Title: Towards Scaling Relations in Relativistic Hydrodynamics and Gravity

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Abstract: Turbulence is ubiquitous in hydrodynamics and its study is dominated by statistical methods and dimensional arguments. Even so analytic results tend to rely heavily on statistical symmetries. I will discuss some such results in non-relativistic turbulence and possible extensions to the relativistic case. The 2+1 dimensionality of our numerical setup allows for gaining insight about 3+1 gravity through the fluid/gravity duality. This work aims to further our understanding of the fluid side in its own right. This partly entails determining the robustness of some recently derived relativistic hydrodynamic scaling relations which may have holographic applications.

Scaling Relations in 2+1 Relativistic Turbulence and 3+1 Gravity

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Motivation

- **Broad**: probe gravitational physics through fluid/gravity duality.
- **Specific**: test analytic results by Fouxon, Oz (2009)

Motivation

Broad:

- Fluid/gravity duality obtainable as a limit from AdS/CFT.
- Also, can obtain it independently in regime where gradient expansion of g_{ab} is valid.
- It holds in particular in the **turbulent regime** on fluid side.
- Gravity side must contain black hole/black brane (gives temperature scale).

More details: Bhattacharyya et al. (2008), Van Raamsdonk (2008)

Motivation

Specific:

- Fluid side is **relativistic hydrodynamics**, *not* Navier-Stokes.
- Most of our knowledge of the 2D turbulent regime is for Navier-Stokes. See [*Boffetta (2012)*] for review.
- Thus, important to extend our understanding of turbulence to relativistic case.

Steps already made in this direction:
[*Fouxon, Oz (2009)*, *Carrasco, Lehner et al. (2012)*]

Also, fluid side has already has pointed us to novel phenomena in gravity, eg. turbulent behaviour, inverse energy cascade
[*Adams, Chesler, Liu (2013)*]

Motivation

Direct astrophysical applications:

- Any place where we have effectively 2+1 fluid behaviour.

Eg: thin accretion discs, shock fronts...

Background: five-thirds law

$$E_{tot} = \int E(k)dk$$

$$[E] = \frac{energy}{mass} = (\frac{length}{time})^2$$

$$[dk] = \frac{1}{length}$$

$$\Rightarrow [E(k)] = \frac{length^3}{time^2}$$

Background: five-thirds law

Ansatz: energy flow ϵ scale-free ($\epsilon \neq \epsilon(k)$), so $E(k) \propto \epsilon^p k^q$, where

$$[\epsilon] = \frac{energy}{mass}\frac{1}{time} = \frac{length^2}{time^3}$$

$$[k] = \frac{1}{length}$$

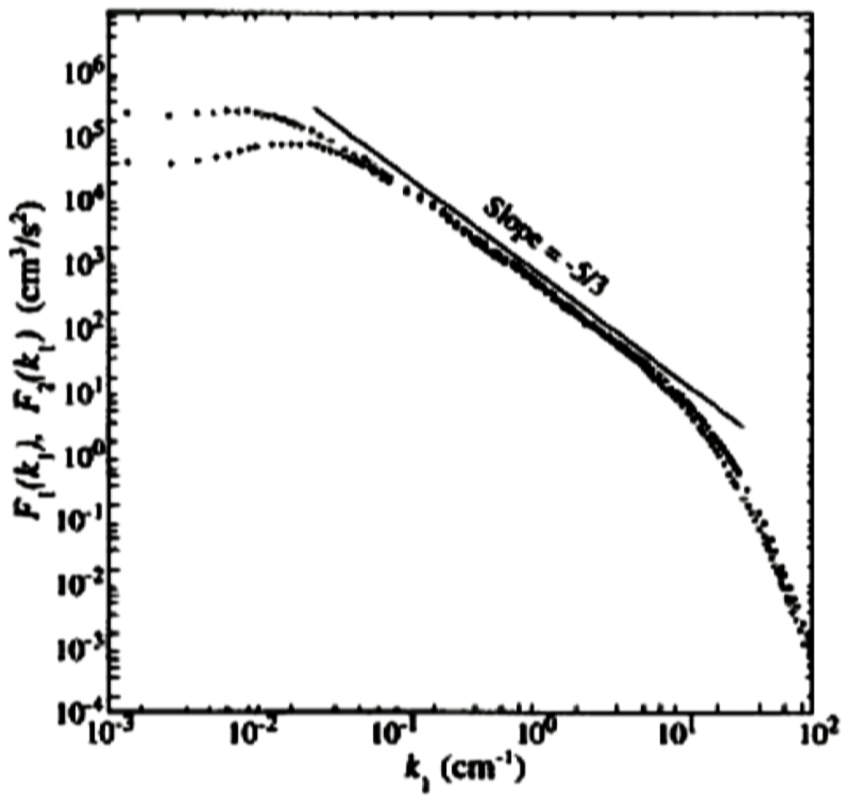
Background: five-thirds law

Now match units on both sides of $E(k) \propto \epsilon^p k^q$:

$$\frac{length^3}{time^2} = \frac{length^{2p-q}}{time^{3p}}$$
$$\Rightarrow p = \frac{2}{3}$$
$$\Rightarrow q = -\frac{5}{3}$$

Thus, $E(k) \propto k^{-5/3}$.
This is associated with the inverse energy cascade in 2+1.

