Title: Long-time behavior of periodically driven isolated interacting quantum systems

Date: May 13, 2014 11:25 AM

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Abstract: We show that generic interacting quantum systems, which are isolated and finite, periodically driven by sudden quenches exhibit three physical regimes. For short driving periods the Floquet Hamiltonian is well approximated by the time-averaged Hamiltonian, while for long periods the evolution operator exhibits properties of random matrices of a Circular Ensemble (CE). In-between, there is a crossover
regime. We argue that, in the thermodynamic limit and for nonvanishing driving periods, the evolution operator always exhibits properties of CE random matrices. Consequently, driving leads to infinite temperature at infinite time and to an unphysical Floquet Hamiltonian.

Long-time behavior of periodically driven isolated interacting quantum system

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Anatoli Polkovnikov (BU)

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LD, M. Rigol ArXiv:1402.5141
M. Bukov, LD, A. Polkovnikov (in preparation)
LD, M. Rigol (in preparation)

Perimeter Institute, Waterloo

May 13th 2014

Why periodically drive a system?

equilibrium

periodic drive

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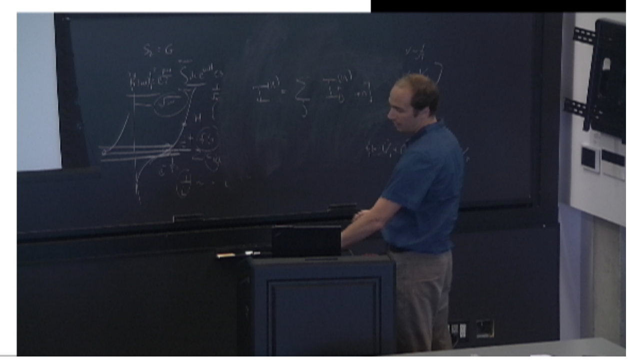
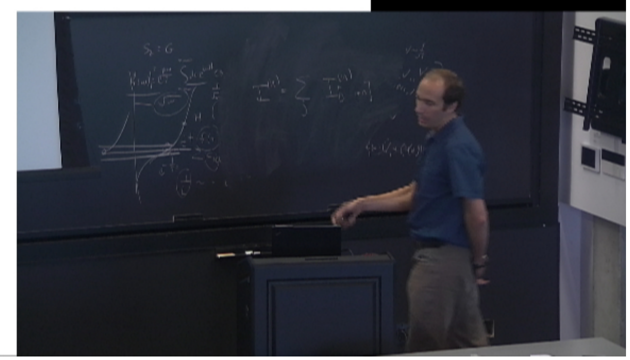
equilibrium   quench   periodic drive



Why periodically drive a system?

→ "fun"

equilibrium quench periodic drive

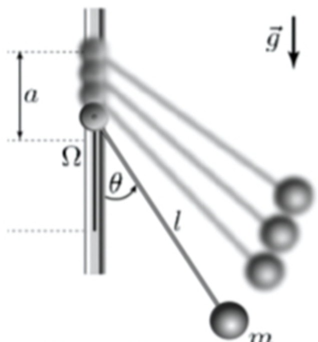


Why periodically drive a system?



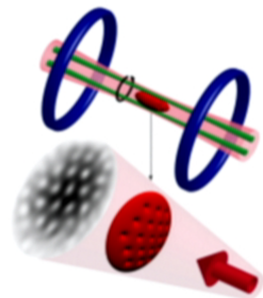
"fun"

equilibrium | quench | periodic drive



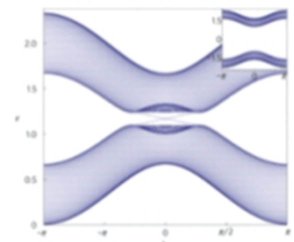
Kapitza pendulum

P.L. Kapitza. Soviet Phys. JETP **21** (1951) 588
L.D. Landau and E. M. Lifshitz. Mechanics (1976)



Vortex Lattices in BEC

J. R. Abo-Shaeer, et al. Science **292**, 476 (2001)
....
A. L. Fetter, Rev. Mod. Phys. **81**, 647 (2009)



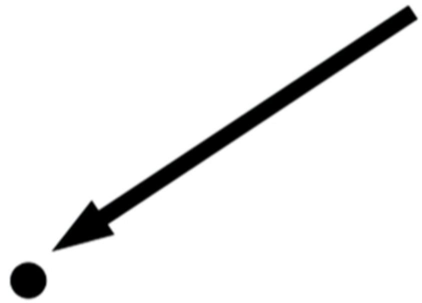
Floquet Topological States

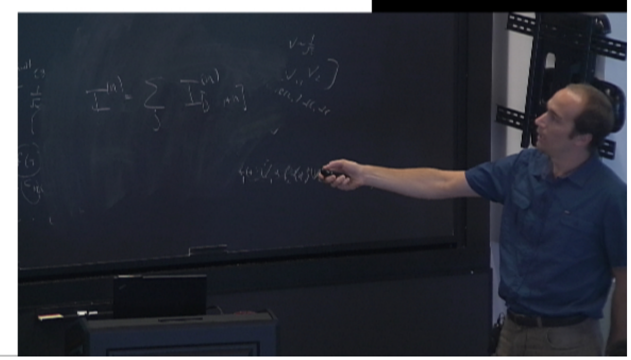
T. Oka and H. Aoki, PRB **79** (2009) 081406(R)
T. Kitagawa et al. , PRB **84** (2011) 235108
N.H. Lindner et al. Nat. Phys. **7** (2011) 490

Why periodically drive a system?

Time-independent, H=const

$$U(t) = e^{-iHt}$$





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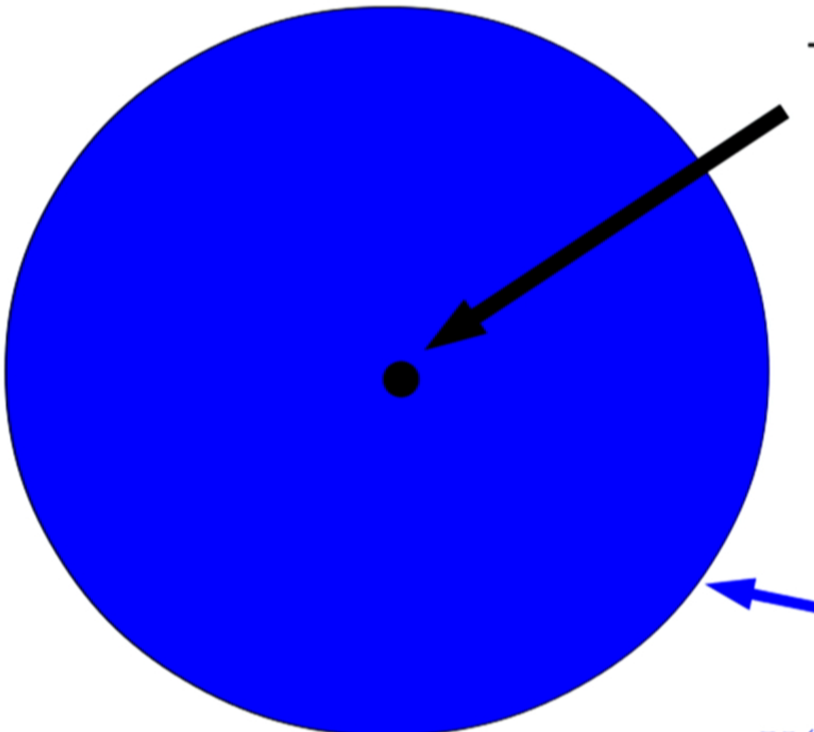

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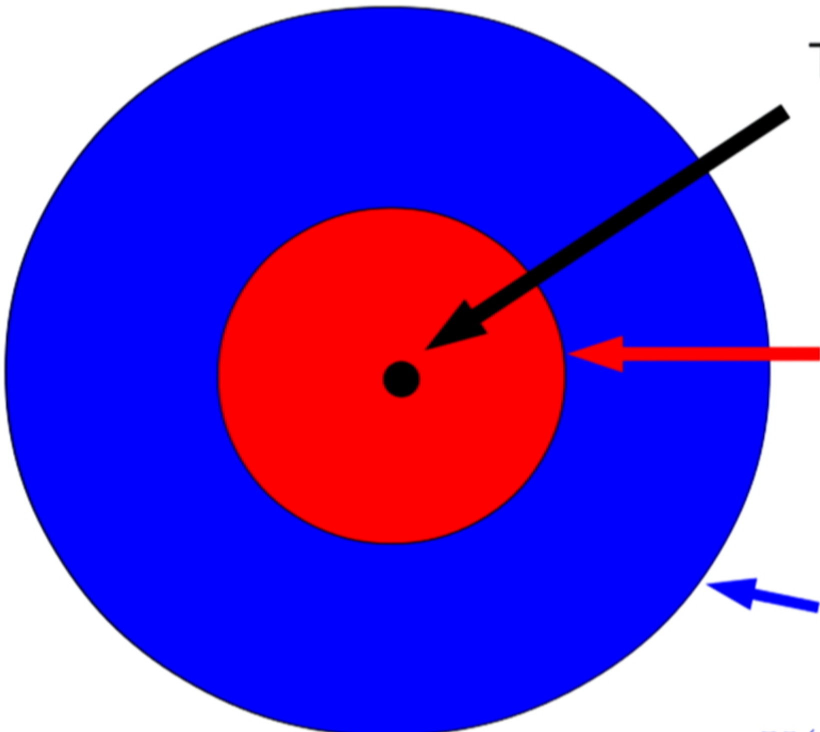
$$U(t) = e^{-iHt}$$

Arbitrary time dependence

$$U(t) = T \exp\left[-i \int_0^t d\tau H(\tau)\right]$$

Why periodically drive a system?



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$$U(t) = e^{-iHt}$$

Time-periodic, H(t)=H(t+T)

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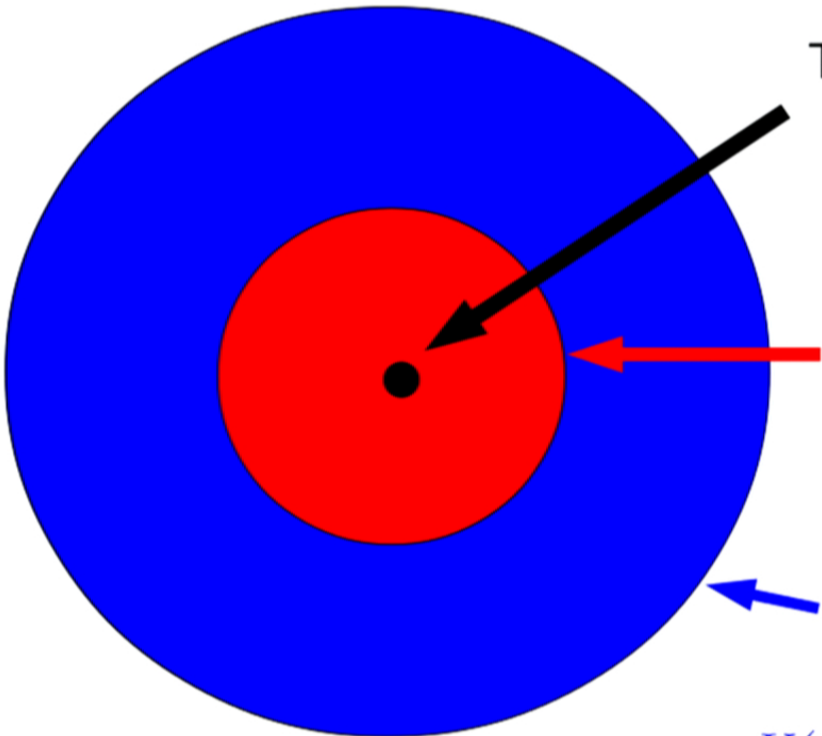

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Warning:
factorization is not unique!

