

Title: Testing Discontinuous Galerkin Methods in the Einstein Toolkit for Numerical Relativity

Date: May 07, 2014 12:25 PM

URL: <http://pirsa.org/14050045>

Abstract: Discontinuous Galerkin finite element (DGFE) methods combine advantages of both finite differences and finite elements approaches. These methods scale extremely well and they have been very successful in computational fluid dynamics. As such we would like to transpose them to the domain of relativistic astrophysics. Recently we have implemented DGFE methods in the Einstein Toolkit a large numerical relativity codebase used by hundreds of scientists around the world. However before DGFE methods can be used in production simulations we must ensure that our implementation is up to the efficiency and accuracy standards of a production codebase. Here we detail our efforts to test our implementation using the Apples with Apples tests (c.f. arXiv:gr-qc/0305023 and arXiv:0709.3559). We briefly introduce DGFE methods explain the Apples with Apples tests and our rationale for using them and discuss results.

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# Testing Discontinuous Galerkin Methods in the Einstein Toolkit for Numerical Relativity

Jonah M. Miller

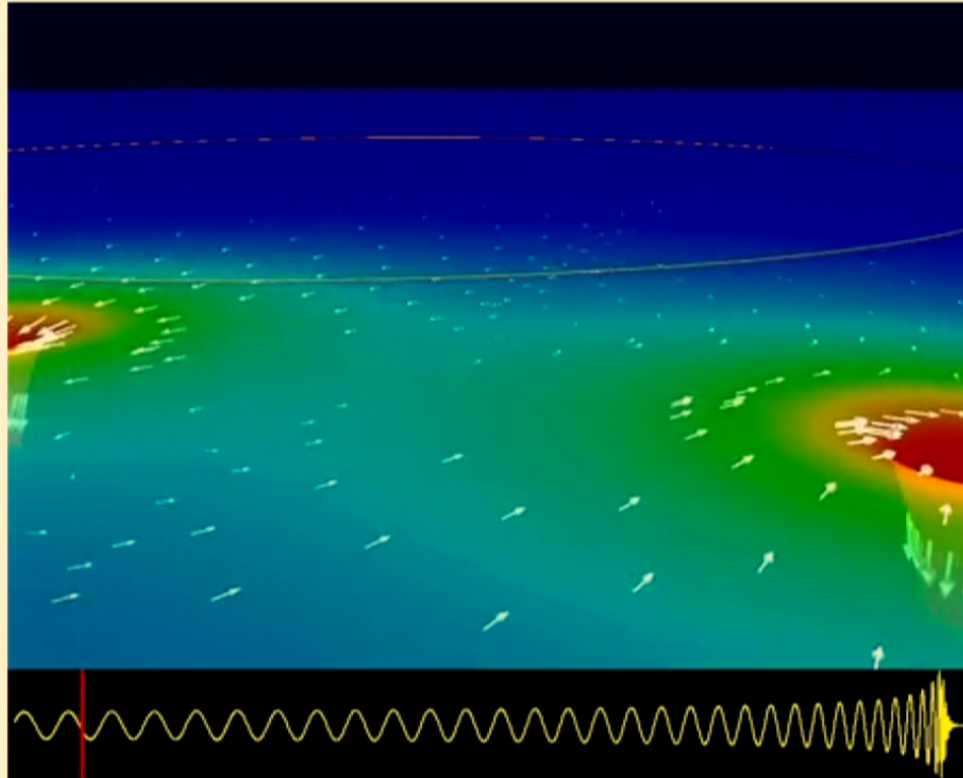
with Erik Schnetter, Ian Hinder, Frederico Guercilena,  
David Radice, and Barry Wardell

Perimeter Institute for Theoretical Physics  
University of Guelph

Compute Ontario Research Day,  
May 2014

# Numerical Relativity

- Numerically solving system of 10 coupled highly nonlinear second-order PDEs



10.1103/PhysRevD.79.024003 or arXiv:0810.1767

# The Einstein Toolkit

- Computational toolbox for relativistic astrophysics



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Over 100 *registered* users

⇒ most registered users contribute

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Over 50 research groups from around the world

⇒ a real international community

# The Einstein Toolkit

- Computational toolbox for relativistic astrophysics



Over 100 *registered* users

⇒ most registered users contribute

Over 50 research groups from around the world

⇒ a real international community

Cited in well over 100 publications

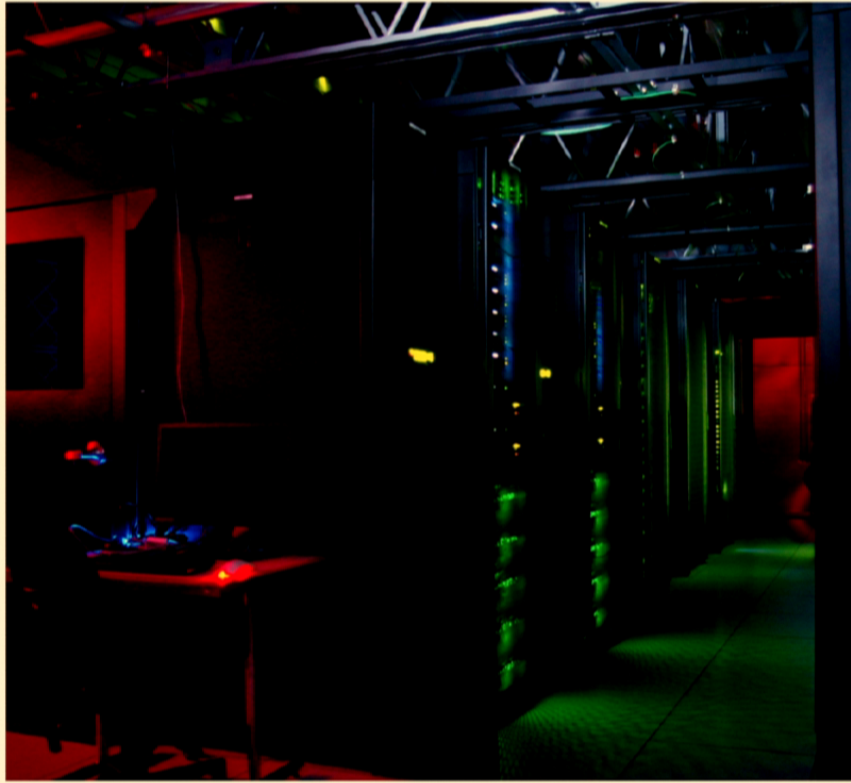
⇒ very successful

# The Search Is On!

- Computational toolbox for relativistic astrophysics



J. Miller (PI)



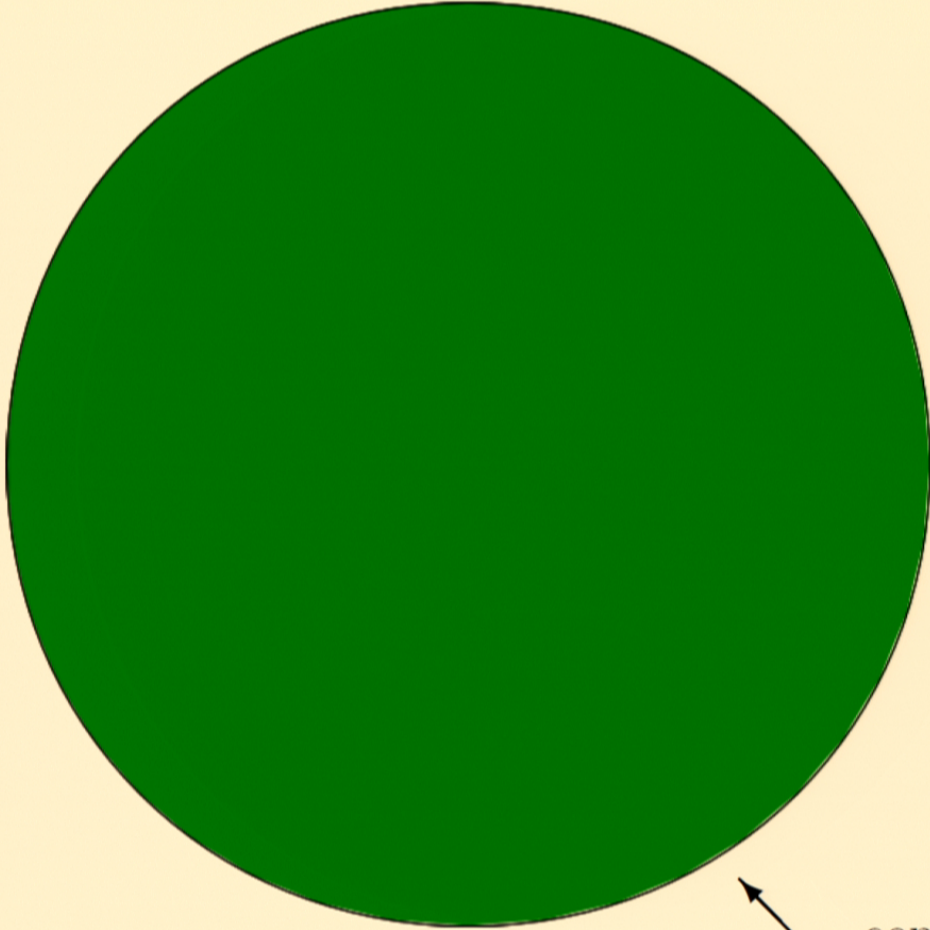
DG Methods

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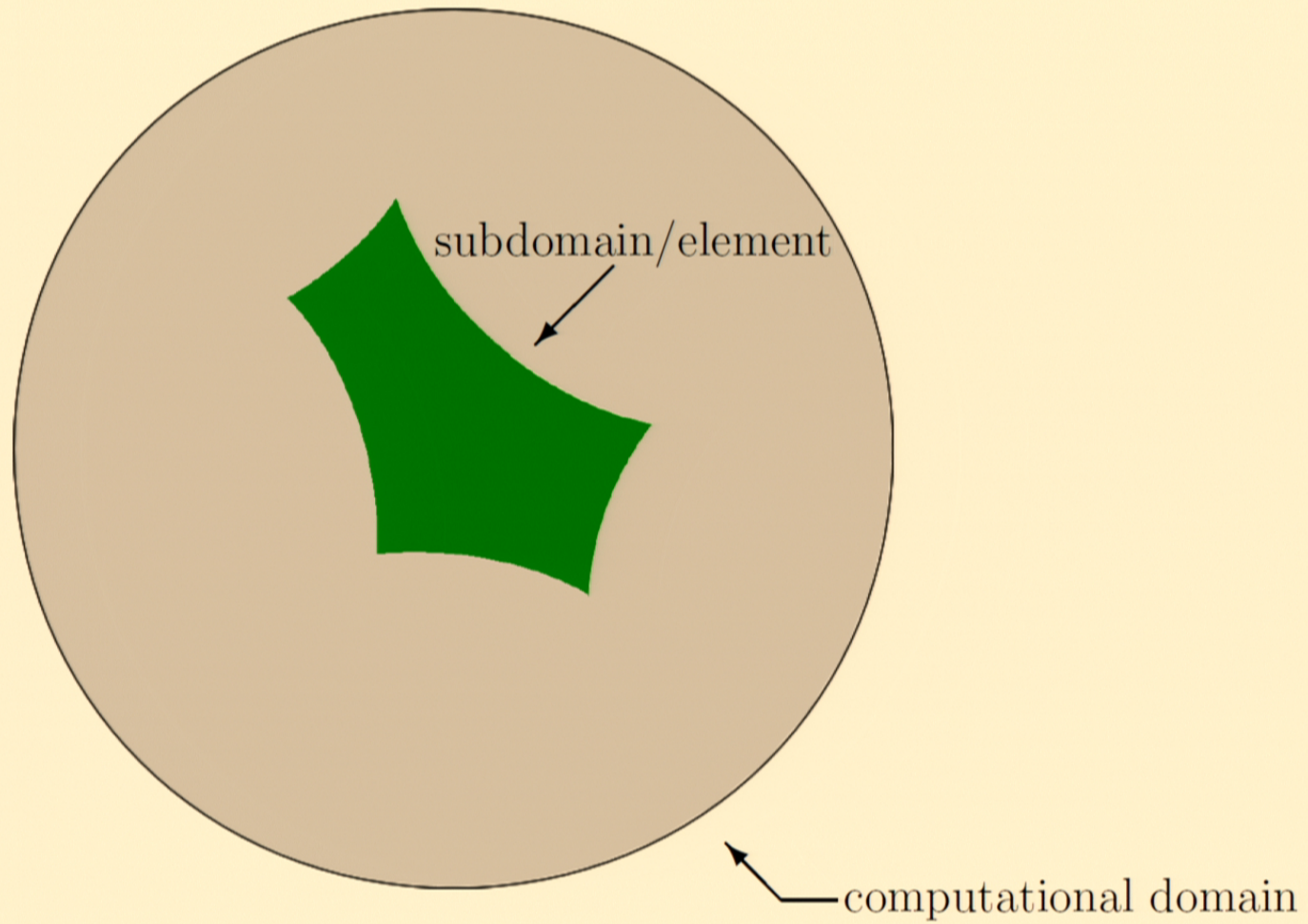


