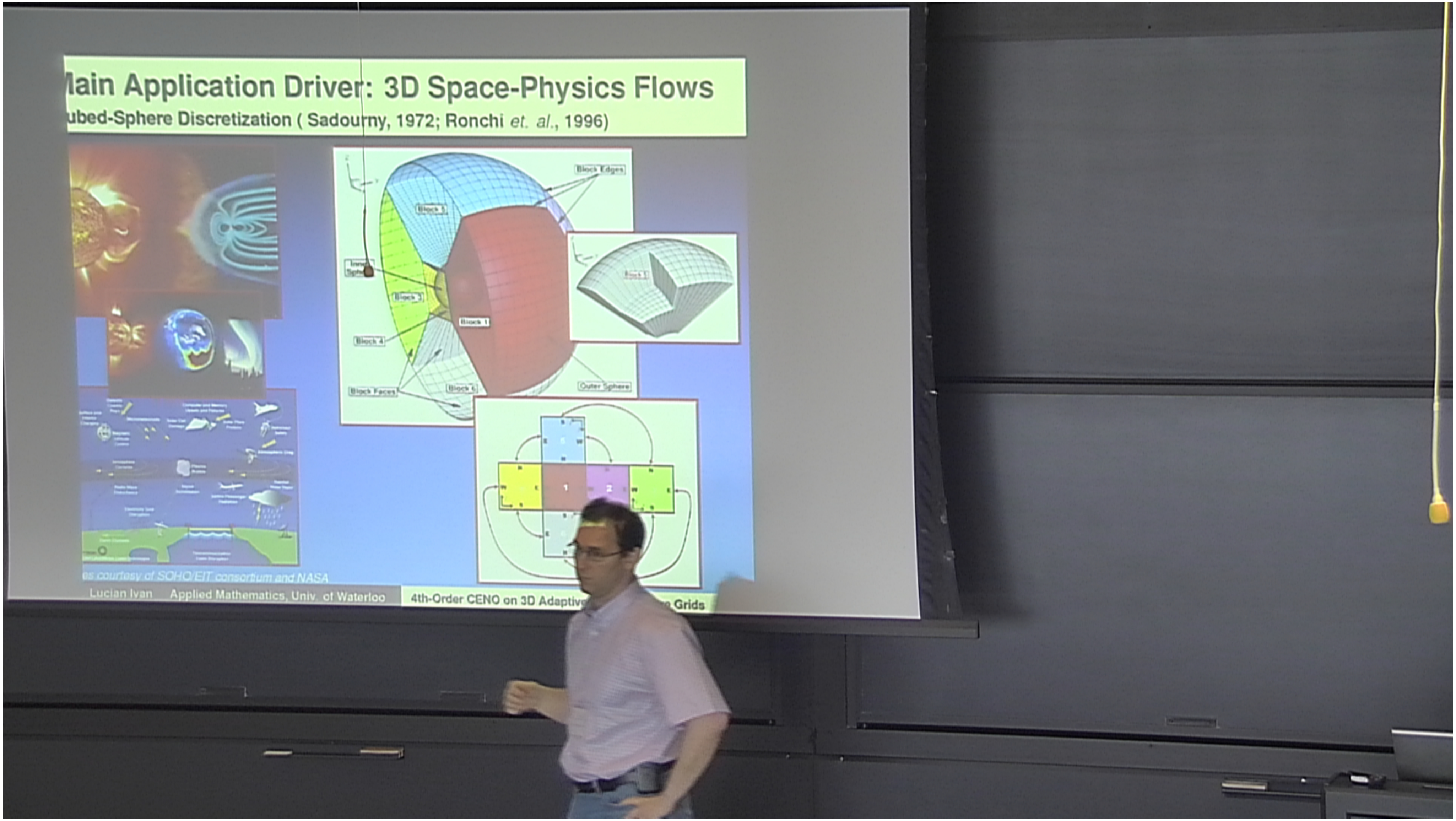
Title: A Fourth-Order Solution-Adaptive CENO Scheme for Space-Physics Flows on Three-Dimensional Multi-Block Cubed-Sphere Grids

Date: May 07, 2014 03:50 PM

URL: http://pirsa.org/14050051

Abstract: Accurate efficient and scalable computational methods are highly desirable for large-scale scientific computing applications especially for problems exhibiting spatial and temporal multi-resolution scales non-trivial geometries and complex boundary conditions (BSc). For global magnetohydrodynamics (MHD) modelling of space-physics problemshigh-performance approaches could significantly reduce the grid requirements to achieve targeted solution accuracies thereby enabling more affordable yet accurate predictions of space-plasma flows. Key challenges encountered relate to providing solenoidal magnetic fields accurate discretizations on spherical domains capturing of MHD shocks and implementing accurate BCs. This talk gives an overview of a fourth-order finite-volume discretization procedure in combination with a parallel solution-adaptive algorithm for the computation of MHD space plasmas on cubed-sphere grids. Numerical results to demonstrate the accuracy and capability of the multidimensional high-order solution-adaptive cubed-sphere computational framework are presented.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ubed-Sphere Discretization (Sadourny, 1972; Ronchi et. al., 1996)
Block Edges
Block 5
Block 3
Block 1
Block 4
Block Faces
Block 6
Outer Sphere
es courtesy of SOHO/EIT consortium and NASA
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Grids

