Title: Light-Cone Effects after Quantum Quenches and Excitations at Finite Entropy

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Abstract:

"Light-cone" effects after quantum quenches and excitations at finite entropy

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in collaboration with

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EPSRC Engineering and Physical Sciences Research Council

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Outline

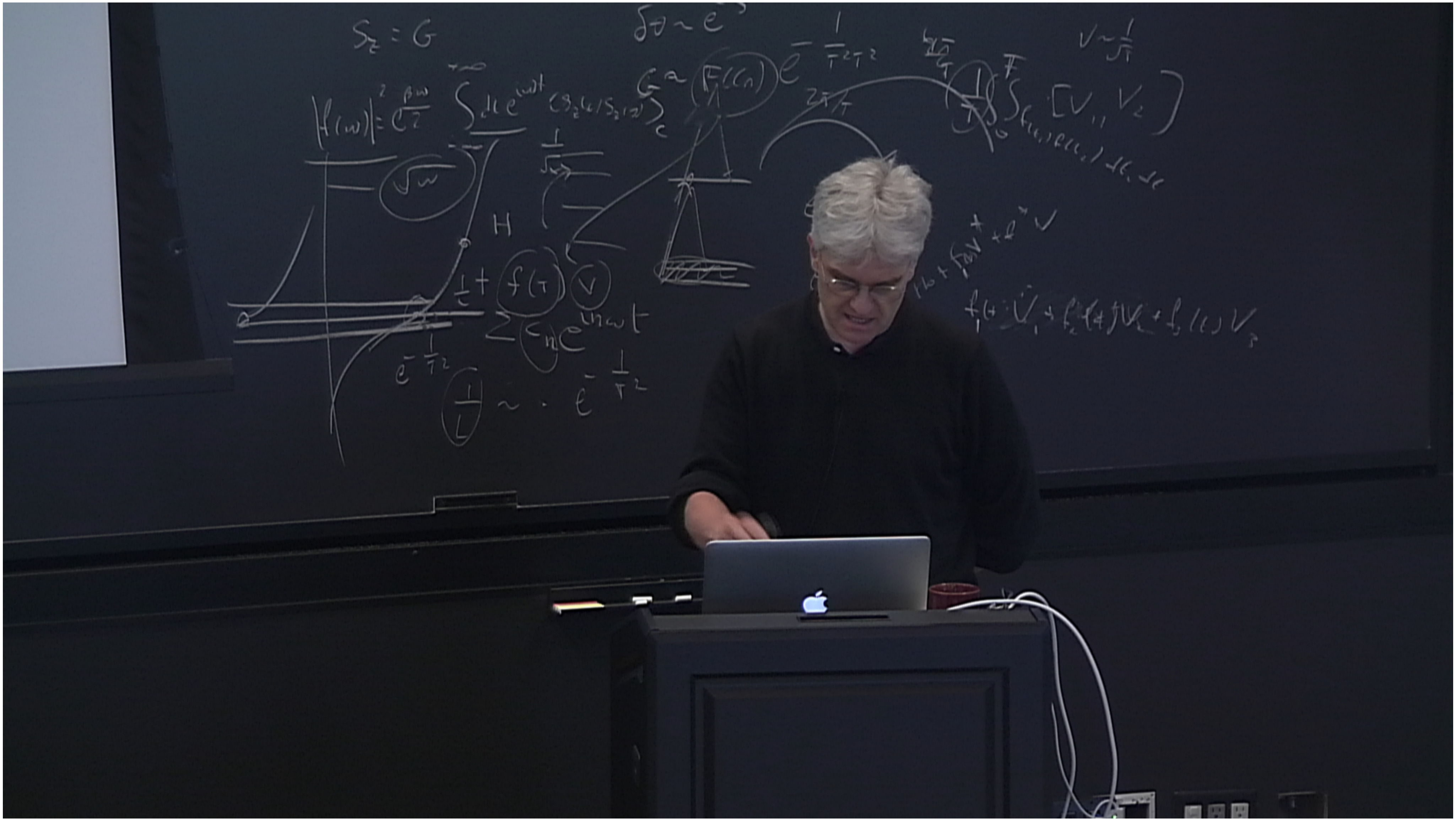
A. Introduction to quantum quenches.

B. "Light-cone" effects.

C. New results for quenches to the Heisenberg chain.

D. Interpretation in terms of "excitations" relative to particular states in the Hilbert space.

E. Some open questions.



Isolated quantum systems out of equilibrium

Idea:

A. Consider a quantum many-particle system with Hamiltonian H (no randomness, translationally invariant)

B. Prepare the system in an initial density matrix $\rho(0)$ that does **not** correspond to an eigenstate. Will assume that $\rho(0)$ fulfils cluster decomposition & is translationally invariant.

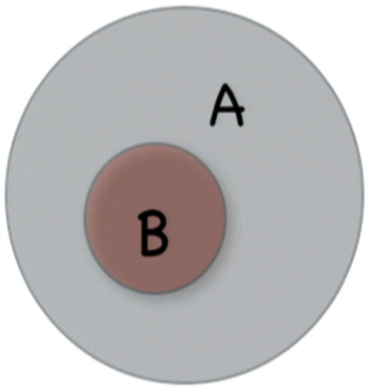
C. Time evolution $\rho(t) = \exp(-iHt)\, \rho(0)\, \exp(iHt)$

D. Study time evolution of local observables $\mathrm{Tr}[\rho(t)\, \mathcal{O}(x)]$ in the thermodynamic limit.

Local Relaxation

Given that we are considering an **isolated** system, does the system relax in some way ?