# Discontinuous Galerkin Methods



— computational domain

# Discontinuous Galerkin Methods



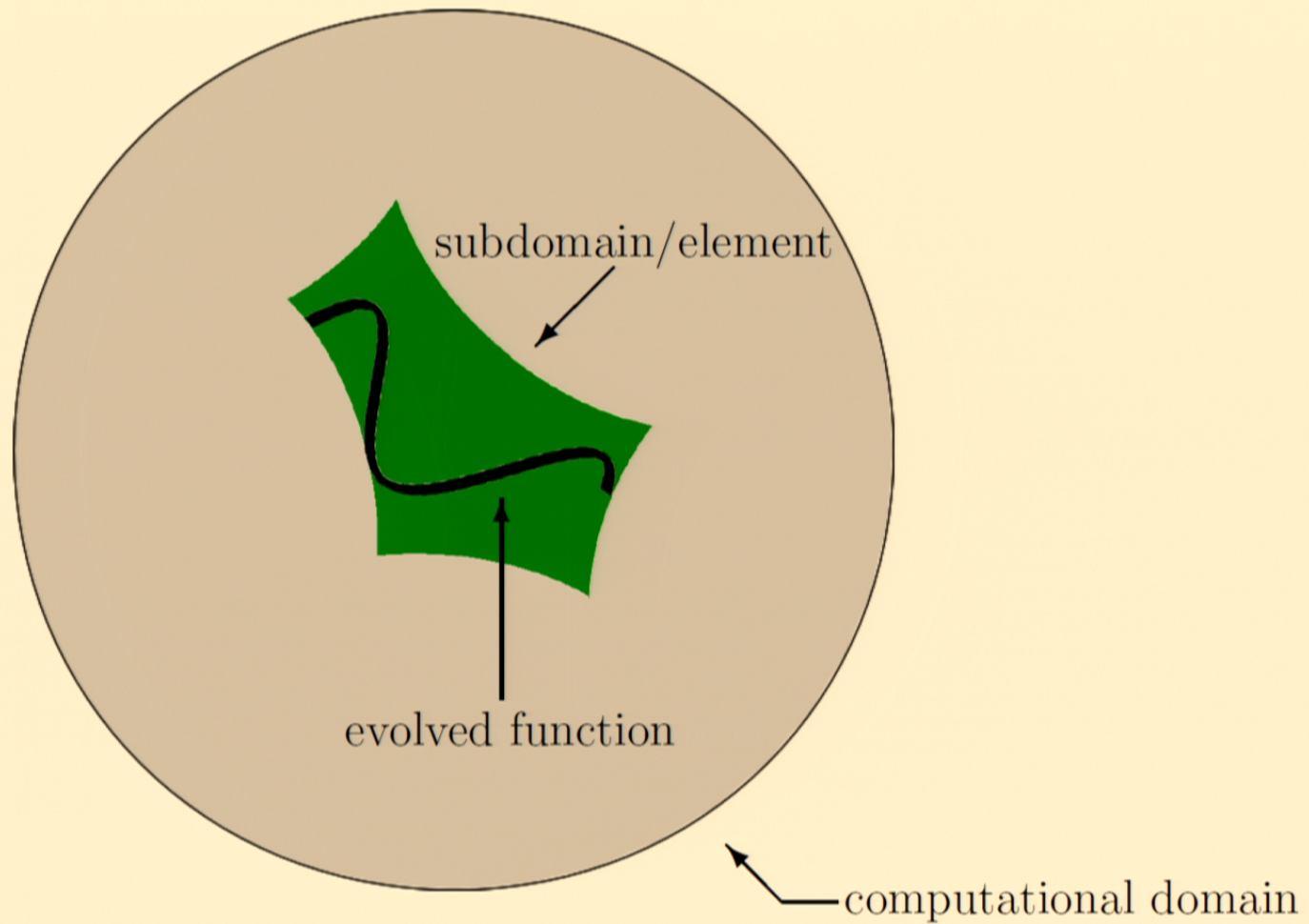
J. Miller (PI)

DG Methods

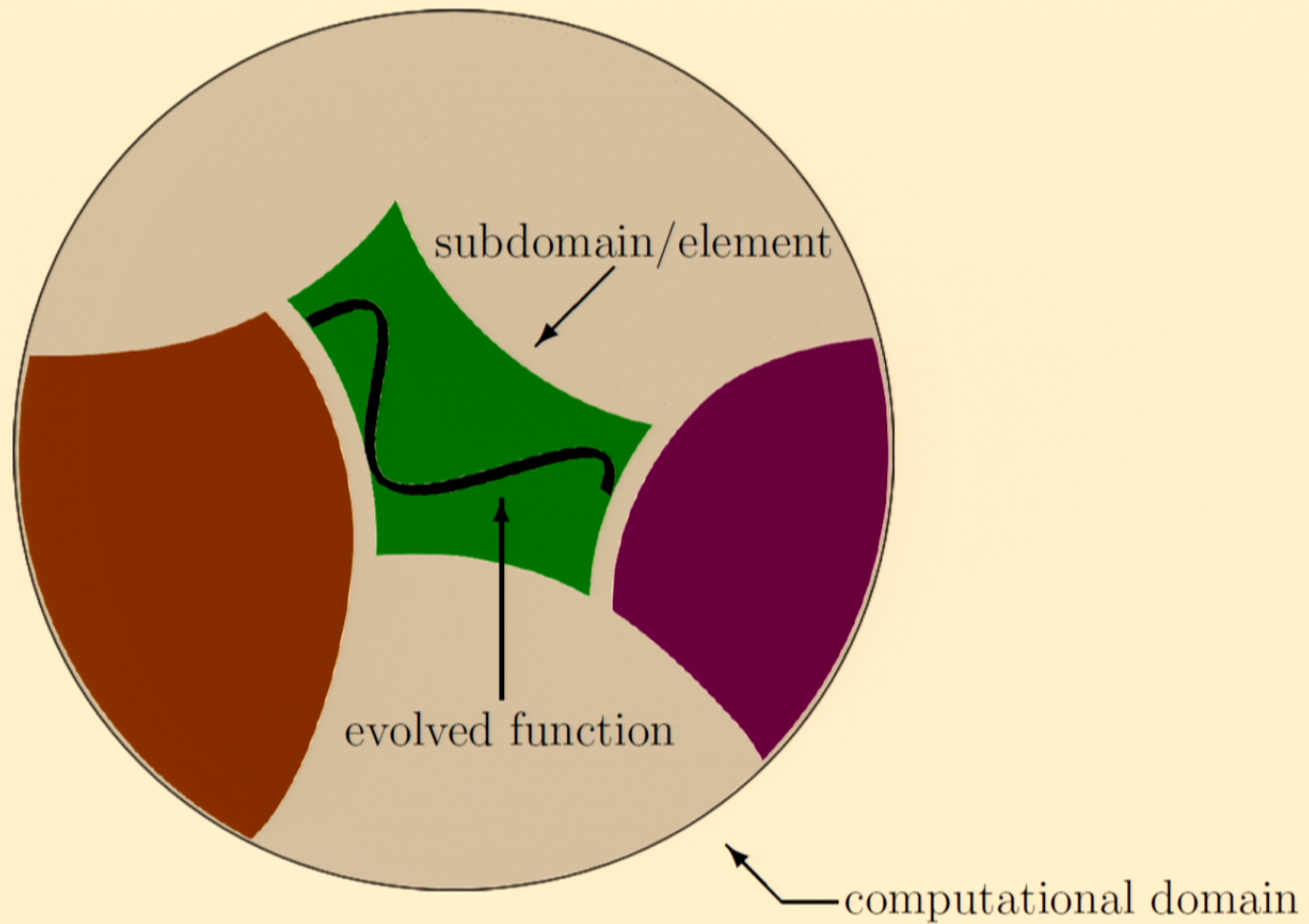
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# Discontinuous Galerkin Methods



