Title: Holographic Path to the Turbulent Side of Gravity

Date: May 07, 2014 04:20 PM

URL: http://pirsa.org/14050038

Abstract: We study the dynamics of a 2 1-dimensional relativistic viscous conformal fluid in Minkowski spacetime. Such fluid solutions arise as duals under the gravity/fluid correspondence to 3 1-dimensional asymptotically antide Sitter (AAdS) black-brane solutions to the Einstein equation. We examine stability properties of shear flows which correspond to hydrodynamic quasinormal modes of the black brane. We find that for sufficiently high Reynolds number the solution undergoes an inverse turbulent cascade to long-wavelength modes. We then map this fluid solution via the gravity/fluid duality into a bulk metric. This suggests a new and interesting feature of the behavior of perturbed AAdS black holes and black branes which is not readily captured by a standard quasinormal mode analysis. Namely for sufficiently large perturbed black objects (with long-lived quasinormal modes) nonlinear effects transfer energy from short- to long-wavelength modes via a turbulent cascade within the metric perturbation. As long-wavelength modes have slower decay this transfer of energy lengthens the overall lifetime of the perturbation. We also discuss various implications of this behavior including expectations for higher dimensions and the possibility of predicting turbulence in more general gravitational scenarios.

Holographic Path to the Turbulent Side of Gravity

Phys. Rev. X4, 011001 (2014), arXiv:1309.7940 [hep-th]

Stephen Green (CITA National Fellow, U. of Guelph)
with Federico Carrasco and Luis Lehner

Compute Ontario Research Day
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Introduction

- AdS / CFT correspondence: Relates quantum gravity in (d+1) dimensions and quantum field theory on the d-dimensional boundary.

- There exists a purely classical limit, where

$$\begin{tikzcd}
\text{(d+1) quantum gravity} \arrow[r] \arrow[d, leftrightarrow, "\text{AdS/CFT}"] & \text{(d+1) perturbed black brane solutions to Einstein's equation in anti-de Sitter} \arrow[d, leftrightarrow, "\text{“gravity/fluid correspondence”}"] \\
\text{d-dimensional QFT} \arrow[r] & \text{d-dimensional fluid}
\end{tikzcd}$$

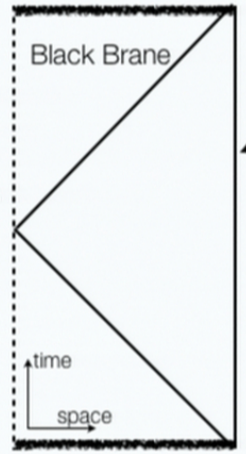
- Can studying fluids can teach us anything about general relativity?

Review of "gravity/fluid correspondence"

- Black Brane spacetime:

Bulk:
Asymptotically AdS black brane

Einstein Equation holds
$G_{AB} - \Lambda g_{AB} = 0$



Boundary:
Impose "boundary metric" = $\eta_{\mu\nu}$ (mirror)

Read off "boundary stress-energy"

$$T_{\mu\nu} \equiv \lim_{r\to\infty} \frac{r^{d-2}}{8\pi G_{d+1}} (K_{\mu\nu} - K\eta_{\mu\nu} - (d-1)\eta_{\mu\nu})$$

- *At late times, the boundary stress-energy takes the form of a relativistic, viscous conformal fluid.* This is derived in a derivative expansion; valid for long-wavelength perturbations. (Bhattacharyya et al, 2008)

- From the boundary stress-energy, can re-construct a bulk metric which solves the Einstein equation in the derivative expansion.

Boundary Fluid

- Resulting boundary stress-energy tensor (to 2nd order in derivatives):

$$T^{[0+1+2]}_{\mu\nu} = \frac{\rho}{d-1}\left(d u_\mu u_\nu + \eta_{\mu\nu}\right) + \Pi_{\mu\nu}$$

where the viscous part is given by

$$\Pi_{\mu\nu} = \underbrace{\boxed{-2\eta\sigma_{\mu\nu}}}_{\text{1st order in derivatives}}$$

