Title: Localization and topology protected quantum coherence at the edge of 'hot' matter

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Abstract: Topological phases are often characterized by special edge states confined near the boundaries by an energy gap in the bulk. On raising temperature, these edge states are lost in a clean system due to mobile thermal excitations. Recently however, it has been established that disorder can localize an isolated many body system, potentially allowing for a sharply defined topological phase even in a highly excited state.I will show this to be the case for the topological phase of a one dimensional magnet with quenched disorder, which features spin one-half excitations at the edges. The time evolution of a simple, highly excited, initial state is used to reveal quantum coherent edge spins. In particular, I will demonstrate, using theoretical arguments and numerical simulation, the coherent revival of an edge spin over a time scale that grows exponentially bigger with system size. This is in sharp contrast to the general expectation that quantum bits strongly coupled to a 'hot' many body system will rapidly lose coherence.

Localization and topology protected quantum coherence at the edge of 'hot' matter

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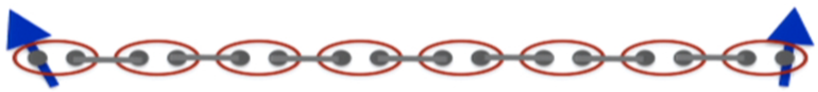
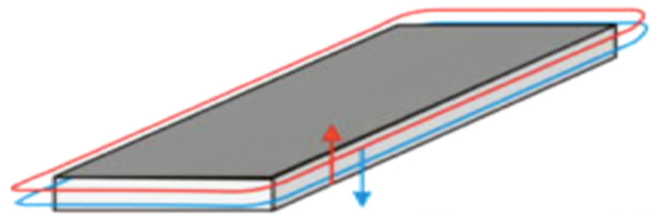
an topological edge states survive at high energies

Integer spin chain



Topological insulators



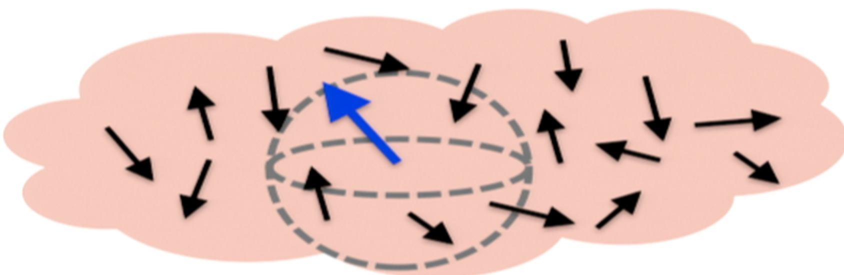
Kane and Mele 05,
Bernevig et. al. 06

Normally edge states are well defined only at T=0 !

At T>0 or finite energy density there is a finite density of
bulk excitations with which edge states can mix and decay.

Unless all excitations are localized!
What is the fate and dynamics of edge states in MBL states?

Can a local quantum degree of freedom retain its coherence (i.e. serve as a q-bit) when it is embedded in a time evolving many-body system?



$$e^{-iHt} \,|\, \Psi_0 \,\rangle$$

Naïve answer: No, because of thermalization!

Once the time evolution thermalized the q-bit with its surroundings all quantum coherence would be lost from the q-bit.

This talk: Show that it is possible!

Possible to retain local-coherence in a generic many-body system, at arbitrary energy density above its ground state.

Outline

- ETH and its breaking in localized systems
- Is the localized state necessarily trivial?
  Distinct loclaized states. Dynamical order
- Topological MBL states.
  - Model with topological spin-1/2 edge states
  - Dynamics of the edge state.

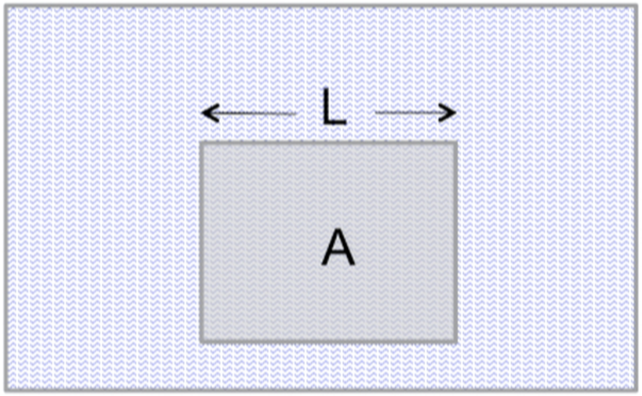
Eigenstate thermalization hypothesis (ETH)