# Discontinuous Galerkin Methods



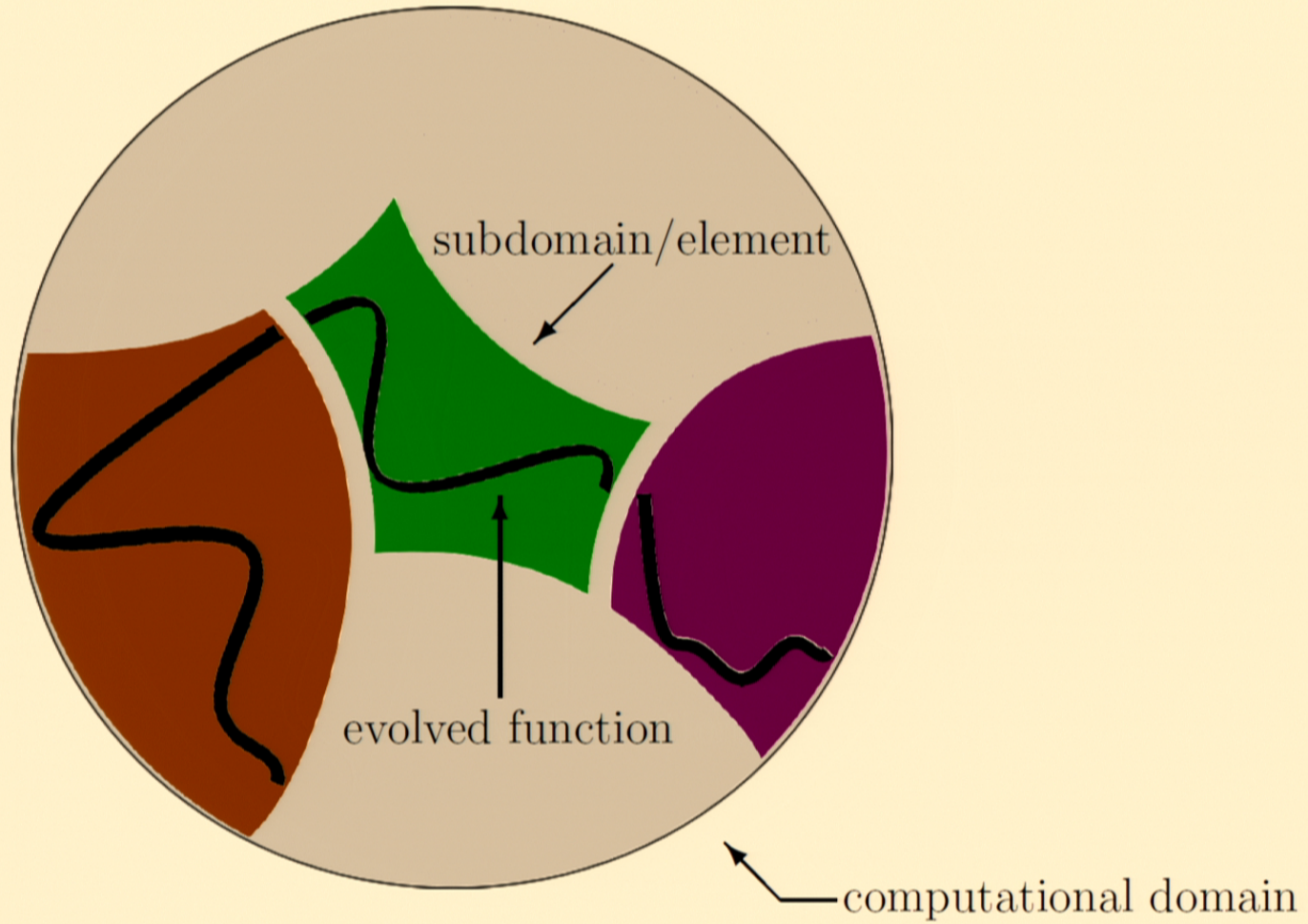
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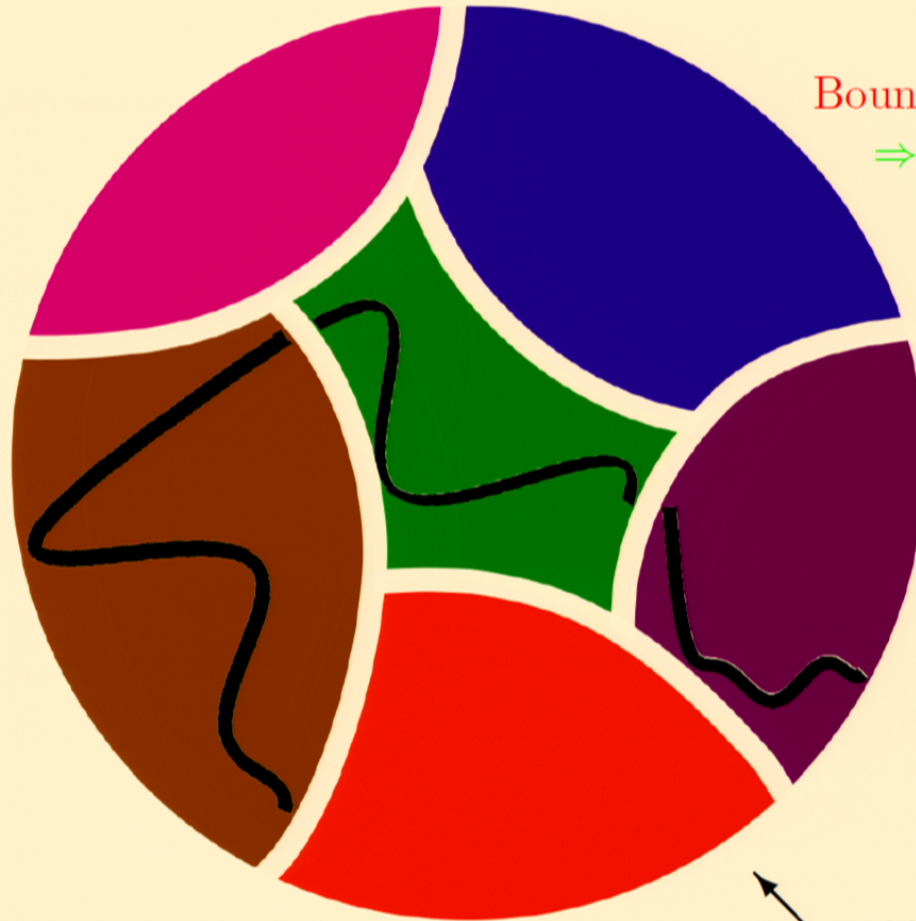
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# Discontinuous Galerkin Methods



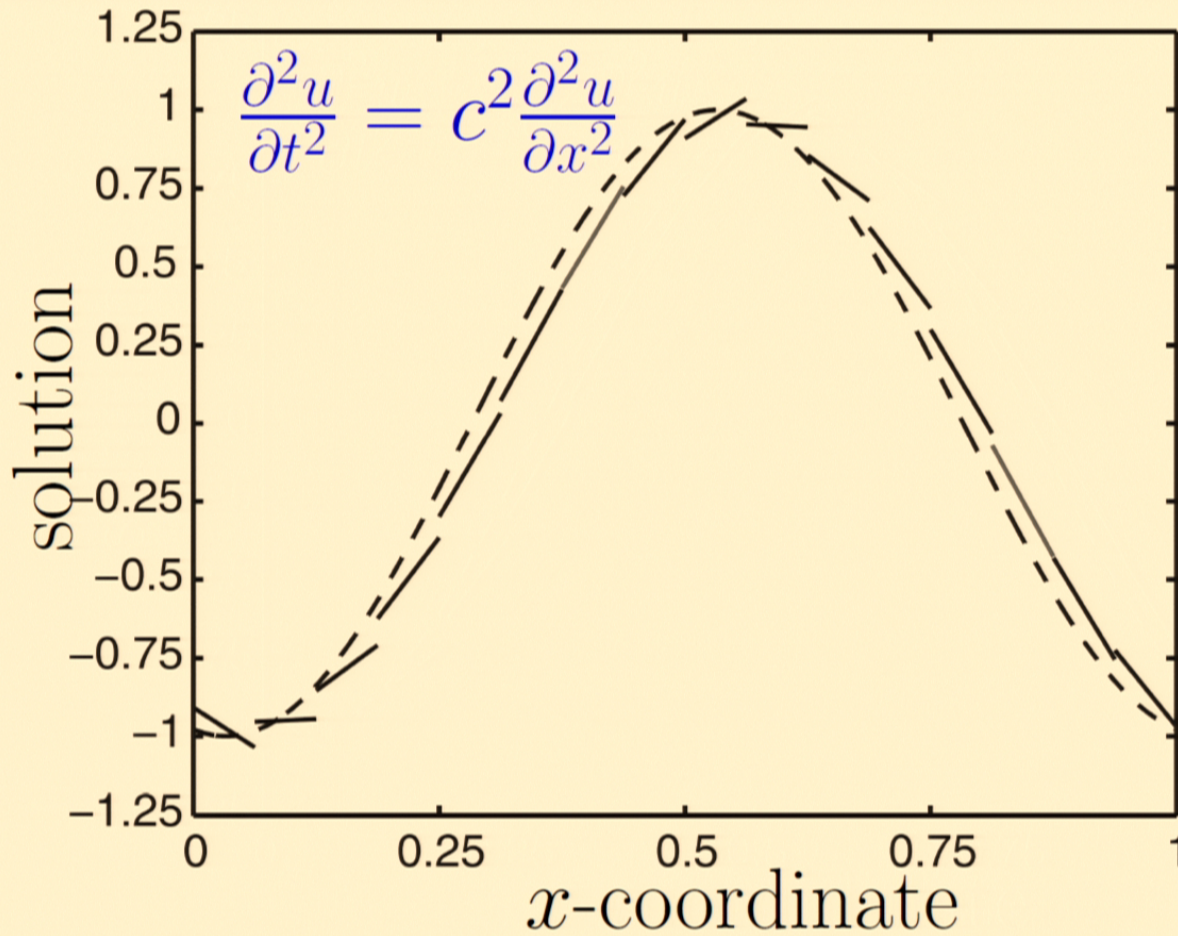
# Discontinuous Galerkin Methods



Boundaries are multivalued  
⇒ impose penalty terms

computational domain

# The Numerical Flux



Hesthaven and Warburton. *Nodal Discontinuous Galerkin Methods*. Springer (2008).

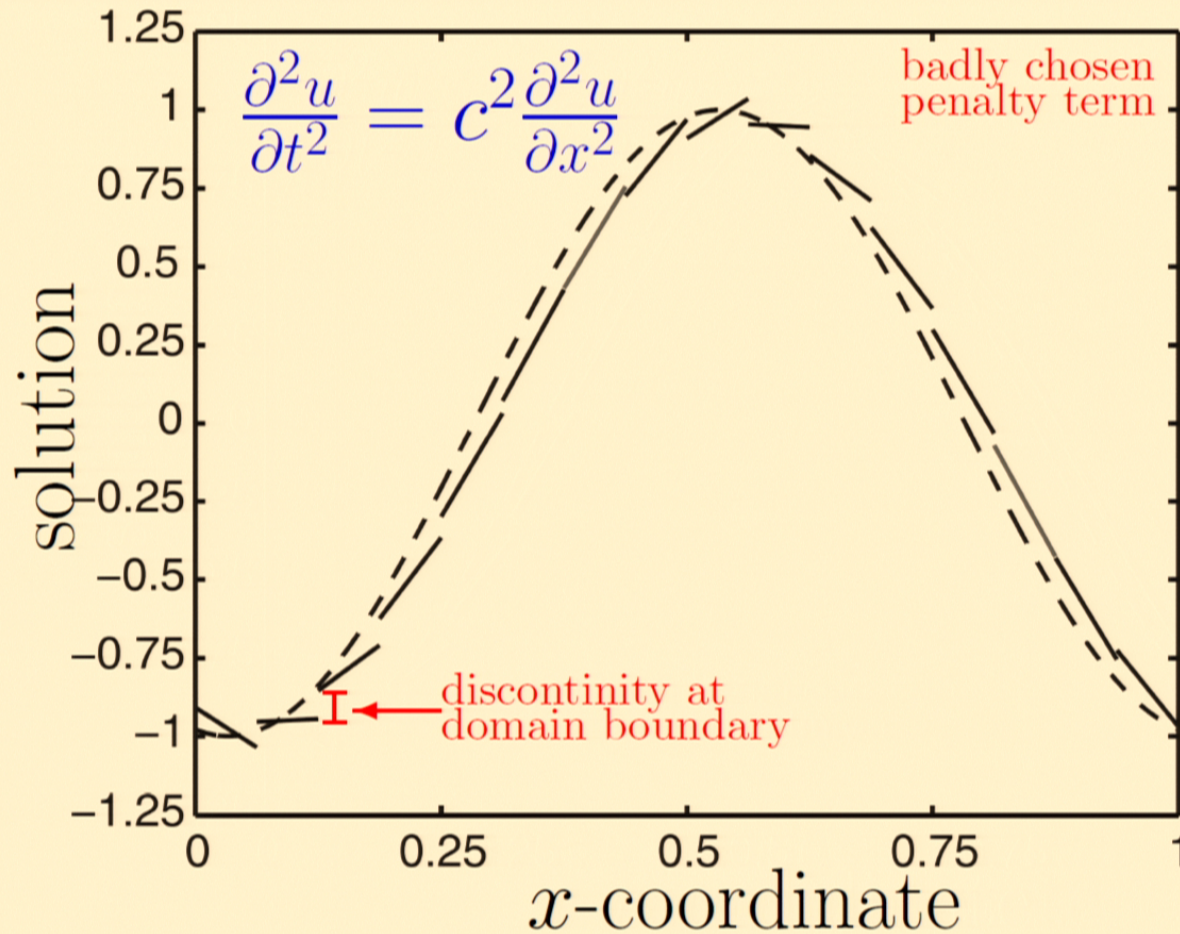
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DG Methods

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Hesthaven and Warburton. *Nodal Discontinuous Galerkin Methods*. Springer (2008).