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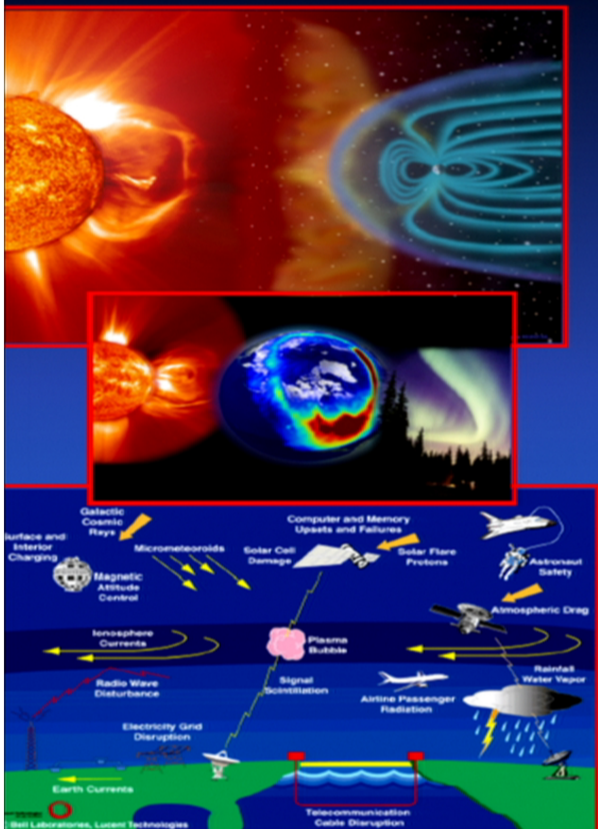

