Title: Many-body mobility edge in a mean-field quantum spin-glass

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Abstract: Isolated, interacting quantum systems in the presence of strong disorder can exist in a many-body localized phase where the assumptions of equilibrium statistical physics are violated. On tuning either the parameters of the Hamiltonian or the energy density, the system is expected to transition into the ergodic phase. While the transition at "infinite temperature" as a function of system parameters has been found numerically but, the transition tuned by energy density has eluded such methods.
In my talk I will discuss the nature of the many-body localization-delocalization (MBLD) transition as a function of energy denisty in the quantum random energy model (QREM). QREM provides a mean-field description of the equilibrium spin glass transition. We show that it further exhibits a many-body mobility edge when viewed as a closed quantum system. The mean-field structure of the model allows an analytically tractable description of the MBLD transition. I will also comment on the nature of the critical states in this mean-field model.
This opens the possibility of developing a mean-field theory of this interesting dynamical phase transition.

Ergodic hypothesis in QM

Beweis des Ergodensatzes und des H-Theorems in der neuen Mechanik.

Von **J. v. Neumann** in Berlin.

(Eingegangen am 10. Mai 1929.)

Es wird gezeigt, wie der scheinbare Widerspruch zwischen dem makroskopischen Ansatz des Phasenraumes und dem Bestehen von Unbestimmtheitsrelationen aufzulösen ist. Danach werden die hauptsächlichsten Begriffsbildungen der statistischen Mechanik quantenmechanisch umgedeutet, der Ergodensatz und das H-Theorem formuliert und (ohne „Unordnungsannahmen") bewiesen. Es folgt eine Diskussion des physikalischen Sinnes der ihren Gültigkeitsbereich festlegenden mathematischen Bedingungen.

Similar ideas from Wigner, Schrodinger, Pauli….

Translation: Tumulka, Eur. Phys. J. H 35, 201–237 (2010)
Commentary: Goldstein, Lebowitz, Tumulka, Zanghi, European Phys. J. H 35, 173 (2010)

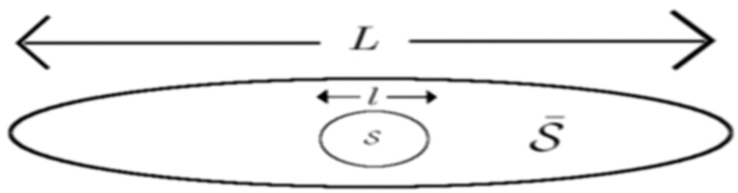
Many-body localization: A quintessential glass

- Can an isolated quantum system serve as its own heat bath?
- The system can locally exchange energy/particles/spin with the rest of the system.

Absence of Diffusion in Certain Random Lattices

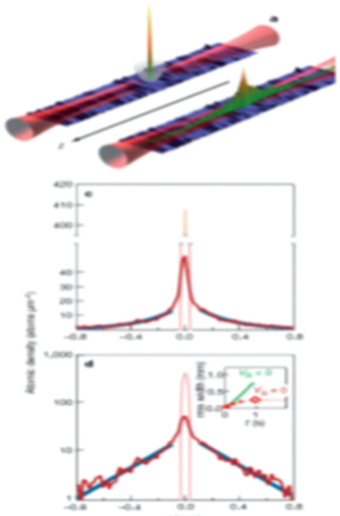
P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

ligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible



- Many-body localization is the breakdown of ergodicity.
- The system *freezes,* retaining the memory of its initial conditions.

Recent experimental developments

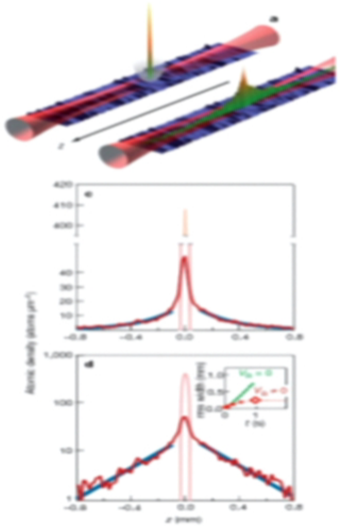


J. Billy et. al., Nature (2008)

Cold atomic systems are highly isolated interacting systems. Absence of lattice phonons which serve as a heat bath in solid state systems

Interactions and disorder are tunable parameters.

Recent experimental developments



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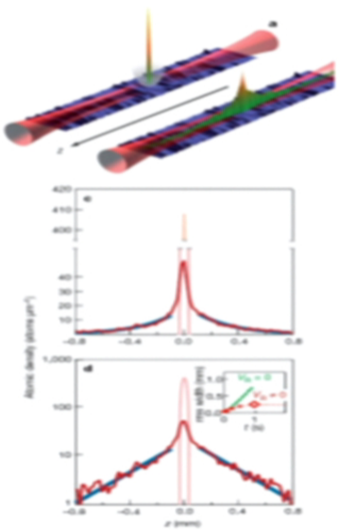

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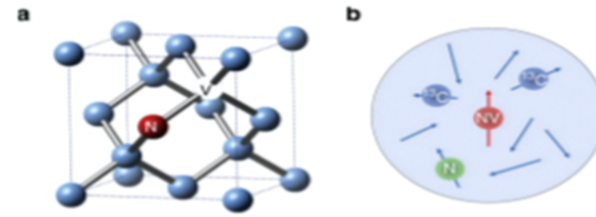


L. Childress et. al., Science (2006)
Guredev Dutt et. al., Science (2007)
N. Bar-Gill, Nature Communications (2012)

Cold atomic systems are highly isolated interacting systems. Absence of lattice phonons which serve as a heat bath in solid state systems

Interactions and disorder are tunable parameters.

