Title: Noncommutative geometry and the symmetries of the standard model

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Abstract: I will describe Connes approach to the standard model based on spectral noncommutative geometry with particular emphasis on the symmetries. The model poses constraints which are satisfied by the standard model group, and does not leave much room for other possibilities. There is however a possibility for a larger symmetry (the ``grand algebra") which may also be instrumental to obtain the correct mass of the Higgs.

Noncommutative geometry

and the symmetries of the standard model

Fedele Lizzi

Università di Napoli Federico II
and Institut de Ciencies del Cosmos, Universitat de Barcelona

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In this seminar I will connect noncommutative geometry and the standard model

The framework I choose is that of the spectral triples, i.e. the approach to geometry based on the spectral properties of the algebra of operators defined on them. The construction enables to generalize ordinary geometry to noncommutative geometry

I will not dwell on the need to generalize geometry. The original noncommutative geometry is the quantum mechanics phase space. The Planck scale, and the need to quantize gravity, indicates that also spacetime may be described by some sort of noncommutative geometry.

For this seminar noncommutative geometry is not the one given by noncommutative coordinates $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, in fact for most of the talk I will have a recognizable spacetime with the usual symmetries.

1

The starting point of Connes' approach to is that geometry and its (noncommutative) generalizations are described by the spectral data of three basic ingredients:

- An algebra $\boxed{\mathcal{A}}$ which describes the topology of spacetime.
- A Hilbert space $\boxed{\mathcal{H}}$ on which the algebra act as operators, and which also describes the matter fields of the theory.
- A (generalized) Dirac Operator $\boxed{D_0}$ which carries all the information of the metric structure of the space, as well as other crucial information about the fermions.

An important role is also played by two other operators: the chirality $\boxed{\gamma}$ and charge conjugation $\boxed{J}$

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There is a profound mathematical result (Gefand-Najmark) which states that the category of commmutative C^*-algebras and that of topological Hausdorff spaces are in one to one correspondence. The algebra being that of continuous complex valued functions on the space.

Connes programme is the transcription of all usual geometrical objects into algebraic terms, so to provide a ready generalization to the case for which the algebra is noncommutative

The points of the space (that can be reconstruced) are pure states, or maximal ideals of the algebra, or irreducible representations. They all coincide in the commutative case.

The geometric aspects are encoded in the Dirac operator.

In the case in which $\boxed{\mathcal{A}}$ is commutative this describes an ordinary topological space. And over the years a dictionary has been built to translate the usual geometrical concepts using these algebraic data.

Bundles are projective modules, forms are built with the commutators between $\boxed{D_0}$ and $\boxed{\mathcal{A}}$ and are represented as operators on $\boxed{\mathcal{H}}$, the distances are defined by the Dirac operator, etc.

The algebraic concepts are more robust than those based on "pointwise" geometry and they survive when the algebra is noncommutative, enabling us to do noncommutative geometry

In the commutative case it is possible to characterize a manifold with properties of the elements of the triple (all five of them)

There is a list of conditions and a theorem (Connes) which proves this.

Since the conditions are all purely algebraic there remain valid in the noncommutative case, defining a noncommutative manifold

In case you want to see them:

1. Dimension There is a nonnegative integer n such that the eigenvalues of D_0 grow os $O(\frac{1}{n})$.
2. Regularity For any $a \in \mathcal{A}$ both a and $[D_0, a]$ belong to the domain of δ^k for any integer k, where δ is the derivation given by $\delta(T) = [|D|, T]$.
3. Finiteness The space $\bigcap_k \mathrm{Dom}(D^k)$ is a finitely generated projective left $\mathcal{A}$ module.
4. Reality There exist J with the commutation relation fixed by the number of dimensions with the property
   (a) Commutant $[a, Jb^* J^{-1}] = 0, \forall a, b$
   (b) First order $[[D, a], b^o = Jb^* J^{-1}] = 0 \ , \forall a, b$
5. Orientation There exists a Hochschild cycle c of degree n which gives the grading γ , This condition gives an abstract volume form.
6. Poincaré duality A Certain intersection form detemrined by D_0 and by the K-theory of $\mathcal{A}$ and its opposite is nondegenrate.