Deutsch 91, Srednicki 94

Generic eigenstates of a thermalizing system “appear” thermal

Reduced density-matrix of
a big subsystem A:

$$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$$

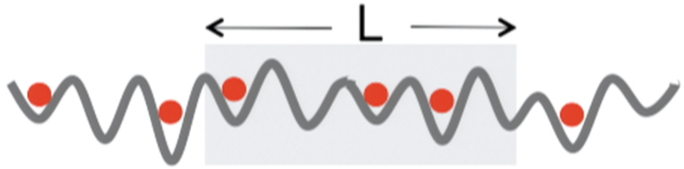


In particular this implies
extensive Von-Neuman entropy:

$$S_A \propto L^d$$

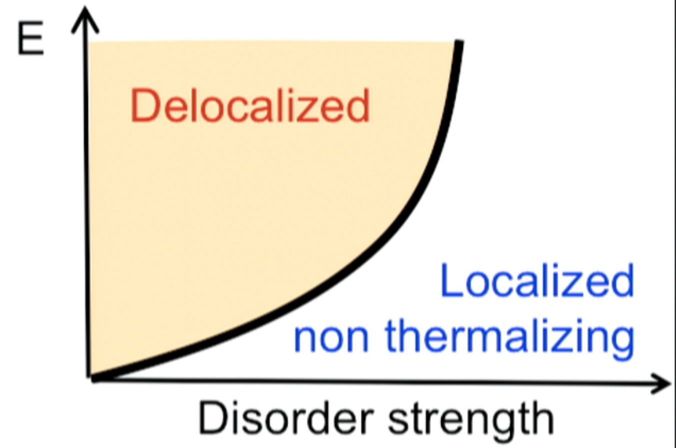
Exception to ETH: many-body localization

Anderson localized eigenstates of non interacting particles:



$$S_A \propto L^{d-1}$$

Such states can be stable to interactions ➡ MBL



Conjectured by Anderson 1958;
Gornyi, Polyakov and Mirlin (2005);
Basko, Aleiner, Altshuler (2006);
Oganesyan & Huse 2007;
Pal & Huse 2010, …

Define MBL as a state at finite energy density with area law entanglement. Bauer&Nayak 2013, …

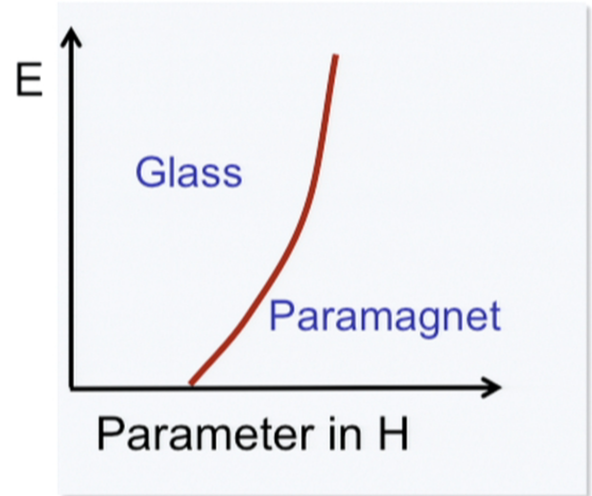
High energy states with ground state like properties!

Localization protected quantum order

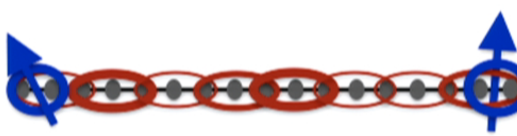
Huse et. al. (2013)

Ground state like entanglement properties enable quantum "phases" in high energy eigenstates.

Distinct localized "phases" separated by **quantum** critical points with universal dynamics. (See Ronen's talk)



Similarly, MBL can even support topological eigenstates!



Impossible to prepare energy eigenstates!
What are the dynamical signatures?

Example: dynamics in a disordered Ising model

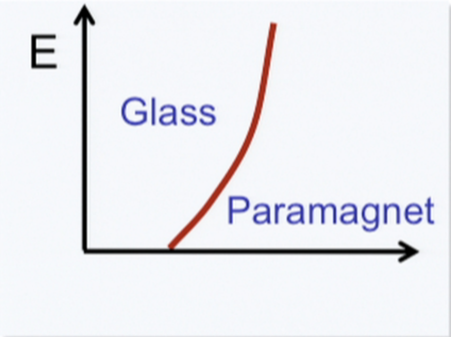
$$H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right]$$

$$|\Psi_0\rangle = \downarrow\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\downarrow\downarrow\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow$$

Evolve with H and measure local spin

In the **glass** phase: $\langle \sigma_i^z(t) \rangle \longrightarrow m_i \neq 0$

Comes out naturally from RG
(Vosk & EA arXiv:1307.3256; see Ronen's talk)



- σ_i^z's are emergent conserved quantities
- Have non vanishing overlaps with true conserved quantities.

Quasi-local integrals of motion

$$H_0 = \sum_i h_i \sigma_i^x$$

$$H_0 = \sum_i J_i^z \sigma_i^z \sigma_{i+1}^z \longrightarrow H = \sum_i \left[J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \dots \right]$$

Strictly-local conserved quantities transform smoothly to quasi-local ones upon adding weak generic interactions

Oganesyan and Huse (2013), Serbyn, Papic & Abanin (2013)

Integrals of motion in Paramagnet:

$$H_0 = \sum_i h_i \sigma_i^x \qquad \sigma_i^x \longrightarrow \tilde{\sigma}_i^x = Z \sigma_i^x + \text{exponential tail}$$

Integrals of motion in Glass:

$$H_0 = \sum_i J_i^z \sigma_i^z \sigma_{i+1}^z \qquad \sigma_i^z \longrightarrow \tilde{\sigma}_i^z = Z \sigma_i^z + \text{exponential tail}$$

Important: in this example the integrals of motion are classical ! σ_i^z and σ_i^x are not conserved at the same time

Classical versus quantum integrals of motion

Integrals of motion in the last example are classical:

σ_i^z and σ_i^x are not conserved at the same time.

