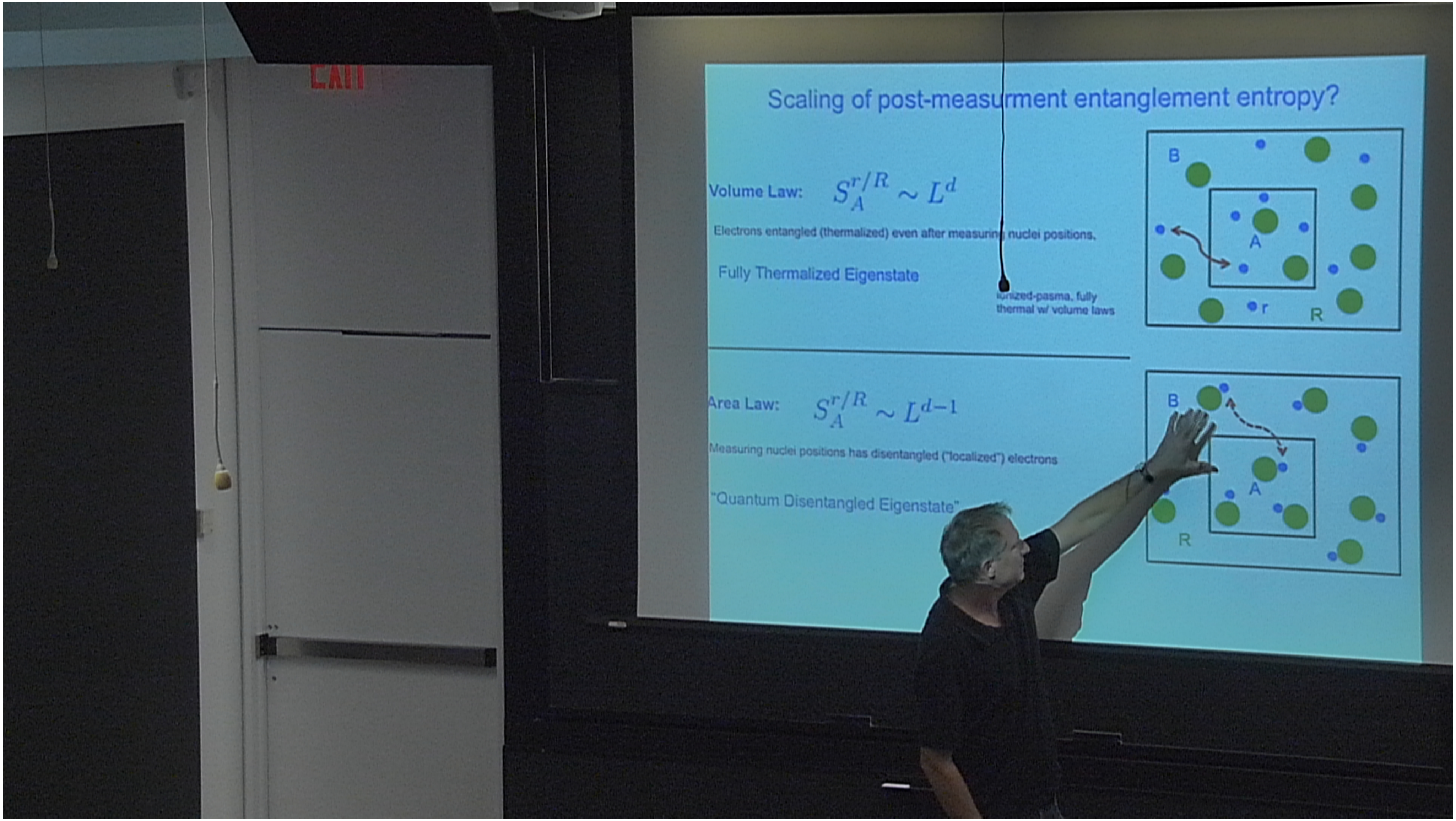
B
A
r
R



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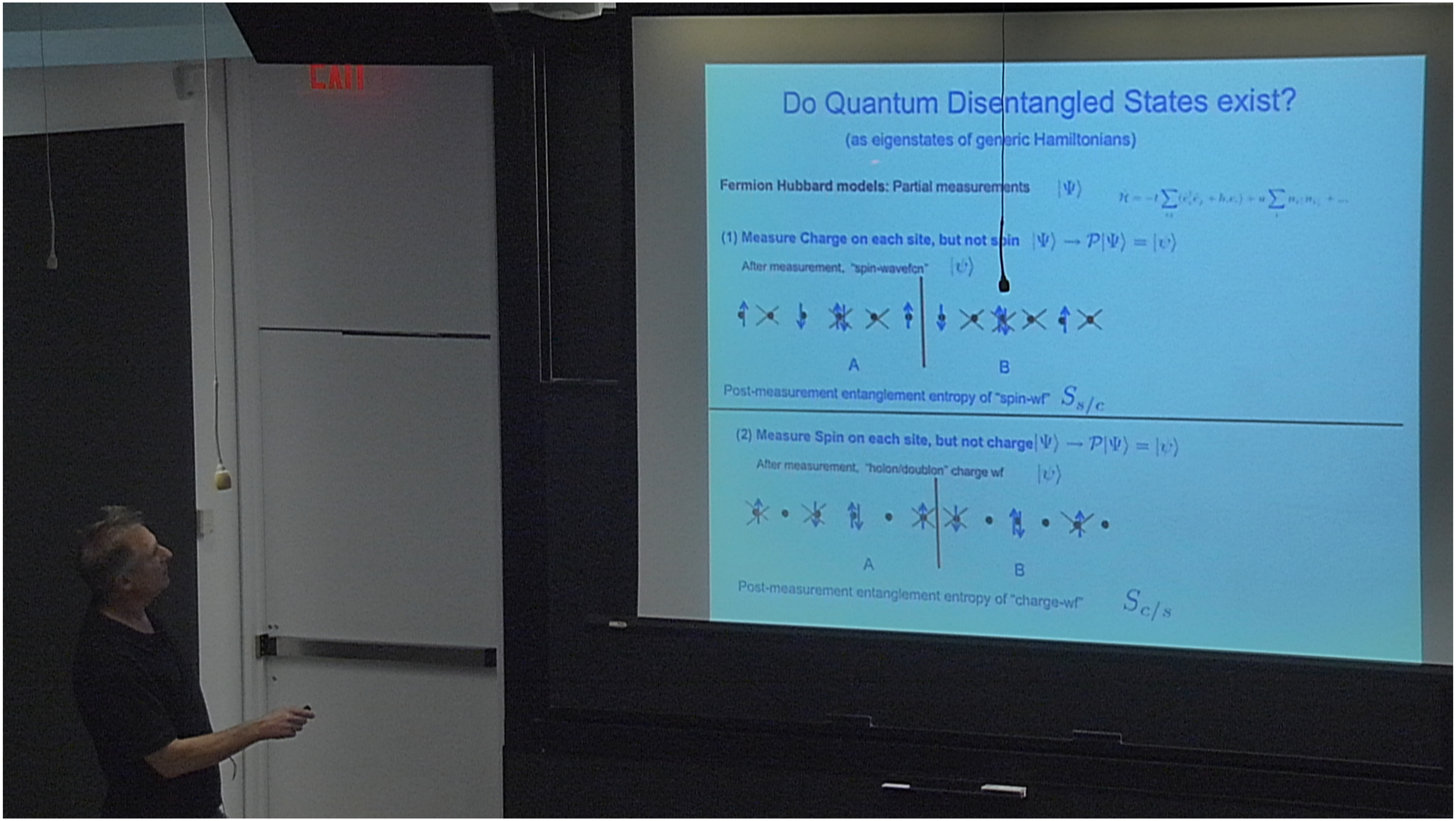
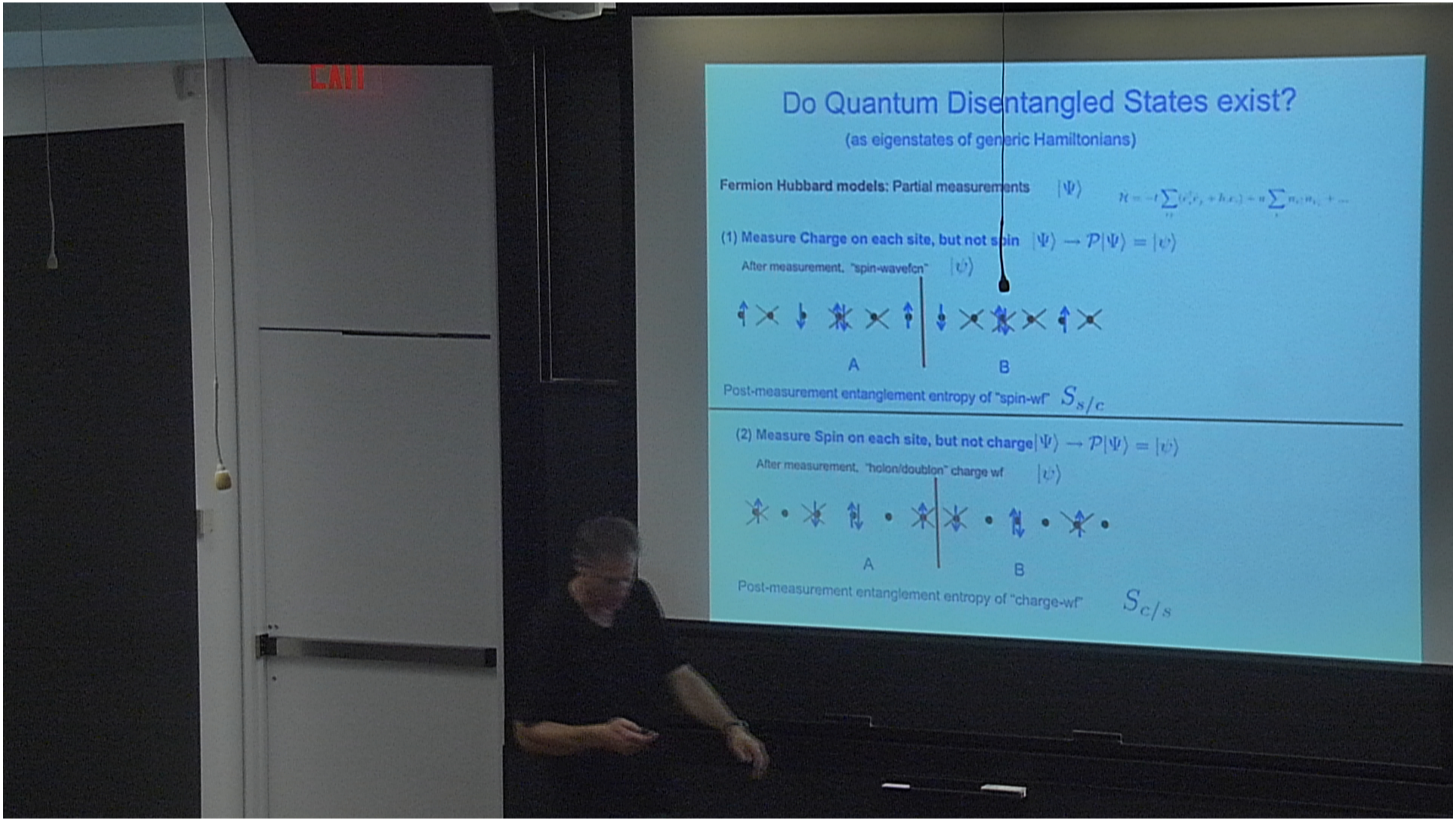
"Quantum Disentangled Eigenstate"
B
A
r
R