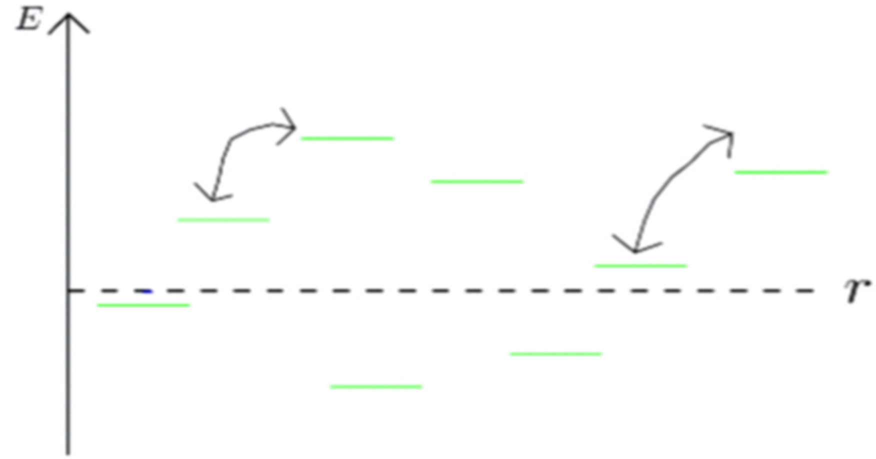
System with long coherence times due to weak coupling to phonons.

High level of control of the local spin degrees of freedom

Single-particle localization

$$H_{\text{non-interacting}} = \sum_{i,j} t_{ij} \left(c_i^\dagger c_j + h.c. \right) + \sum_i \mu_i c_i^\dagger c_i$$

$$\mu_i \in [-W, W]$$



- Eigenstates in the absence of hopping are localized on individual sites.

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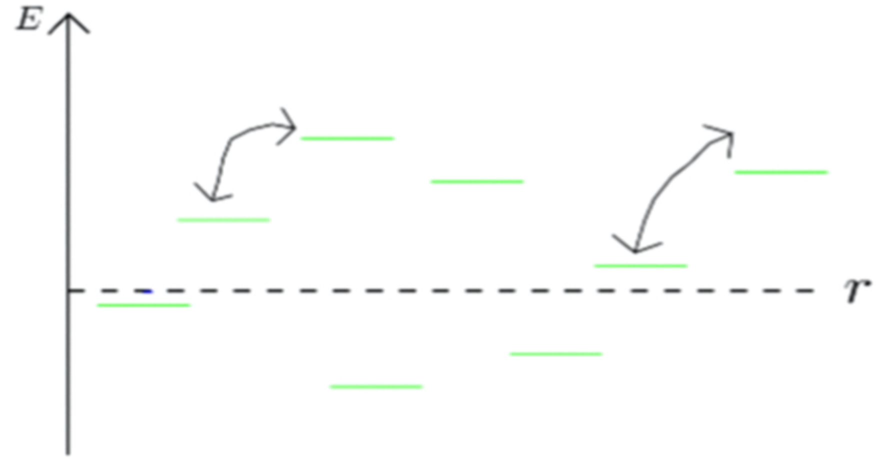
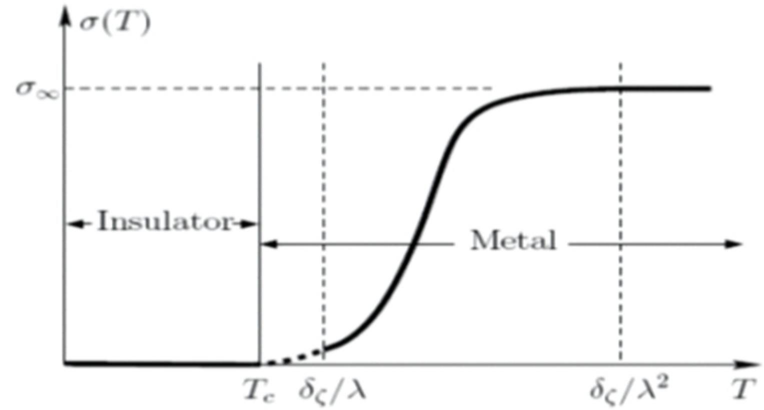


- Eigenstates in the absence of hopping are localized on individual sites.
- On turning on hopping (t) weakly, at strong disorder the perturbative correction to the wavefunction is small.
- The delocalization is marked by the proliferation of resonances as hopping amplitude increases.

