

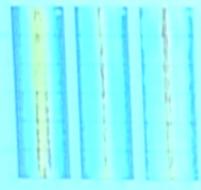
Title: Dynamical analogue quantum simulators

Date: May 12, 2014 11:25 AM

URL: <http://pirsa.org/14050068>

Abstract: Complex quantum systems out of equilibrium are at the basis of a number of long-standing questions in physics. This talk will be concerned on the one hand with recent progress on understanding how quantum many-body systems out of equilibrium eventually come to rest, thermalise and cross phase transitions, on the other hand with dynamical analogue quantum simulations using cold atoms [1-4]. In an outlook, we will discuss the question of certification of quantum simulators, and will show how this problem also arises in other related settings, such as in Boson samplers [5,6]. [1] S. Braun, M. Friesdorf, S. S. Hodgman, M. Schreiber, J. P. Ronzheimer, A. Riera, M. del Rey, I. Bloch, J. Eisert, U. Schneider, arXiv:1403.7199.
[2] M. Kliesch, M. Kastoryano, C. Gogolin, A. Riera, J. Eisert, arXiv:1309:0816.
[3] S. Trotzky, Y.-A. Chen, A. Flesch, I. P. McCulloch, U. Schollwoeck, J. Eisert, I. Bloch, Nature Physics 8, 325 (2012).
[4] A. Riera, C. Gogolin, M. Kliesch, J. Eisert, in preparation (2014).
[5] C. Gogolin, M. Kliesch, L. Aolita, J. Eisert, in preparation (2014) and arXiv:1306.3995.
[6] S. Aaronson, A. Arkhipov, arXiv:1309.7460.

Dynamical analogue quantum simulators Probing quantum many-body systems out of equilibrium



Jens Eisert

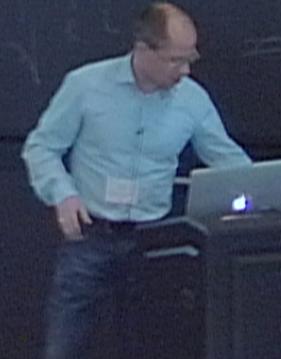
TUM Universität Bayreuth



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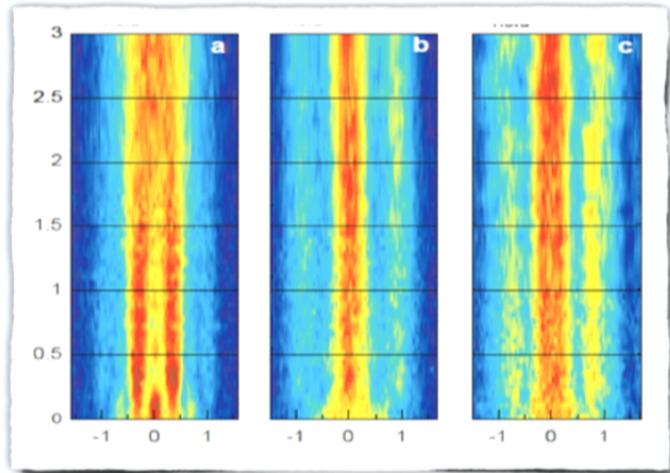
Quantum many-body dynamics, Program, Institute für Theorie der Kondensationsphänomene, Institut für Theoretische Physik, Universität Regensburg, 11.-12.12.2014

Montagvormittag mit M. Friesen, C. Guglielmi, R. Kosfeld, A. Krause, S. Krämer, S. Krätzschmar, J. Lippert, A. Pfeifer, J. Schmid, S. Schmid, S. Stöckl



Dynamical analog(ue) quantum simulators

Probing quantum many-body systems out of equilibrium



Jens Eisert
Freie Universität Berlin

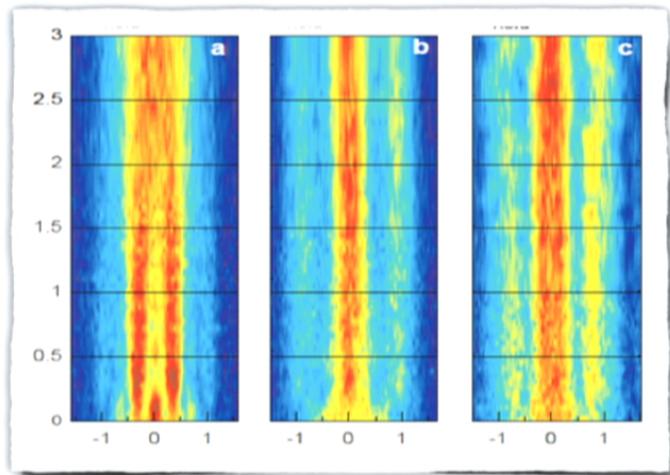


Quantum many-body dynamics, Perimeter Institute, May 12, 2014

Mentions joint work with M. Friesdorf, C. Gogolin, M. Kliesch, A. Riera, M. del Rey, S. Braun, S. Trotzky, I. McCulloch, A. Flesch, Y.-U. Chen, U. Schneider, and I. Bloch

Dynamical analog(ue) quantum simulators

Probing quantum many-body systems out of equilibrium



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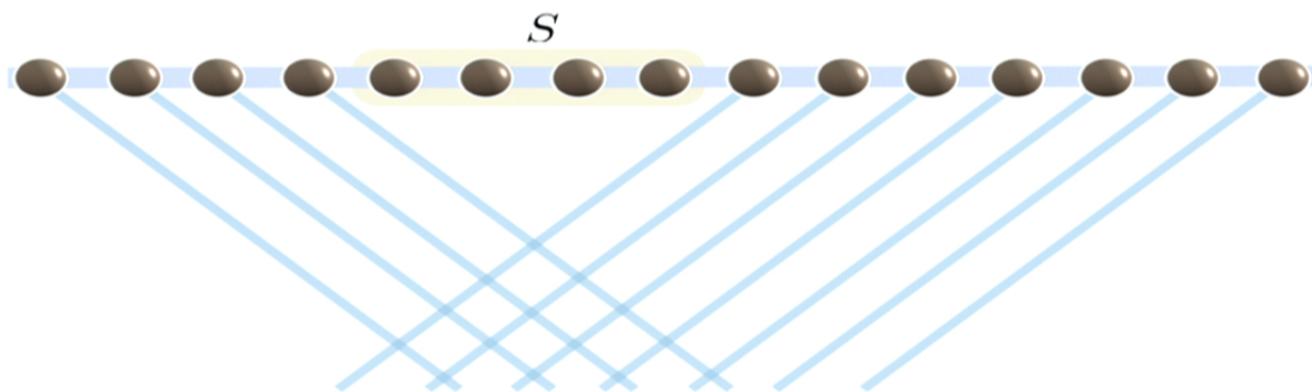
Freie Universität Berlin



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Equilibration (to generalised Gibbs ensemble) with time scales