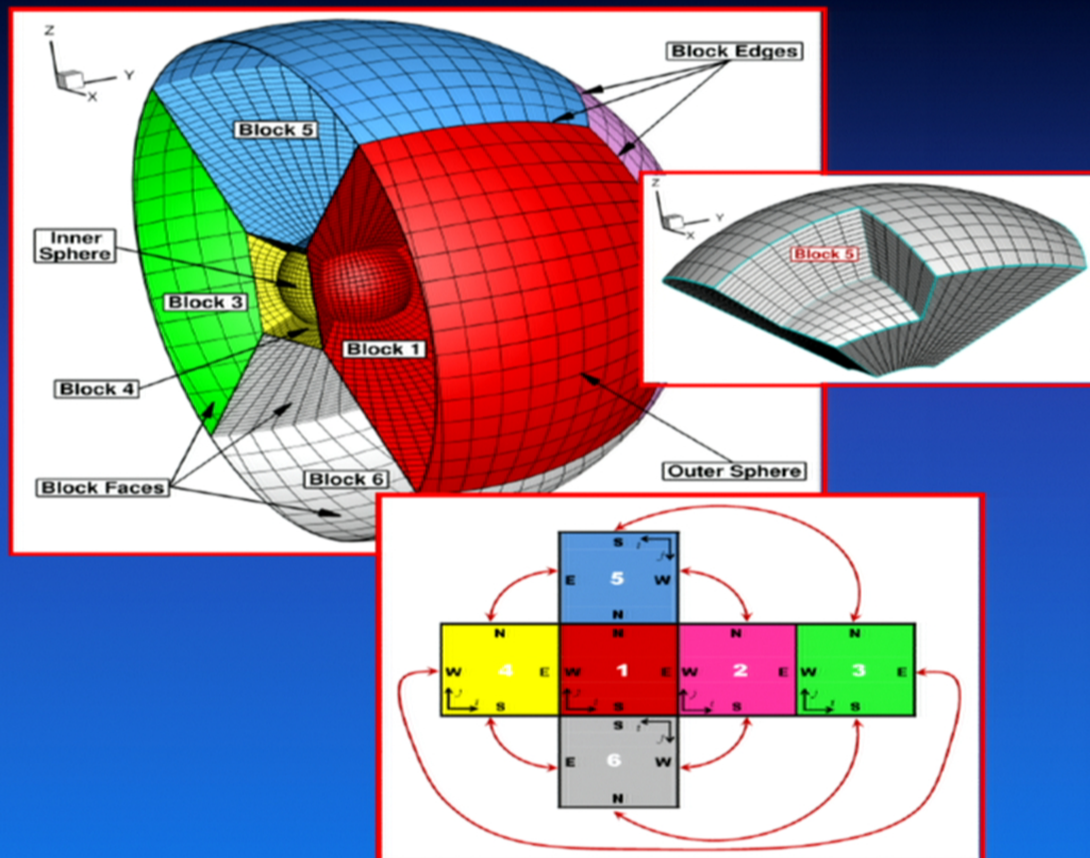

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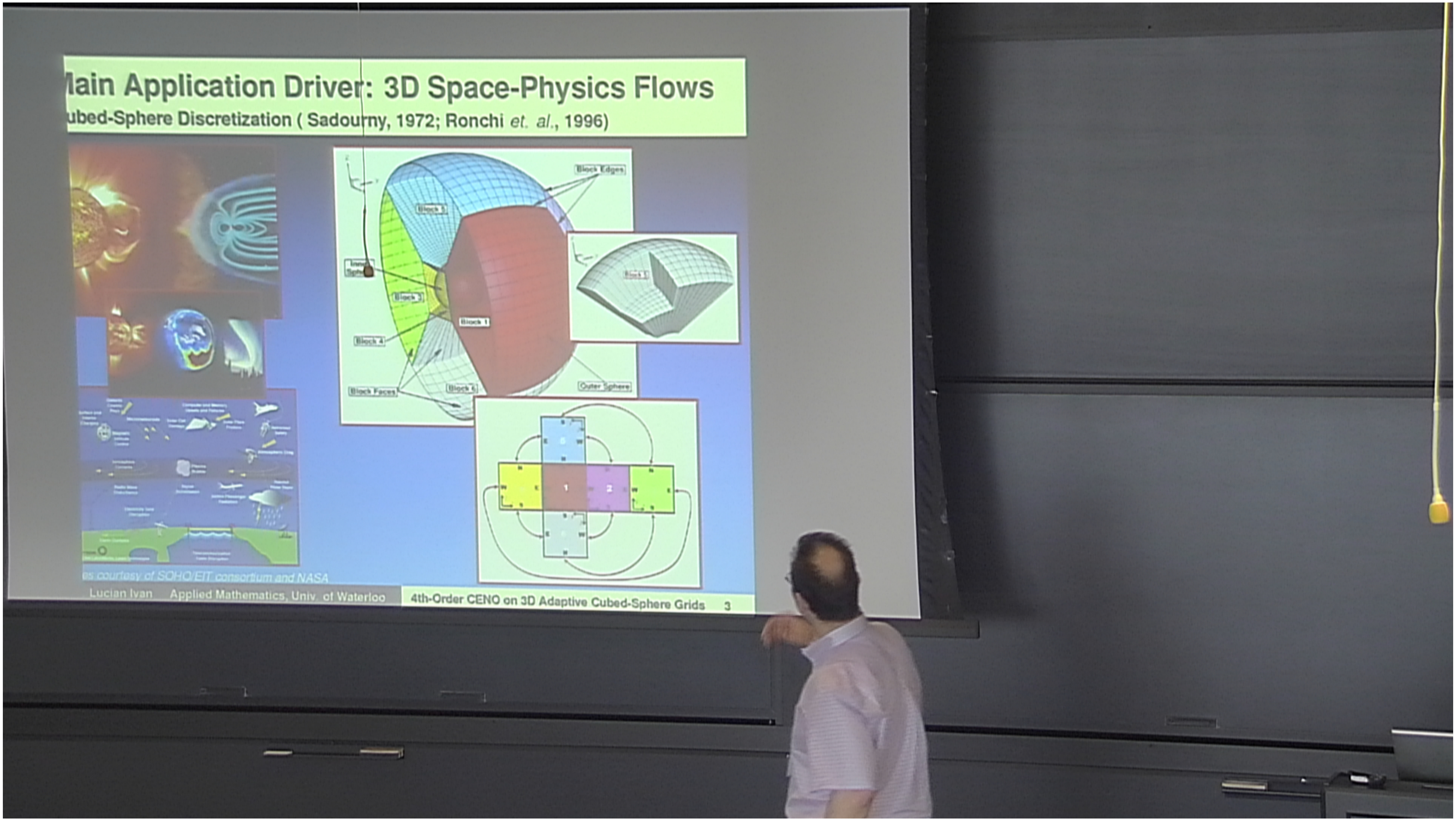