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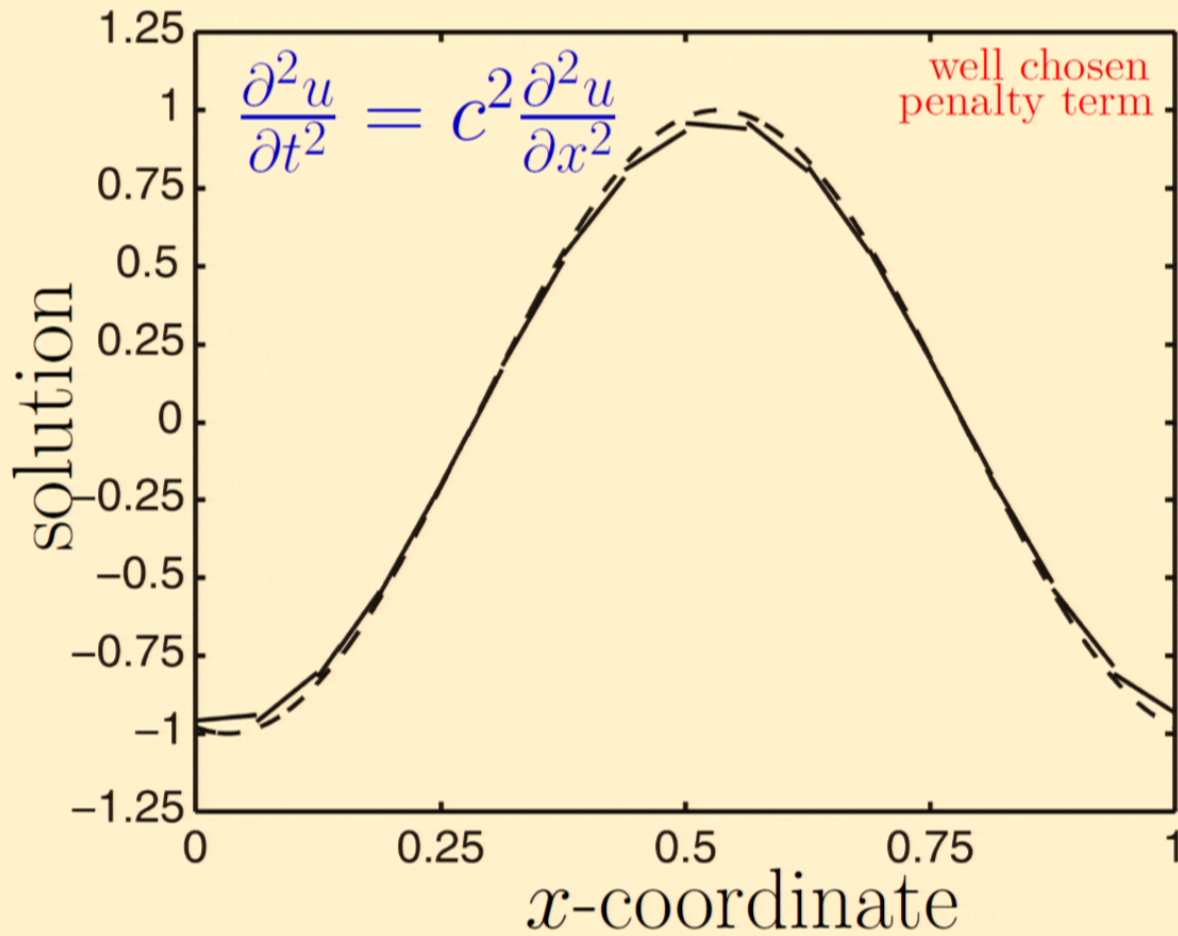
DG Methods

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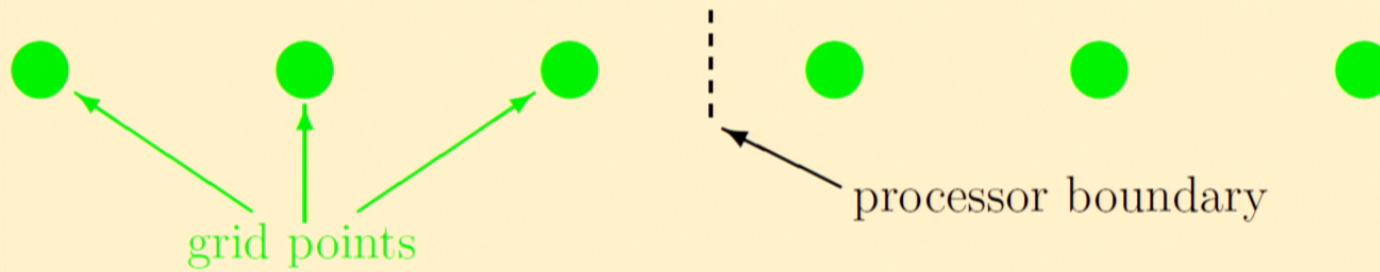
J. Miller (PI)

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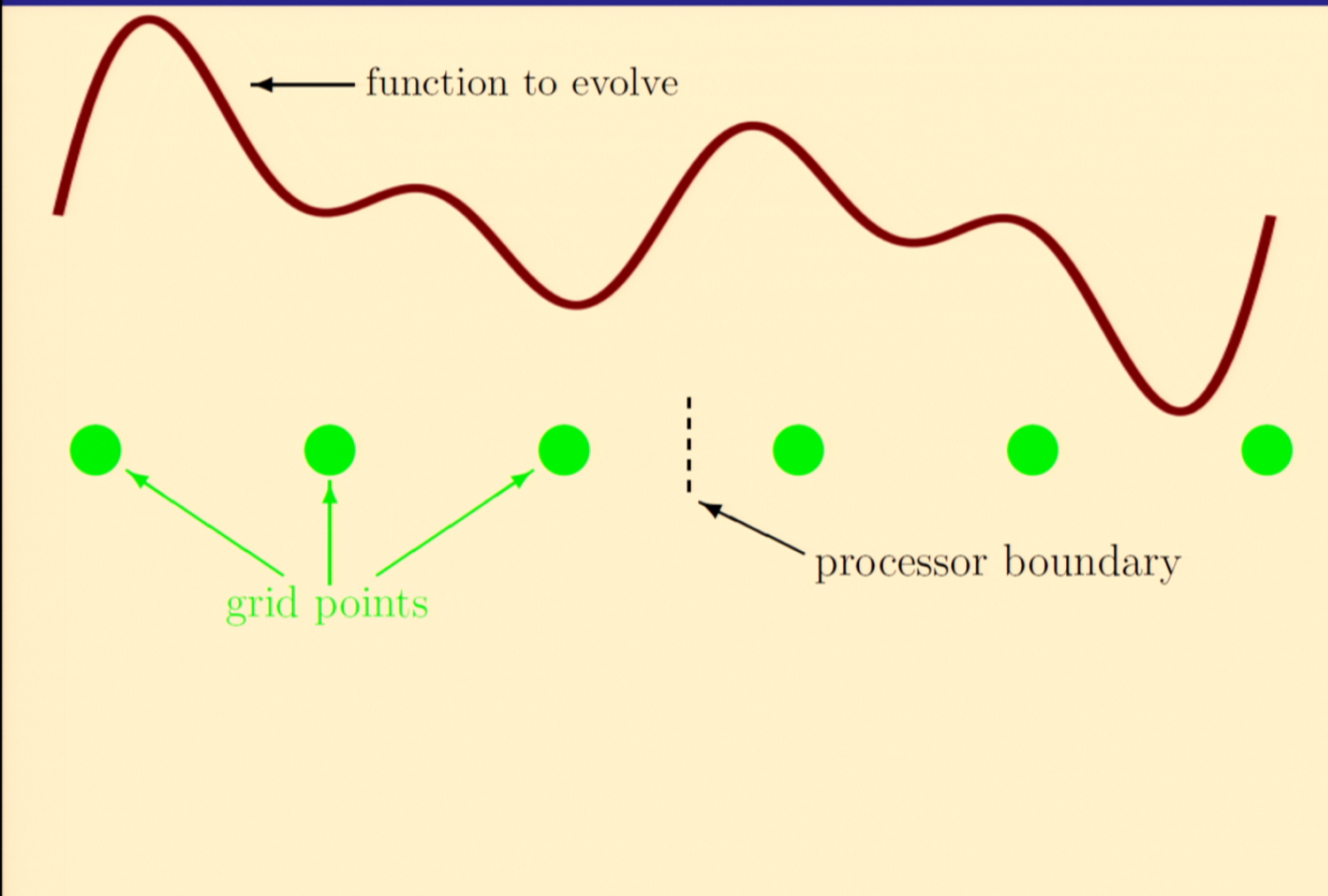
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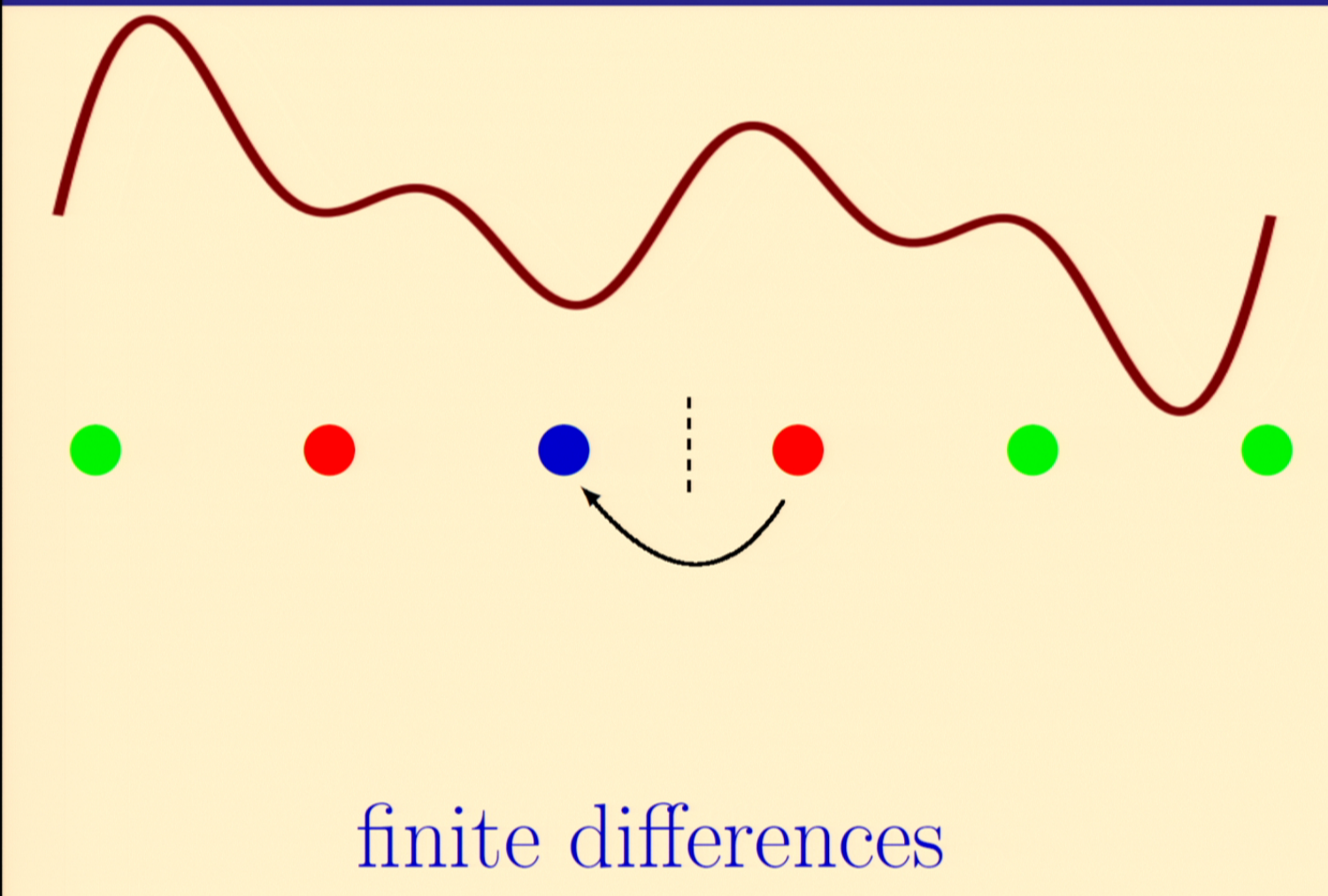
# Scalability: Finite Differences Vs. Discontinuous Galerkin



