Title: Universal dynamics and topological order in many-body localized states

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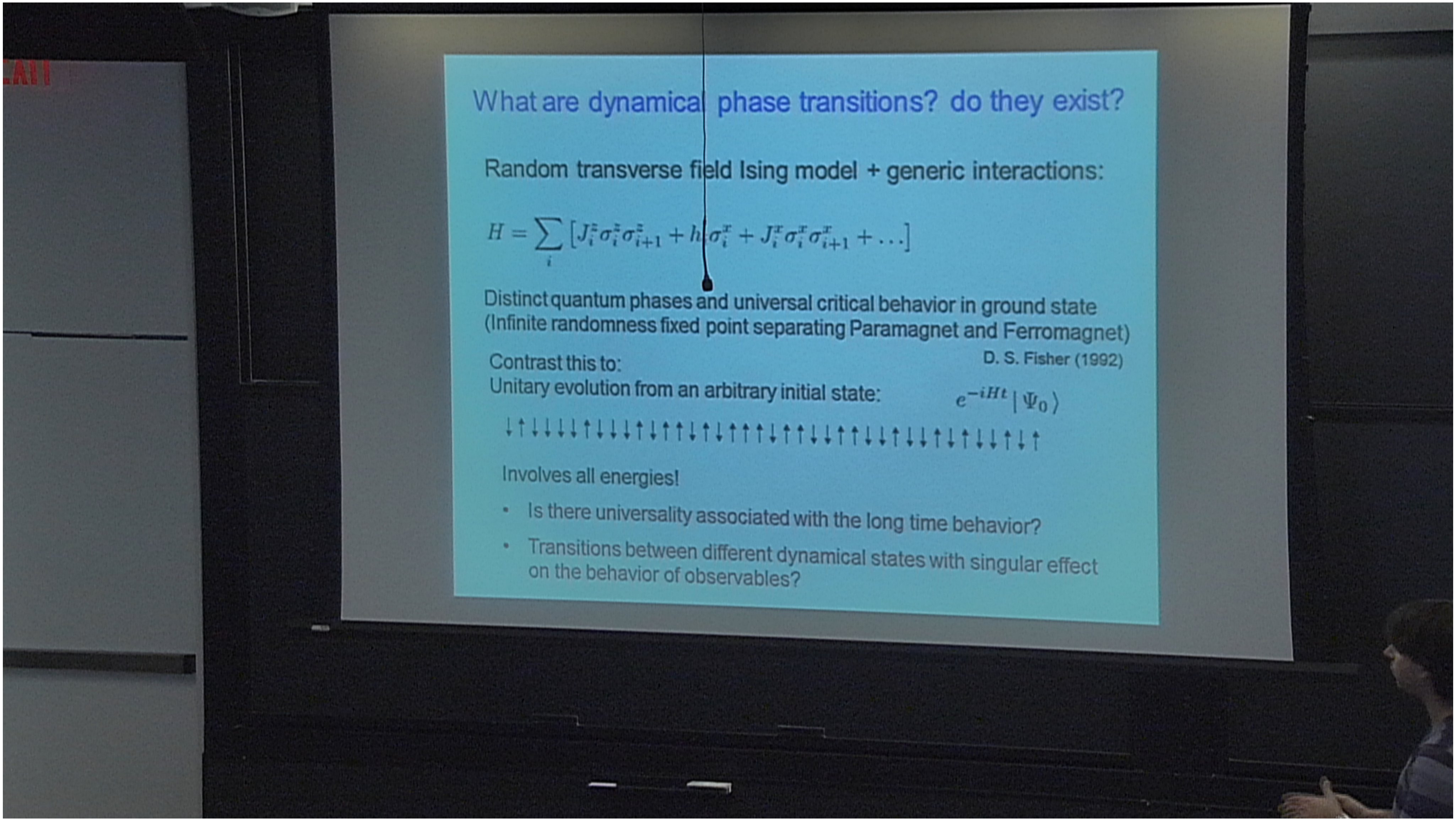
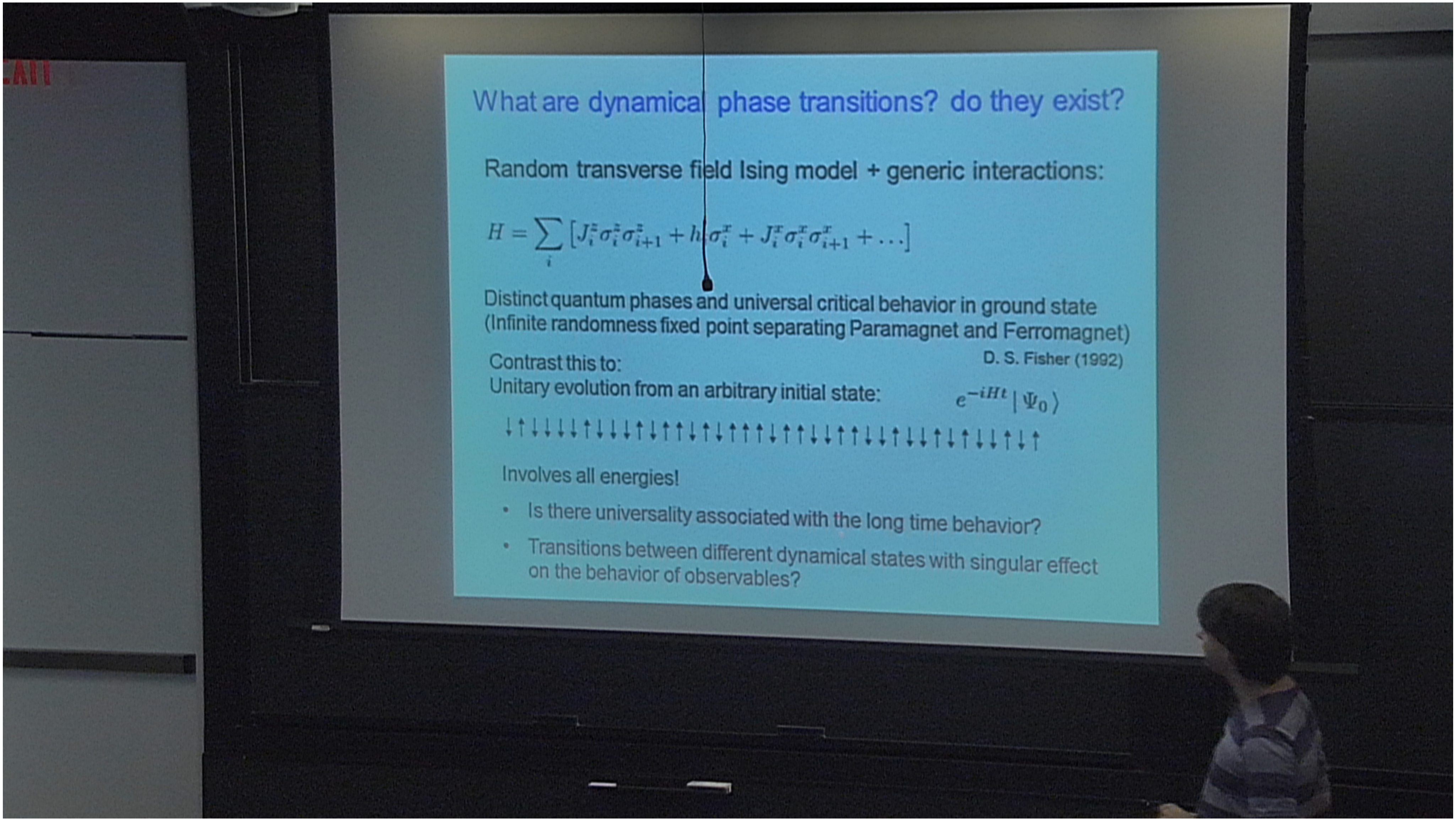
Abstract: It has been argued recently that, through a phenomenon of many-body localization, closed quantum systems subject to sufficiently strong disorder would fail to thermalize. In this talk I will describe a real time renormalization group approach, which offers a controlled description of universal dynamics in the localized phase. In particular it explains the ultra-slow entanglement propagation in this state and identifies the emergent conserved quantities which prevent thermalization. The RG analysis also shows, that far from being a trivial dead state, the MBL state admits phase transitions between distinct dynamical phases. For example, I will discuss the universal aspects of a transition between a paramagnetic localized state to one which exhibits spin-glass order. Finally, I will present a development of the RG scheme, defined on an effective coarse grained model, which allows to capture the transition from a many-body localized to a thermalizing state.


What are dynamical phase transitions? do they exist?
Random transverse field Ising model + generic interactions:
$H = \sum_i [J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \dots]$
Distinct quantum phases and universal critical behavior in ground state
(Infinite randomness fixed point separating Paramagnet and Ferromagnet)
D. S. Fisher (1992)
Contrast this to:
Unitary evolution from an arbitrary initial state:
$e^{-iHt} | \Psi_0 \rangle$
Involves all energies!
Is there universality associated with the long time behavior?
Transitions between different dynamical states with singular effect on the behavior of observables?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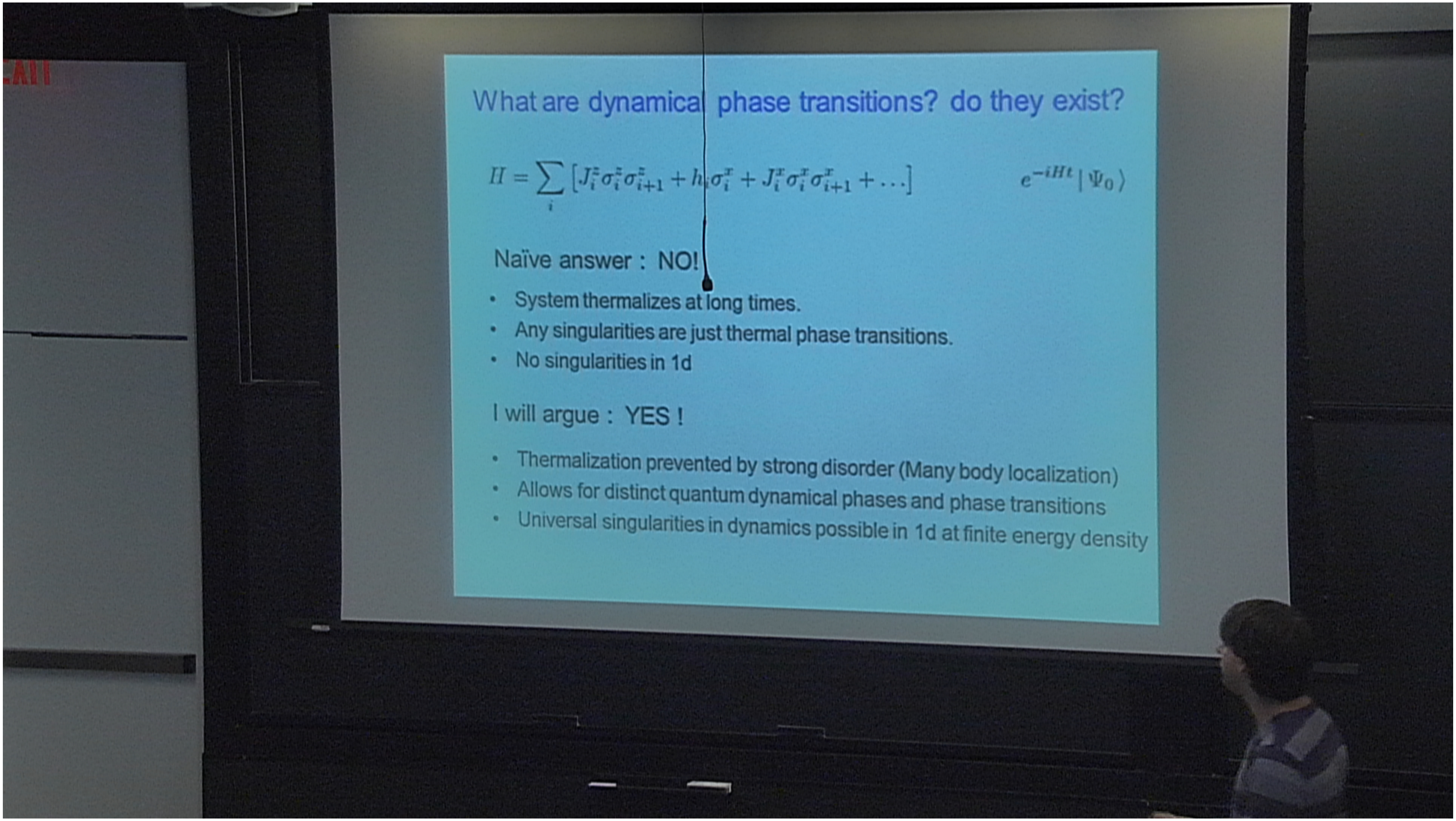
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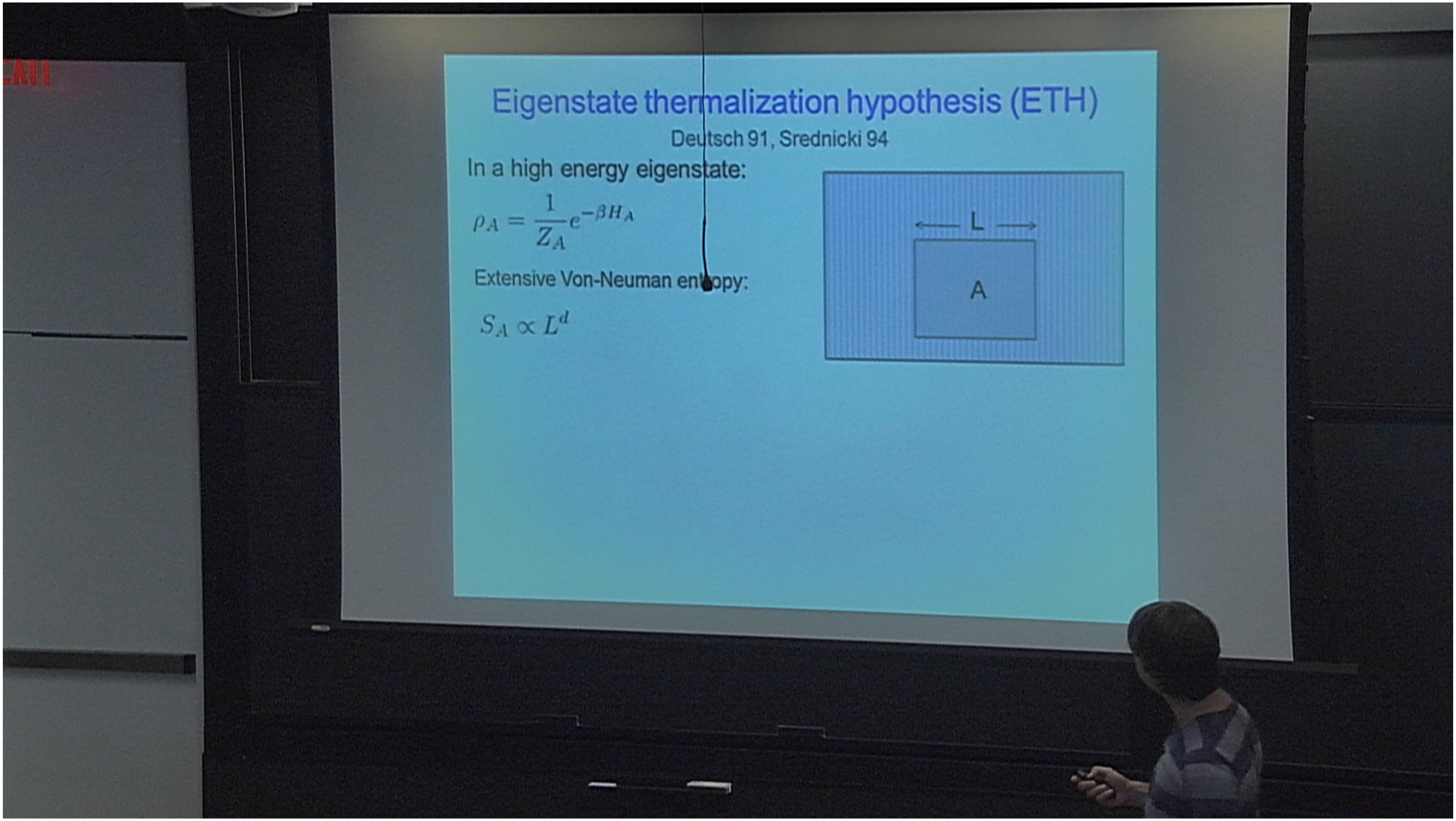
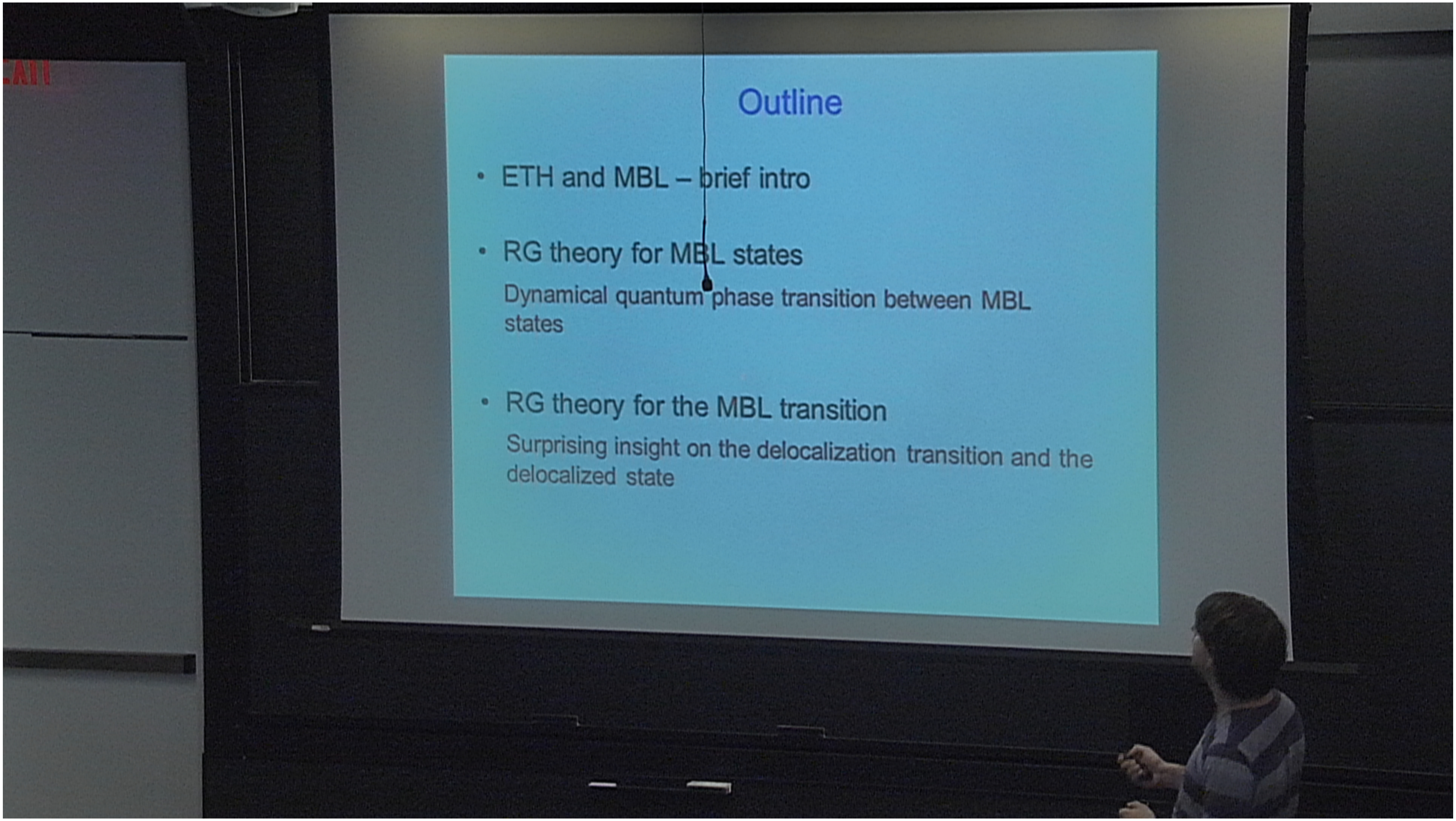
$e^{-iHt} | \Psi_0 \rangle$
Naïve answer : NO!
System thermalizes at long times.
Any singularities are just thermal phase transitions.
No singularities in 1d
I will argue : YES !
Thermalization prevented by strong disorder (Many body localization)
Allows for distinct quantum dynamical phases and phase transitions
Universal singularities in dynamics possible in 1d at finite energy density