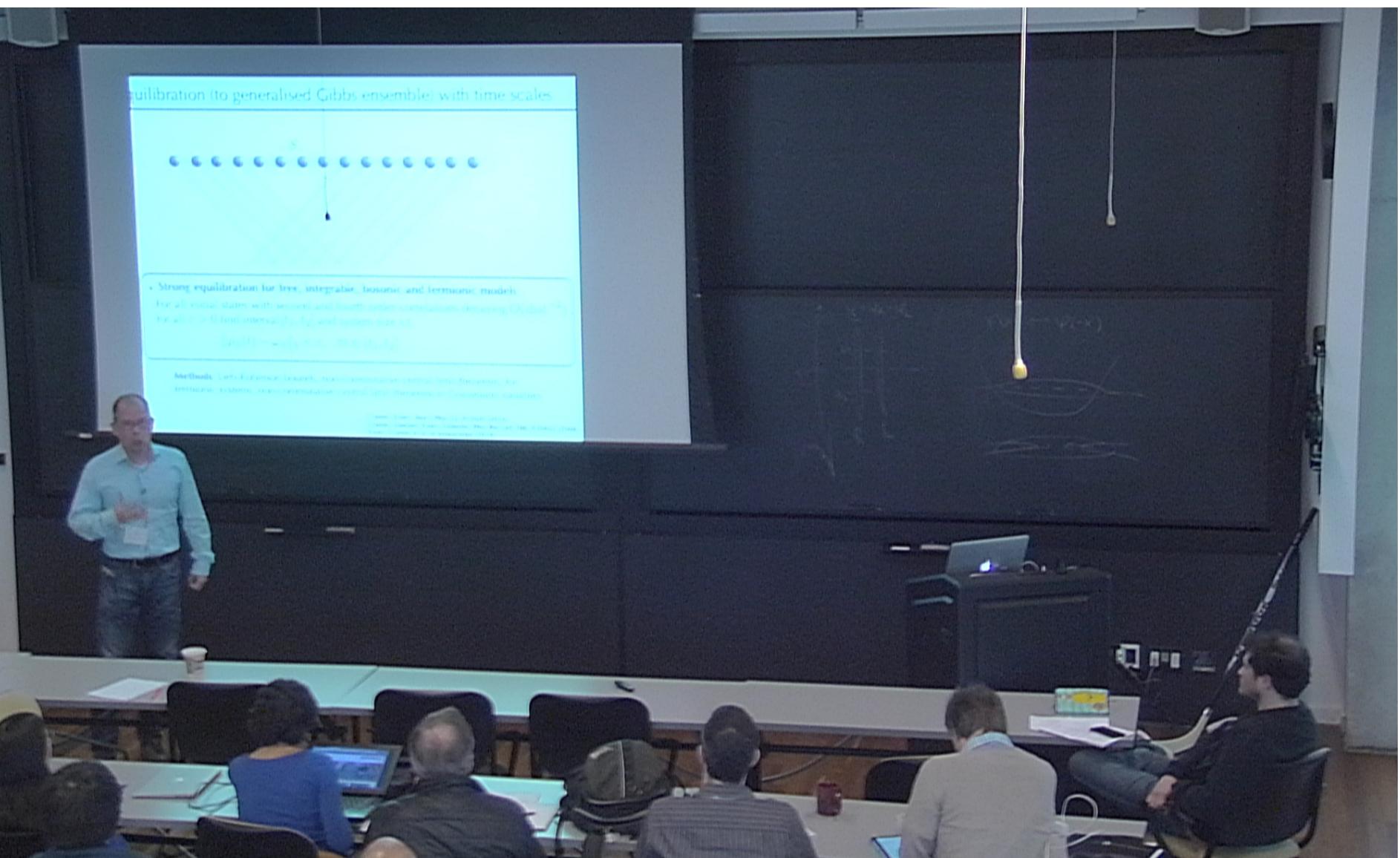
- **Strong equilibration for free, integrable, bosonic and fermionic models**

For all initial states with second and fourth order correlations decaying $O(\text{dist}^{-2})$, for all $\varepsilon > 0$ find interval $[t_1, t_2]$ and system size s.t.

$$\|\rho_S(t) - \omega_S\|_1 < \varepsilon, \quad \forall t \in [t_1, t_2]$$

Methods: Lieb-Robinson bounds, non-commutative central limit theorems, for fermionic systems, non-commutative central limit theorems in Grassmann variables

Cramer, Eisert, *New J Phys* **12**, 055020 (2010)
Cramer, Dawson, Eisert, Osborne, *Phys Rev Lett* **100**, 030602 (2008)
Eisert, Cramer et al, in preparation (2014)



quilibration in expectation

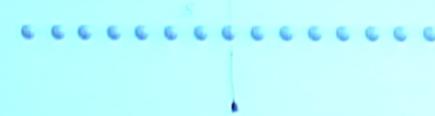


- **Equilibration in expectation:** For a Hamiltonian with non-degenerate energy gaps

$$\mathbb{E}(\|\rho_S(t) - \omega_S\|_1) \leq \frac{1}{2} \sqrt{\frac{d_S^2}{d^{\text{eff}}}} , \quad d^{\text{eff}} = \frac{1}{\sum_k |\langle E_k | \psi_0 \rangle|^4}$$

Linden, Popescu, Short, Winter, *Phys Rev E* **79**, 061103 (2009)
Short, Farrelly, arXiv:1110.5759
Reimann, *Phys Rev Lett* **101**, 190403 (2008)
Gogolin, Mueller, Eisert, *Phys Rev Lett* **106**, 040401 (2011)

