Title: Designing exotic circuitry with non-Abelian anyons

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Abstract: Non-Abelian anyons are widely sought for the exotic fundamental physics they harbor as well as for their possible applications for quantum information processing. Currently, there are numerous blueprints for stabilizing the simplest type of non-Abelian anyon, a Majorana zero energy mode bound to a vortex or a domain wall. One such candidate system, a so-called "Majorana wire" can be made by judiciously interfacing readily available materials; the experimental evidence for the viability of this approach is presently emerging.
 Following this idea, we introduce a device fabricated from conventional fractional quantum Hall states, s-wave superconductors and insulators with strong spin-orbit coupling. Similarly to a Majorana wire, the ends of our â€œquantum wireâ€• would bind "parafermions", exotic non-Abelian anyons which can be viewed as fractionalised Majorana zero modes.
 I will discuss their properties and describe how such parafermions can be used to construct new and potentially useful circuit elements which include current and voltage mirrors, transistors for fractional charge currents and "flux capacitors".

Anyonics: Designing exotic circuitry with non-Abelian anyons

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David Clarke, UC Riverside → Caltech

UC RIVERSIDE
UNIVERSITY OF CALIFORNIA

Support:
DARPA QuEST

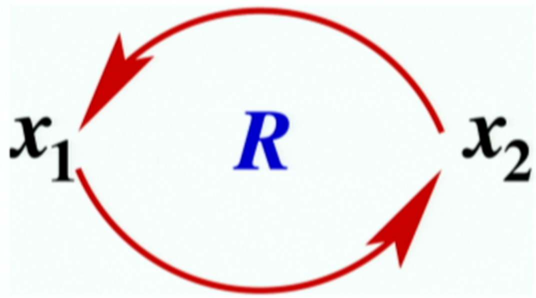
PI 2014

Quantum Exchange Statistics

- Two quantum-mechanical particles are located at x_1 & x_2

  What happens to their wavefunction when two indistinguishable particles are exchanged?

W. Pauli



$$\hat{\mathrm{R}}\Psi(\mathbf{x}_1,\mathbf{x}_2)=\Psi(\mathbf{x}_2,\mathbf{x}_1)$$

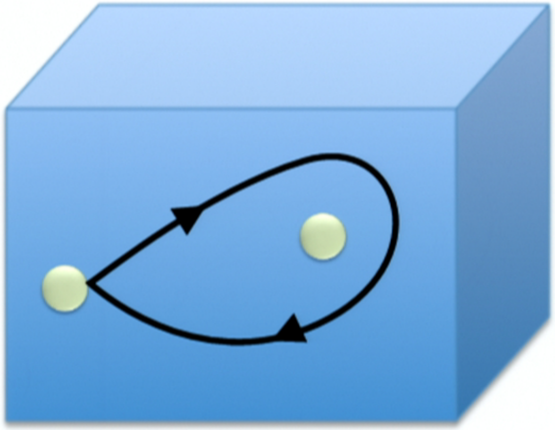
Fundamental question:

What's the relation between $\Psi(\mathbf{x}_2,\mathbf{x}_1)$ & $\Psi(\mathbf{x}_1,\mathbf{x}_2)$?

Quantum Exchange Statistics

d = 3

Only bosons & fermions (no knots in 3+1 dimensions)



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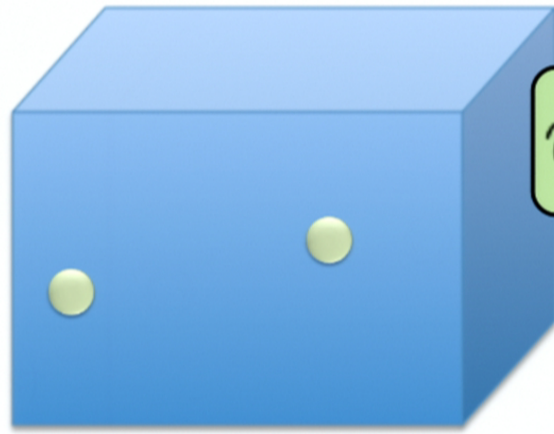

$$\psi \longrightarrow \pm\psi$$

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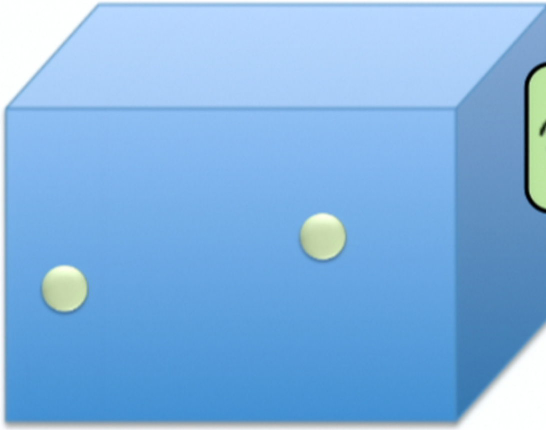