Eigenstate thermalization hypothesis (ETH)
Deutsch 91, Srednicki 94
In a high energy eigenstate:
$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$
Extensive Von-Neuman entropy:
$S_A \propto L^d$
L
A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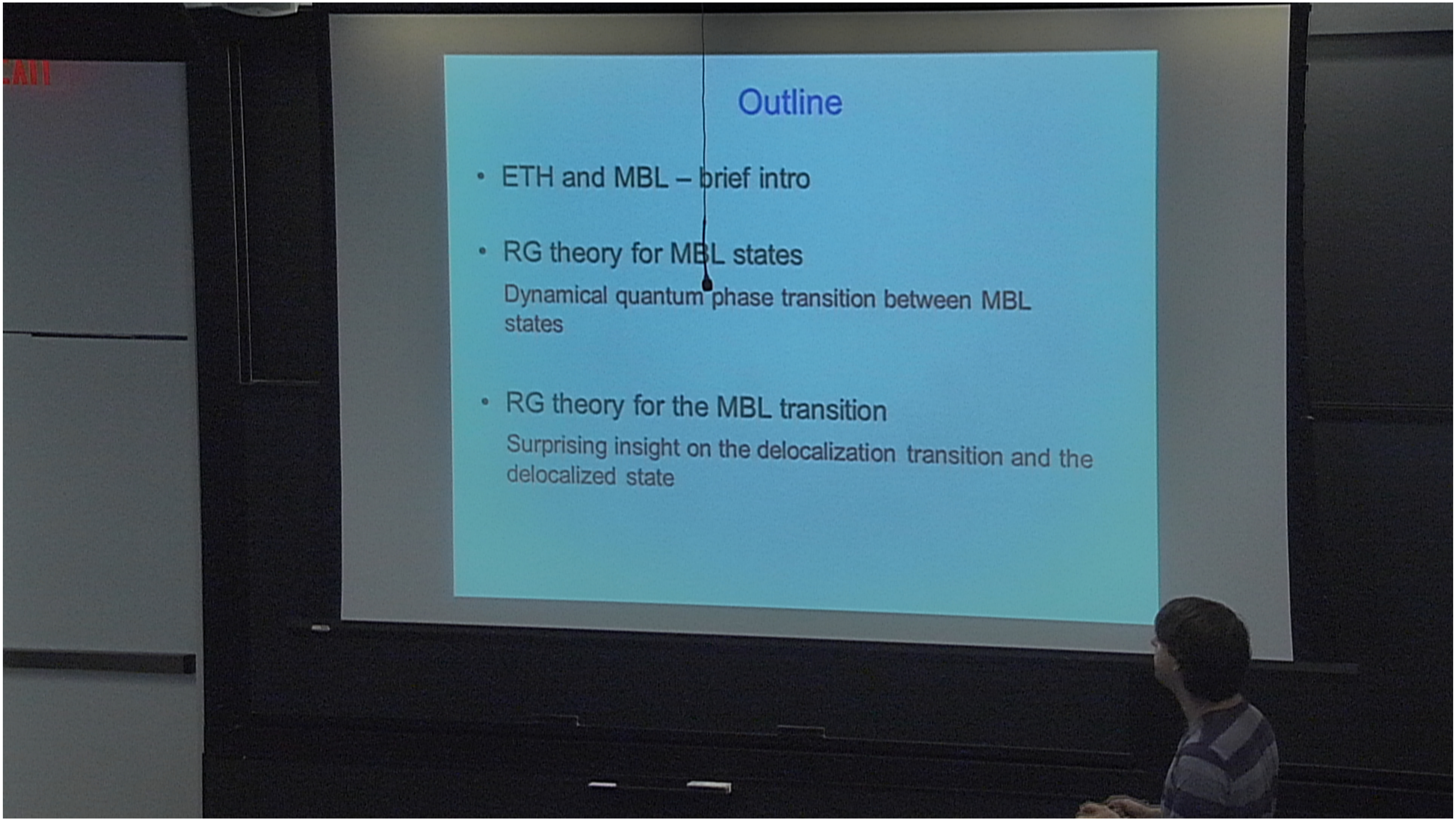
• ETH and MBL – brief intro
• RG theory for MBL states
Dynamical quantum phase transition between MBL states
• RG theory for the MBL transition
Surprising insight on the delocalization transition and the delocalized state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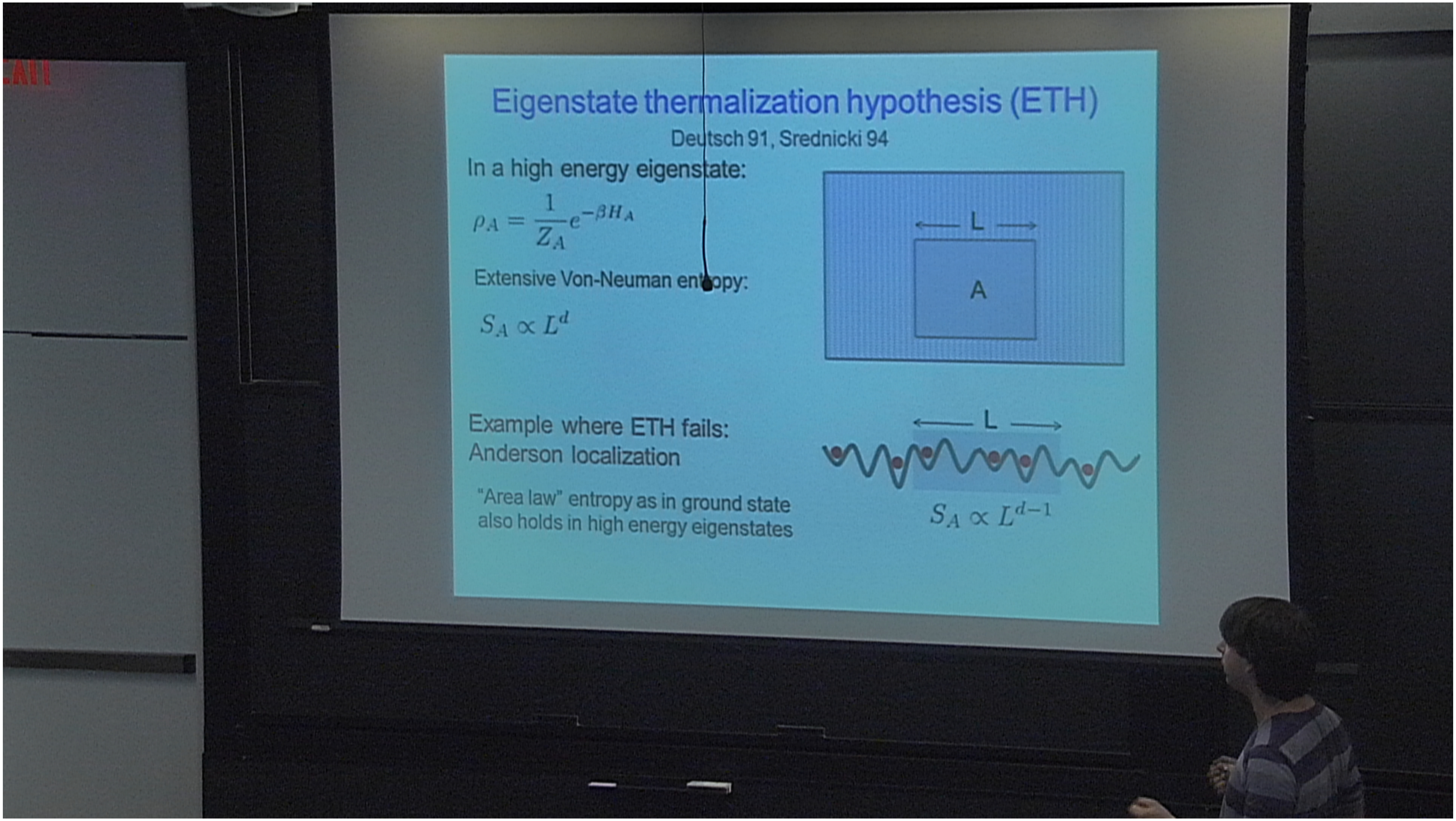
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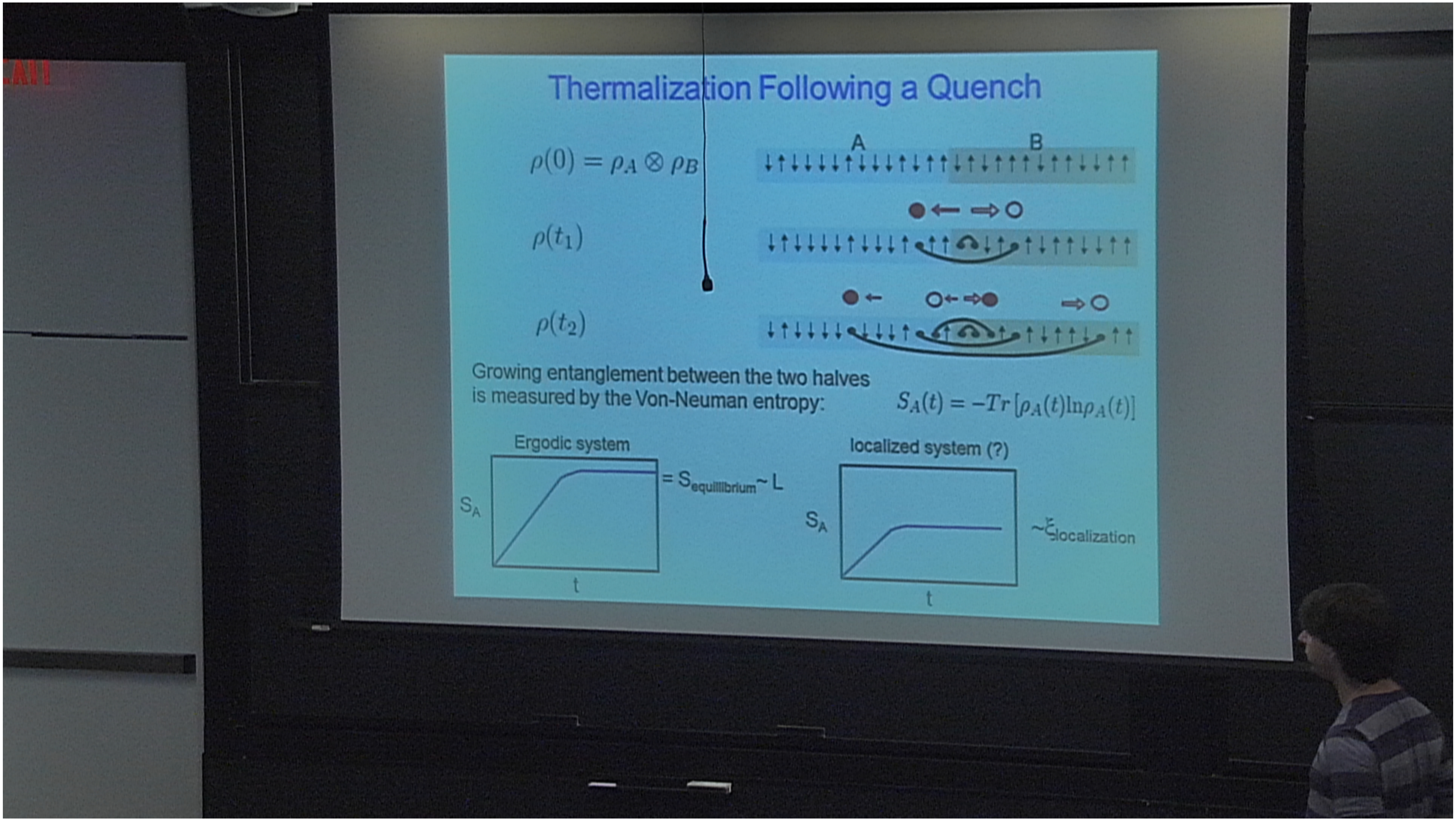
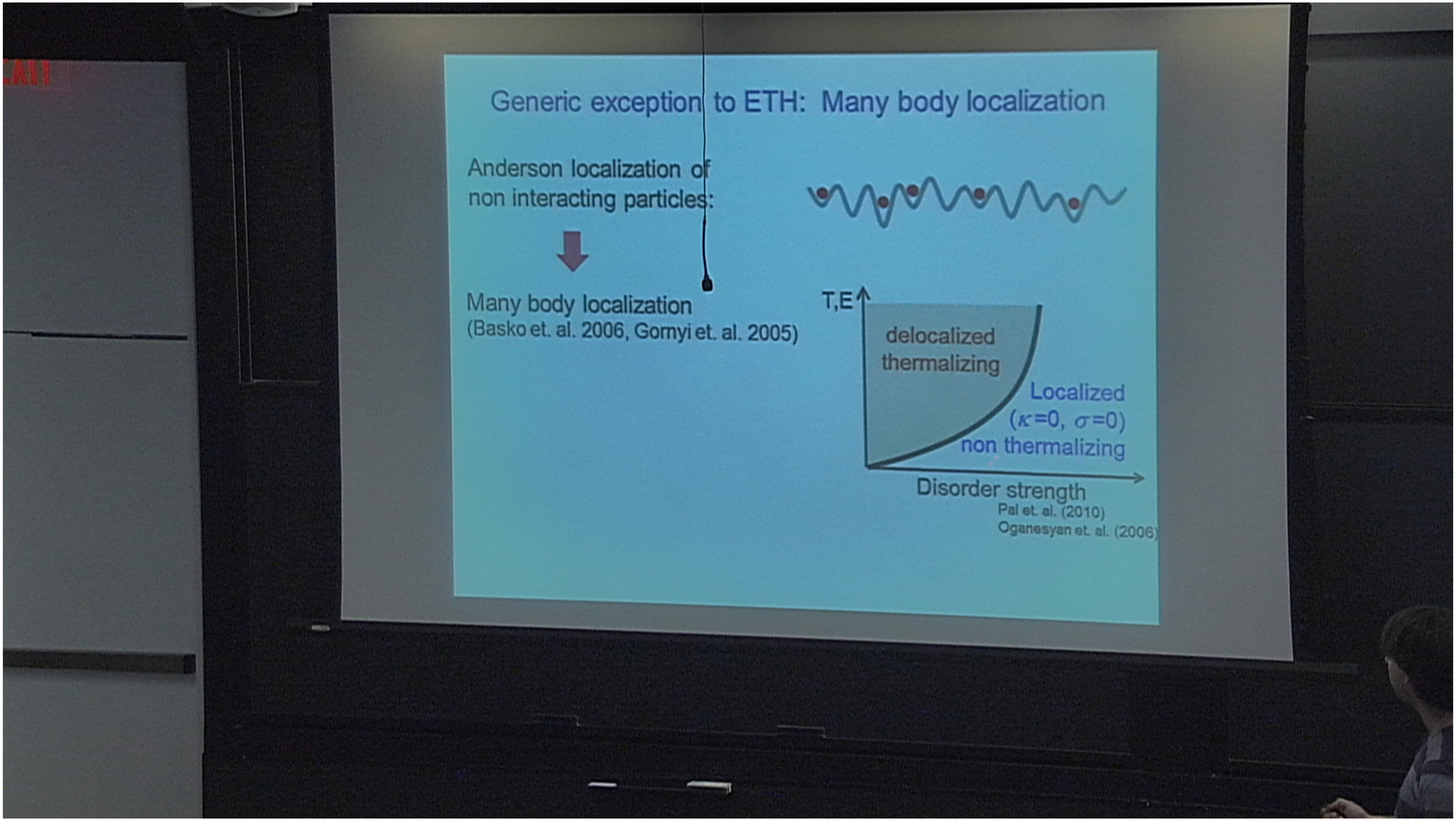
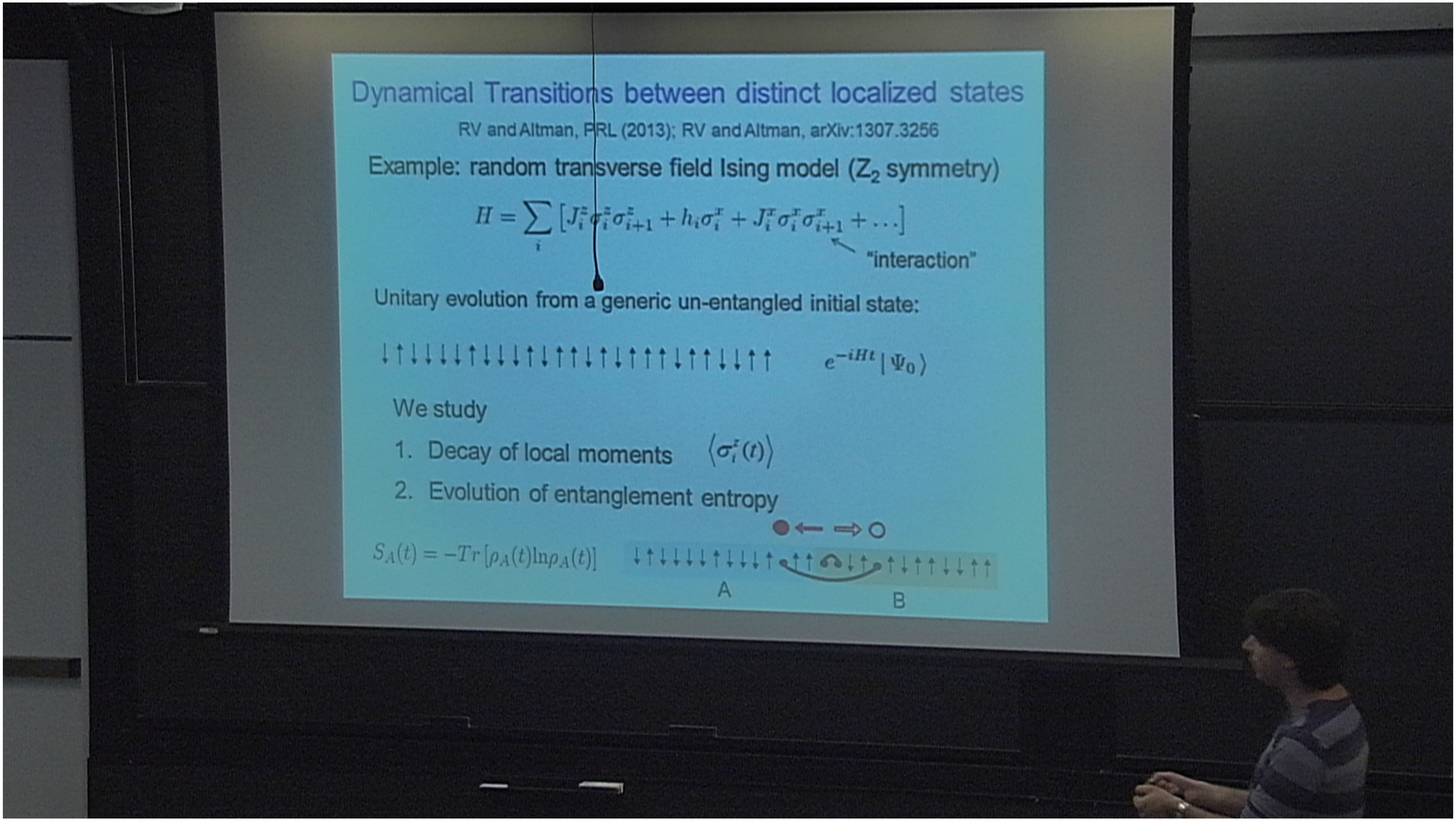
Example where ETH fails:
Anderson localization
"Area law" entropy as in ground state
also holds in high energy eigenstates
L
$S_A \propto L^{d-1}$


Thermalization Following a Quench
$\rho(0) = \rho_A \otimes \rho_B$
A
B
$\rho(t_1)$
$\rho(t_2)$
Growing entanglement between the two halves
is measured by the Von-Neuman entropy:
$S_A(t) = -Tr\left[\rho_A(t)\ln\rho_A(t)\right]$
Ergodic system
S_A
t
$= S_{equilibrium} \sim L$
localized system (?)
S_A
t
$\sim \xi_{localization}$


Generic exception to ETH: Many body localization
Anderson localization of
non interacting particles:
Many body localization
(Basko et. al. 2006, Gornyi et. al. 2005)
T,E
delocalized
thermalizing
Localized
(κ=0, σ=0)
non thermalizing
Disorder strength
Pal et. al. (2010)
Oganesyan et. al. (2006)


Dynamical Transitions between distinct localized states
RV and Altman, PRL (2013); RV and Altman, arXiv:1307.3256
Example: random transverse field Ising model (Z_2 symmetry)
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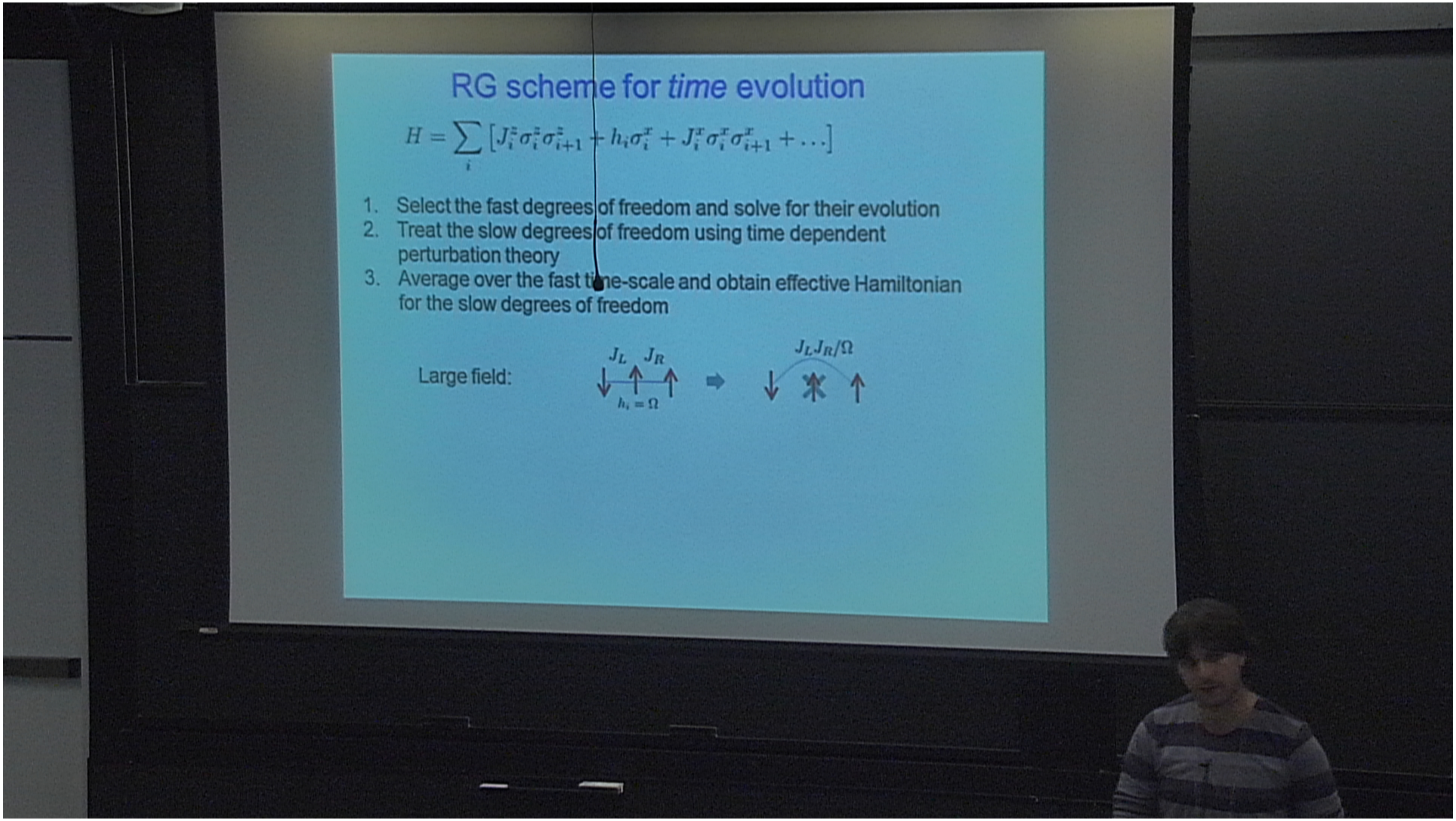
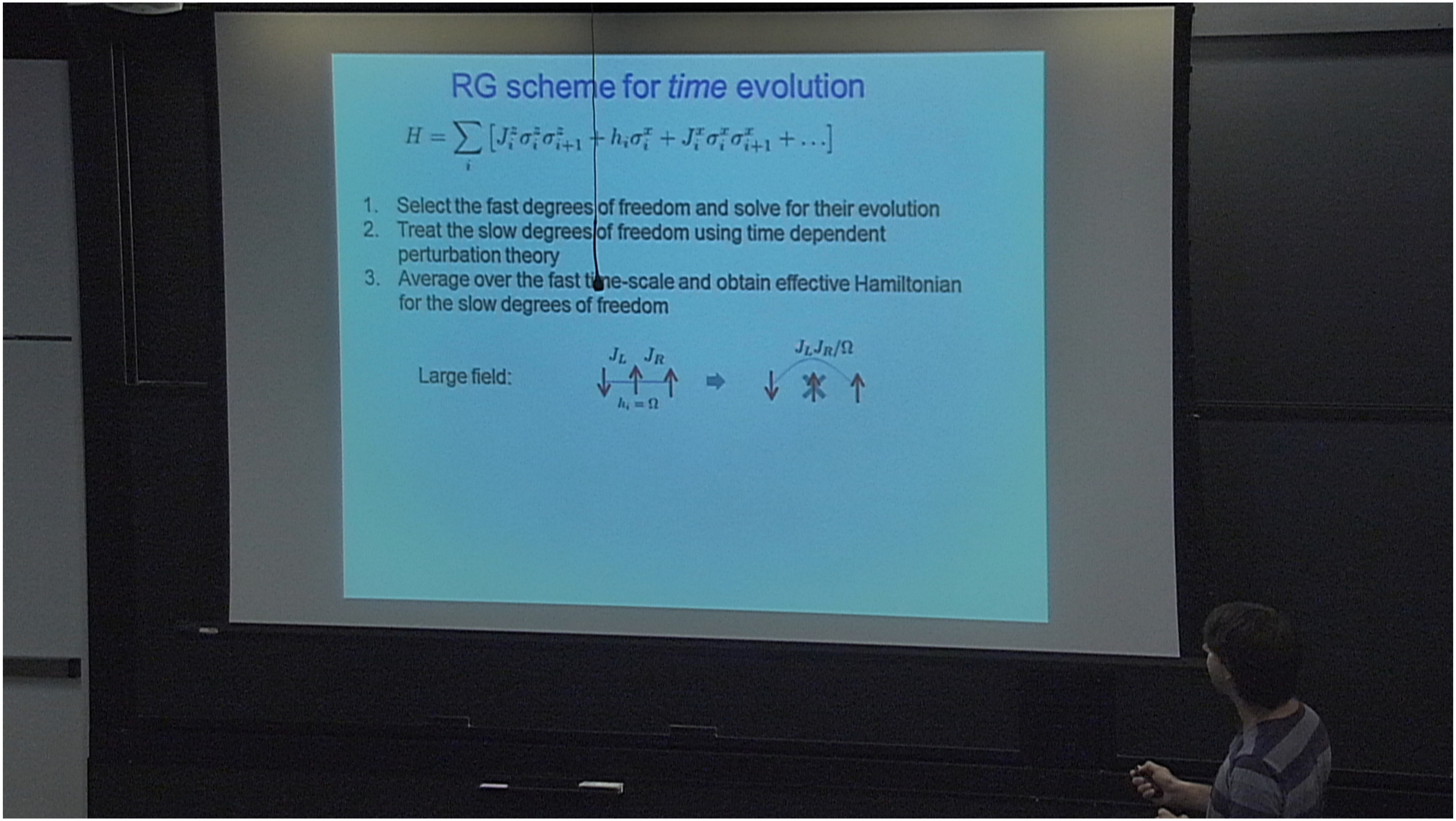
"interaction"
Unitary evolution from a generic un-entangled initial state:
$e^{-iHt} | \Psi_0 \rangle$
We study
1. Decay of local moments $\langle \sigma_i^z(t) \rangle$
2. Evolution of entanglement entropy
$S_A(t) = -Tr[\rho_A(t) \ln \rho_A(t)]$
A
B


RG scheme for *time* evolution
$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right]$
1. Select the fast degrees of freedom and solve for their evolution
2. Treat the slow degrees of freedom using time dependent perturbation theory
3. Average over the fast time-scale and obtain effective Hamiltonian for the slow degrees of freedom
Large field:
J_L J_R
$h_i = \Omega$
$J_L J_R / \Omega$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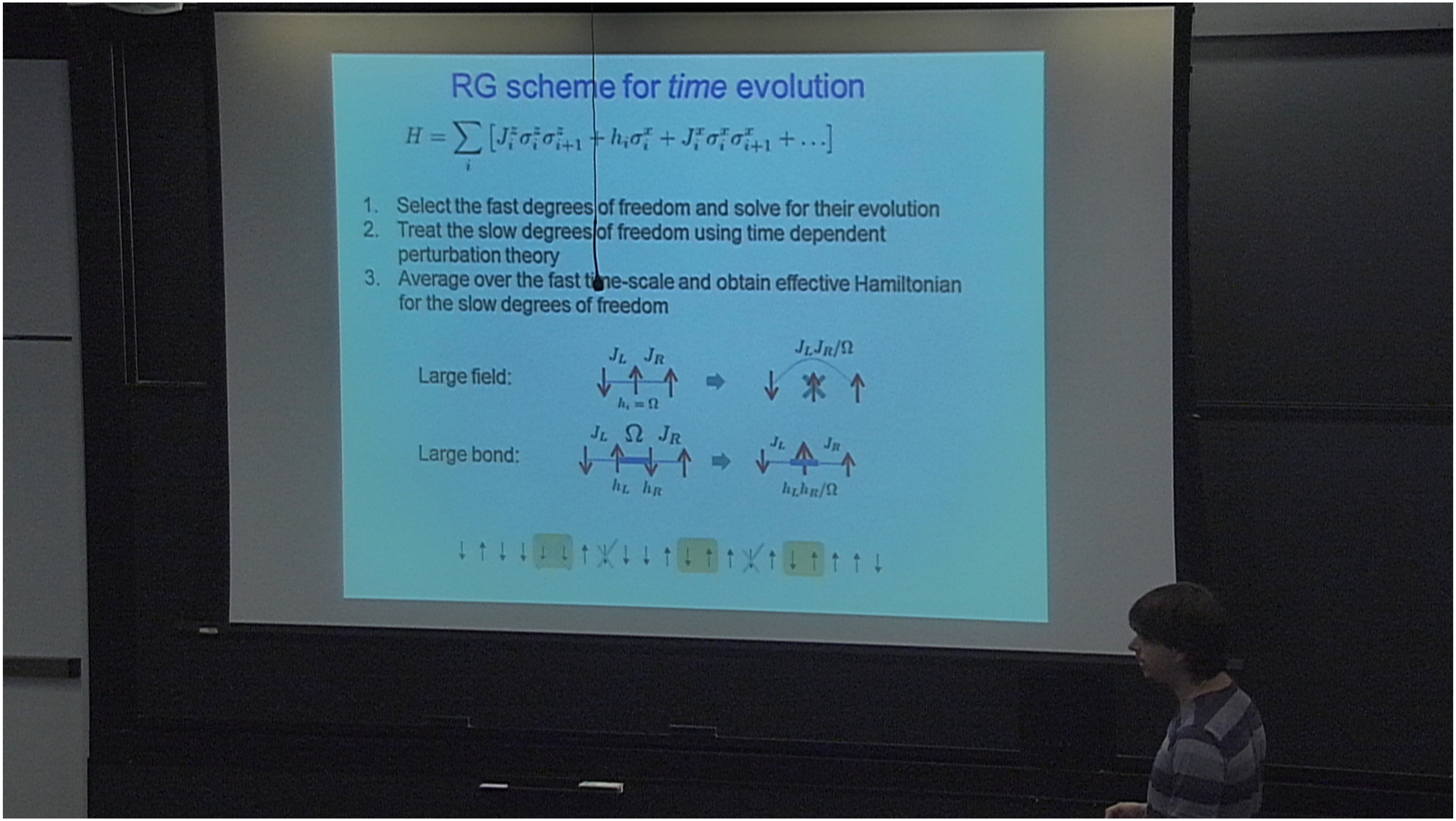
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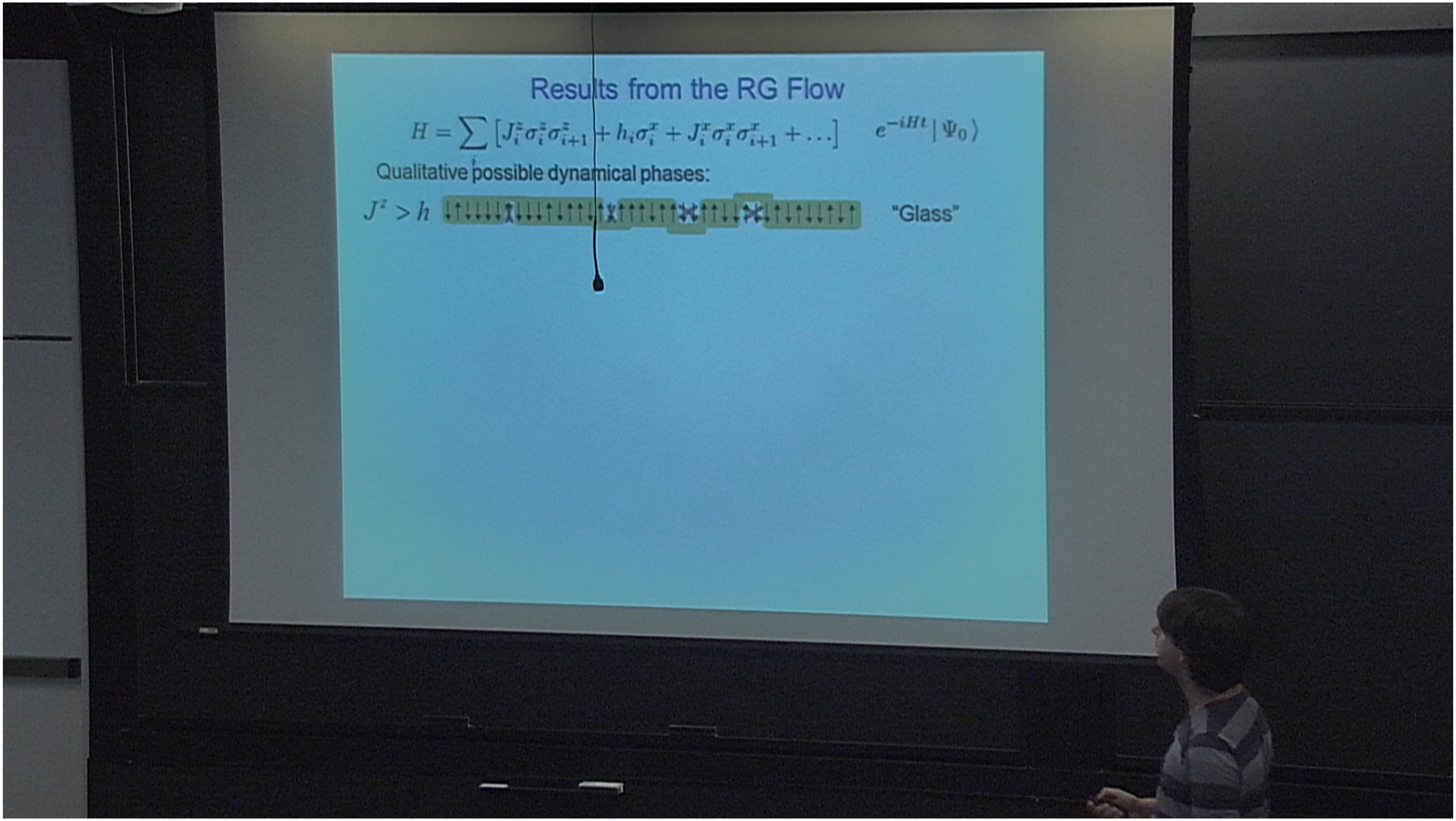
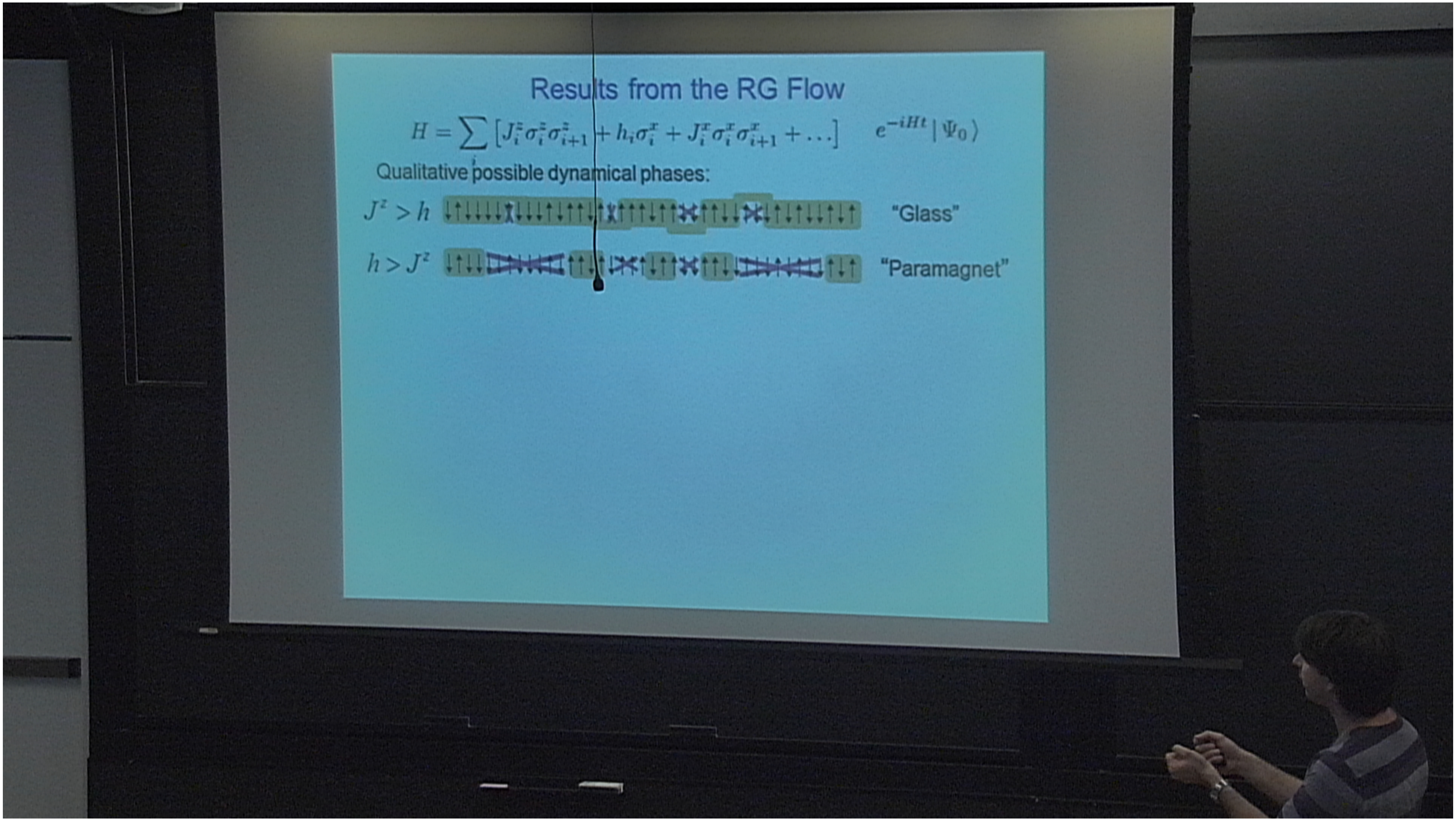
Large field:
J_L J_R
$h_i = \Omega$
$J_L J_R / \Omega$
Large bond:
J_L Ω J_R
h_L h_R
J_L J_R
$h_L h_R / \Omega$