Arbitrary time dependence

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"drawing not to scale"

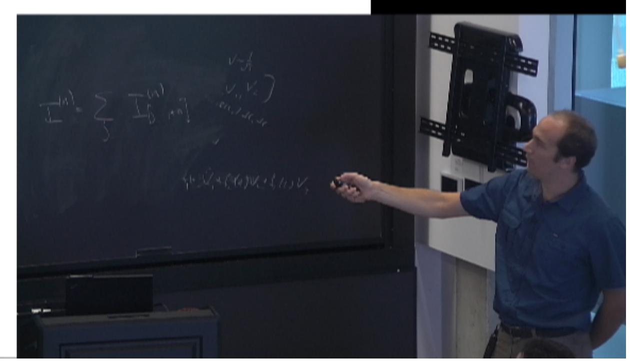
Outline

1. Understanding the Floquet theorem
2. Long-time behavior of periodically driven isolated interacting quantum systems
3. Conclusions and outlook

Floquet Theorem: $U(t) = P(t)e^{-iH_F t}$

Periodic envelope + Floquet Hamiltonian

1. Since $P(T) = 1$, then $U(nT) = e^{-iH_F nT}$
   as in t-independent Hamiltonians



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   Factorization is valid at any time

3. Envelope $P(t)$ is the novelty w.r.t. usual evolution
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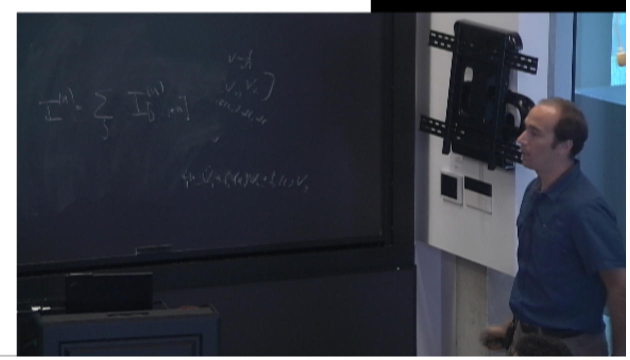

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For example, if $H_F = 0$ all (topological) properties are encoded into the envelope $P(t)$ (see Victor Galitski)

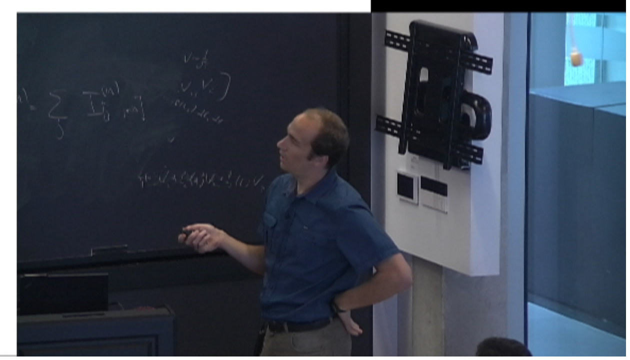
Question the paradigm:
(topological) properties of H_F ⟺ (topological) properties of the system

Floquet Theorem: $U(t) = P(t)e^{-iH_F t}$

Plug Floquet ansatz into SE: $i\partial_t U(t) = H(t)U(t)$

Do some algebra:

$$i\left(\partial_t P(t)\, e^{-iH_F t} - iP(t)\, H_F\, e^{-iH_F t}\right) = H(t)\, P(t)\, e^{-iH_F t}$$



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And rearrange the terms to obtain:

$$H_F = P^\dagger(t)\, H(t)\, P(t) - iP^\dagger(t)\, (\partial_t P(t))$$

This equation is always exact and give connection between $H(t)$ and H_F.

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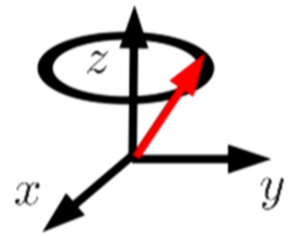
Example: single spin in rotating B-field



In the lab reference frame:

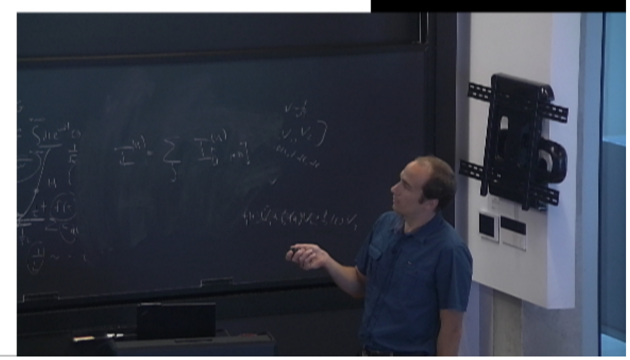
$$H(t) = B_z \sigma_z + B_{\|} \left(\cos(\Omega t) \sigma_x + \sin(\Omega t) \sigma_y \right)$$

In the rotating reference frame:

$$\begin{aligned} \hat{H}^{\text{rot}}_{\phi} &= \hat{R}^{\dagger}_{\phi}(t) \hat{H}(t) \hat{R}_{\phi}(t) - i \hat{R}^{\dagger}_{\phi}(t) \partial_t \hat{R}_{\phi}(t) \\ &= B_z \sigma_z + B_{\|} (\cos\phi \, \sigma_x - \sin\phi \, \sigma_y) - \frac{\Omega}{2} \sigma_z \end{aligned}$$

where R(t) transforms into the rotating frame:

$$\hat{R}_{\phi}(t) = \exp\left[-i \frac{\sigma_z}{2} (\Omega t + \phi) \right]$$



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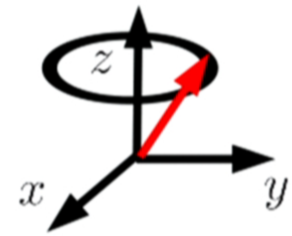

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The evolution in lab frame is:

$$\hat{U}(t,0) = \hat{R}_\phi(t)\; e^{-i\hat{H}_\phi^{\text{rot}}\,t}\; \hat{R}_\phi^\dagger(0)$$

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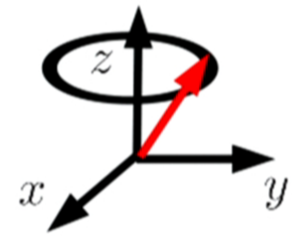

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$$\hat{U}(t,0) = \hat{R}_{\phi}(t)\ e^{-i\hat{H}^{\text{rot}}_{\phi}\,t}\ \hat{R}^{\dagger}_{\phi}(0) \rightarrow \boxed{U(t) = \hat{R}_0(t)\ e^{-i\hat{H}^{\text{rot}}_0\,t}}$$

Choosing $\phi = 0$ we can identify: $\hat{H}_F \equiv \hat{H}^{\text{rot}}_0, \quad \hat{P}(t) \equiv \hat{R}_0(t)$

It's all about the right reference frame!

1. If you choose the right reference frame the evolution becomes trivial (Floquet theorem says it is always possible):

$$H_F = P^\dagger(t)\, H(t)\, P(t) - iP^\dagger(t)\, (\partial_t P(t))$$

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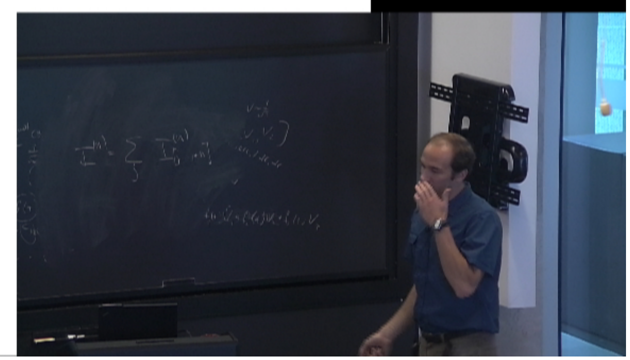

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3. $P(t)$ is analogue to the generating function of a canonical transformation in classical mechanics: $(q, p, H) \to (Q, P, K)$
For example if, $G \equiv G_1(q, Q, t)$ then $p = \frac{\partial G_1}{\partial q},\ P = -\frac{\partial G_1}{\partial Q},\ K = H + \frac{\partial G_1}{\partial t}$

It's all about the right generating function!

Finding the right generating function is non-trivial.
1) S. Fishman et al. PRL **49**, 509 (1982).
2) T. P. Grozdanov and M. J. Rakovic, PRA **38**, 1739 (1988)
3) M. M. Maricq, PRB **25**, 6622 (1982)

These methods are iterative and based on simultaneous expansion of $P(t)$ and H_F.

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For example [3]: $P(t) = \sum_{n=0} P^{(n)}(t),\ H_F = \sum_{n=0} H_F^{(n)},\ H_F^{(0)} = 0,\ P^{(0)} = 1$

Use Fourier transforms: $H(t) \equiv \sum_\alpha H_\alpha e^{i\alpha\Omega t},\ P^{(n)}(t) = \sum_\alpha P_\alpha^{(n)} e^{i\alpha\Omega t}$

The formal solution is: $H_F^{(n)} = \sum_\alpha H_{-\alpha} P_\alpha^{(n-1)} - \sum_{k=1}^{n-1} P_0^{(k)} H_F^{(n-k)}$

$$P^{(n)}(t) = \sum \sum_{\alpha+\beta\neq 0} \frac{1 - \exp[i(\alpha+\beta)\Omega t]}{(\alpha+\beta)\Omega} H_\alpha P_\beta^{(n-1)} - \sum_{k=1}^{n-1} \sum_{\beta\neq 0} \frac{1 - \exp(i\beta\Omega t)}{\beta\Omega} P_\beta^{(k)} H_F^{(n-k)}$$

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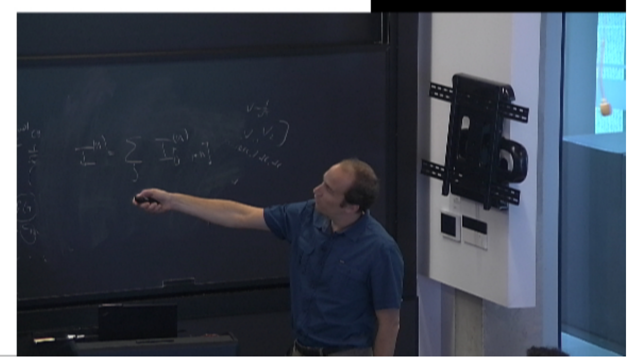
The first few terms are....

The I.C. are: $H_F^{(0)} = 0,\ P^{(0)} = 1$

First corrections:

$$\begin{cases} H_F^{(1)} = \sum_\alpha H_{-\alpha} P_\alpha^{(0)} = H_0 \equiv \frac{1}{T}\int_0^T d\tau\, H(\tau) \\ P^{(1)}(t) = \sum_{k\neq 0} \frac{1-\exp[ik\Omega t]}{k\Omega} H_k = -i\int_0^t \left(H(\tau) - H_F^0\right) d\tau \end{cases}$$



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Second corrections:

$$\begin{cases} H_F^{(2)} = \sum_{k=1}^{\infty} \frac{1}{k\Omega}\left([H_k, H_{-k}] + [H_0, H_k] - [H_0, H_{-k}]\right) \\ \quad = \frac{1}{2Ti}\int_0^T dt_1 \int_0^{t_1} dt_2\, [H(t_1), H(t_2)] \\ P^{(2)}(t) = \ldots\ldots \end{cases}$$

It quickly becomes cumbersome...