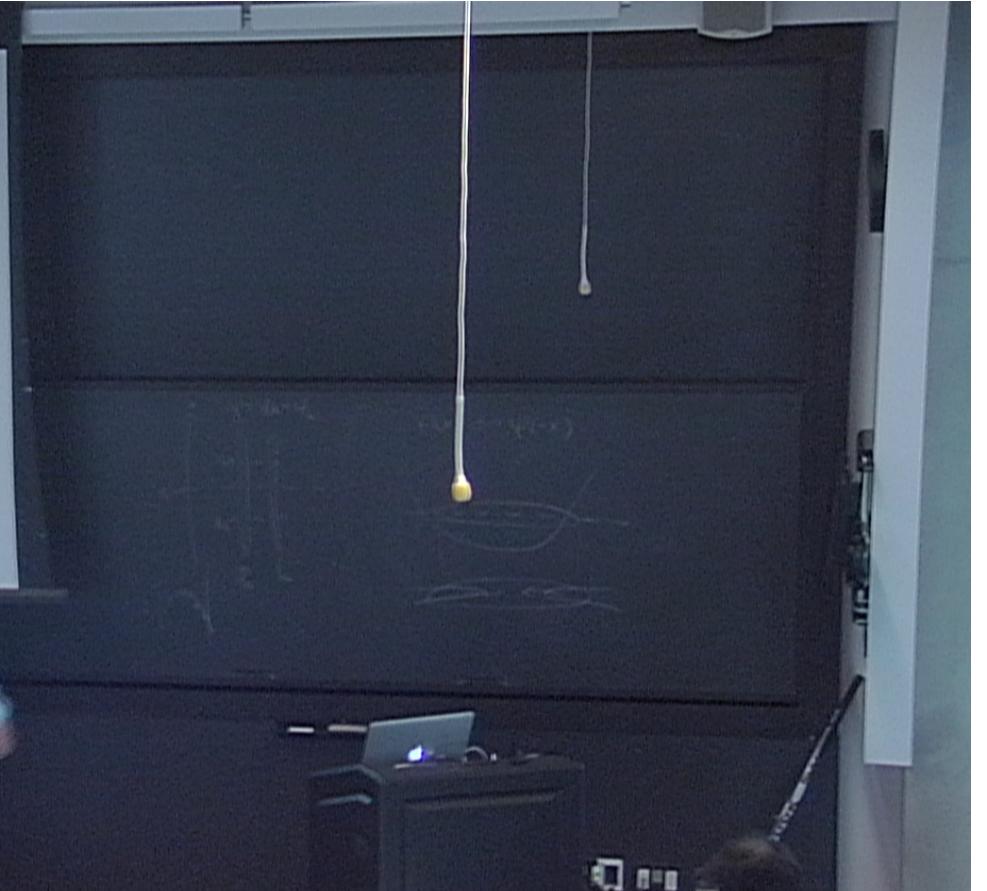
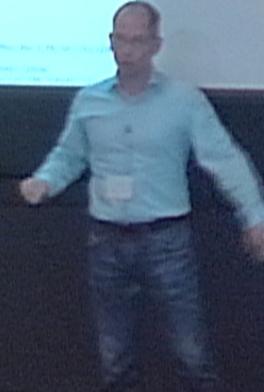
Equilibration in expectation



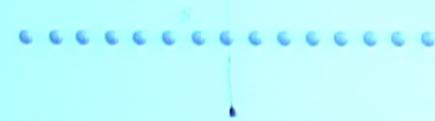
• Equilibration in expectation: For a Hamiltonian with non-degenerate energy levels

$$\text{Expectation value: } \langle S \rangle = \frac{1}{N} \sum_{i=1}^N S_i$$

Source: Wikipedia, David Wallace, Max Tegmark, The Feynman Lectures on Physics, Volume III, Chapter 1, Section 10, Nondegenerate Energy Levels, Nondegenerate Ground States, Nondegenerate Excited States, Nondegenerate Resonances, Nondegenerate Bound States, Nondegenerate Unbound States.



Equilibration in expectation



- Equilibration in expectation: For a Hamiltonian with non-degenerate energy levels

$$\text{Equilibrium distribution: } \frac{e^{-\beta E_i}}{\sum e^{-\beta E_j}}$$

Source: Physics 201: Modern Physics by Michael D. Scully, Marlan O. Scully, and M. S. Zubairy
Section 10.2: Equilibrium Distribution Functions



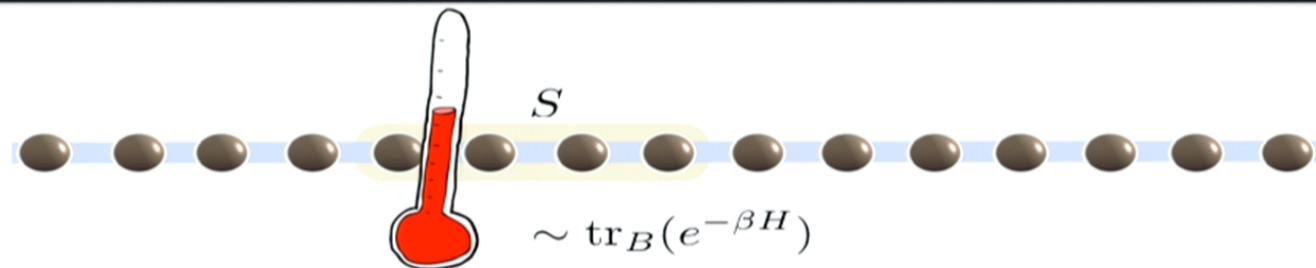
- **Lesson:** Systems generically locally "appear relaxed", although dynamics is entirely unitary
- Proven in *strong sense* for general states in *integrable limit* of Bose-Hubbard model
- True in slightly weaker sense generically

- **Time scales for non-integrable models?**

- **Lesson:** Systems generically locally "appear relaxed", although dynamics is entirely unitary
- Proven in *strong sense* for general states in *integrable limit* of Bose-Hubbard model
- True in slightly weaker sense generically

- **Time scales for non-integrable models?**

Strategies towards thermalisation



Assumptions about $|E_k\rangle$

"Eigenstate thermalization hypothesis"

Assumptions about $\langle E_k | \psi_0 | E_k \rangle$

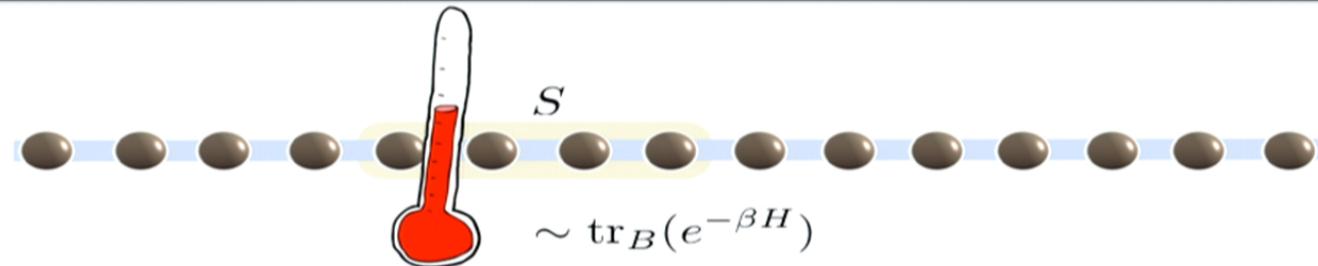
This approach

Thermalisation

Klich, *Phys Rev A* **43**, 2046 (1991)
Innicki, *Phys Rev E* **50**, 888 (1994)
Dunjko, Olshanii, *Nature* **452**, 854 (2008)

Riera, Gogolin, Eisert, *Phys Rev Lett* **108**, 080402 (2012)
Riera, Kliesch, Gogolin, Eisert, *in preparation* (2014)

Thermalization



• Progress on thermalisation question

- Consider **1D local Hamiltonians** H of n sites, translationally invariant,
- Initial states with a flat energy distribution in $[nE, n(E + \Delta)]$ locally equilibrate towards a Gibbs state, even if they are **initially far from equilibrium**: can bound

$$\|\text{tr}_B(e^{-\beta H})/Z - \omega_S\|_1 \leq \delta$$

with δ that depends on Δ , E , and basic spectral properties of $H = \sum_j h_j$

Methods: Spectral perturbation theory, concentration of measure and typicality tools

Riera, Gogolin, Eisert, *Phys Rev Lett* **108**, 080402 (2012)
Riera, Gogolin, Eisert, in preparation (2014)