- It can relax **locally** (in space).



- Entire System: $A \cup B$
- Take A infinite, B finite
- Ask questions only about B:

Expectation values of **local** ops: $Tr[\rho(t) O_B(x)]$

Physical Picture: A acts like a bath for B.

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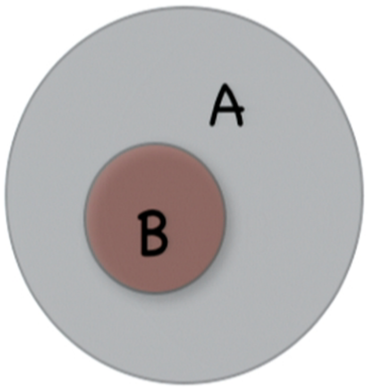


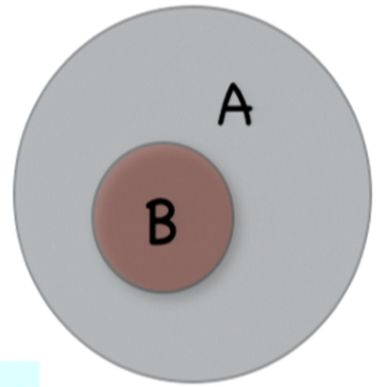
No time-averaging involved !!!

Physical Picture: A acts like a bath for B.

Subsystems and Reduced Density Matrices

Density matrix of entire system: $\rho(t)$

Reduced density matrix: $\rho_B(t) = \mathrm{tr}_A\, \rho(t)$



ρ_B contains **all** local correlation functions in B:

$$\rho_B(t) = \frac{1}{2^\ell} \sum_{\alpha_1,\ldots,\alpha_\ell} \mathrm{Tr}\left[\rho(t)\ \sigma_1^{\alpha_1}\ldots\sigma_\ell^{\alpha_\ell}\right]\ \sigma_1^{\alpha_1}\ldots\sigma_\ell^{\alpha_\ell}$$

α_j=0,x,y,z

for B=[1,…,ℓ] in a spin-1/2 quantum spin chain

Definition of the Stationary State

If $\lim_{t \to \infty} \rho_B(t) = \rho_B(\infty)$ exists for any finite subsystem B:

$\rightarrow$ system approaches a stationary state; averages of all local operators become time-independent.

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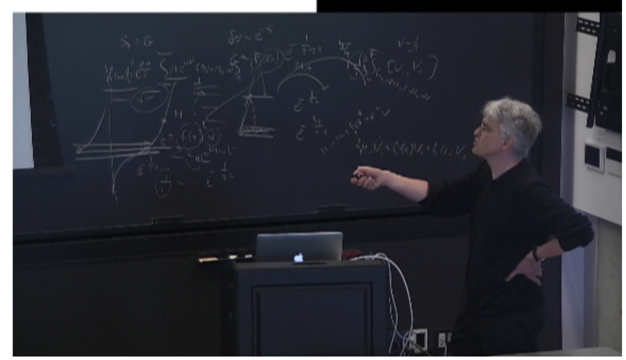
→ system approaches a stationary state; averages of all local operators become time-independent.

Question: What physical properties determine the ensemble $\rho_B(\infty)$?



Thermalization

(Deutsch '91, Srednicki '94)

Belief: "generic" system thermalize at infinite times.

Define a **Gibbs ensemble** for the entire system $A \cup B$

$$\rho_G = \exp(-\beta H(h))/Z$$

β fixed by: $\mathrm{tr}[\rho_G H(h)] = \mathrm{Tr}[\rho(0) H(h)]$

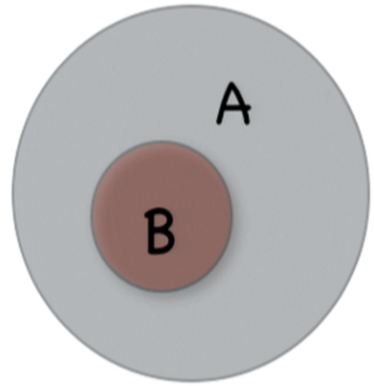
Reduced density matrix for subsystem B: $\rho_{G,B} = \mathrm{tr}_A \, \rho_G$

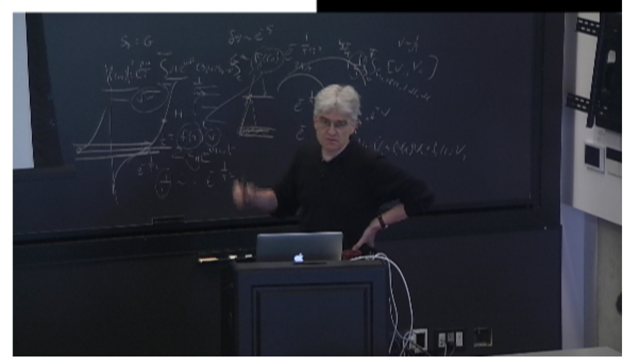
The system **thermalizes** if for any finite subsystem B