Quantum Disentangled State for atomic fluid
An atomic fluid eigenstate is Quantum-Disentangled if,
Measuring (heavy) nuclei, disentangles (light) electrons, Area law: $S_A^{r/R} \sim L^{d-1}$
Measuring (light) electrons does not disentangle (heavy) nuclei, Volume law: $S_A^{R/r} \sim L^d$
Quantum Disentangled States: General defn
Volume law entangled eigenstate (E >0) $S_A^{\Psi} \sim L^d$
Perform (some) global partial-measurement, wf of un-measured d.o.f. $|\Psi\rangle \to |\psi\rangle$
Entanglement-entropy of post-measurement wf $|\psi\rangle \to S_A^{\psi}$
Quantum Disentangled State: post-measurement wf has area law $S_A^{\psi} \sim L^{d-1}$
Partial Measurment induces full locality
Thermal State: wf post-measurment still entangled (non-local, volume law) $S_A^{\psi} \sim L^d$


Do Quantum Disentangled States exist?
(as eigenstates of generic Hamiltonians)
Fermion Hubbard models: Partial measurements $|\Psi\rangle$
(1) Measure Charge on each site, but not spin $|\Psi\rangle \rightarrow \mathcal{P}|\Psi\rangle = |\psi\rangle$
After measurement, "spin-wavefcn" $|\psi\rangle$
A
B
Post-measurement entanglement entropy of "spin-wf" $S_{s/c}$
(2) Measure Spin on each site, but not charge $|\Psi\rangle \rightarrow \mathcal{P}|\Psi\rangle = |\psi\rangle$
After measurement, "holon/doublon" charge wf $|\psi\rangle$
A
B
Post-measurement entanglement entropy of "charge-wf" $S_{c/s}$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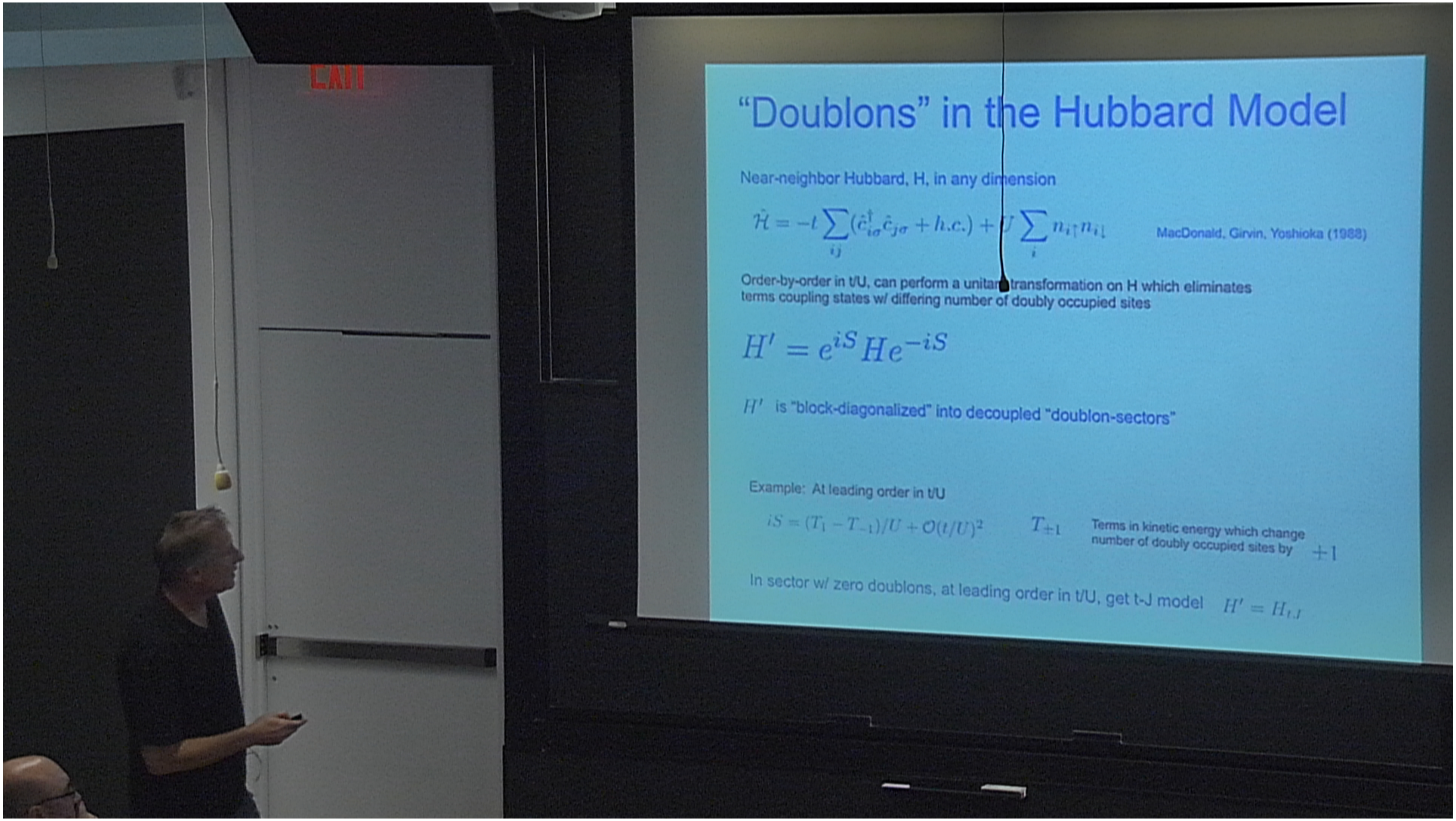
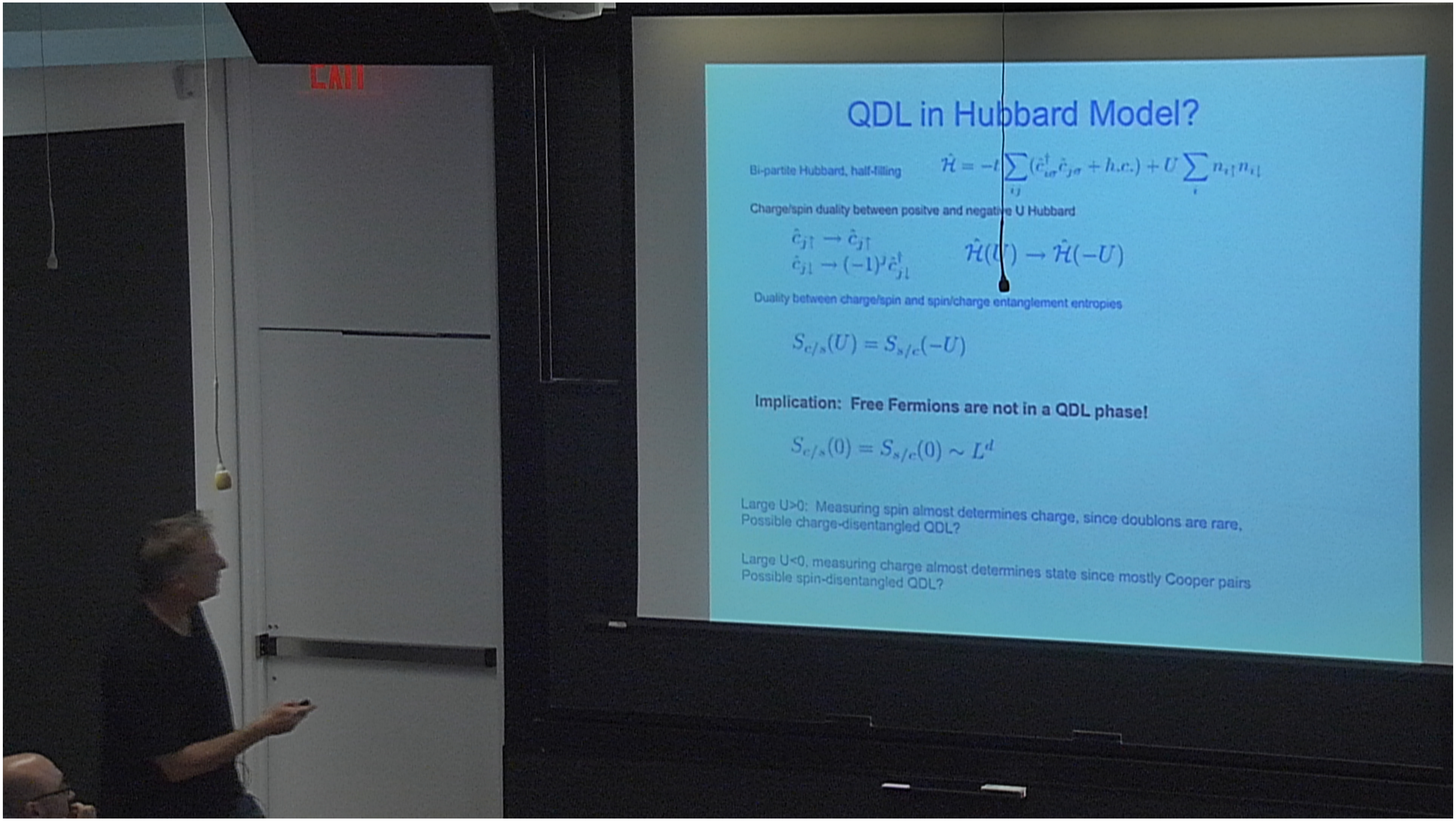
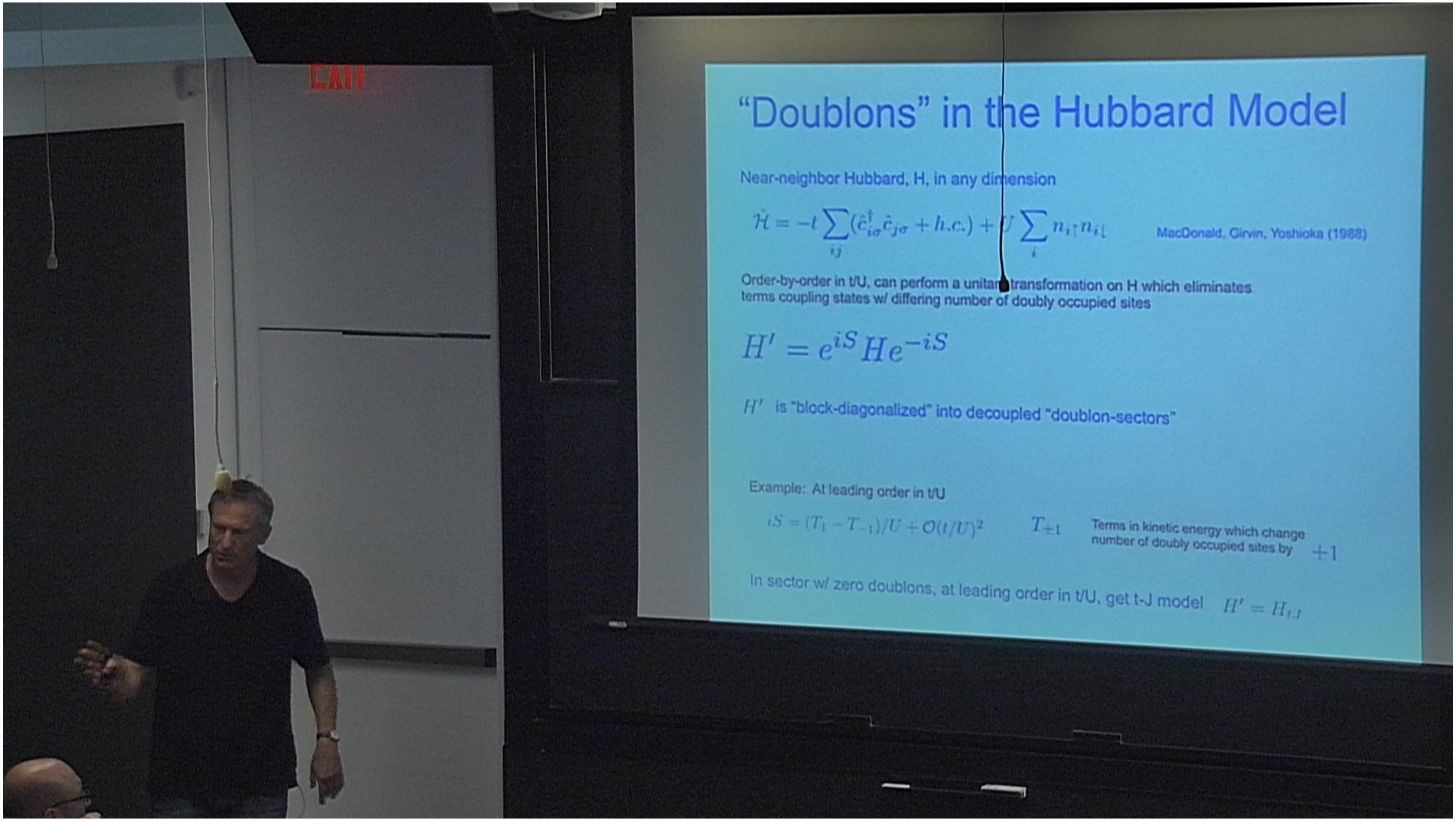