Basko, Aleiner & Altshuler (BAA)
Annals of Physics (2006)

- Weakly interacting fermions in d-dimensions with quenched disorder. (Perturbation Theory)

- Isolated system without an external bath. (with electron-phonon coupling)



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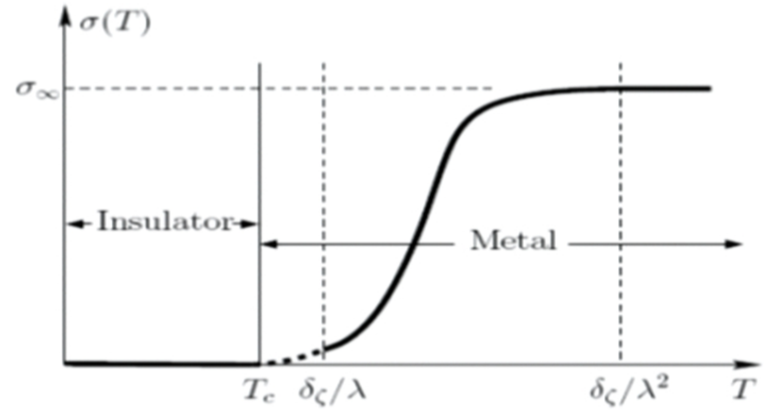


- Isolated system without an external bath. (with electron-phonon coupling)

- D.C. conductivity $\sigma(T)= 0$ below energy densities corresponding to a finite T_c (temperature well-defined only in the metallic phase) - Phase transition.

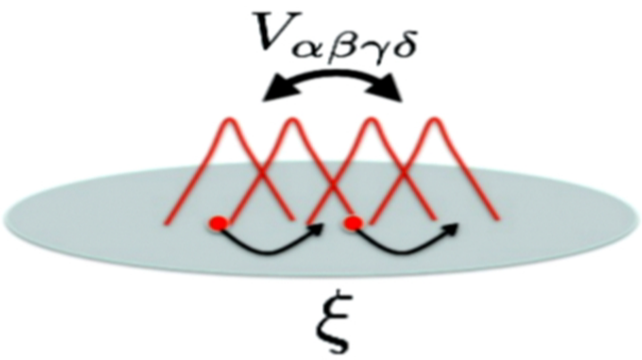
- No thermodynamic signatures, only a dynamical transition.

General line of reasoning of BAA

- All single-particle states are localized. $|\phi(r)|^2 \sim e^{-r/\xi}$

- Interactions are short-ranged. Hamiltonian expressed in terms of the localized orbitals

$$\hat{H} = \sum_{\alpha} \xi_\alpha \hat{c}^\dagger_\alpha \hat{c}_\alpha + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} \hat{c}^\dagger_\alpha \hat{c}^\dagger_\beta \hat{c}_\gamma \hat{c}_\delta.$$



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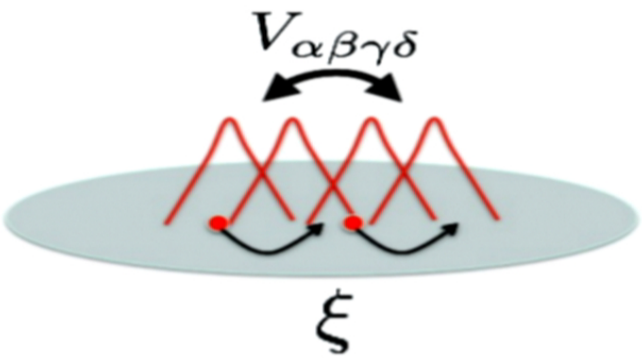

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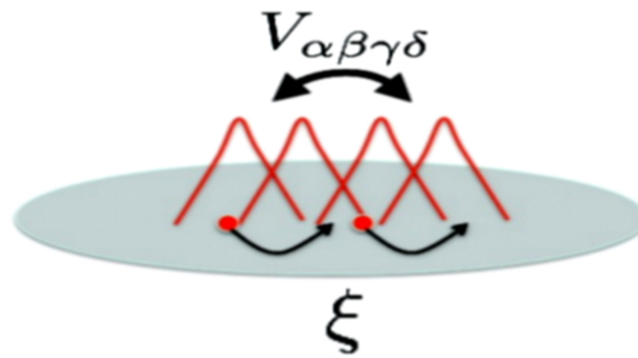

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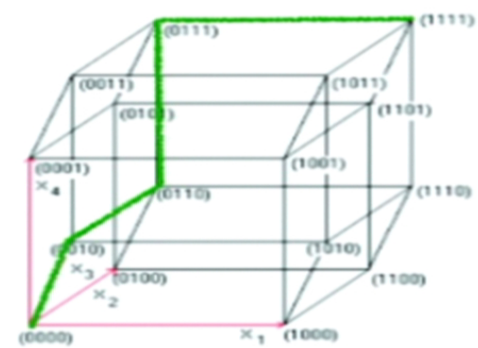
- Basis states of the Fock space represent the sites of the N-dimensional hypercube (for N particles).

- The “interaction” produces hops along the edges of the hypercube.



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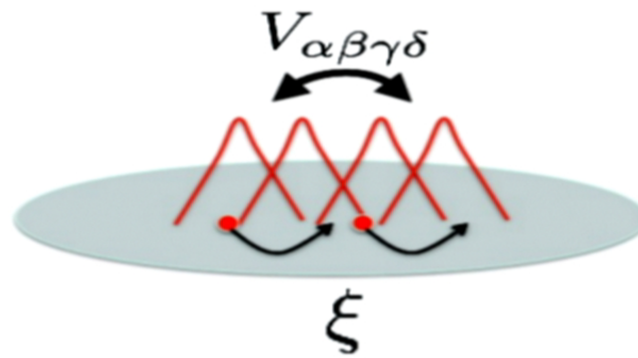

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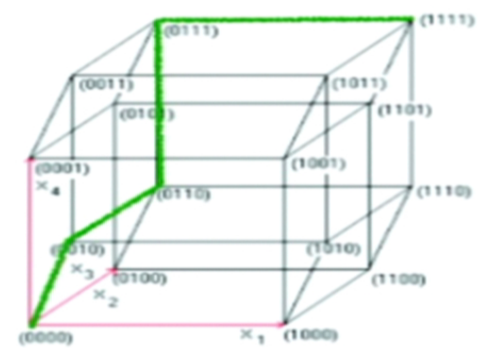


