Title: Anatoli Polkovnikov - Deriving effective Floquet Hamiltonians in interacting systems using the Magnus expansion

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Abstract:

Deriving effective Floquet Hamiltonians in interacting systems using the Magnus expansion

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Marin Bukov BU

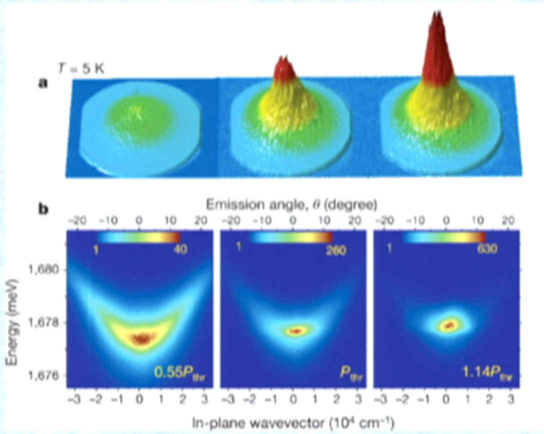
Quantum many-body dynamics, Perimeter Institute, 05/13/2014

Plan

- Periodic vs. chaotic driving in isolated systems
- Getting nontrivial Floquet Hamiltonians from the Magnus expansion.
- Many-body localization in the energy space.

(Quantum) Driven systems. Many different applications. Limited understanding.

Exciton-Polariton condensates in a driven-dissipiative system



J. Kasprzak et. al, Nature, 443, 409 (2006)

Light induced Superconductivity in a Stripe-ordered Cuprate
D. Fausti, et all, Science 331, 189 (2011)

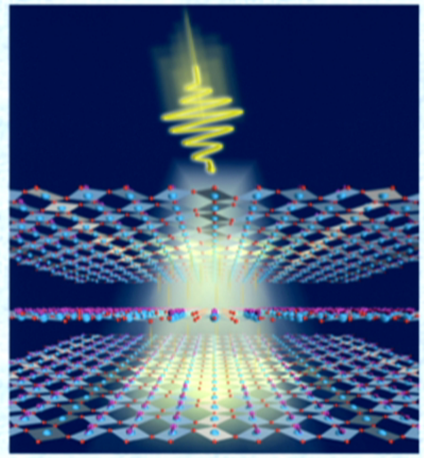
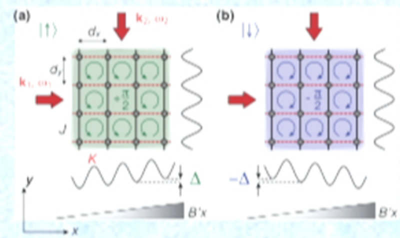


Image taken from A. Cavalleri web page

Realization of Hofstadter-Harper Hamiltonian
M. Aidelsburger, … I. Bloch. 2013
H. Miyake, … W. Ketterle, 2013

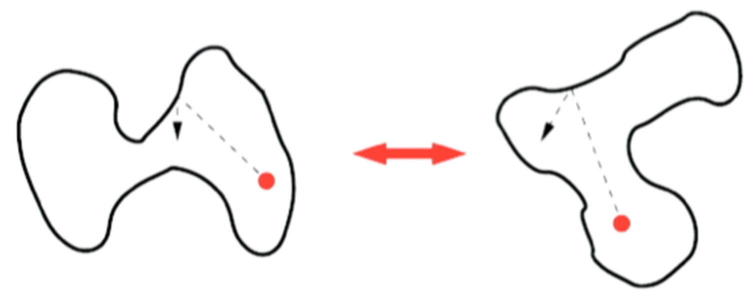
$$\hat{H}_{1,1} = -\sum_{m,n}\left(K e^{\pm i\phi_{m,n}} \hat{a}^{\dagger}_{m+1,n}\hat{a}_{m,n} + J \hat{a}^{\dagger}_{m,n+1}\hat{a}_{m,n}\right) + \text{H.c.}, \quad (1)$$



In certain cases we can understand steady states as effective equilibrium with respect to some effective Hamiltonian. In certain cases it is impossible.

Incoherent driving: heating, entropy increase, second law…

Example: particle in a chaotic cavity.
C. Jarzynski, 1992; L. D'Alessio P. Krapivsky 2011



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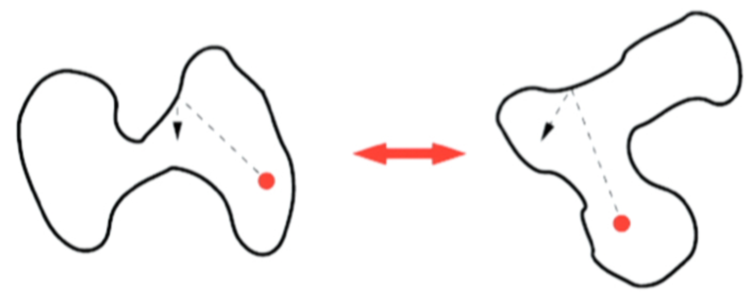
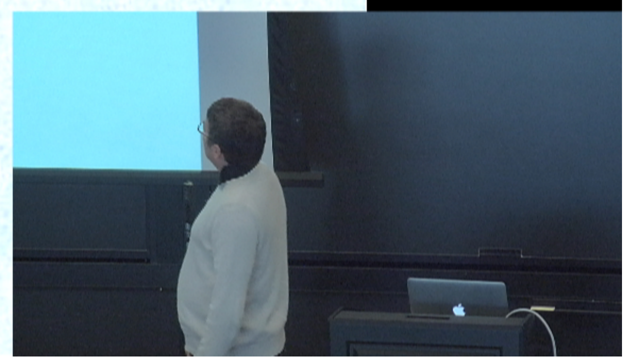


Long time limit: universal energy distribution

$$W(E) \propto \exp[-f(t)\sqrt{E}]$$

Many-particles: Gaussian distribution due to central limit theorem. But non-thermal variance and lower entropy.



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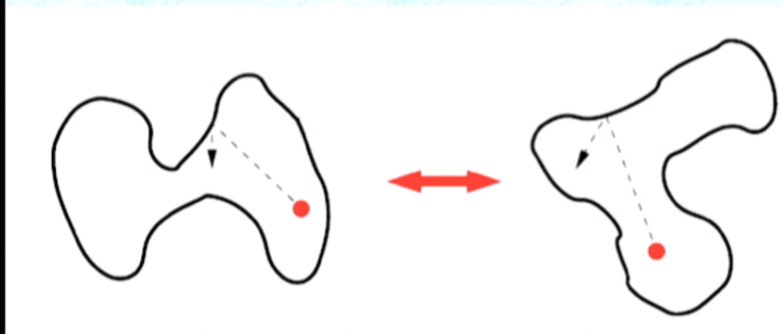

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Can be extended to interacting systems
G. Bunin, L. D'Alessio, Y. Kafri, A.P. 2011

- Universal non-Gibbs distribution
- Dynamical phase transitions as a function of the heating protocol (to the superheated regime).
- Can prepare arbitrarily narrow distributions.

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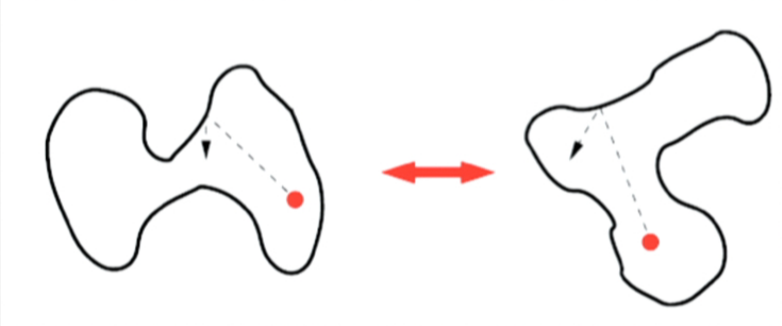

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- Dynamical phase transitions as a function of the heating protocol (to the superheated regime).
- Can prepare arbitrarily narrow distributions.

Physical reason: for initial stationary states can use fluctuation relations (Einstein, Onsager, Jarzynski, Crooks, …)

$$2w = \beta\delta w^2 + \partial_E \delta w^2$$

Works for both single and many-particle systems; β – microcanonical temperature

Periodic (coherent) driving. Fluctuation and other relations break. Laws of thermodynamics can not be justified (locally in time).