Five-Thirds Law: readily observed in nature



Champagne (1978)

More non-relativistic scaling results

Another canonical result is the scaling of *third-order structure functions*:

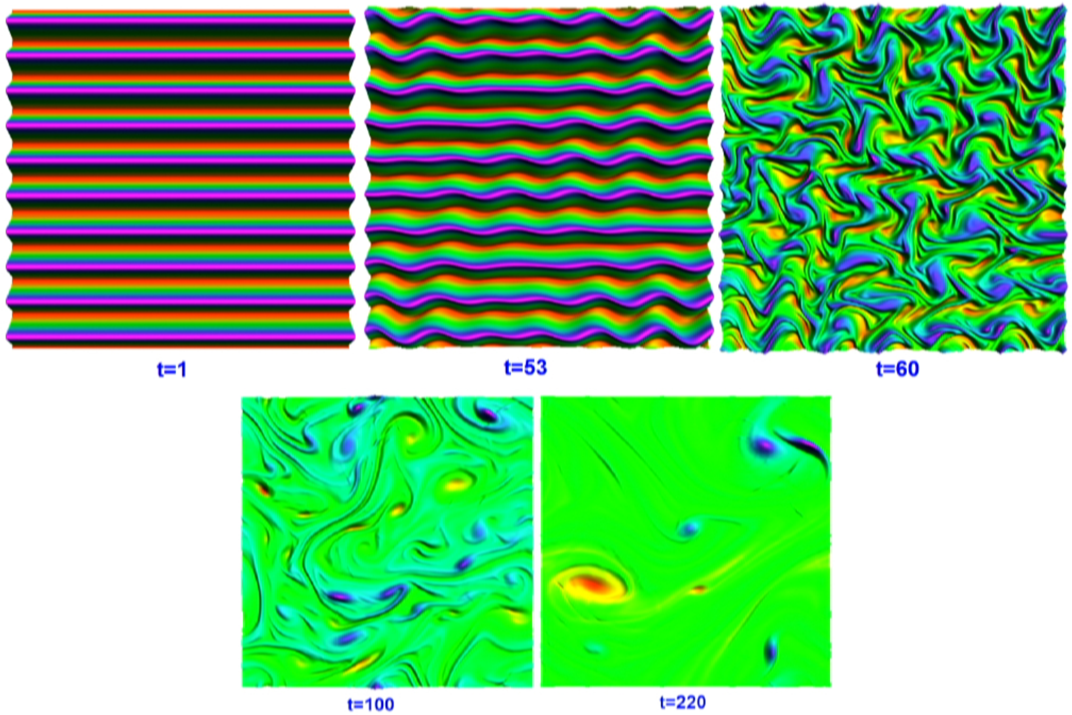
- $\left\langle [v_L(\vec{r}) - v_L(0)]^3 \right\rangle = -\frac{4}{5}\epsilon r$ in 3d
- $\left\langle [v_L(\vec{r}) - v_L(0)]^3 \right\rangle = \frac{3}{2}\epsilon r$ in 2d

These are suited to non-relativistic case because they are Galilean invariants.

Moving to relativity:

- 5/3 law: whose k?
- scaling relations: want to speak of T_{ab}, not $\vec{v}$.

Relativistic case: inverse energy cascade occurs



Carrasco, Lehner et al. (2012)

Note: same qualitative behaviour with viscosity
[*Green, Carrasco, Lehner (2013)*]

Fouxon, Oz results...

$$\langle T_{0L}(0)\,T_{LL}(\vec{r})\rangle = \frac{\epsilon}{2} r$$

- Derivation assumes *forced* steady-state turbulence, $\partial_a T^{ab} = f^b$, where f is a *random, homogeneous and isotropic field*.
- Also assumes the distance r is far away from forcing and viscous scales... Must localize f in spectral space.

Note that this scaling relation is not even Lorentz-invariant! This is part of the difficulty of relativistic turbulence...

Simulation setup

- Spacetime dimension = 2+1 ... computationally inexpensive, useful for 4D gravity.
- Perfect fluid ... $T^{ab} = (P + \rho)u^a u^b + P\eta^{ab}$.
- Equations of motion given by conservation of T^{ab}, $\partial_a T^{ab} = f^b$.
- Edges of computational grid are periodically identified ... i.e. a 2-torus.
- Use Landau frame.

Note: viscid case does not change general behaviour in the turbulent regime, only makes this regime smaller.