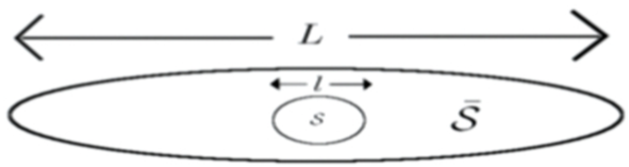
Eigenstate thermalization hypothesis

Deutsch, Phys. Rev A (1991), Srednicki, Phys. Rev. E (1994), M. Rigol et. al., Nature (2008)

A subsytem thermalizes as t→∞ and L→∞ if

$$\rho_S(t) \to \rho_S^{\mathrm{eq}}(\mu, \beta) \equiv \mathrm{Tr}_{\bar{S}} \left(e^{-\beta(H-\mu N)} \right)$$

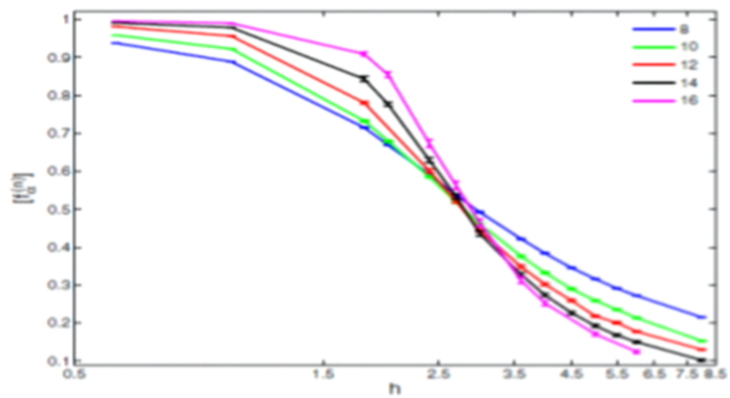
with the initial energy density ε(T) deciding the temperature for arbitrary initial conditions.



$$\rho(t) = \sum_{m,n} \rho_{mn}(0) e^{-i(E_m - E_n)t} |m\rangle\langle n|$$

Evidence of “infinite temperature” localization

$$H = \sum_{i=1}^{L} [h_i \hat{S}_i^z + J\hat{\vec{S}}_i \cdot \hat{\vec{S}}_{i+1}]$$



$$\hat{M}_1 = \sum_j \hat{S}_j^z \exp(i2\pi j/L)$$

$$f_\alpha^{(n)} = 1 - \frac{\langle n|\hat{M}_1^\dagger|n\rangle\langle n|\hat{M}_1|n\rangle}{\langle n|\hat{M}_1^\dagger \hat{M}_1|n\rangle}$$

- For nearest-neighbor interactions the many-body localization-delocalization (MBLD) transition was numerically observed at infinite temperature (states in the middle of the band) by tuning disorder strength.

Pal and Huse, PRB (2010)

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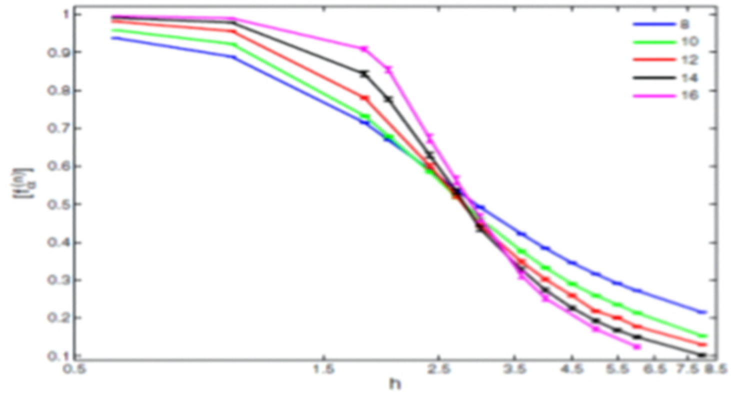

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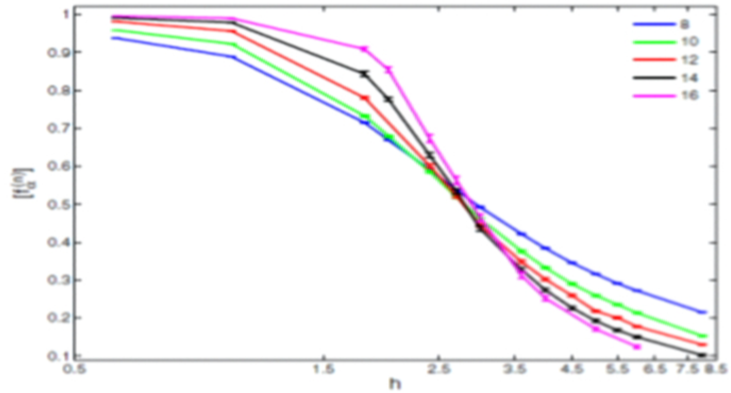