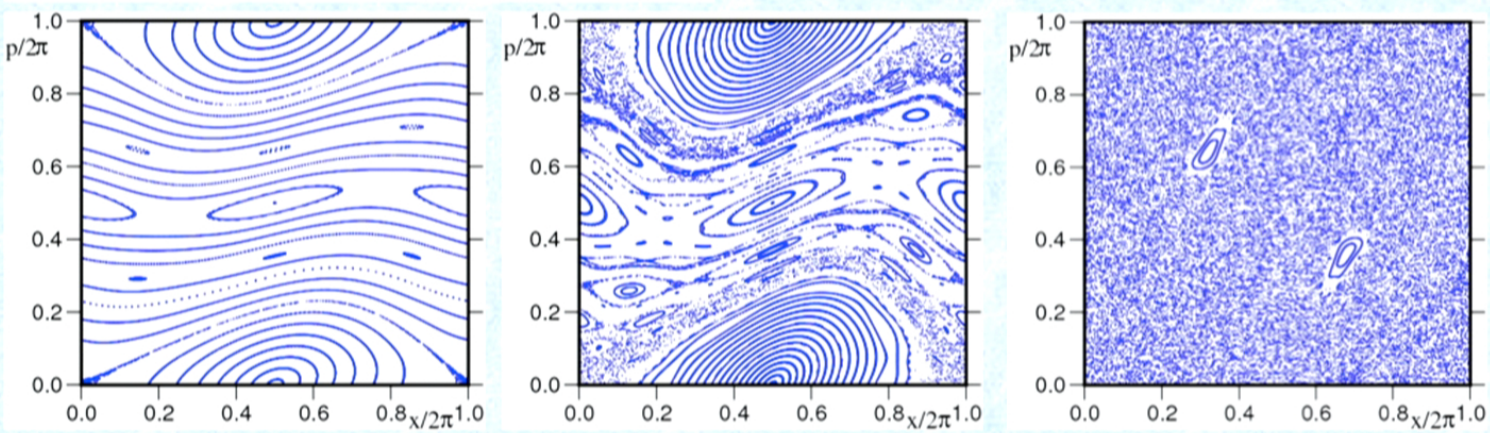
Intuition: chaotic (ergodic) systems. Periodicity does not matter and the system will heat up because it forgets it memory.

Non-ergodic systems: coexistence of regular and chaotic regimes

Kicked rotor: realization of the standard Chirikov's map

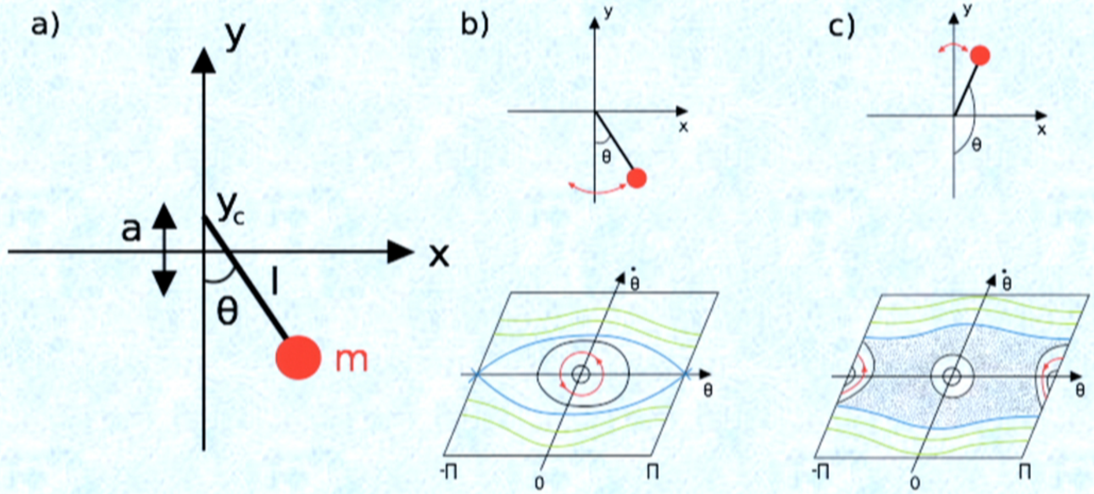
$$H(p,x,t) = \frac{p^2}{2} + K\cos(x)\sum_n \delta(t-n)$$

Transition from regular (localized) to chaotic (delocalized) motion as K increases. Chirikov, 1971



K=0.5 K=K_g=0.971635 K=5 (images taken from scholarpedia.org)

Another example: Kapitza pendulum (emerged from particle accelerators, 1951)



$$\ddot{\theta} = -\left(\omega_0^2 + \frac{a}{l}\gamma^2 \cos(\gamma t)\right)\sin\theta$$

Stable inverted equilibrium for $\frac{a}{l}\frac{\gamma}{\omega_0} > \sqrt{2}$

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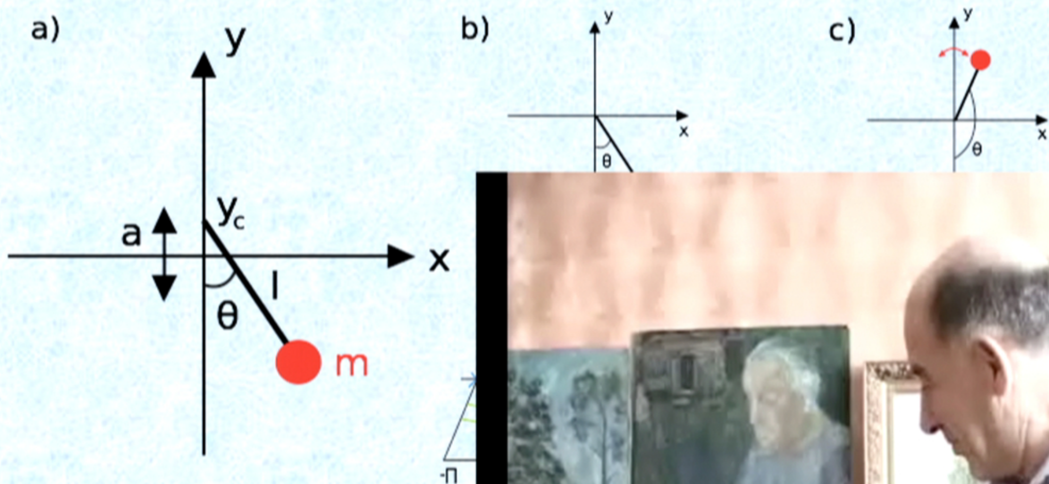

$$\ddot{\theta} = -\left(\omega_0^2 + \frac{a}{l}\gamma^2 \cos\left(\gamma t\right.\right.$$

Stable inverted equilibrium for

Stability: experimentally proven

Singer sewing machine and by Ar

Theoretically proven by Arnold u

theorem

How do we solve equations like this?

Classical systems. Original Kapitza idea: separate fast and slow variables (Landau Lifshitz, volume 1)

$$\ddot{\theta} = -\left(\omega_0^2 + \frac{a}{l}\gamma^2 \cos\left(\gamma t\right)\right) \sin\theta$$

$$\theta(t) = X(t) + \xi(t),\ \xi(t) = A(t)\cos(\gamma t)$$

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After simple manipulations find: X(t) moves in the effective potential

$$U_{\rm eff} = U + \frac{\overline{f^2}}{2m\gamma^2},\ f = \frac{a}{l}\gamma^2 \sin(X)\cos(\gamma t)$$

$$m\ddot{X} = -\frac{\partial U_{\rm eff}}{\partial X},\ U_{\rm eff} = -\omega_0^2\cos(X) + \left(\frac{a\gamma}{2l}\right)^2 \sin^2 X$$

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Can explain the new inverted equilibrium but can not explain transition to the chaotic behavior.

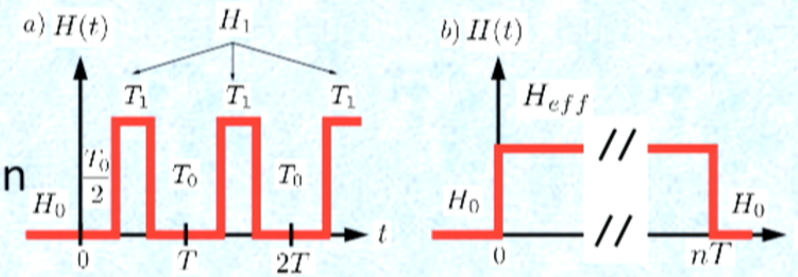
Quantum Systems. Floquet Theory.

$$|\psi(nT)\rangle = [U(T)]^n\, |\psi_0\rangle, \quad U(T) = \exp[-iH_2T_2]\exp[-iH_1T_1] = \exp[-iH_FT]$$

$$|\psi(nT)\rangle = \exp[-iH_FnT]|\psi_0\rangle$$

Time evolution is like a single quench to the Floquet Hamiltonian

$$H_F = H_0 + V,\ V = H_F - H_0.$$



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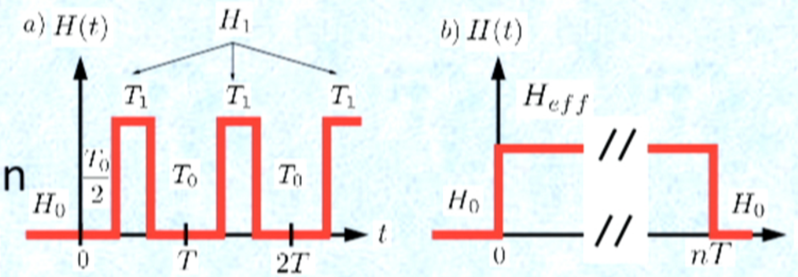

Only works if the Floquet Hamiltonian is local and bounded.

$$H_F = i\hbar\log[U(T)] = i\hbar\log[\exp[-iH_2T_2/\hbar]\exp[-iH_1T_1/\hbar]]$$

Evaluating this log is a very hard problem. Similar to finding free energy in Stat. Mech.