Mueller, Adlam, Masanes, Wiebe, arXiv:1312.7420
Tasaki, *Phys Rev Lett* **80**, 1373 (1998)
Popescu, Short, Winter, *Nature Phys* **2**, 754 (2006)
Goldstein, Lebowitz, Tumulka, Zanghi, *Phys Rev Lett* **96**, 050403 (2006)

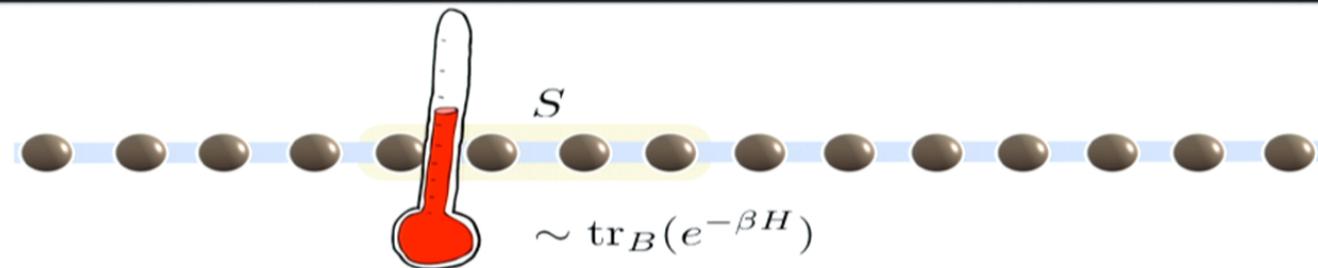
Normalizat



↳ Lesson: very weakly coupled systems with same initial energy thermalize



thermalization



- **Lesson:** Very weakly coupled systems with narrow initial energies thermalise

- Criteria for models *not* thermalising: If effective entanglement in eigenbasis

$$R(\psi_0) = \sum_k |\langle E_k | \psi_0 \rangle|^2 \| \text{tr}_B (|E_k\rangle \langle E_k| - \psi_0^S) \|_1$$

is small, infinite memory of initial condition remains

Gogolin, Mueller, Eisert, Phys Rev Lett 106, 040401 (2011)

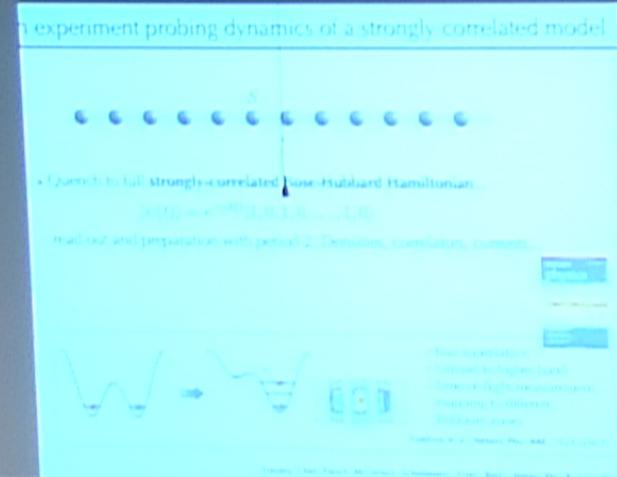
- Present in many-body localised models

Pal, Huse, Phys Rev B 82, 174411 (2010)
Bauer, Nayak, J Stat Mech, P09005 (2013)

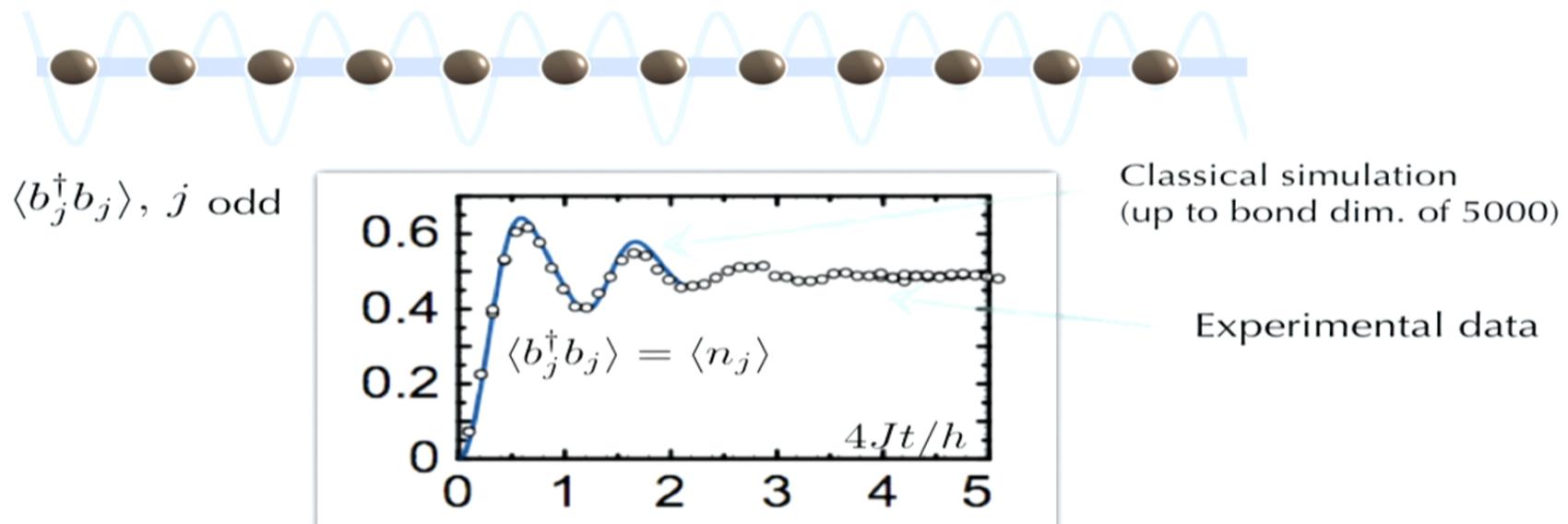
- Robust many-body translationally invariant models?