Is there a better and convergent expansion?

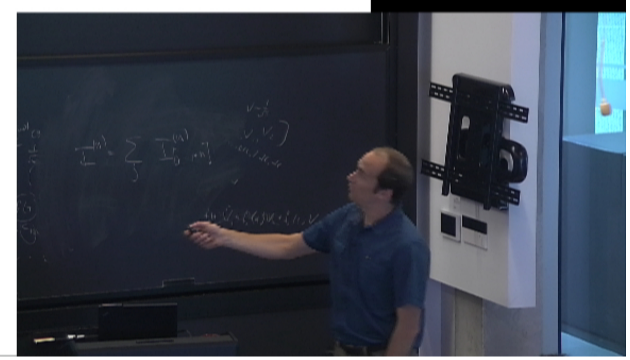
2. Long-time behavior of periodically driven isolated interacting quantum systems

LD, M. Rigol ArXiv:1402.5141

Long-time depends only on H_F

1. $P(t)$ is continuous and periodic $\rightarrow$ bounded

2. The stability/instability transition is solely determined by H_F



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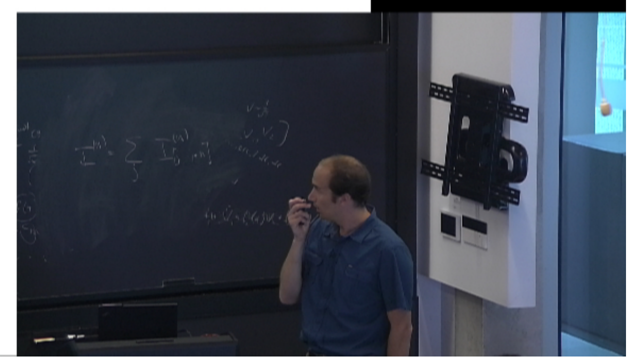
$$H(t) = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 + \frac{\alpha}{2}\left(\hat{x}\hat{p} + \hat{p}\hat{x}\right)\delta_T(t), \quad \alpha \in \mathbb{R}, \quad \delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



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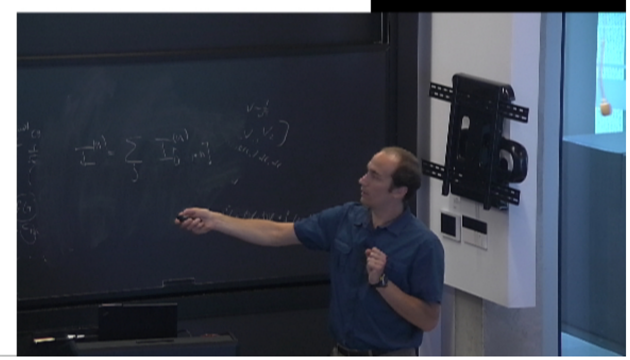
Classically the Floquet map is:

$$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} M \end{pmatrix} \begin{pmatrix} x \\ p \end{pmatrix}$$

$$\lambda_\pm = \cosh\alpha\cos(\omega T) \pm \sqrt{\cosh\alpha^2\cos(\omega t)^2 - 1}$$



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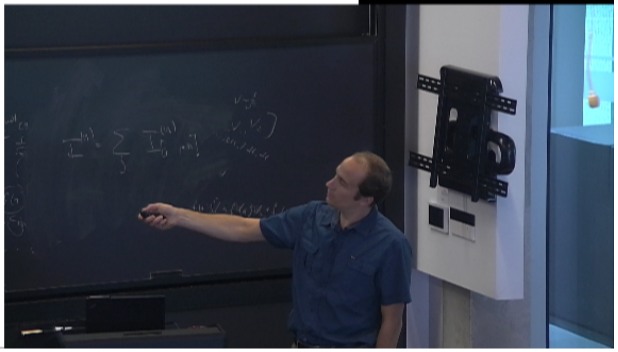

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	STABLE	MARGINAL	UNSTABLE
CL	$\lambda_\pm = \exp[\pm i\Omega]$	$\lambda_\pm = \pm 1$	$\lambda_\pm = \exp[\pm\mu]$
QM	$\Omega^2 > 0$,normalizable WF	$\Omega^2 = 0$,planes waves	$\Omega^2 < 0$,non-normalizable

Stability-to-Instability transition

1. H_F does **NOT** need to be "physical" (for example unbounded from below)

2. Signature of transition both in eigenvalues and eigenvectors of H_F

OUR GOAL:
Find signature of transition in $U(T)$ for finite, interacting quantum systems

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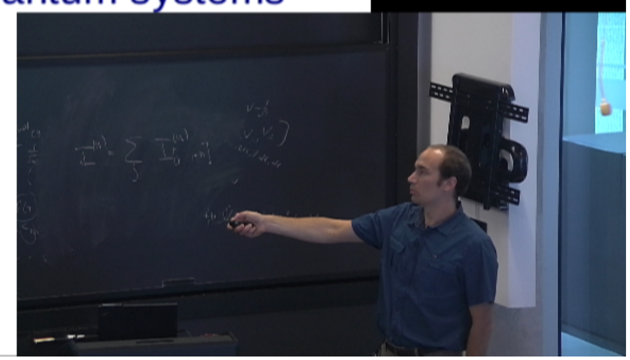
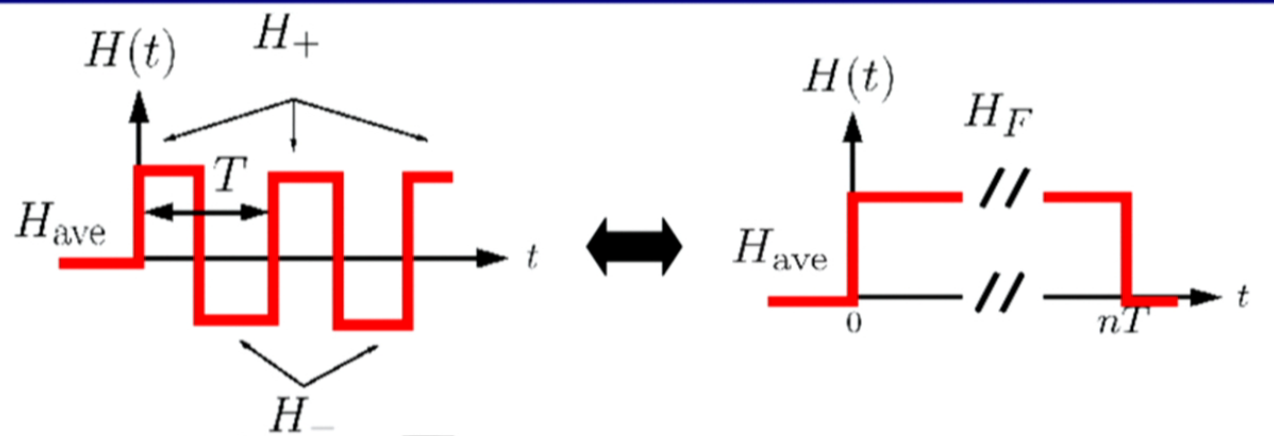


Model: interacting spin chain



$$\hat{H}_{\pm} = (J \pm \delta J)\hat{H}_{nn} + J' \sum_j \sigma_j^z \sigma_{j+2}^z$$

$$J = 1,\ \delta J = 0.2,\ J' = 0.8$$

$$\hat{H}_{nn} = \sum_j \left[\sigma_j^z \sigma_{j+1}^z - \frac{1}{2}\left(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y \right) \right]$$

$$H_+,\ H_-,\ H_{\rm ave} \in \mathrm{ETH}$$

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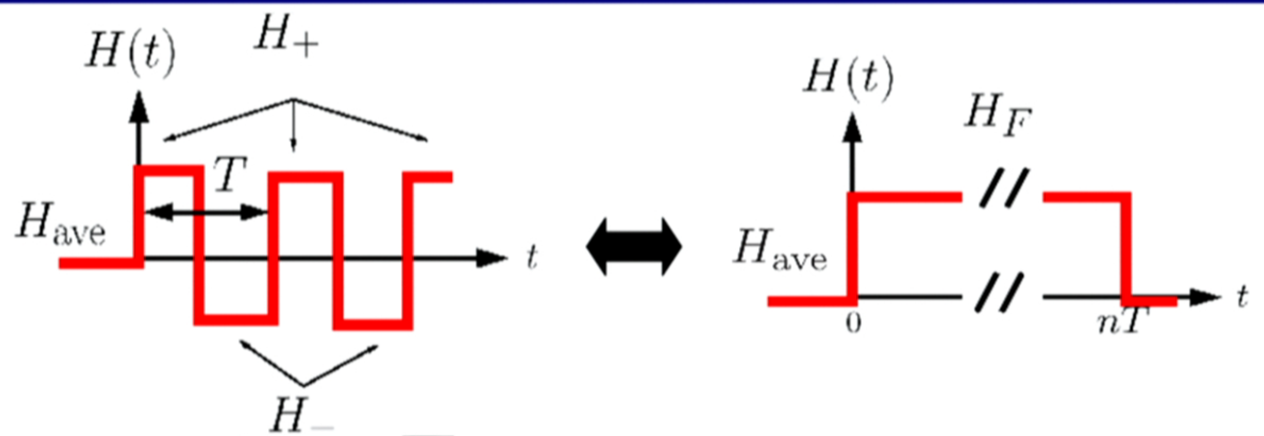