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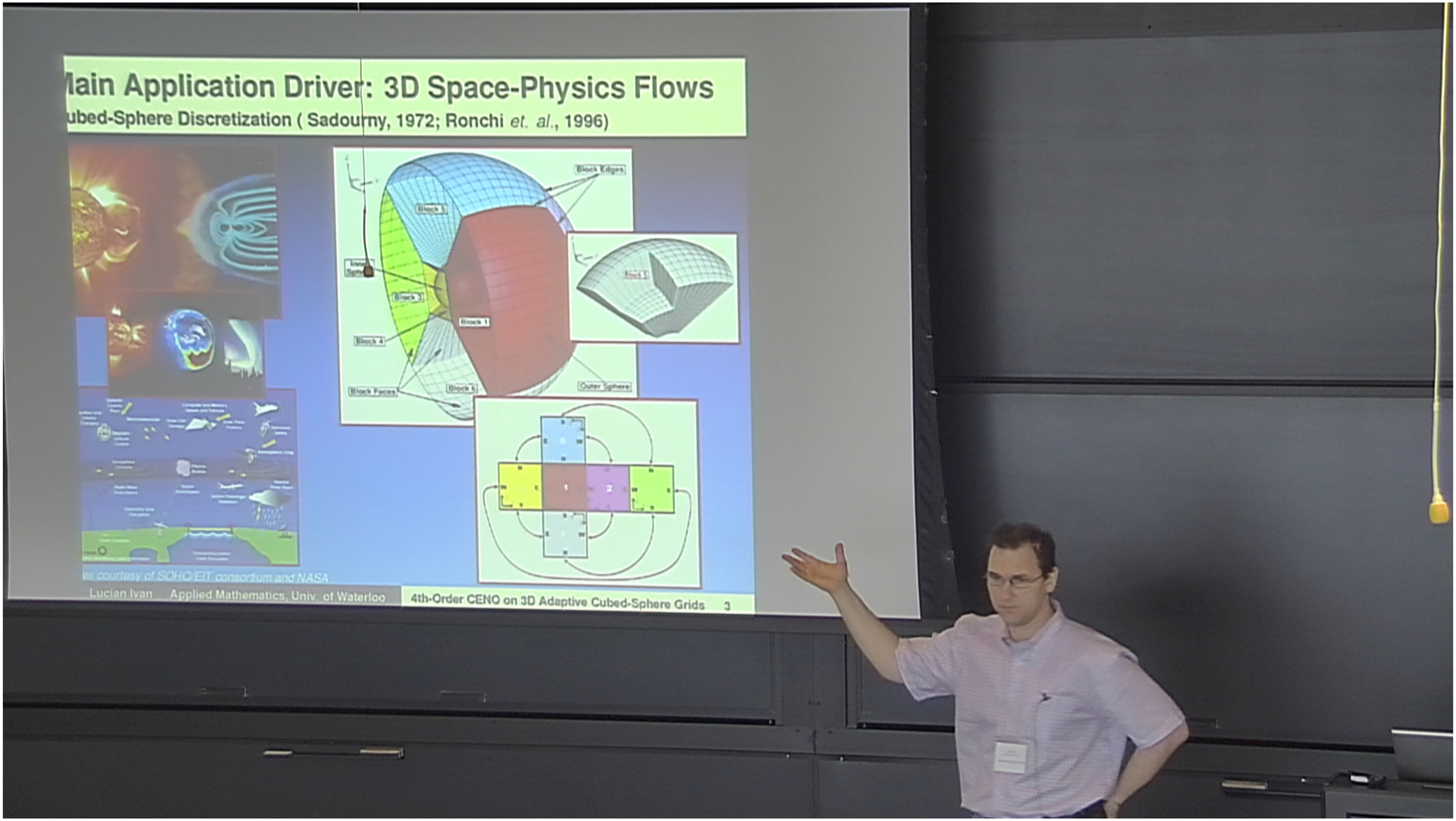
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lagnetohydrodynamics (MHD) Model

pproaches to Deal with the Divergence Constraint Condition, $\nabla \cdot \vec{B} = 0$

Powell Source Term (Powell *et. al.*, 1999)

$$\mathbf{S}_{\text{num}} = -\nabla \cdot \vec{B} \left[\ 0,\ \ \vec{B},\ \ \vec{V},\ \ \vec{V} \cdot \vec{B}\ \right]^{\mathrm{T}}$$

- 8-wave MHD system that is symmetric and Galilean invariant
  $\lambda_{1,2} = v_x \pm c_{fx},\ \lambda_{3,4} = v_x \pm c_{Ax},\ \lambda_{5,6} = v_x \pm c_{sx},\ \lambda_{7,8} = v_x$
- Numerical error in $\nabla \cdot \vec{B}$ is convected out of the domain by $\lambda_8 = v_x$

Divergence Correction Technique: Generalized Lagrange Multiplier (GLM)-MHD (Dedner *et al.*, 2002)

$$\frac{\partial \vec{B}}{\partial t} + \nabla \cdot (\vec{V}\vec{B} - \vec{B}\vec{V}) + \nabla \psi = 0$$

$$\frac{\partial \psi}{\partial t} + c_h^2 \nabla \cdot \vec{B} = -\frac{c_h^2}{c_p^2} \psi$$