Results from the RG Flow
$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right]$
$e^{-iHt} | \Psi_0 \rangle$
Qualitative possible dynamical phases:
$J^z > h$
"Glass"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$h > J^z$
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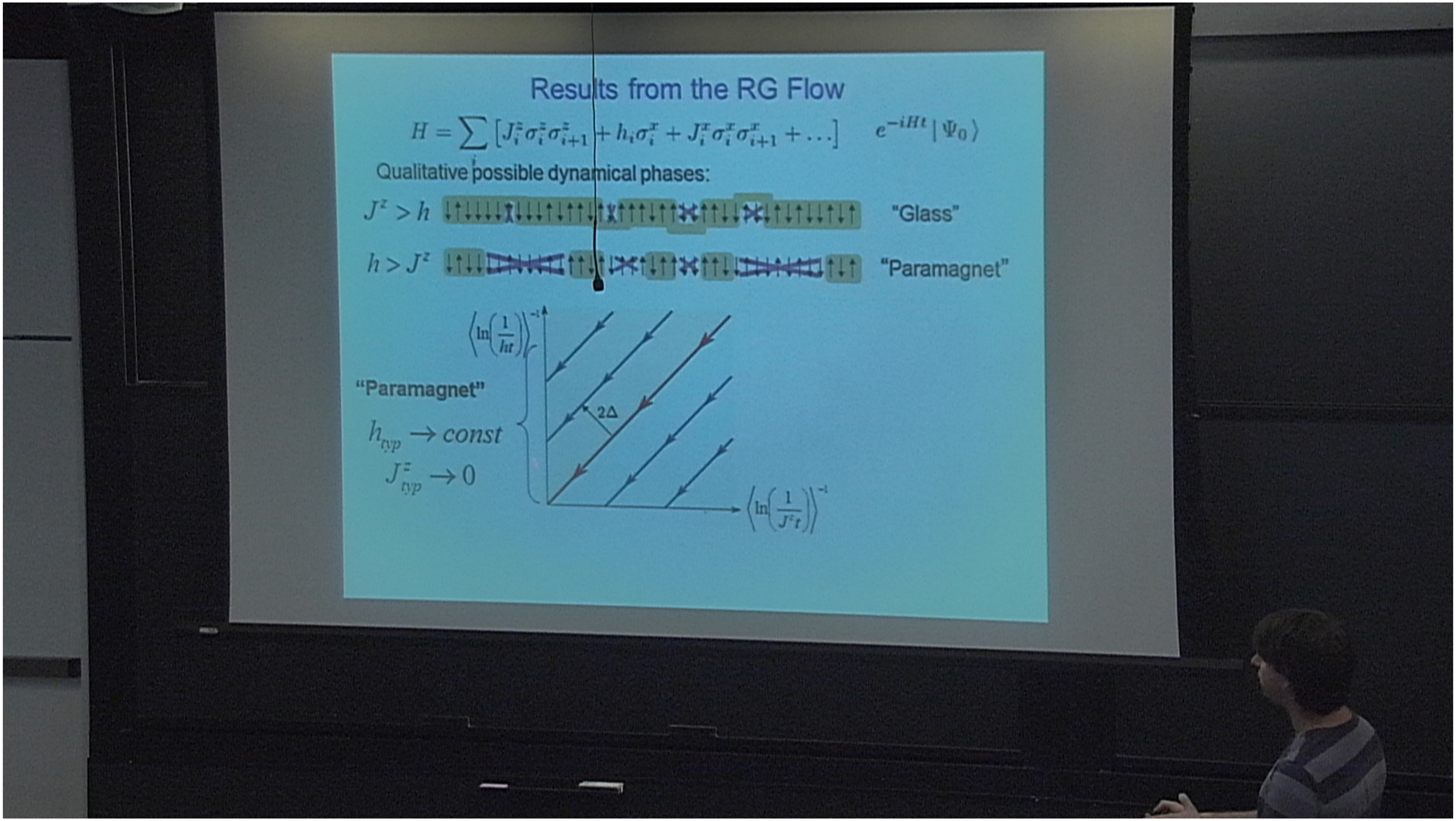

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Qualitative possible dynamical phases:
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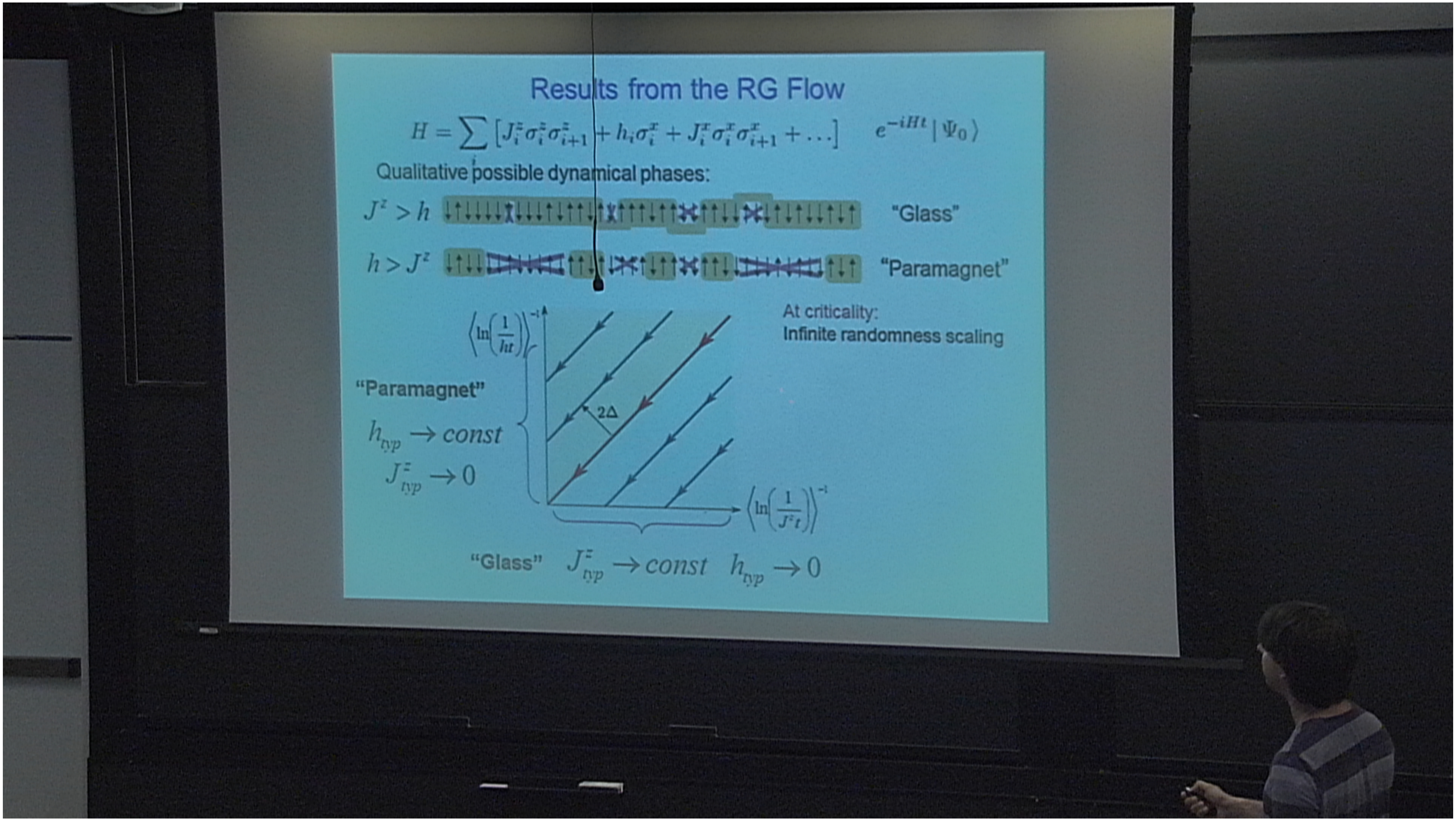
"Glass"
$h > J^z$
"Paramagnet"
$\langle \ln(\frac{1}{ht}) \rangle^{-1}$
"Paramagnet"
$h_{typ} \to const$
$J_{typ}^z \to 0$
2Δ
$\langle \ln(\frac{1}{J^z t}) \rangle^{-1}$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At criticality:
Infinite randomness scaling
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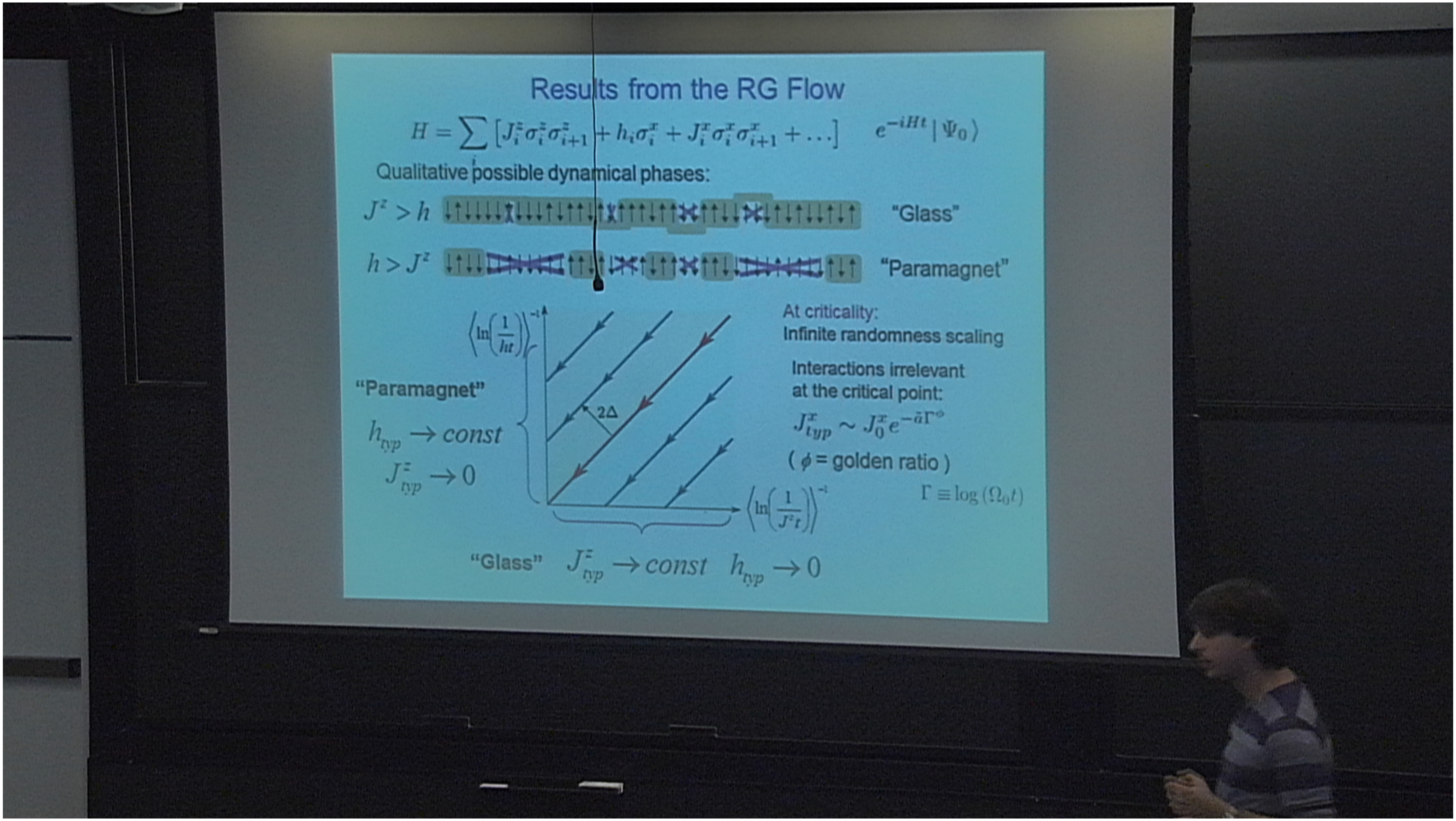
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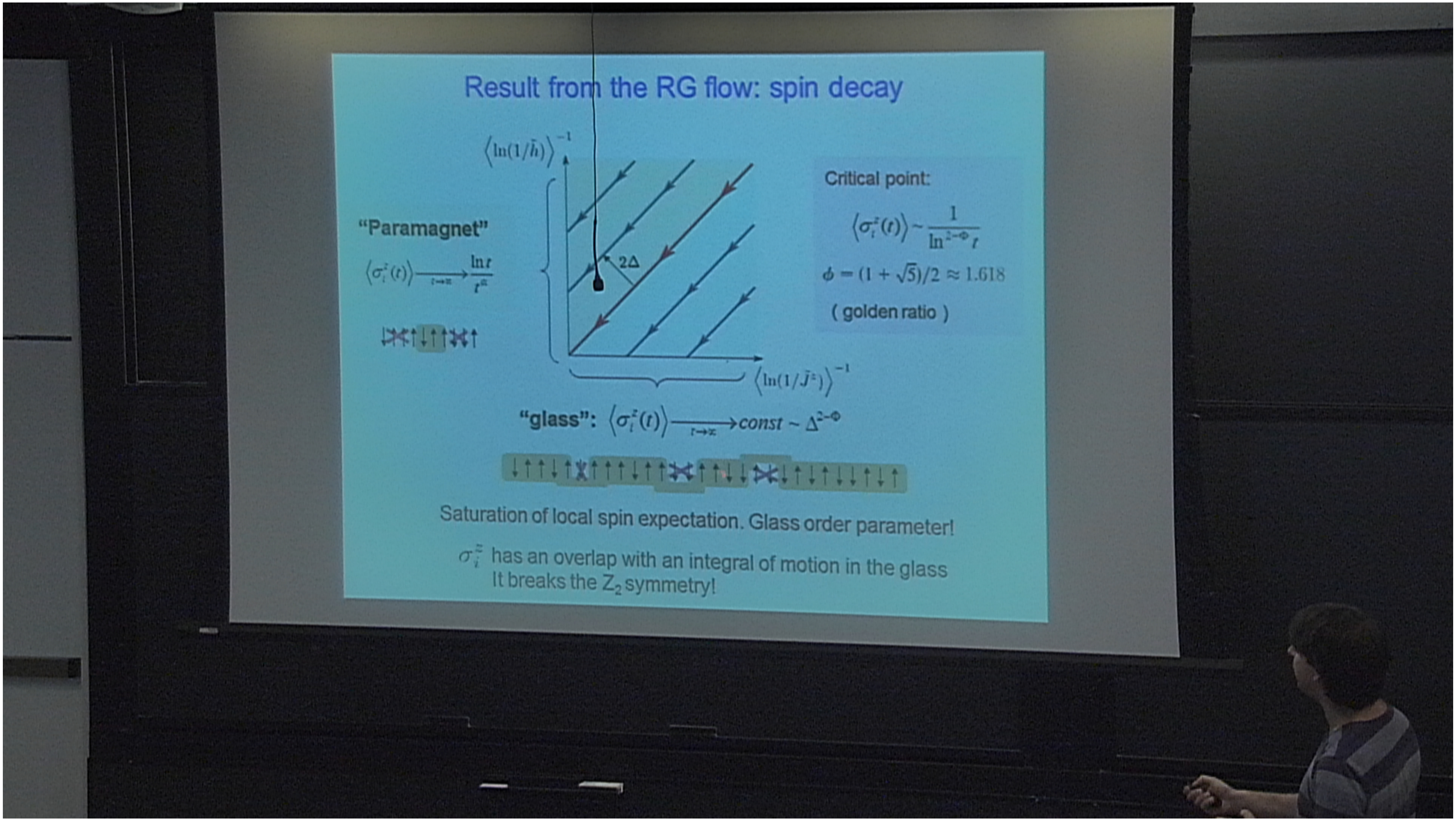
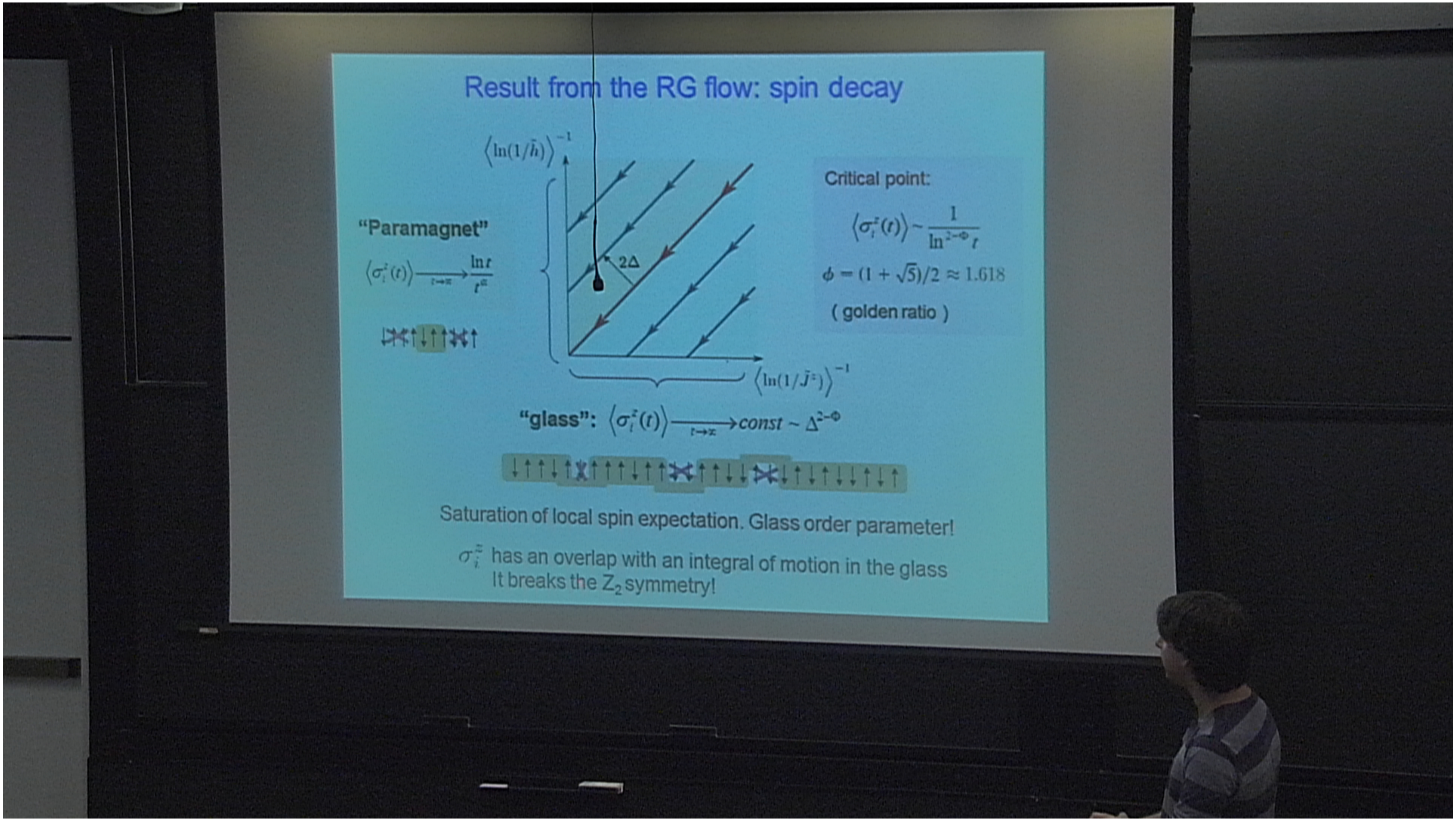
2Δ
$\left\langle \ln\left(\frac{1}{J^z t}\right) \right\rangle^{-1}$
At criticality:
Infinite randomness scaling
Interactions irrelevant
at the critical point:
$J^x_{typ} \sim J^x_0 e^{-\tilde{a}\Gamma^\phi}$
(ϕ = golden ratio)
$\Gamma \equiv \log(\Omega_0 t)$
"Glass" $J^z_{typ} \to const$ $h_{typ} \to 0$


Result from the RG flow: spin decay
"Paramagnet"
$\langle \sigma_i^z(t) \rangle \xrightarrow[t\to\infty]{} \frac{\ln t}{t^{\alpha}}$
$\langle \ln(1/\tilde{h}) \rangle^{-1}$
2Δ
$\langle \ln(1/\tilde{J}^z) \rangle^{-1}$
Critical point:
$\langle \sigma_i^z(t) \rangle \sim \frac{1}{\ln^{2-\Phi} t}$
$\phi = (1+\sqrt{5})/2 \approx 1.618$
(golden ratio)
"glass": $\langle \sigma_i^z(t) \rangle \xrightarrow[t\to\infty]{} const \sim \Delta^{2-\Phi}$
Saturation of local spin expectation. Glass order parameter!
σ_i^z has an overlap with an integral of motion in the glass
It breaks the Z_2 symmetry!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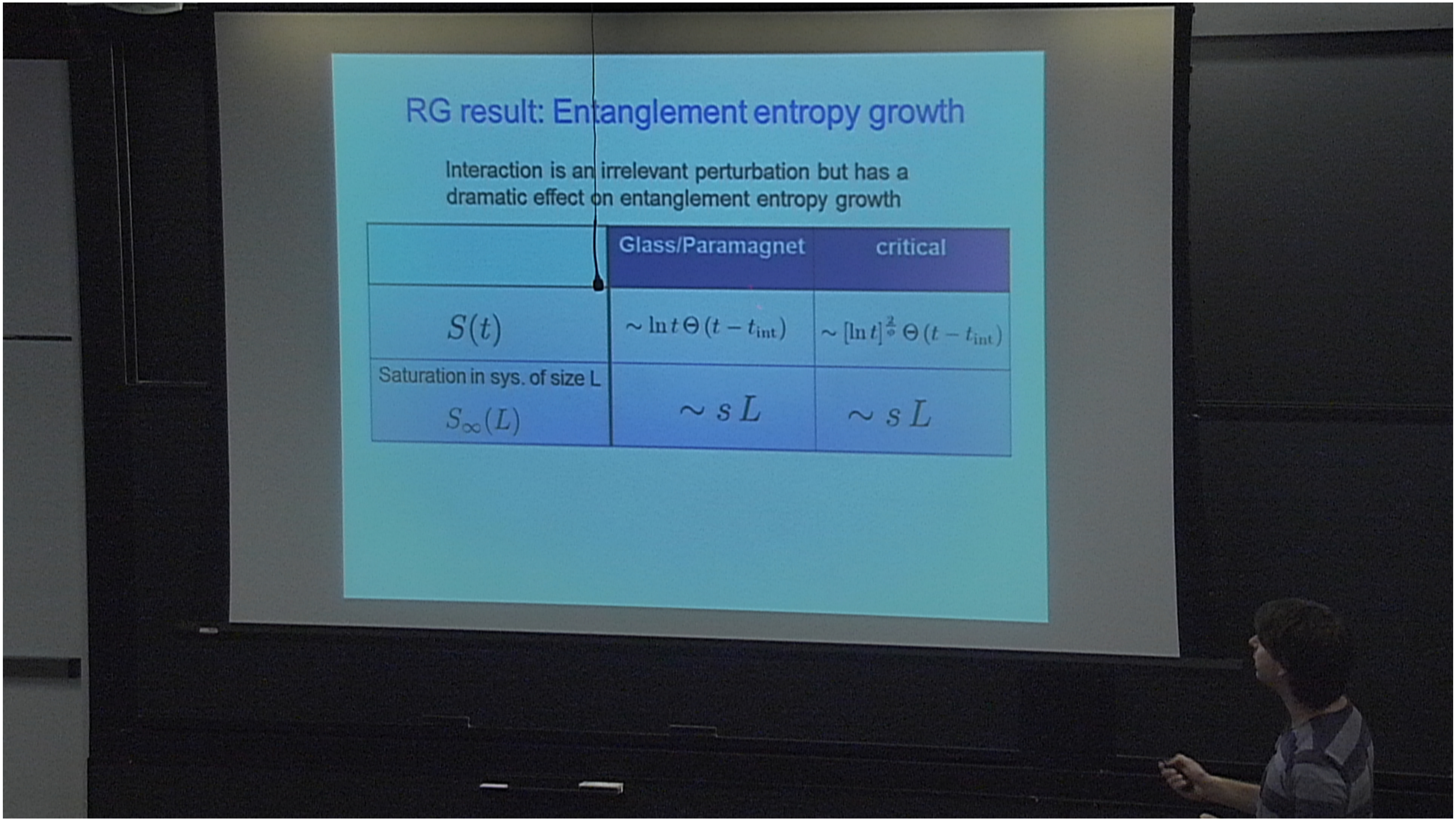
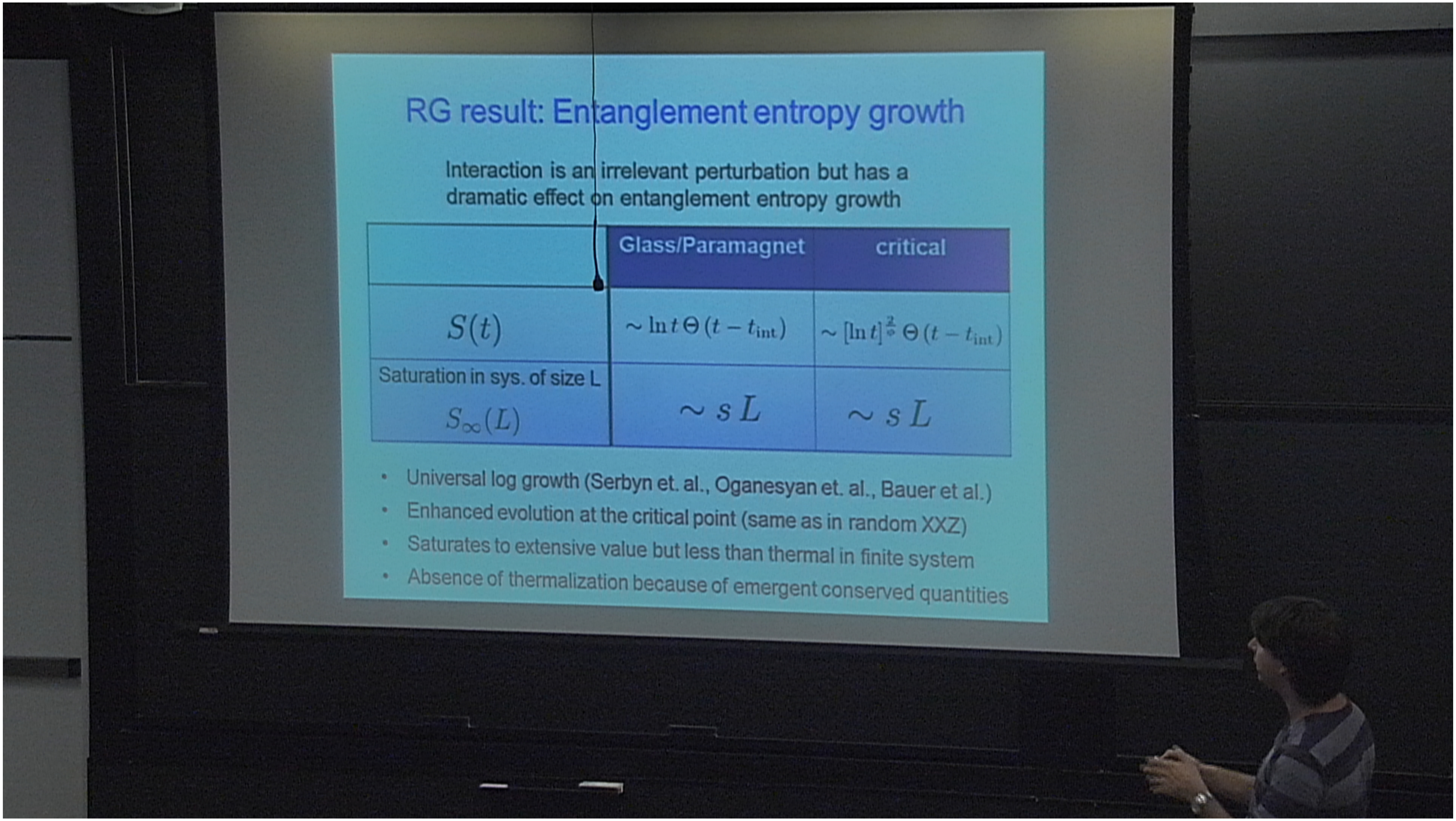