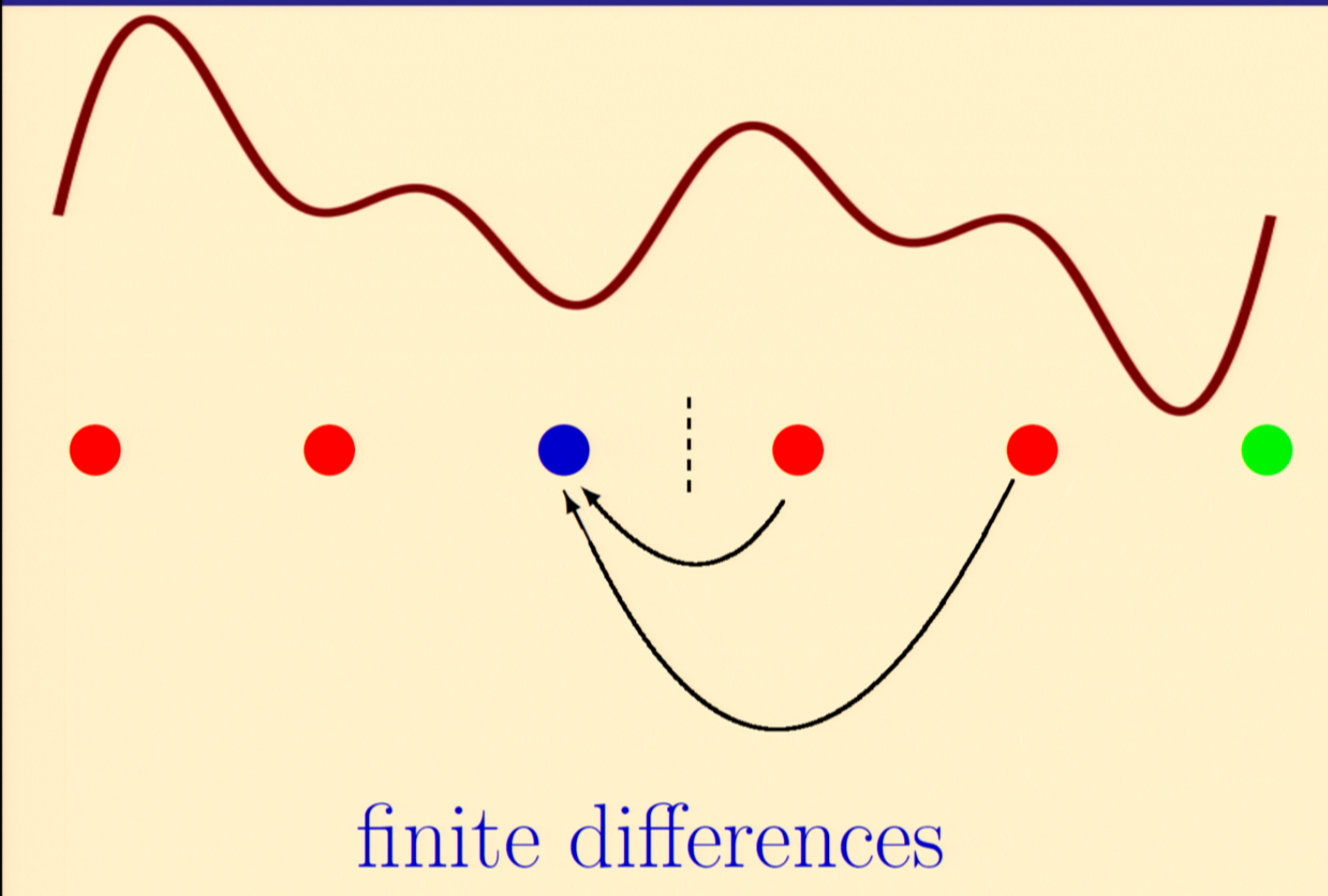
# Scalability: Finite Differences Vs. Discontinuous Galerkin



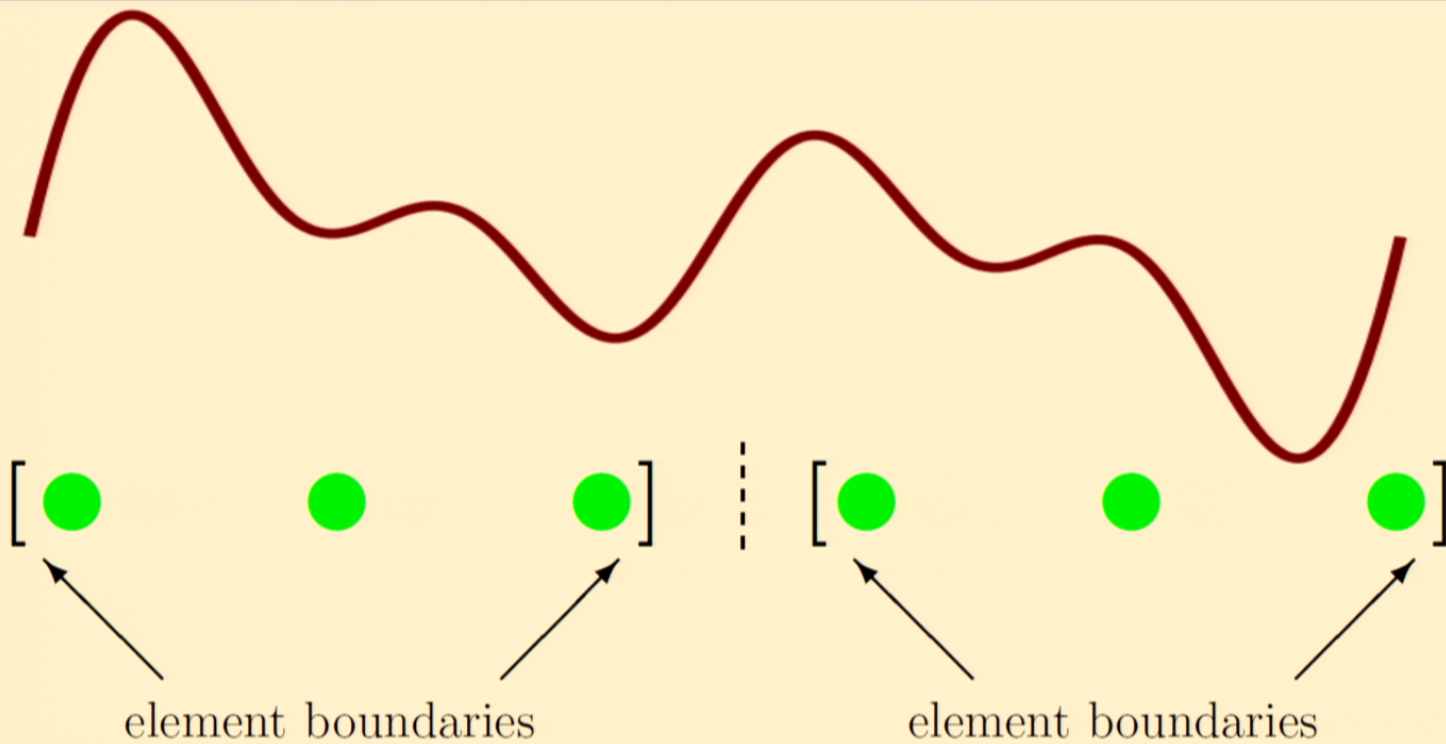
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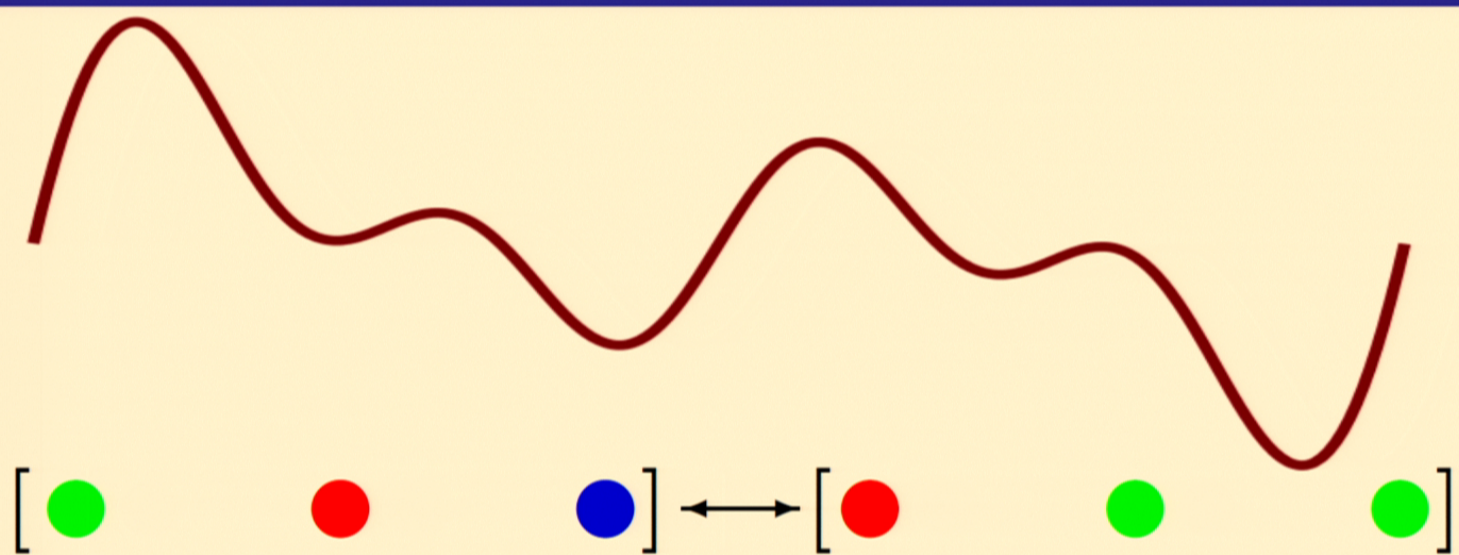