$$\rho_B(\infty) = \rho_{G,B}$$

cf
Goldstein et al '05
Barthel&Schollwöck '08
Cramer, Eisert et al '08

A acts as a heat bath with T_{eff}

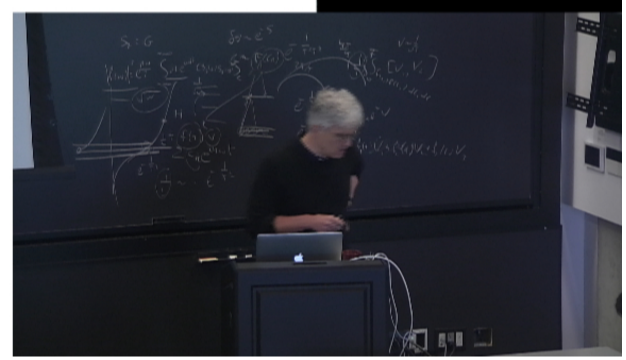




Generalized Gibbs Ensemble

Rigol, Dunjko, Yurosvki & Olshanii '07

Integrable systems don't thermalize but are described by a GGE.



Generalized Gibbs Ensemble

Let I_m be **local** (in space) integrals of motion $[I_m, I_n]=[I_m, H(h)]=0$

Define GGE density matrix by: $\rho_{gG}=\exp(-\Sigma\ \lambda_m I_m)/Z_{gG}$

λ_m **fixed** by $\lim_{L\to\infty}\frac{1}{L}\mathrm{Tr}\left[\rho_{gG}\ I_m\right]=\lim_{L\to\infty}\frac{1}{L}\mathrm{Tr}\left[\rho(0)\ I_m\right]$

Reduced density matrix of B: $\rho_{gG,B}=\mathrm{tr}_A[\rho_{gG}]$

The system is described by a GGE if for any finite subsystem B

$$\rho_B(\infty)=\rho_{gG,B}$$

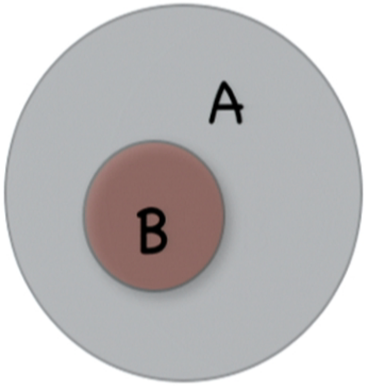
Barthel&Schollwöck '08

Cramer, Eisert et al '08

Calabrese,Essler&Fagotti '12

Generalized Gibbs Ensemble

A is not a standard heat bath:
∞ information about the initial state is retained.



Microcanonical Version

Cassidy,Clark&Rigol '11; Poszgay '11
Caux&Essler '13

ρ_{gG}=exp(- Σ λ_m I_m)/Z_{gG}

saddle point ↓

ρ_{rep}=$|\Phi_s\rangle\langle\Phi_s|$

} ensembles are the same **locally** in space

"Representative State" $|\Phi_s\rangle$= simultaneous eigenstate of all local conservation laws such that

$$i_n \equiv \lim_{L\to\infty} \frac{1}{L} \mathrm{Tr}[\rho(t=0) I_n] = \lim_{L\to\infty} \frac{1}{L} \frac{\langle \Phi_s | I_n | \Phi_s \rangle}{\langle \Phi_s | \Phi_s \rangle}.$$

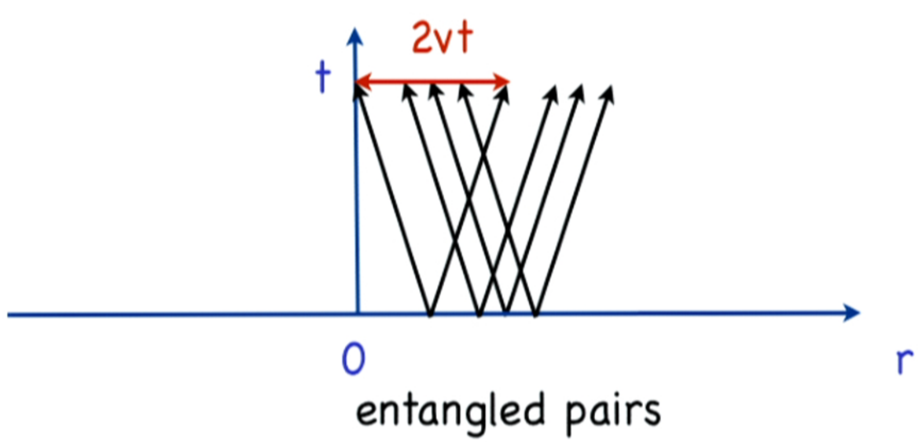
Light-Cone (Horizon) Effects

Calabrese&Cardy '05, '06 certain quenches to CFTs in 1+1 dimensions:

$$\langle\Phi(r,t)\Phi(0,t)\rangle_{\text{conn}} = \langle\Phi(r,t)\Phi(0,t)\rangle - \langle\Phi(0,t)\rangle^2 \propto \begin{cases} 0 & \text{for } t < r/2\,, \\ e^{-x\pi r/2\tau_0} - e^{-x\pi t/\tau_0} & \text{for } t > r/2\,, \end{cases}$$