2nd order

$$\boxed{+2\eta\tau_\Pi\left(\langle u^\alpha\partial_\alpha\sigma_{\mu\nu}\rangle + \frac{1}{d-1}\sigma_{\mu\nu}\partial_\alpha u^\alpha\right) + \langle\lambda_1\sigma_{\mu\alpha}\sigma_\nu{}^\alpha + \lambda_2\sigma_{\mu\alpha}\omega_\nu{}^\alpha + \lambda_3\omega_{\mu\alpha}\omega_\nu{}^\alpha\rangle}$$

shear

vorticity

Transport coefficients all functions of the density.
In particular, shear viscosity $\eta = \frac{s}{4\pi} \propto T^{d-1} \propto \rho^{(d-1)/d}$

- There is also a dual bulk metric, which is expressed in terms of $u_\mu,\ \rho$

Fluid equations of motion

- From stress-energy conservation:

$$0 = u^\mu \partial_\mu \rho + \frac{d}{d-1}\rho \partial_\mu u^\mu - u^\mu \partial^\nu \Pi_{\mu\nu}$$
$$0 = \frac{d}{d-1}\rho u^\mu \partial_\mu u^\alpha + \frac{1}{d-1}\partial^\alpha \rho - \frac{d}{(d-1)^2} u^\alpha \rho \partial_\mu u^\mu + \frac{1}{d-1} u^\alpha u^\mu \partial^\nu \Pi_{\mu\nu} + P^{\alpha\mu}\partial^\nu \Pi_{\mu\nu}$$

- **Technical point:** Need to promote $\Pi_{\mu\nu}$ to an independent field, and substitute $-2\eta\sigma_{\mu\nu} \to \Pi_{\mu\nu}$ to obtain

$$\Pi_{\mu\nu} = -2\eta\sigma_{\mu\nu} - \tau_\Pi \left(\langle u^\alpha \partial_\alpha \Pi_{\mu\nu} \rangle + \frac{d}{d-1}\Pi_{\mu\nu}\partial_\alpha u^\alpha \right) + \langle \frac{\lambda_1}{\eta^2}\Pi_{\mu\alpha}\Pi_\nu{}^\alpha - \frac{\lambda_2}{\eta}\Pi_{\mu\alpha}\omega_\nu{}^\alpha + \lambda_3 \omega_{\mu\alpha}\omega_\nu{}^\alpha \rangle$$

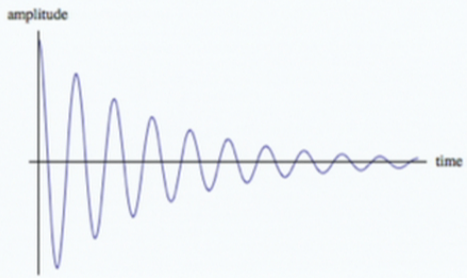
- We will perform numerical simulations in d=3; Take dynamical variables to be

$$\mathcal{U} \equiv (\rho, u_x, u_y, \Pi_{xx}, \Pi_{xy})$$

- Put equations in form $\partial_t \mathcal{U} = \mathcal{F}(\mathcal{U}, \partial_i \mathcal{U})$