- Solve an extra transport equation for the GLM, ψ
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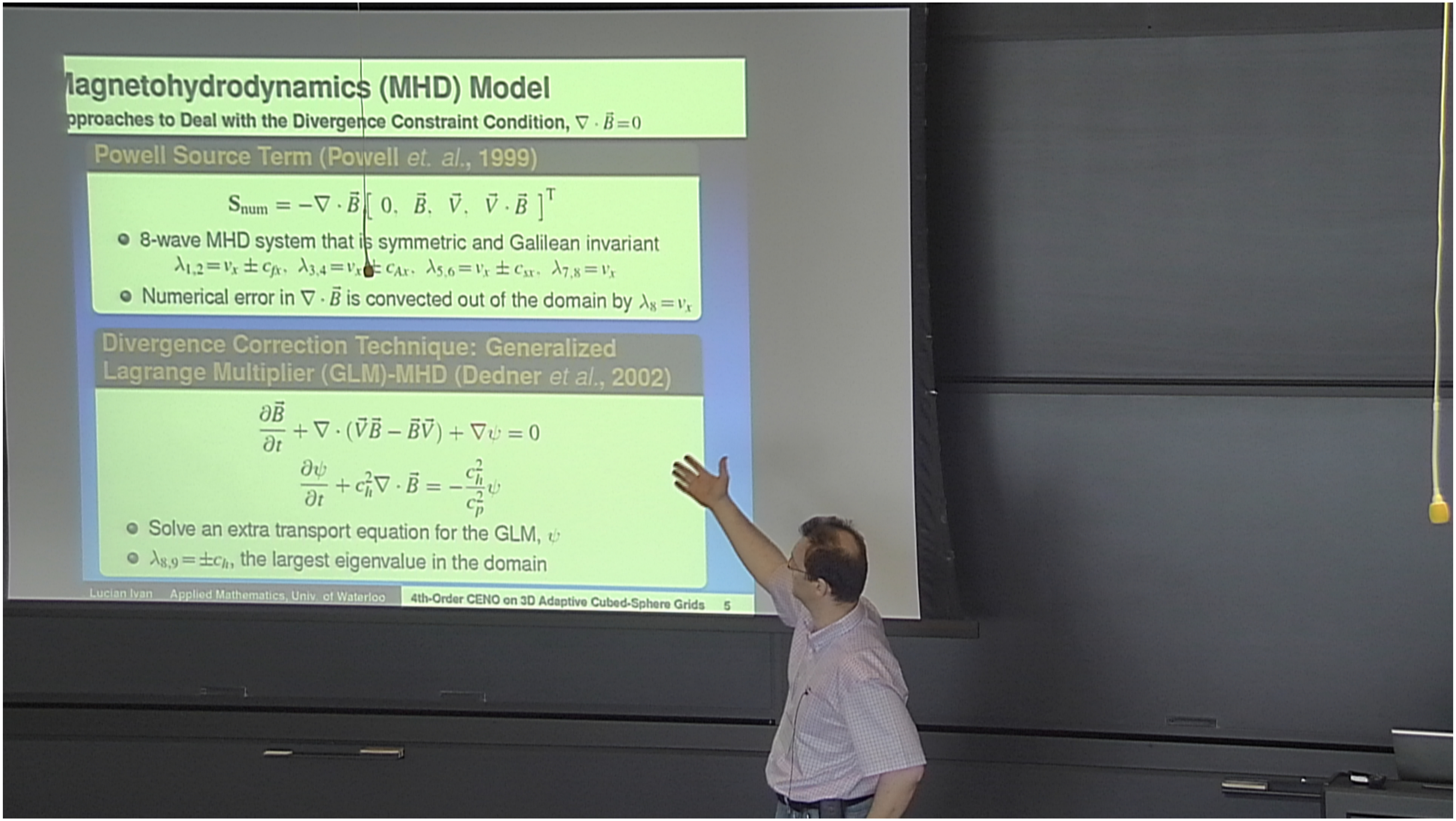
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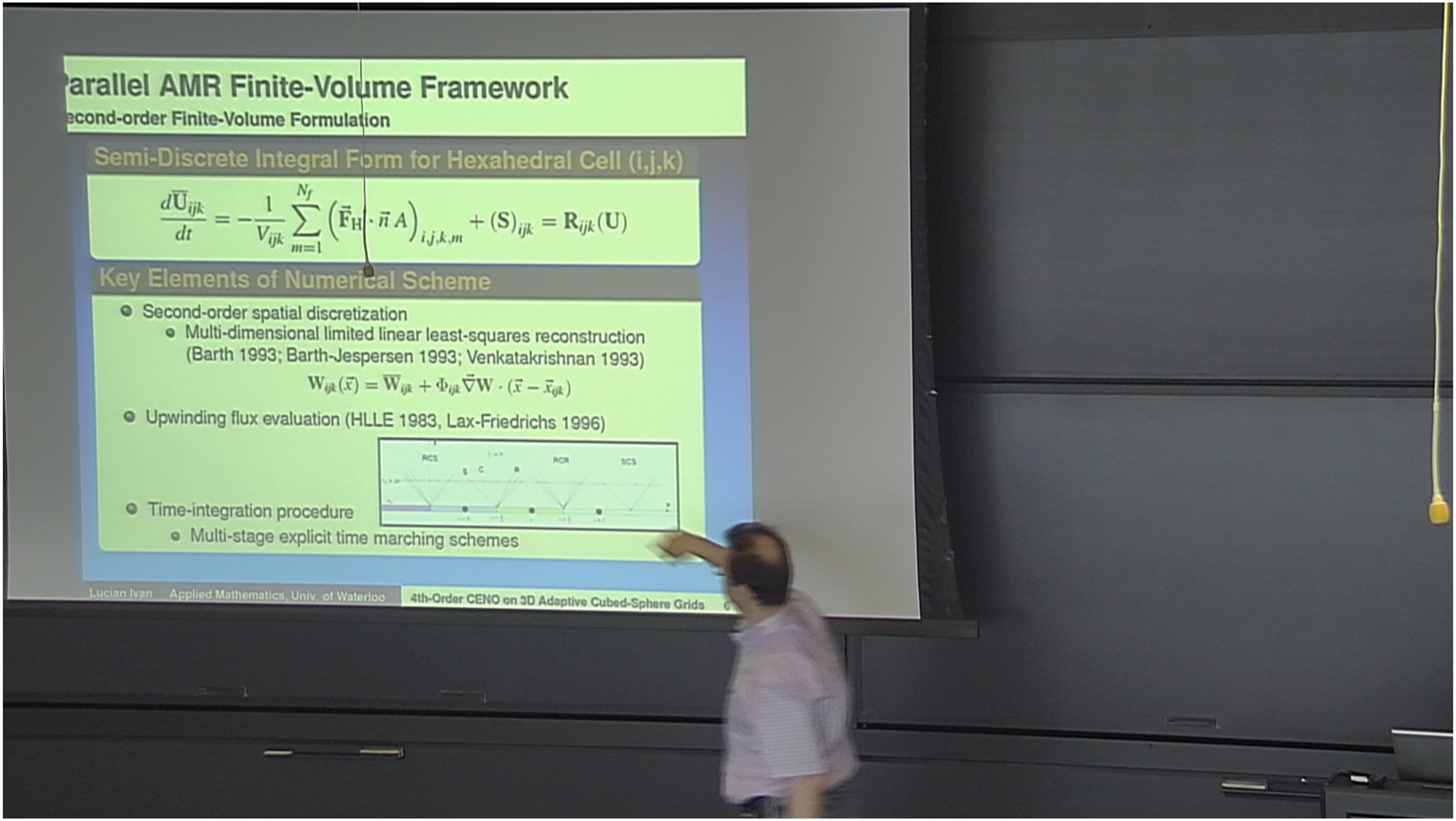
arallel AMR Finite-Volume Framework
econd-order Finite-Volume Formulation
Semi-Discrete Integral Form for Hexahedral Cell (i,j,k)
$\frac{d\overline{\mathbf{U}}_{ijk}}{dt} = -\frac{1}{V_{ijk}} \sum_{m=1}^{N_f} \left(\vec{\mathbf{F}}_{\mathrm{H}} \cdot \vec{n}\, A \right)_{i,j,k,m} + (\mathbf{S})_{ijk} = \mathbf{R}_{ijk}(\mathbf{U})$
Key Elements of Numerical Scheme
Second-order spatial discretization
Multi-dimensional limited linear least-squares reconstruction (Barth 1993; Barth-Jespersen 1993; Venkatakrishnan 1993)