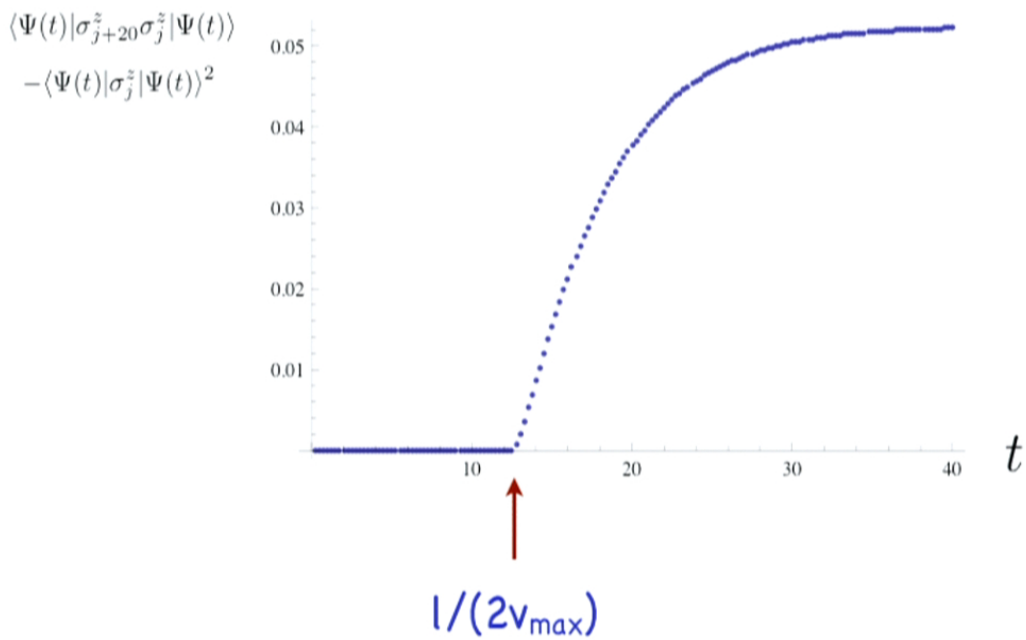
Horizon effect in connected correlators at r=2vt

"Quasiparticle" picture:



Transverse field Ising chain order parameter 2-point function

$$H(h) = -J\sum_j \sigma_j^z \sigma_{j+1}^z + h\sum_j \sigma_j^x$$



quench
h_0=0.2 → h=0.8

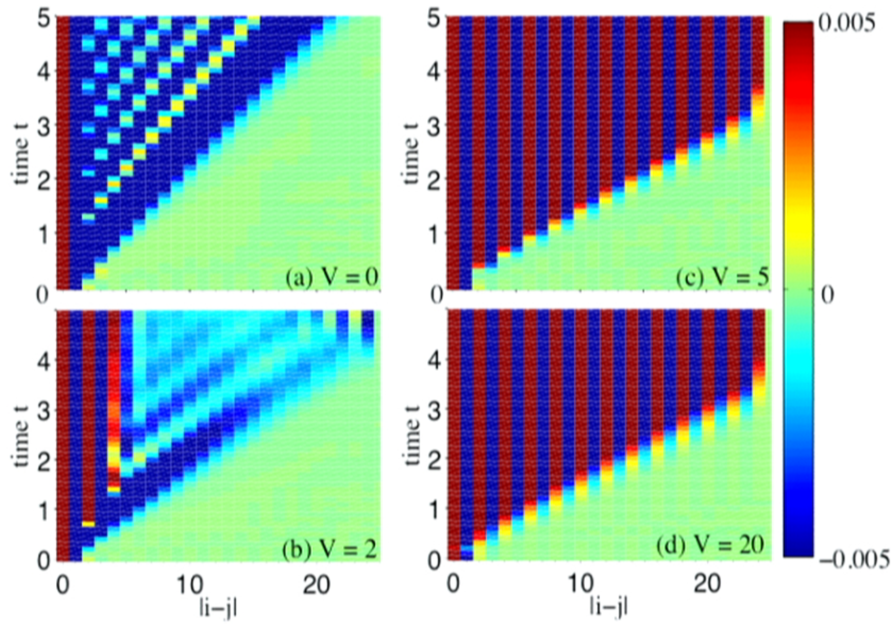
l=20 fixed

Density-density correlator for spinless fermions

$$\hat{H} = -t_{\mathrm{h}} \sum_j \left(c^\dagger_{j+1} c_j + h.c. \right) + V \sum_j n_j n_{j+1} \,,$$

Manmana et al '07

$$C_{i,j}(t) = \langle n_i(t) n_j(t) \rangle - \langle n_i(t) \rangle \langle n_j(t) \rangle \,,$$