Spins in the random Ising model can serve as protected classical bits but not as q-bits

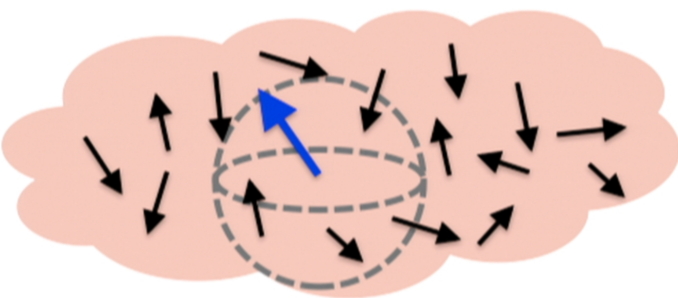
We need:

local conserved quantities, which can serve as quantum bits!

Localization and topology protected coherence at the edge of "hot" matter

Y. Bahri, R. Vosk, E.A. and A. Vishwanath arXiv:1307.4092

Can a local quantum degree of freedom retain its coherence when it is embedded in a time evolving many-body system?



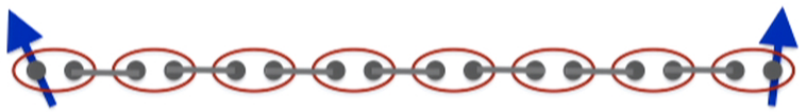
$$e^{-iHt} \,|\, \Psi_0 \,\rangle$$

Digression: symmetry protected topological states (usual ground state picture)

Example: spin-1 chains:

$$H = \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \lambda \left(\mathbf{S}_i \cdot \mathbf{S}_{i+1}\right)^2$$

Haldane 83, AKLT 87



Spin-1/2 edge states

- Bulk is gapped and invariant under the symmetry.
- Ground state degeneracy due to edges.
- Edge states transform non trivially under symmetry (projective rep.)

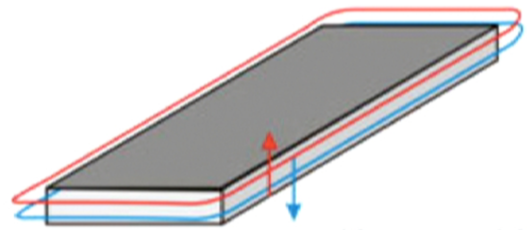
Pollmann, Turner, Berg, Oshikawa (2010); Chen, Gu, Wen (2011)

e.g. with spin symmetry:
microscopic constituents transform as spin-1 while edge states transform as spin-1/2 (proj. rep. of SO(3))

Digression: symmetry protected topological states (usual ground state picture)

Integer spin chain

Topological insulators



Kane and Mele 05, Bernevig et. al. 06

Normally edge states are well defined only at T=0 !

At T>0 or finite energy density there is a finite density of bulk excitations with which edge states can mix and decay.

This can be avoided if all excitations are localized!
Can a localized state have topological character?