$\mathbf{W}_{ijk}(\vec{x}) = \overline{\mathbf{W}}_{ijk} + \Phi_{ijk} \vec{\nabla} \mathbf{W} \cdot (\vec{x} - \vec{x}_{ijk})$
Upwinding flux evaluation (HLLE 1983, Lax-Friedrichs 1996)
RCS
S
C
R
RCR
SCS
Time-integration procedure
Multi-stage explicit time marching schemes
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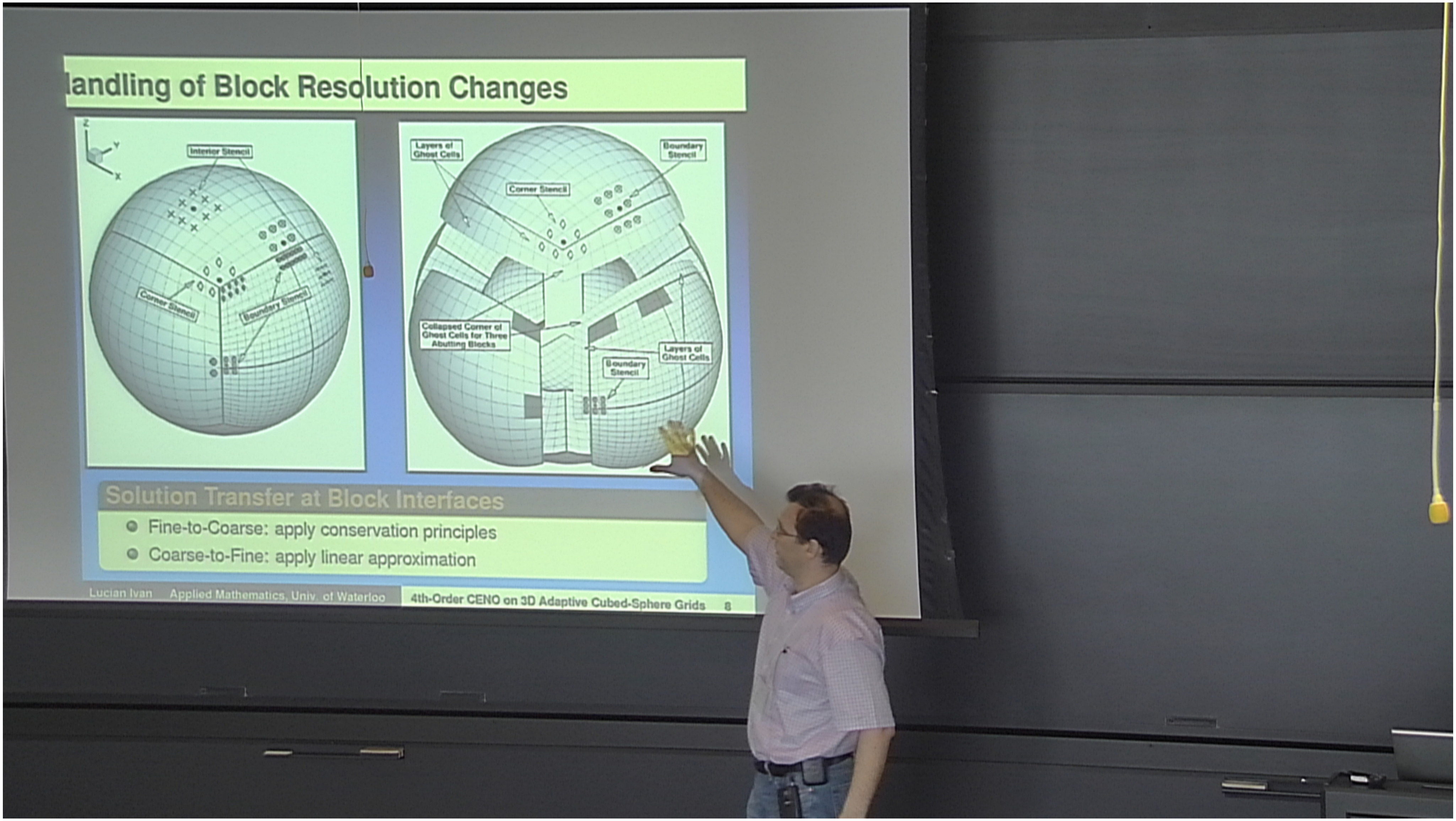
Handling of Block Resolution Changes
Interior Stencil
Corner Stencil
Boundary Stencil
Layers of Ghost Cells
Corner Stencil
Boundary Stencil
Collapsed Corner of Ghost Cells for Three Abutting Blocks
Layers of Ghost Cells
Boundary Stencil
Solution Transfer at Block Interfaces
Fine-to-Coarse: apply conservation principles
Coarse-to-Fine: apply linear approximation