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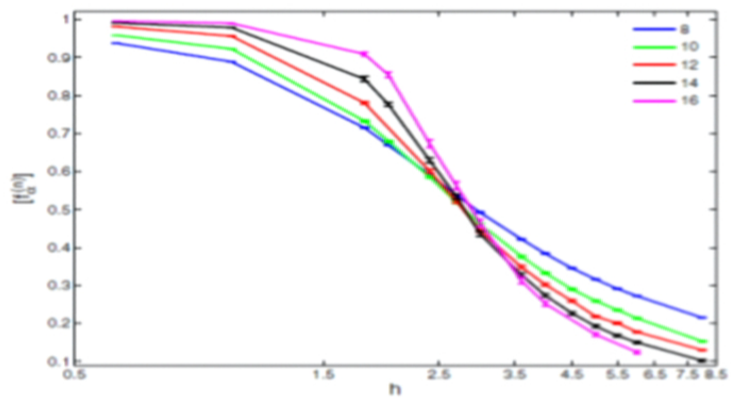

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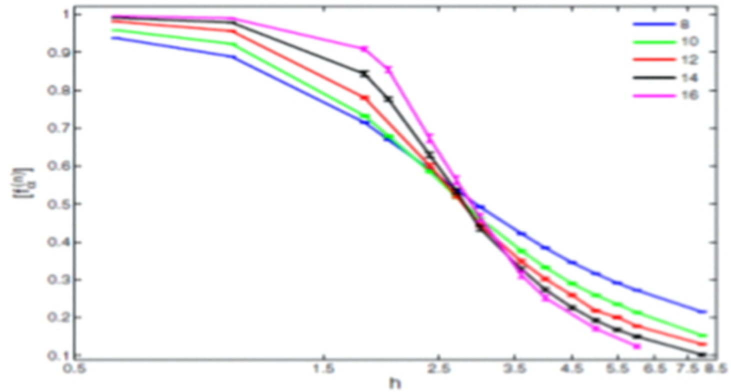

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Pal and Huse, PRB (2010)

A selfconsistent theory of localization

R Abou-Chacra†, P W Anderson‡§ and D J Thouless†

† Department of Mathematical Physics, University of Birmingham, Birmingham, B15 2TT

‡ Cavendish Laboratory, Cambridge, England and Bell Laboratories, Murray Hill, New Jersey, 07974, USA

- Single-particle localization on a Bethe lattice provides a mean-field description.

- No loops (high-volume growth space) but some features of the finite-dimensional problem can be captured in a controllable manner.

Mean-field model for MBL

- Is there a mean-field description of the MBL transition?
- Mean field model for spin-glasses may be a possible candidate.

Laumann, Pal and Scardicchio, arXiv:1404.2276

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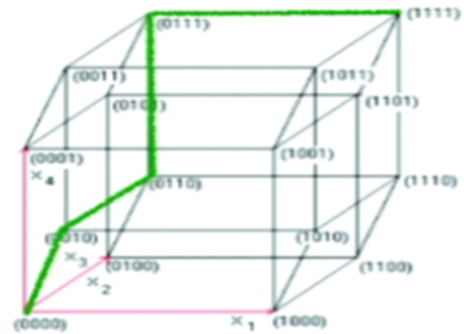
$$P(E) = \frac{1}{\sqrt{\pi N}} e^{-\frac{E^2}{N}}$$



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$$\sum_{i_1,\ldots,i_p} A_{i_1,\ldots,i_p} \sigma_{i_1}^z \ldots \sigma_{i_p}^z$$

- The transverse field produces hops along the edges of the hypercube.

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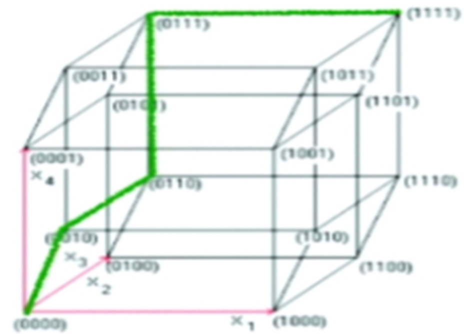

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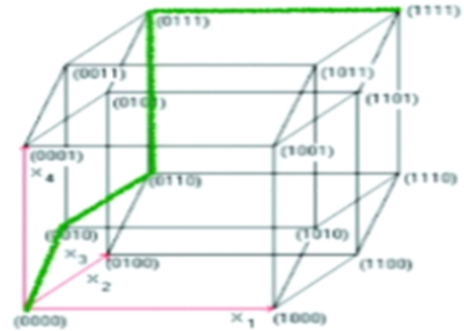

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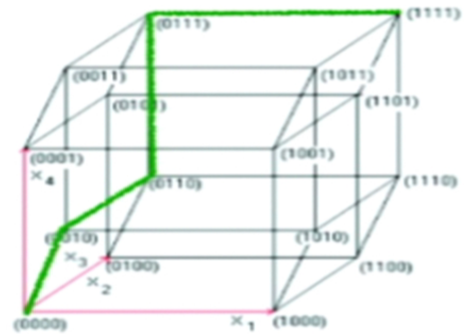

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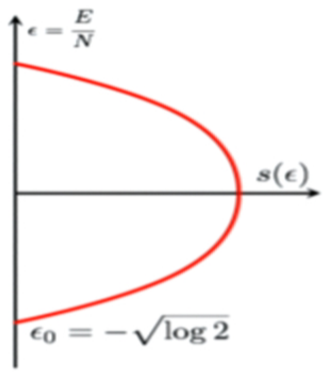
Thermodynamic phase diagram (Γ=0)

$$\overline{n(E)} = 2^N P(E) \sim e^{Ns(E/N)}$$

$$s(\epsilon) = \log 2 - \epsilon^2$$

Microcanonical



Derrida 1981

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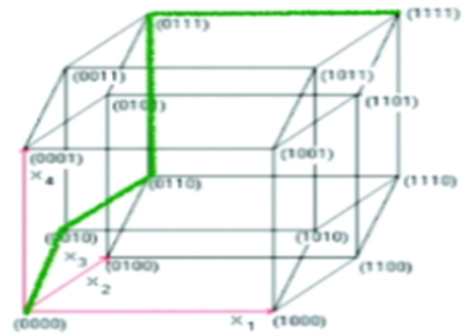