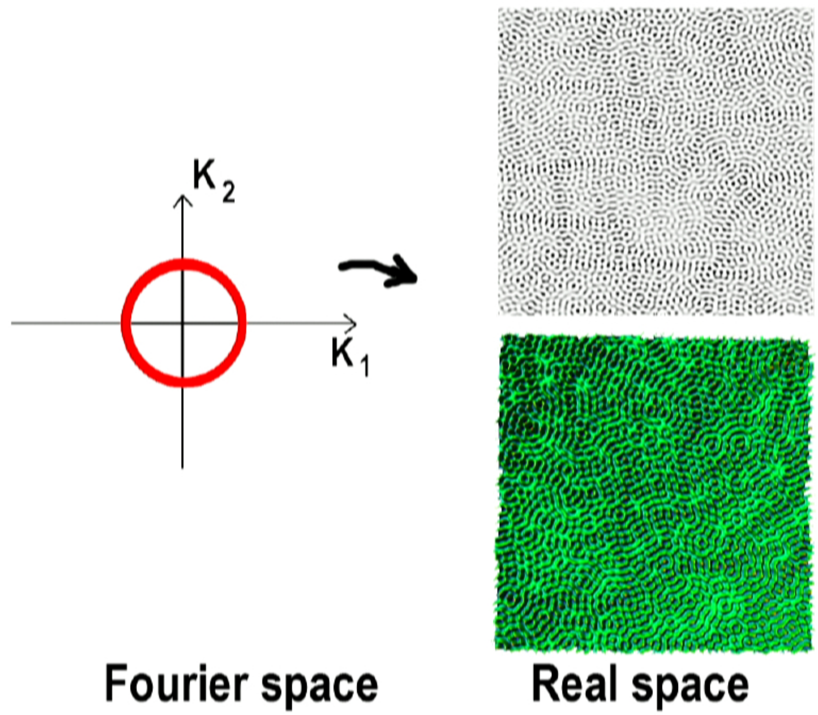
Lack of scale implies an equation of state...

To be useful for fluid/gravity duality, we impose conformal invariance of the equations of motion. This is achieved with a perfect fluid if and only if $T^a_a = 0$ [*Wald (1984)*]. Thus,

$$
\begin{aligned}
0 = T^a_a &= (P + \rho)u_a u^a + P\delta^a_a \\
&= -(P + \rho) + 3P \\
\Rightarrow P &= \frac{1}{2}\rho.
\end{aligned}
$$

How we construct f

Build it in Fourier space, then Fourier transform to real space...
Real condition: $f^*(\vec{k}) = f(-\vec{k})$



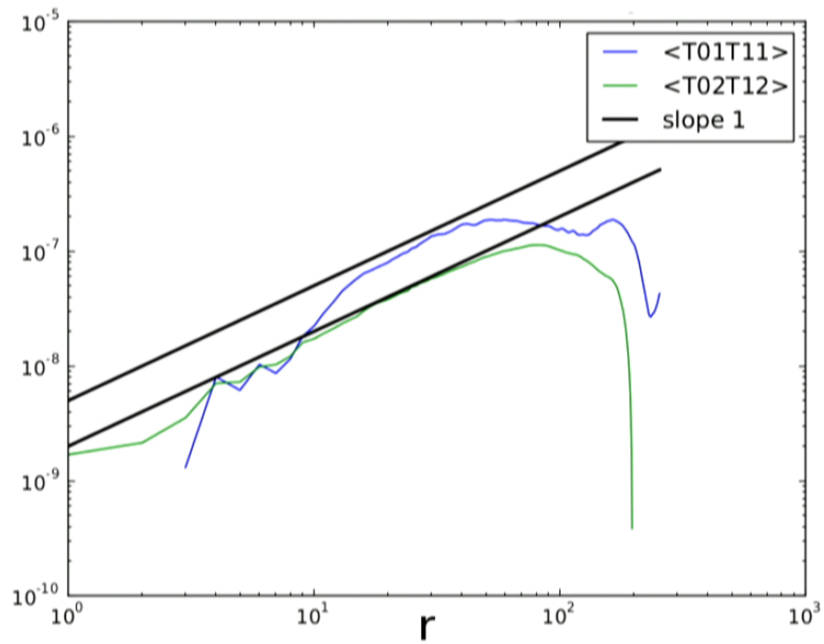
Ensemble averaging

- Ensemble averages are impractical to compute without statistical symmetries.
- Homogeneity and isotropy means $\langle . \rangle$ is a **spatial average**.
- Stationarity in time means $\langle . \rangle$ is a **time average**.
- We must use **both** to resolve the scaling relation.

See [*Uriel Frisch (1996)*] for more details.

Results

Averaged over all of space and over **100 times** separated by one large-eddy turnover time:



Also derived linear scaling with slightly **weaker assumptions**, suggesting that this result is robust.

Summary and directions

Previous work:

- Showed turbulent inverse energy cascade in 2+1.
- Generalized concept of enstrophy.
- Derived scaling relations.
- Fluid-gravity predicts analogs of these on gravity side.
  eg. inverse cascade and turbulence in Carrasco et al. (2012)
  $\rightarrow$ seen in Chesler et al. (2013)

This work:

- Numerically measured $\langle T_{0j}(0) T_{ij}(\vec{r})\rangle \propto r$.
- Showed linear scaling persists with weaker assumptions.

Future work:

- Translate scaling relations into gravity side?
- Find relativistic analog of 5/3 law?
- Explore more relativistic flows.