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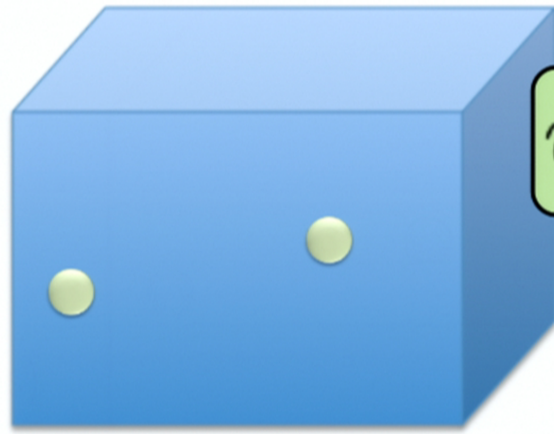
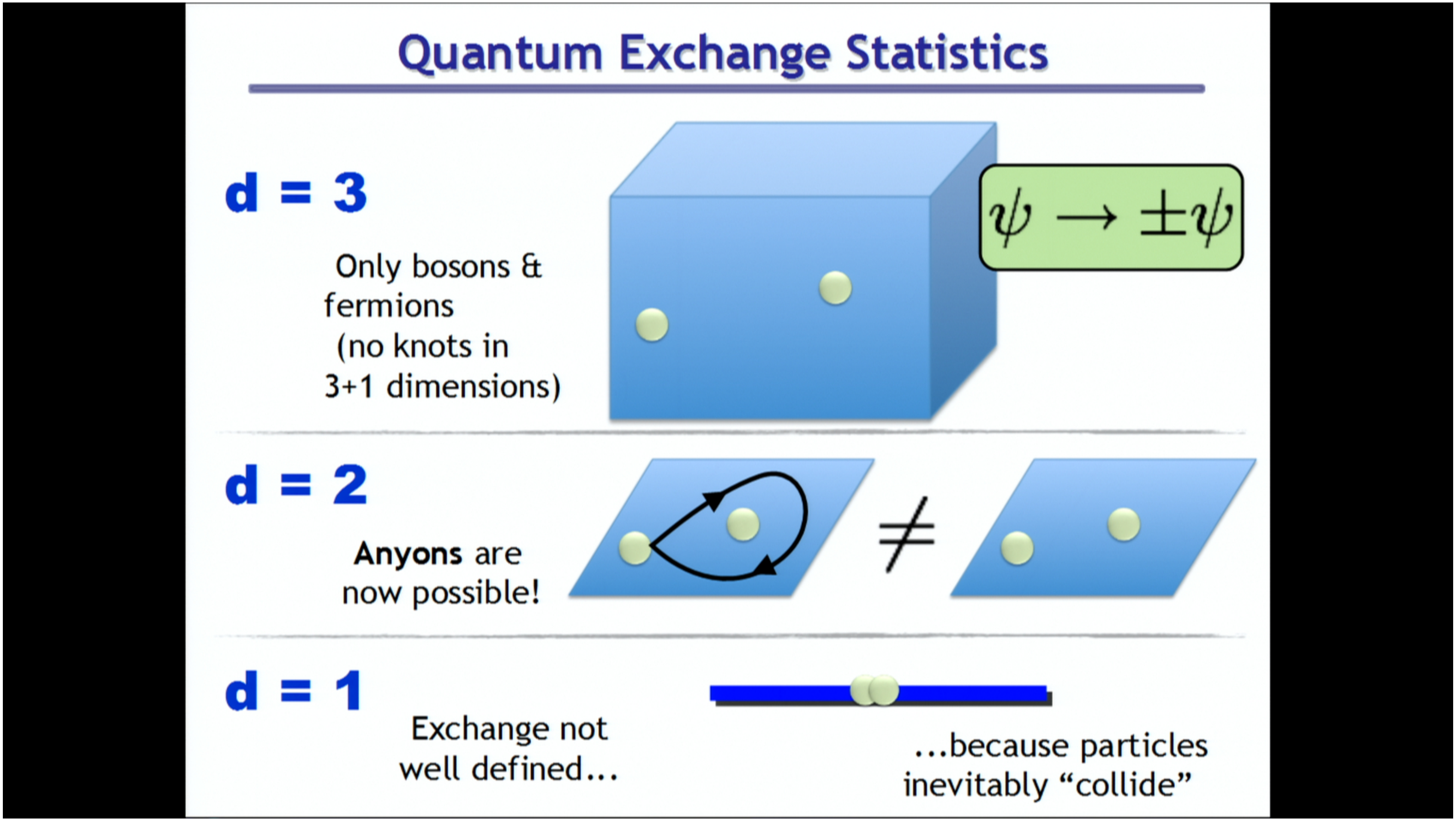


$$\psi \rightarrow \pm\psi$$

d = 2

Anyons are now possible!


Quantum Exchange Statistics
d = 3
Only bosons & fermions (no knots in 3+1 dimensions)
$\psi \to \pm\psi$
d = 2
Anyons are now possible!
≠
d = 1
Exchange not well defined...
...because particles inevitably "collide"

Quantum Exchange Statistics

d = 2

Anyons are now possible!

(Leinaas and Myrrheim 1979, Wilczek 1982)

Abelian anyons: a wave function acquires a *phase* upon exchanging the particles (not just ±1).
These anyons are called *abelian* because the order of exchange does not matter.