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Remarkably, if one applies this to the algebra of functions valued in diagonal $\boxed{2 \times 2}$ matrices one finds the Higgs Lagrangian of a $\boxed{U(1) \times U(1) \to U(1)}$ breaking, in which the Higgs is the "vector" boson corresponding to the internal degree of freedom.

In this case the space is only "almost" noncommutative, in the sense that there still is an underlying spacetime, and and internal noncommutative but finite dimensional algebra

In these cases the algebra is of the kind $\boxed{\mathcal{A} = C(\mathbb{R}^4 \otimes \mathcal{A}_F}$, where $\boxed{\mathcal{A}_F}$ is ma finite dimensional (matrix) algebra.

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Describe not only a geometry, but also the behaviour of the fields, and their couplings to the geometry of spacetime (gravity). Treating on an equal footing the *external geometry (spacetime), and the inner one, gauge degrees of freedom*

The approach is based on the spectral action. The algebra is almost commutative, the Hilbert space is that of fermion matter fields, and the Dirac operator contains all information on the metric of spacetime, as well as the masses , couplings and mixings of fermions.

As Dirac operator we take $\boxed{D_0 = \partial\!\!\!/ + \phi\!\!\!/ \otimes \mathbb{I} + \gamma_5 \otimes D_F}$

$\boxed{D_F}$ is a finite matrix containing masses (mixings) of the fermions

Its covariant version $\boxed{D_A = D_0 + A + JAJ}$, where $\boxed{A}$ is a one-form, we obtain the gauge vector bosons, and the Higgs boson which is like the internal component of the vector bosons

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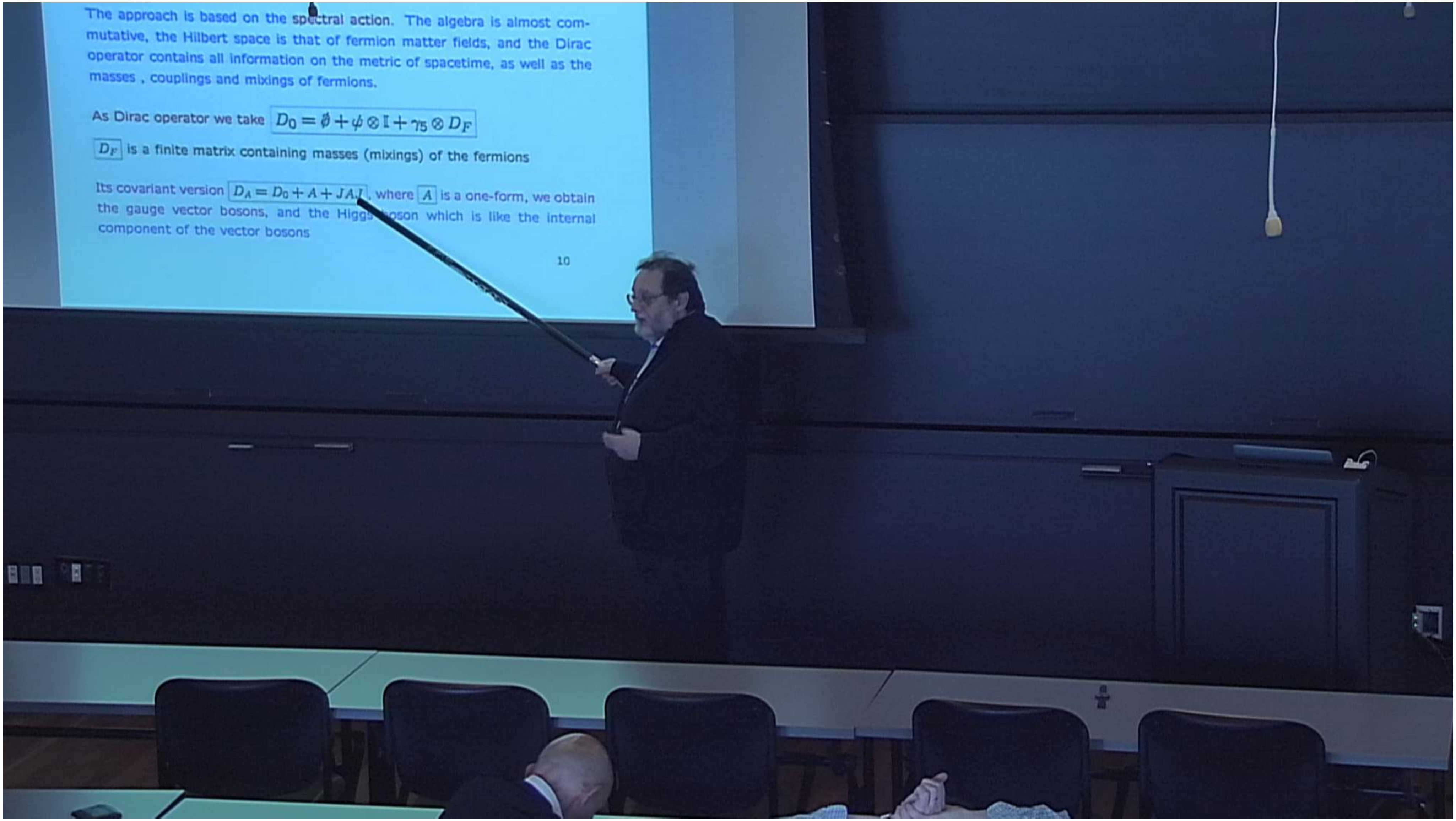
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The spectral action contains two part, one is the bosonic action, which is a regularized trace:

$$S_B = \mathrm{Tr}\,\chi\left(\frac{D_A}{\Lambda}\right)$$

where χ is a cutoff function, for example the characteristic function of the interval $[0,1]$, in this case the action is just the number of eigenvalues of the Laplacian which are below the scale Λ

Then there is a "standard" fermionic action $\langle\Psi|\, D_A\, |\Psi\rangle$ which needs to be regularized, in the usual way (one can use the same cutoff)

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In the work of Chamseddine, Connes and Marcolli the renormalization group flow is done by considering as boundary condition the unification of the three interaction coupling constants at $\boxed{\Lambda}$. This is approximately true.

The various couplings and parameters are then found at low energy via the renormalization flow

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Technically the bosonic spectral action is a sum of residues and can be expanded in a power series in terms of Λ^{-1} as

$$S_B = \sum_n f_n \, a_n(D^2/\Lambda^2)$$

where the f_n are the momenta of χ

$$\begin{aligned} f_0 &= \int_0^\infty \mathrm{d}x \, x\chi(x) \\ f_2 &= \int_0^\infty \mathrm{d}x \, \chi(x) \\ f_{2n+4} &= (-1)^n \partial_x^n \chi(x)\bigg|_{x=0} \quad n \geq 0 \end{aligned}$$

the a_n are the Seeley-de Witt coefficients which vanish for n odd. For D^2 of the form

$$D^2 = -(g^{\mu\nu}\partial_\mu\partial_\nu \mathbb{1} + \alpha^\mu \partial_\mu + \beta)$$

Defining (in term of a generalized spin connection containing also the gauge fields)

$$
\begin{aligned}
\omega_\mu &= \frac{1}{2} g_{\mu\nu} \left(\alpha^\nu + g^{\sigma\rho} \Gamma^\nu_{\sigma\rho} \mathbb{1} \right) \\
\Omega_{\mu\nu} &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu + [\omega_\mu, \omega_\nu] \\
E &= \beta - g^{\mu\nu} \left(\partial_\mu \omega_\nu + \omega_\mu \omega_\nu - \Gamma^\rho_{\mu\nu} \omega_\rho \right)
\end{aligned}
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\begin{aligned}
a_0 &= \frac{\Lambda^4}{16\pi^2} \int \mathrm{d}x^4 \sqrt{g}\, \mathrm{tr}\, \mathbb{1}_F \\
a_2 &= \frac{\Lambda^2}{16\pi^2} \int \mathrm{d}x^4 \sqrt{g}\, \mathrm{tr} \left(-\frac{R}{6} + E \right) \\
a_4 &= \frac{1}{16\pi^2} \frac{1}{360} \int \mathrm{d}x^4 \sqrt{g}\, \mathrm{tr} \, (-12 \nabla^\mu \nabla_\mu R + 5R^2 - 2R_{\mu\nu} R^{\mu\nu} \\
&\quad +2R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} - 60RE + 180E^2 + 60\nabla^\mu \nabla_\mu E + 30\Omega_{\mu\nu} \Omega^{\mu\nu}
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tr is the trace over the inner indices of the finite algebra $\mathcal{A}_F$ and in Ω and E are contained the gauge degrees of freedom including the gauge stress energy tensors and the Higgs, which is given by the inner fluctuations of D