Magnus (short period) expansion – like high temperature expansion

$$H(t) = H_0 + f(t)V$$

$$H_F = i\hbar \log \left[\mathcal{T}_t \exp \left(-\frac{i}{\hbar} \int_0^T \mathrm{d}t H(t) \right) \right]$$

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Expand in powers of T

$$H_F = H_F^{(0)} + H_F^{(1)} + H_F^{(2)} + \ldots$$

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$$\begin{aligned} H_F^{(2)} = & - \frac{1}{6\hbar^2}[H_0,[H_0,V]]\frac{1}{T}\int_{0<t_1<t_2<t_3<T} dt_1 dt_2 dt_3 (f(t_1) + f(t_3) - 2f(t_2)) \\ & + \frac{1}{6\hbar^2}[V,[H_0,V]]\frac{1}{T}\int_{0<t_1<t_2<t_3<T} dt_1 dt_2 dt_3 (f(t_2)f(t_3) + f(t_2)f(t_1) - 2f(t_1)f(t_2)) \end{aligned}$$

Expansion in commutators. Consider symmetric protocols

$$H_F = H_F^0 + \frac{T^2}{\hbar^2}(f_1[H_0,[H_0,V]] + f_2[V,[H_0,V]]) + \dots$$

- Each term in the expansion is extensive and local (like in high temperature expansion)
- Higher order terms are suppressed by the period T but become more and more non-local.
- Competition between suppression of higher order term and their non-locality – similar to many-body localization.
- The expansion is well defined classically if we change commutators to the Poisson brackets.

Two important questions.

1) How can we get non-trivial (non time averaged Hamiltonians) in the high frequency limit? This limit is like a North Star. Can not reach it but gives guidance.

2) Is the Magnus expansion convergent? If yes then the system defies thermodynamics and does not heat up to infinite temperature?
Connected to many body localization, ergodicity, ...

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Getting non-trivial (non time averaged Hamiltonians) in the high frequency limit.

$$H_F = H_F^0 + \frac{T^2}{\hbar^2}(f_1[H_0,[H_0,V]] + f_2[V,[H_0,V]]) + \dots$$

1. Scale amplitude with frequency: V~1/T. Extract the subseries

$$H_F = H_F^0 + c_1\frac{T^2}{\hbar^2}[[H_0,V],V] + c_2\frac{T^4}{\hbar^4}[[[[H_0,V],V],V],V]\dots$$

2. Require that this series can be either resummed or it terminates.

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Example: quadratic kinetic energy. Arbitrary driven potential.

$$H_0 = \sum_j \frac{p_j^2}{2m} + U(\{\theta_j\}) = -\sum_j \frac{\hbar^2}{2m}\partial^2_{\theta_j} + U(\{\theta_j\}), \quad V = \frac{2\pi}{T}v(\{\theta_j\})f(t)$$

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Recover the Kapitza limit (both for quantum and classical systems)

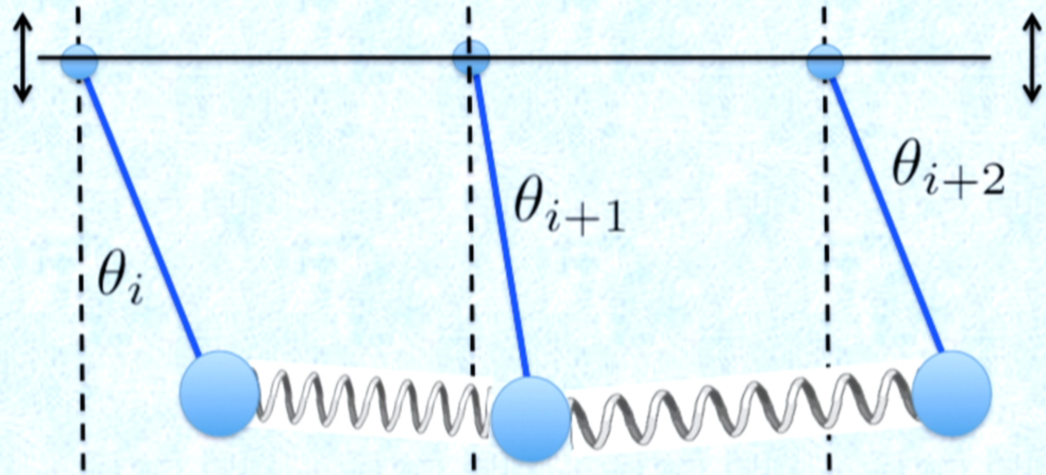
$$H_0 = \frac{p^2}{2} - \omega_0^2\cos(\theta), \quad V = \gamma\lambda cos(\theta)\cos(\gamma t)$$

$$H_F = H_0 + \frac{\lambda^2}{4}\sin^2\theta$$

Inverted potential for $\lambda > \sqrt{2}\omega_0$

Extensions to coupled systems.

Coupled driven oscillators: realization of the Sine-Gordon model



$$H_F = \sum_j \frac{p_j^2}{2} - k\cos(\theta_j - \theta_{j+1}) - \frac{\lambda^2}{8}\cos(2\theta_j)$$

At large *k* this becomes Sine-Gordon (Frenkel-Kontorova) model

Another possibility

$$H_F = H^0 + c_1 \frac{T^2}{\hbar^2}[[H_0, V], V] + c_2 \frac{T^4}{\hbar^4}[[[[H_0, V], V], V], V] \ldots$$

H_0 is an arbitrary interacting Hamiltonian, V is a sum of local, single-particle terms, V is local (not a sum).

$$V = f(t)\sum_j \Delta_j n_j, \quad V = f(t)\sum_j \vec{B}_j \vec{S}_j, \quad V = f(t) n_1 n_2, \ldots$$

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North star – renormalization of hopping.

First subleading corrections: interaction-dependent hopping, second nearest neighbour hopping, effective static potential.

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$[[S_x^1 S_x^2, S_z^1], S_z^1] = S_x^1 S_x^2$ External magnetic field dresses spin-spin interactions

Single-particle driving. Common trick: gauge transformations like in e/m

$$H = -J \sum_{ij} (a_j^\dagger a_i + a_i^\dagger a_j) + n_i v_{ij} n_j + f(t) \sum_j \Delta_j n_j$$

Gauge transformation

$$a_j = \tilde{a}_j \mathrm{e}^{-iF(t)\Delta_j}, \quad F(t) = \int_0^t f(t')dt'$$

Heisenberg equations $i\dot{a}_j = [a_j, H] \Rightarrow i\dot{\tilde{a}}_j + f(t)\Delta_j \tilde{a}_j = [\tilde{a}_j \tilde{H}]$

Single-particle driving. Common trick: gauge transformations like in e/m

$$H = -J\sum_{ij}(a_j^\dagger a_i + a_i^\dagger a_j) + n_i v_{ij} n_j + f(t)\sum_j \Delta_j n_j$$

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$$\tilde{H} = -J\sum_{ij}\left[\tilde{a}_i^\dagger \tilde{a}_j \mathrm{e}^{iF(t)(\Delta_i - \Delta_j)} + h.c.\right] + \tilde{n}_i v_{ij} \tilde{n}_j$$

$\tilde{H}_0 = \overline{\tilde{H}}$ Is an infinite resummation of the Magnus series. Hopping renormalization.

Example

$$H = -J\sum_{ij}(a_j^\dagger a_i + a_i^\dagger a_j) + n_i v_{ij} n_j + f(t)\sum_j \Delta_j n_j, \quad f(t) = \cos(\omega t)$$

$$F(t) = \frac{1}{\omega}\sin(\omega t)$$

$$\tilde{H} = -J\sum_{ij}\left[\tilde{a}_i^\dagger \tilde{a}_j \mathrm{e}^{iF(t)(\Delta_i - \Delta_j)} + h.c.\right] + \tilde{n}_i v_{ij}\tilde{n}_j$$

$$\tilde{H}_0 = \overline{\tilde{H}} = -J\sum_{ij} J_0\left(\frac{\Delta_i - \Delta_j}{\omega}\right)(\tilde{a}_i^\dagger \tilde{a}_j + \tilde{a}_j^\dagger \tilde{a}_i) + \tilde{n}_i v_{ij}\tilde{n}_j$$

Theory: Dunlap and Kenkre (1986), Experiment Struck et. al. (2011)

Example

$$H = -J \sum_{ij} (a_j^\dagger a_i + a_i^\dagger a_j) + n_i v_{ij} n_j + f(t) \sum_j \Delta_j n_j, \quad f(t) = \cos(\omega t)$$

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$$\tilde{H} = -J \sum_{ij} \left[\tilde{a}_i^\dagger \tilde{a}_j \mathrm{e}^{iF(t)(\Delta_i - \Delta_j)} + h.c. \right] + \tilde{n}_i v_{ij} \tilde{n}_j$$

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Theory: Dunlap and Kenkre (1986), Experiment Struck et. al. (2011)

Slight modification: $V(t) = \sum_j \Delta j \cos(\omega t + \phi) + \omega j$

$$\tilde{H}_0 = \overline{\tilde{H}} = -J \sum_{ij} J_1 \left(\frac{\Delta_i - \Delta_j}{\omega} \right) (\mathrm{e}^{i\phi} \tilde{a}_i^\dagger \tilde{a}_j + \mathrm{e}^{-i\phi} \tilde{a}_j^\dagger \tilde{a}_i) + \tilde{n}_i v_{ij} \tilde{n}_j$$

Can be used to create flux lattices (Harper Hamiltonian):
M. Aidelsburger, … I. Bloch. 2013; H. Miyake, … W. Ketterle, 2013

Corrections are often a mess.