RG result: Entanglement entropy growth
Interaction is an irrelevant perturbation but has a dramatic effect on entanglement entropy growth
Glass/Paramagnet
critical
$S(t)$
$\sim \ln t\, \Theta(t - t_{\rm int})$
$\sim [\ln t]^{\frac{2}{\phi}}\, \Theta(t - t_{\rm int})$
Saturation in sys. of size L
$S_\infty(L)$
$\sim s\, L$
$\sim s\, L$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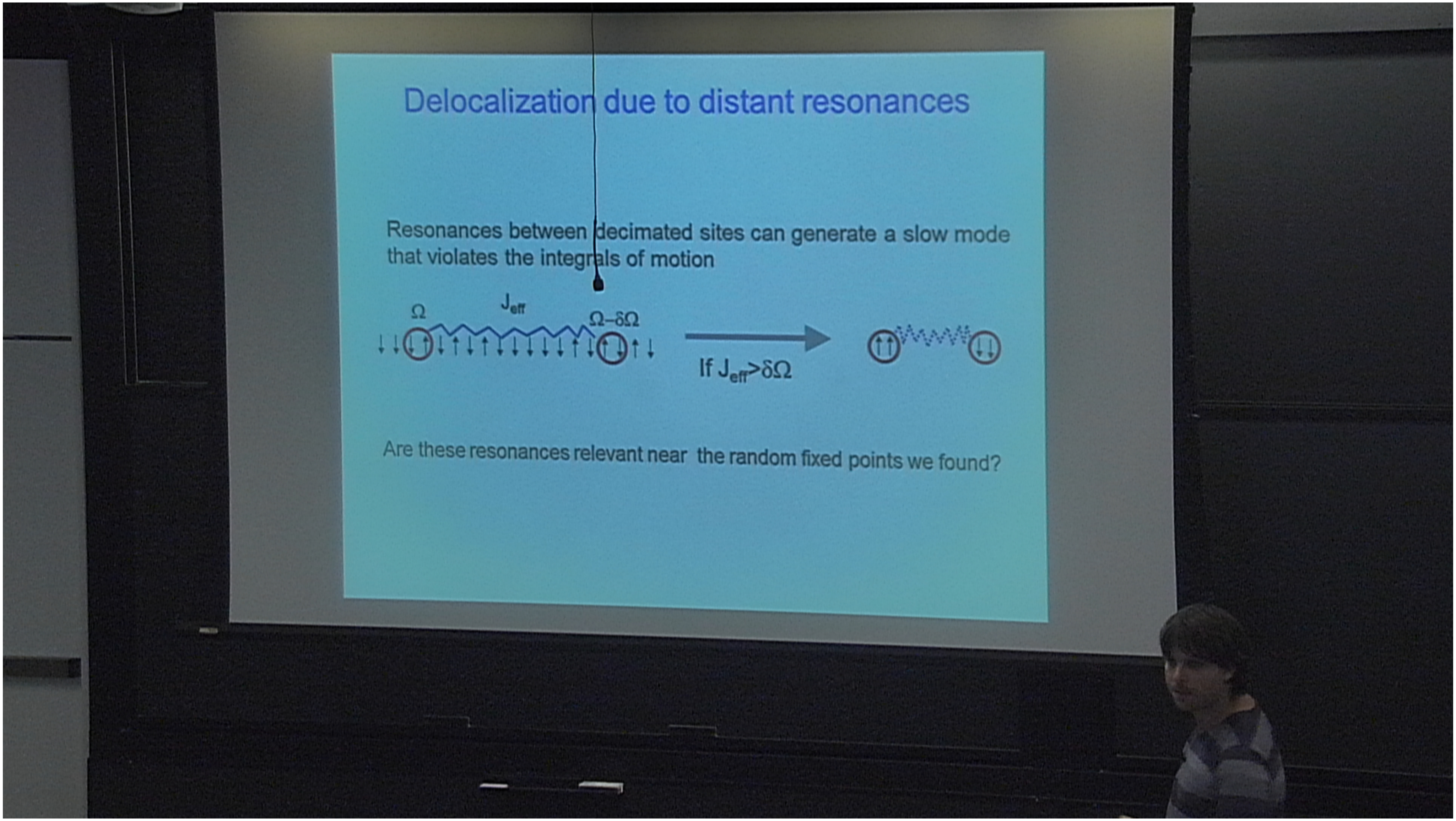
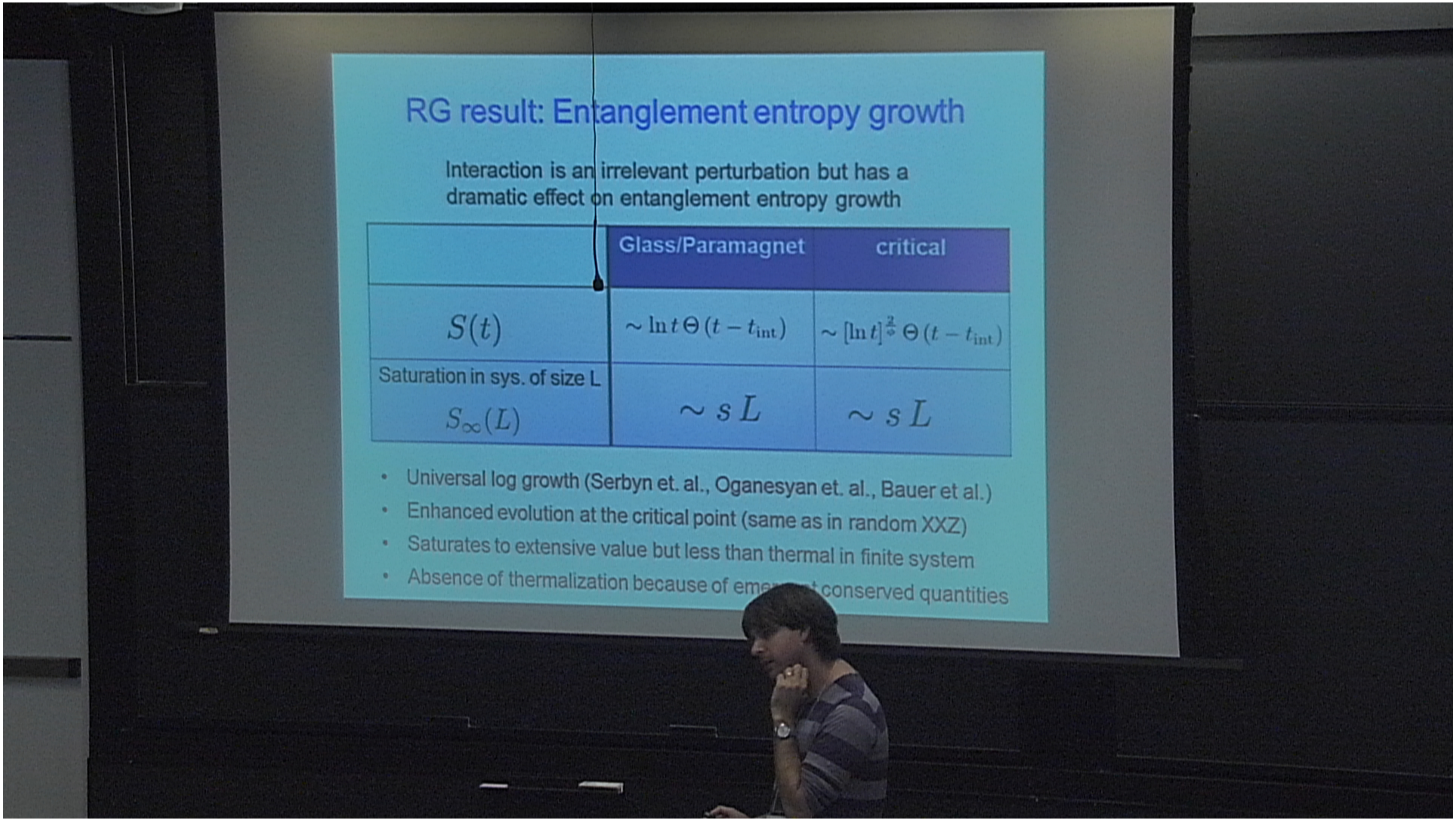
Universal log growth (Serbyn et. al., Oganesyan et. al., Bauer et al.)
Enhanced evolution at the critical point (same as in random XXZ)
Saturates to extensive value but less than thermal in finite system
Absence of thermalization because of emergent conserved quantities


Delocalization due to distant resonances
Resonances between decimated sites can generate a slow mode that violates the integrals of motion
Ω
J_eff
Ω−δΩ
If J_eff>δΩ
Are these resonances relevant near the random fixed points we found?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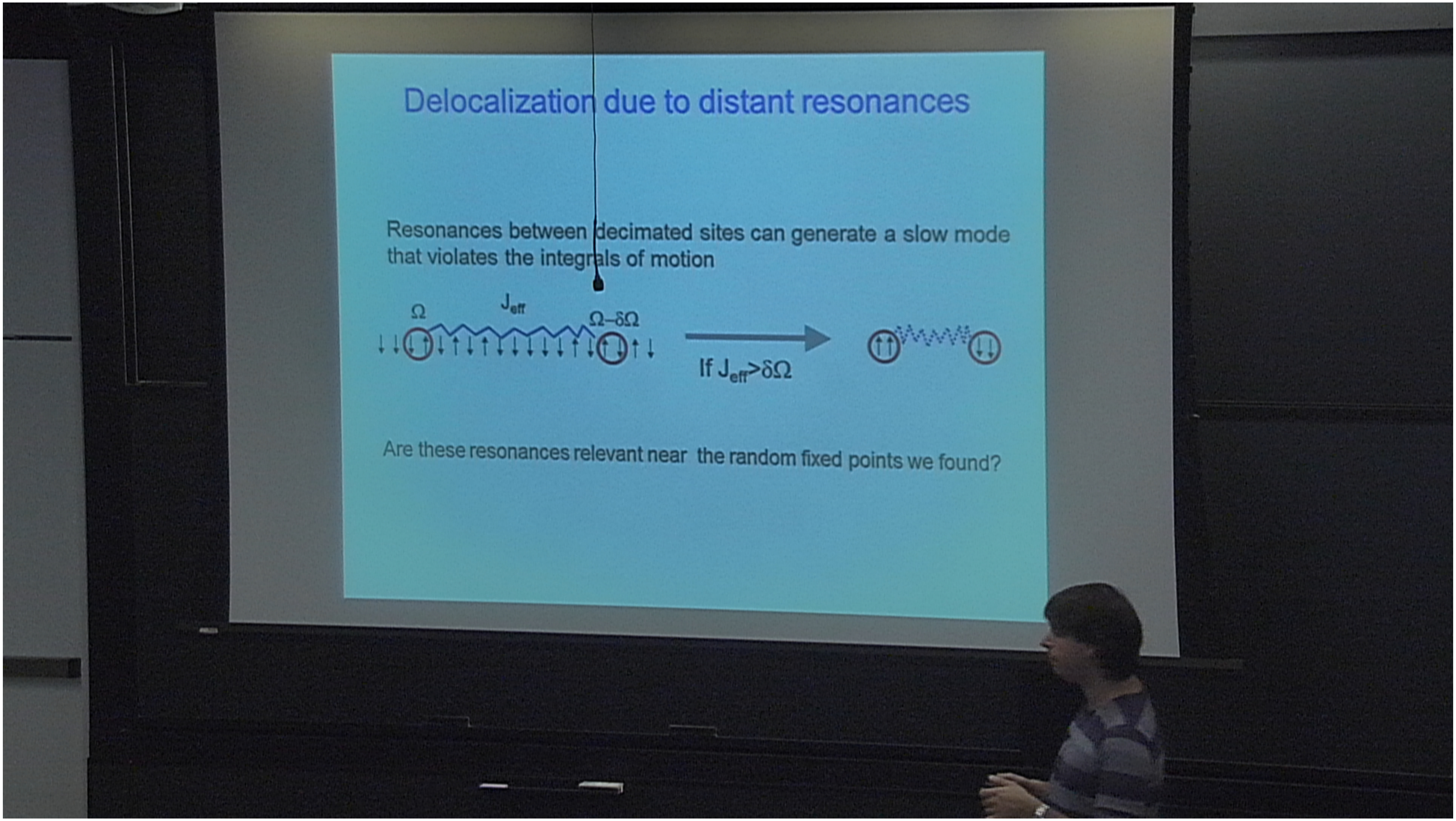
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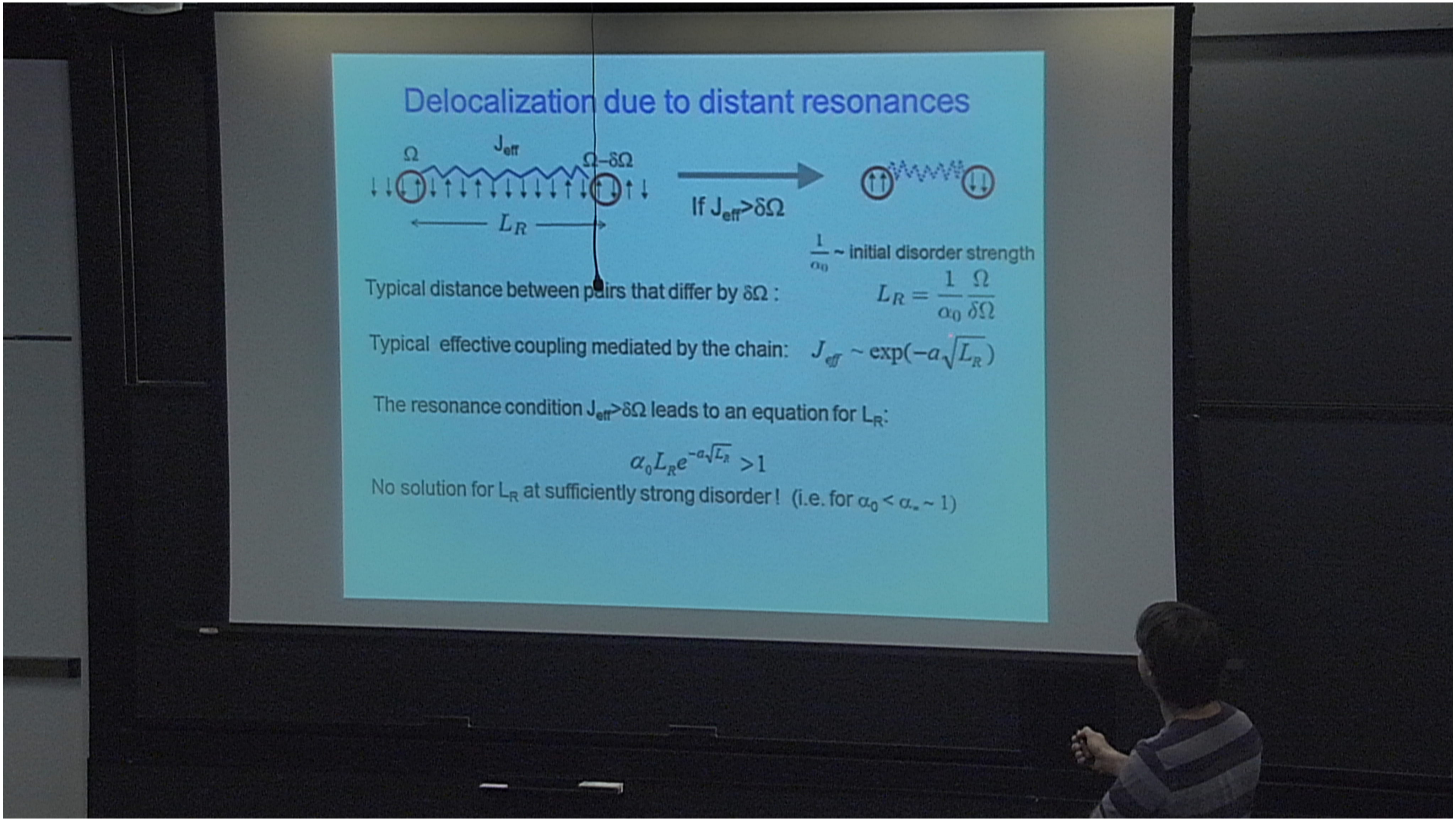
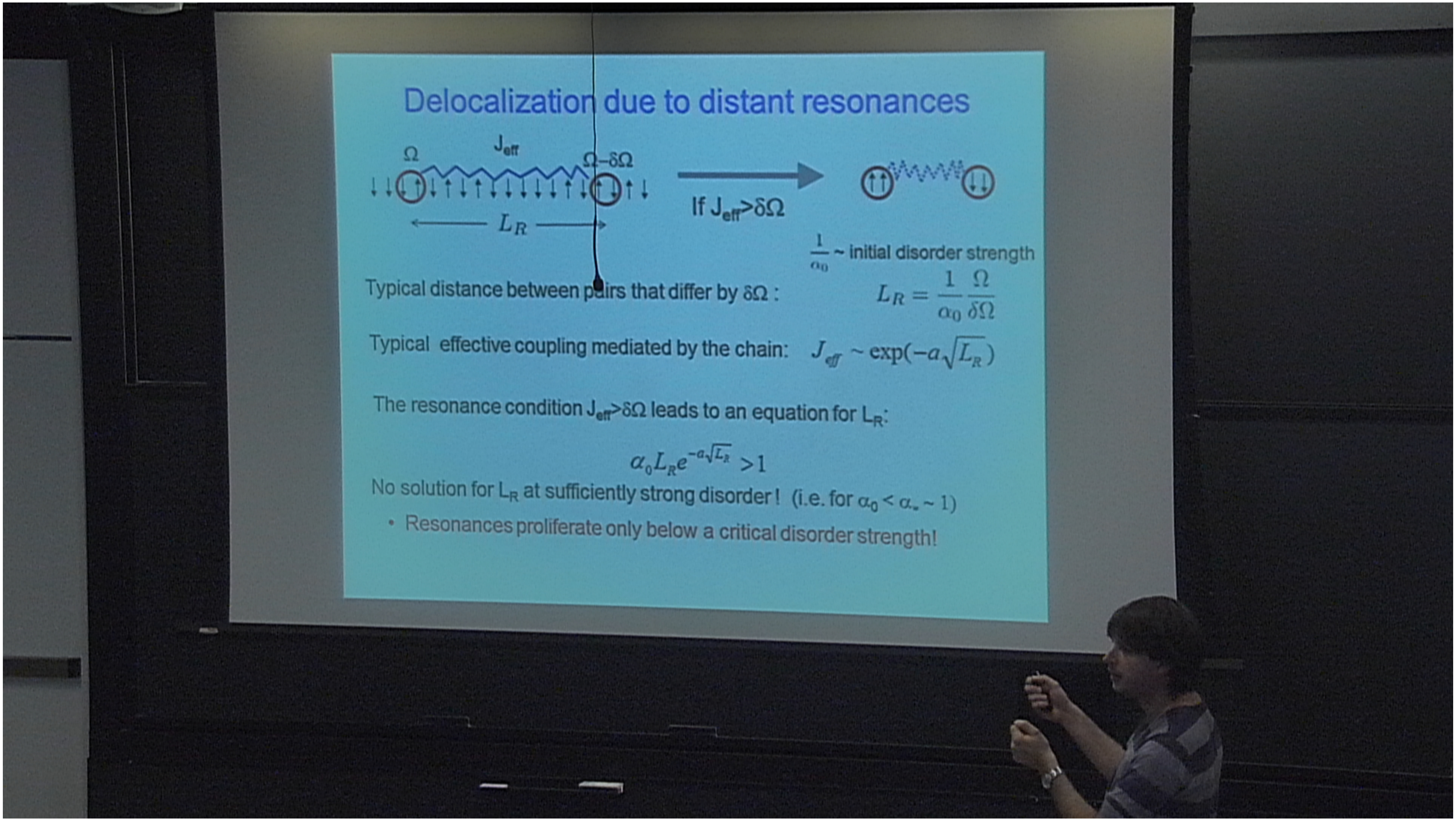
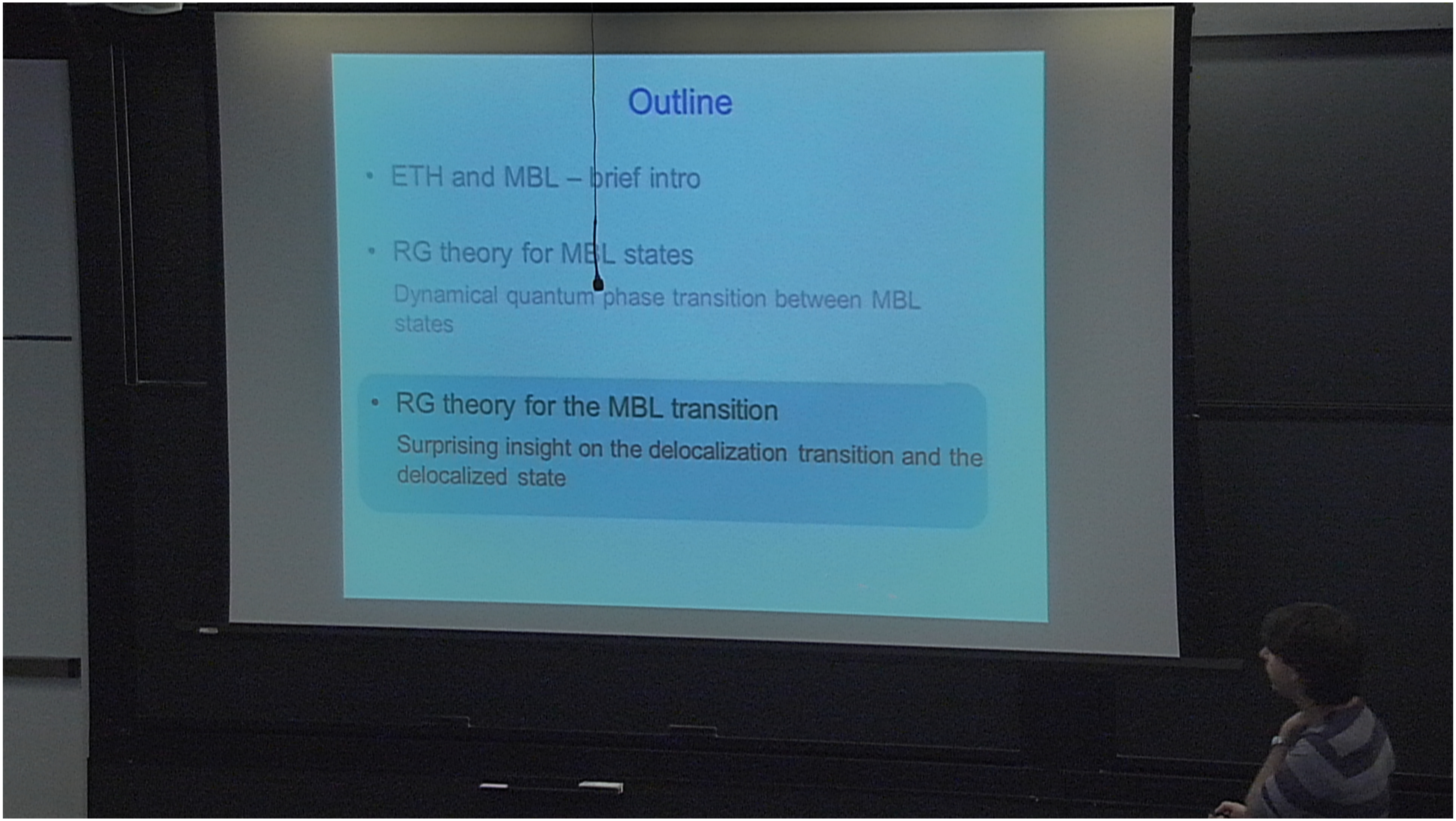

Delocalization due to distant resonances
Ω
J_{eff}
Ω−δΩ
L_R
If J_{eff}>δΩ
$\frac{1}{\alpha_0}$ ~ initial disorder strength
Typical distance between pairs that differ by δΩ :
$L_R = \frac{1}{\alpha_0}\frac{\Omega}{\delta\Omega}$
Typical effective coupling mediated by the chain:
$J_{eff} \sim \exp(-a\sqrt{L_R})$
The resonance condition J_{eff}>δΩ leads to an equation for L_R:
$\alpha_0 L_R e^{-a\sqrt{L_R}} > 1$
No solution for L_R at sufficiently strong disorder! (i.e. for $\alpha_0 < \alpha_* \sim 1$)