Expectations from gravity

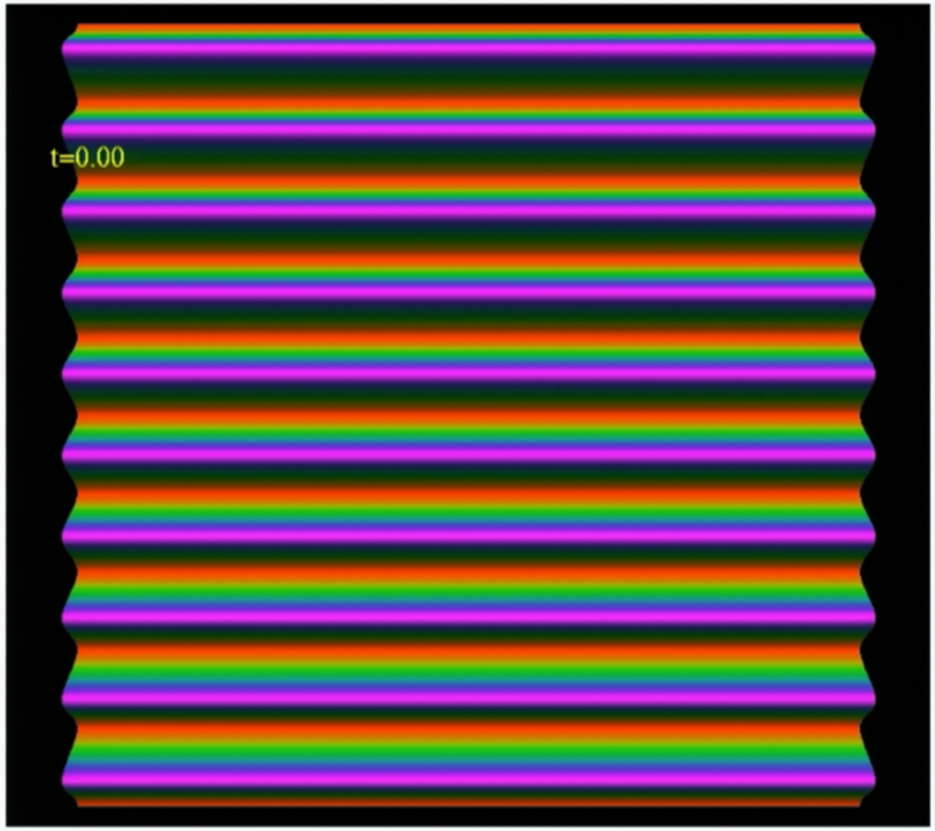
- Black hole / black brane perturbations typically studied analytically in *linear approximation*: Find *quasinormal modes*.



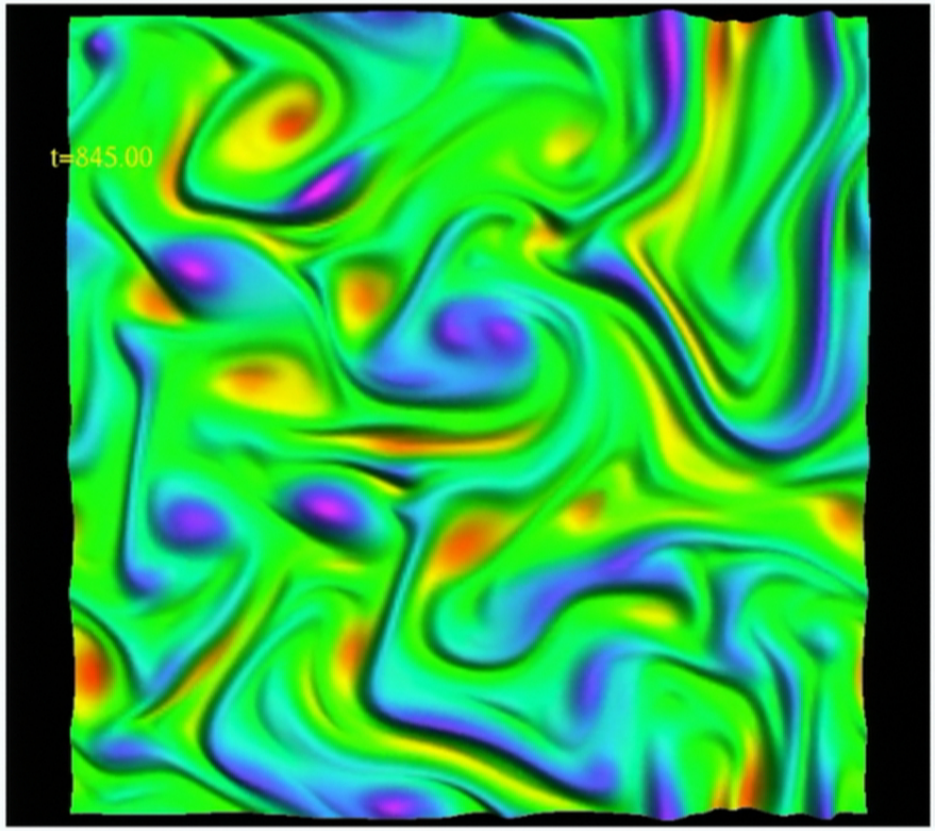
- Fluids at high Reynolds number characterized by *turbulence*, which is a *nonlinear effect*.

We use the fluid/gravity correspondence to study black hole perturbations nonlinearly, via the dual fluid. We choose initial data corresponding to shear mode, and study its evolution under the full nonlinear fluid equations. We also include tiny random perturbations, in order to assess stability.