Leading corrections to the Harper Hamiltonian (M. Bukov)

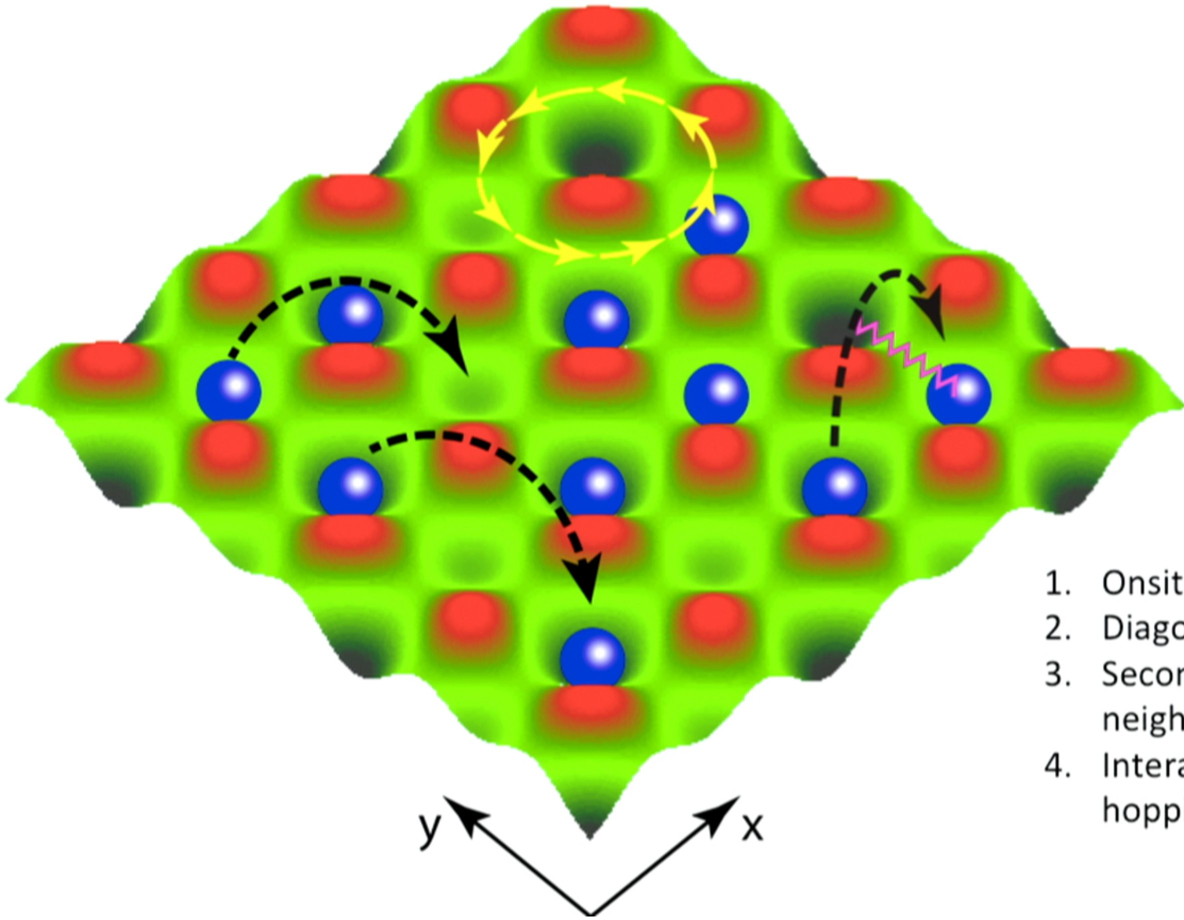
$$H_F^{(2)} = \frac{1}{2Ti\hbar}\int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H[t_2)]$$

$$\begin{aligned}H_F^{(2)} = \; & -\sum_{m,n}\left(\frac{J_x^2}{\Omega}{}^{\rightarrow}C^n_{m,m+2}(z)a^\dagger_{m+2,n}a_{mn} + \frac{J_y^2}{\Omega}{}^{\uparrow}C^{n,n+2}_m(z)a^\dagger_{m,n+2}a_{mn} + \text{h.c.}\right)\\ & -\sum_{m,n}\left(\frac{J_xJ_y}{\Omega}{}^{\nearrow}D^{n,n+1}_{m,m+1}(z)a^\dagger_{m+1,n+1}a_{mn} + \frac{J_yJ_y}{\Omega}{}^{\nwarrow}D^{n,n+1}_{m,m-1}(z)a^\dagger_{m-1,n+1}a_{mn} + \text{h.c.}\right)\\ & +\sum_{m,n}\left(\frac{J_x^2}{\Omega}{}^{\rightarrow}E^n_{m,m+1}(z)(n_{m,n} - n_{m+1,n}) + \frac{J_y^2}{\Omega}{}^{\uparrow}E^{n,n+1}_m(z)(n_{mn} - n_{m,n+1})\right)\\ & -\sum_{m,n}\left(\frac{J_xU}{\Omega}B^n_{m,m+1}(z)a^\dagger_{m+1,n}a_{mn}(n_{mn} - n_{m+1,n} - 1)\right.\\ & \left.\qquad + \frac{J_yU}{\Omega}B^{n,n+1}_m(z)a^\dagger_{m,n+1}a_{mn}(n_{mn} - n_{m,n+1} - 1) + \text{h.c.}\right).\end{aligned}$$

$$z = \frac{\Delta}{\Omega}$$

Leading corrections (down by T):

$$H_F^{(1)} = \frac{1}{2Ti\hbar}\int_0^T dt_1 \int_0^{t_1} dt_2 [H(t_1), H[t_2)]$$



1. Onsite potentials
2. Diagonal hoppings
3. Second nearest neighbour hoppings
4. Interaction dependent hoppings

Driven Dirac type Hamiltonians

$$H(t) = \mathbf{p} \cdot \boldsymbol{\sigma} + A\Omega^2 \cos(\Omega t) V(\mathbf{r}),$$

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Perform a Gauge transformation

$$U^{\mathrm{I}}(t) = \exp\left(-iA\Omega\sin(\Omega t)U(\boldsymbol{r})\right).$$

$$H^{\mathrm{I}}_{\mathrm{rot}}(t) = \; U^{\mathrm{I}}(t)\boldsymbol{p}\left(U^{\mathrm{I}}(t)\right)^{\dagger}\cdot\boldsymbol{\sigma} = \; \boldsymbol{p}\cdot\boldsymbol{\sigma} + A\Omega\sin(\Omega t)\boldsymbol{\nabla}V\cdot\boldsymbol{\sigma}.$$

Obtain time dependent magnetic field

Perform another Gauge transformation, time average

$$U^{\mathrm{II}}(t) = \; \exp\left(iA\cos(\Omega t)W(\boldsymbol{r})\hat{\boldsymbol{n}}\cdot\boldsymbol{\sigma}\right), \quad W(\boldsymbol{r})\hat{\boldsymbol{n}} = \boldsymbol{\nabla}V(\boldsymbol{r})$$

$$\overline{H^{\mathrm{II}}_{\mathrm{rot}}(t)} = \; \frac{1}{2}\{\mathcal{J}_0\left(2AW(\boldsymbol{r})\right), \boldsymbol{p}\cdot\boldsymbol{\sigma} - (\hat{\boldsymbol{n}}\cdot\boldsymbol{p})(\hat{\boldsymbol{n}}\cdot\boldsymbol{\sigma})\}_+ + (\hat{\boldsymbol{n}}\cdot\boldsymbol{p})(\hat{\boldsymbol{n}}\cdot\boldsymbol{\sigma}),$$

Can generate all sorts of spin-orbit type interactions. Rotating normal gives additional freedom.

Can easily understand the scaling from the Magnus expansion.