Delocalization due to distant resonances
Ω
J_eff
Ω–δΩ
L_R
If J_eff>δΩ
1/α0 ~ initial disorder strength
Typical distance between pairs that differ by δΩ :
L_R = (1/α0)(Ω/δΩ)
Typical effective coupling mediated by the chain: J_eff ~ exp(−a√L_R)
The resonance condition J_eff>δΩ leads to an equation for L_R:
α0 L_R e^(−a√L_R) > 1
No solution for L_R at sufficiently strong disorder! (i.e. for α0 < α* ~ 1)
• Resonances proliferate only below a critical disorder strength!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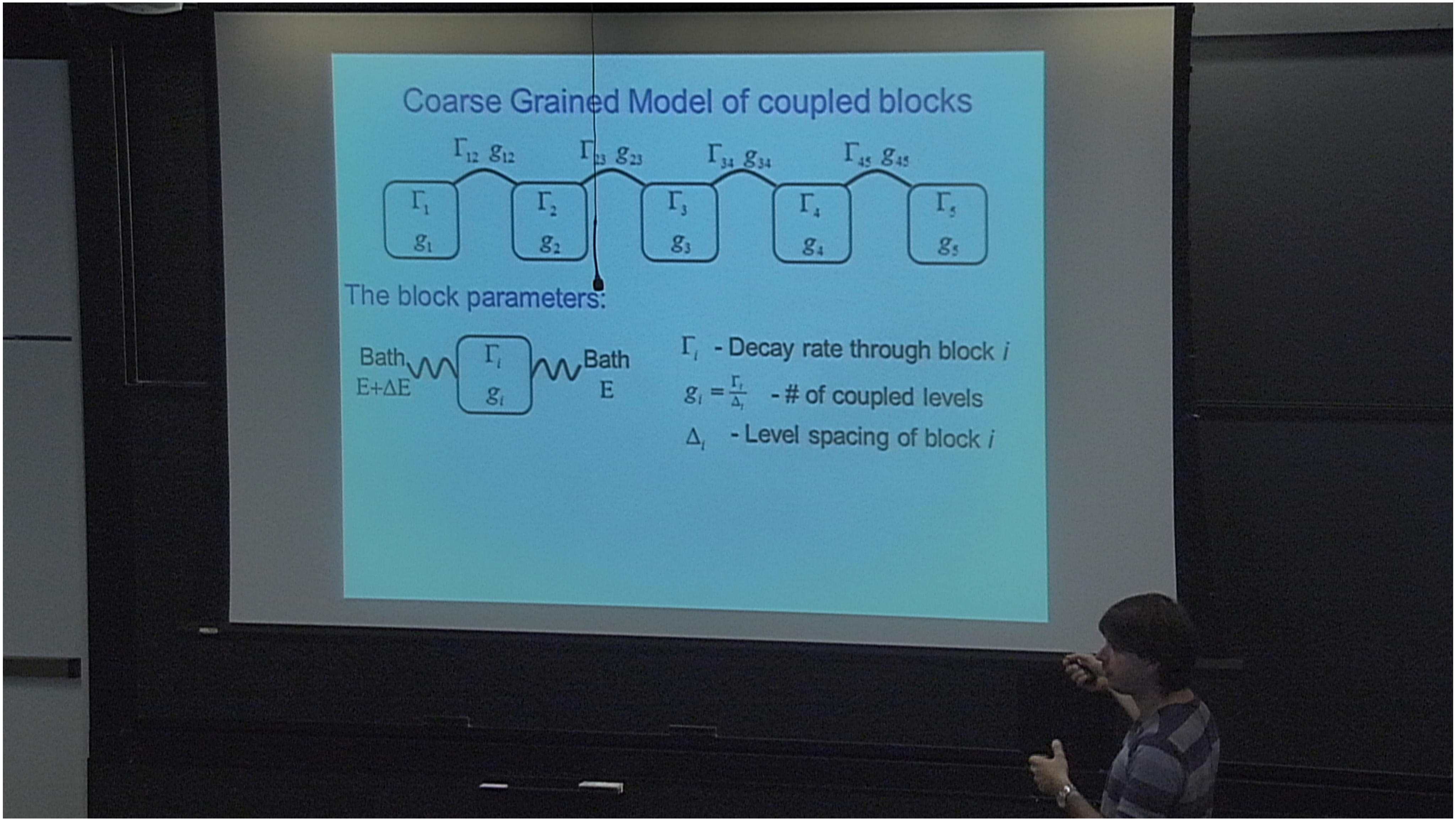
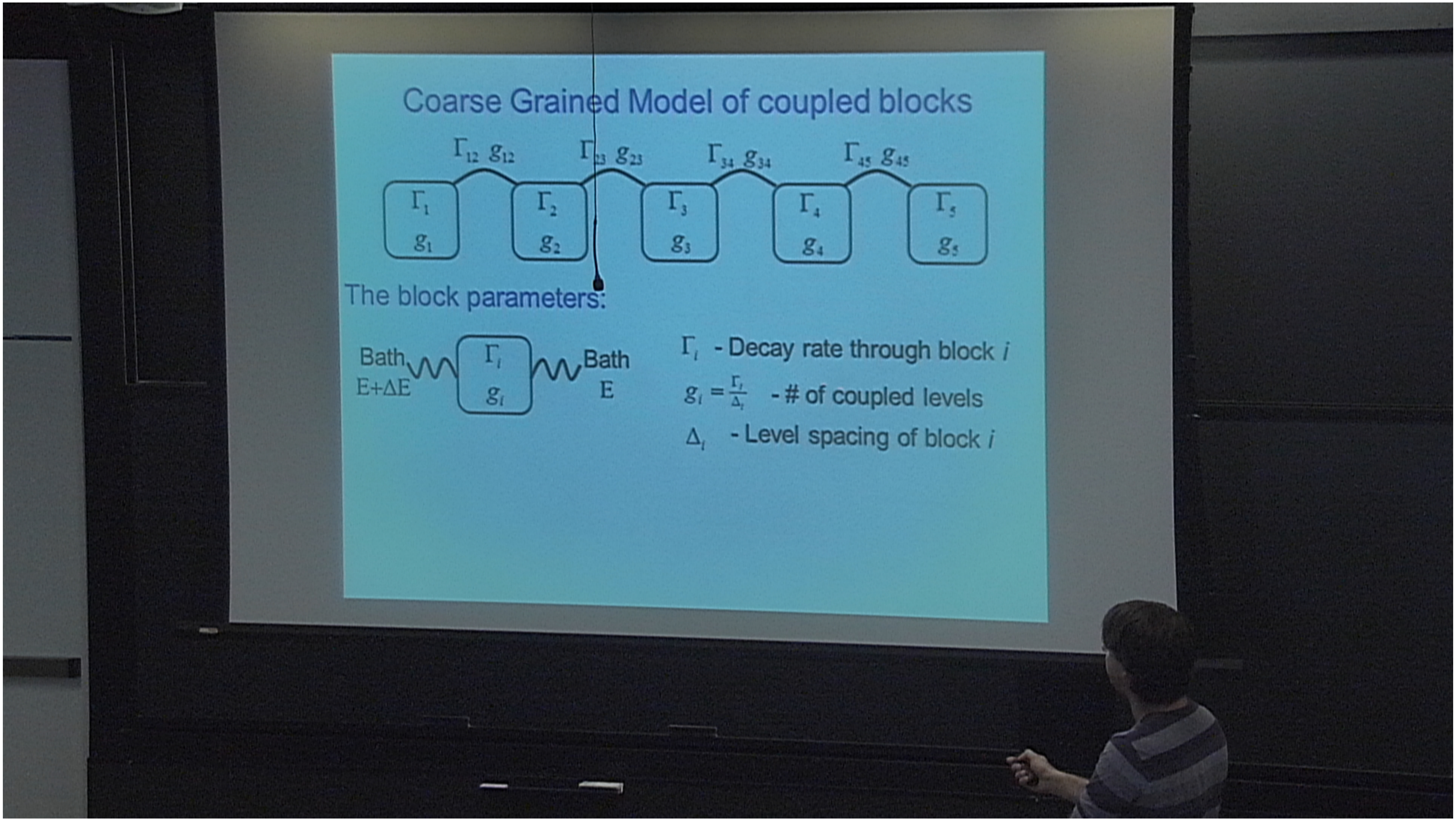
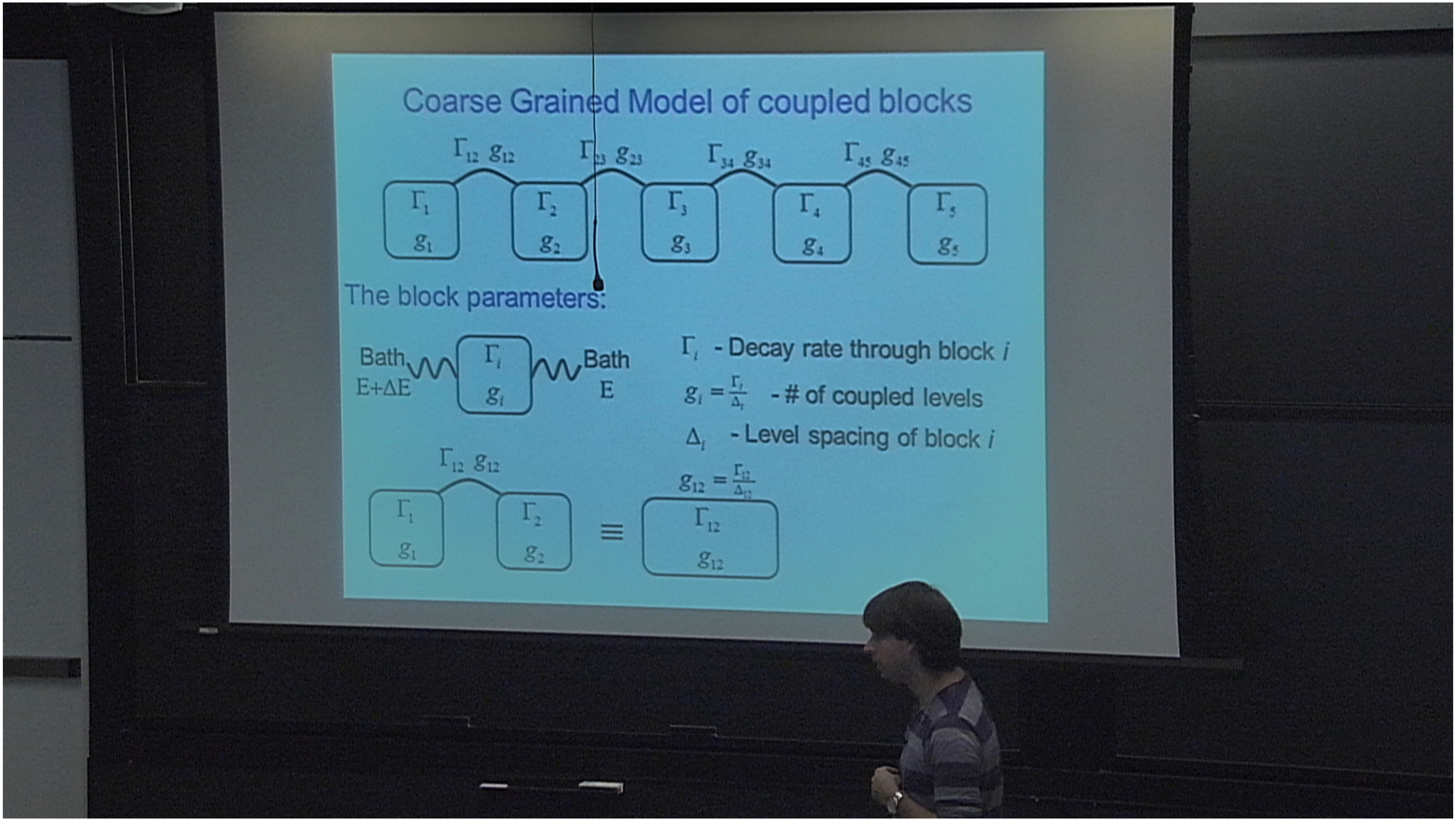
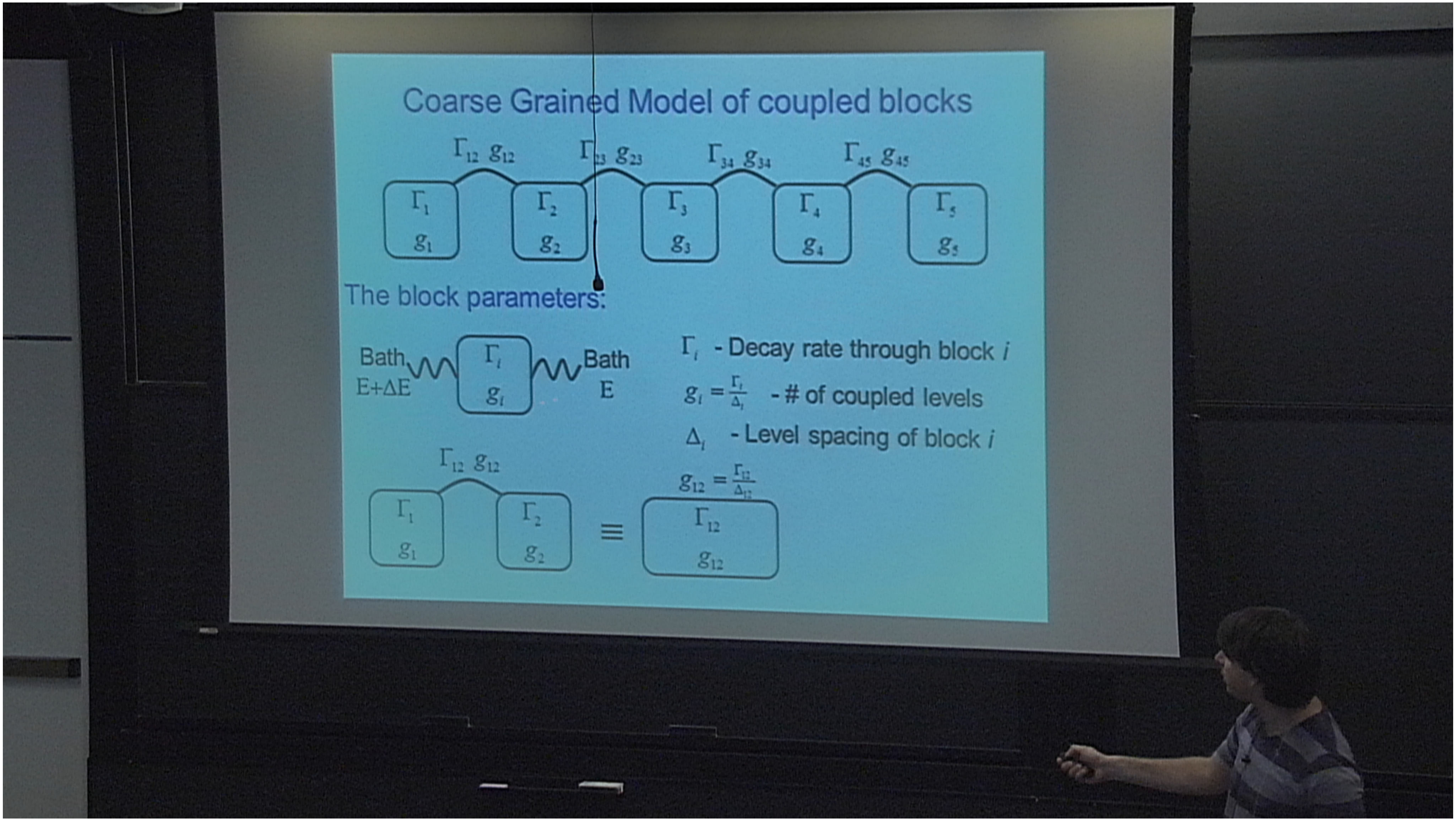


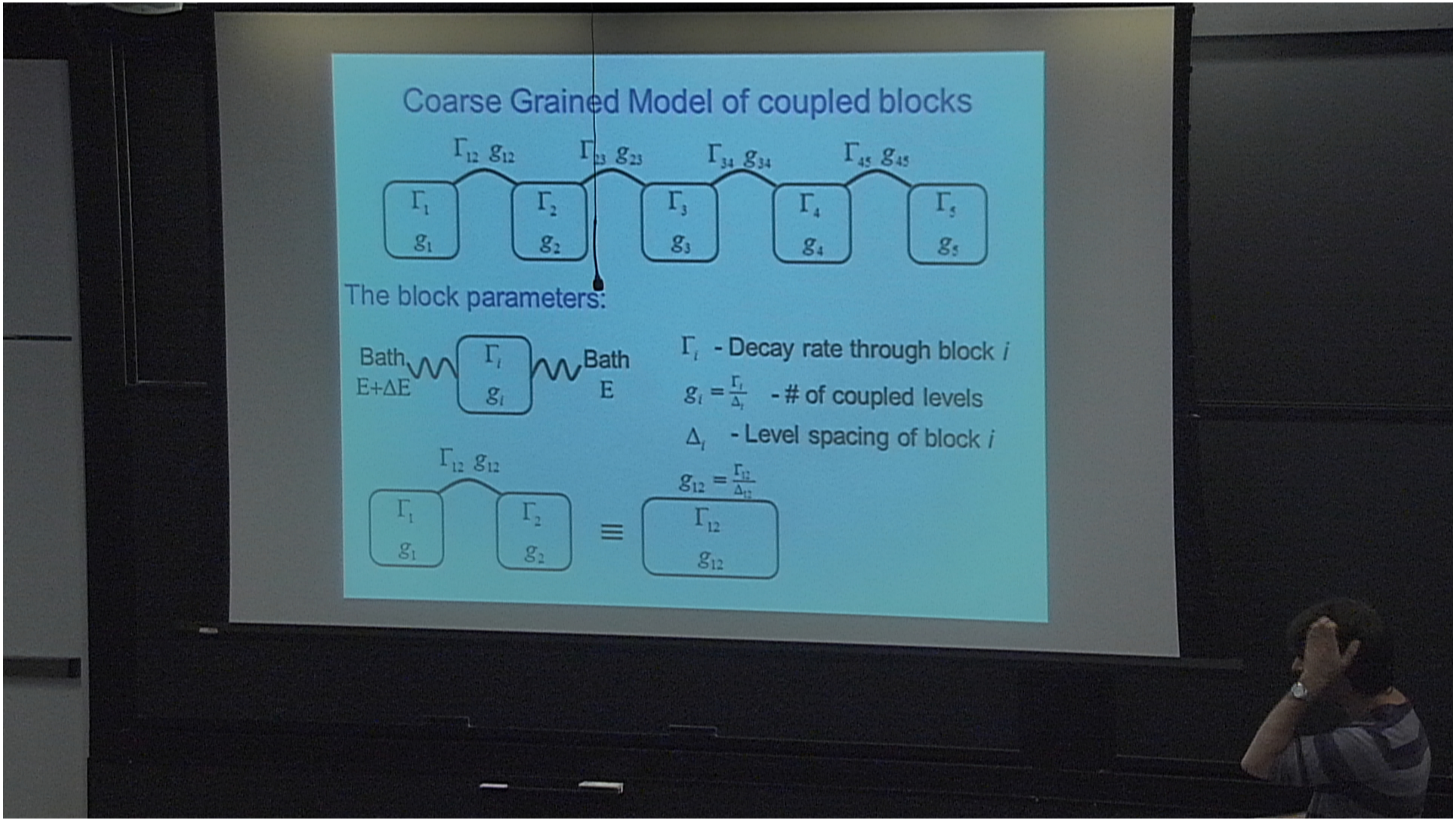
Coarse Grained Model of coupled blocks
Γ_{12} g_{12}
Γ_{23} g_{23}
Γ_{34} g_{34}
Γ_{45} g_{45}
Γ_1 g_1
Γ_2 g_2
Γ_3 g_3
Γ_4 g_4
Γ_5 g_5
The block parameters:
Bath
E+ΔE
Γ_i g_i
Bath
E
Γ_i - Decay rate through block *i*
$g_i = \frac{\Gamma_i}{\Delta_i}$ - # of coupled levels
Δ_i - Level spacing of block *i*


Coarse Grained Model of coupled blocks
Γ12 g12
Γ23 g23
Γ34 g34
Γ45 g45
Γ1 g1
Γ2 g2
Γ3 g3
Γ4 g4
Γ5 g5
The block parameters:
Bath E+ΔE
Γi gi
Bath E
Γi - Decay rate through block i
gi = Γi/Δi - # of coupled levels
Δi - Level spacing of block i



Coarse Grained Model of coupled blocks
The block parameters:
Bath E+ΔE
Bath E
Γ_i - Decay rate through block i
$g_i = \frac{\Gamma_i}{\Delta_i}$ - # of coupled levels
Δ_i - Level spacing of block i
$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$


Coarse Grained Model of coupled blocks
Γ_{12} g_{12}
Γ_{23} g_{23}
Γ_{34} g_{34}
Γ_{45} g_{45}
Γ_1 g_1
Γ_2 g_2
Γ_3 g_3
Γ_4 g_4
Γ_5 g_5
The block parameters:
Bath $E+\Delta E$
Γ_i g_i
Bath E
Γ_i - Decay rate through block *i*
$g_i = \frac{\Gamma_i}{\Delta_i}$ - # of coupled levels
Δ_i - Level spacing of block *i*
Γ_{12} g_{12}
Γ_1 g_1
Γ_2 g_2
$\equiv$
$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$
Γ_{12} g_{12}


Coarse Grained Model of coupled blocks
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The block parameters:
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Bath
E
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Γ_{12} g_{12}
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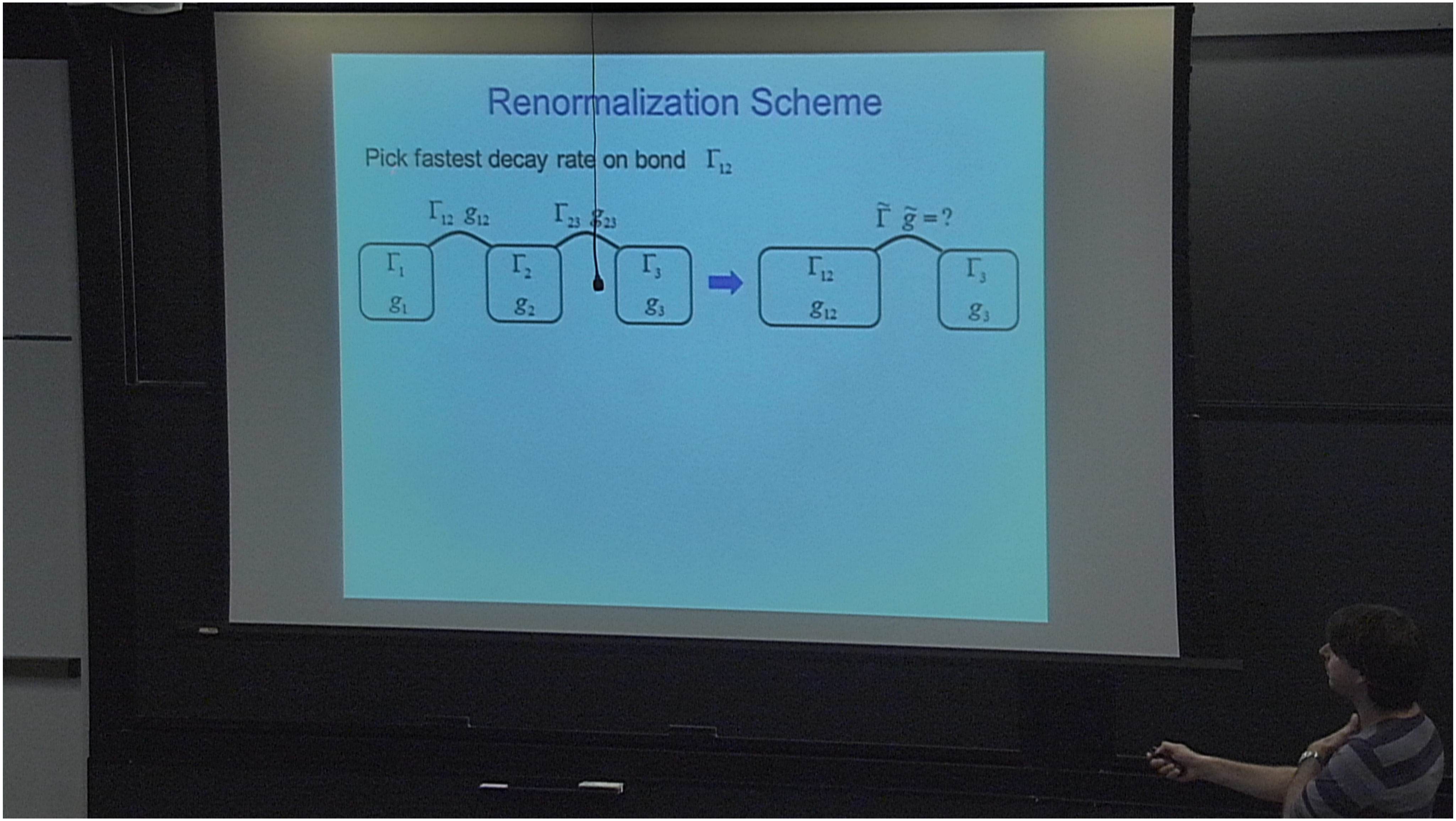
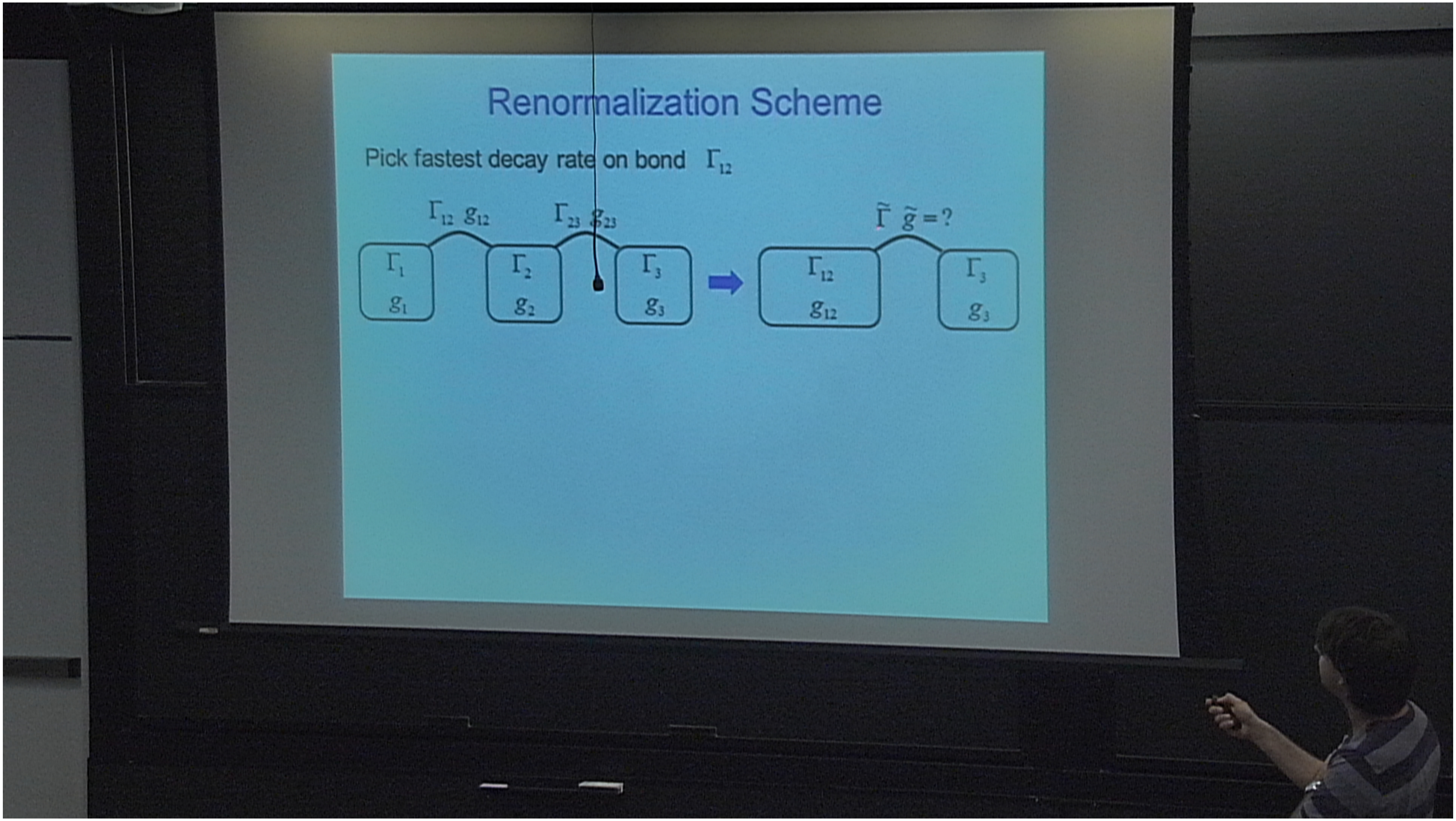
Γ_2 g_2
≡
$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$
Γ_{12} g_{12}