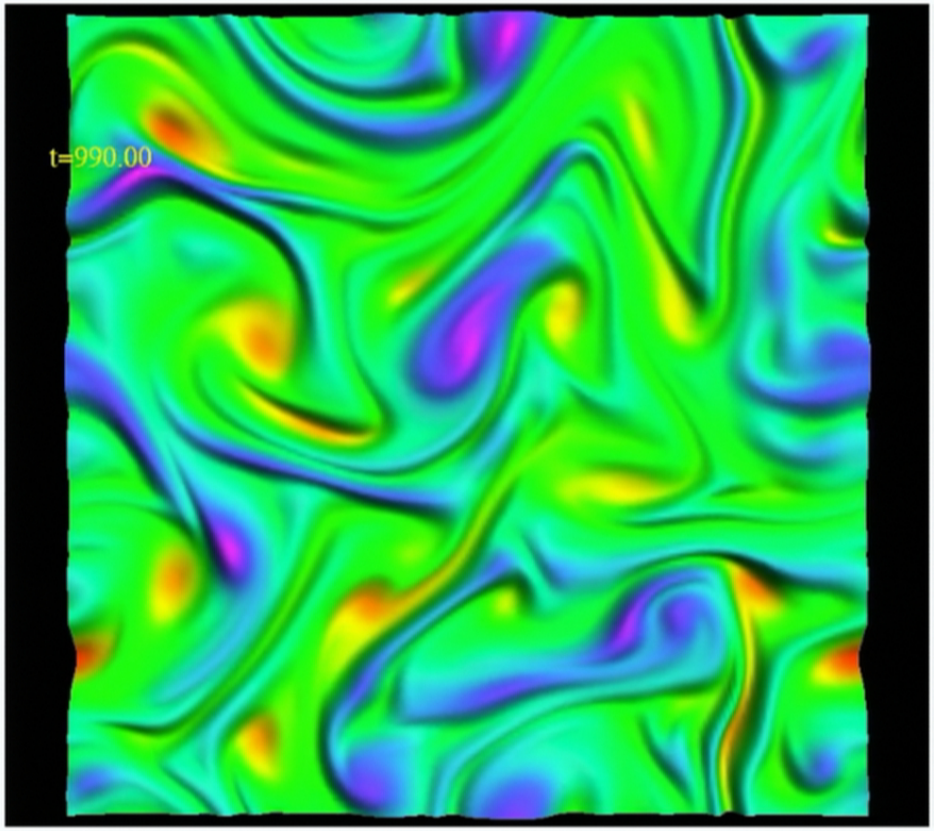
Typical turbulent run



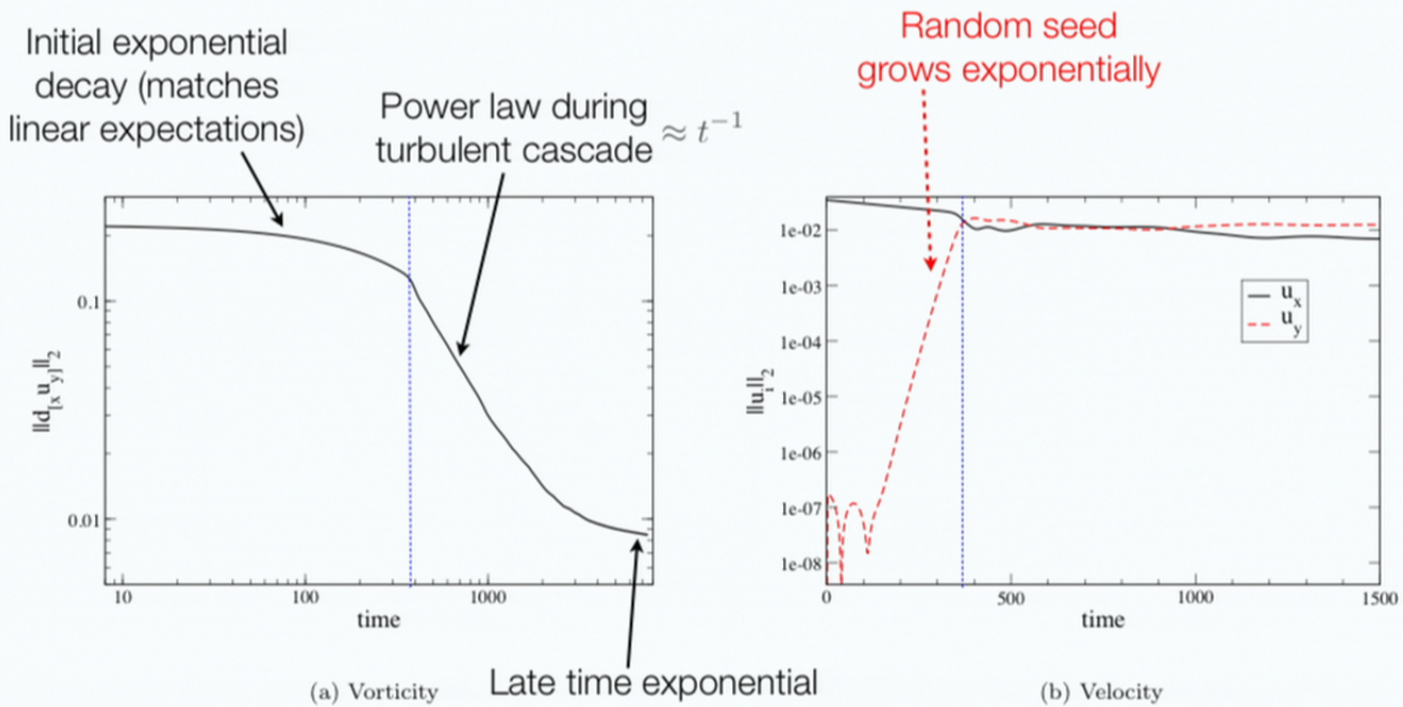
Typical turbulent run



Typical turbulent run



What's going on?



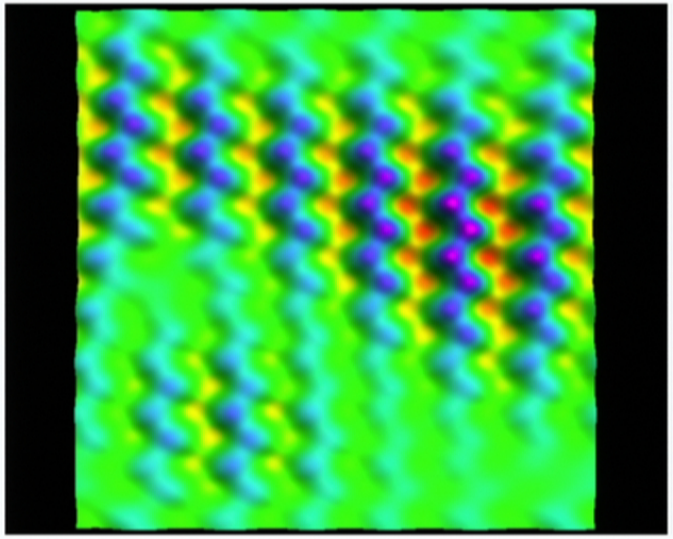
(a) Vorticity

(b) Velocity

Unstable mode

- u_y is used for monitoring the unstable mode. Here we plot the u_y field during the initial growth period.