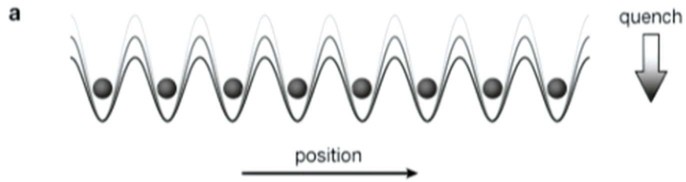


quench
V_0=10 → V

Cold atom Experiments

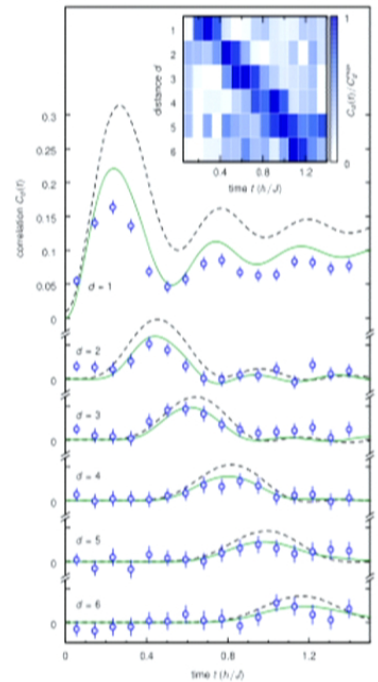
Cheneau et al '12

Barmettler et al '12



quench

$U_0/J=40 \rightarrow U/J=9$



$$\hat{H} = \sum_j \left\{ -J\left(\hat{a}_j^\dagger \hat{a}_{j+1} + \mathrm{h.c.}\right) + \frac{U}{2}\hat{n}_j(\hat{n}_j - 1)\right\},$$

occupation parity 2-point function

$$C_d(t) = \langle \hat{s}_j(t)\hat{s}_{j+d}(t)\rangle - \langle \hat{s}_j(t)\rangle\langle \hat{s}_{j+d}(t)\rangle ,$$

$$\hat{s}_j(t) = \exp(i\pi[\hat{n}_j(t) - \bar{n}])$$

Cold atom Experiments

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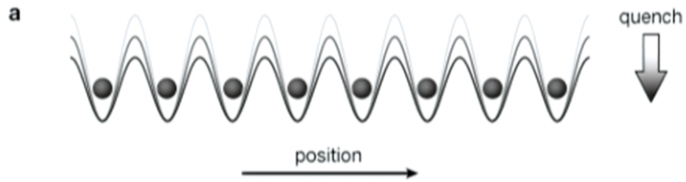

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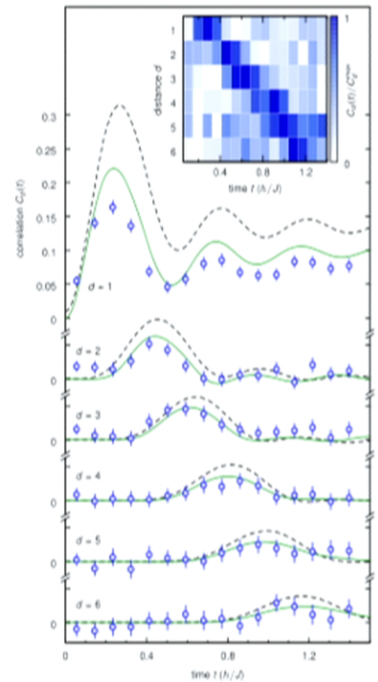

Why does this happen although there is no "speed limit" in non-relativistic quantum theory?

Lieb-Robinson bounds

Lieb&Robinson '72

Consider some non-relativistic quantum spin system

$$\| [O_A(t), O_B(0)] \| \leq c N_{min} \|O_A\| \, \|O_B\| \, \exp\left(-\frac{L - v|t|}{\xi}\right),$$

From this it follows that Bravyi, Hastings& Verstraete '06

$$\langle \mathcal{O}_A(t) \mathcal{O}_B(t) \rangle_{\rm conn} < \bar{c}(|A| + |B|) e^{-(L-2vt)/\xi}$$

$\bar{c}, \xi, v$ are constants

There is a kind of "speed limit" for "sizeable" connected correlations to emerge.

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There is a kind of "speed limit" for "sizeable" connected correlations to emerge.

Question: what is the light-cone propagation velocity after quantum quenches in non-relativistic systems?

Answer for free theories: $\max_{j,p}\left|\frac{\partial\epsilon_j(p)}{\partial p}\right|$

$\epsilon_j(p)$ dispersion relation of particle species j

What about interacting theories?

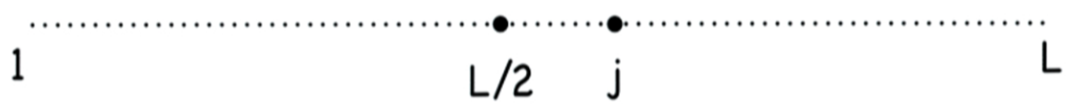
Quantum quenches in the spin-1/2 Heisenberg XXZ chain

$$H(\Delta) = J \sum_{i=1}^{L-1} \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right).$$

Initial density matrix: $\rho(t=0) = Z_\beta^{-1} \exp[-\beta H(\Delta_i)]\,, \quad \beta = \frac{1}{k_{\rm B}T},$