Localized topological model with spin-1/2 edge states

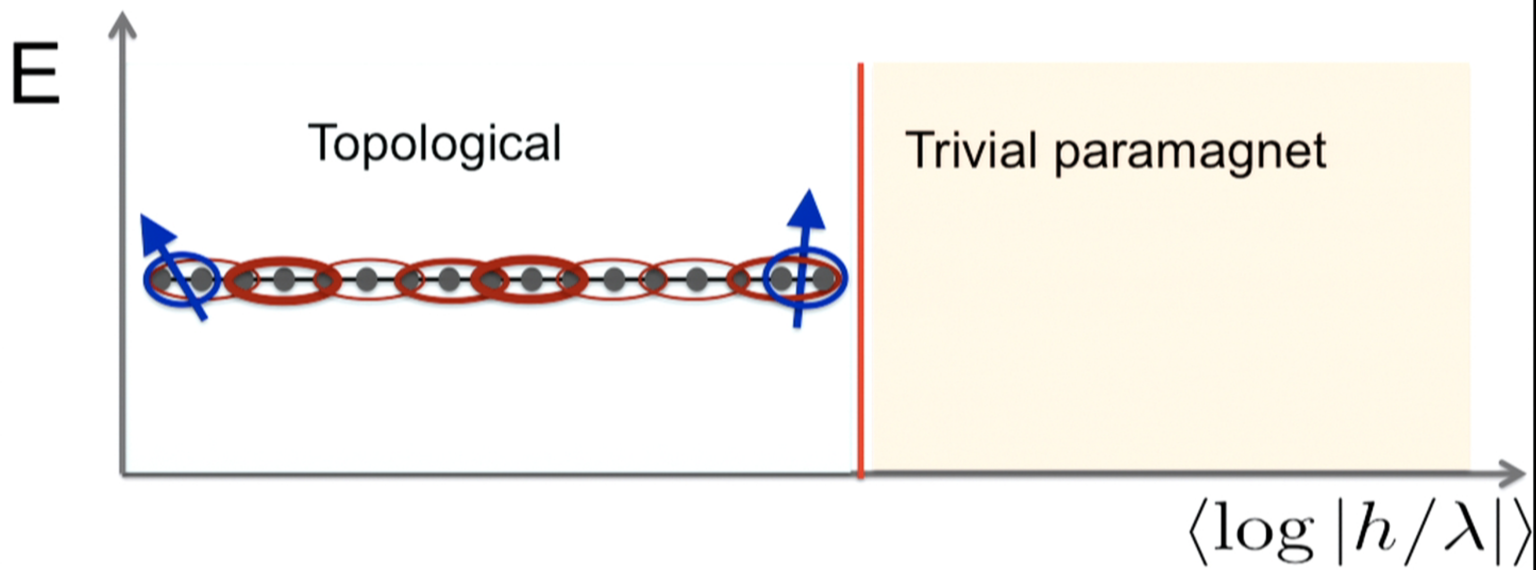
$$H = \sum_i \left[\lambda_i \sigma^z_{i-1} \sigma^x_i \sigma^z_{i+1} + h_i \sigma^x_i + V_i \sigma^x_i \sigma^x_{i+1} \right]$$

V<<h,λ needed to make the model generic.
If V=0 it can be mapped to free fermions

$Z_2 \times Z_2$ Sym.

$\sigma^z_{odd} \to -\sigma^z_{odd}$

$\sigma^z_{even} \to -\sigma^z_{even}$



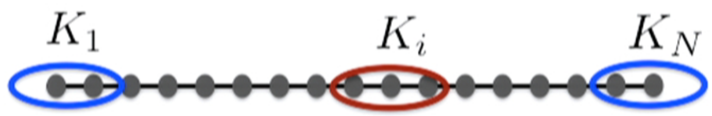
Topological state: Idealized "cluster" model

$$H_{\text{ideal}} = \sum_{i=2}^{N-1} \lambda_i \sigma_{i-1}^z \sigma_i^x \sigma_{i+1}^z \equiv \sum_{i=2}^{N-1} \lambda_i K_i \qquad [K_i, K_j] = 0$$

Spectrum consists of all "Cluster states".

Labeled by K_i eigenvalues $\kappa_i = \pm 1$

$$|\psi_\alpha\rangle = |\kappa_1^\alpha, \ldots, \kappa_N^\alpha\rangle$$



$$E_\alpha = \sum_{i=2}^{N-1} \lambda_i \kappa_i^\alpha$$

4 fold degenerate spectrum for open BC !

Edge: Local edge operators transform between the degenerate states. Form spin-1/2 algebra. (projective rep. of Z_2 x Z_2)

$$\Sigma_L^{x,y} = \sigma_1^{x,y} \sigma_2^z$$

$$\Sigma_L^z = \sigma_1^z$$

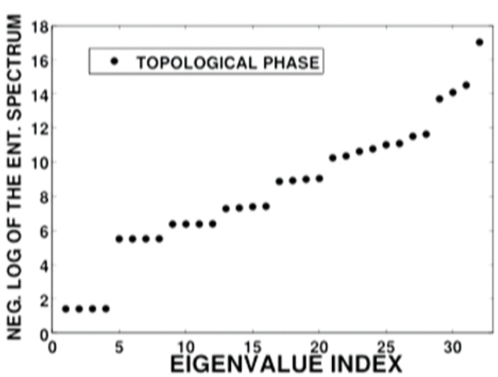
Eigenstate Order Parameters (Also in fully interacting model)

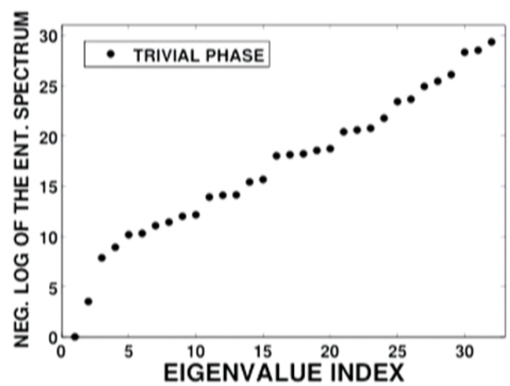
1. String correlations: $O_{\rm st}(i,j) = \langle \sigma_i^z \sigma_{i+1}^y \left(\prod_{k=i+2}^{j-2} \sigma_k^x\right) \sigma_{j-1}^y \sigma_j^z \rangle$