# Scalability: Finite Differences Vs. Discontinuous Galerkin



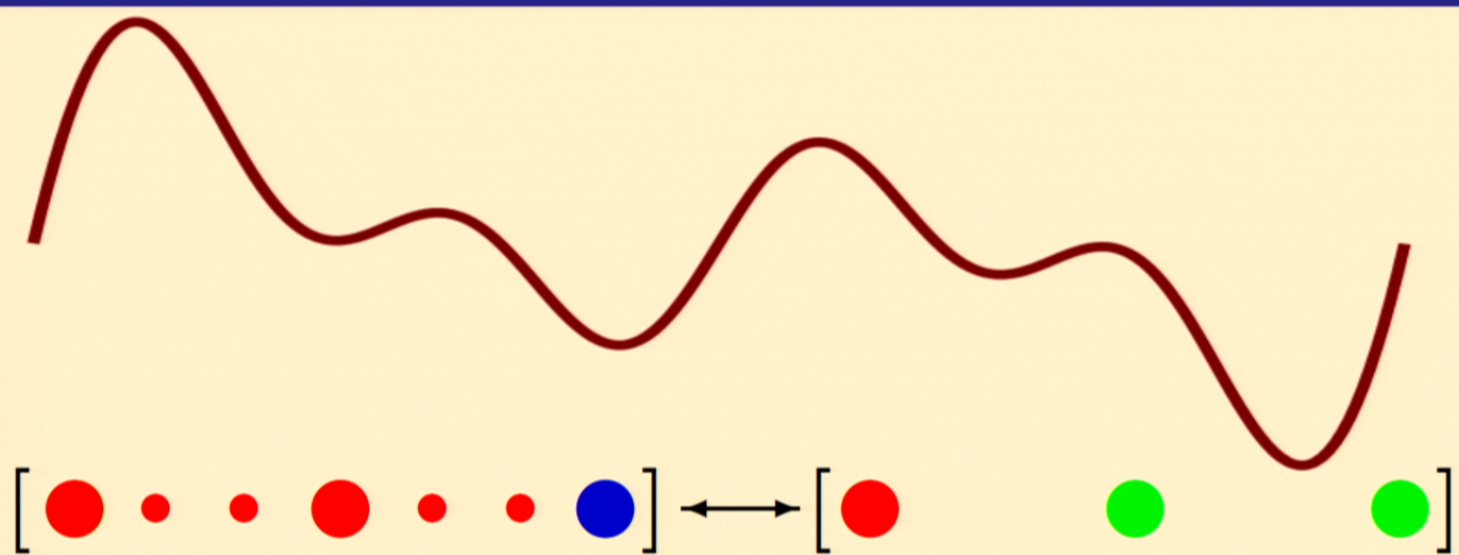
## discontinuous Galerkin

# Scalability: Finite Differences Vs. Discontinuous Galerkin



discontinuous Galerkin

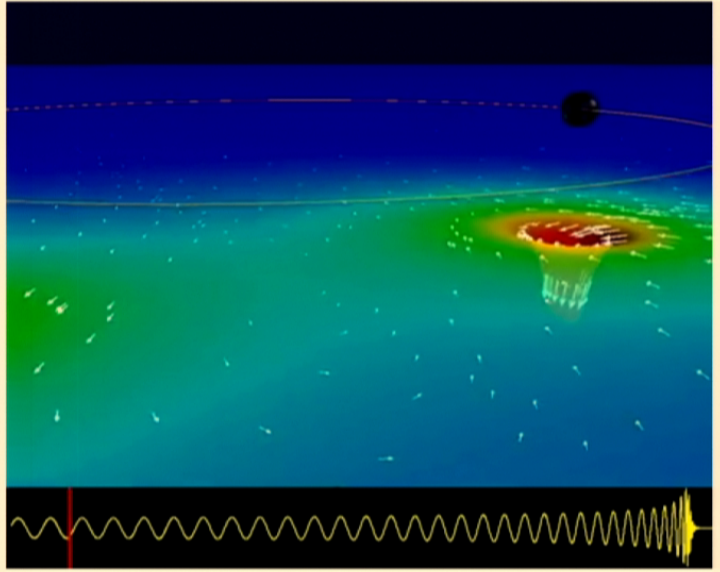
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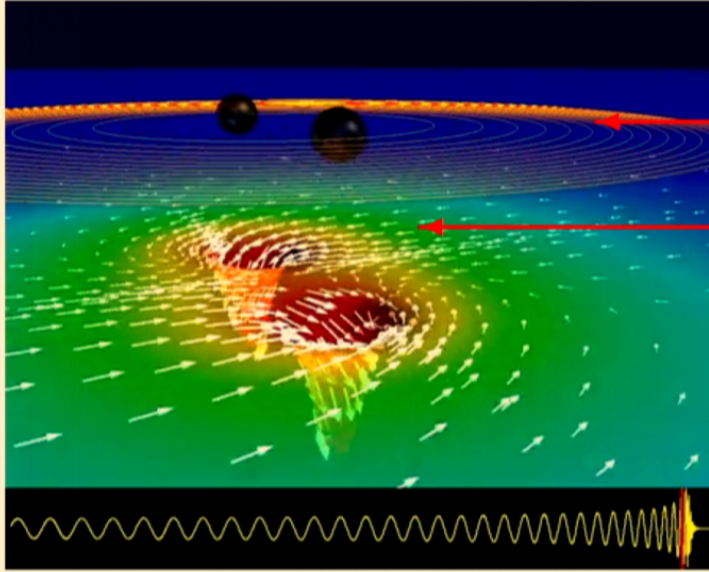
discontinuous Galerkin



# The Apples With Apples Tests



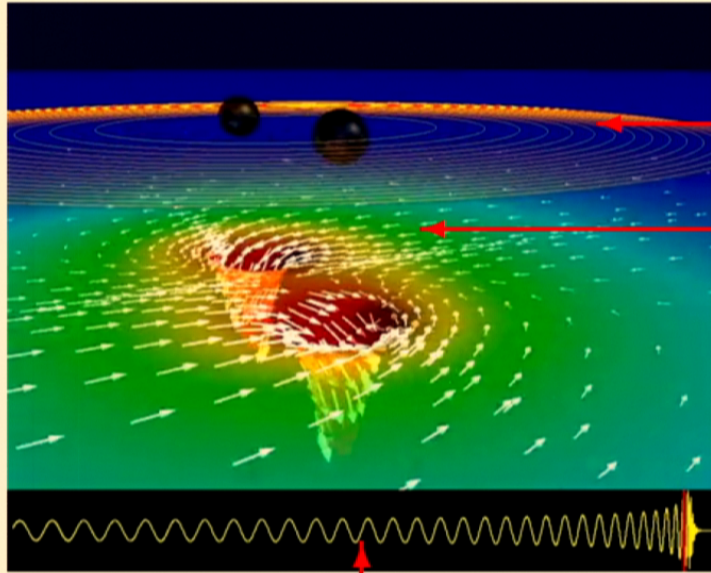
# The Apples With Apples Tests



physical system

highly nonlinear equations

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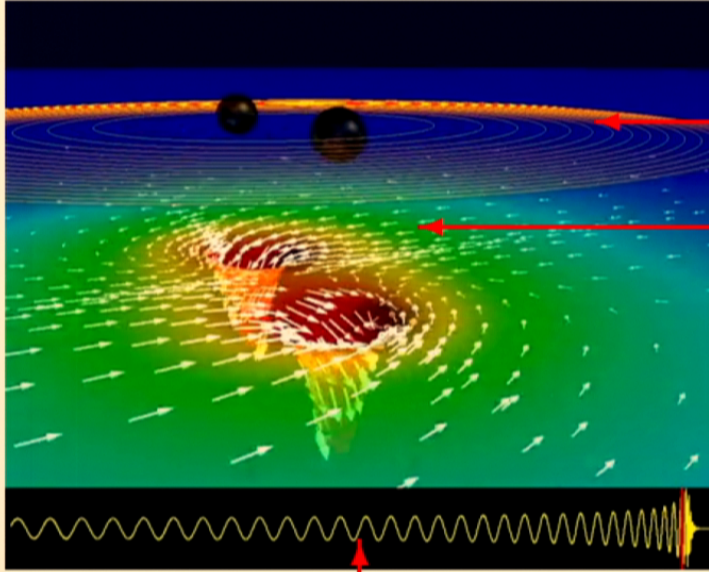


physical system

highly nonlinear equations

measurable quantity to extract

# The Apples With Apples Tests



physical system

highly nonlinear equations

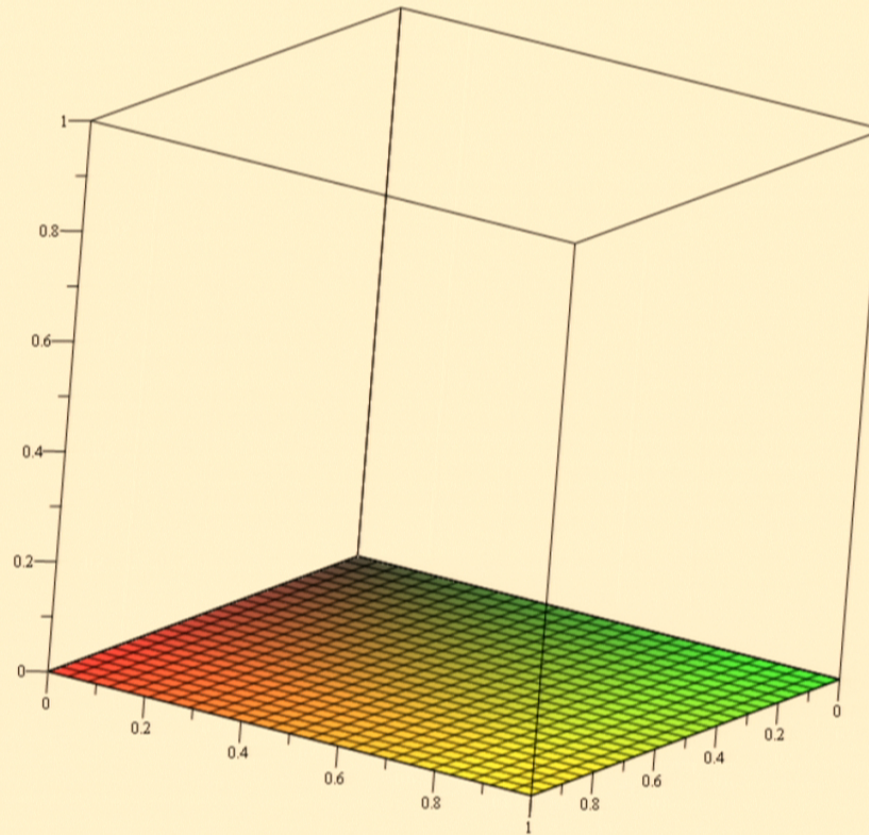
measurable quantity to extract

Let's compare...