Compare Matthew's, Arijet's talks

- **Many questions wide open**



Classical matrix product state simulation

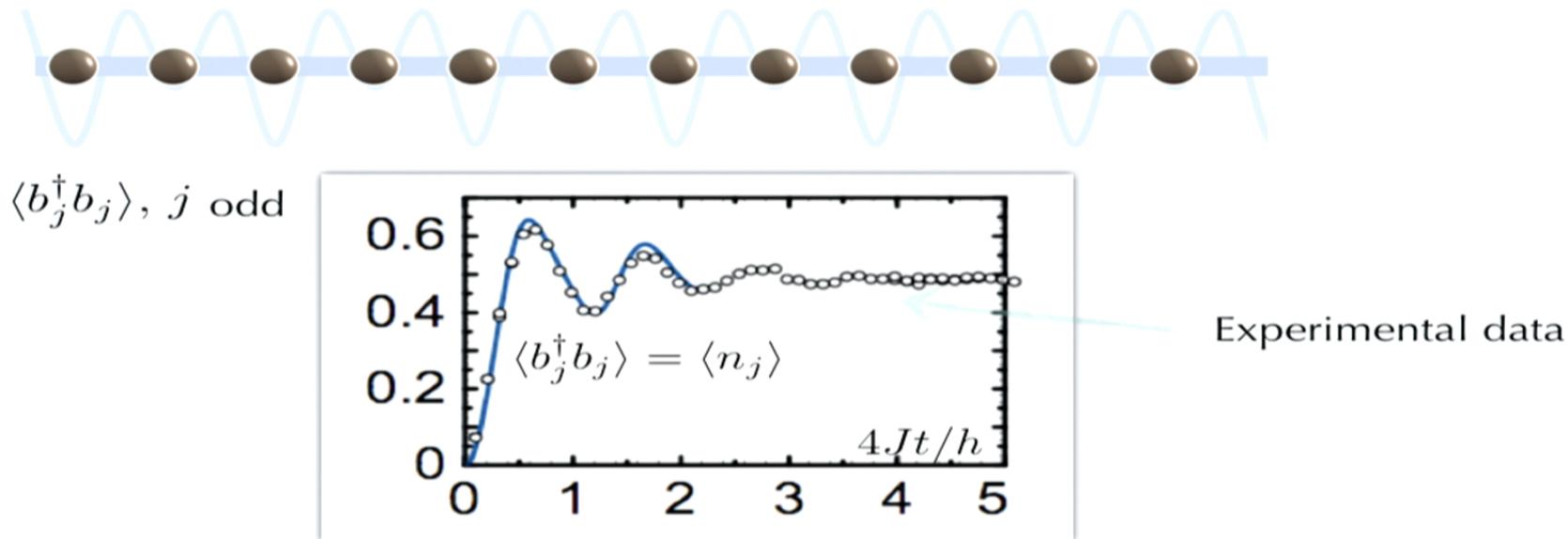


- **Observation 4: Short times matrix-product state (MPS) simulation**

- **Short times:** Practically to machine precision with t-DMRG (5000 MPS-dim)
- Variational method based on MPS
- **Long times:** Exponential blow-up of bond dimension in time

Schollwoeck, *Rev Mod Phys* **77**, 259 (2005)
Verstraete, Cirac, Murg, *Adv Phys* **57**, 143 (2008)
Eisert, Cramer, Plenio, *Rev Mod Phys* **82**, 277 (2010)

uantum simulation

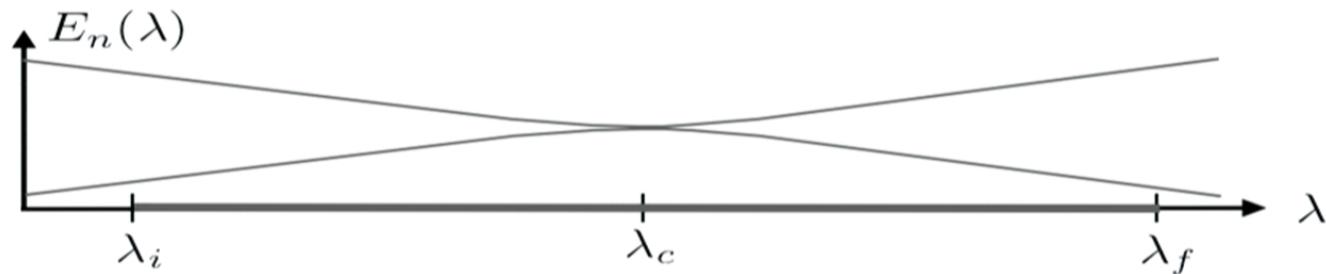


- **Observation 5: Long time dynamics of many-body dynamics in experiment**
- Can accurately probe dynamics for long times (exp vs poly decay, ...)
- "Outperforms" **best available classical simulation** on supercomputers

Trotzky, Chen, Flesch, McCulloch, Schollwoeck, Eisert, Bloch, *Nature Phys* **8**, 325 (2012)

Crossing critical lines

- **Gapped phases:** "Adiabatic theorem ensures equilibrium"
- **Crossing a critical point:** Never sufficiently slow



bble Zurek narrative

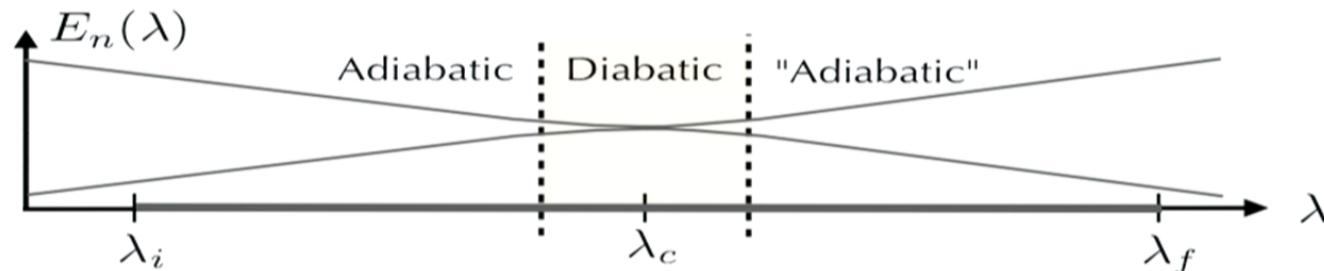
Crude intuitive picture: Dynamics are simplified to

- **Adiabatic regime** $\frac{d\Delta}{dt} \ll \Delta$
- **Diabatic regime**

Universality close to critical point $\xi \sim |\lambda - \lambda_c|^{-\nu}$

$$\Delta \sim |\lambda - \lambda_c|^{z\nu}$$

Predicts **power laws** for correlation lengths in quench time



Zurek, Dorner, Zoller, *Phys Rev Lett* **95**, 105701 (2005)
Dziarmaga, Rams, *New J Phys* **12**, 055007 (2010)
Del Campo, De Chiara, Morigi, Plenio, Retzker, *Phys Rev Lett* **105**, 075701 (2010)