In each disorder realization: $O_{\rm st}(i,j) = \pm 1.$

Glass string order: $\Psi_{sg} = \overline{O_{\rm st}^2}$

2. Doubly degenerate entanglement spectrum





Quasi local integrals of motion

Assuming many-body localization, local integrals of motion transform smoothly to quasi-local ones

$$K_i \to \tilde{K}_i = \tilde{Z} K_i + \dots$$

$$\Sigma_L^\alpha \to \tilde{\Sigma}_L^\alpha = \tilde{Z} \Sigma_L^\alpha + \dots$$

(Exponential tail)

$$\text{edge x bulk} \longrightarrow \widetilde{\text{edge}} \text{ x } \widetilde{\text{bulk}}$$

$Z_2 \times Z_2$ symmetry is still realized projectively on the new edge

$\tilde{\Sigma}_L^\alpha$ is a true quantum integral of motion.
All three spin components are conserved

Persistence of the edge spin in dynamics

Practically we can observe only the strictly local operators Σ_L^α

Can we see the conservation of the quasi-local edge spin $\tilde{\Sigma}_L^\alpha$?

Use: $$\Sigma_L^\alpha = Z_\alpha \tilde{\Sigma}_L^\alpha + \sum_n c_n \tilde{\Sigma}_L^\alpha \tilde{B}_n$$

Part of each component of the edge spin is protected from decay.

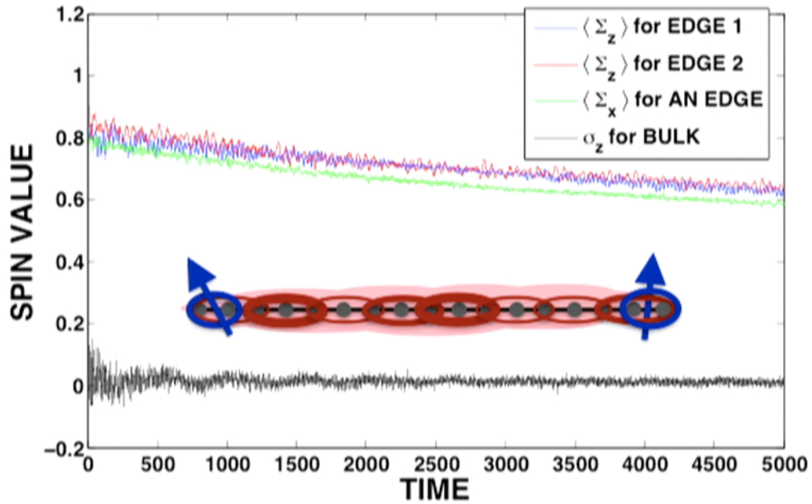
Persistence of the edge spin: exact diagonalization (6 to 11 spins)

Prepare simple initial states (classical, high E):

$|\Psi_0\rangle = \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$ or $|\Psi_0\rangle = \rightarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$

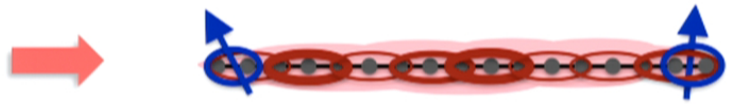
$\langle\Sigma_L^z\rangle_0 = 1$ $\langle\Sigma_L^x\rangle_0 = 1$

$$e^{-iHt} \mid \Psi_0 \rangle$$

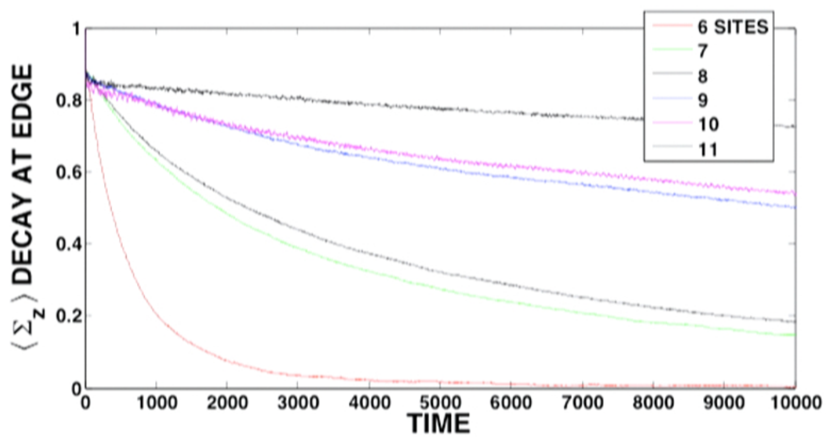


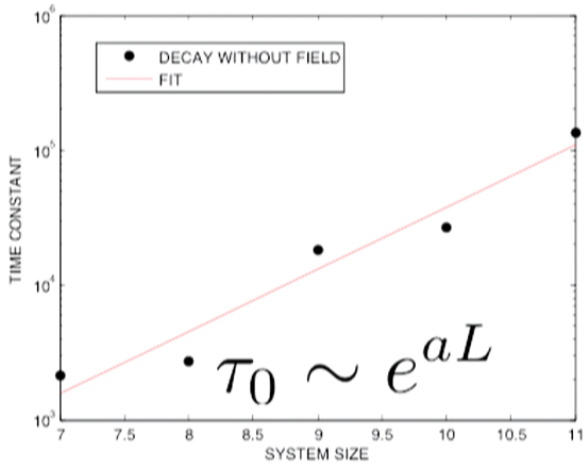
Decay of the edge spin is due to finite size

$|\Psi_0\rangle = \nwarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow$



Interactions mediate coupling between edges: $H_{\text{edge}} = J_m^{\alpha\beta}\tilde{\Sigma}_L^{\alpha}\tilde{B}_m\tilde{\Sigma}_R^{\beta}$





Can the edge spin be manipulated coherently?