Dicke (optical lattice) limit

$$H = H_0 + \lambda(t)V + \lambda^*(t)V^\dagger$$

$$H_F^{(1)} = \frac{1}{2T}[V, V^\dagger] \int_0^T dt_1 \int_0^{t_1} dt_2 (\lambda(t_1)\lambda^*(t_2) - c.c.)$$

Get nontrivial limit if scale $V \sim \sqrt{\Omega}$

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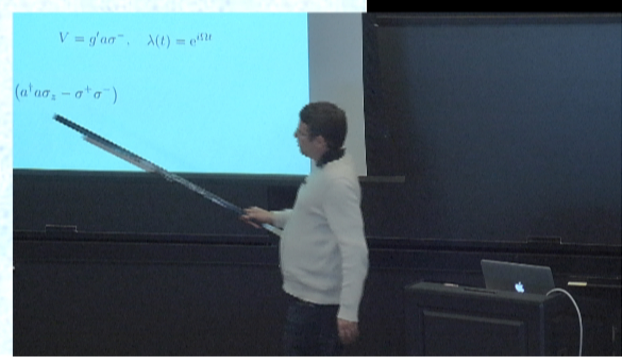
Example: corrections to the Dicke model

$$H_0 = g_0(a^\dagger \sigma^- + a\sigma^+) \qquad V = g' a\sigma^-, \quad \lambda(t) = \mathrm{e}^{i\Omega t}$$

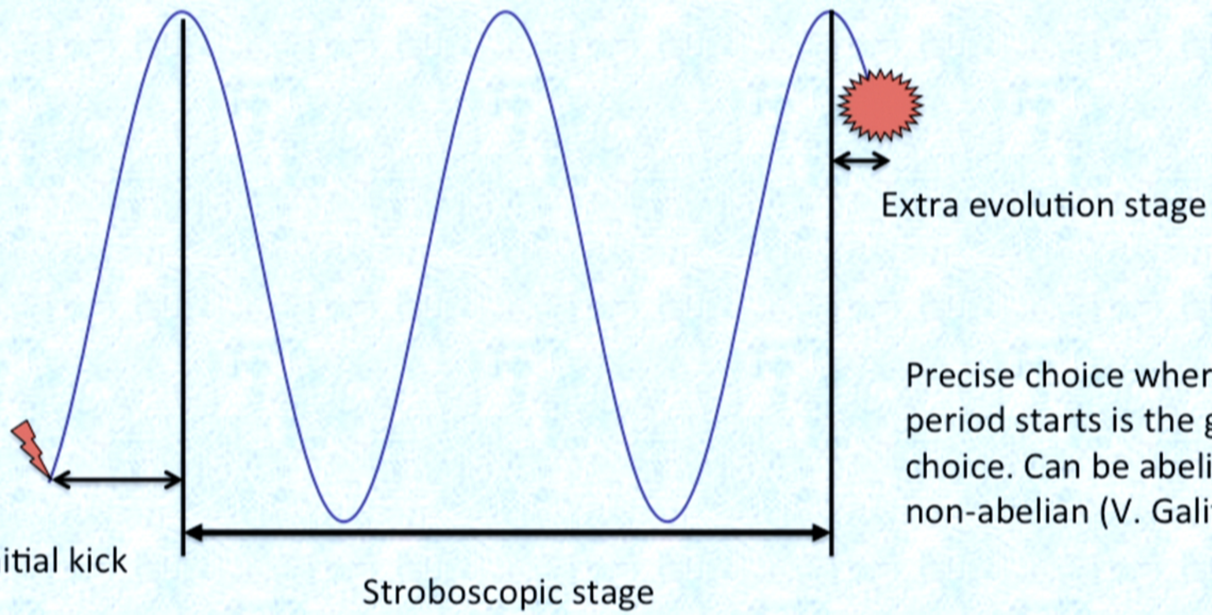
$$H_F^{(1)} = \frac{g'^2}{\Omega}\left(a^\dagger a \sigma_z - \sigma^+\sigma^-\right)$$



The Floquet Hamiltonian describes the stroboscopic evolution

$$|\psi(nT)\rangle = [U(T)]^n\,|\psi_0\rangle, \quad U(T) = \exp[-iH_F T]$$

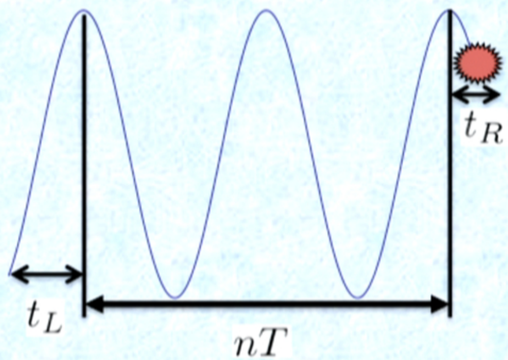
Does it describe other times?



Precise choice where the period starts is the gauge choice. Can be abelian and non-abelian (V. Galitski,…)

Also N. Goldman and J. Dalibard

The situation is not as bad as it seems. Consider the evolution operator



$$U(0,t) = U_R \mathrm{e}^{-iH_F t} U_L, \quad U_L = \mathrm{e}^{iH_F t_L}\, T_t \exp\left[-\int_0^{t_L} H(t')dt'\right]$$

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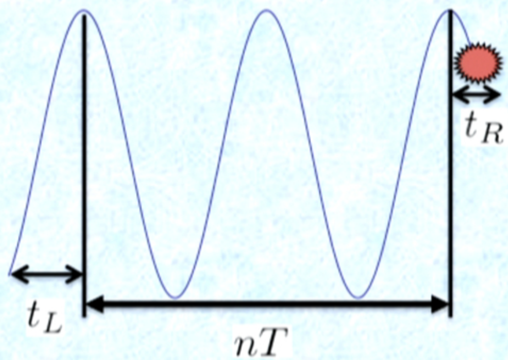

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$$\langle\psi_0|U^\dagger \Omega U|\psi_0\rangle = \langle\psi_0|U_L^\dagger \mathrm{e}^{iH_F t} U_R^\dagger \Omega U_R \mathrm{e}^{-iH_F t} U_L|\psi_0\rangle = \mathrm{Tr}\left[\tilde{\rho}\mathrm{e}^{iH_F t}\tilde{\Omega}\mathrm{e}^{-iH_F t}\right]$$

$$\tilde{\rho} = \overline{U_L|\psi_0\rangle\langle\psi_0|U_L^\dagger}, \quad \tilde{\Omega} = \overline{U_R^\dagger \Omega U_R}$$

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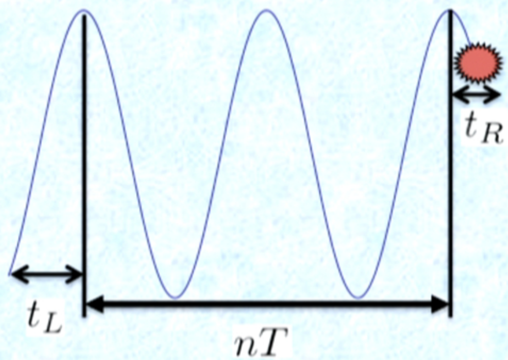

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$$\tilde{\rho} = \overline{U_L|\psi_0\rangle\langle\psi_0|U_L^\dagger}, \quad \tilde{\Omega} = \overline{U_R^\dagger \Omega U_R}$$

The Floquet description is OK, just need to modify the initial density matrix and the observable.

Convergence of Magnus expansion and localization in the energy space

$$U(T) = \exp[-iH_F T] = \log[\exp[-iH_2 T_2] \exp[-iH_1 T_1]]$$

Floquet energies determined modulo. $2\pi/T$

Like in Bloch theorem there is infinite folding in thermodynamic limit.

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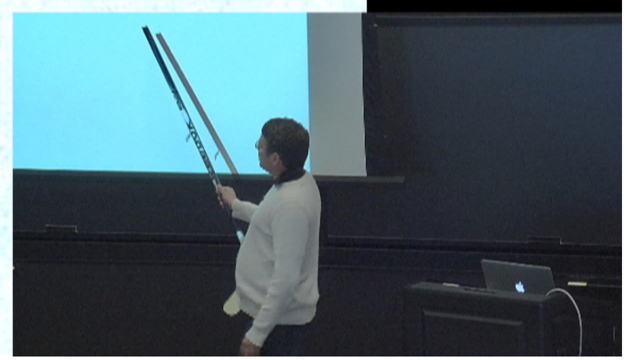
Magnus expansion: unfolded local Hamiltonian in each order

$$H_F = \frac{i}{T}\log[\exp[-iH_1 T_1]\exp[-iH_2 T_2]] = \frac{1}{T}\int_0^T dt H(t) - \frac{i}{2T}\int_0^T dt_1 \int_0^T dt_2\, [H(t_1), H(t_2)] + \dots$$



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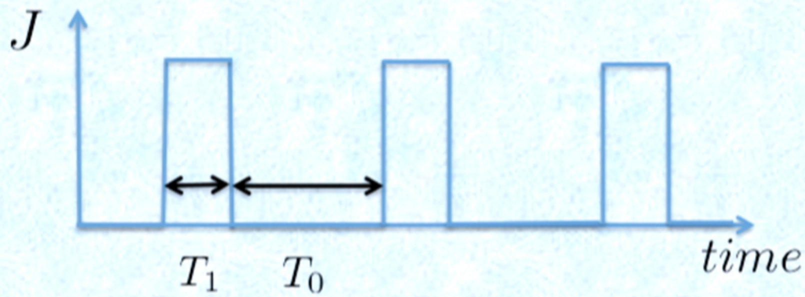
Convergence of the Magnus expansion indicates that the Folding is irrelevant and Floquet Hamiltonian is extensive and local (folding is artificial).

Divergence of the Magnus expansion indicates folding, infinite heating, … If the series is divergent it might take a long time to heat at fast driving (asymptotic series).

Specific model: classical or quantum spin chain (with L. D'Alessio)

$$H = -h\sum_j s_j^z - J\left[g\sum_j s_j^z s_{j+1}^z + \sum_j (s_j^+ s_{j+1}^- + s_j^- s_{j+1}^+)\right]$$



Start in the ground state of the noninteracting system. Follow the noninteracting energy.