EXIT
"Doublons" in the Hubbard Model
Near-neighbor Hubbard, H, in any dimension
$\hat{\mathcal{H}} = -t \sum_{ij} (\hat{c}^{\dagger}_{i\sigma} \hat{c}_{j\sigma} + h.c.) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}$
MacDonald, Girvin, Yoshioka (1988)
Order-by-order in t/U, can perform a unitary transformation on H which eliminates terms coupling states w/ differing number of doubly occupied sites
$H' = e^{iS} H e^{-iS}$
H' is "block-diagonalized" into decoupled "doublon-sectors"
Example: At leading order in t/U
$iS = (T_1 - T_{-1})/U + \mathcal{O}(t/U)^2$
$T_{\pm 1}$
Terms in kinetic energy which change number of doubly occupied sites by ± 1
In sector w/ zero doublons, at leading order in t/U, get t-J model $H' = H_{tJ}$


QDL in Hubbard Model?
Bi-partite Hubbard, half-filling
$\hat{\mathcal{H}} = -t \sum_{ij} (\hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$
Charge/spin duality between positve and negative U Hubbard
$\hat{c}_{j\uparrow} \to \hat{c}_{j\uparrow}$
$\hat{c}_{j\downarrow} \to (-1)^j \hat{c}^\dagger_{j\downarrow}$
$\hat{\mathcal{H}}(U) \to \hat{\mathcal{H}}(-U)$
Duality between charge/spin and spin/charge entanglement entropies
$S_{c/s}(U) = S_{s/c}(-U)$
Implication: Free Fermions are not in a QDL phase!
$S_{c/s}(0) = S_{s/c}(0) \sim L^d$
Large U>0: Measuring spin almost determines charge, since doublons are rare. Possible charge-disentangled QDL?
Large U<0, measuring charge almost determines state since mostly Cooper pairs Possible spin-disentangled QDL?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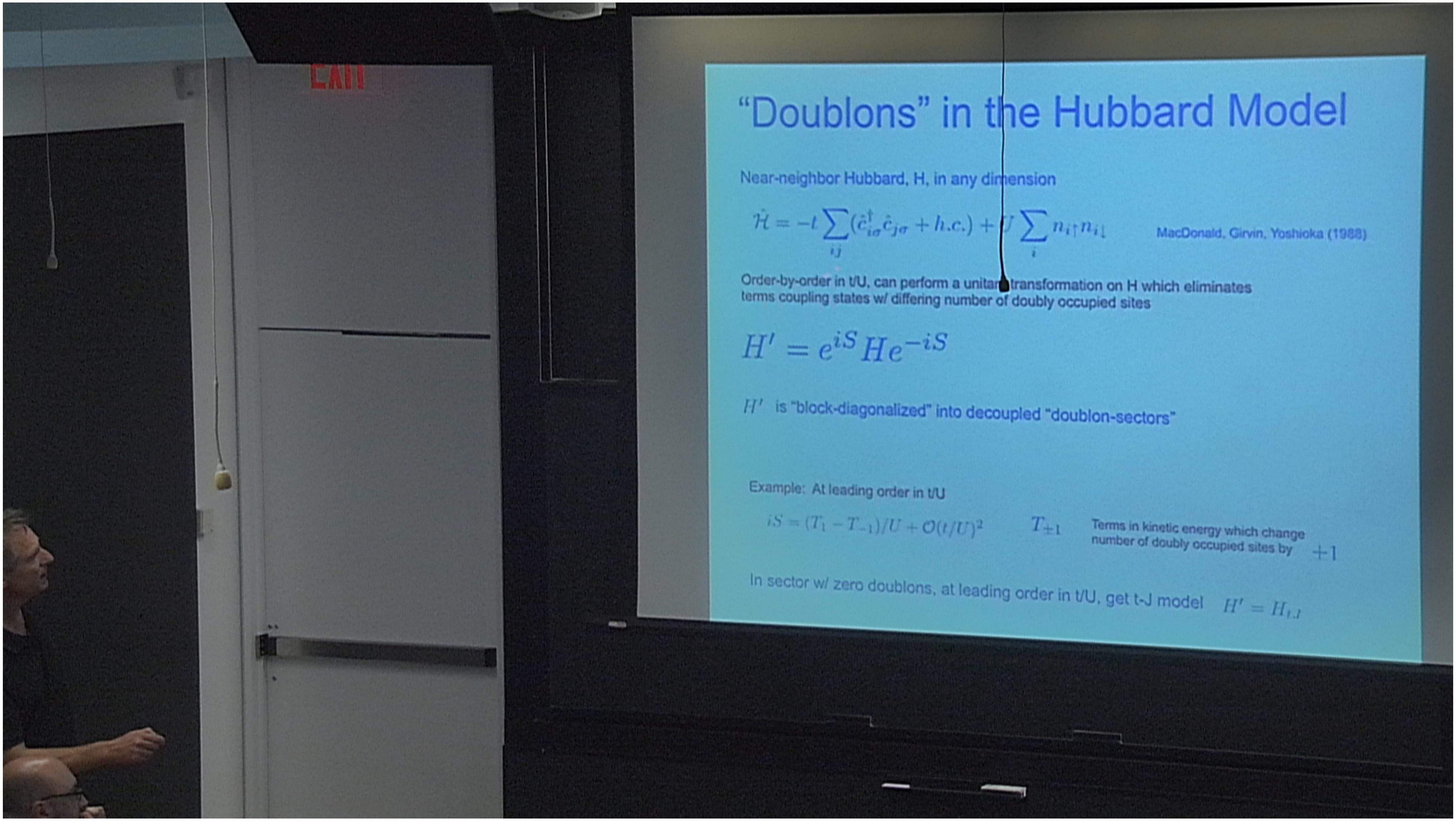
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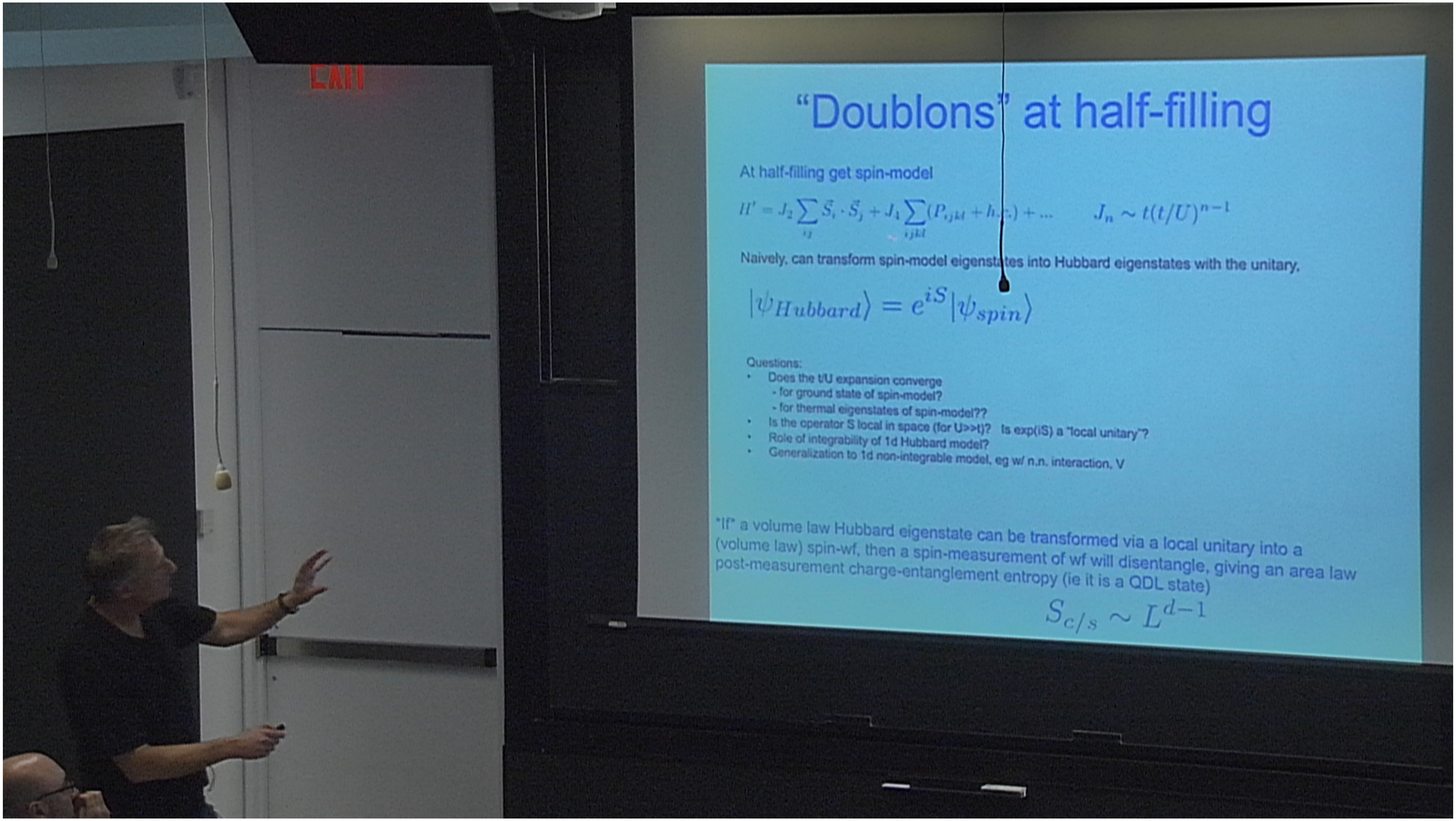
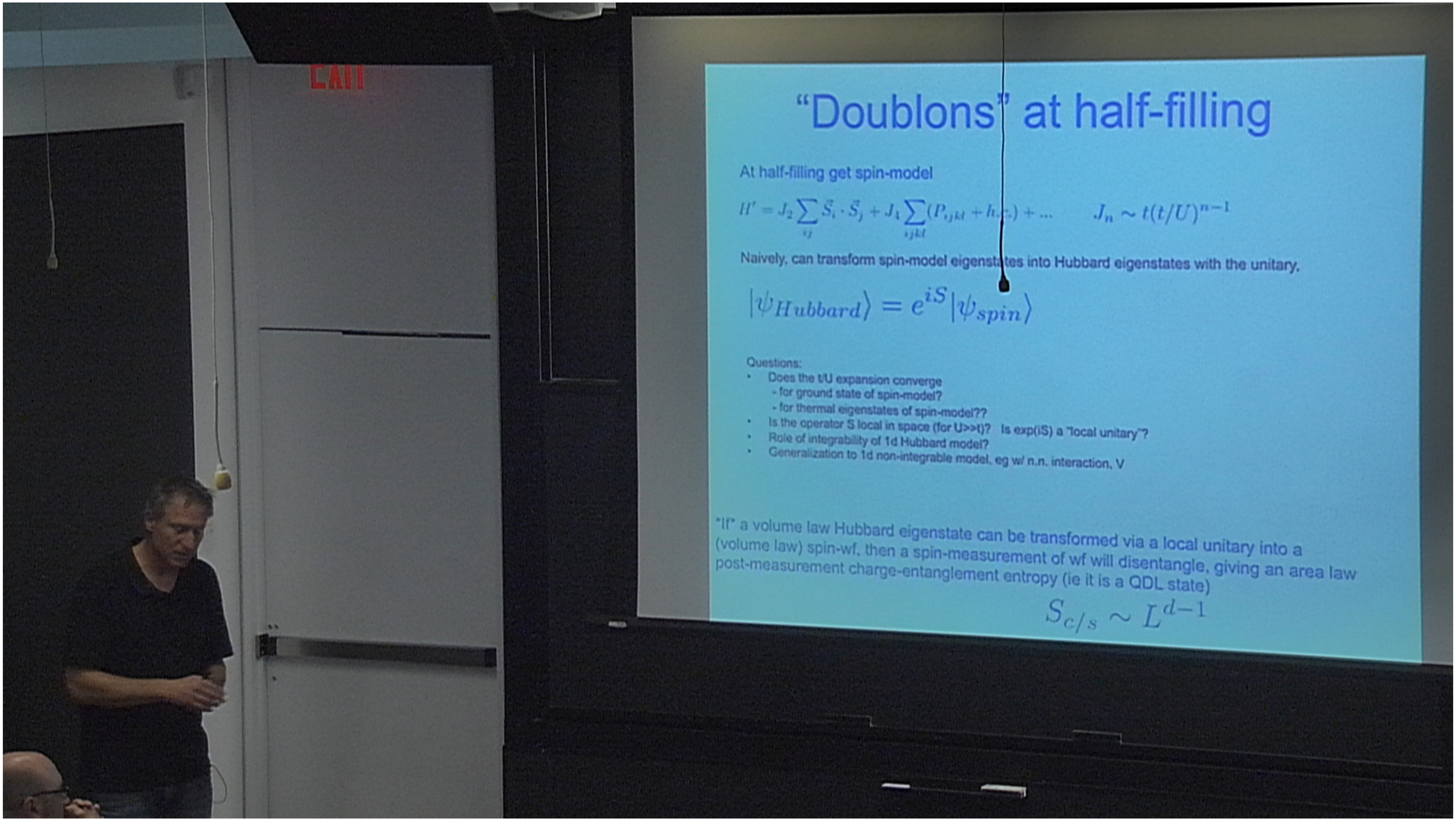