bble Zurek narrative

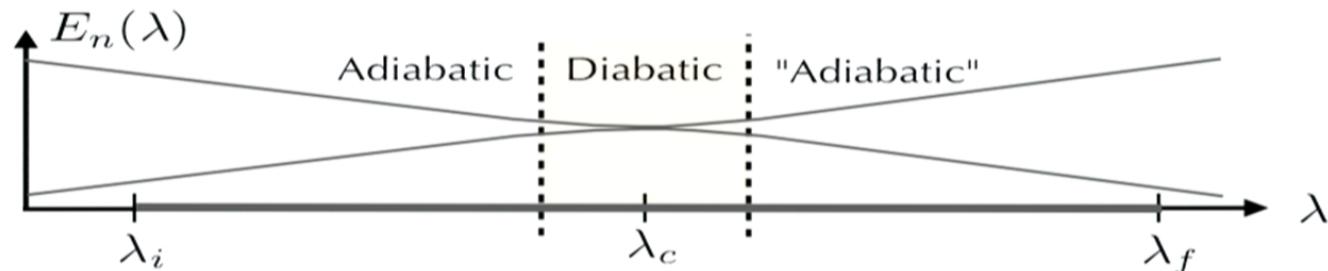
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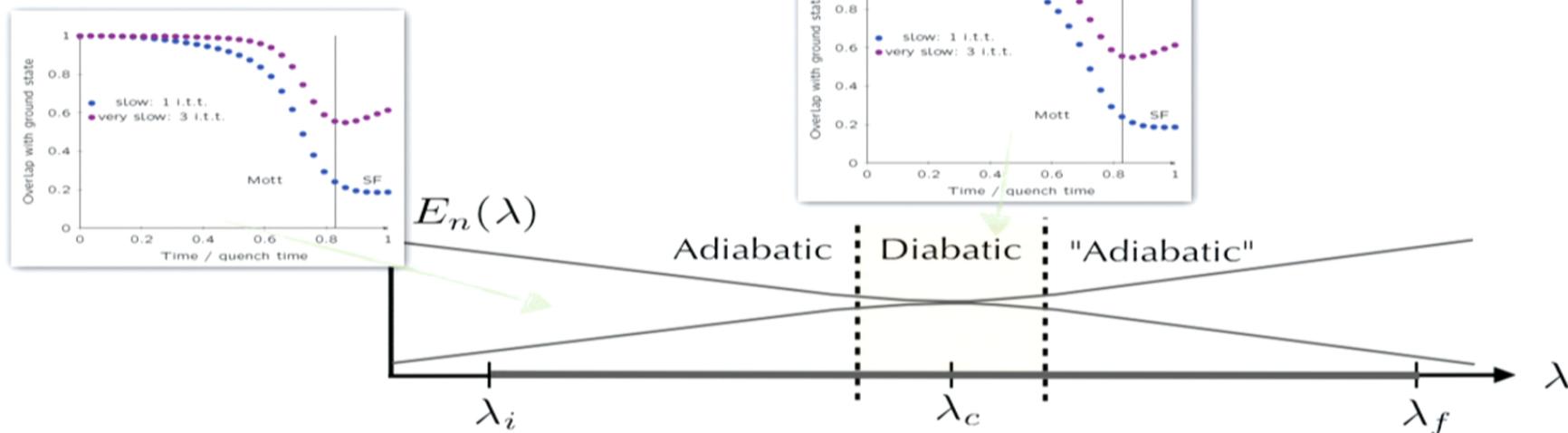
Zurek, Dorner, Zoller, *Phys Rev Lett* **95**, 105701 (2005)
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bble Zurek narrative

Crude picture is often too crude

Huge literature, no complete understanding

Inequivalent approaches: Free models, adiabatic perturbation theory, scaling collapse



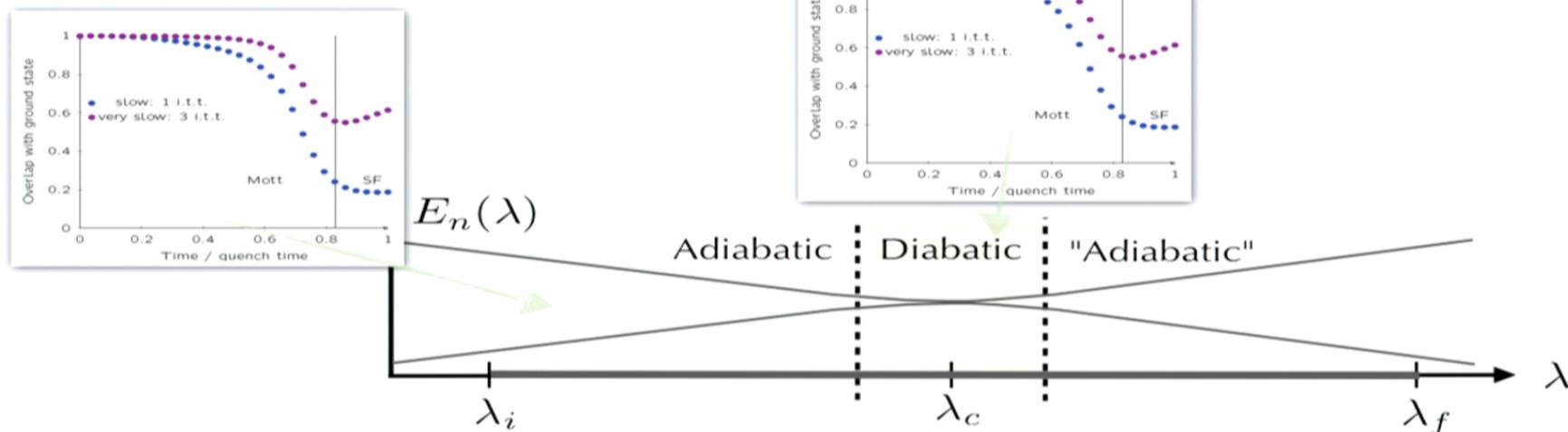
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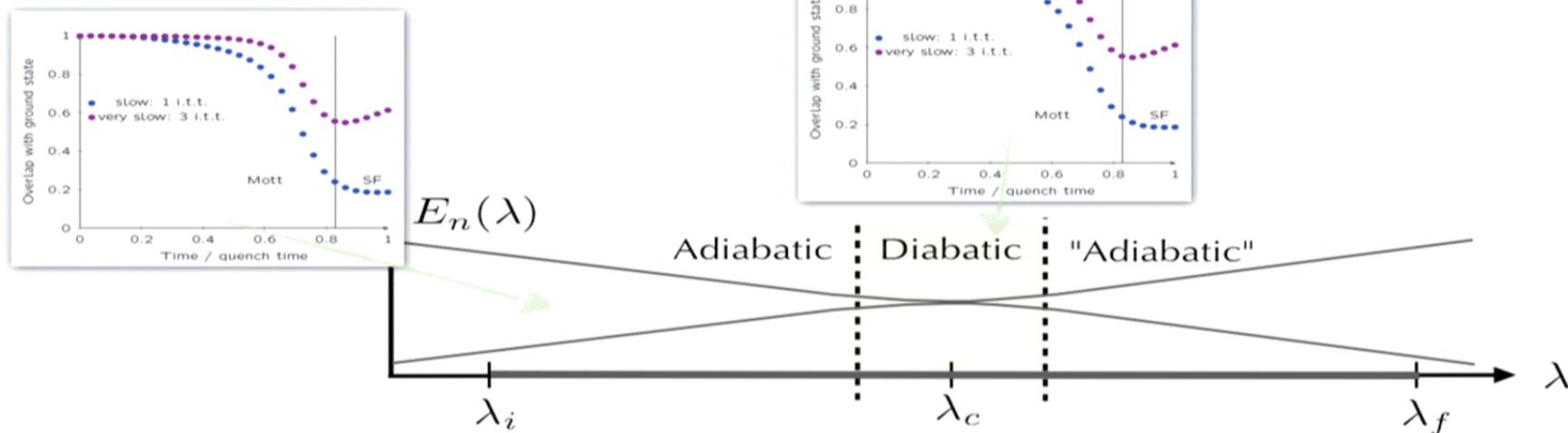
Zurek, Dorner, Zoller, *Phys Rev Lett* **95**, 105701 (2005)
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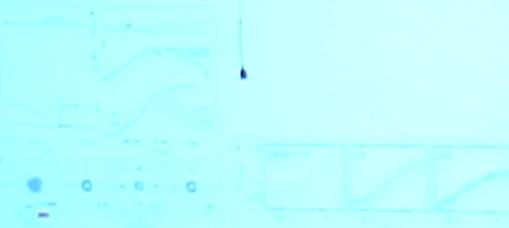
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Dziarmaga, Rams, *New J Phys* **12**, 055007 (2010)
Del Campo, De Chiara, Morigi, Plenio, Retzker, *Phys Rev Lett* **105**, 075701 (2010)