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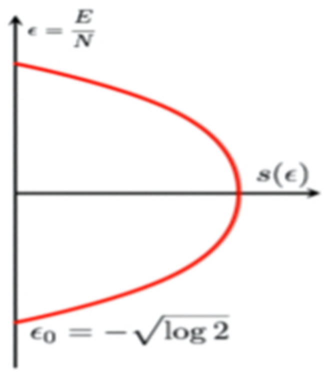
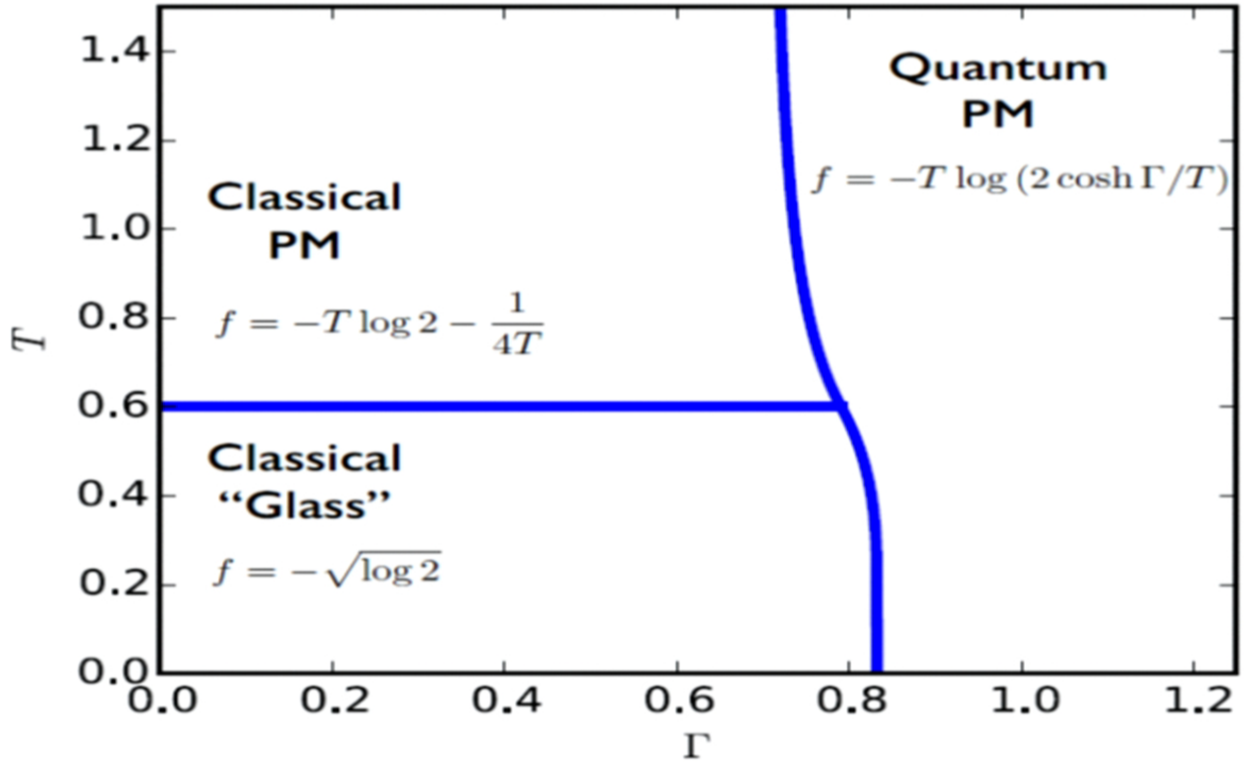