External field coupled to edge locally.
Breaks protecting symmetry!

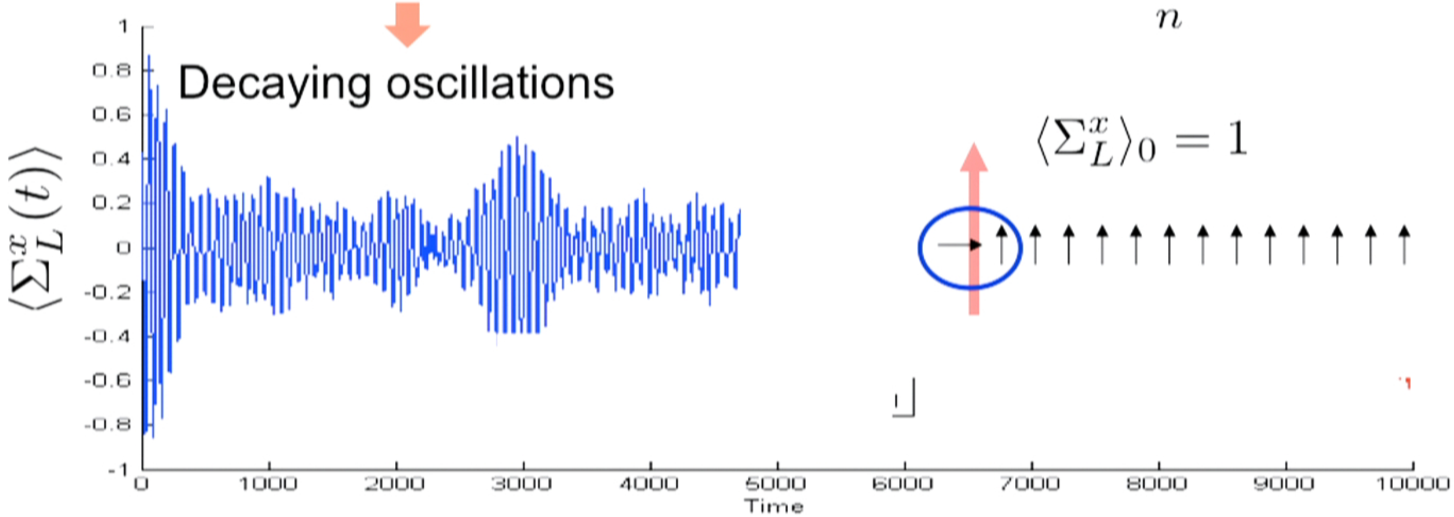
$$H_{\text{edge}} = g\Sigma_L^z$$

Leads to edge bulk coupling
in presence of interactions

$$\approx gZ\tilde{\Sigma}_L^z + g\sum_n c_n \tilde{\Sigma}_L^z \tilde{B}_n$$

Decaying oscillations

$$\langle \Sigma_L^x \rangle_0 = 1$$



Can we retrieve the coherence in a spin echo ?
(applying strictly local operations)

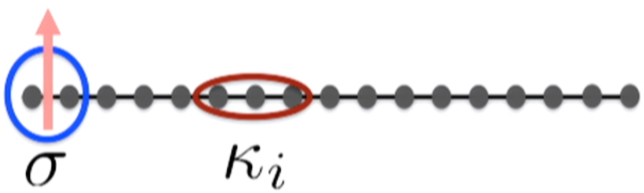
Edge spin echo protocol

Evolve to time t_R with the Hamiltonian: $H(g) = H_0 + g\Sigma_L^z$

Reverse edge field at time t_R and evolve to time $2t_R$ with $H(-g)$

Derivation of the edge echo: preliminaries

Basis of the quasi-local integrals of motion in presence of the field g:



$$|\{\tilde{\sigma}, \tilde{\kappa}_2, \tilde{\kappa}_3, \ldots\}; g\rangle \equiv |\tilde{\sigma}, \tilde{\alpha}; g\rangle$$

<u>Simplification (for presentation):</u>
as initial state take an eigenstate of $\tilde{\Sigma}_L^x$ in presence of g (instead of Σ_L^x)

$$|\psi_0\rangle = \sum_{\sigma,\alpha} c_\alpha |\tilde{\sigma}, \tilde{\alpha}; g\rangle$$