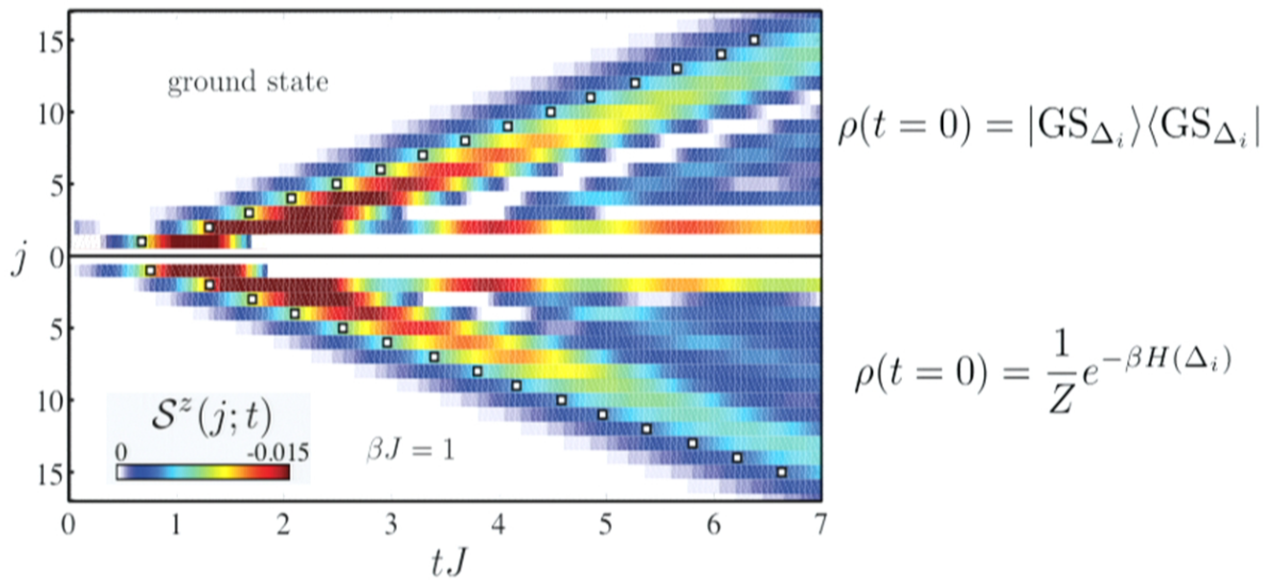
Gibbs distribution for $H(\Delta_i)$ at temperature T

Observable: $\mathcal{S}^z(j;t) = \langle S^z_{L/2}(t) S^z_j(t)\rangle - \langle S^z_{L/2}(t)\rangle\langle S^z_j(t)\rangle$



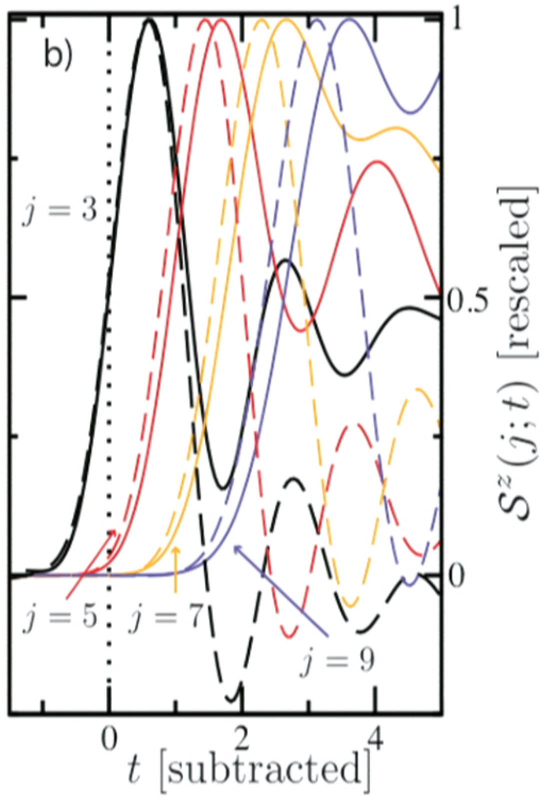
Compute this numerically using METTS (Minimally Entangled Typical Thermal States) MPS methods. White '09

Results for quenches $\Delta_i = 4 \longrightarrow \Delta = \cos(\pi/4)$



Nice light-cone effect.

Observation: Light-cone velocity **depends on initial state** (and not just final Hamiltonian):



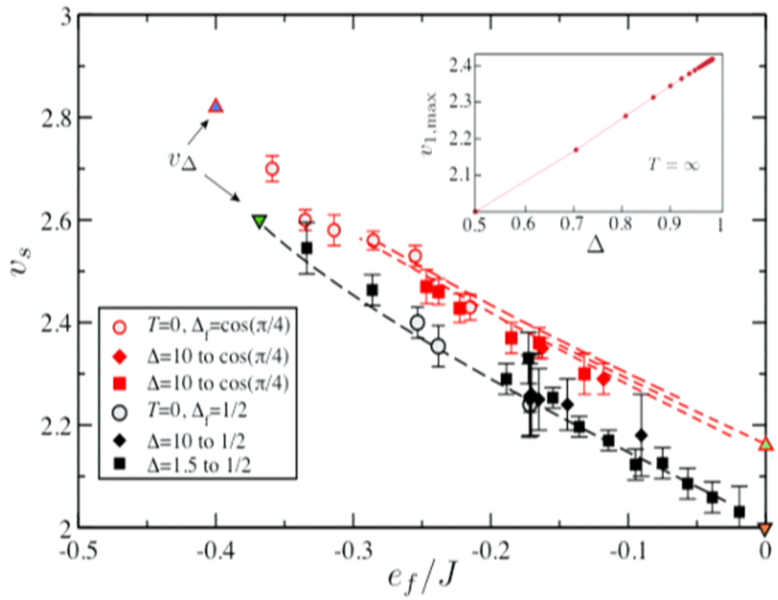
dashed: T=0
solid: T=J

Increasing T decreases the velocity.

Question: what properties of the initial state determine v?