"Doublons" at half-filling
At half-filling get spin-model
$H' = J_2 \sum_{ij} \vec{S}_i \cdot \vec{S}_j + J_4 \sum_{ijkl} (P_{ijkl} + h.c.) + \ldots \qquad J_n \sim t(t/U)^{n-1}$
Naively, can transform spin-model eigenstates into Hubbard eigenstates with the unitary,
$|\psi_{Hubbard}\rangle = e^{iS} |\psi_{spin}\rangle$
Questions:
Does the t/U expansion converge
- for ground state of spin-model?
- for thermal eigenstates of spin-model??
Is the operator S local in space (for U>>t)? Is exp(iS) a "local unitary"?
Role of integrability of 1d Hubbard model?
Generalization to 1d non-integrable model, eg w/ n.n. interaction, V
"If" a volume law Hubbard eigenstate can be transformed via a local unitary into a (volume law) spin-wf, then a spin-measurement of wf will disentangle, giving an area law post-measurement charge-entanglement entropy (ie it is a QDL state)
$S_{c/s} \sim L^{d-1}$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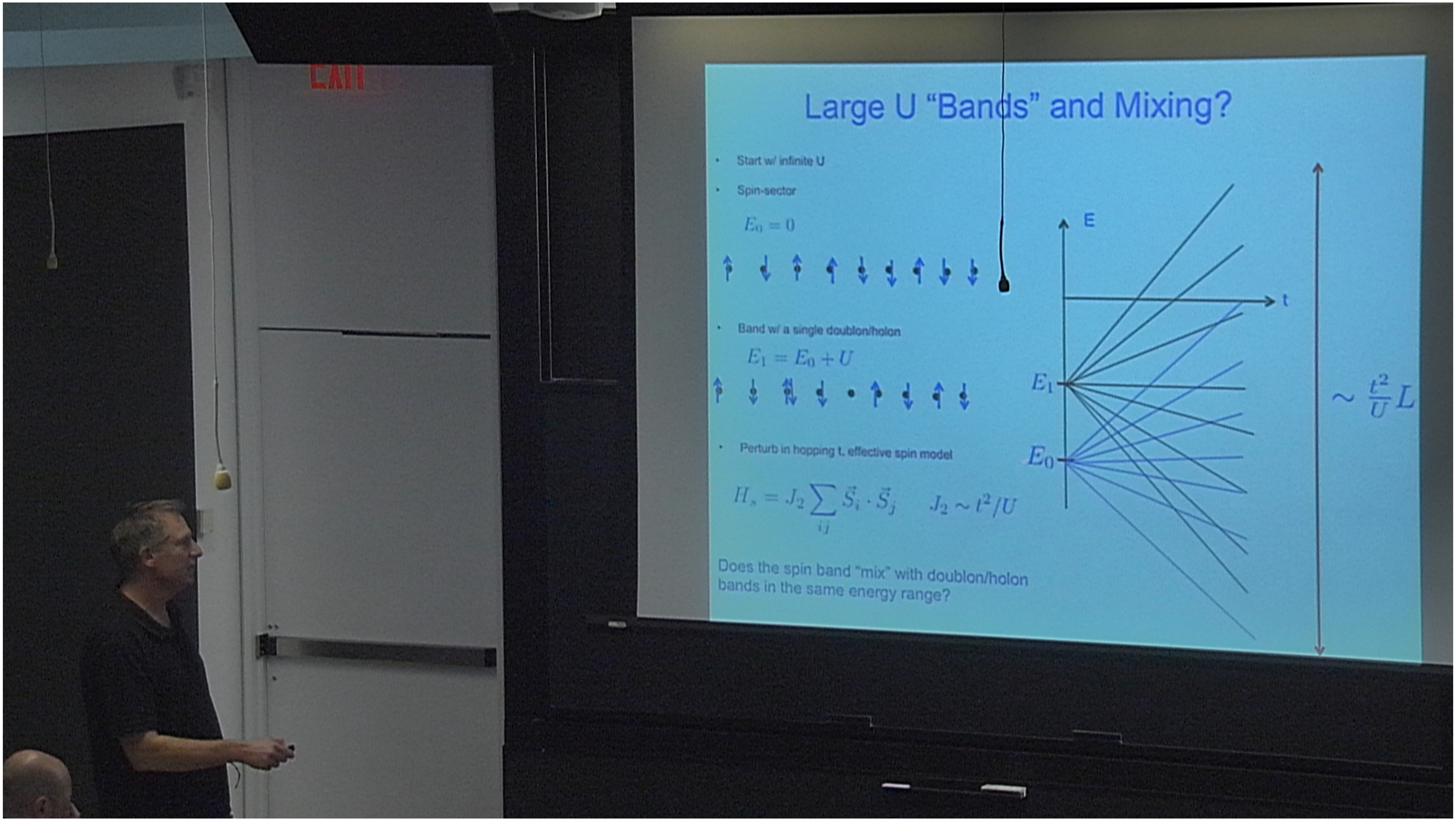
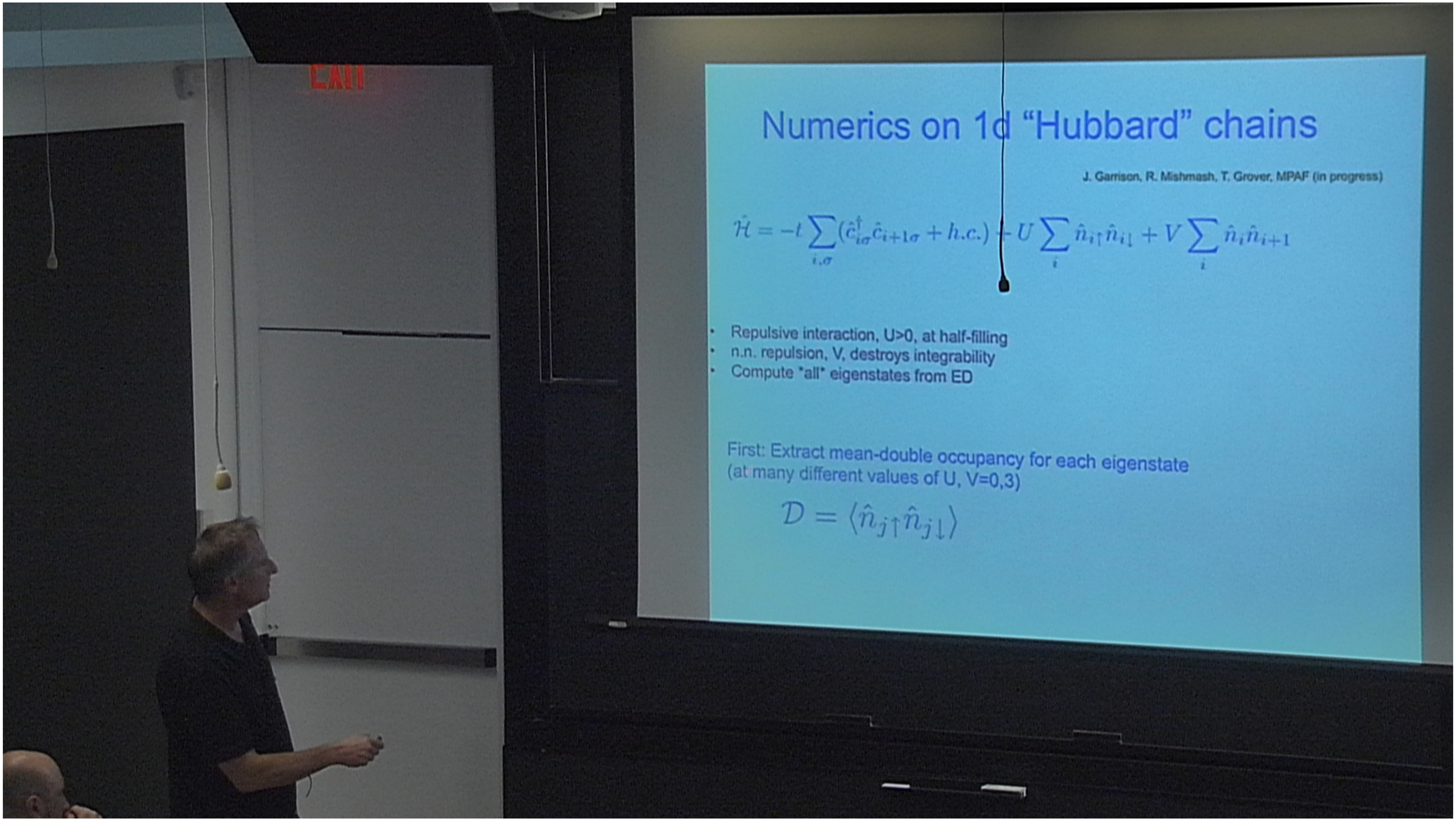
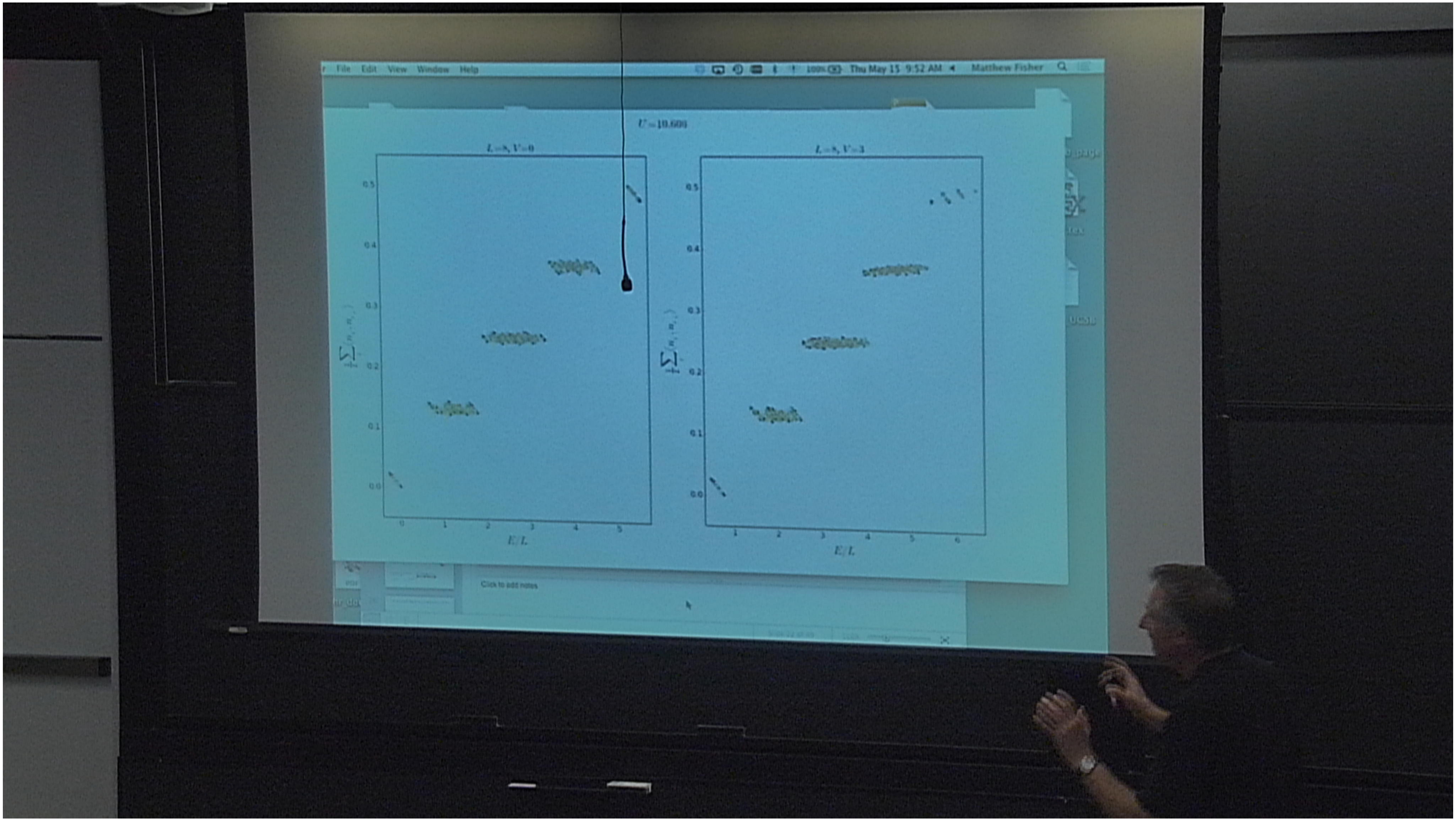