in experiment

Experiment in Immanuel Bloch/Uli Schneid's lab: Mott insulator quench



Heavy numerics, DMRG and exact diagonalisation in 1D

Uwe Römer, Immanuel Bloch, Uli Schneid, Numerical simulation of quantum quenches

Careful study of effects of trap - excellent agreement with data

From: U. Schneid, I. Bloch, U. Römer, Phys. Rev. Lett. 106, 040401 (2011)



Experiment

Experiment in Immanuel Bloch/Uli Schneid's lab: Mott insulator quench

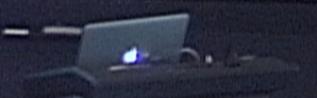
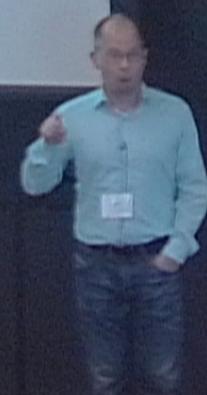


Heavy numerics, DMRG and exact diagonalization in 1D

1D wave function at different times, 100x100 lattice, 1000 sites, 1000000000 iterations

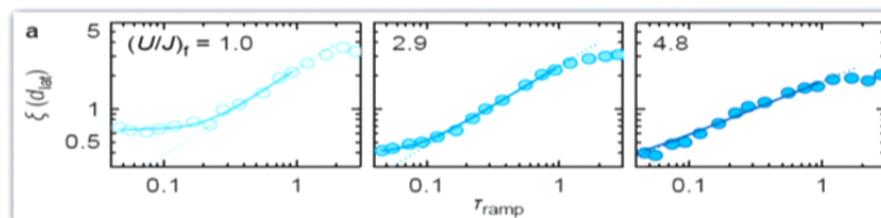
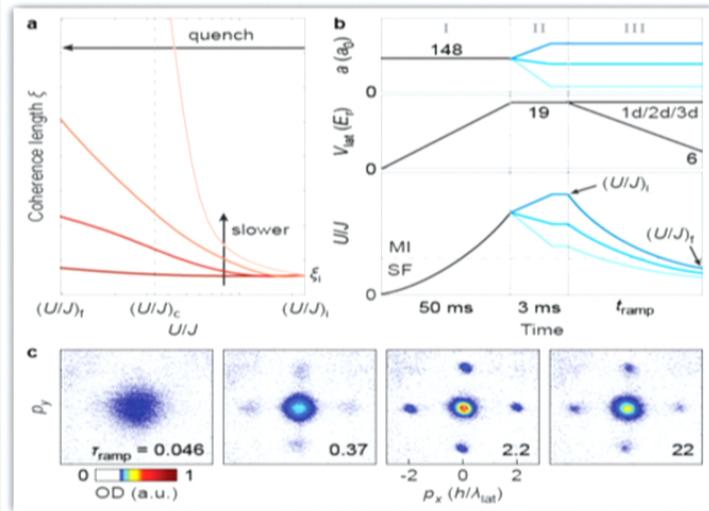
Careful study of effective Majorana - excellent agreement with data

From: J. K. Pachos, M. Turner, M. Bauer, P. Bonderson, R. Wil�de, B. Zeng, S. Das Sarma, arXiv:1008.4365



n experiment

Experiment in Immanuel Bloch/Uli Schneider's lab: Mott to superfluid quench



Heavy numerics: DMRG and **exact diagonalisation** in 1D

15 sites, truncation at 9 bosons, 2.581.186 basis states, 5.5 GB size of involved matrices)

Careful study of effects of trap ..., excellent **agreement** with data

Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, arXiv:1403.7199

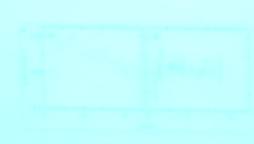
dynamical analogue quantum simulator

Finds accurate power laws over wide range of parameters,
but very different β from naive reading of Kibble-Zurek



KZM prediction: $\alpha = 1/3D$

(possibly log corrections)



$\alpha = 0.422D$ $\alpha = 1.333D$

Source: [reference]

Author: [reference]

License: [reference]

Version: [reference]

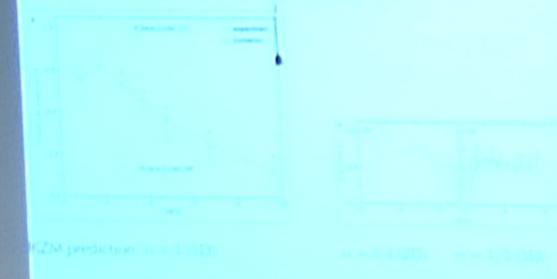
DOI: [reference]

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dynamical analogue quantum simulator

Finds accurate power laws over wide range of parameters.
but very different to linear scaling of Kitaev-Zurek.