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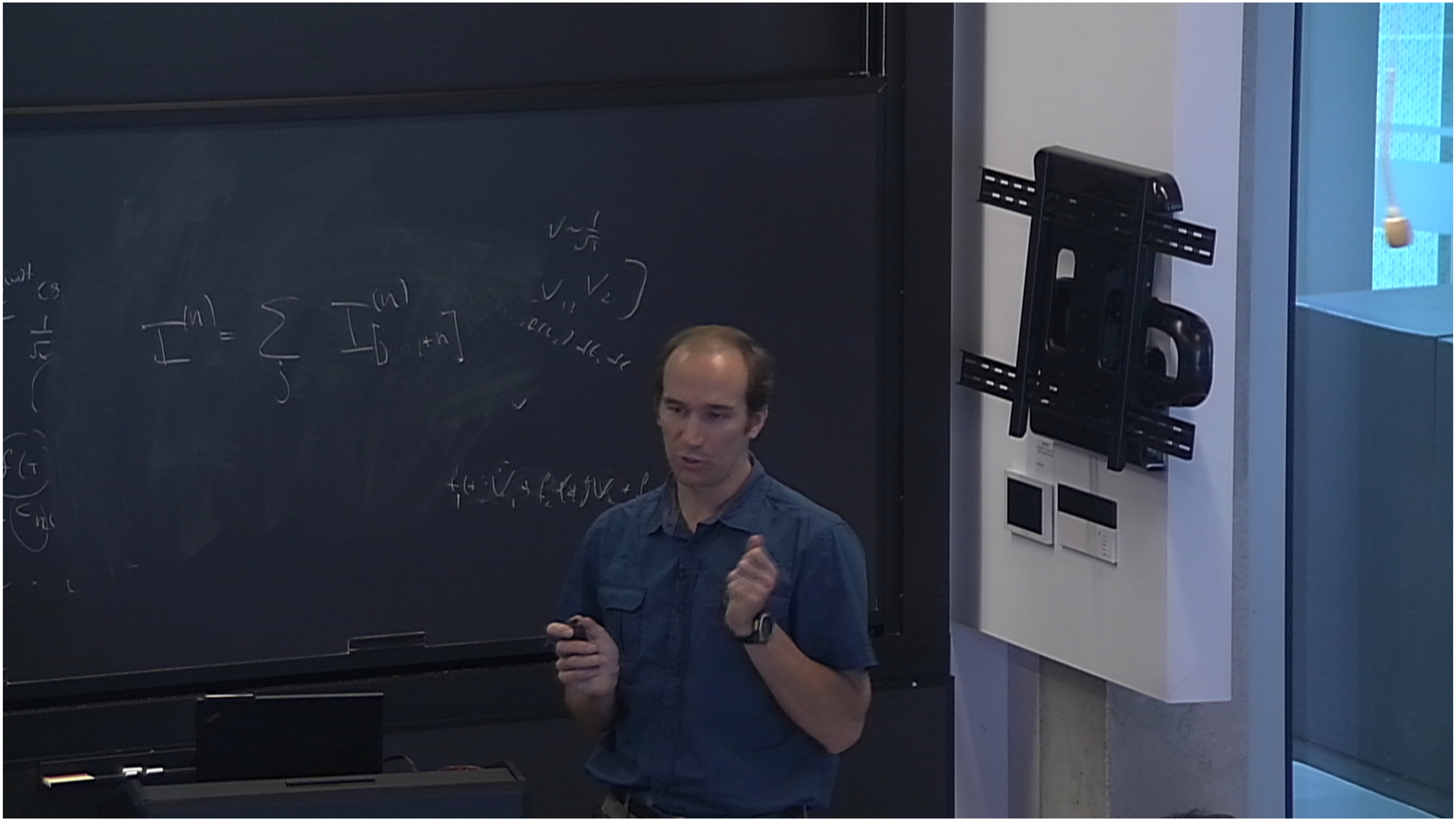
Exact Diag.

Evolution operator: $\hat{U}(T) = e^{-iH_- \frac{T}{2}} e^{-iH_+ \frac{T}{2}} = e^{-iH_F T} = \sum_n |\phi_n\rangle e^{-i\theta_n} \langle \phi_n|$

Floquet Hamiltonian: $\hat{H}_F \equiv \sum_n |\phi_n\rangle \epsilon_n \langle \phi_n|, \quad \theta_n = mod(\epsilon_n T, 2\pi)$



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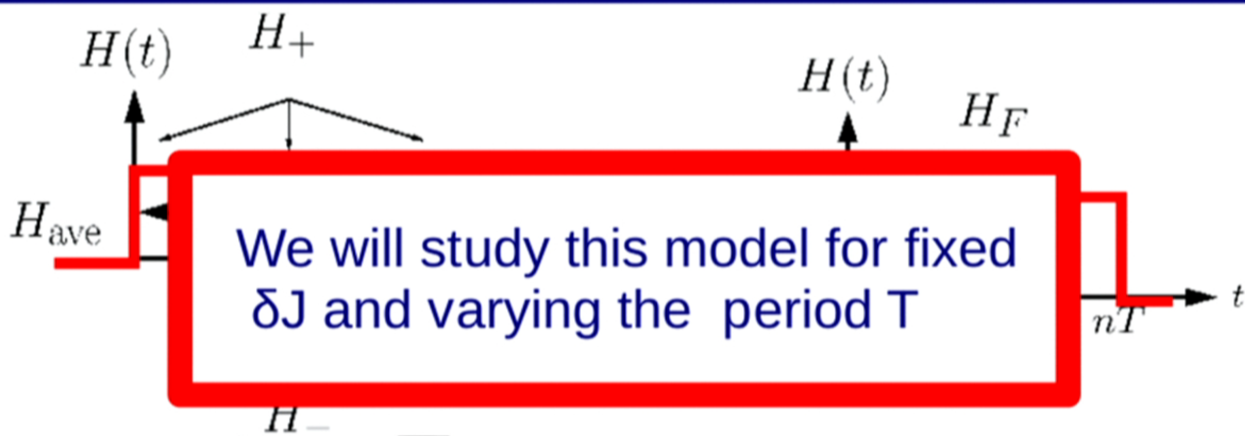

We will study this model for fixed δJ and varying the period T

$$\hat{H}_{\pm} = (J \pm \delta J)\hat{H}_{nn} + J' \sum_j \sigma_j^z \sigma_{j+2}^z \qquad J=1,\ \delta J = 0.2,\ J' = 0.8$$

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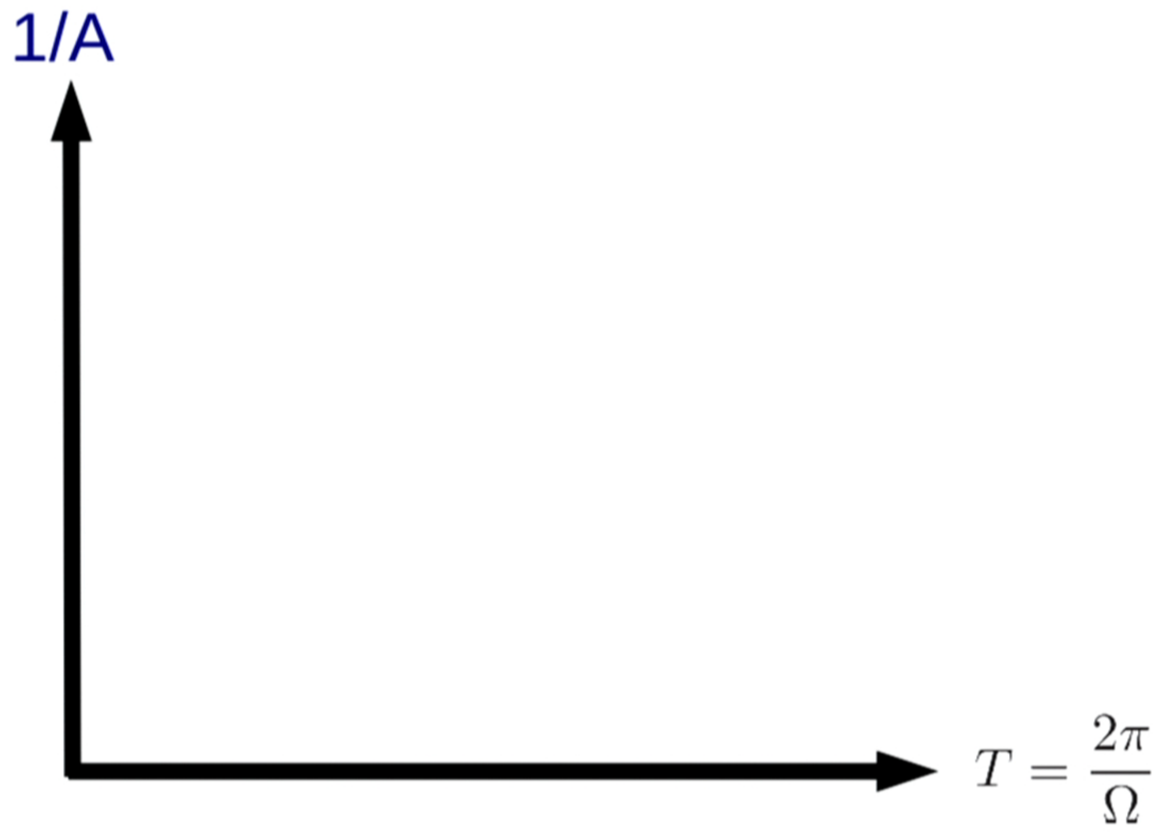
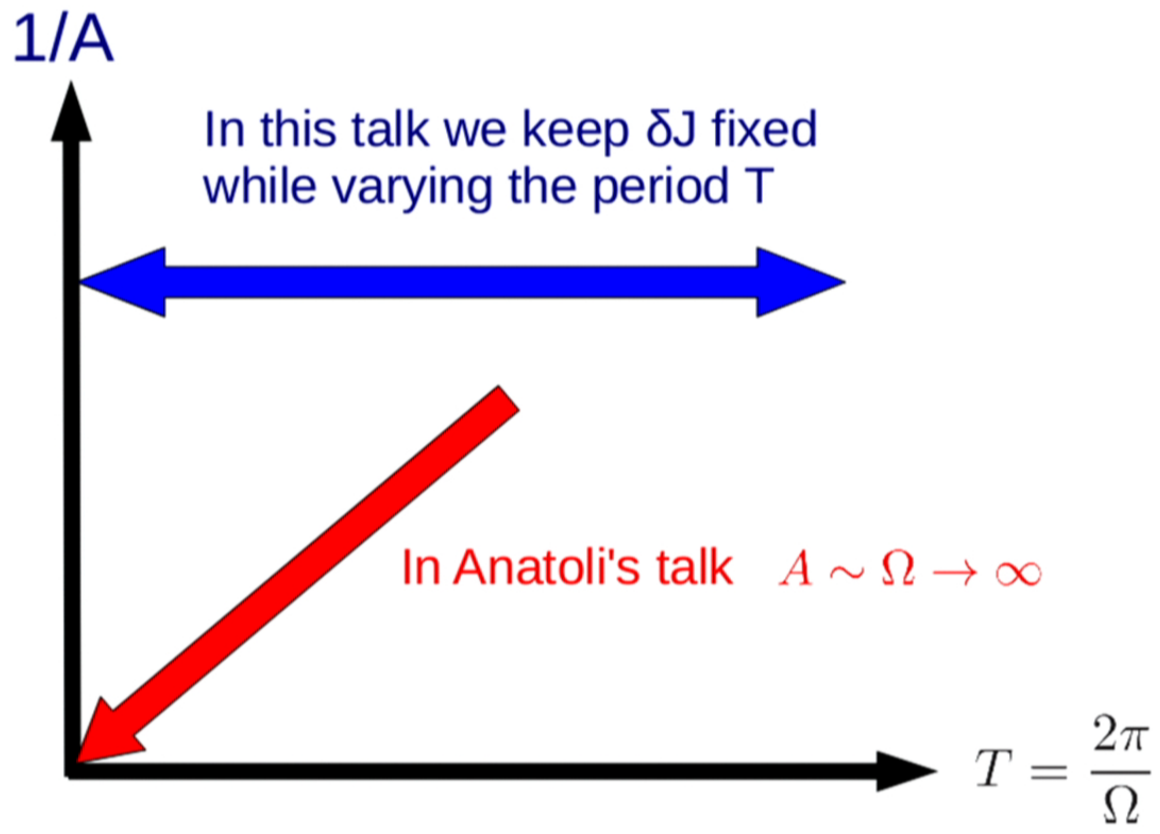
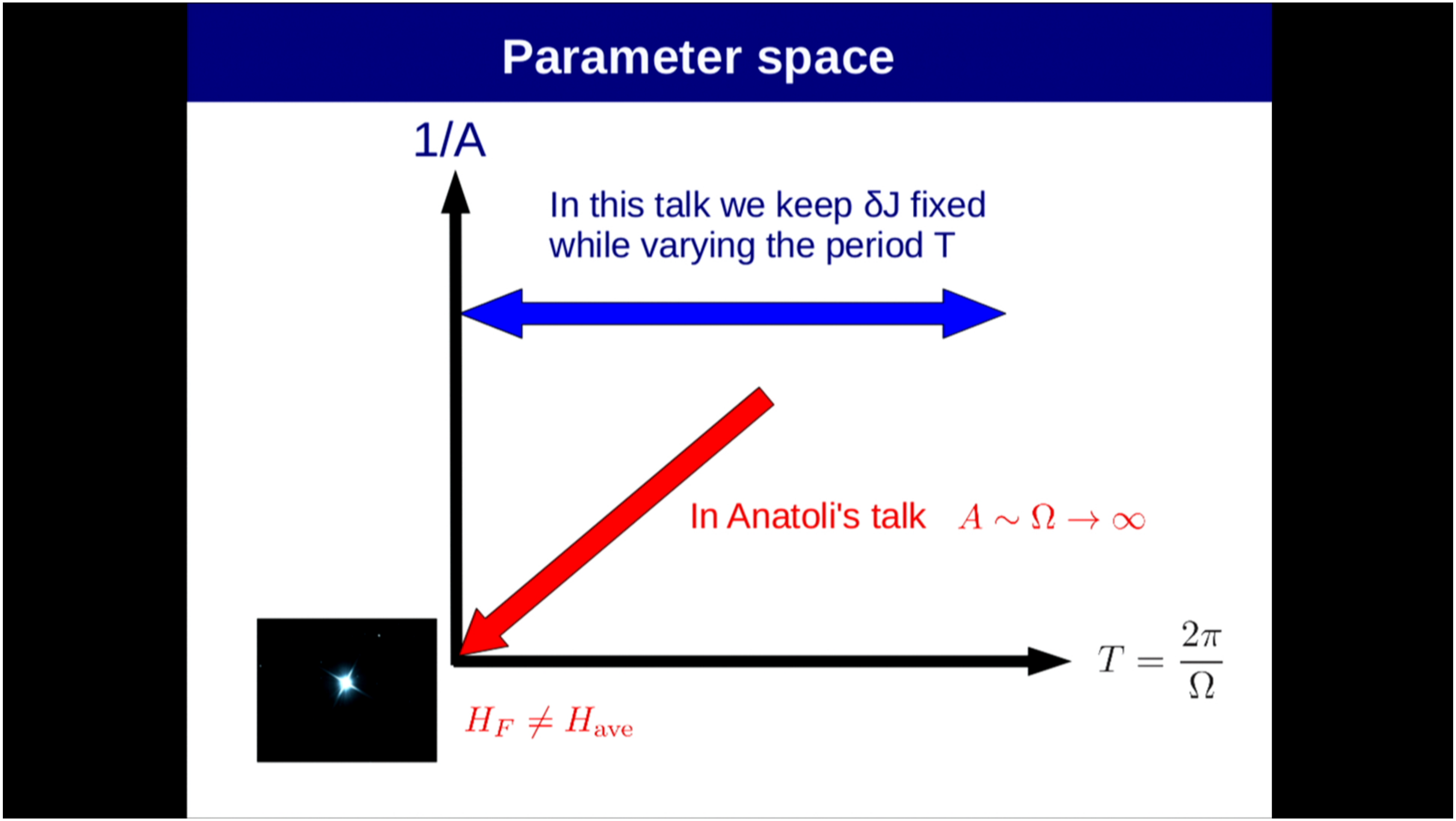
Same eigenvectors but different (folded) eigenvalues