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Now the matter is just cranking a machine. As algebra one takes an "almost commutative geometry", i.e. the product of the usual spacetime times a finite dimensional noncommutative algebra

We start from the algebra, a tensor product $\boxed{\mathcal{A} = C(\mathbb{R}^4) \otimes \mathcal{A}_F}$, with the finite $\boxed{\mathcal{A}_F = \mathsf{Mat}(\mathbb{C})_3 \oplus \mathbb{H} \oplus \mathbb{C}}$

The unitaries of the algebra correspond to the symmetries of the standard model: $\boxed{SU(3) \oplus SU(2) \oplus U(1)}$

A unimodularity condition takes care of the extra U(1)

This algebra must be represented as operators on a Hilbert space, which also has a continuos infinte dimesional part (spinors on spacetime) times a finite dimensional one: $\boxed{\mathcal{H} = \mathsf{sp}(\mathbb{R}) \otimes \mathcal{H}_F}$. The grading given by $\boxed{\gamma}$ splits it into a left and right subspace: $\boxed{\mathcal{H}_L \oplus \mathcal{H}_R}$

As I said the Dirac operator contains all data relative to the fermions, but no information on the Higgs mass (actually vev and quartic coupling coefficient) which can be calculated from the fermion mass parameters (Yukawa couplings). These in turn are dominated by the top quark coupling.

Hence we have a "prediction" for the Higgs mass.

The prediction is 170 GeV. As you know the actual mass is 126 GeV.

Now it depends how you consider this theory. if you take it as a mature fully formed theory then the result is wrong. If you take it (as I do) as a tool to investigate the standard model starting from first principles, then I think it is remarkable that a theory based on pure mathematical result gets reasonable numbers

I took the measurement of the Higgs as a reason to understand in which direction one has to improve on the theory

I will now try to understand in this framework the origin the standard model algebra, and see if it may shed light on the mass of the Higgs.

$$\mathcal{A}_{sm} = \mathbb{C} \oplus \mathbb{H} \oplus \mathbb{M}_3(\mathbb{C}),$$

$\mathbb{H}$ are the quaternions, which we represent as 2×2 matrices

It is possible to have this emerge from the most general algebra which satisfies the condition of being a noncommutative manifold

The manifold conditions I flashed earlier are purly algebraic. Therefore they can be applied to finite dimensional (matrix) algebras. The resul is that only one kind of algebras are allowed:

$$\mathcal{A}_{\mathcal{F}} = \mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C}) \qquad a \in \mathbb{N}.$$

Let us look in detail to a vector in the Hilbert space:

$$\Psi^{\mathrm{CI}m}_{s\dot{s}\alpha}(x) \in \mathcal{H} = L^2(\mathcal{M}) \otimes \mathsf{H}_F = sp(L^2(\mathcal{M})) \otimes \mathcal{H}_F$$

Note the difference between $\mathcal{H}_F$, which is 96 dimensional, and H_F which is 384 dimensional. The meaning of the indices is as follows:

$$\Psi^{\mathrm{cI}m}_{s\dot{s}\alpha}(x)$$

$\begin{matrix} s = r, l \\ \dot{s} = \dot{0}, \dot{1} \end{matrix}$ are the spinor indices. They are not internal indices in the sense that the algebra $\mathcal{A}_F$ acts diagonally on it. They take two values each, and together they make the four indices on an ordinary Dirac spinor.

$$\Psi^{cIm}_{s\dot{s}\alpha}(x)$$

$\alpha = 1 \ldots 4$ is the flavour index. It runs over the set u_R, d_R, u_L, d_L when $\mathrm{I} = 1, 2, 3$, and ν_R, e_R, ν_L, e_L when $\mathrm{I} = 0$. It repeats in the obvious way for the other generations.