Renormalization Scheme
Pick fastest decay rate on bond Γ_{12}
Γ_{12} g_{12}
Γ_{23} g_{23}
Γ_1 g_1
Γ_2 g_2
Γ_3 g_3
$\tilde{\Gamma}$ $\tilde{g} = ?$
Γ_{12} g_{12}
Γ_3 g_3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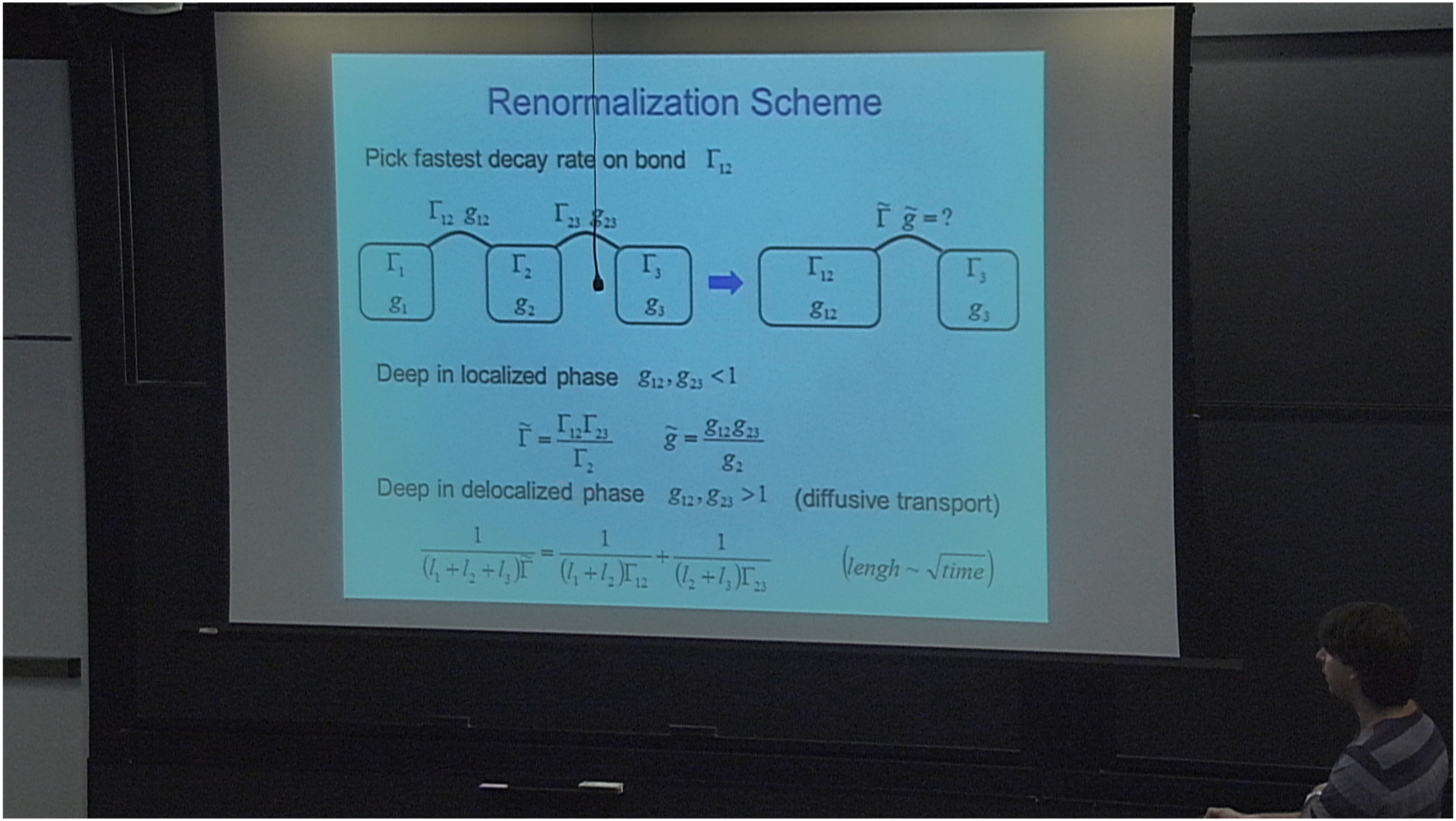
Renormalization Scheme
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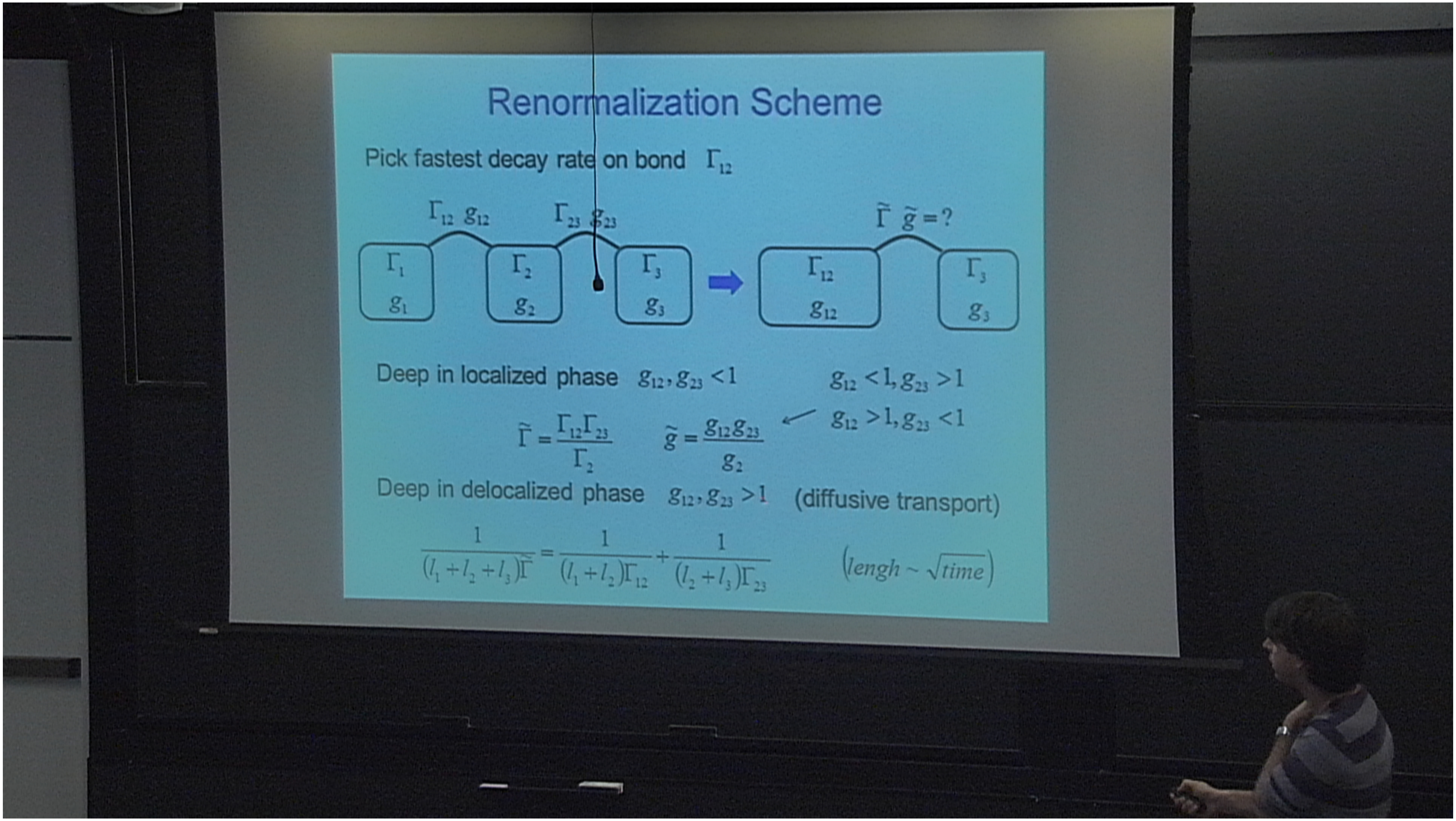
Γ_3 g_3
$\tilde{\Gamma}$ $\tilde{g} = ?$
Γ_{12} g_{12}
Γ_3 g_3
Deep in localized phase $g_{12}, g_{23} < 1$
$\tilde{\Gamma} = \frac{\Gamma_{12}\Gamma_{23}}{\Gamma_2}$ $\tilde{g} = \frac{g_{12}g_{23}}{g_2}$
Deep in delocalized phase $g_{12}, g_{23} > 1$ (diffusive transport)
$\frac{1}{(l_1+l_2+l_3)\tilde{\Gamma}} = \frac{1}{(l_1+l_2)\Gamma_{12}} + \frac{1}{(l_2+l_3)\Gamma_{23}}$
$(lengh \sim \sqrt{time})$


Renormalization Scheme
Pick fastest decay rate on bond Γ_{12}
Γ_{12} g_{12}
Γ_{23} g_{23}
Γ_1 g_1
Γ_2 g_2
Γ_3 g_3
$\tilde{\Gamma}$ $\tilde{g} = ?$
Γ_{12} g_{12}
Γ_3 g_3
Deep in localized phase $g_{12}, g_{23} < 1$
$g_{12} < 1, g_{23} > 1$
$g_{12} > 1, g_{23} < 1$
$\tilde{\Gamma} = \frac{\Gamma_{12}\Gamma_{23}}{\Gamma_2}$ $\tilde{g} = \frac{g_{12}g_{23}}{g_2}$
Deep in delocalized phase $g_{12}, g_{23} > 1$ (diffusive transport)
$\frac{1}{(l_1 + l_2 + l_3)\tilde{\Gamma}} = \frac{1}{(l_1 + l_2)\Gamma_{12}} + \frac{1}{(l_2 + l_3)\Gamma_{23}}$
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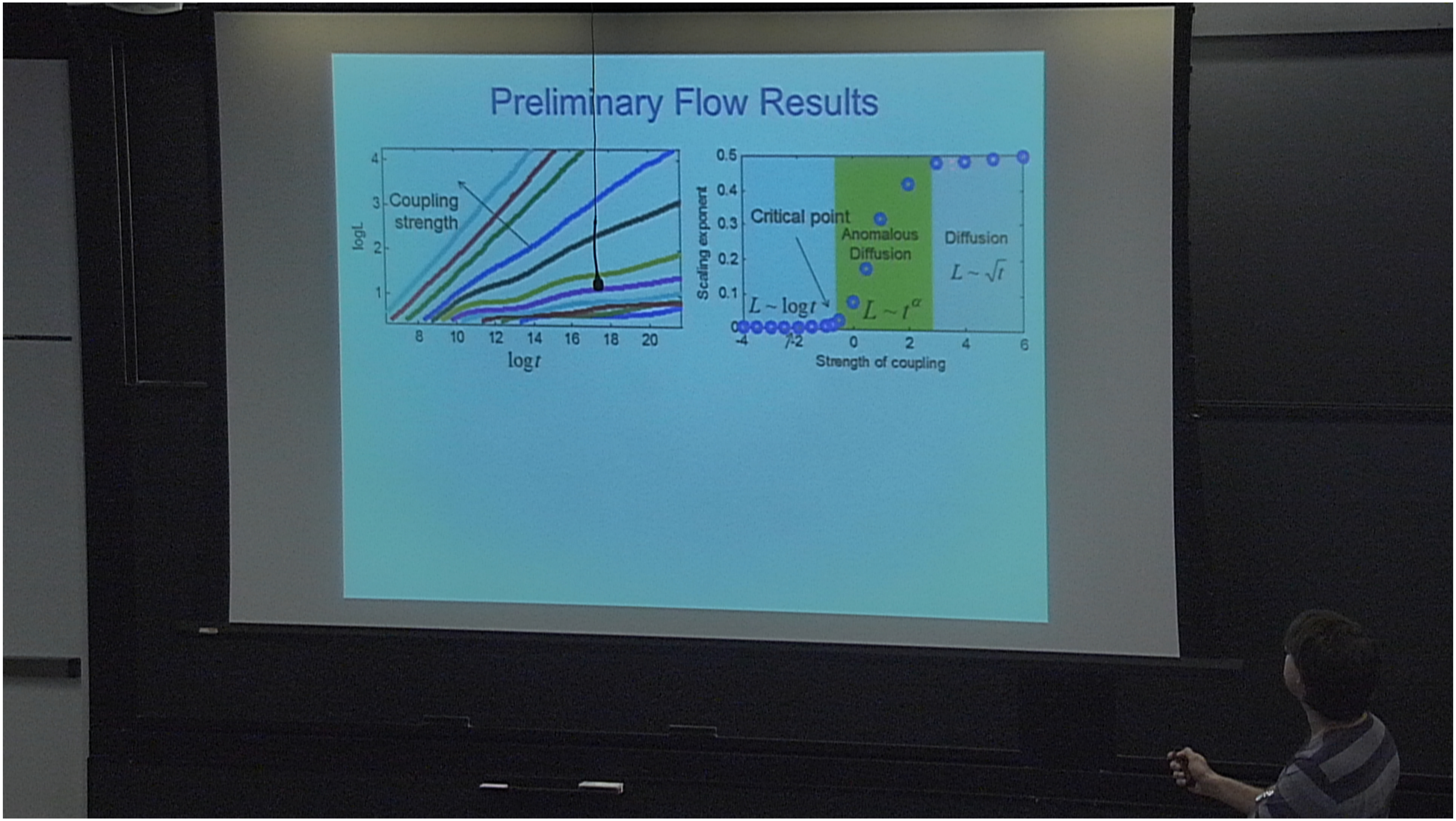
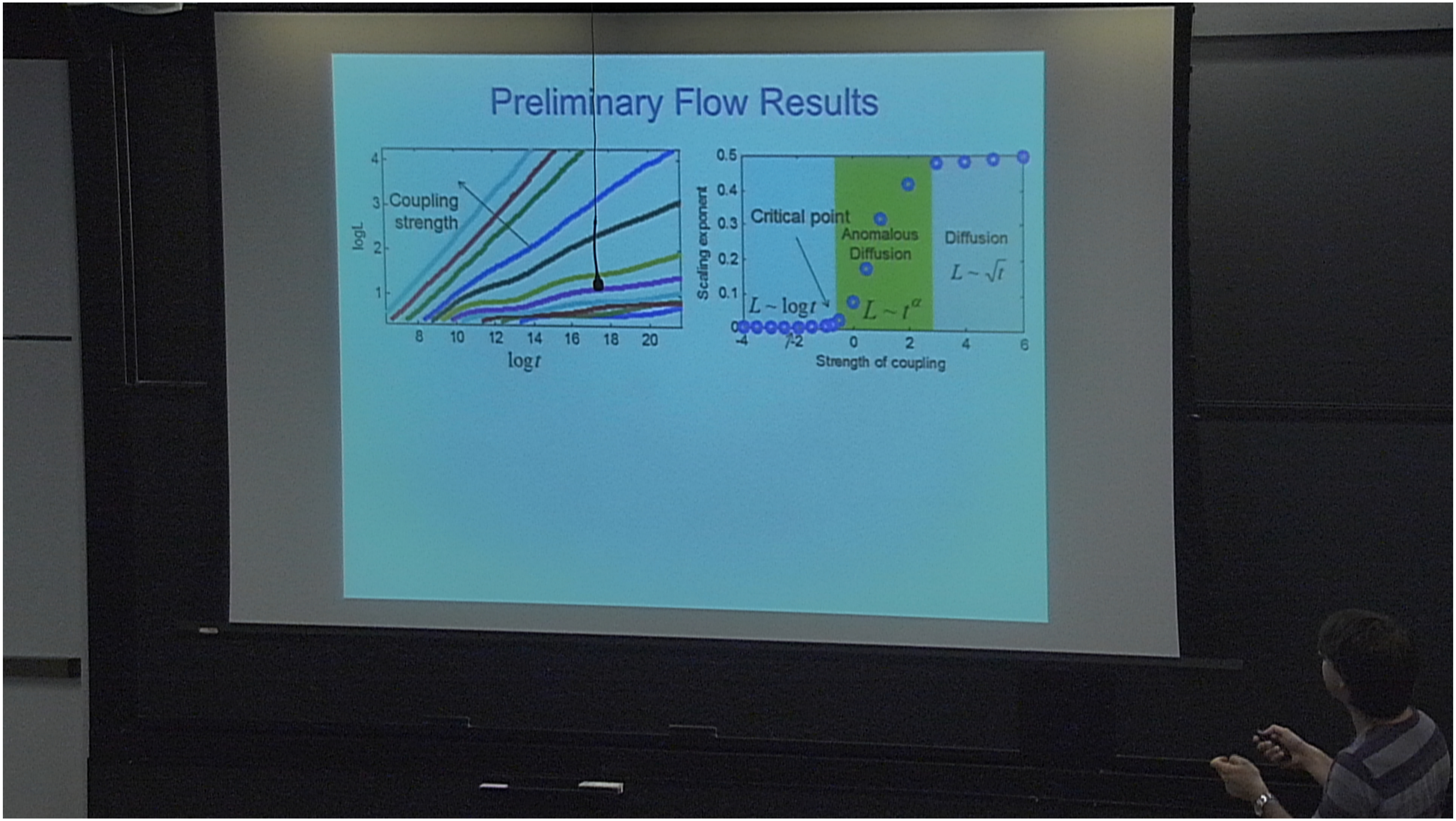
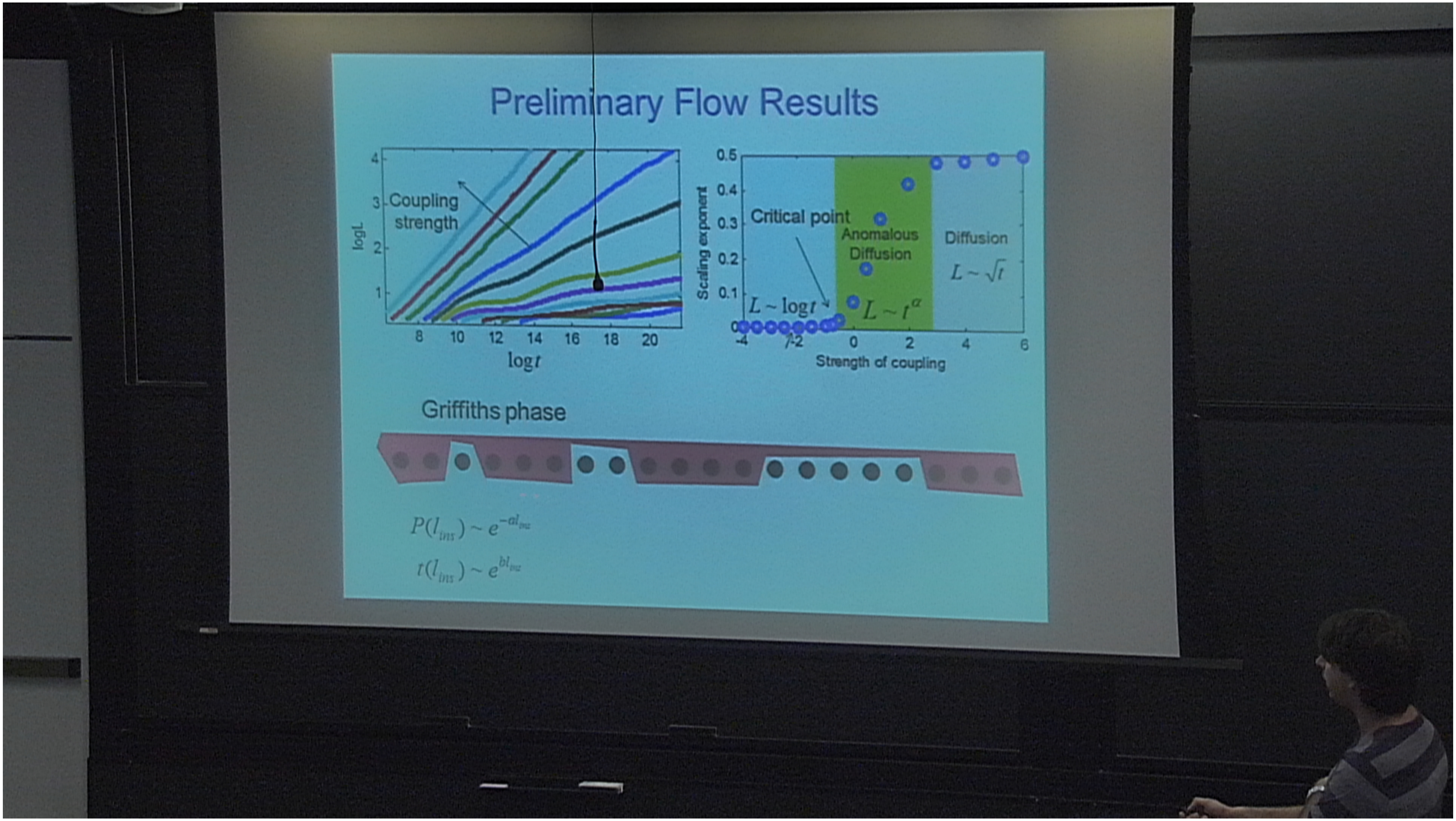

Preliminary Flow Results
Coupling strength
logL
logt
Critical point
Anomalous Diffusion
Diffusion
$L \sim \sqrt{t}$
$L \sim \log t$
$L \sim t^{\alpha}$
Scaling exponent
Strength of coupling


Preliminary Flow Results
Coupling strength
logL
logt
Scaling exponent
Critical point
Anomalous Diffusion
Diffusion
L ~ √t
L ~ log t
L ~ t^α
Strength of coupling


Preliminary Flow Results
Coupling strength
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Diffusion
$L \sim \sqrt{t}$
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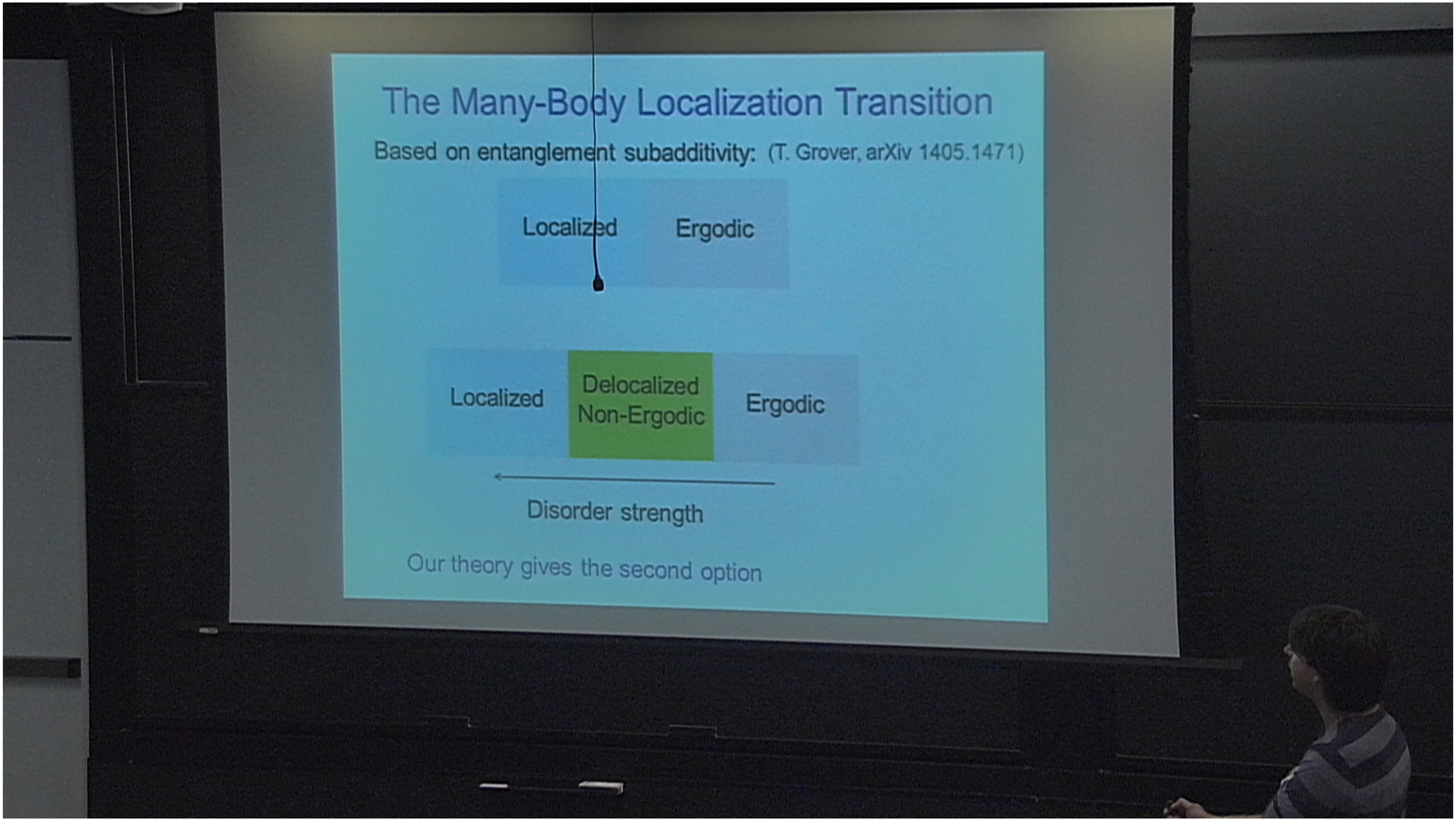
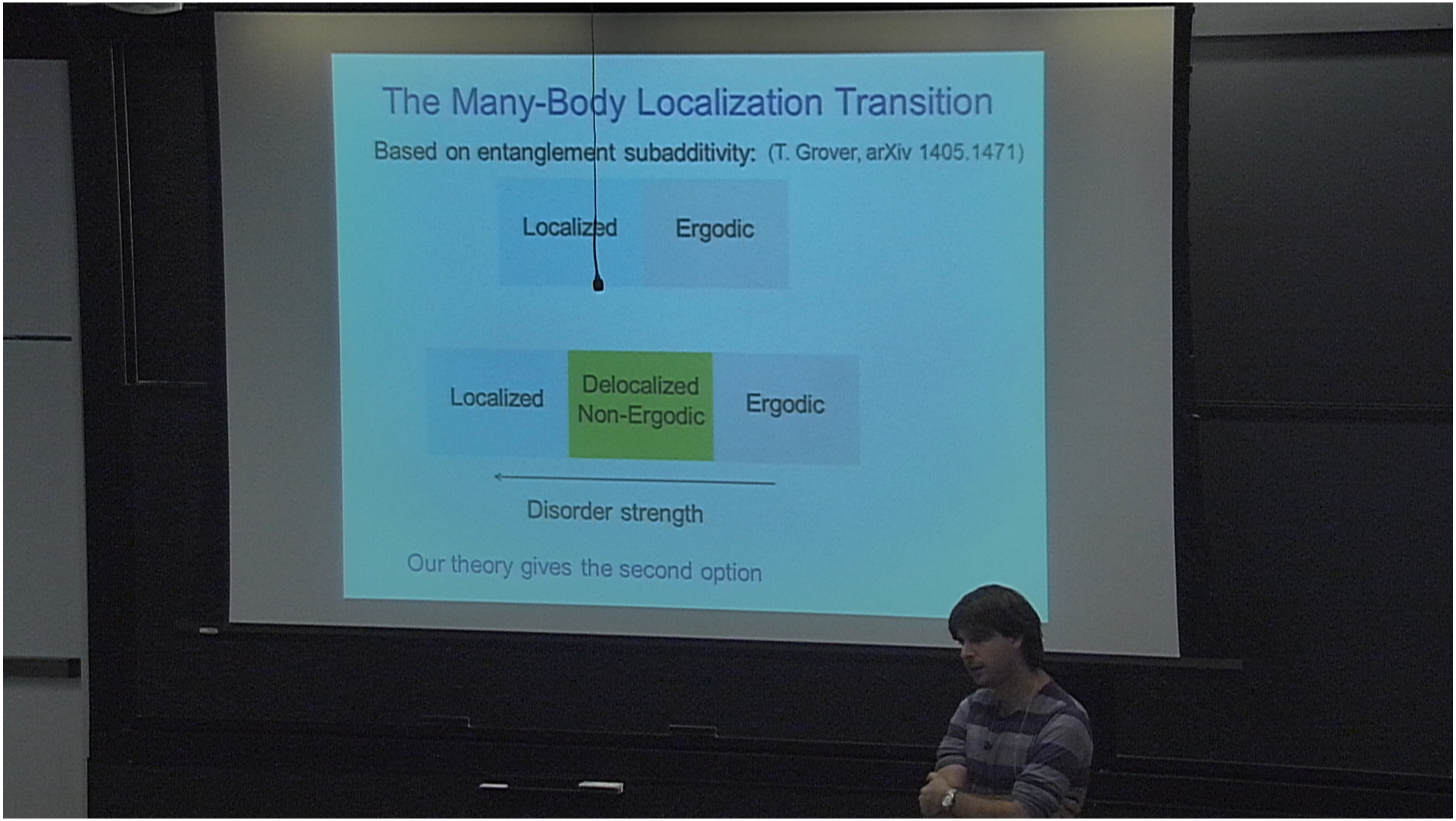
$L \sim t^{\alpha}$
Strength of coupling
Griffiths phase
$P(l_{ins}) \sim e^{-a l_{ins}}$
$t(l_{ins}) \sim e^{b l_{ins}}$