- It is also a shear mode, but with longer wavelength than the initial flow.



Reynolds number

- The initial decaying shear flow has an associated Reynolds number, which we define as

$$R \equiv \frac{\rho\lambda}{\eta} \max u_x \propto \rho^{1/3} \lambda \max u_x$$

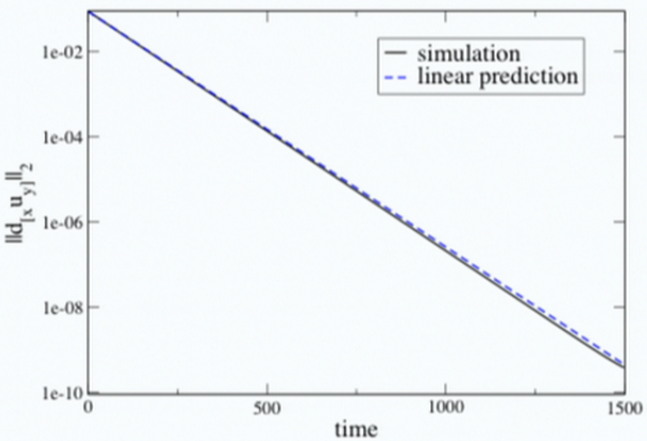
- This is a function of time, since the flow is decaying due to viscosity.