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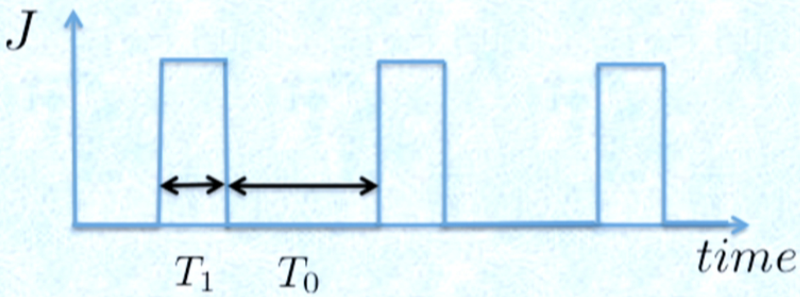

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$$\log[\exp[X]\exp[Y]] = X + Y + \frac{1}{2}[X,Y] + \frac{1}{12}[X,[X,Y]] + \ldots + O(Y^2)$$

$$X = ihT_0\sum_j s_j^z, \quad Y = iT_1 J\left[g\sum_j s_j^z s_{j+1}^z + \sum_j (s_j^+ s_{j+1}^- + s_j^- s_{j+1}^+)\right]$$

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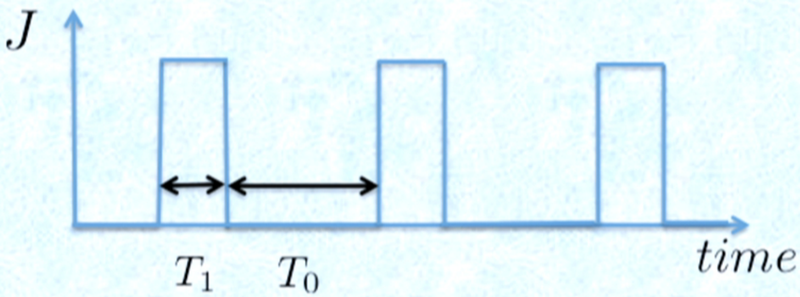

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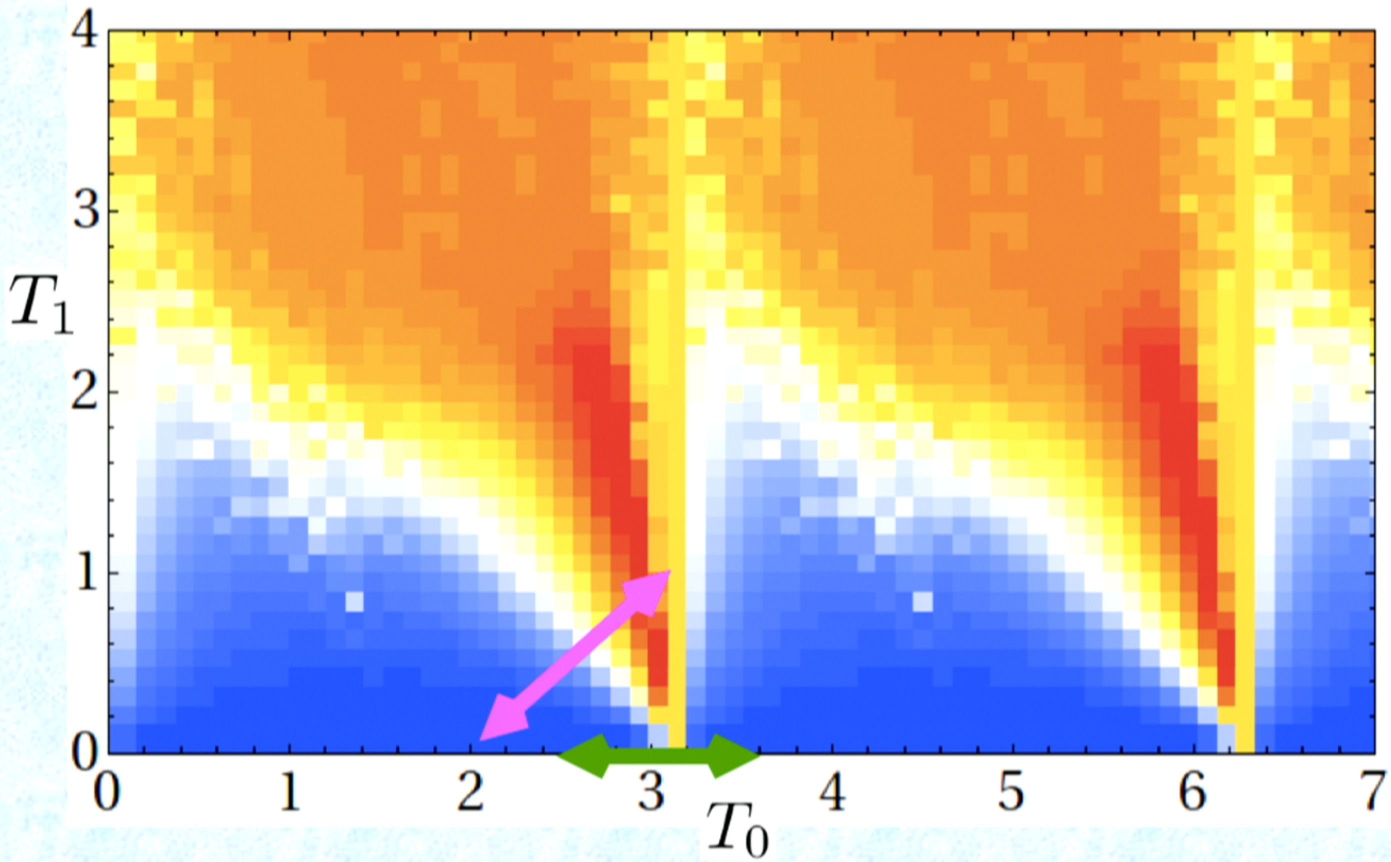
$$H_F = \overline{H} + J(g-1)\frac{T_1}{2(T_1+T_0)}\left(hT_0\cot(hT_0) - 1\right)\sum_j(\sigma_j^z\sigma_{j+1}^z - \sigma_j^y\sigma_{j+1}^y) + \ldots + O(J^2T_1^2)$$

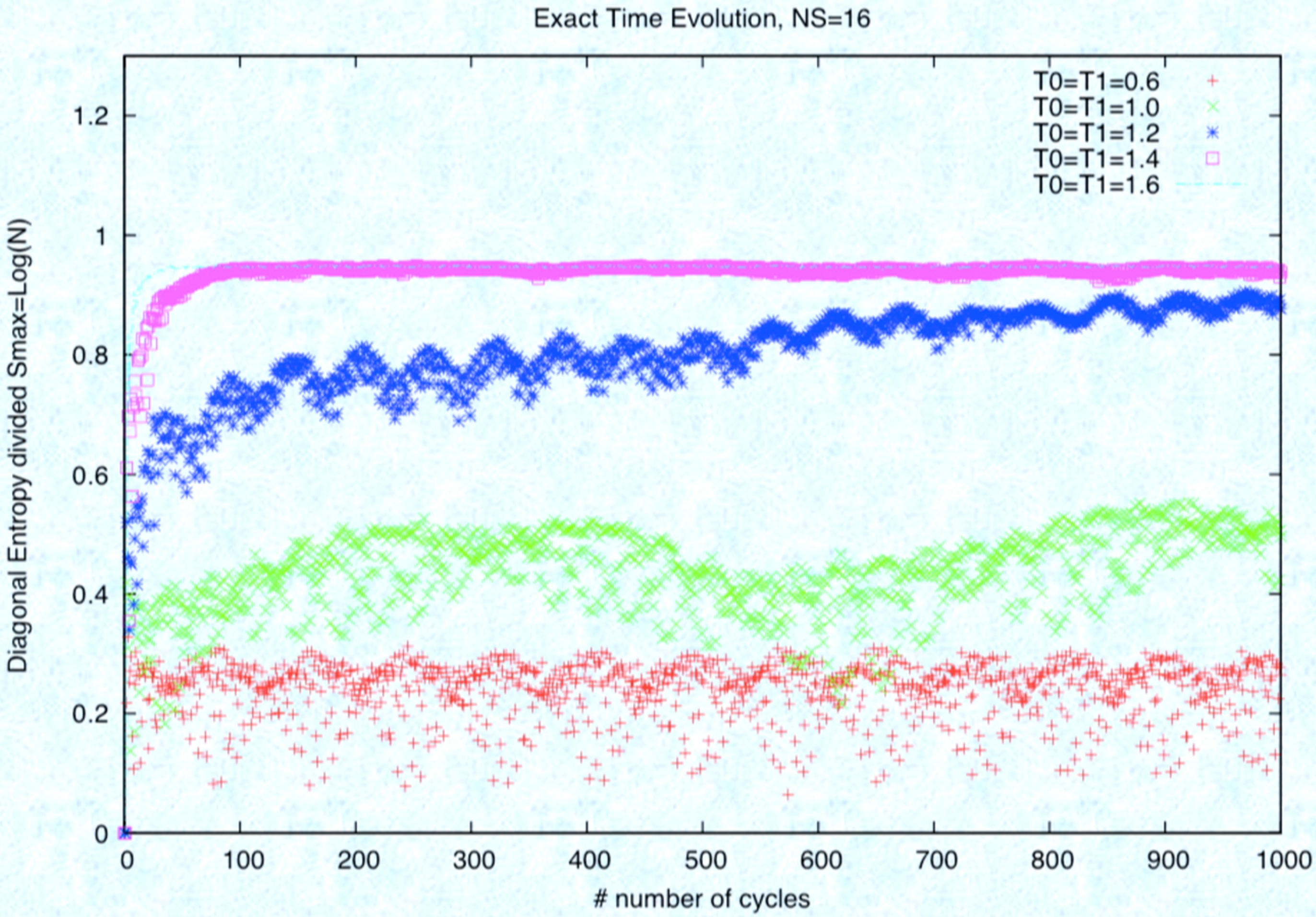
Singularity (phase transition?) at $hT_0 = \pi$