The Many-Body Localization Transition
Based on entanglement subadditivity: (T. Grover, arXiv 1405.1471)
Localized
Ergodic
Localized
Delocalized
Non-Ergodic
Ergodic
Disorder strength
Our theory gives the second option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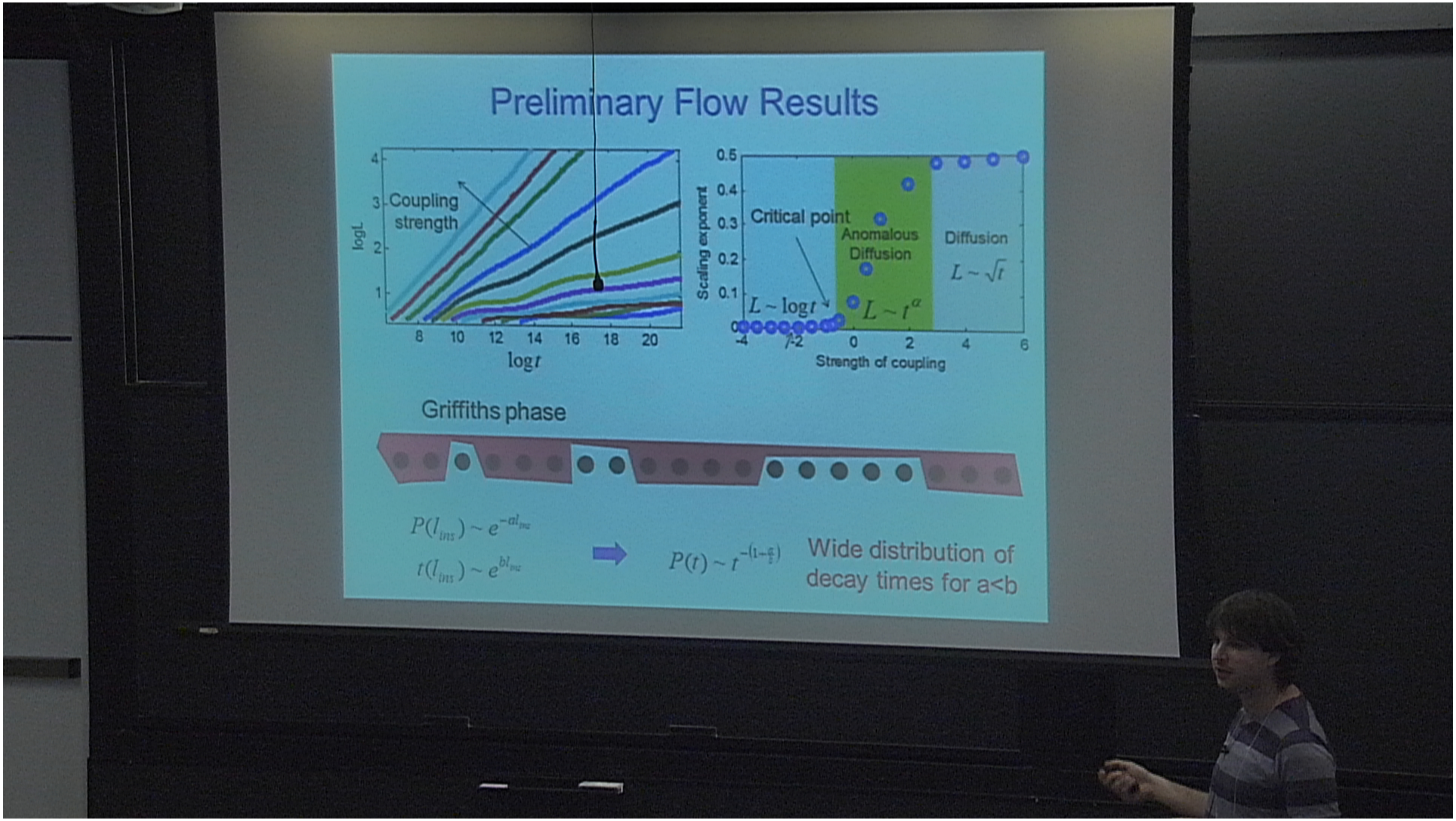
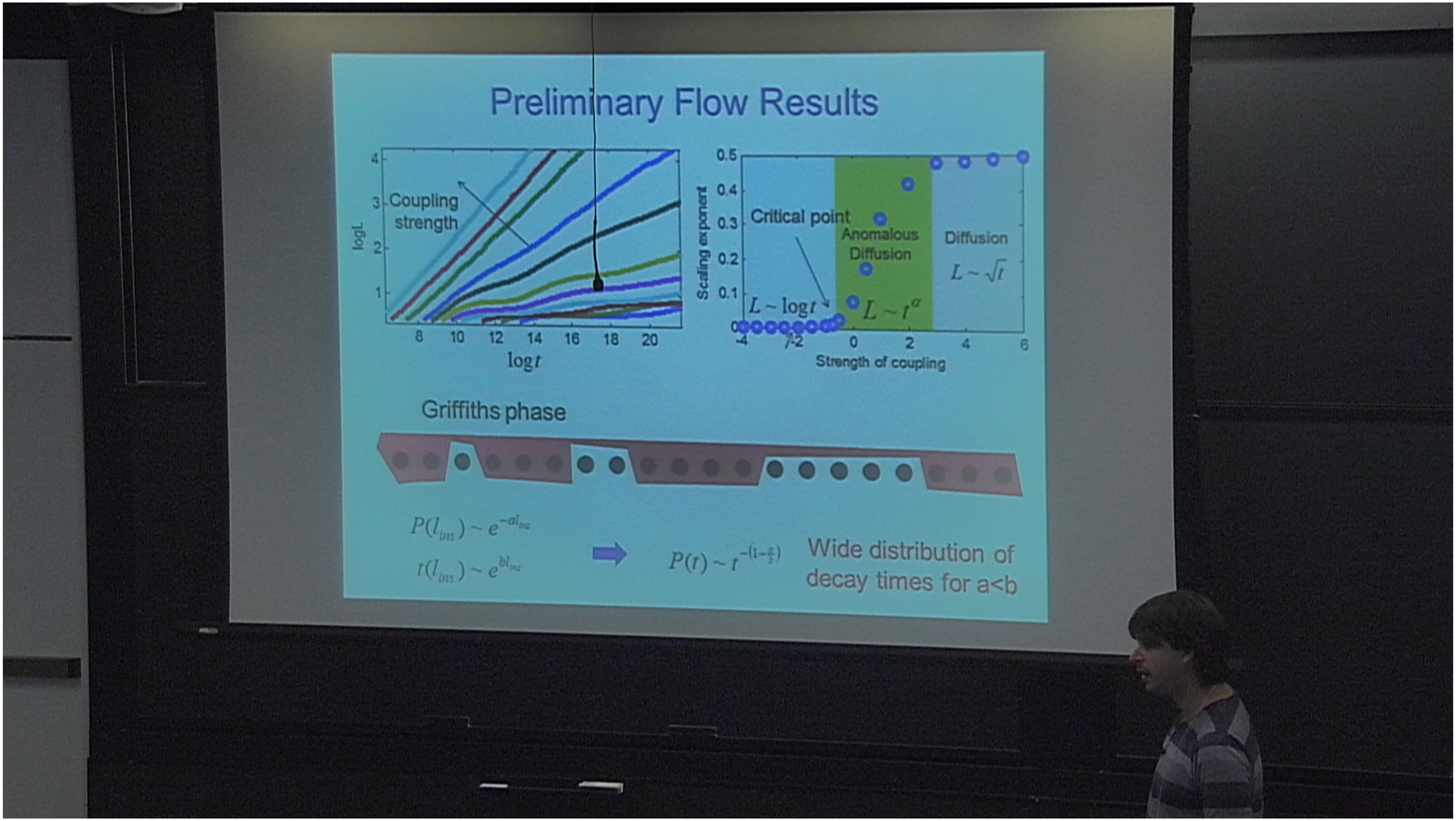
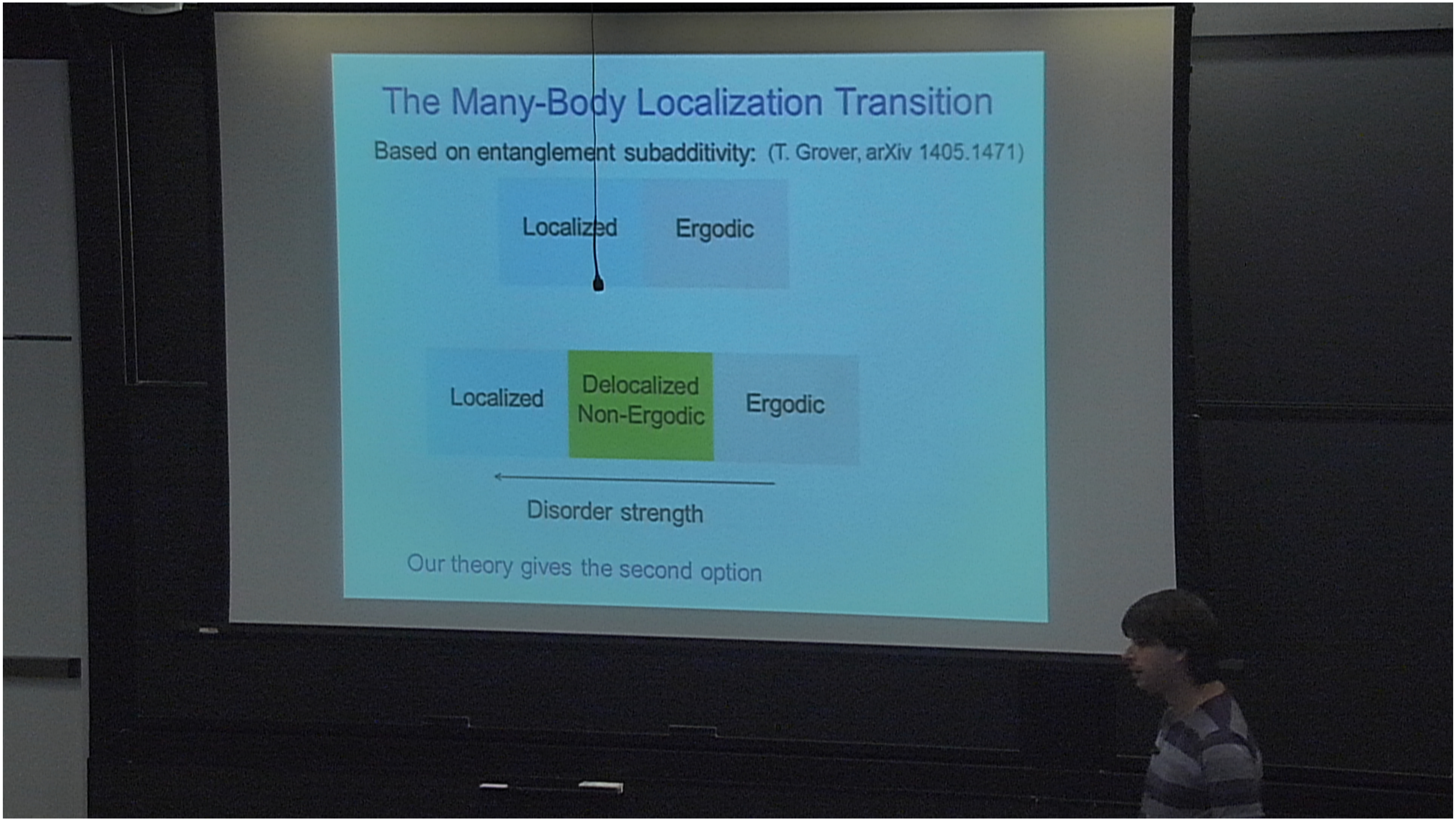


Preliminary Flow Results
Coupling strength
logL
logt
Critical point
Anomalous Diffusion
Diffusion
Scaling exponent
Strength of coupling
Griffiths phase
Wide distribution of decay times for a<b



Preliminary Flow Results
Coupling strength
logL
logt
Critical point
Anomalous Diffusion
Diffusion
$L \sim \sqrt{t}$
$L \sim \log t$
$L \sim t^{\alpha}$
Scaling exponent
Strength of coupling
Griffiths phase
$P(l_{ins}) \sim e^{-al_{ins}}$
$t(l_{ins}) \sim e^{bl_{ins}}$
$P(t) \sim t^{-(1-\frac{a}{b})}$
Wide distribution of decay times for a<b


The Many-Body Localization Transition
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Localized
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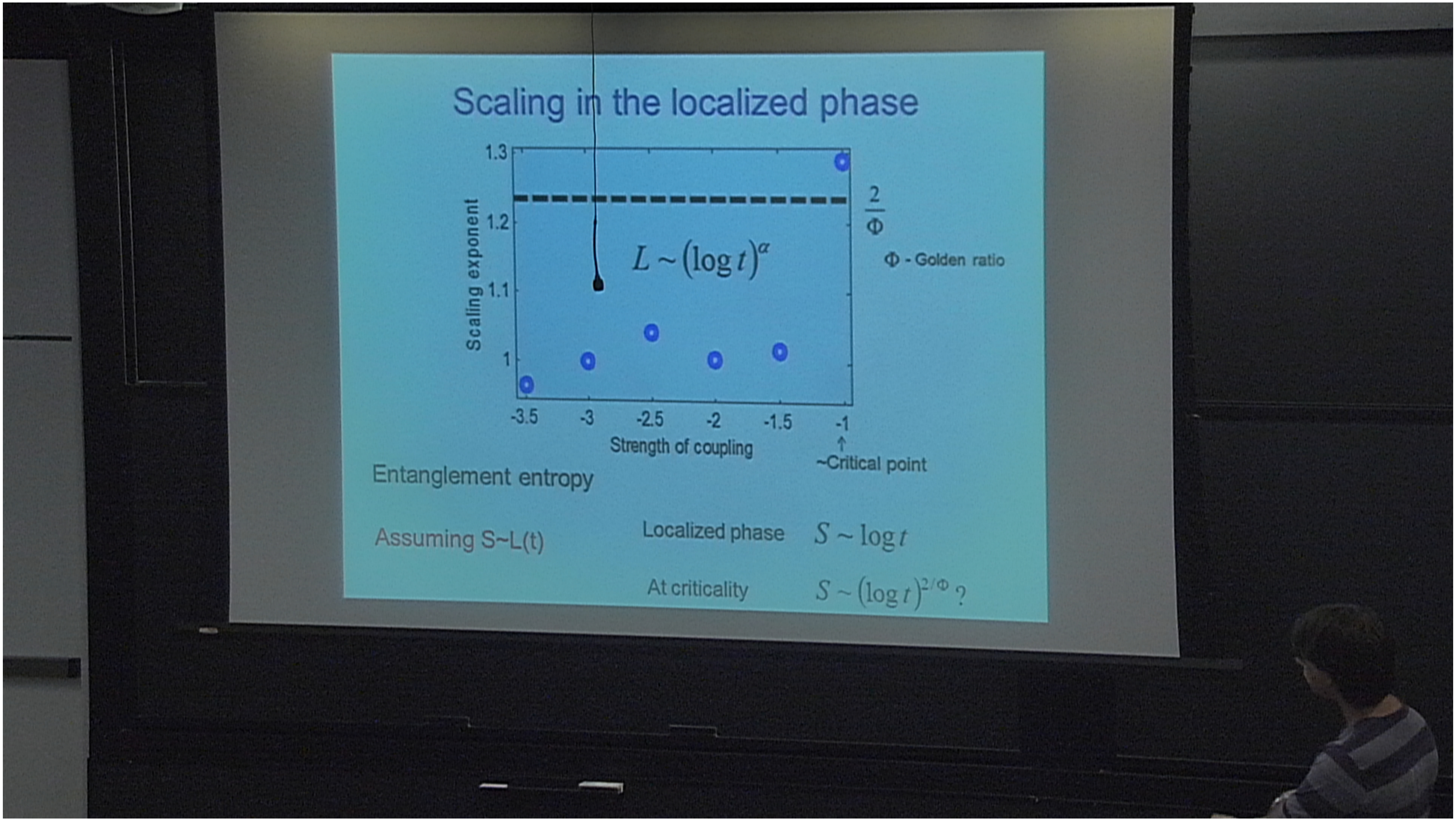
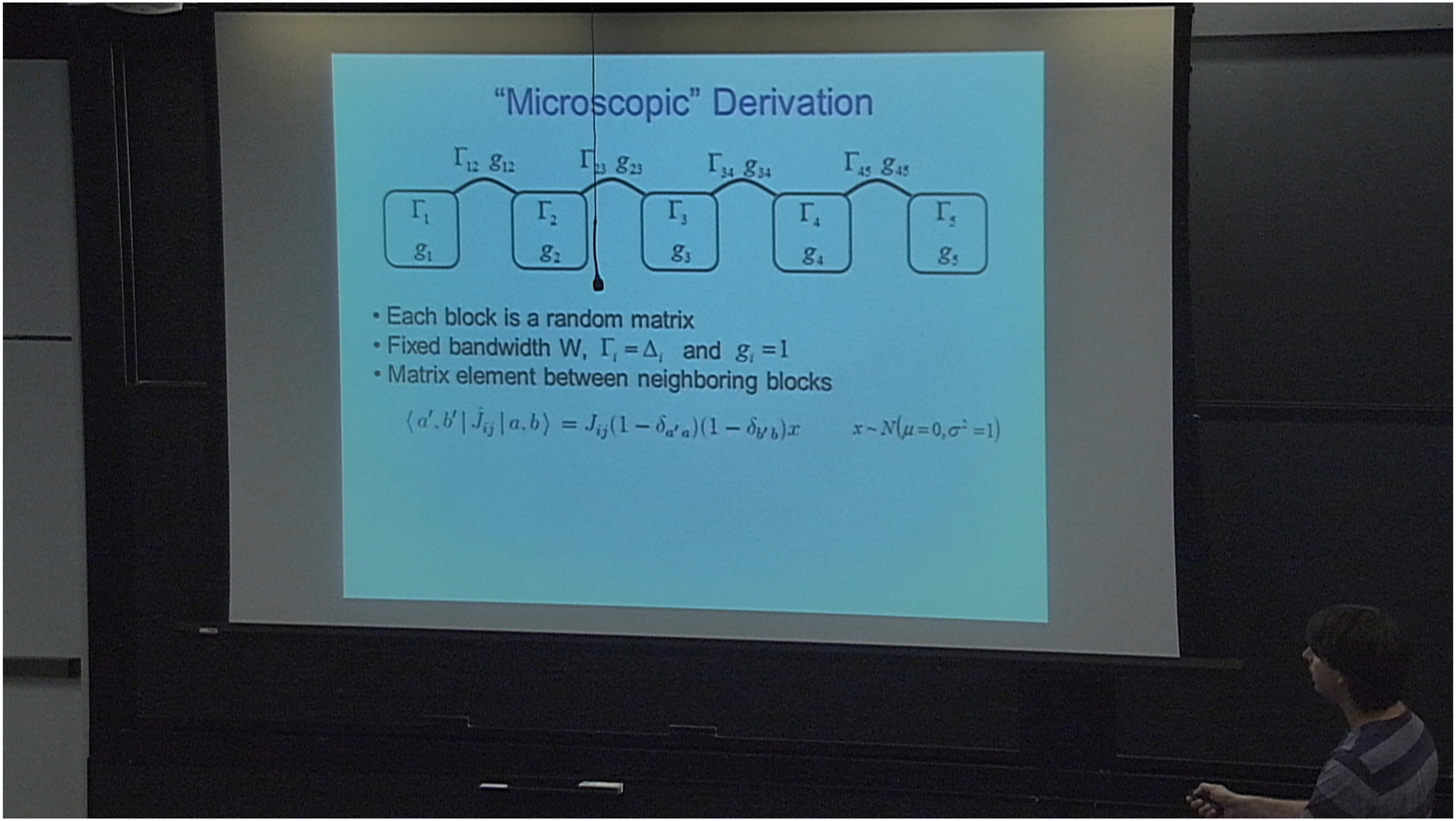
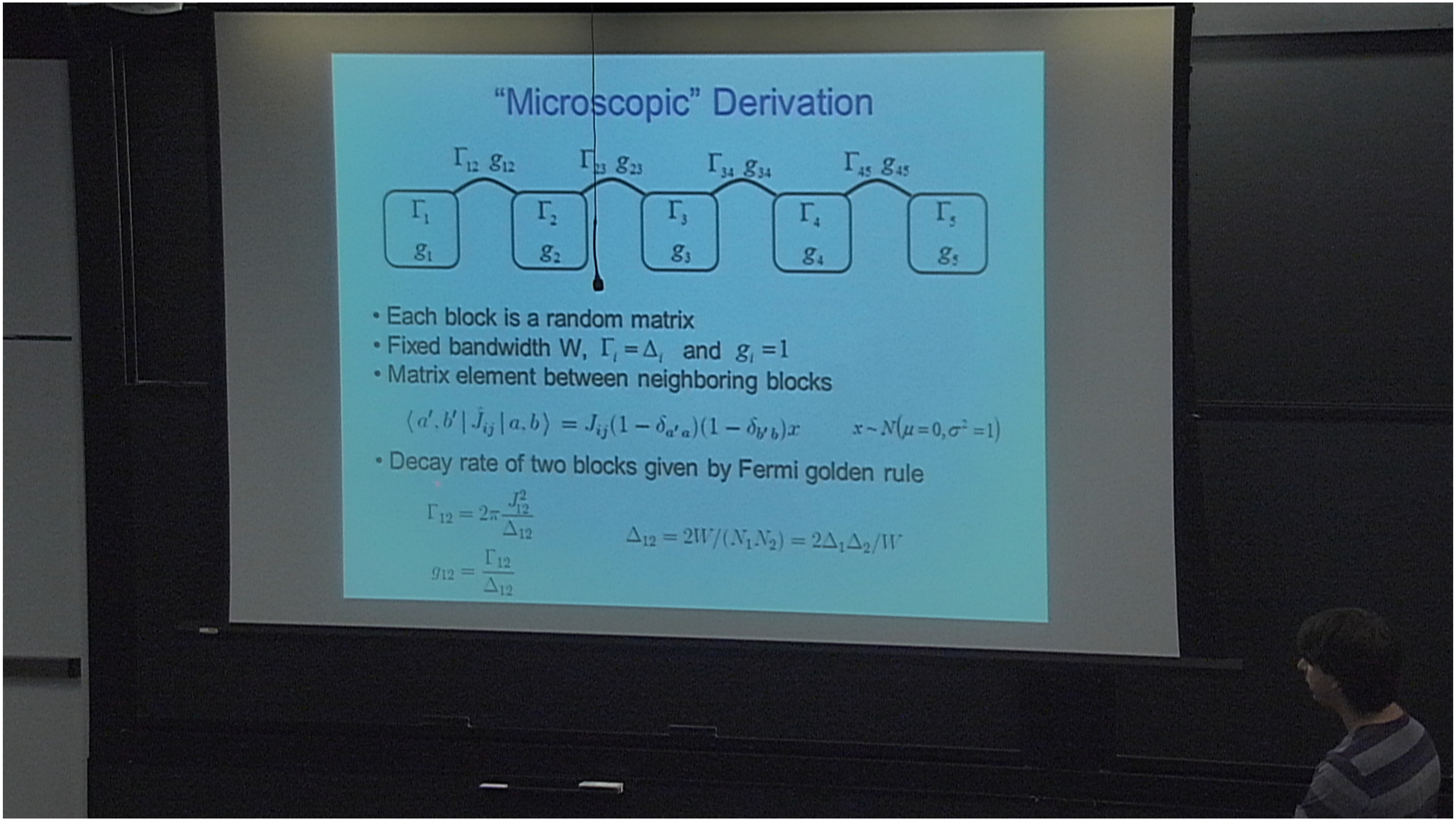

Scaling in the localized phase
1.3
1.2
1.1
1
Scaling exponent
-3.5
-3
-2.5
-2
-1.5
-1
Strength of coupling
~Critical point
$\frac{2}{\Phi}$
Φ - Golden ratio
$L \sim (\log t)^{\alpha}$
Entanglement entropy
Assuming S~L(t)
Localized phase $S \sim \log t$
At criticality $S \sim (\log t)^{2/\Phi}$?


"Microscopic" Derivation
$\Gamma_{12}\ g_{12}$
$\Gamma_{23}\ g_{23}$
$\Gamma_{34}\ g_{34}$
$\Gamma_{45}\ g_{45}$
Γ_1 g_1
Γ_2 g_2
Γ_3 g_3
Γ_4 g_4
Γ_5 g_5
• Each block is a random matrix
• Fixed bandwidth W, $\Gamma_i = \Delta_i$ and $g_i = 1$
• Matrix element between neighboring blocks
$\langle a', b' \,|\, \hat{J}_{ij} \,|\, a, b \rangle = J_{ij}(1-\delta_{a'a})(1-\delta_{b'b})x \qquad x \sim N(\mu = 0, \sigma^2 = 1)$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• Each block is a random matrix
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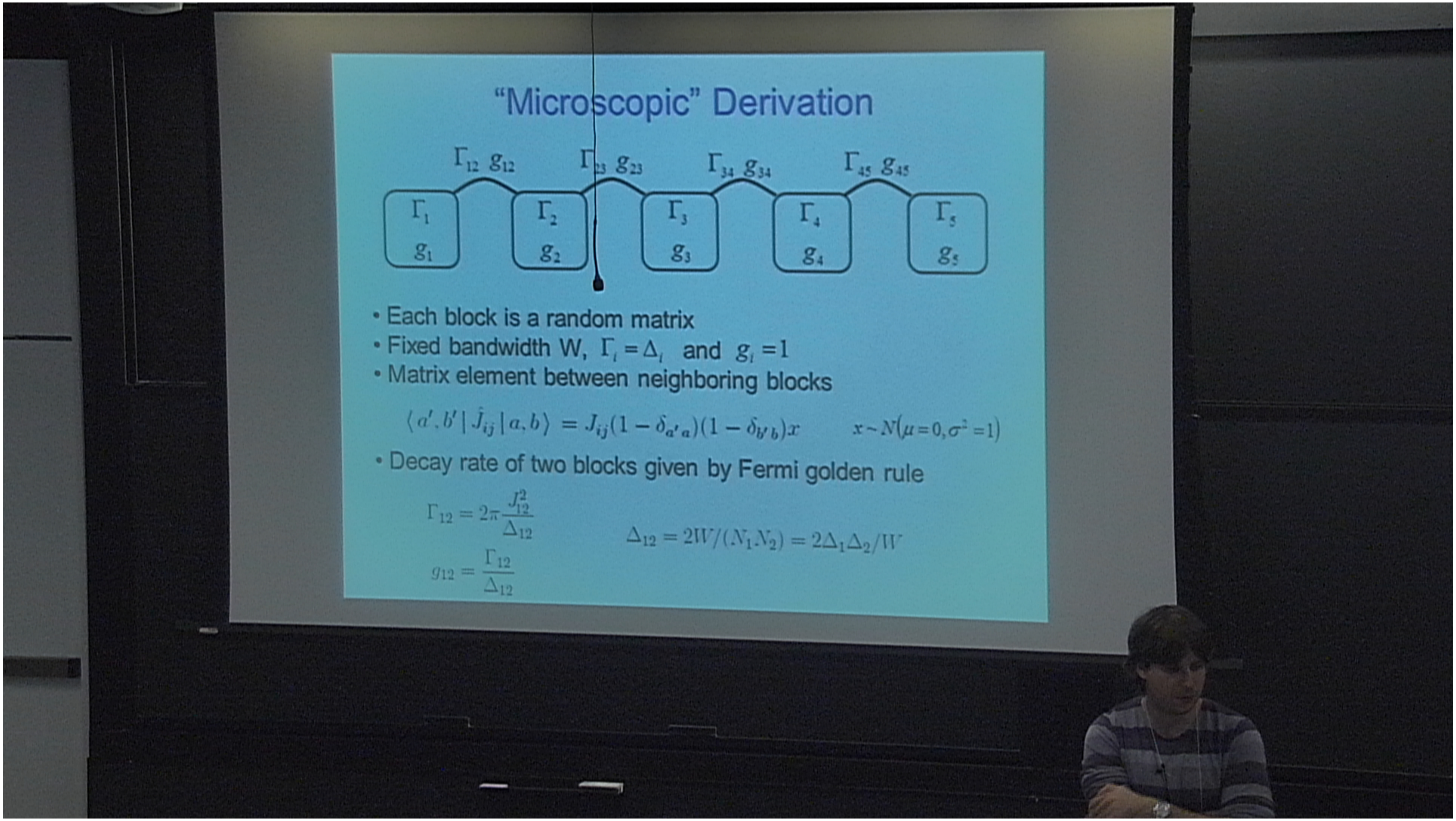
$\langle a', b' \,|\, \hat{J}_{ij} \,|\, a, b \rangle = J_{ij}(1-\delta_{a'a})(1-\delta_{b'b})x$ $x \sim N(\mu = 0, \sigma^2 = 1)$
• Decay rate of two blocks given by Fermi golden rule
$\Gamma_{12} = 2\pi \frac{J_{12}^2}{\Delta_{12}}$
$\Delta_{12} = 2W/(N_1 N_2) = 2\Delta_1\Delta_2/W$
$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$