- Large R: unstable flow; turbulence
  Small R: stable flow; laminar

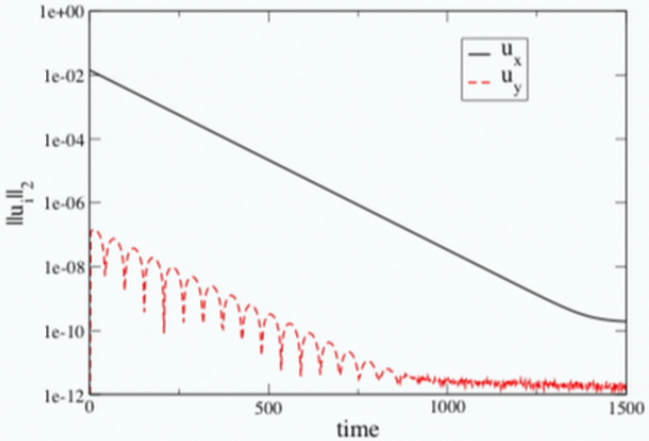
Critical Reynolds number
R_c

Laminar flow

- The random seed perturbation does not grow. u_y remains small.



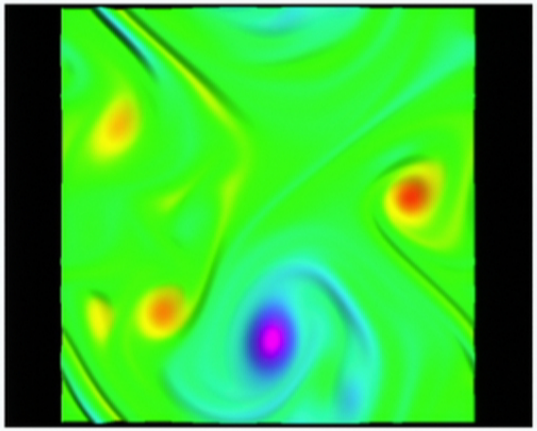
(a) Vorticity



(b) Velocity

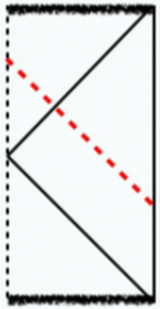
Mapping from boundary to bulk

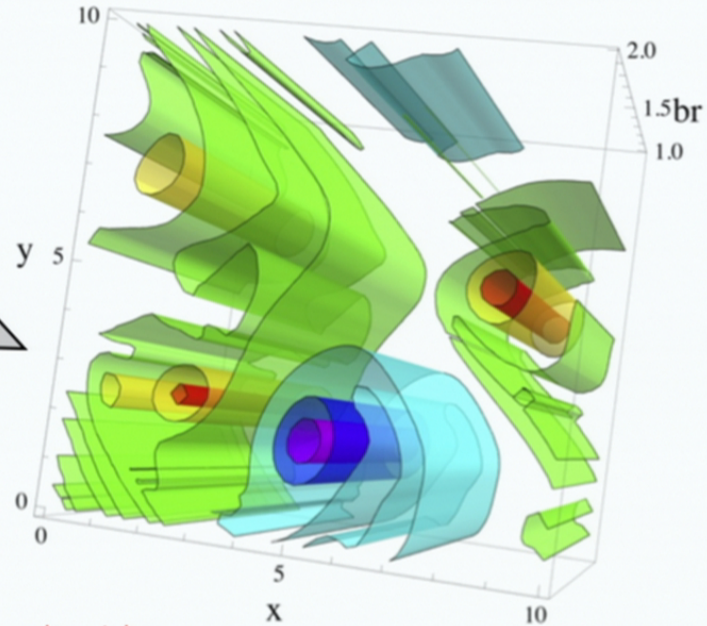
- Fluid/gravity correspondence includes a direct mapping from boundary quantities to a bulk metric...