Large U "Bands" and Mixing?
Start w/ infinite U
Spin-sector
$E_0 = 0$
Band w/ a single doublon/holon
$E_1 = E_0 + U$
Perturb in hopping t, effective spin model
$H_s = J_2 \sum_{ij} \vec{S}_i \cdot \vec{S}_j \quad J_2 \sim t^2/U$
Does the spin band "mix" with doublon/holon bands in the same energy range?
E
t
E_1
E_0
$\sim \frac{t^2}{U} L$


EXIT
Numerics on 1d "Hubbard" chains
J. Garrison, R. Mishmash, T. Grover, MPAF (in progress)
$\hat{\mathcal{H}} = -t\sum_{i,\sigma}(\hat{c}^{\dagger}_{i\sigma}\hat{c}_{i+1\sigma} + h.c.) + U\sum_i \hat{n}_{i\uparrow}\hat{n}_{i\downarrow} + V\sum_i \hat{n}_i\hat{n}_{i+1}$
Repulsive interaction, U>0, at half-filling
n.n. repulsion, V, destroys integrability
Compute *all* eigenstates from ED
First: Extract mean-double occupancy for each eigenstate
(at many different values of U, V=0,3)
$\mathcal{D} = \langle \hat{n}_{j\uparrow}\hat{n}_{j\downarrow} \rangle$