Parameter space

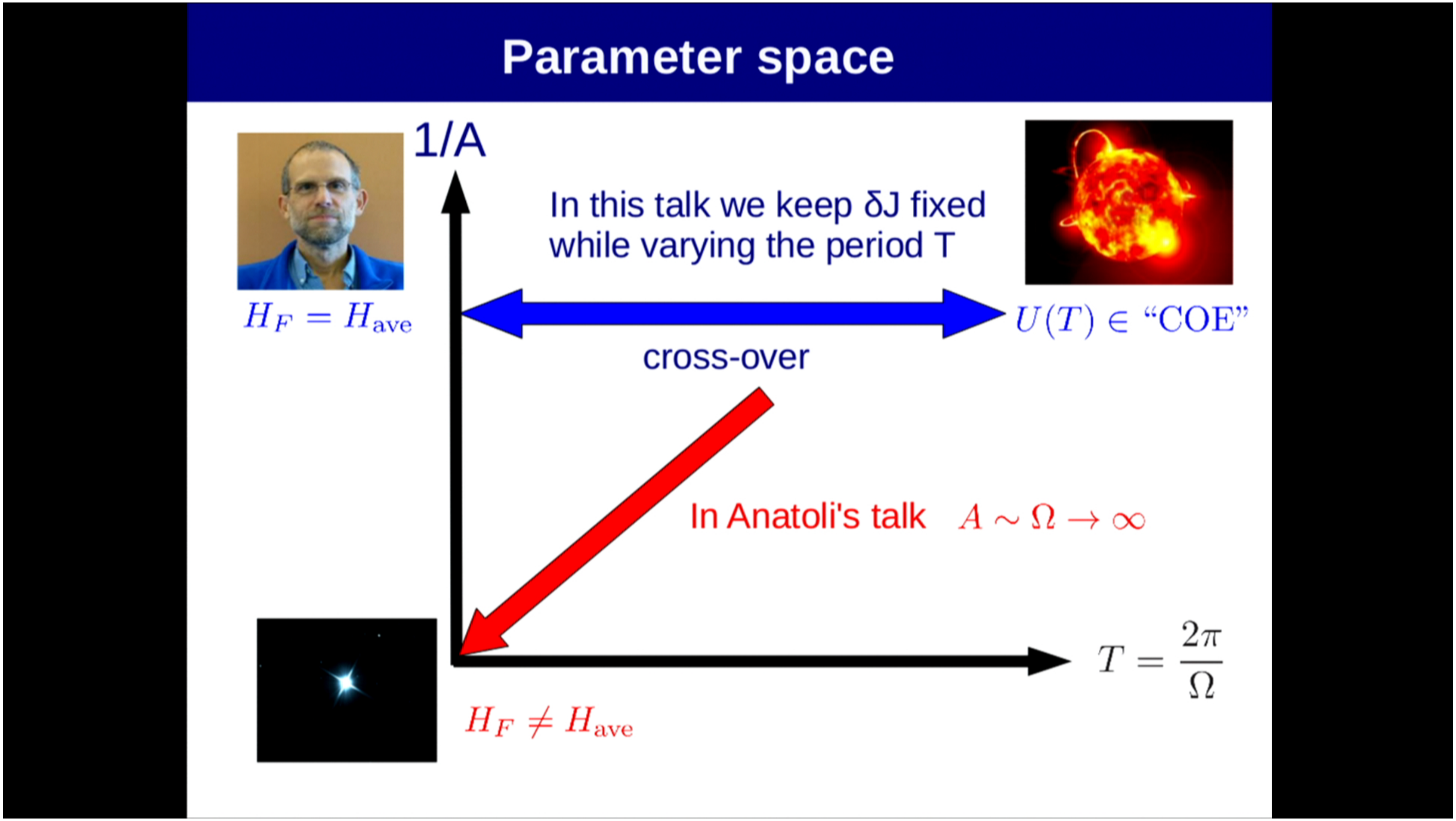


Parameter space





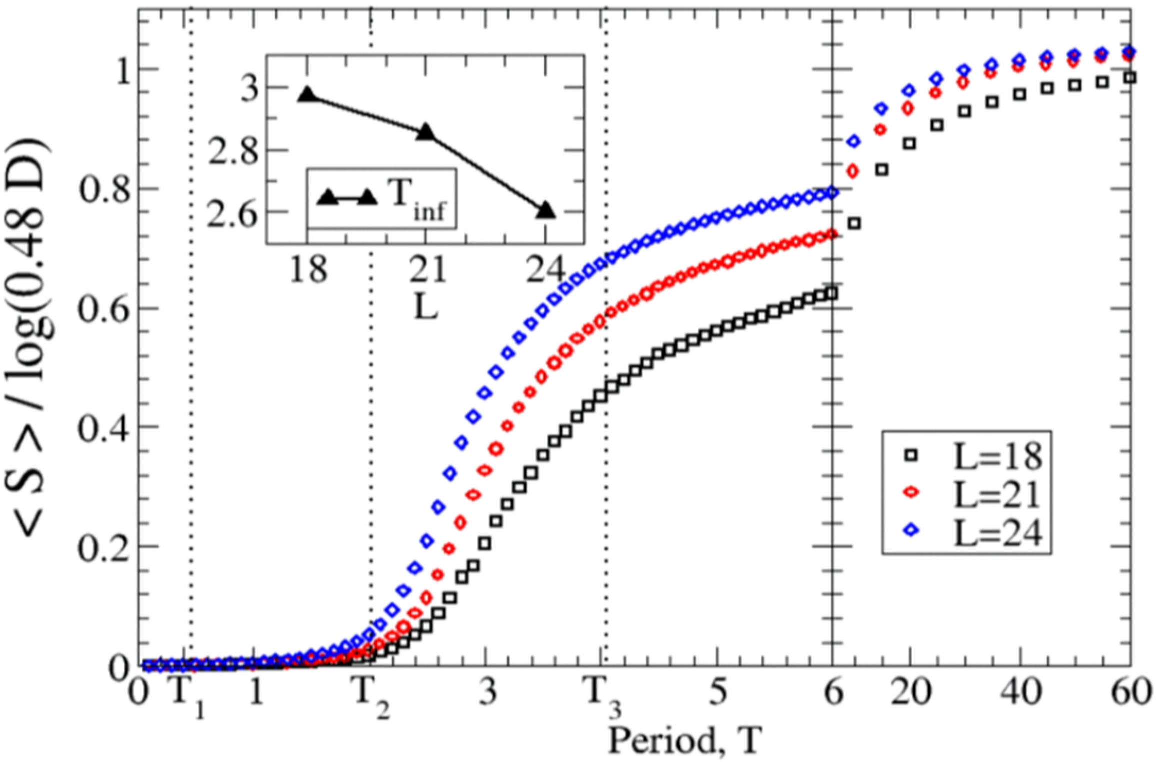
Parameter space
1/A
In this talk we keep δJ fixed
while varying the period T
In Anatoli's talk $A \sim \Omega \to \infty$
$T = \frac{2\pi}{\Omega}$
$H_F \neq H_{\text{ave}}$


Parameter space
1/A
In this talk we keep δJ fixed
while varying the period T
$H_F = H_{\text{ave}}$
$U(T) \in$ "COE"
cross-over
In Anatoli's talk $A \sim \Omega \to \infty$
$T = \frac{2\pi}{\Omega}$
$H_F \neq H_{\text{ave}}$

1) Eigenvectors statistics

Information entropy of the eigenvectors of H_F in the base of $H_{\rm ave}$

$$|\phi_n\rangle = \sum_{m=1}^{D} c_m^n |m_{\rm ave}\rangle, \quad \hat{H}_{\rm ave}|m_{\rm ave}\rangle = \epsilon_m^{\rm ave}|m_{\rm ave}\rangle, \quad S_n = -\sum_{m=1}^{D} |c_m^n|^2 \ln |c_m^n|^2$$



2) Phase repulsion

Let us assume $H_F = H_{\text{ave}}$ then:

$$\hat{U}(T) = e^{-iH_{\text{ave}}T} = \sum_m |m_{\text{ave}}\rangle e^{-i\theta_m^{\text{ave}}} \langle m_{\text{ave}}|, \quad \theta_n^{\text{ave}} = mod(T\epsilon_n^{\text{ave}}, 2\pi)$$

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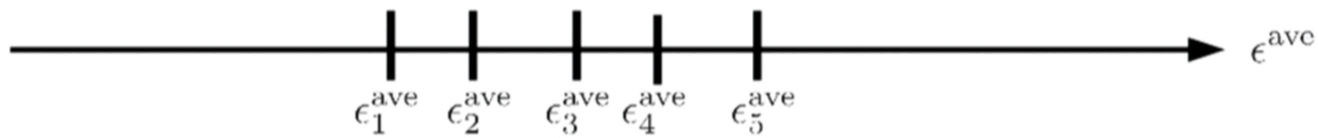
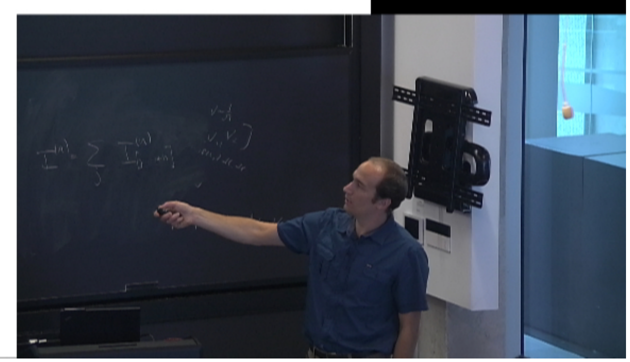
What would be the statistics of $\theta_n^{\rm ave}$?