"Microscopic" Derivation
Γ_{12} g_{12}
Γ_{23} g_{23}
Γ_{34} g_{34}
Γ_{45} g_{45}
Γ_1 g_1
Γ_2 g_2
Γ_3 g_3
Γ_4 g_4
Γ_5 g_5
• Each block is a random matrix
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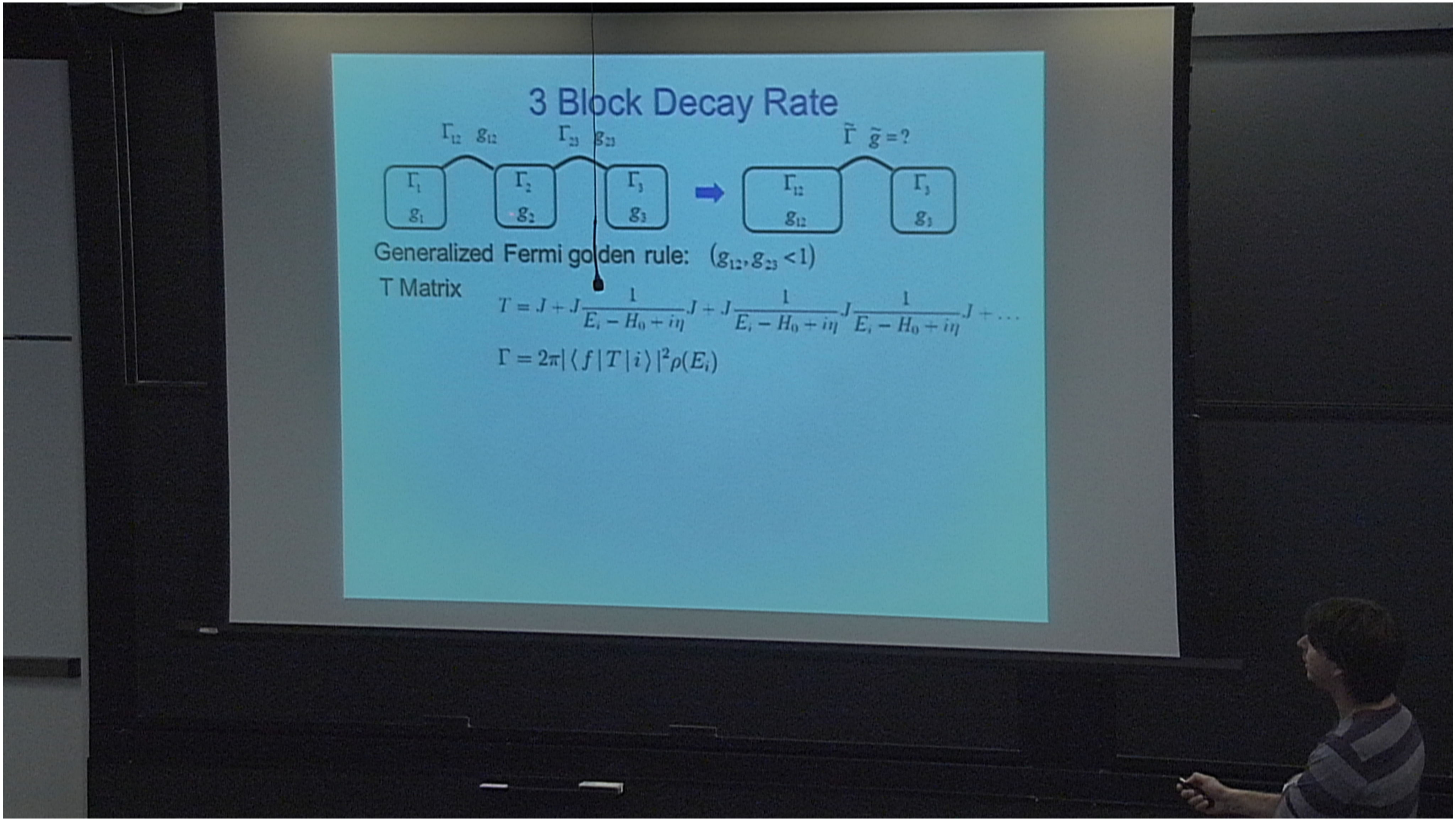
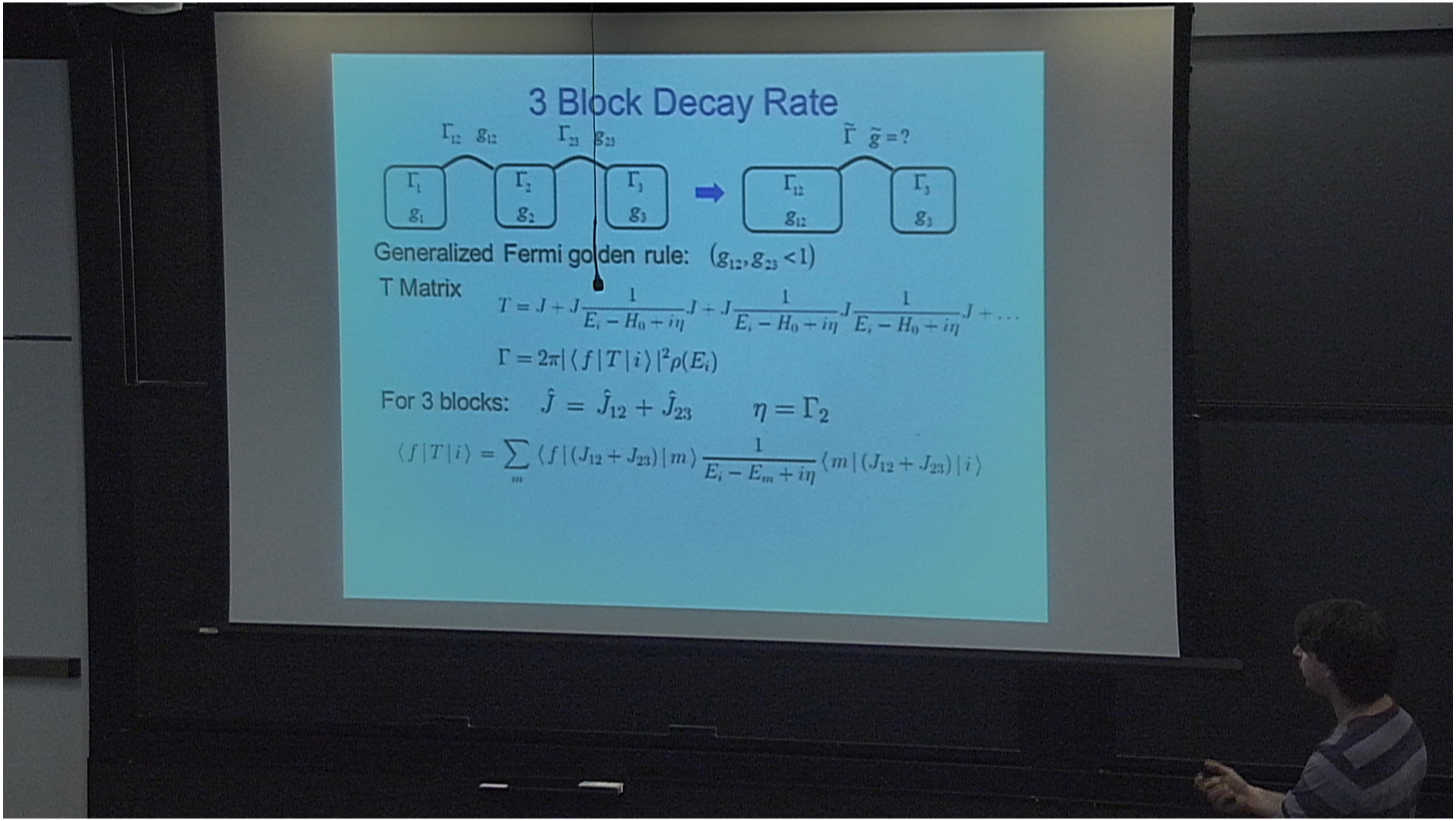
• Decay rate of two blocks given by Fermi golden rule
$\Gamma_{12} = 2\pi \frac{J_{12}^2}{\Delta_{12}}$
$\Delta_{12} = 2W/(N_1 N_2) = 2\Delta_1 \Delta_2 / W$
$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$



3 Block Decay Rate
Γ12 g12
Γ23 g23
Γ1 g1
Γ2 g2
Γ3 g3
Γ̃ g̃ = ?
Γ12 g12
Γ3 g3
Generalized Fermi golden rule: (g12, g23 < 1)
T Matrix
T = J + J 1/(Ei − H0 + iη) J + J 1/(Ei − H0 + iη) J 1/(Ei − H0 + iη) J + ...
Γ = 2π|⟨f|T|i⟩|²ρ(Ei)


3 Block Decay Rate
Γ_{12} g_{12}
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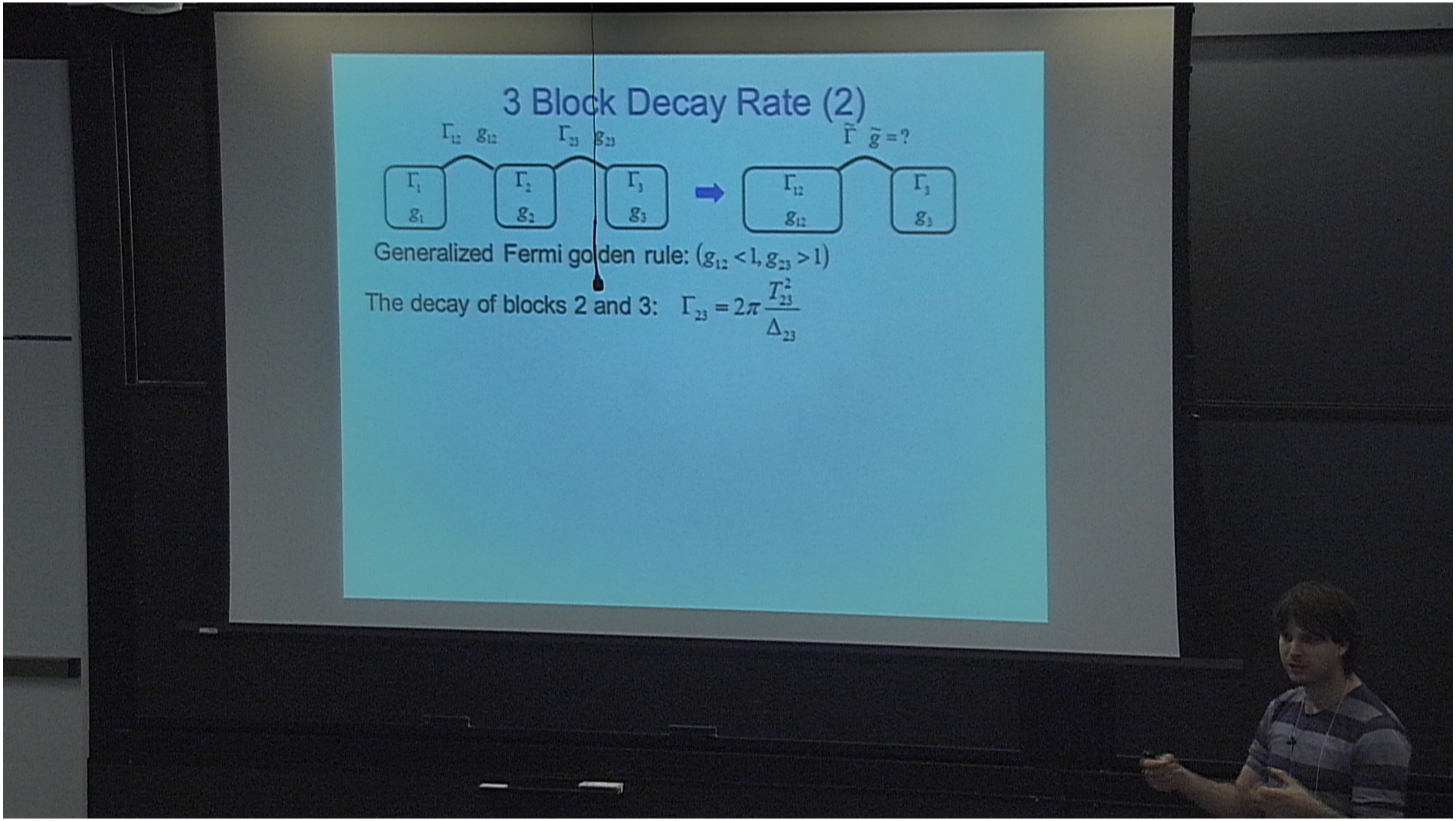
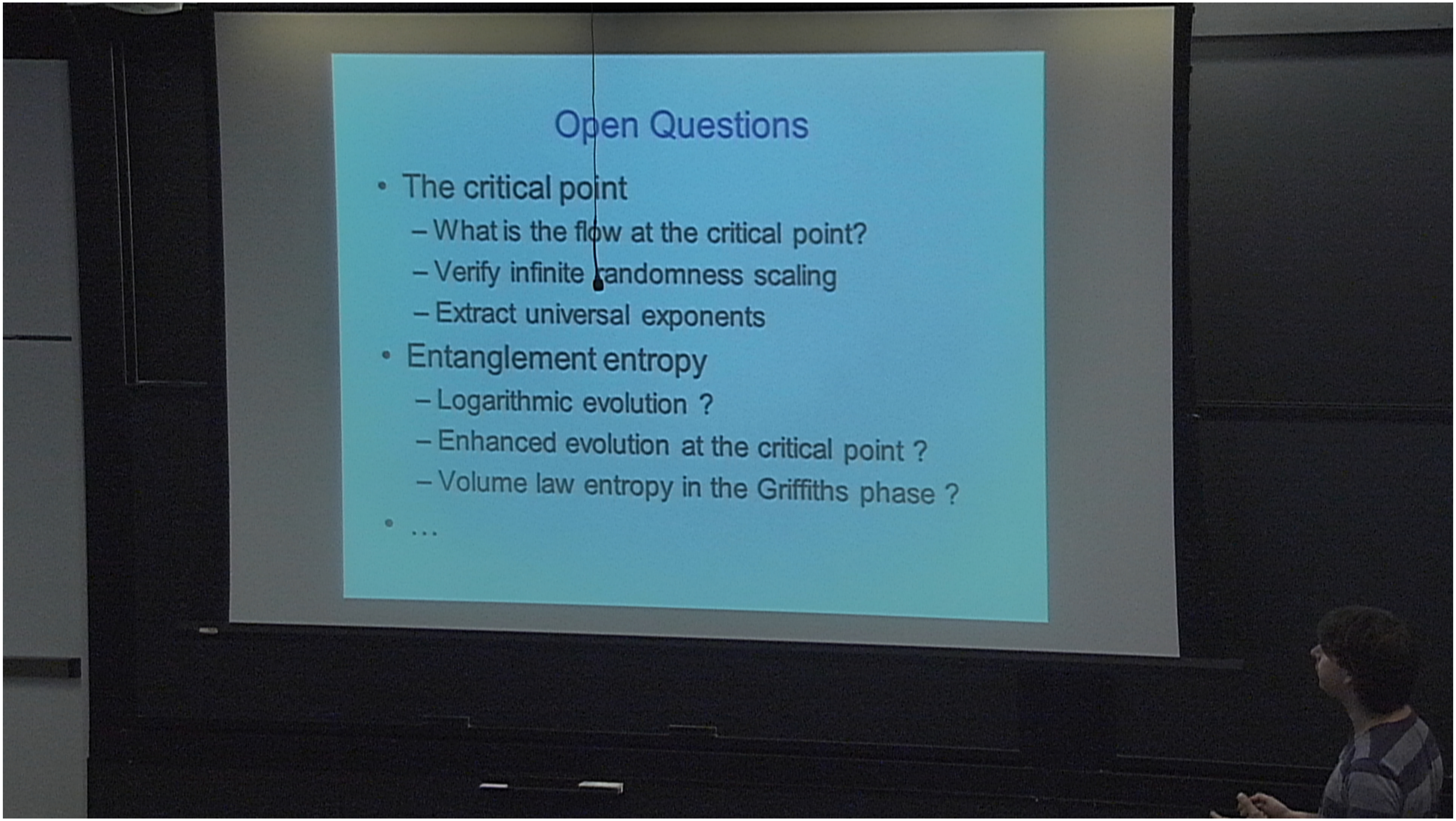
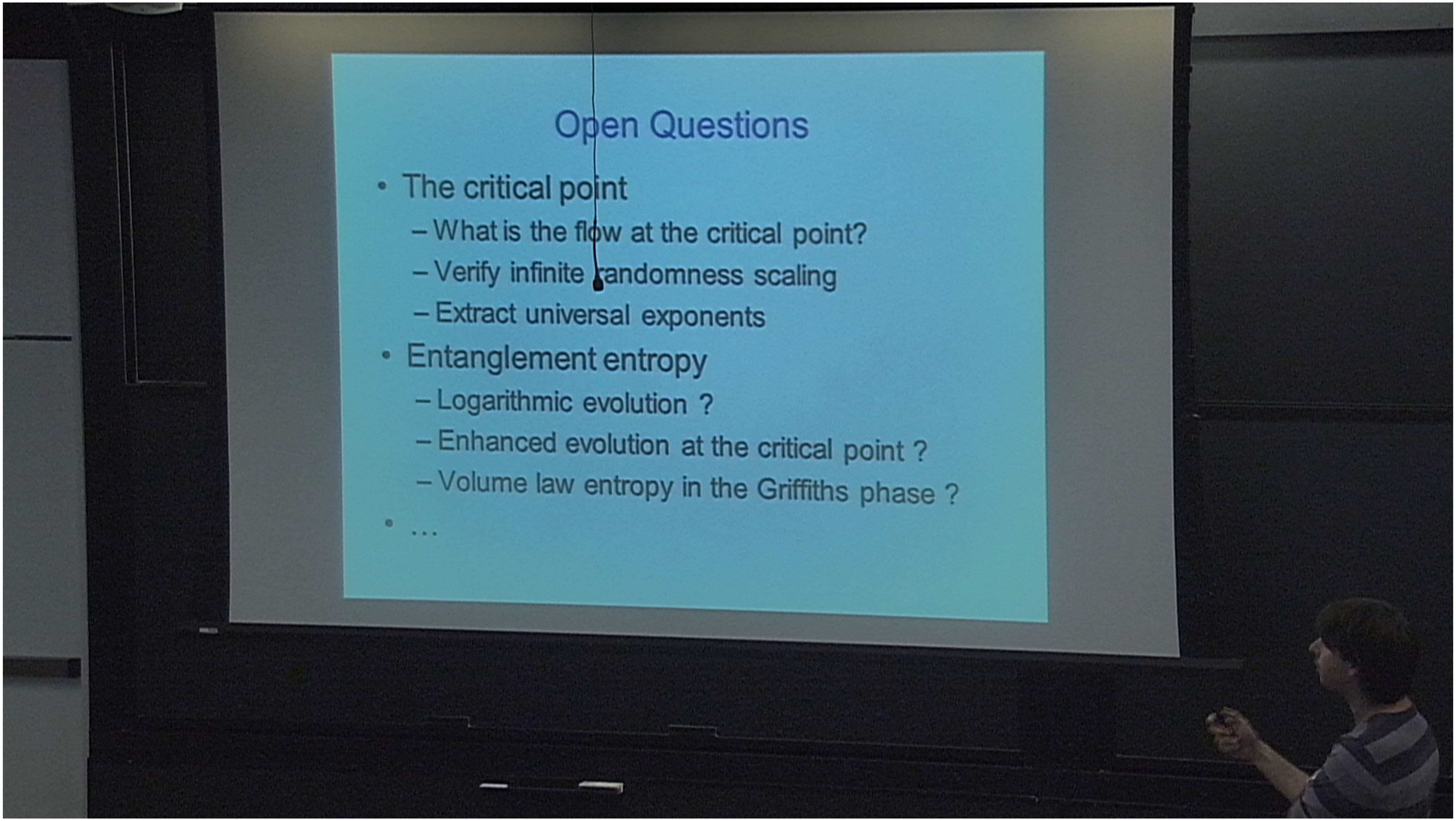
Γ_1 g_1
Γ_2 g_2
Γ_3 g_3
$\tilde{\Gamma}$ $\tilde{g}=?$
Γ_{12} g_{12}
Γ_3 g_3
Generalized Fermi golden rule: $(g_{12}, g_{23} < 1)$
T Matrix
$T = J + J\frac{1}{E_i - H_0 + i\eta}J + J\frac{1}{E_i - H_0 + i\eta}J\frac{1}{E_i - H_0 + i\eta}J + \ldots$
$\Gamma = 2\pi |\langle f | T | i \rangle|^2 \rho(E_i)$
For 3 blocks: $\hat{J} = \hat{J}_{12} + \hat{J}_{23}$ $\eta = \Gamma_2$
$\langle f | T | i \rangle = \sum_m \langle f | (J_{12} + J_{23}) | m \rangle \frac{1}{E_i - E_m + i\eta} \langle m | (J_{12} + J_{23}) | i \rangle$


3 Block Decay Rate (2)
Γ_{12} g_{12}
Γ_{23} g_{23}
Γ_1 g_1
Γ_2 g_2
Γ_3 g_3
$\tilde{\Gamma}$ $\tilde{g} = ?$
Γ_{12} g_{12}
Γ_3 g_3
Generalized Fermi golden rule: $(g_{12} < 1, g_{23} > 1)$
The decay of blocks 2 and 3: $\Gamma_{23} = 2\pi \frac{T_{23}^2}{\Delta_{23}}$


Open Questions
• The critical point
– What is the flow at the critical point?
– Verify infinite randomness scaling
– Extract universal exponents
• Entanglement entropy
– Logarithmic evolution ?
– Enhanced evolution at the critical point ?
– Volume law entropy in the Griffiths phase ?
• …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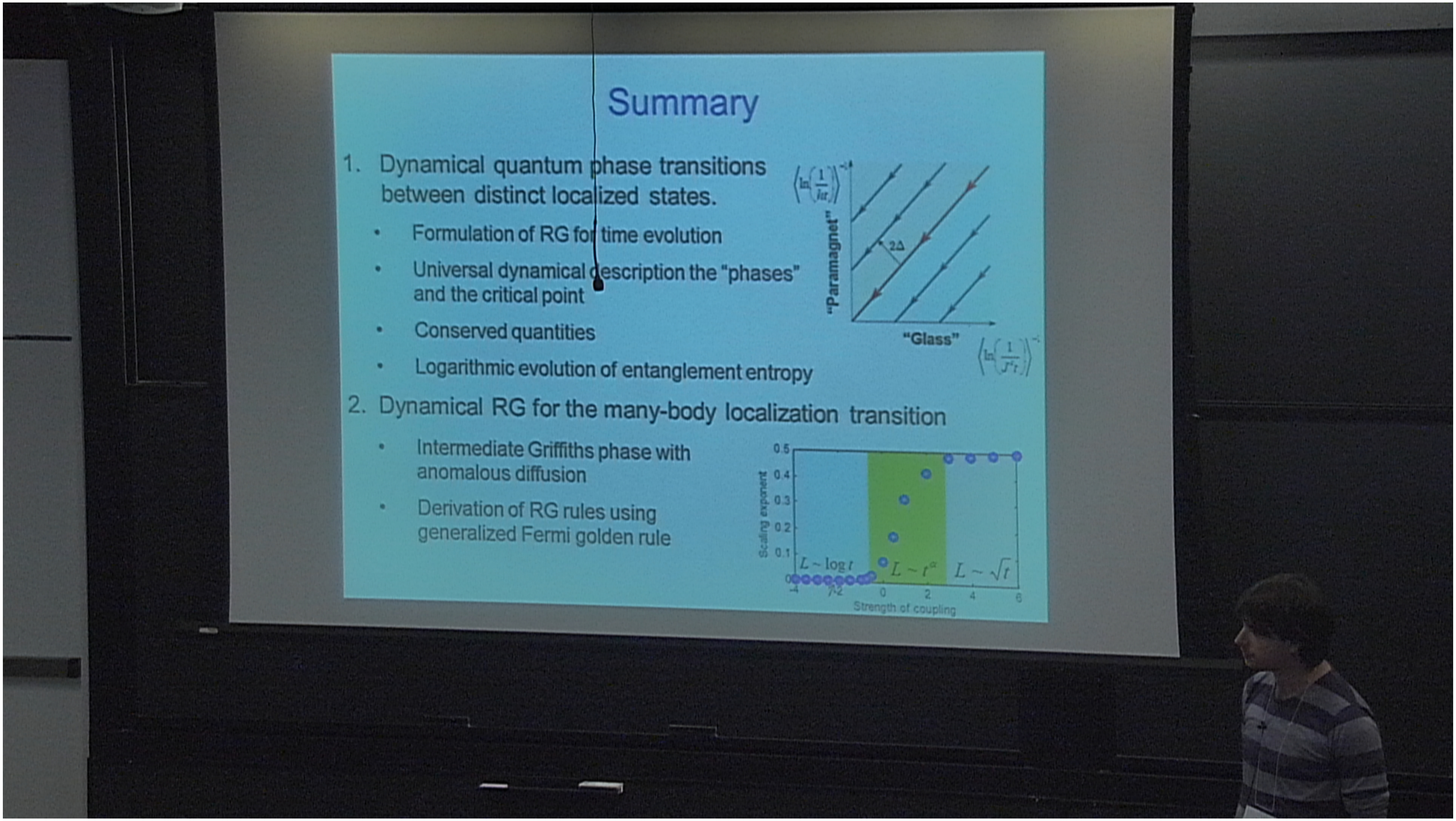
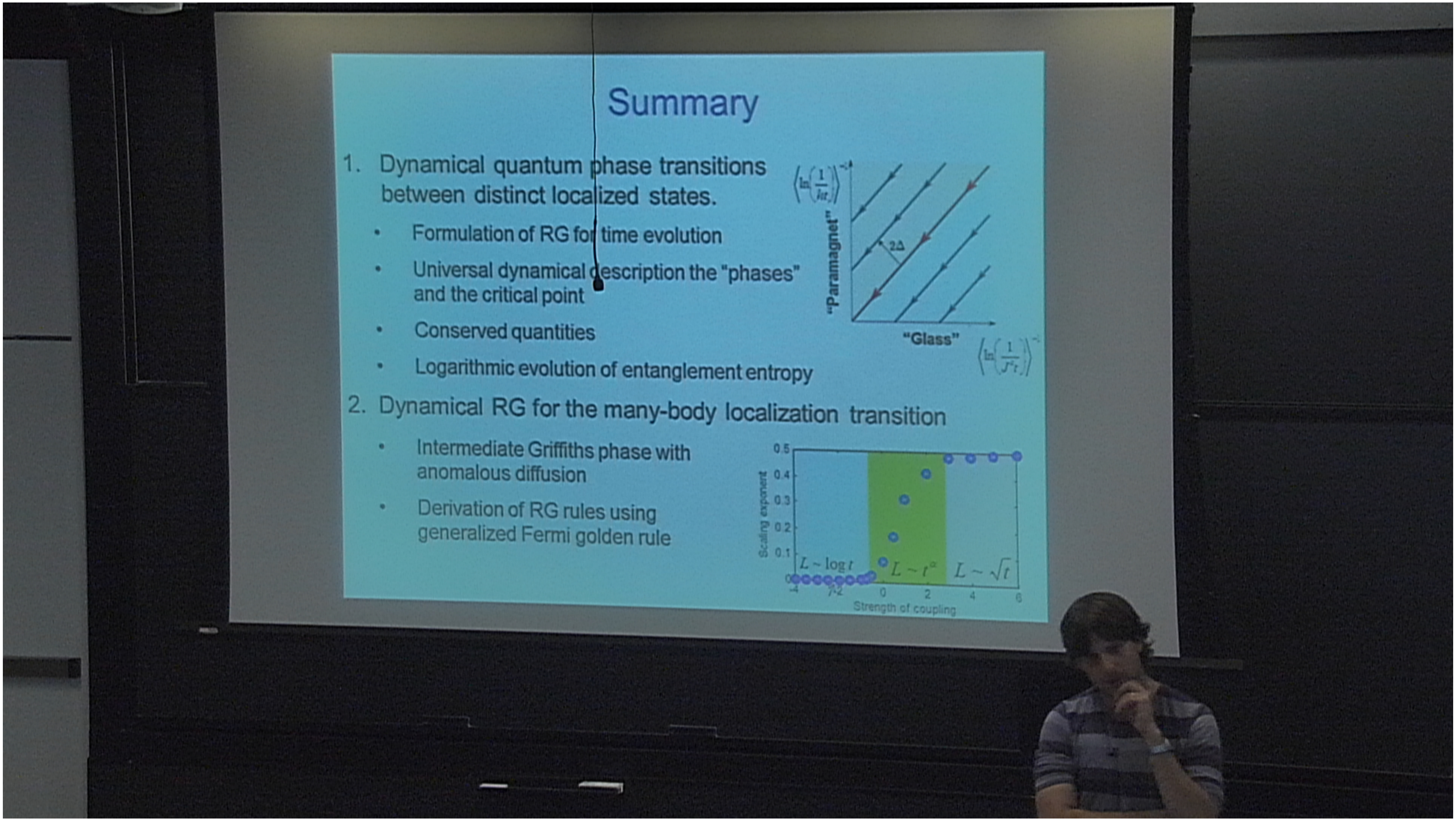

Summary
1. Dynamical quantum phase transitions between distinct localized states.
• Formulation of RG for time evolution
• Universal dynamical description the "phases" and the critical point
• Conserved quantities
• Logarithmic evolution of entanglement entropy
"Paramagnet"
"Glass"
2Δ
2. Dynamical RG for the many-body localization transition
• Intermediate Griffiths phase with anomalous diffusion
• Derivation of RG rules using generalized Fermi golden rule
Scaling exponent
Strength of coupling
L ~ log t
L ~ t^α
L ~ √t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