Observation: good data collapse if we plot velocity as a function of final state energy

$$e_f = \frac{\mathrm{Tr}[H(\Delta_f)\rho(t=0)]}{L}.$$



The XXZ chain is **integrable**. Stationary state should be described by an appropriate GGE. For |Δ|<1 this should include both "ultra-local" and "semi-local" conservation laws

$$Q^{(n)} = \sum_j f^{(n)}_{\alpha_j,\ldots,\alpha_{j+n}} \sigma_j^{\alpha_j} \sigma_{j+1}^{\alpha_{j+1}} \ldots \sigma_{j+n}^{\alpha_{j+n}}$$

Lieb '67 Sutherland '70, Baxter '72, Lüscher '76, Faddeev&Takhtajan '84

$$J^{(m)} = \sum_j \sum_{n=1} g^{(m,n)}_{\alpha_j,\ldots,\alpha_{j+n}} \sigma_j^{\alpha_j} \sigma_{j+1}^{\alpha_{j+1}} \ldots \sigma_{j+n}^{\alpha_{j+n}}$$

Prosen&Ilievski '13

These are difficult to handle.

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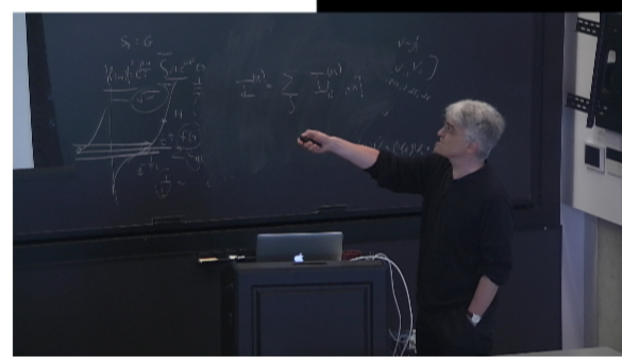

Caux& Essler '13

Time evolution of observables after quench to integrable models prepared in state $|\psi\rangle$

$$\lim_{L\to\infty}\langle\mathcal{O}(t)\rangle = \lim_{L\to\infty}\left[\frac{\langle\Psi|\mathcal{O}(t)|\Phi_s\rangle}{2\langle\Psi|\Phi_s\rangle} + \Phi_s \leftrightarrow \Psi\right].$$

"Representative State" $|\Phi_s\rangle$= simultaneous eigenstate of all local conservation laws such that

$$i_n \equiv \lim_{L\to\infty}\frac{1}{L}\mathrm{Tr}[\rho(t=0)I_n] = \lim_{L\to\infty}\frac{1}{L}\frac{\langle\Phi_s|I_n|\Phi_s\rangle}{\langle\Phi_s|\Phi_s\rangle}.$$

$|\Phi_s\rangle$ is constructed from a generalized TBA

Has been done so far only for simple ρ (t=0): Caux& Essler '13 Ising

Caux et al '13, '14 Lieb-Liniger, XXZ

Bertini, Schuricht&Essler '14 sine-Gordon

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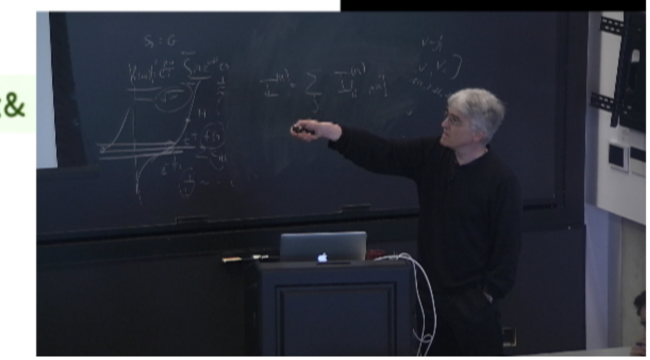

Main idea: light cone velocity determined by "excitations" over the representative state.

Motivate this by Lehmann representation $H(\Delta)|n\rangle = E_n|n\rangle$

$$\langle\Psi|\mathcal{O}(t)|\Phi_s\rangle = \sum_n \langle\Psi|n\rangle\langle n|\mathcal{O}|\Phi_s\rangle e^{-i(E_n - E_{\Phi_s})t},$$

important states are those with $E_n - E_{\Phi_s} = \mathcal{O}(1)$

want
"excitation spectrum"
around this state

Free theories: (think of a Fermi gas)

- single species, dispersion ε (p) $H = \sum_p \epsilon(p)\hat{n}(p)$
- I_n **linearly** related to n(k) $\rho_{\rm GGE} = \frac{1}{Z}\exp\big(\sum_k \lambda_k \hat{n}(k)\big)$

Fagotti& Essler '13

- can describe $|\Phi_s\rangle$ by densities of particles and holes $\rho^{p,h}(k)$

$$\rho^p(k) = \frac{{\rm Tr}\left[\rho_{\rm GGE}\ \hat{n}(k)\right]}{2\pi} = \frac{\langle\Phi_s|\hat{n}(k)|\Phi_s\rangle}{2\pi} \qquad \rho^p(k) = \frac{1}{2\pi} - \rho^h(k) = \frac{(2\pi)^{-1}}{1+e^{\lambda_k}}$$

- **Excitations= particles and holes with energies $\pm\varepsilon$ (p)**
- velocity= $\max_p|\varepsilon'(p)|$

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- **Excitations= particles and holes with energies $\pm\,\varepsilon$ (p)**
- velocity= $\max_p|\,\varepsilon\,'(p)|$

Integrable theories like XXZ:

- several (M) species ("strings") of elementary excitations

M depends on Δ. cf one type of excitation (spinon) over GS

- can describe $|\Phi_s\rangle$ by particle/hole densities $\rho_j^{p,h}(k)$, $j = 1, 2, \ldots, M$

particle/hole densities related in non-trivial way.