EXIT
File Edit View Window Help
Thu May 15 9:50 AM
Matthew Fisher
Hubbard_V_Double_movie
$L=8, V=0$
$L=8, V=3$
E/L
E/L
Click to add notes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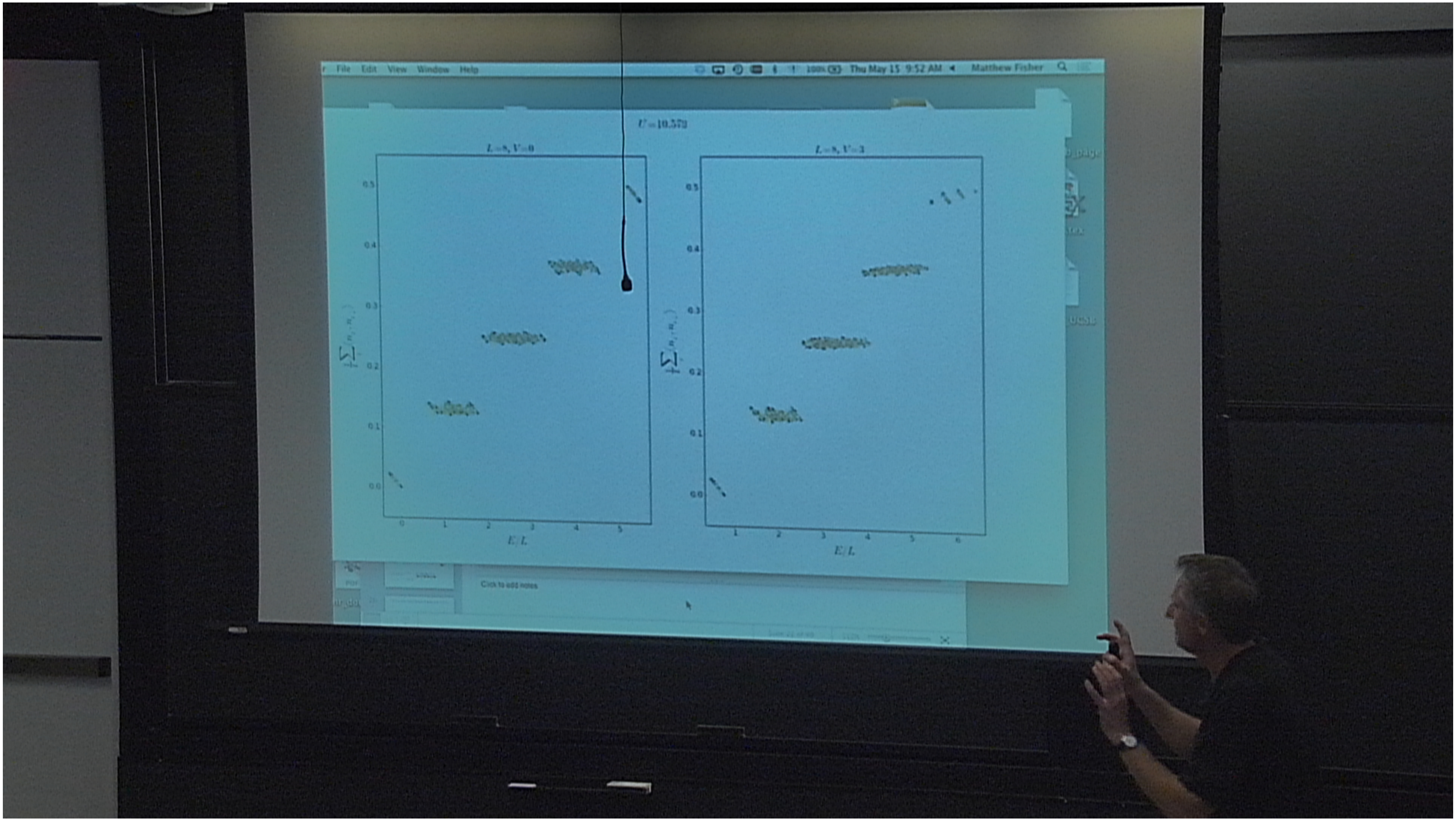
Thu May 15 9:52 AM
Matthew Fisher
E/L
E/L
Click to add notes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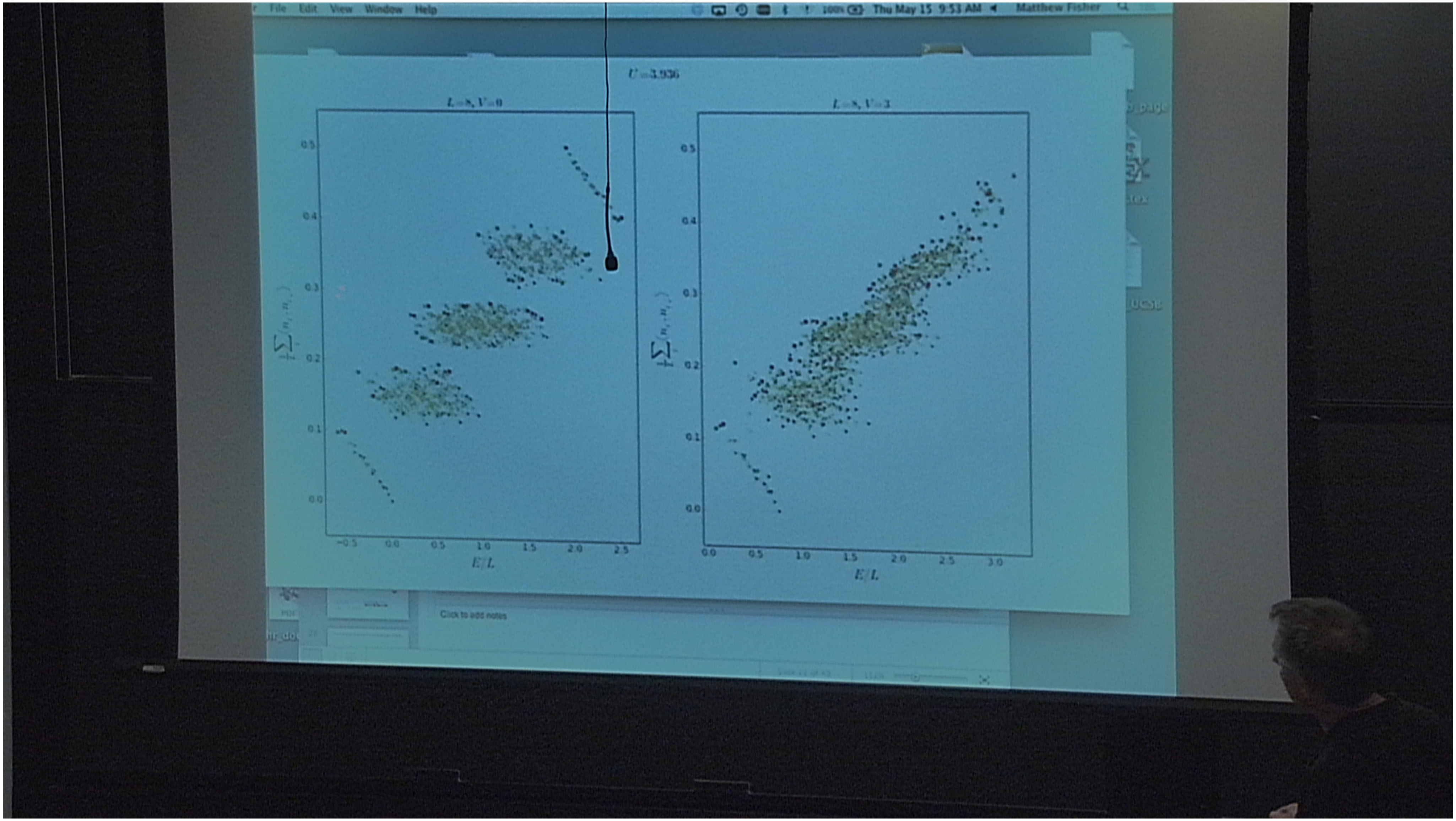
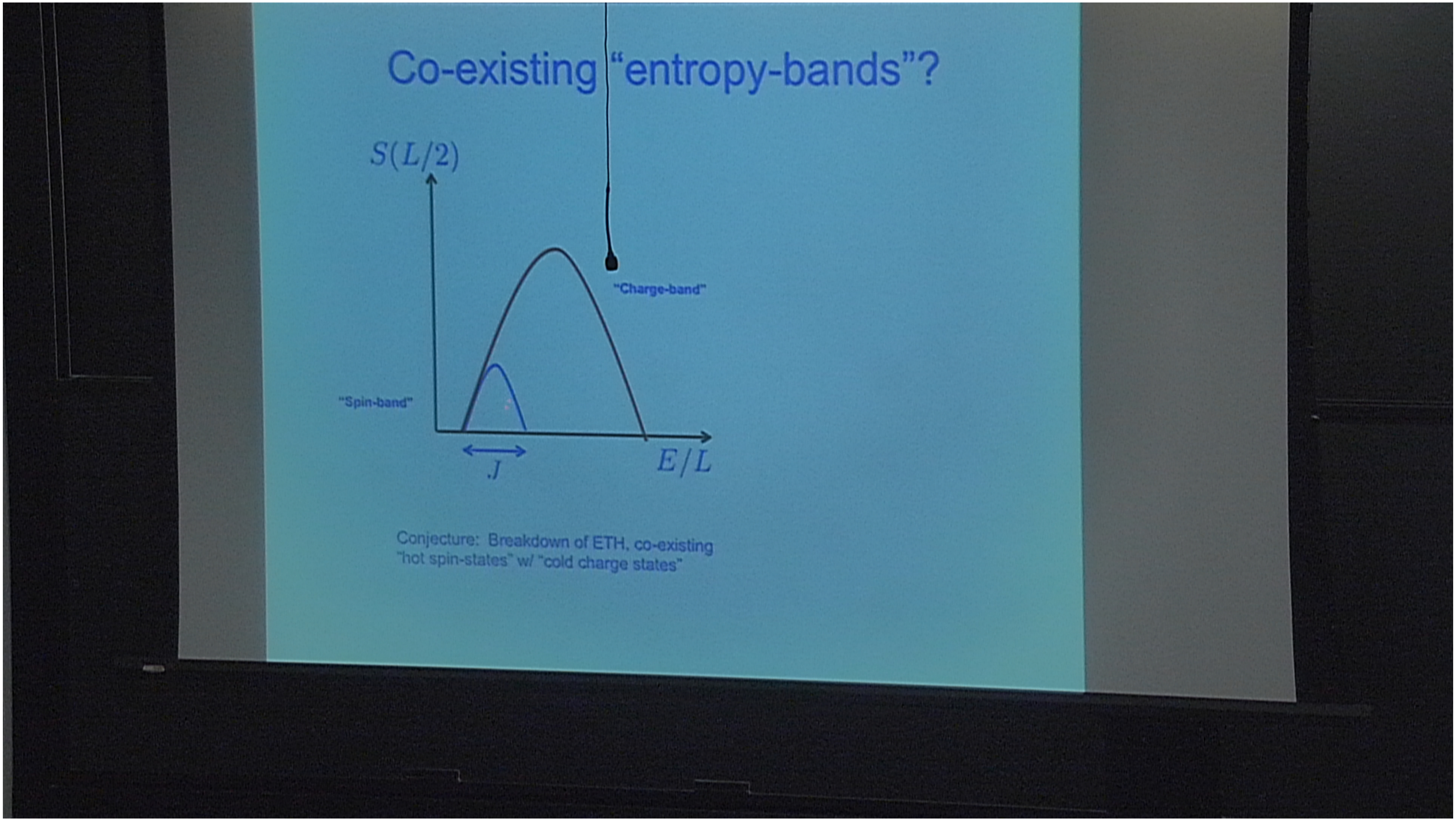
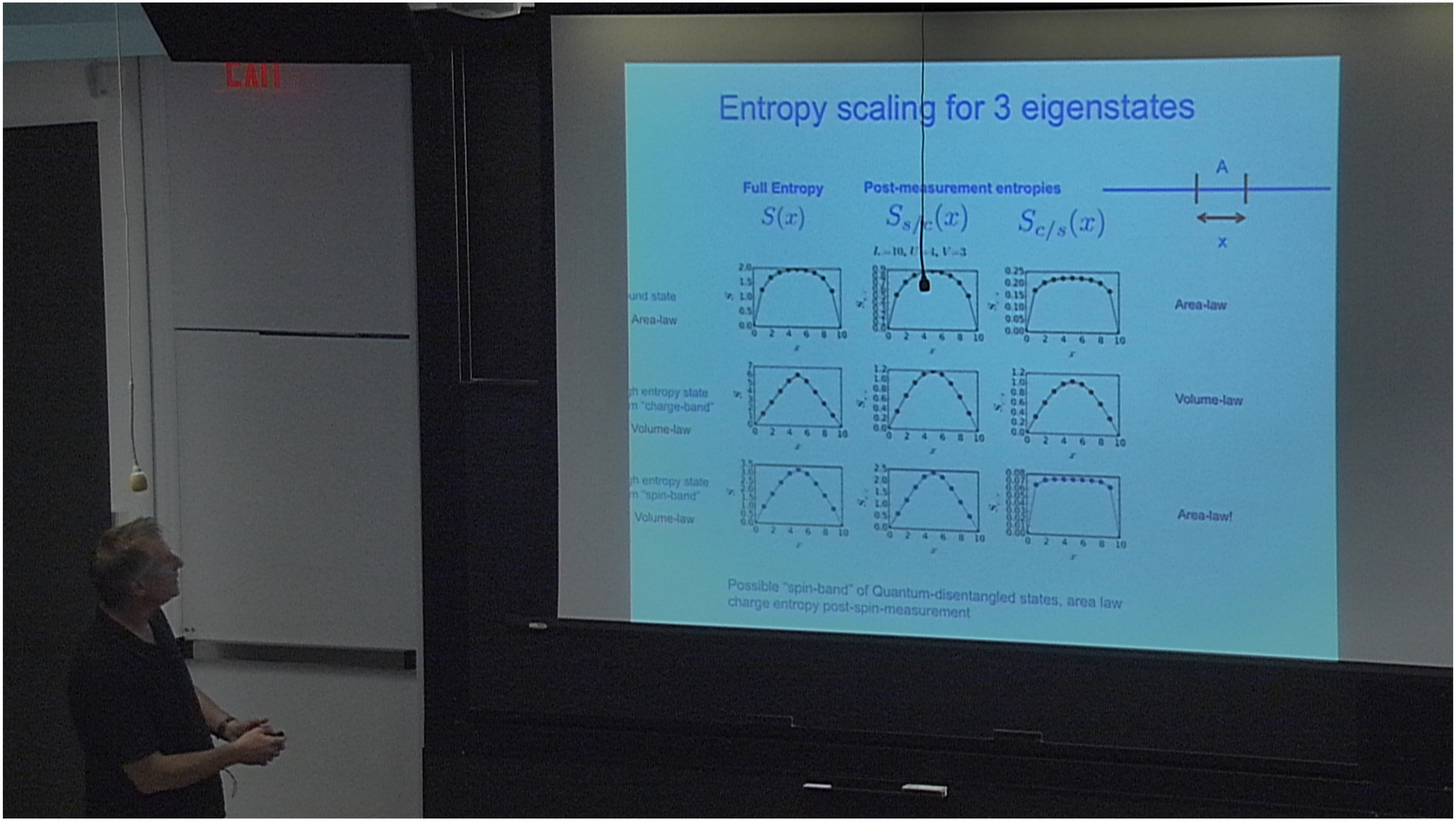
Thu May 15 9:52 AM
Matthew Fisher
L = 8, V = 0
L = 8, V = 3
E/L
E/L
Click to add notes


$U=3.936$
$L=8, V=0$
$L=8, V=3$
E/L
E/L

Full and post-measurement entanglement entropy
0 site Hubbard chain
n.n. repulsion, V=3
Full Entanglement entropy
$S(L/2)$
Post-spin-measurement entropy of "charge-wf"
$S_{c/s}(L/2)$
"Charge-band"
"Spin-band" -
Post-spin-measurment, "charge-wf" low entanglement

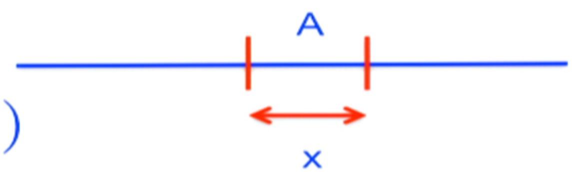

Co-existing “entropy-bands”?
$S(L/2)$
"Charge-band"
"Spin-band"
J
E/L
Conjecture: Breakdown of ETH, co-existing
"hot spin-states" w/ "cold charge states"