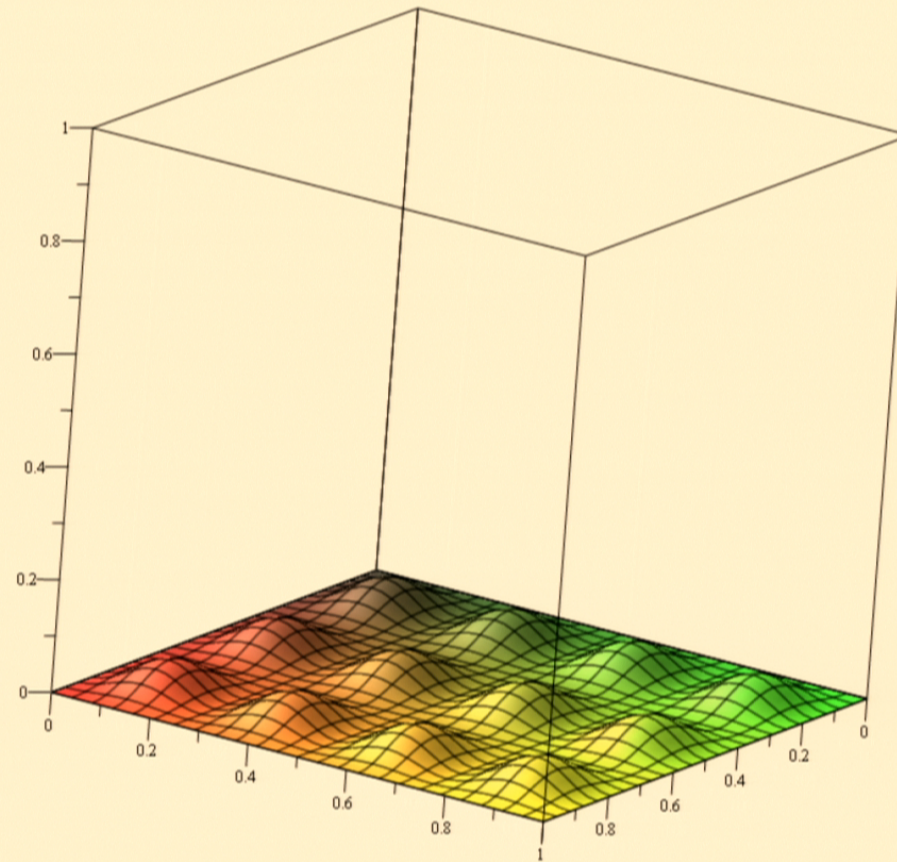
# The Robust Stability Test

evolved spacetime

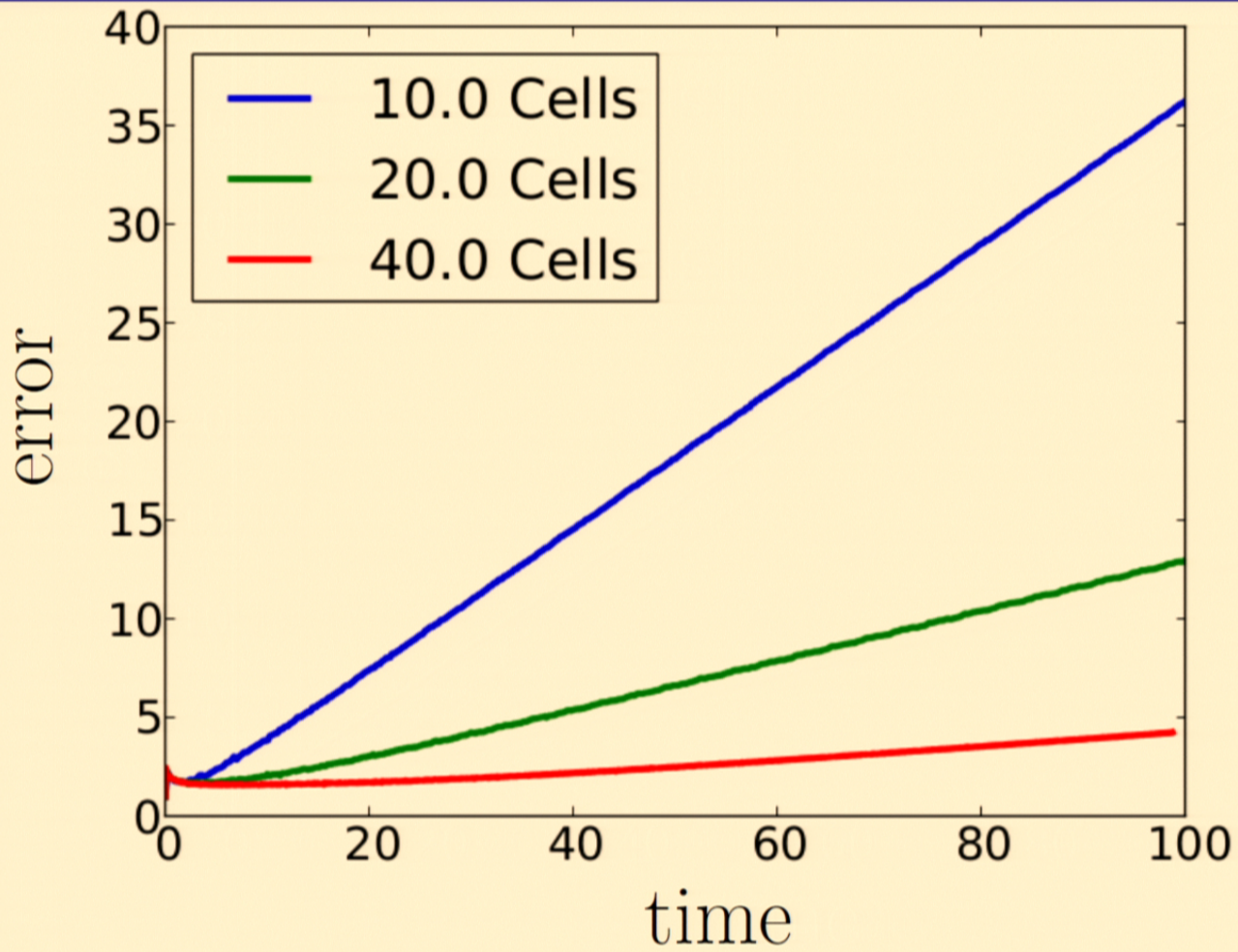


# The Robust Stability Test

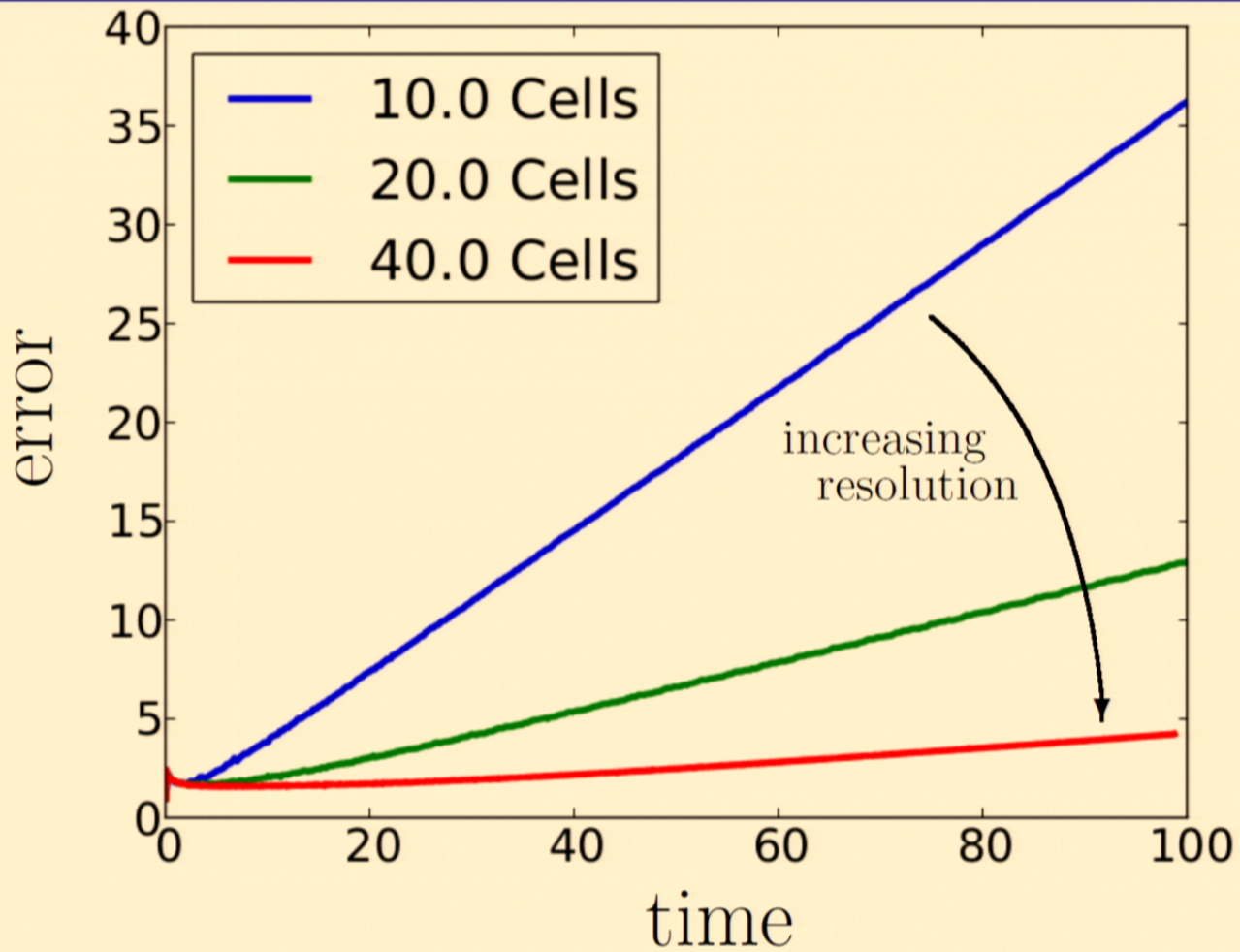
evolved spacetime



# The Robust Stability Test



# The Robust Stability Test





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# Coordinates are Dynamical Variables



Image: Abell 2218 galaxy cluster. Credit: NASA.

# Coordinates are Dynamical Variables

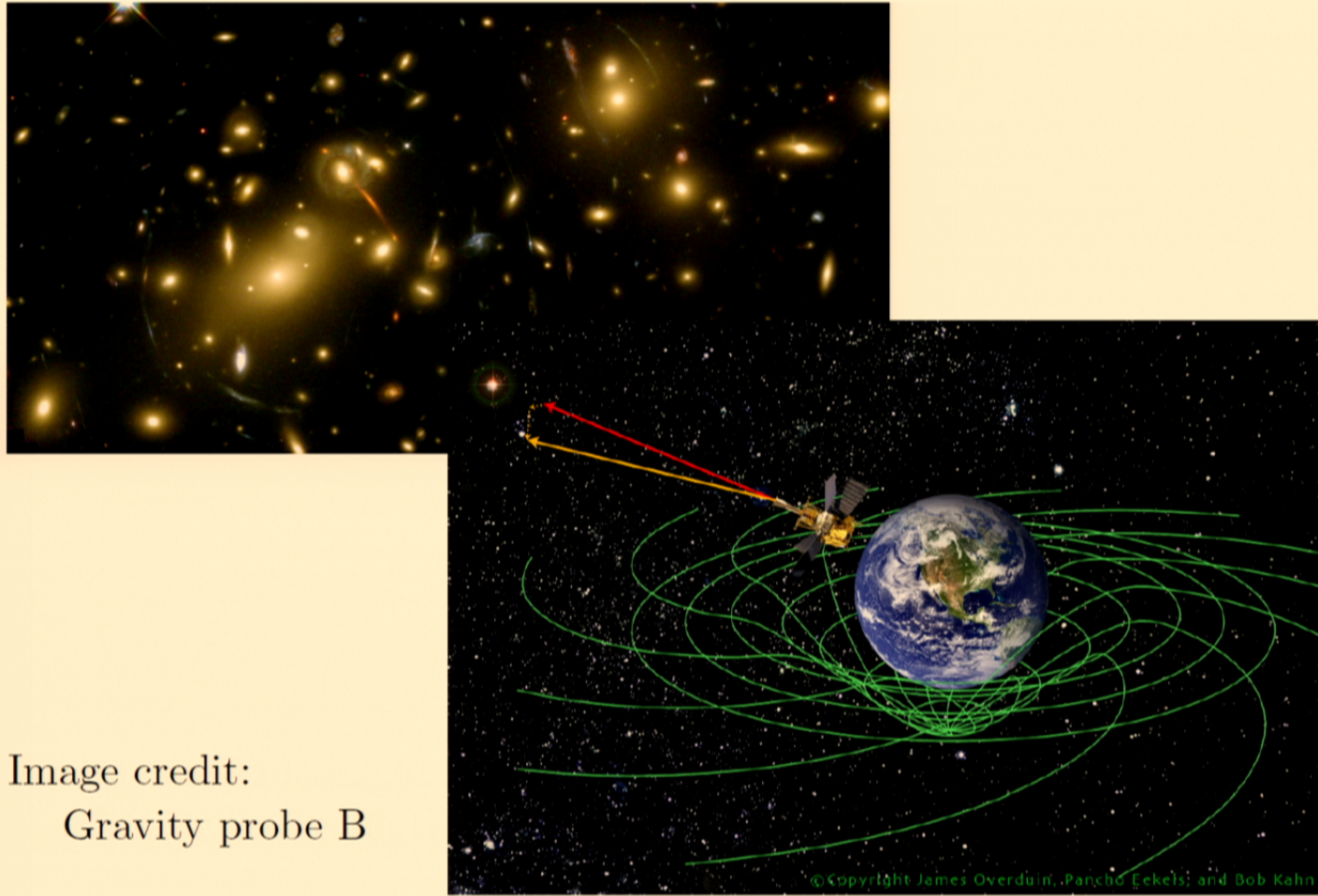
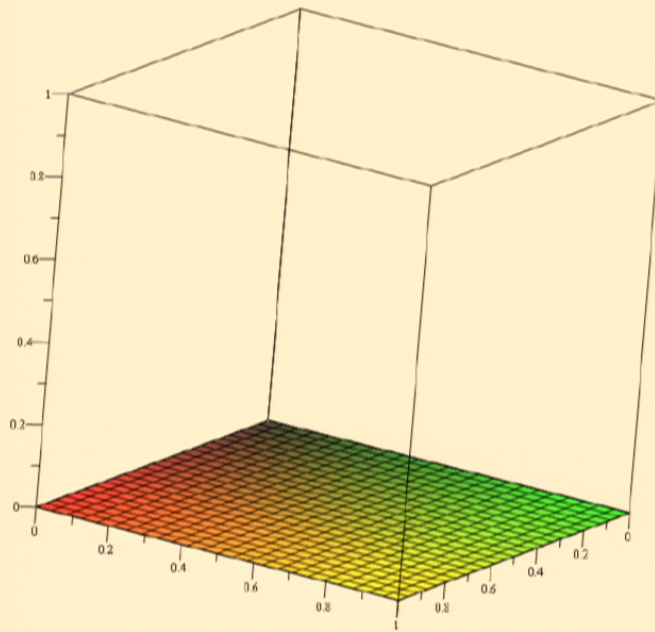
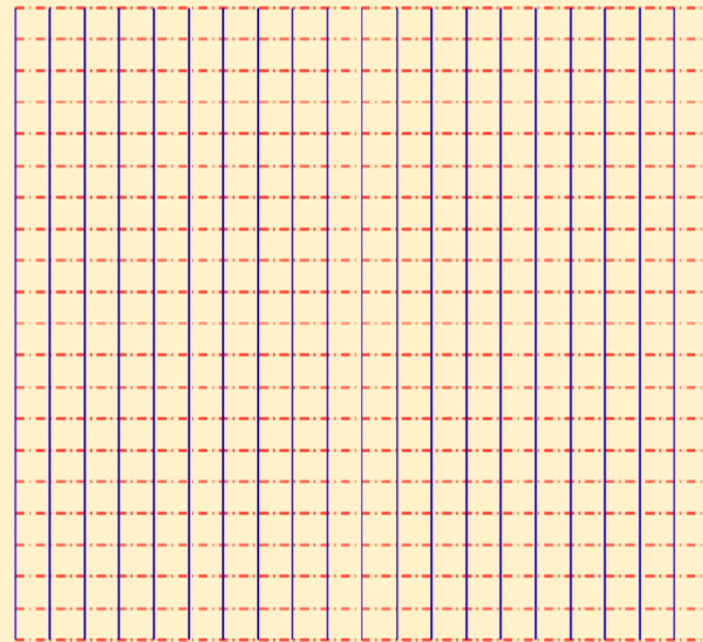