Quantum spin chain: energy in the infinite time limit



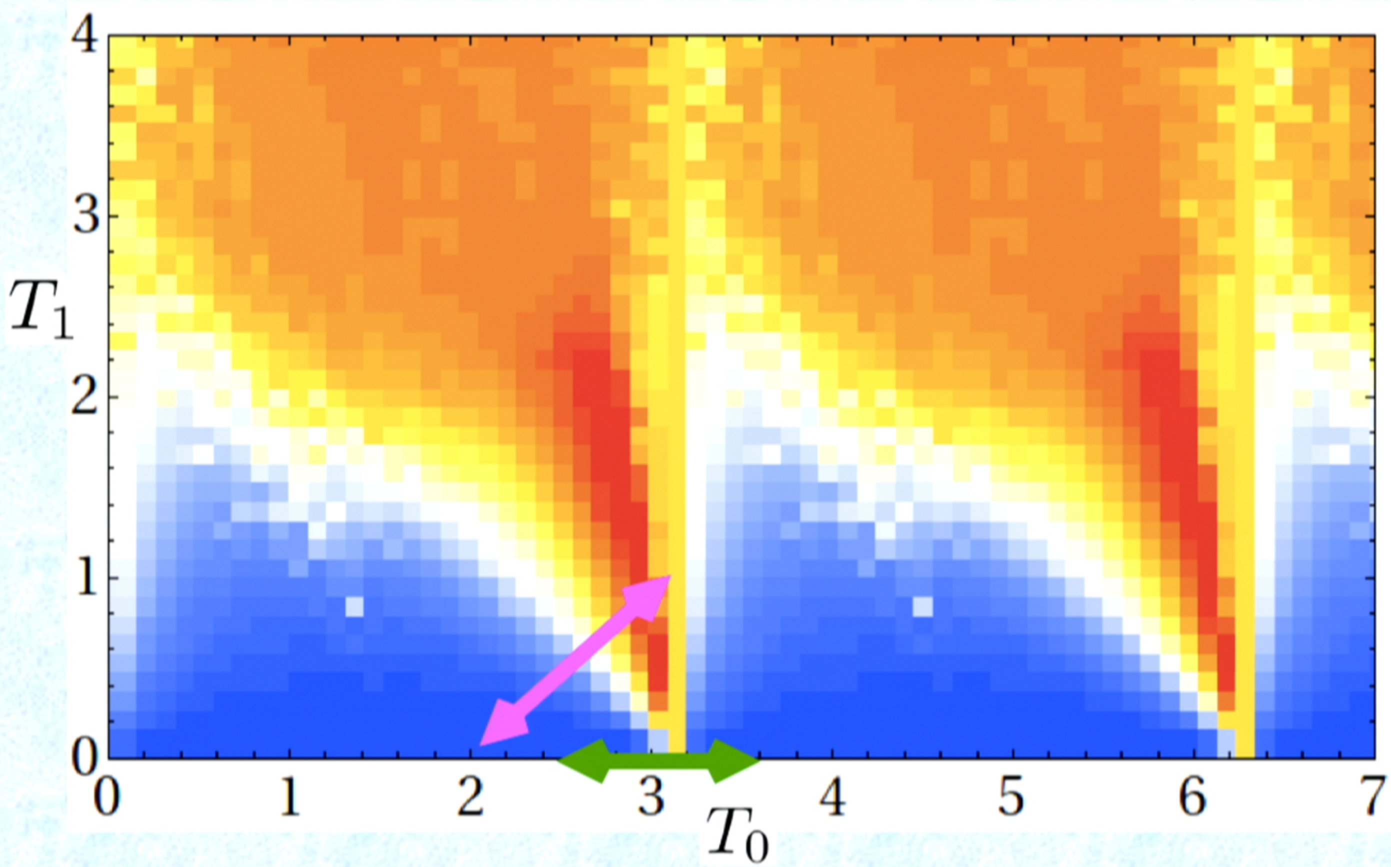
Two different regimes. Is it a crossover or a transition?

Entropy (log of number if occupied states)



Potential implications for driven dissipative systems

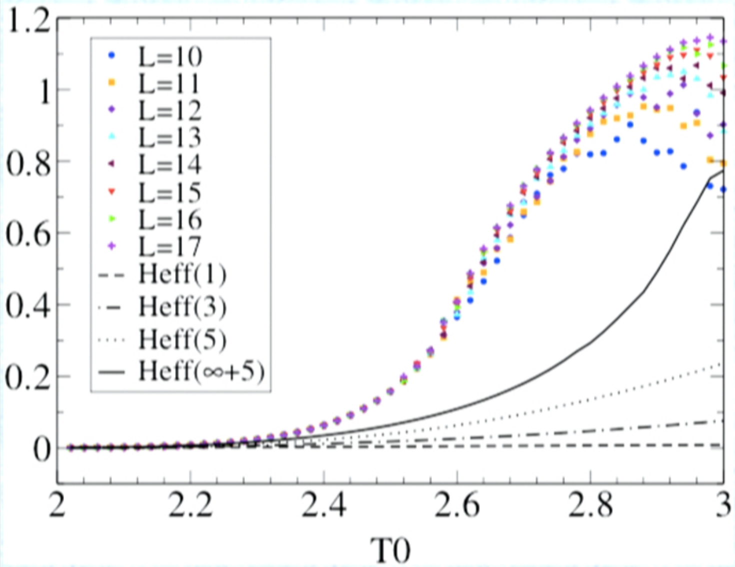
Quantum spin chain: energy in the infinite time limit



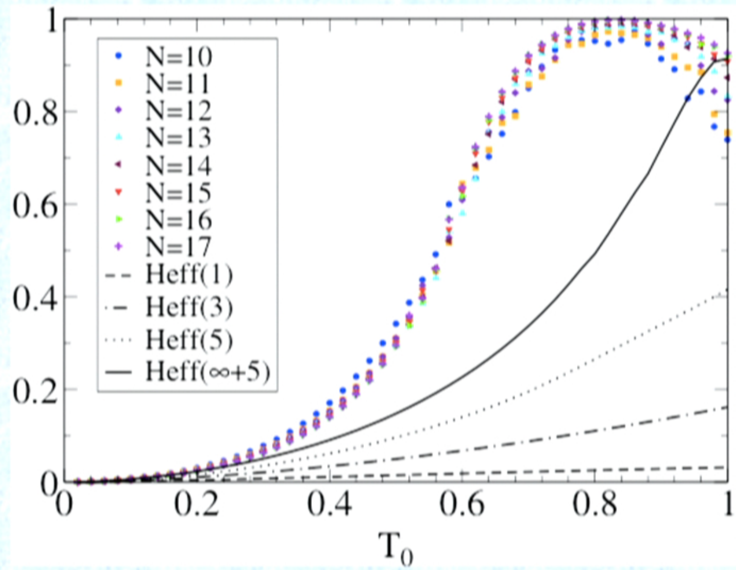
Two different regimes. Is it a crossover or a transition?

Quantum spin chain (comparison with Magnus expansion)

Energy



Entropy



Clear evidence for the phase transition as a function of the driving period.

Almost no size dependence in the localized regime. Similar behavior in classical spin chains.

Can be different in other systems, more in L. D'Alessio talk.