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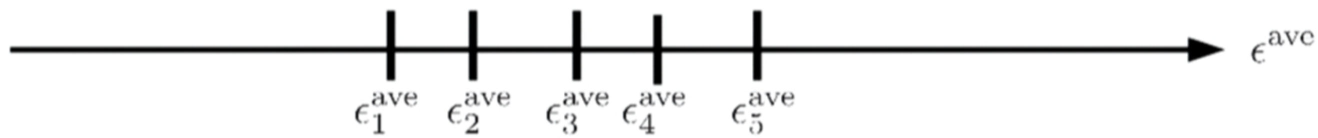
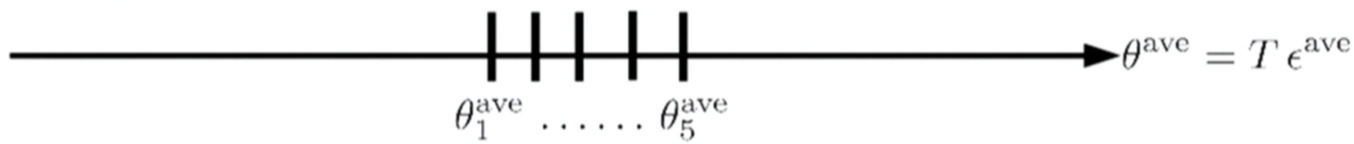
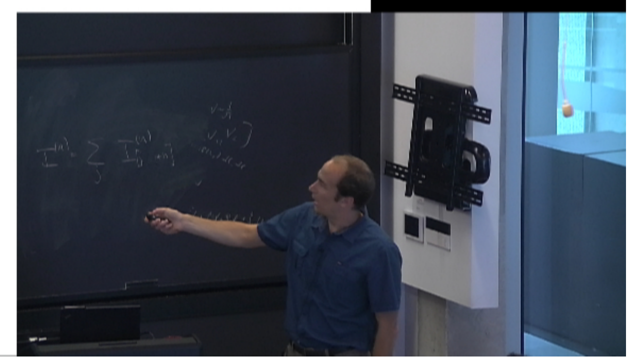
What would be the statistics of θ_n^{ave} ?



1) Short T (shorter than 1/bandwidth)





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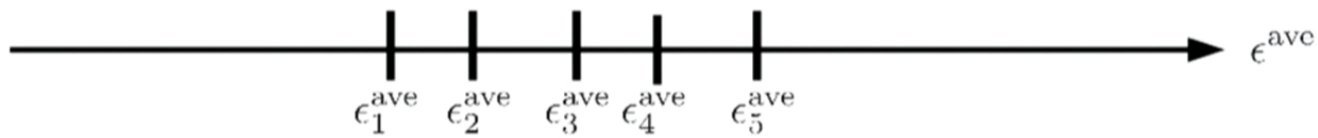
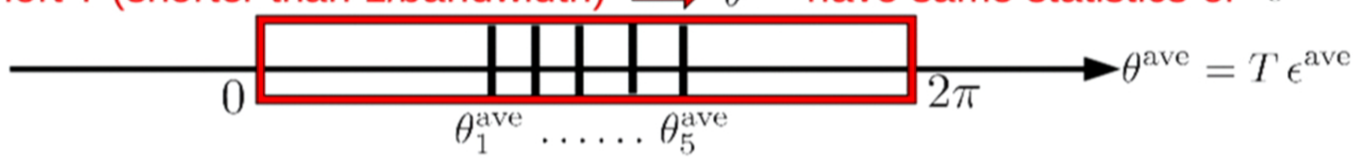


1) Short T (shorter than 1/bandwidth) ➡ $\theta^{\rm ave}$ have same statistics of $\epsilon^{\rm ave}$



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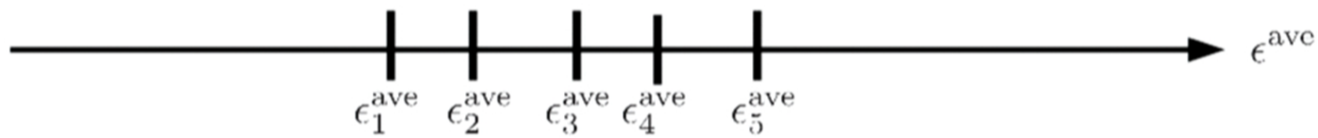

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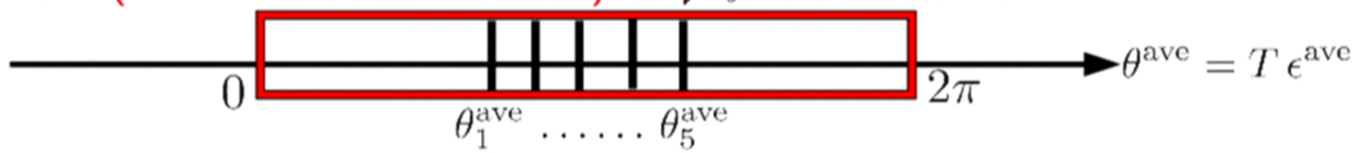
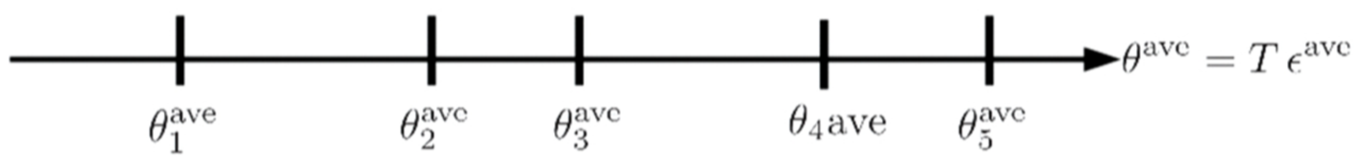


2) Long T (longer than ~1/bandwidth)



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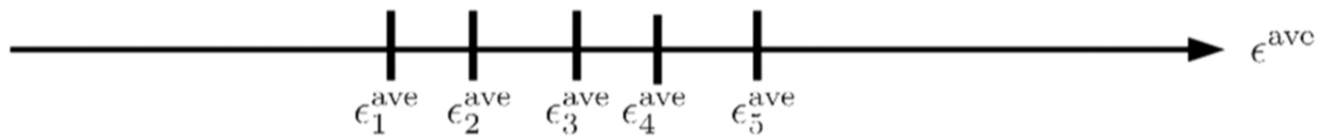

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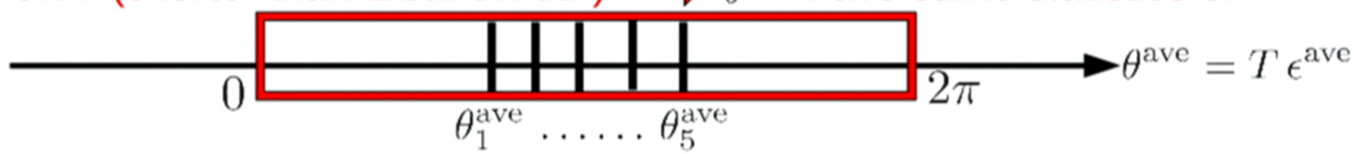


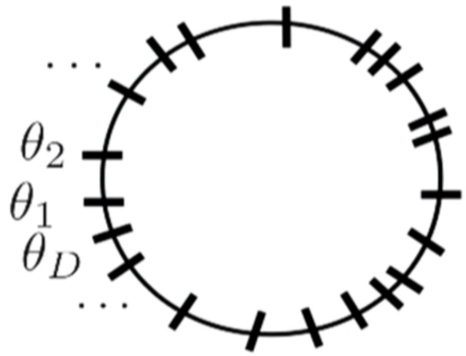
2) Long T (longer than ~1/bandwidth) ➡ $\theta^{\rm ave}$ are **always** Poisson-like

2) Phase repulsion

Let us assume U(T) is "COE":

$$\sum_n |\phi_n\rangle e^{-i\theta_n} \langle\phi_n| = \hat{U}(T) \in COE$$

3) The phases are NATURALLY defined into $(0\,,\,2\pi)$ and repeal
(see F. Haake "Quantum Signatures of Chaos", (Springer,Berlin, 3rd ed. 2010))



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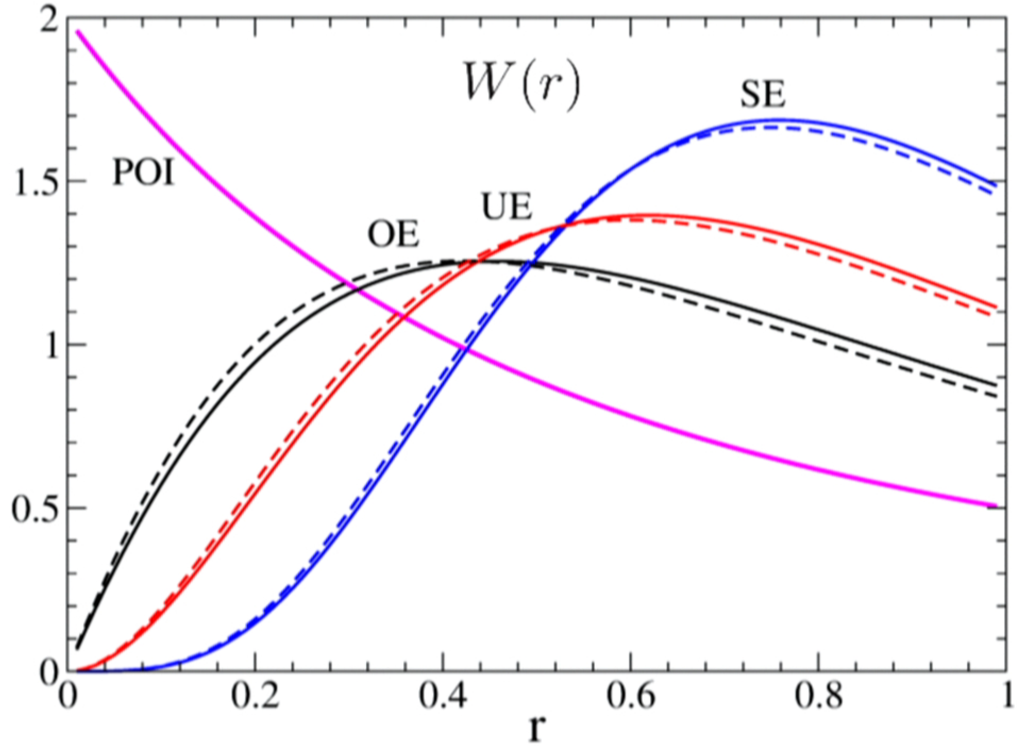
$$W(r),\ \ r = \frac{\min(\delta_n, \delta_{n+1})}{\max(\delta_n, \delta_{n+1})} \in (0,1)$$

V. Oganesyan at al. PRB **75**, 155111(2007).
G. Biroli at al. arXiv:1211.7334v2.
Y. Y. Atas et al. PRL **110**, 084101 (2013).

where $\delta_n = \theta_n - \theta_{n+1}$ are the "spacing" in the folded, i.e. $(0\ ,\ 2\pi)$, spectrum.