Evolve to the reversal time:

$$|\psi(t_R)\rangle = \sum_{\sigma,\alpha} c_\alpha e^{-iE(\tilde{\sigma},\tilde{\alpha},g)t_R} |\tilde{\sigma}, \tilde{\alpha}; g\rangle$$

Field reversal g to –g (local quench)

Evolution after quench at t_R is simple in the basis: $|\tilde{\sigma}, \tilde{\alpha}; -g\rangle$

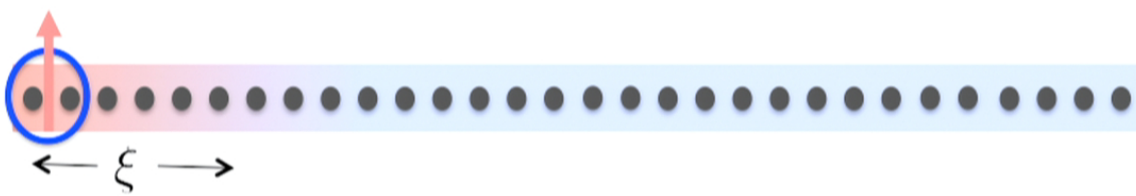
Basis change:

$$|\tilde{\sigma}, \tilde{\alpha}; g\rangle = Z_{\sigma\alpha}|\tilde{\sigma}, \tilde{\alpha}; -g\rangle + \ldots$$

$$Z_{\sigma\alpha} = \langle\tilde{\sigma}, \tilde{\alpha}; -g|\tilde{\sigma}, \tilde{\alpha}; g\rangle \approx \prod_i \left(1 - F_\alpha(V,h) g^2 e^{-x_i/\xi_i}\right) e^{i\phi_{\sigma\alpha}} \approx e^{-F_\alpha(V,h) g^2 \xi/a} e^{i\phi_{\sigma\alpha}}$$

Only d.o.f within ξ of the edge are affected by changing the edge field

Incomplete orthogonality catastrophe due to localization!



Note: by symmetry

$$Z_{\sigma,\alpha}(g) = Z_{-\sigma,\alpha}(-g) = [Z_{-\sigma,\alpha}(g)]^*$$

Final evolution (t_R to $2t_R$) in field -g

$$|\psi(t_R)\rangle = \sum_{\sigma,\alpha} c_\alpha e^{-iE(\tilde{\sigma},\tilde{\alpha},g)t_R} |\tilde{\sigma},\tilde{\alpha};g\rangle$$

$$\downarrow$$

$$|\psi(2t_R)\rangle \approx \sum_{\sigma,\alpha} c_\alpha Z_{\sigma\alpha} e^{-i[E(\tilde{\sigma},\tilde{\alpha},g)+E(\tilde{\sigma},\tilde{\alpha},-g)]t_R} |\tilde{\sigma},\tilde{\alpha};-g\rangle + \dots$$

$$\approx \sum_{\sigma\alpha} c_\alpha Z_{\sigma\alpha} e^{-iE_+(\tilde{\alpha},g^2)t_R} |\tilde{\sigma},\tilde{\alpha};-g\rangle$$

This is because by symmetry:

$$E(\tilde{\sigma},\tilde{\alpha},g) = E^{(0)}(\tilde{\alpha}) + E^{(1)}(\tilde{\alpha})\sigma g + E^{(2)}(\tilde{\alpha})g^2 + \dots = E_+(\tilde{\alpha},g^2) + \sigma g E_-(\tilde{\alpha},g^2)$$

Convert back to the original +g basis:

$$|\psi(2t_R)\rangle \approx \sum_{\sigma\alpha} c_\alpha |Z_{\sigma\alpha}|^2 e^{-iE_+(\tilde{\alpha},g^2)t_R} |\tilde{\sigma},\tilde{\alpha};g\rangle$$