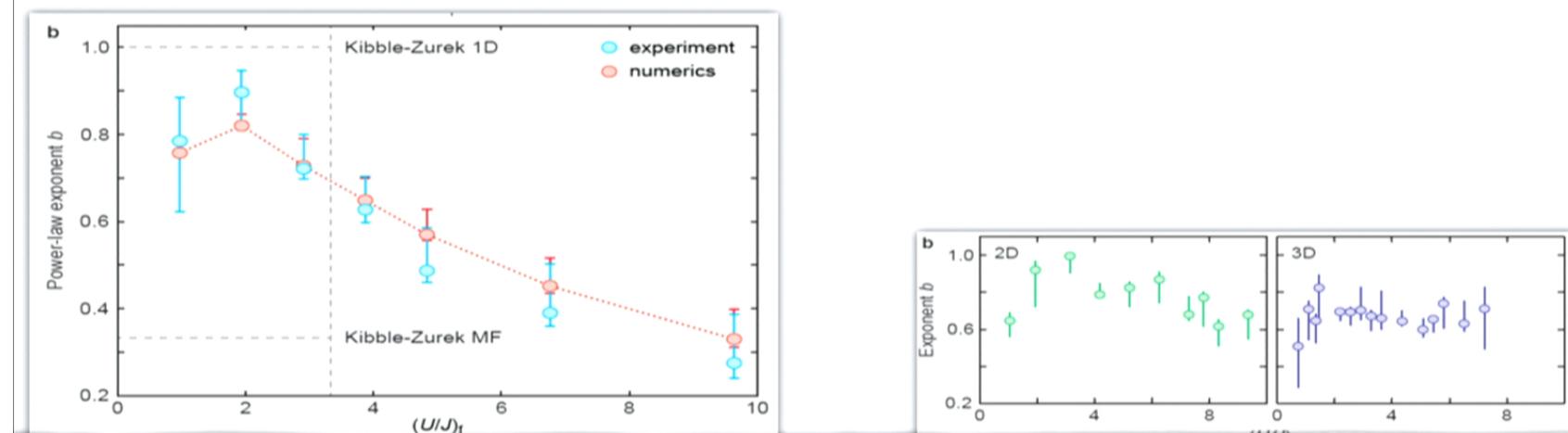
Source: T. C. Hales, J. M. Marzuola, P. R. W. McNamara, D. J. Perez, R. S. Richter, J. R. Rosales



dynamical analogue quantum simulator

Finds **accurate power laws** over wide range of parameters

But **very different** (!) from naive reading of Kibble-Zurek



- **Lesson:** Finds power laws, but different from naive reading of Kibble-Zurek picture
- **"Quantum simulation":** Build trust in correctness of simulation in 1D, experiment allows for assessment of 2D, 3D, alternative schedules etc

Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, arXiv:1403.7199

5. Summary and some musings

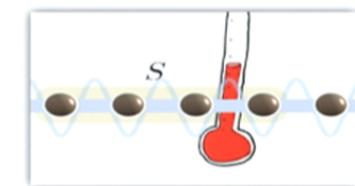
Gogolin, Kliesch, Aolita, Eisert, arXiv:1306.3995
Gogolin, Kliesch, Aolita, Eisert, in preparation (2014)

Summary

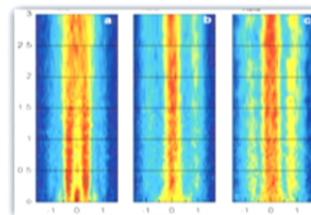
This talk: Quantum simulations of equilibration and thermalisation processes



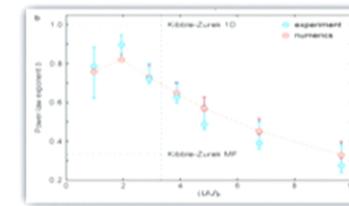
- Equilibration



- Thermalization



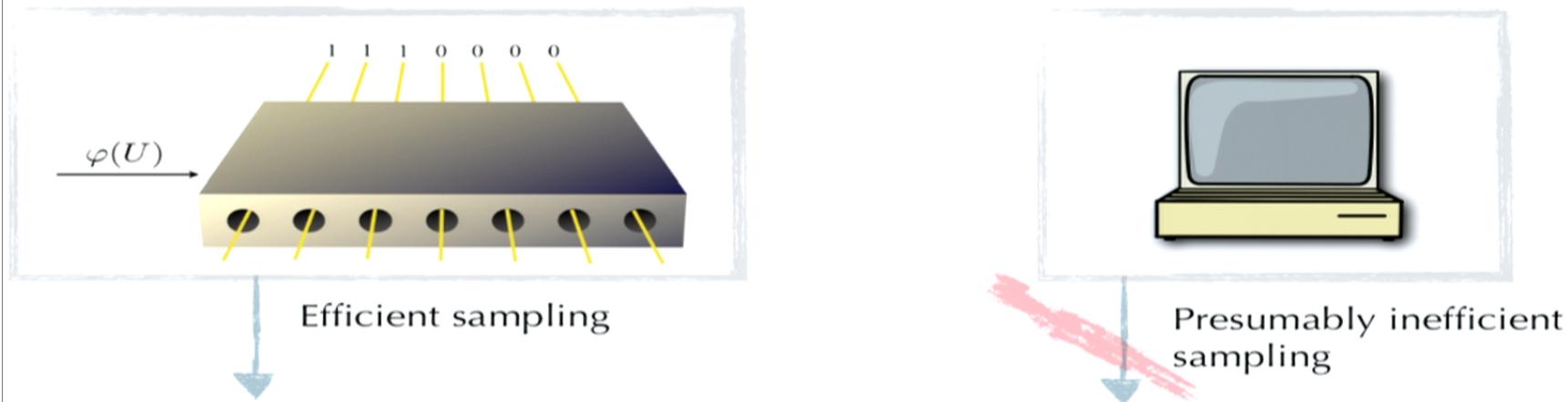
- Experimental fast quenches



- "Crime story"
of slow quenches

Qualifiers in terms of complexity classes?

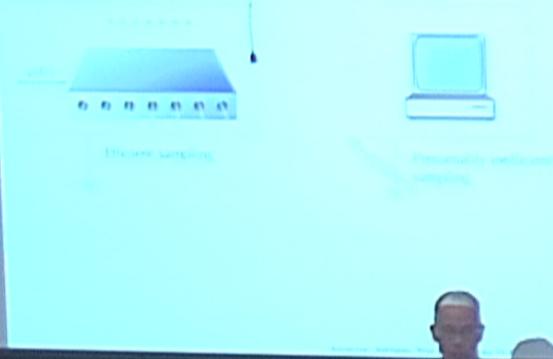
Great **experiments on boson sampling**: Cannot be efficiently classically sampled (to constant error in 1-norm), unless collapse of polynomial hierarchy



Aaronson, Arkhipov, Proc of ACM Symp Th Comp, STOC (2011)

Qualifiers in terms of complexity classes?

Causal experiments on boson sampling: Cannot be efficiently classically sampled to constant error in 1-exponent, unless collapse of polynomial hierarchy.



QMA

Classical certification?

Quantum experiments on boson sampling: Cannot be efficiently classically sampled. No constant error in 1-norm, unless collapse of polynomial hierarchy.



Efficient sampling



Classical efficient certification unlikely: Efficiently classically sample distributions that cannot be distinguished from true distribution with constant error rate.

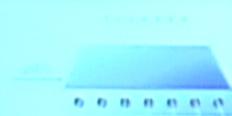


Efficiently precompute
classical distributions



Classical certification?

Current experiments on boson sampling: Cannot be efficiently classically sampled to constant error in 1-second, unless collapse of polynomial hierarchy.



Efficient sampling



Efficiently preparable
classical distributions

Classical efficient certification unlikely: Efficiently classically sample distributions that cannot be distinguished from the distribution with constant error from any

Efficient algorithm, faster than $\text{poly}(n)$ steps.
Source: Beals, Durr, Flamm, et al., in preparation, 2014.
Burgisser, Spielman, Vannieuwenhoven, 2013.