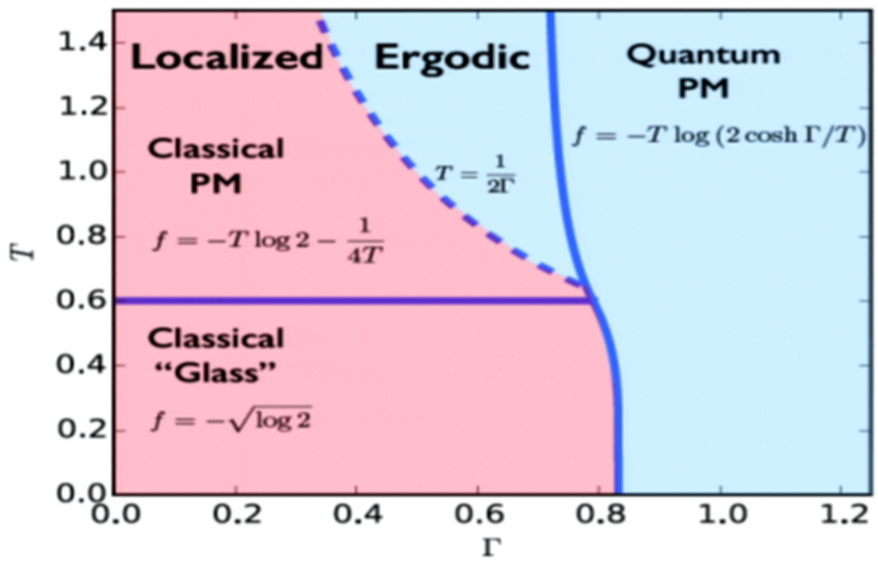
Derrida 1981

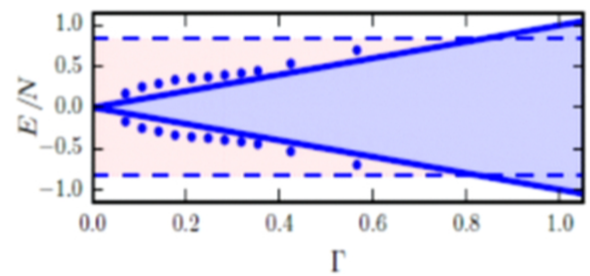
Canonical phase diagram



Goldschmidt, PRB (1990)
Jorg, Krzakalal, Kurchan, Maggs, PRL (2008)

Dynamical phase diagram





Laumann, Pal and Scardicchio, arXiv:1404.2276

Local z-magnetization

- The interactions in the QREM are non-local but the z-magnetization retains its locality.

$$H = E(\{\sigma_i^z\}) - \Gamma \sum_i \sigma_i^x$$

$$|[H, \sigma_i^z]| = \Gamma \sim O(1)$$

$$|[H, \sigma_i^x]| \sim O(N)$$

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$$\frac{\delta M(n)}{\delta n} \approx \frac{dM(\epsilon)}{d\epsilon}\frac{\delta\epsilon}{\delta n} \approx M'(\epsilon)e^{-Ns(\epsilon)}$$

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Signature of the many-body mobility edge

- Spectral statistics diagnosis of MBL

$$r_\alpha^{(n)} = \min\{\delta_\alpha^{(n)}, \delta_\alpha^{(n+1)}\} / \max\{\delta_\alpha^{(n)}, \delta_\alpha^{(n+1)}\}$$

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- Ergodic phase has GOE level statistics: $[r] \approx 0.53$
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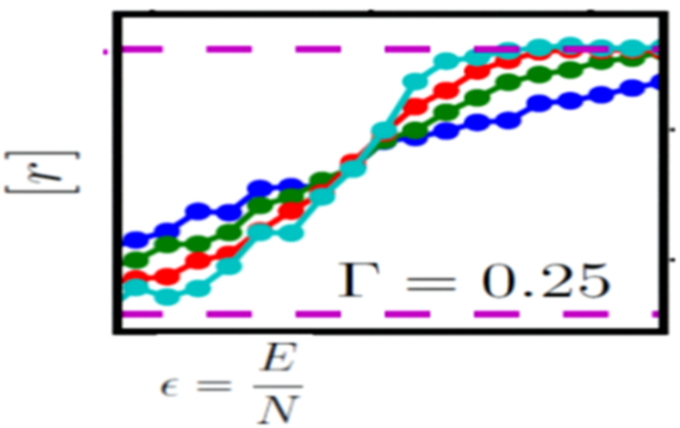
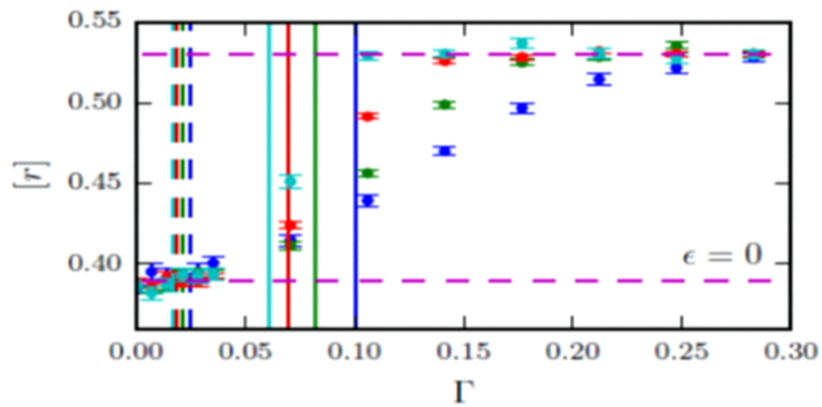
Change in spectral statistics with energy density



Delocalization at "infinite temperature"



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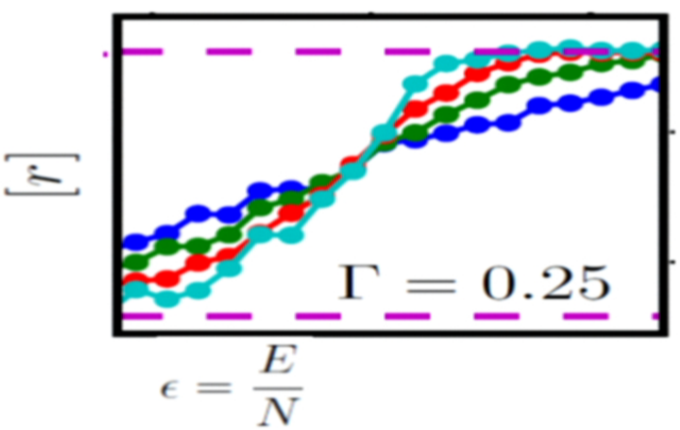