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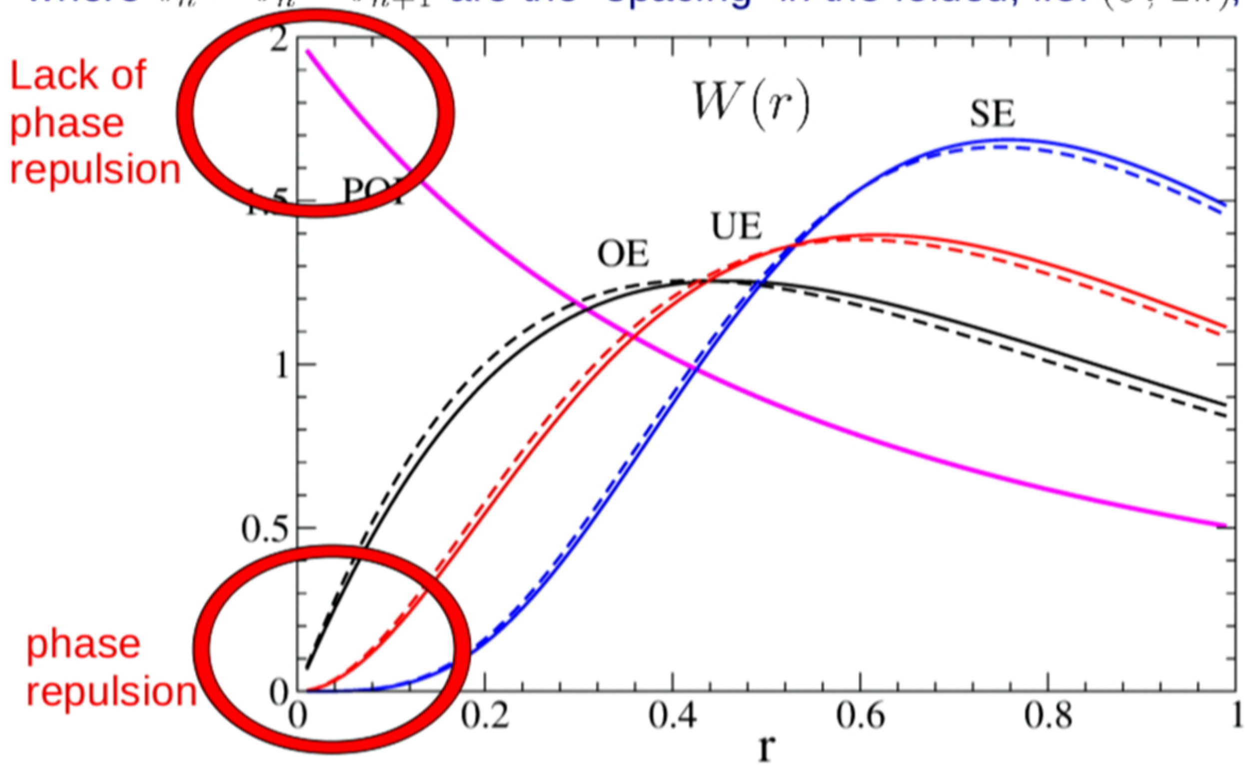
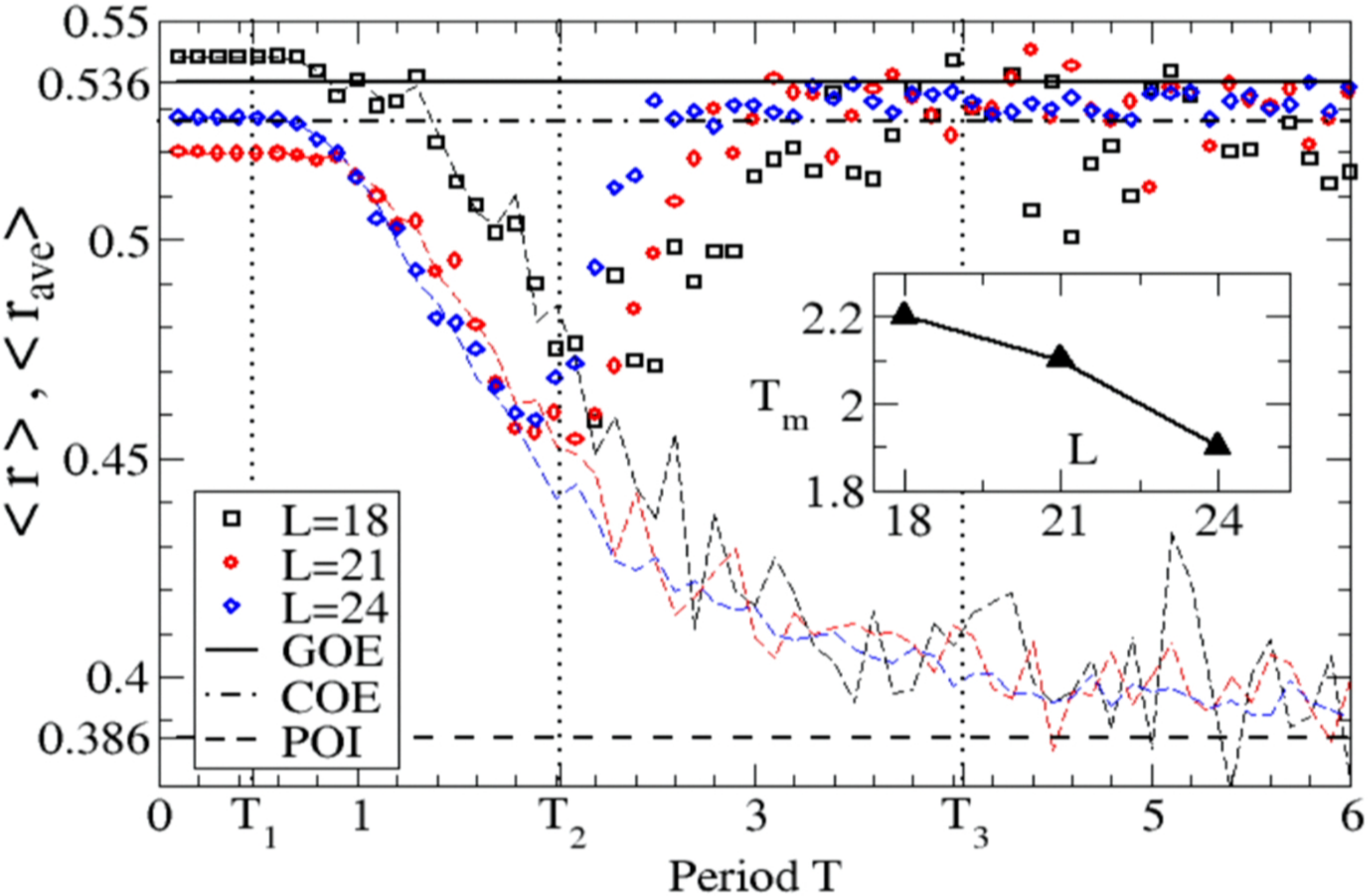
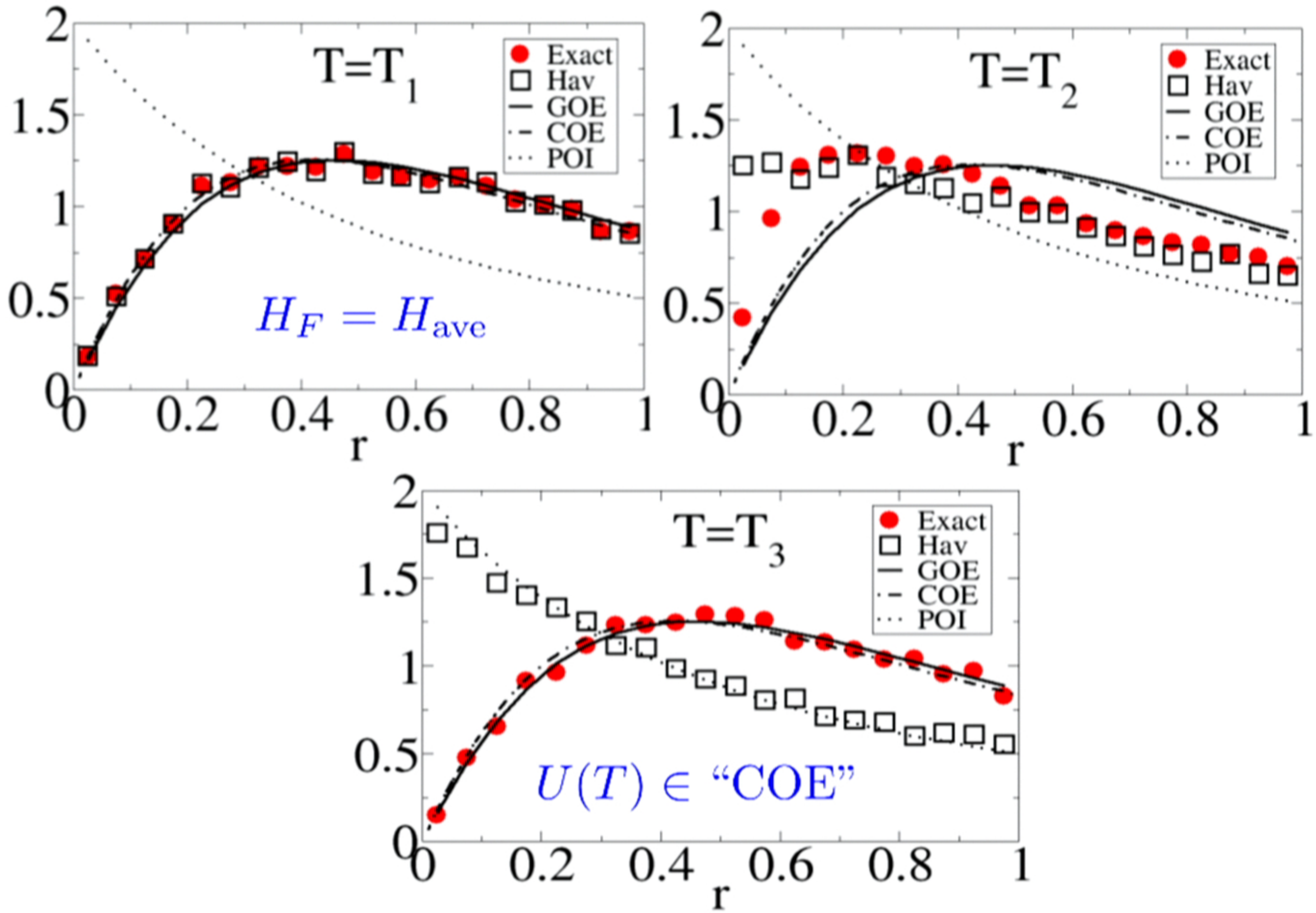


$$\langle r\rangle_{\mathrm{GOE}} \approx \langle r\rangle_{\mathrm{COE}} \approx 0.536, \quad \langle r\rangle_{\mathrm{POI}} \approx 0.386$$