We can similarly write down the Dirac operator

$$\boxed{D = \not{\partial} \otimes \mathbb{I}_{96} + \gamma^5 \otimes D_F}$$

$$\boxed{D_F = \begin{pmatrix} 0_{8N} & \mathcal{M} & \mathcal{M}_R & 0_{8N} \\ \mathcal{M}^\dagger & 0_{8N} & 0_{8N} & 0_{8N} \\ \mathcal{M}_R^\dagger & 0_{8N} & 0_{8N} & \bar{\mathcal{M}} \\ 0_{8N} & 0_{8N} & \mathcal{M}^T & 0_{8N} \end{pmatrix}}.$$

$\boxed{\mathcal{M}}$ contains the Dirac-Yukawa couplings. It links left with right particles. $\boxed{\mathcal{M}_R = \mathcal{M}_R^T}$ contains Majorana masses and links righr particles with right antiparticles. $\boxed{\mathcal{M} = \begin{pmatrix} M_u & 0_{4N} \\ 0_{4N} & M_d \end{pmatrix} \qquad \mathcal{M}_R = \begin{pmatrix} M_R & 0_{4N} \\ 0_{4N} & 0_{4N} \end{pmatrix}}$ where $\boxed{M_u}$ contaінins the masses of the up, charm and top quarks and the neutrinos (Dirac mass), $\boxed{M_R}$ contains the Majorana neutrinos masses and $\boxed{M_d}$ the remaining quarks and electrons, muon and tau masses, including mixings

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Consider the case of $\boxed{\mathbb{M}_a(\mathbb{H}) \oplus \mathbb{M}_{2a}(\mathbb{C})}$ for the case $\boxed{a = 4}$

In this case we need a $\boxed{2 \cdot (2 \cdot 4)^2 = 128}$ dimensional space, which for 3 generations gives a $\boxed{384}$ dimensional Hilbert space.

I need a representation of the algebra $\boxed{\mathbb{M}_4(\mathbb{H}) \oplus \mathbb{M}_8(\mathbb{C})}$ acting on the spinors I gave earlier, and the order zero conditions

I do not want to go into technical details (I could show slides with all indices in gory detail. . .).

The fundamental point is that spinor indices and the internal gauge indices are mixed.

The two part of the algebra act on the indices like

$$\Psi^{c\mathbf{I}m}_{s\dot{s}\alpha}(x)$$

Quaternions act on the blue indices, and complex numbers act on the red indices.

We are in a phase in which the Euclidean structure of space time has not yet emerged.

The fermions are not yet fermions

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This grand algebra, and a corresponding $\boxed{D}$ operator, have "more room" to operate. Although the Hilbert space is the same, we abandoned the factorization of the internal indices, giving us more entries to accommodate the Majorana masses

Hence we can put a Majorana mass for the neutrino and at the same time satisfy the order one condition. Then the one form corresponding to this $\boxed{D_\nu}$ will give us the by now famous field $\boxed{\sigma}$, which can only appear before the transition to the geometric spacetime. But we must abandon he boundedness of the algebra.

The natural scale for this mass is to be above a transition which gives the geometric structure. Therefore it is natural that it may be at a high scale. How high we can discuss

The grand symmetry is no ordinary gauge symmetry, there is never a $\boxed{SU(8)}$ in the game for example

It represents a phase in which the internal noncommutative geometry contains also the spin structure, even the Lorentz (Euclidean) structure of space time in a mixed way

The differentiation between the spin structure of spacetime, and the internal gauge theory comes as a breaking of the symmetry, triggered by $\boxed{\sigma}$, which now appears naturally has having to do with the geometry of spacetime.

What sort of spacetime do we have with this grand symmetry? Should we dare more and go non associative as well?