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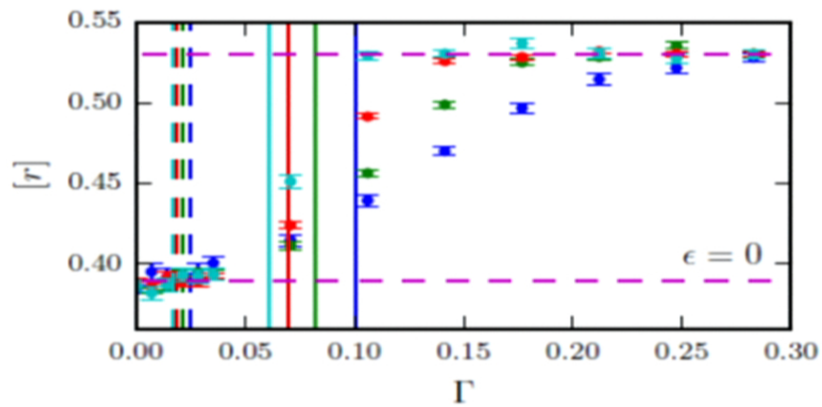

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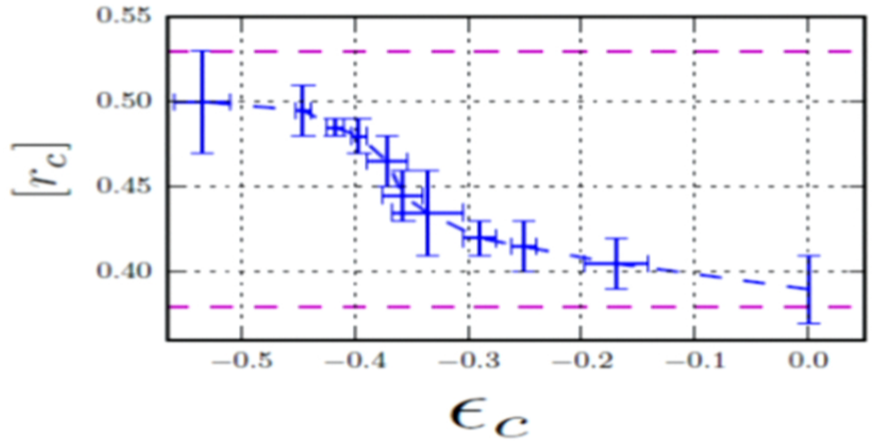
Critical line



Perturbative estimate of mobility edge

- For an initial state localized on site a of the hypercube, the amplitude on site b can be expressed perturbatively as a sum over all paths- Forward-scattering approximation (neglect loops).

$$\psi_b \simeq \Gamma^n \sum_{p \in \Pi_n} \prod_{i \in p} \frac{1}{E_a - E_i}$$

Altshuler, Gefen, Kamenev and Levitov, PRL (1997)

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- Given the amplitude at site n-1, the amplitude at n is

$$\psi_n = \frac{\Gamma}{\delta_n} \psi_{n-1}$$

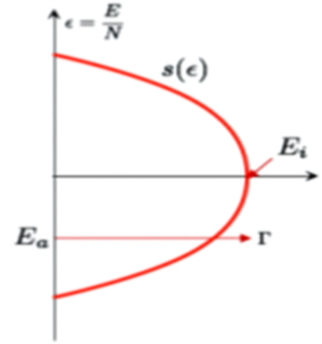
- Condition for resonant-delocalization is when the probability to find a resonance as n $\rightarrow$ N becomes 1

Altshuler, Gefen, Kamenev and Levitov, PRL (1997)

MBLD transition at finite temperature

- At an extensive energy ($E_a = \varepsilon N$), resonances are rare and all paths add democratically until a resonance is reached (neglecting correlations between paths).

$$\begin{aligned}\psi_n &= \frac{\Gamma}{\delta_n}\psi_{n-1} \\ &= \frac{\Gamma}{\delta_n} n! \left(\frac{\Gamma}{\epsilon N}\right)^n\end{aligned}$$



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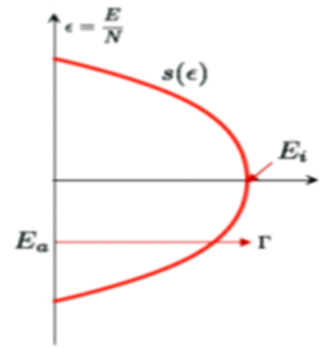


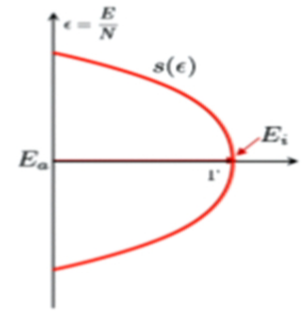
- On estimating the probability for the resonance ($N \to \infty$) provides a critical value of Γ.

$$\Gamma_c = \epsilon + \sqrt{2}\epsilon^2 + \frac{4}{3}\epsilon^3 + \ldots$$

Laumann, Pal and Scardicchio, arXiv:1404.2276

Delocalization at infinite temperature

- For an infinite temperature state in the middle of the band (ε=0) resonances are much more likely.



- Consider paths going up to n steps and estimate the delocalization of the greediest path amongst all possible end points.

Laumann, Pal and Scardicchio, arXiv:1404.2276

Open questions

- Analytically tractable mean-field theory of the transition at finite energy density.
- Signatures of MBL in amorphous materials showing dynamical freezing.