- Excitations= particles and holes; energies $\pm\epsilon_j(p)[\{\rho_\ell^{p,h}(k)\}]$

- velocity $v_{\max} = \max_{j,p}\left|\frac{\partial\epsilon_j(p)}{\partial p}\right|$

Depend on $|\Phi_s\rangle$ ("dressing") !!

GGE hypothesis: $|\Phi_s\rangle$ is **fixed** by the "initial data"

$$\lim_{L\to\infty} \frac{1}{L} \mathrm{Tr}\left[\rho(t=0) I_n\right] = \lim_{L\to\infty} \frac{1}{L} \frac{\langle \Phi_s | I_n | \Phi_s \rangle}{\langle \Phi_s | \Phi_s \rangle}$$

v_{max} is determined by expectation values of all conservation laws in the initial density matrix!

Technical Procedure:

"initial data" (numerics) $\lim_{L\to\infty} \frac{1}{L} \mathrm{Tr}\left[\rho(t=0) I_n\right]$

locally in space $|\Phi_s\rangle\langle\Phi_s| \longleftrightarrow \rho_{\mathrm{GGE}} = \frac{1}{Z} \exp\left(\sum_\ell \lambda_\ell I_\ell\right)$

do **T**hermodynamic **B**ethe **A**nsatz for "Hamiltonian" $\sum_\ell \lambda_\ell I_\ell$

↓ saddle point

$\rho_j^{p,h}(k, \{\lambda_\ell\}) \longleftrightarrow |\Phi_s\rangle$

↓

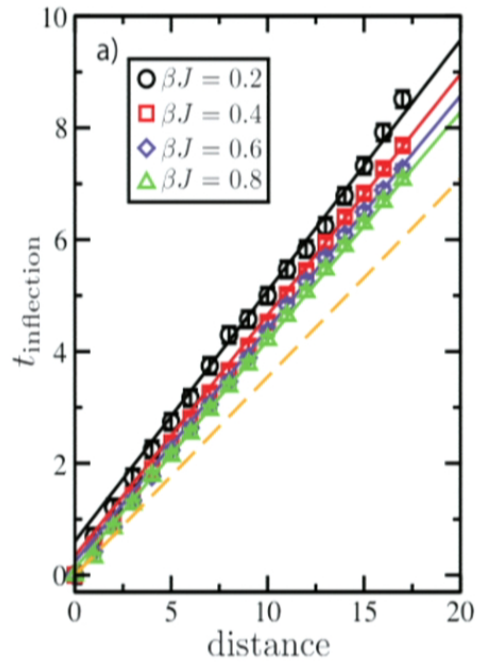
$\lim_{L\to\infty} \frac{1}{L} \frac{\langle\Phi_s|I_n|\Phi_s\rangle}{\langle\Phi_s|\Phi_s\rangle}$ $\overset{?}{=}$ $\lim_{L\to\infty} \frac{1}{L} \mathrm{Tr}\left[\rho(t=0) I_n\right]$

no → adjust $\{\lambda_\ell\}$ ↑

yes → That's a Bingo!

Comparison to METTS results

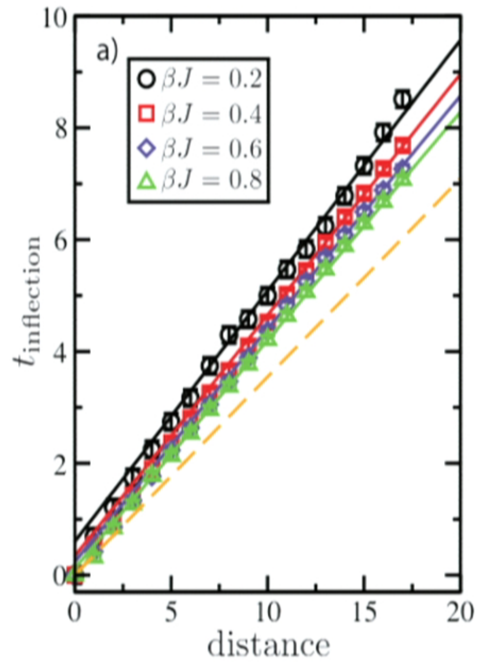
quenches $\Delta_i = 4 \longrightarrow \Delta = \cos(\pi/4)$



→ consistent

Comparison to METTS results

quenches $\Delta_i = 4 \longrightarrow \Delta = \cos(\pi/4)$



→ consistent

Summary and Outlook

1. Light-cone effects after quantum quenches are much more complicated/interesting than perhaps previously thought.

2. Related to properties of "excitations" around particular Hamiltonian eigenstates high up in the spectrum.

3. Integrability allows description of such "excitations".

4. Refined treatment (retaining more conservation laws in GGE) highly desirable.

5. Signatures of slower types of excitations? (✔)

6. What about non-integrable models ("excitations" will have finite lifetimes)? Lars Bonnes & Andreas Läuchli