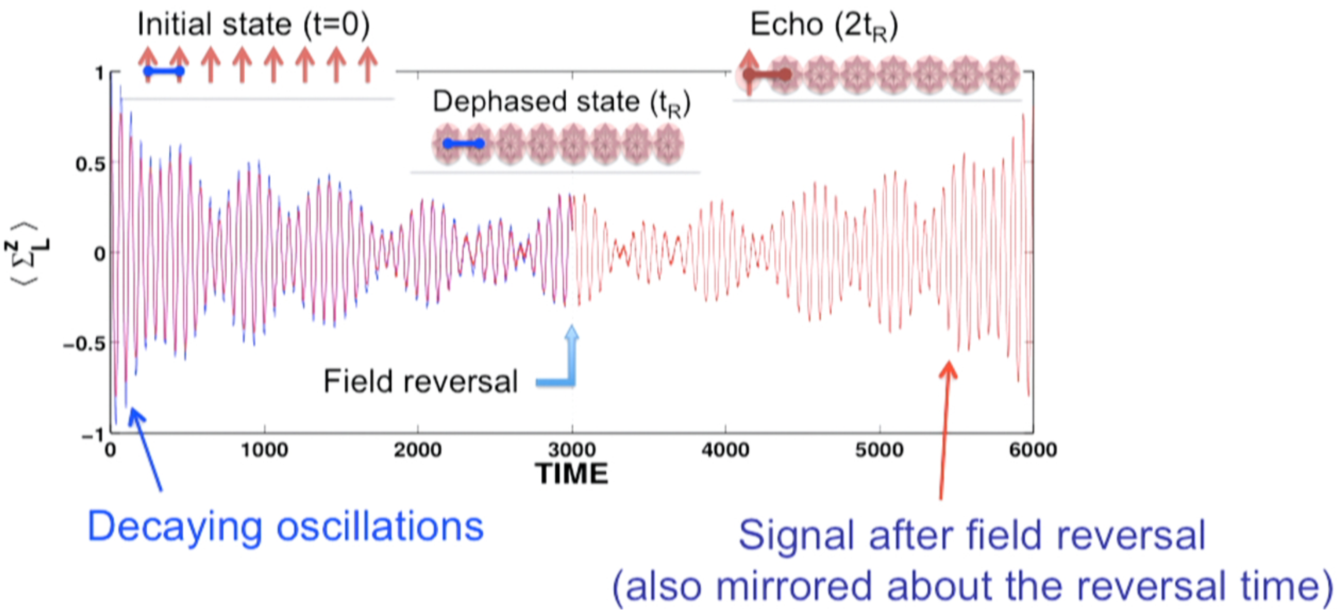
$$\Rightarrow \langle \Sigma_L^x(2t_R)\rangle \approx \sum_{\alpha} |c_\alpha|^2 |Z_\alpha|^2 \sim e^{-Ag^2\xi}$$

Observe finite long time echo as long as we are in the topological phase.

Edge spin echo: numerical result

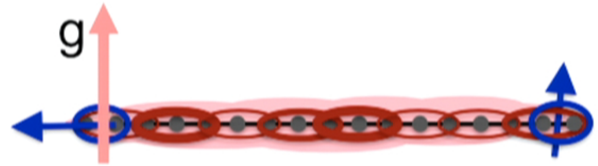
$$H = H_{\rm bulk} + gZ\tilde{\Sigma}^z_L + g\sum_n c_n \tilde{\Sigma}^z_L \tilde{B}_n$$

Reversing the field should reverse the oscillations

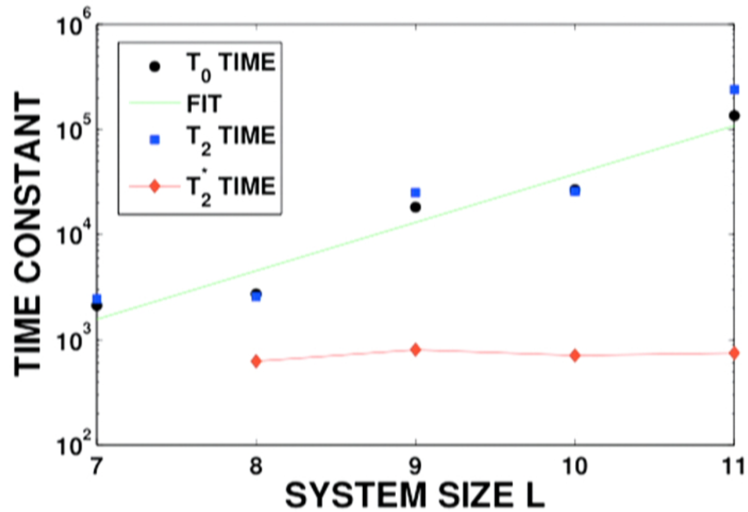


Summary of coherence times

$$H_{\text{edge}} \approx gZ\tilde{\Sigma}_L^z + g\sum_n c_n \tilde{\Sigma}_L^z \tilde{B}_n$$

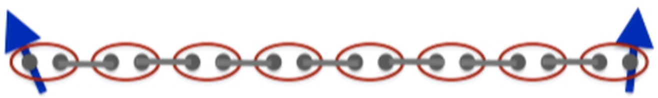


Degradation of the echo (T_2) is exponentially long in system size L and matches exactly with the spin relaxation at zero field!

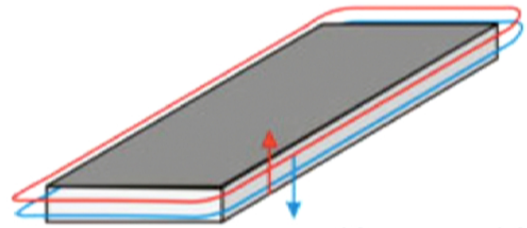


Can topological edge states survive at high energies

Integer spin chain



Topological insulators



Kane and Mele 05, Bernevig et. al. 06

Normally edge states are well defined only at T=0 !

At T>0 or finite energy density there is a finite density of bulk excitations with which edge states can mix and decay.

Summary

- **Many-body localized states can be topological**
  - Topology and localization combine to protect quantum edge states at high energy.

- Simpler realizations with only 2-spin interactions?
- High energy edge-bulk decoupling in 2d / 3d ?