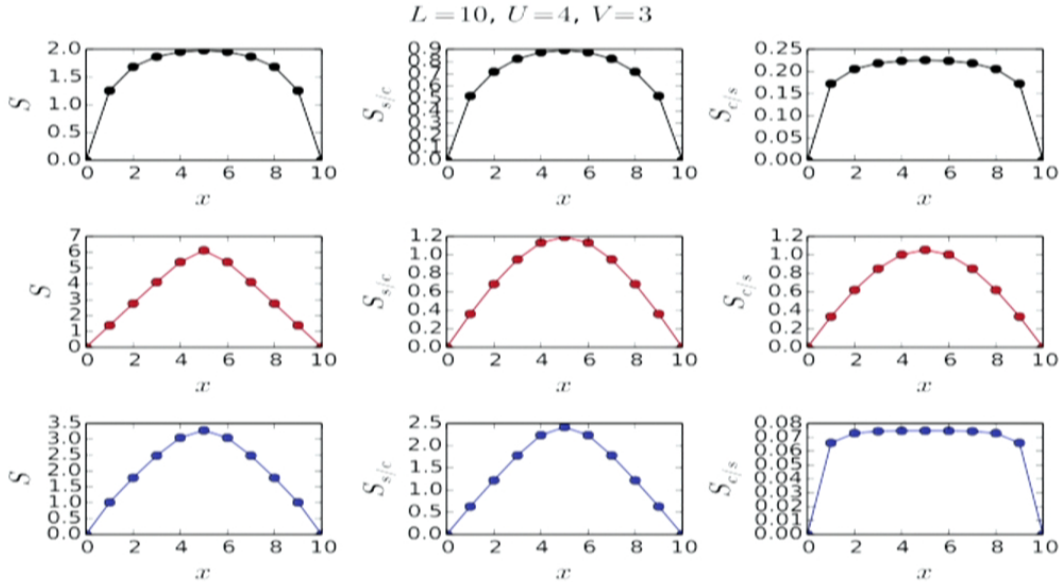

Entropy scaling for 3 eigenstates
Full Entropy
Post-measurement entropies
$S(x)$
$S_{s/c}(x)$
$S_{c/s}(x)$
A
x
Area-law
Volume-law
Area-law!
Possible "spin-band" of Quantum-disentangled states, area law charge entropy post-spin-measurement

Entropy scaling for 3 eigenstates

Full Entropy $S(x)$

Post-measurement entropies $S_{s/c}(x)$ $S_{c/s}(x)$





und state
Area-law

gh entropy state
m "charge-band"
Volume-law

gh entropy state
m "spin-band"
Volume-law

Area-law

Volume-law

Area-law!

Possible "spin-band" of Quantum-disentangled states, area law charge entropy post-spin-measurement

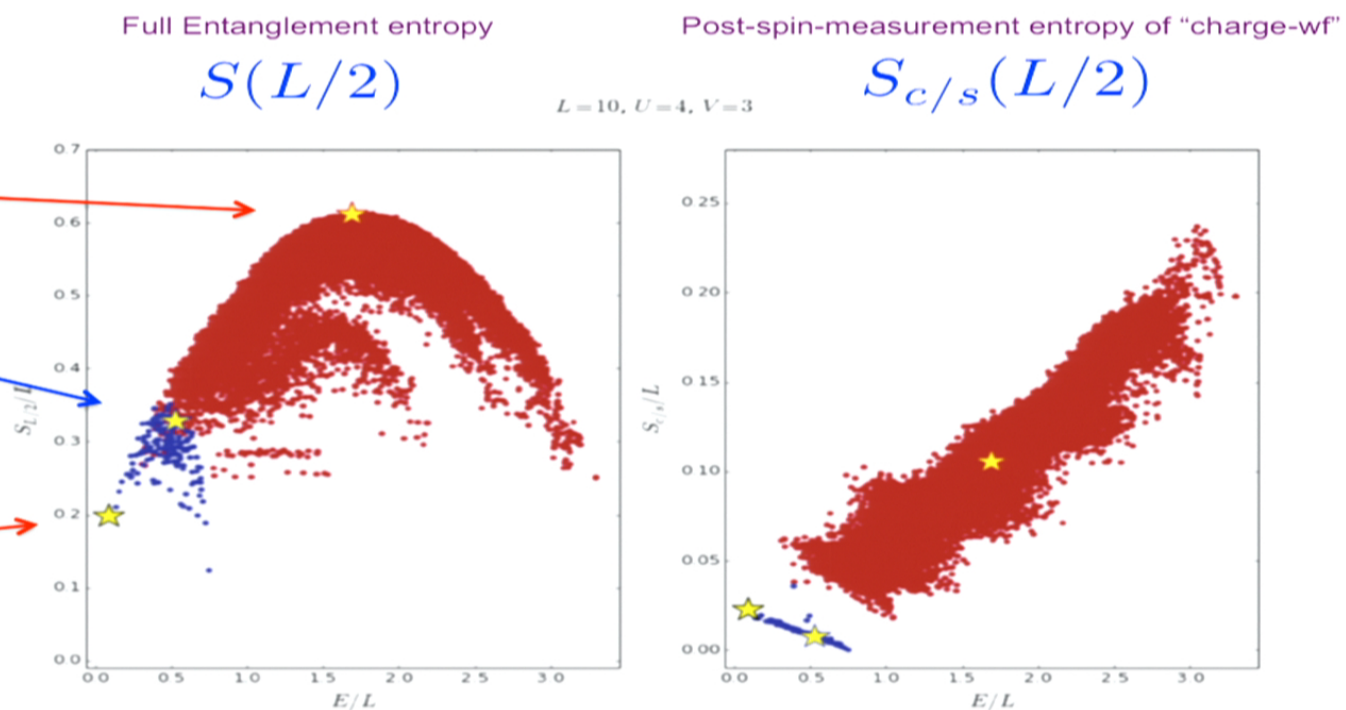
Focus on 3 eigenstates

l0 site Hubbard chain
n.n. repulsion V=3

gh entropy state,
harge-band"

h entropy state
pin-band"

ound state



Summary

Possible "Hidden" Locality in (some) "Thermal" eigenstates

New diagnostic for many-body wf: **Partial-measurement**,
followed by post-measurment entanglement entropy of un-measured dof

Quantum Disentangled States: Volume-law eigenstates w/
post-measurement area-law entanglement entropy – **"induced-locality"**

Possible Breakdown of ETH ("atomic" and "ionized" states co-exist
at same energies)

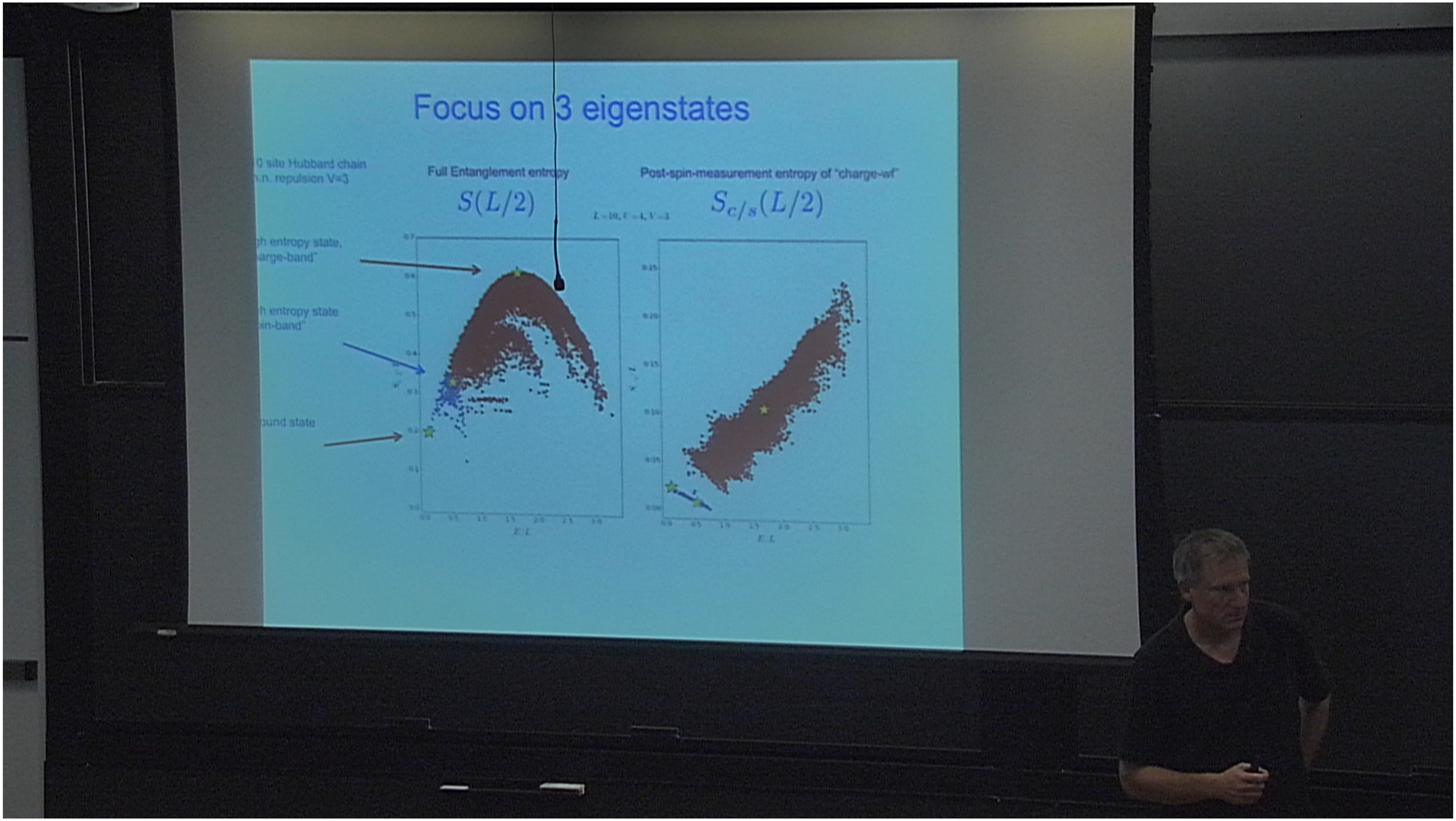
Challenge:

Identify Hamiltonians which exhibit Quantum-disentangled eigenstates?
(eg heavy/light particles)

Dynamics/transport in "Low-entanglement" "atomic" bands?

Other diagnostics to reveal hidden structure in volume-law entangled states?


at same energies)
Challenge:
Identify Hamiltonians which exhibit
(eg heavy/light particles)
Dynamics/transport in "Low-entangleme
Other diagnostics to reveal hidden



Focus on 3 eigenstates
0 site Hubbard chain
n.n. repulsion V=3
Full Entanglement entropy
$S(L/2)$
Post-spin-measurement entropy of "charge-wf"
$S_{c/s}(L/2)$
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