Image credit:  
Gravity probe B

# The Shifted Gauge Wave Test

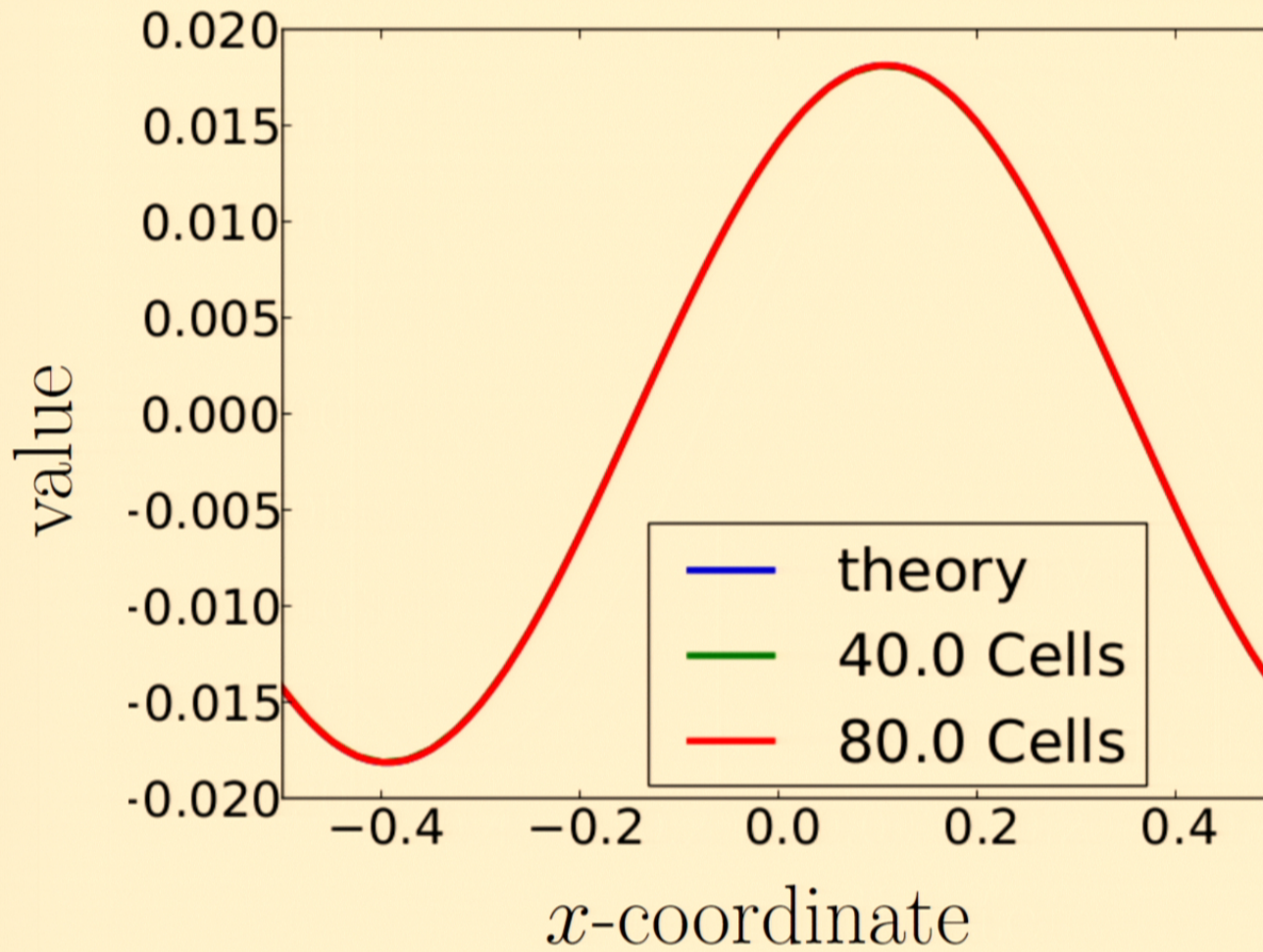


evolved spacetime



coordinate system

# The Shifted Gauge Wave Test

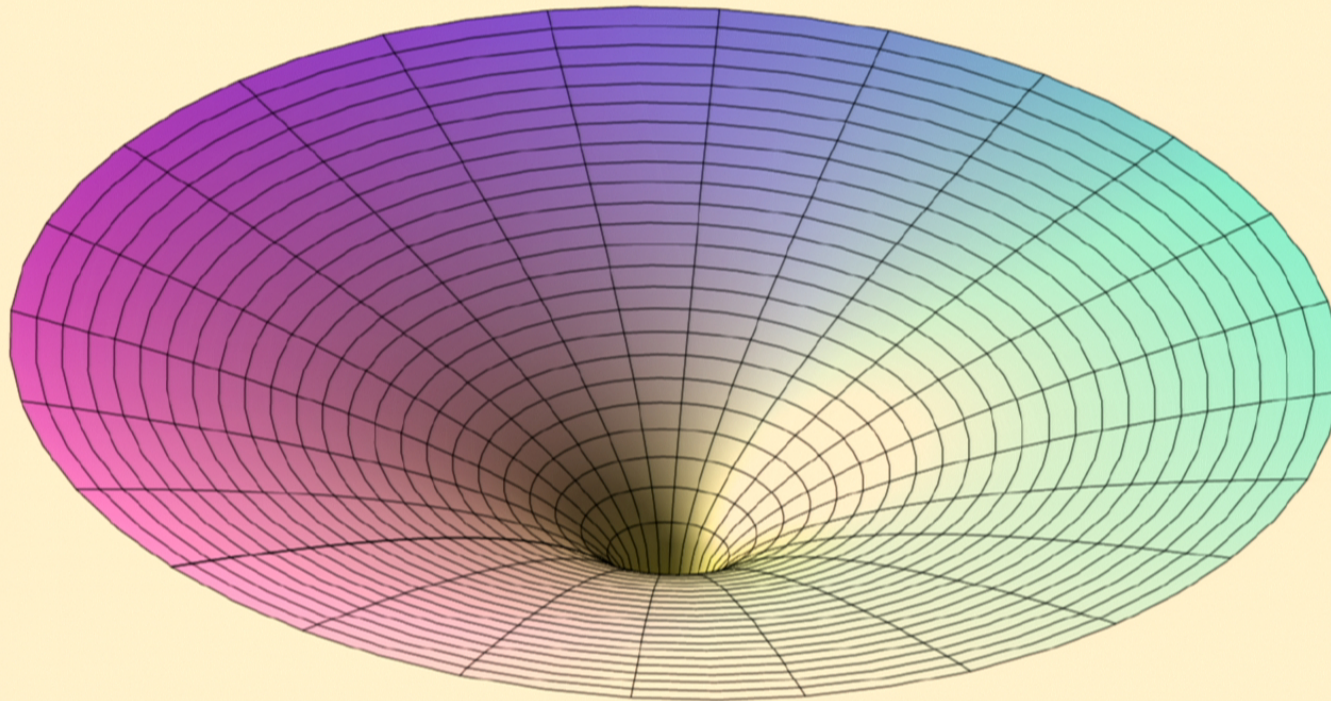


# A Static Black Hole

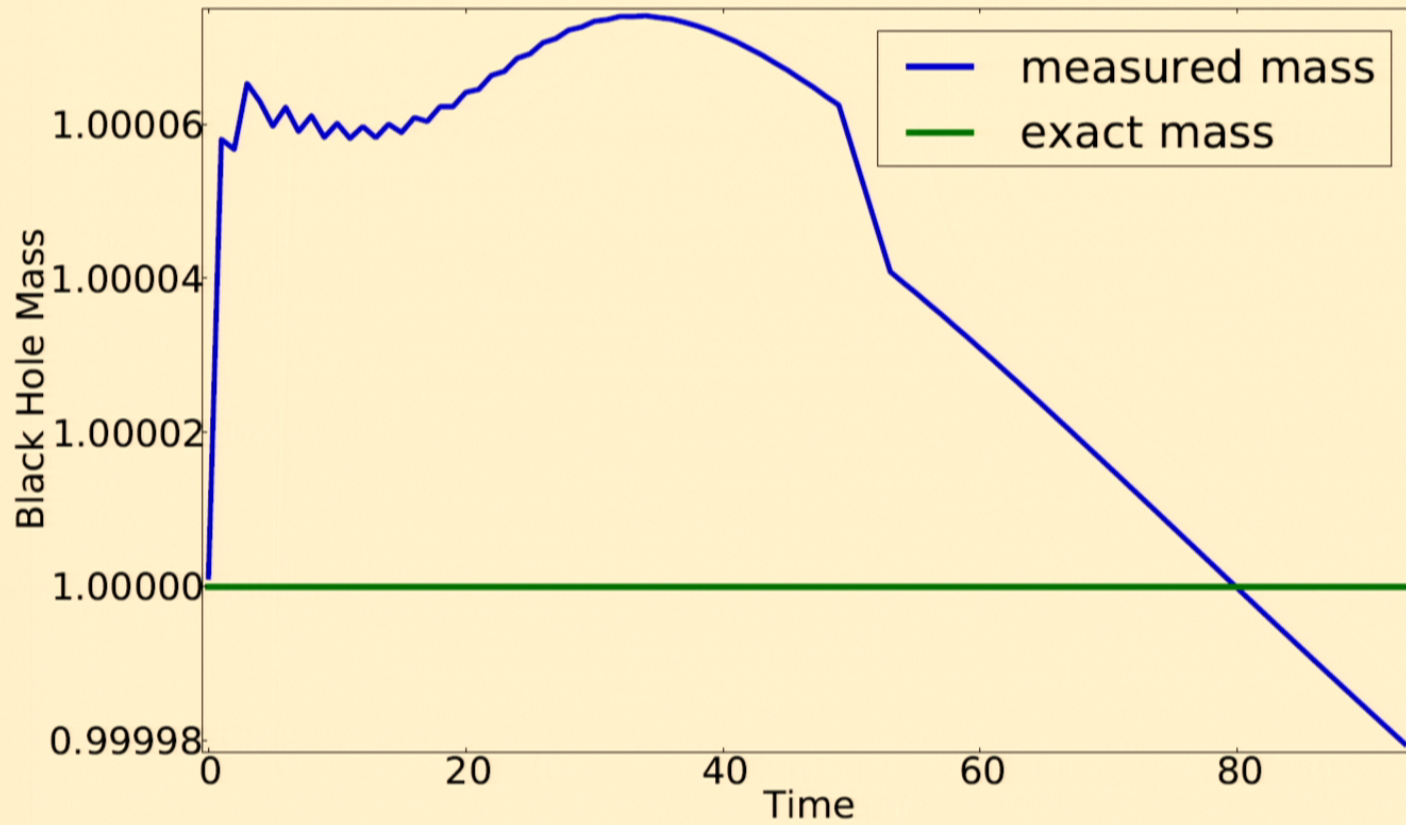


Image credit: NASA/JPL-Caltech

# A Static Black Hole



# A Static Black Hole



## Next Steps

- Port singularity handling to DGFE
- Add fluid dynamics
- Code optimization
- Performance tests
- Relativistic astrophysics!





## Thanks to...

- Thanks to my thesis committee members, Eric Poisson and Luis Lehner
- Thanks to the Einstein Toolkit community for developing a great toolbox
- This work was supported by NSERC
- This work used SHARCNET resources
- This work used XSEDE resources
- Thanks for listening!