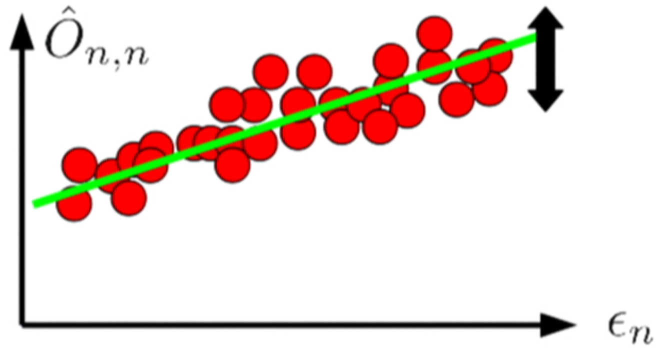


Full distribution W(r)



3) $\langle \phi_n | H_{\rm ave} | \phi_n \rangle$ vs θ_n

1. It measures eigenstate-to-eigenstate variation (similar to ETH-plot)



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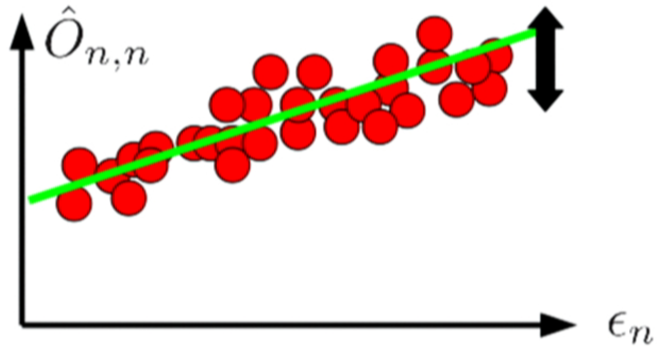

2. If $H_F = H_{\text{ave}}$ there is a simple relation:

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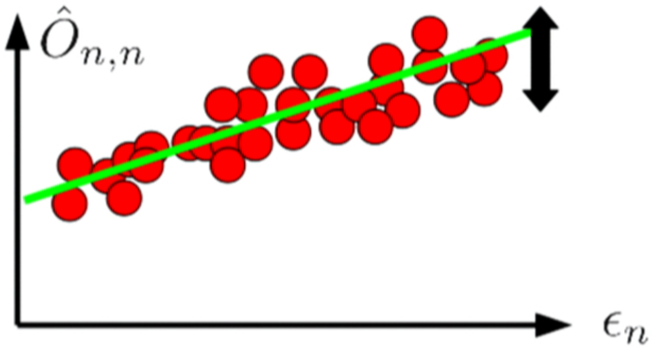

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$$\theta_n = mod(T\epsilon_n, 2\pi)$$

3. If U(T) is "COE" then $|\phi_n\rangle$ are random vectors

➡ $\langle \phi_n | H_{\rm ave} | \phi_n \rangle$ vs θ_n is flat

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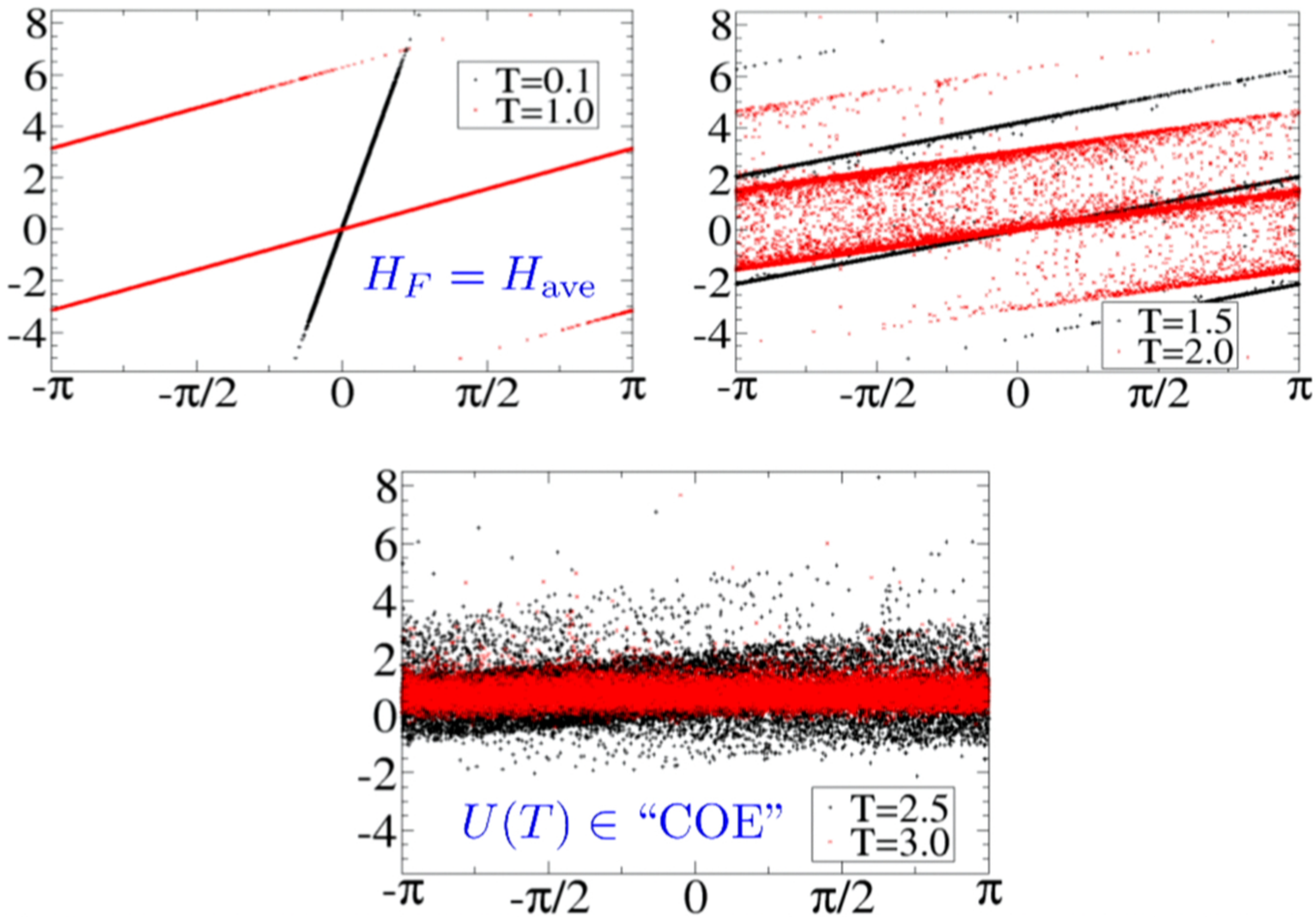
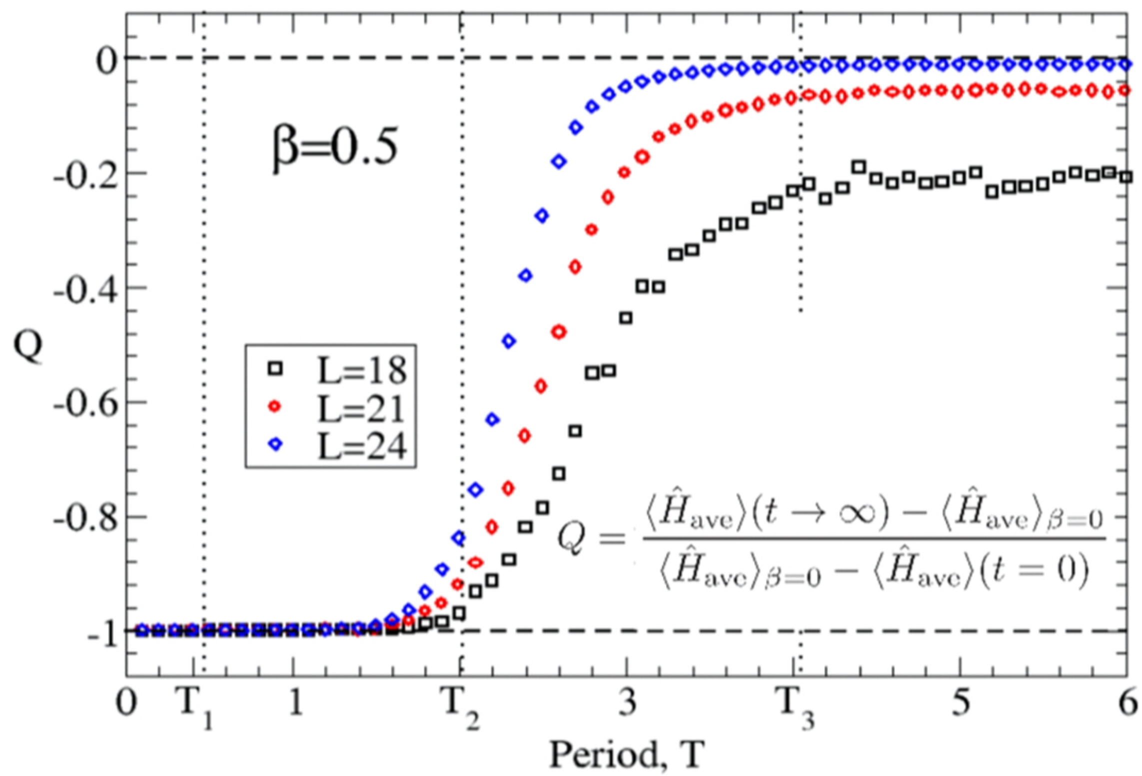


Energy absorption

Infinite energy at infinite time independently on initial conditions

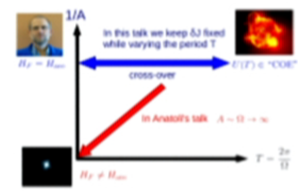
$$\langle \hat{H}_{\rm ave} \rangle (t = NT) = \langle \psi_0 | \left(\hat{U}^\dagger_{\rm cycle} \right)^N \hat{H}_{\rm ave} \left(\hat{U}_{\rm cycle} \right)^N | \psi_0 \rangle \approx \sum_n |\langle \psi_0 | \phi_n \rangle|^2 \langle \phi_n | \hat{H}_{\rm ave} | \phi_n \rangle$$



Conclusions

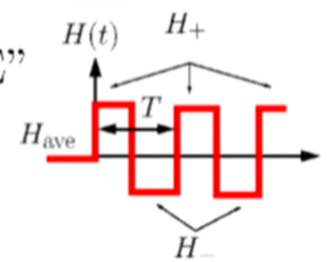
1. Periodically driven systems are **"exciting"**

2. The periodic envelope, P(t), is a generating function of canonical transformations. It can be important.



3. Long-time behavior depends only on H_F which can display phase transitions or crossovers

4. In our model there is a crossover: $\hat{H}_{\mathrm{ave}} \Leftrightarrow$ "COE"
   In the thermodynamic limit always "COE".



Outlook

How are the Floquet eigenstates occupied?

1. coupling to leads, see T. Kitagawa et al. , PRB **84** 235108 (2011)
2. GGE, A. Lazarides et al. PRL **112**, 150401 (2014)
3. dynamical ramps, LD, M. Rigol (in preparation)