Driving by a local perturbation. Connection with MBL

Pedro Ponte, A. Chandran, Z. Papic, and D. Abanin

Use the following identity

$$e^{-i\hat{V}T_1} = \frac{1+i\hat{G}}{1-i\hat{G}}, \ \hat{G} = -\tan\frac{\hat{V}T_1}{2}$$

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Map the Floquet problem $\mathrm{e}^{-i\hat{H}_0 T_0}\mathrm{e}^{-iVT_1}|\chi_i\rangle = \mathrm{e}^{-i\omega_i T}|\chi_i\rangle$

to the following problem $\left(\tan\frac{\hat{H}_0 T_0 - \omega_i T}{2} - \hat{G}\right)|\chi_i\rangle = 0.$

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Interpret as a hopping problem in Hilbert space of H_0

$$\theta_n \chi_n^{(i)} = \sum_{m\neq n} G_{mn}\chi_m^{(i)}, \quad \theta_n = \tan\left(\frac{E_0^n T_0 - \omega_i T}{2}\right) - G_{nn}$$

States are extended if the hopping is sufficiently strong

Argument by P. Ponte et. al. (applies within the Floquet band)

$$\theta_n \chi_n = \sum_{m \neq n} G_{mn} \chi_m, \quad \theta_n = \tan\left(\frac{E_0^n T_0 - \omega_i T}{2}\right) - G_{nn}$$

Assume H_0 is ergodic. ETH (Mark Srednicki, 1988):

$$G_{mn} \approx \mathrm{e}^{-S(\bar{E})/2} f(\omega_{nm}) \sigma_{nm}, \ \delta\theta_n \approx \mathrm{e}^{-S}$$

In large ergodic systems – always delocalize (with MBL do not)

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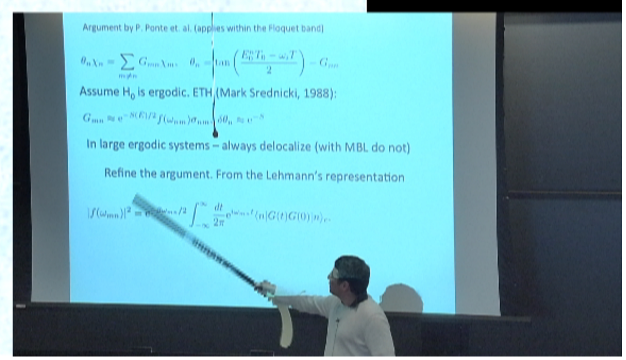
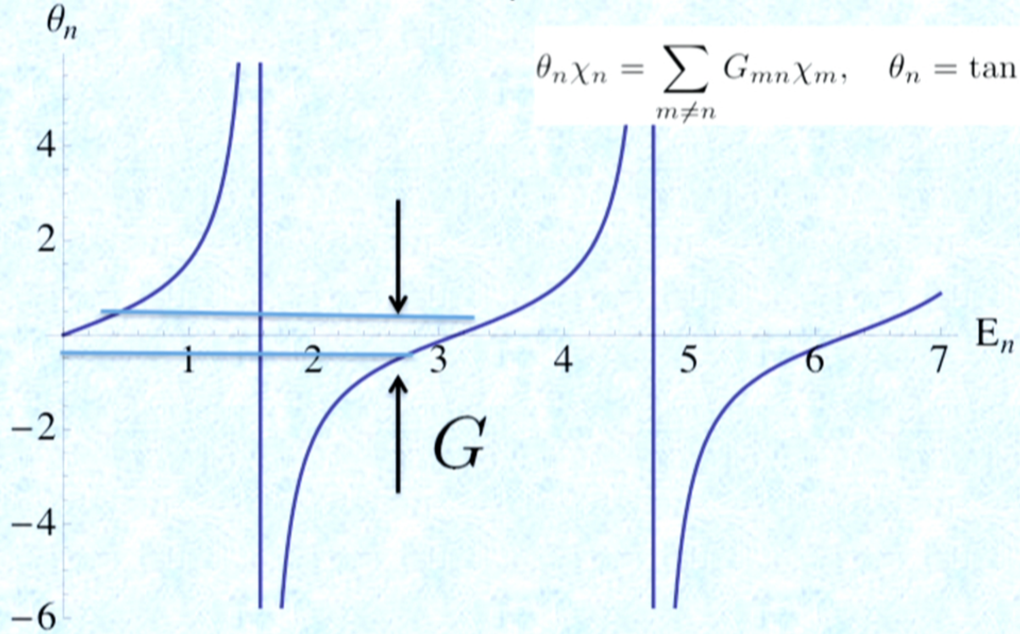
Refine the argument. From the Lehmann's representation

$$|f(\omega_{mn})|^2 = \mathrm{e}^{-\beta\omega_{mn}/2} \int_{-\infty}^{\infty} \frac{dt}{2\pi} \mathrm{e}^{i\omega_{mn} t} \langle n|G(t)G(0)|n\rangle_c.$$



Short period limit

$$\theta_n \chi_n = \sum_{m \neq n} G_{mn} \chi_m, \quad \theta_n = \tan\left(\frac{E_0^n T_0 - \omega_i T}{2}\right) - G_{nn}$$



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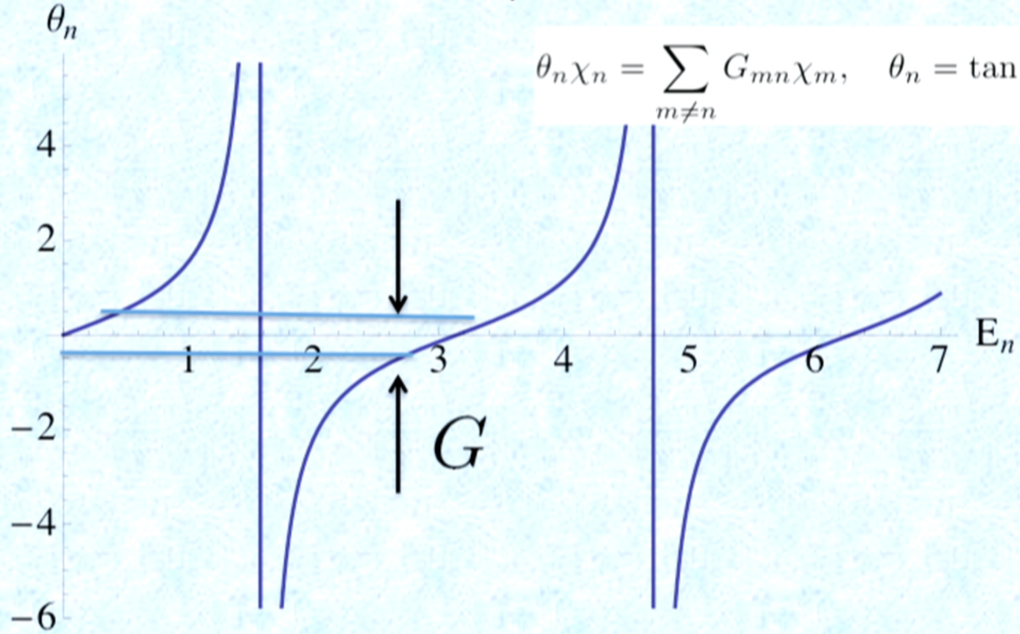

$$\delta\theta \sim \exp[-S] \sim \exp[-\alpha L], \quad G \sim \exp[-S/2] \exp\left[-\frac{\pi^2}{2\Gamma^2 T^2}\right]$$

Conclusion: in the thermodynamic limit for ergodic systems and local driving always heat up. Heating time can be very long (diverges faster than exponential)

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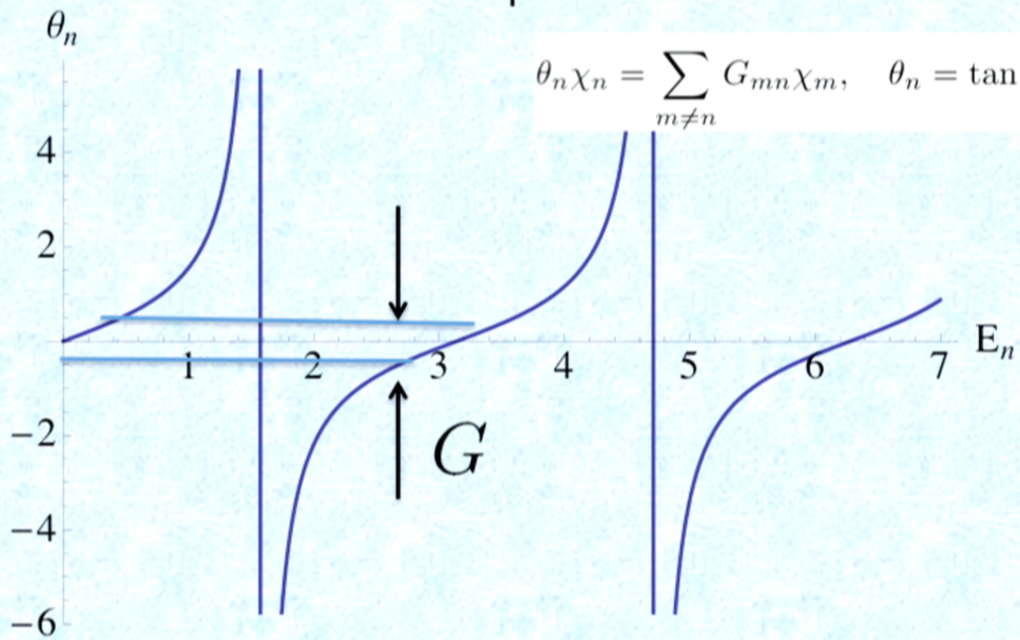

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Conclusion: in the thermodynamic limit for ergodic systems and local driving always heat up. Heating time can be very long (diverges faster than exponential)

$$\delta\theta \sim G \;\Rightarrow\; T \sim \frac{1}{\sqrt{\alpha L}\,\Gamma}$$

Prediction for the localization crossover.

Conclusions.

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4. For local driving heating seems unavoidable in ergodic systems at least. For extended driving there are signatures of much stronger localization (possibly transition).

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