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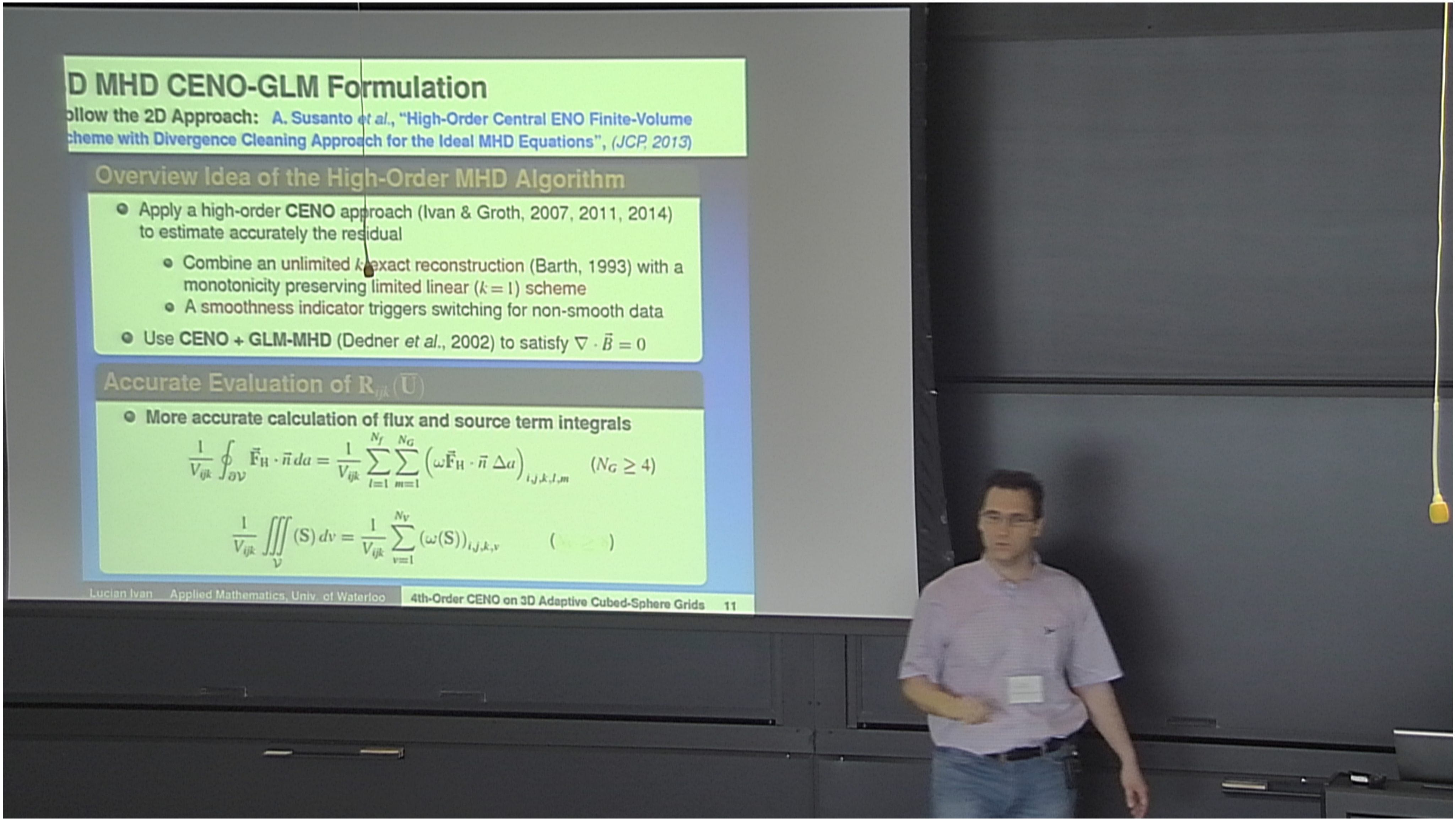
D MHD CENO-GLM Formulation
ollow the 2D Approach: A. Susanto et al., "High-Order Central ENO Finite-Volume
cheme with Divergence Cleaning Approach for the Ideal MHD Equations", (JCP, 2013)
Overview Idea of the High-Order MHD Algorithm
Apply a high-order CENO approach (Ivan & Groth, 2007, 2011, 2014) to estimate accurately the residual
Combine an unlimited k-exact reconstruction (Barth, 1993) with a monotonicity preserving limited linear ($k = 1$) scheme
A smoothness indicator triggers switching for non-smooth data
Use CENO + GLM-MHD (Dedner et al., 2002) to satisfy $\nabla \cdot \vec{B} = 0$
Accurate Evaluation of $\mathbf{R}_{ijk}(\overline{\mathbf{U}})$
More accurate calculation of flux and source term integrals