(e) $t = 2500$

vorticity

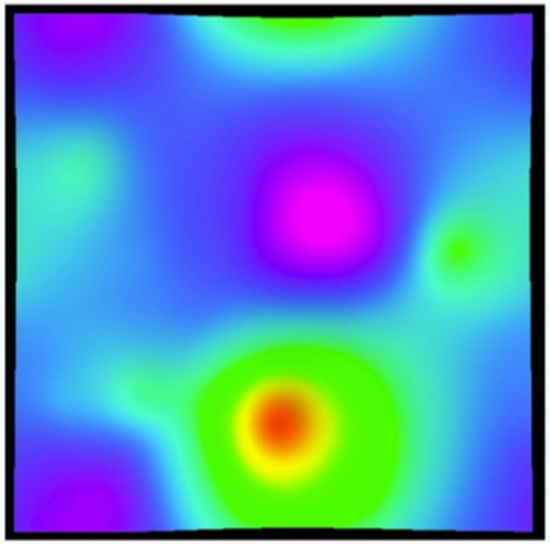




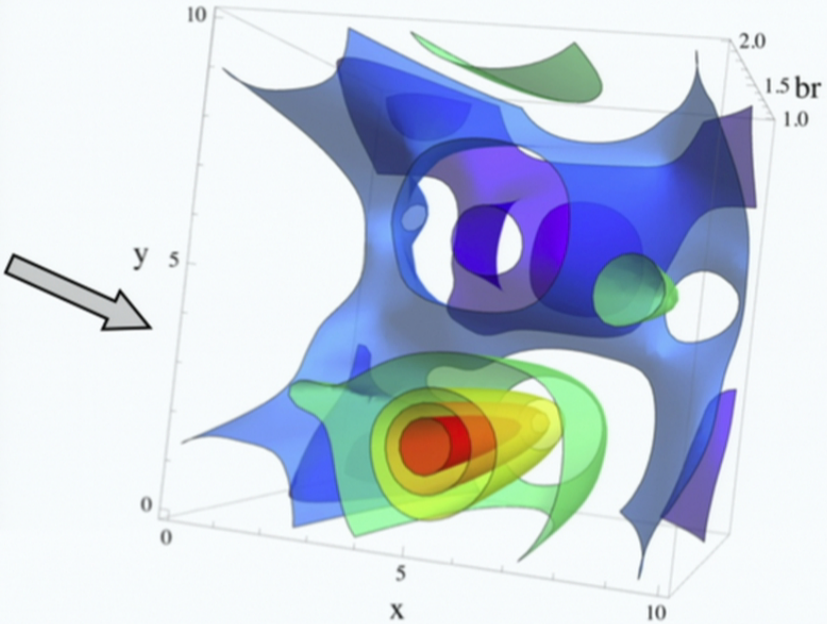
(b) $r^7 {}^*C_{ABCD}C^{ABCD}$

Funnels run along tubes connecting boundary and horizon

Mapping from boundary to bulk



density



(a) $r^6\, C_{ABCD} C^{ABCD}$

Bulk black branes

- Translated to the black brane language, our results imply that, for d=3:

For $R > R_c$, hydrodynamic shear quasinormal modes are unstable to small perturbations. This leads to turbulence, and a transfer of energy to longer wavelength modes.

- We have the correspondence:

Fluid side		**Gravity side**
Laminar flow	$\longrightarrow$	Quasinormal mode regime
Turbulent flow	$\longrightarrow$	New, turbulent phase [observed directly by Adams, Chesler and Liu (2013)]

- Gravitational Reynolds number? $R_{GR} \propto T_{\text{Hawking}} \left\| h_{AB} \left(\frac{\partial}{\partial r} \right)^B \right\| L$

Extensions / Open Questions

- Higher dimensions: Direct cascade to small scales.
- Black holes rather than black branes: Expect similar behavior.
- Beyond AdS: Other cases of long-lived quasinormal modes / slow dissipation / high Reynolds number might lead to turbulence. In particular, near-extremal Kerr black holes (Yang, Zimmerman, Lehner 2014).

Summary

- We have studied conditions for the onset of turbulence in relativistic viscous conformal fluids in d=2+1. Determined numerically the critical Reynolds number for shear flows.

- The fluid/gravity correspondence then implies that the quasinormal mode description for black branes breaks down at high Reynolds number. Turbulent phase entered instead.

- The onset of turbulence at high R is closely related to the presence of long-lived quasinormal modes.

The End