$\frac{1}{V_{ijk}} \oint_{\partial \mathcal{V}} \vec{\mathbf{F}}_{\mathrm{H}} \cdot \vec{n}\, da = \frac{1}{V_{ijk}} \sum_{l=1}^{N_f} \sum_{m=1}^{N_G} \left(\omega \vec{\mathbf{F}}_{\mathrm{H}} \cdot \vec{n}\, \Delta a \right)_{i,j,k,l,m} \quad (N_G \geq 4)$
$\frac{1}{V_{ijk}} \iiint_{\mathcal{V}} (\mathbf{S})\, dv = \frac{1}{V_{ijk}} \sum_{v=1}^{N_V} (\omega(\mathbf{S}))_{i,j,k,v}$
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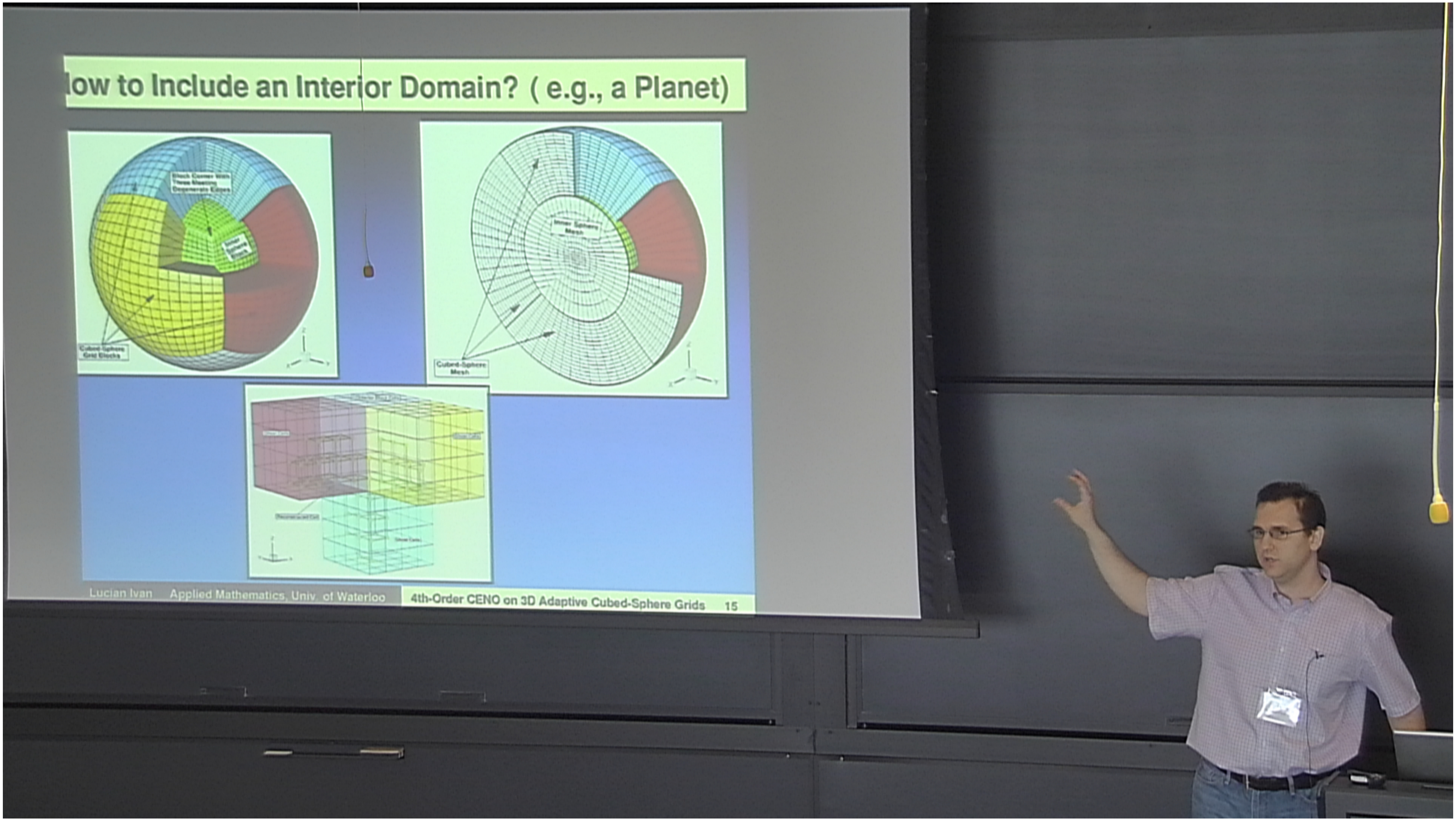

low to Include an Interior Domain? (e.g., a Planet)
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ligh-Order Numerical Results

o-Density Vortex Problem in Solid-Core Cubed-Sphere Grid

$, = 9, \vec{V} = (1, 1, 2), \vec{X}_i = (-2, -2.5, -3), \vec{X}_f = (1, 0.5, 3), t = 3$