Quantum Exchange Statistics

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Anyons are now possible!

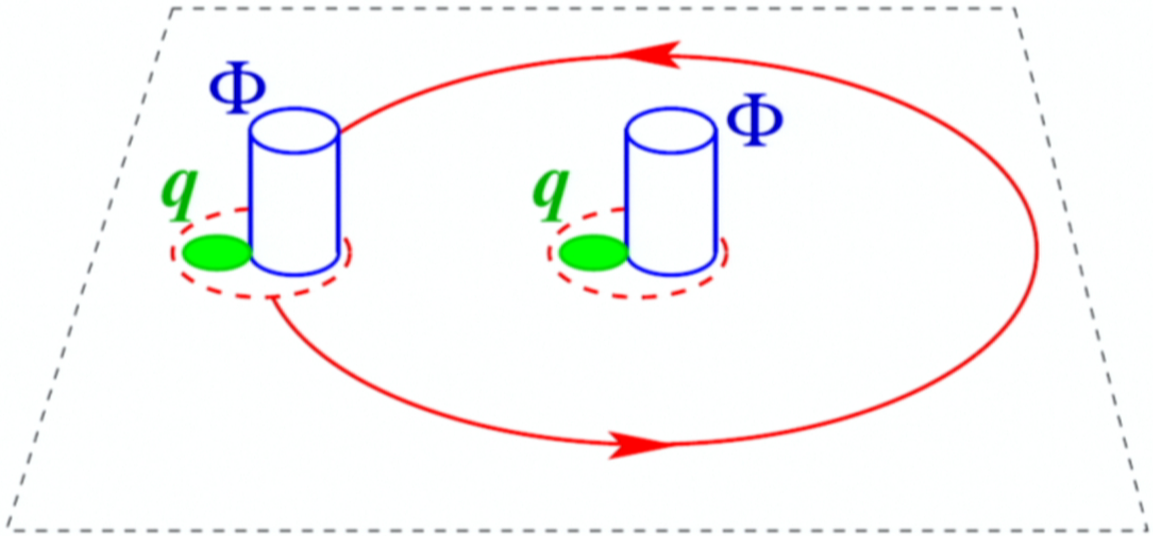
(Leinaas and Myrrheim 1979, Wilczek 1982)

Abelian anyons: a wave function acquires a *phase* upon exchanging the particles (not just ±1).
These anyons are called *abelian* because the order of exchange does not matter.

Abelian anyons have been observed in Fractional Quantum Hall systems.

Example: Toy model of Abelian anyons

Charge q - flux Φ composites (Wilczek '82)



The Aharonov-Bohm phase $2\theta = q\Phi + q\Phi = 2q\Phi$
(in the units $\hbar = c = 1$)

E.g., for $q = \dfrac{e}{n}$, $\Phi = \Phi_0 = \dfrac{2\pi}{e}$, the statistical angle $\theta = \dfrac{2\pi}{n}$.

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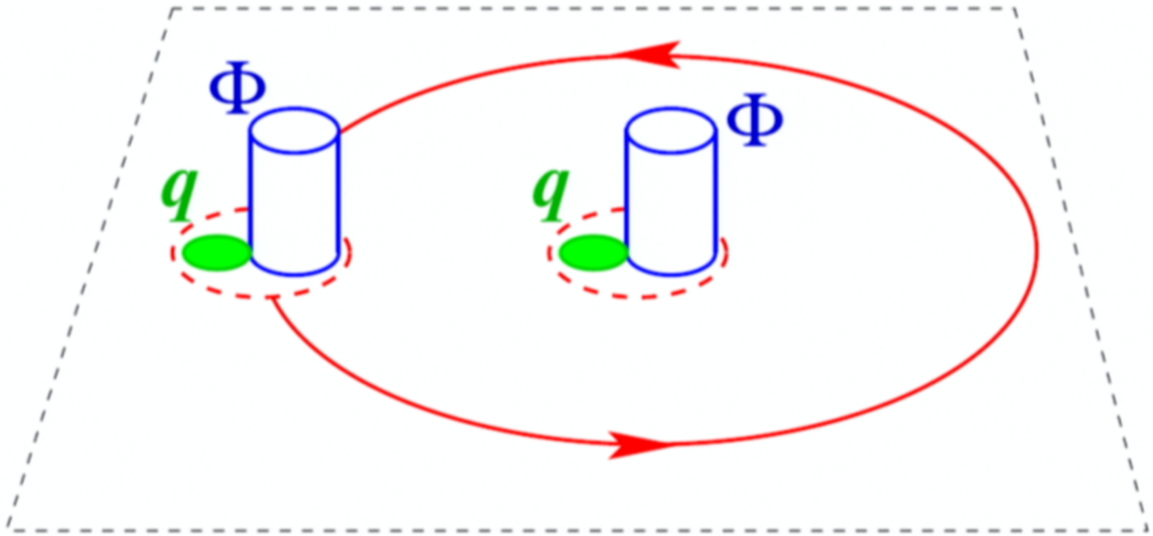

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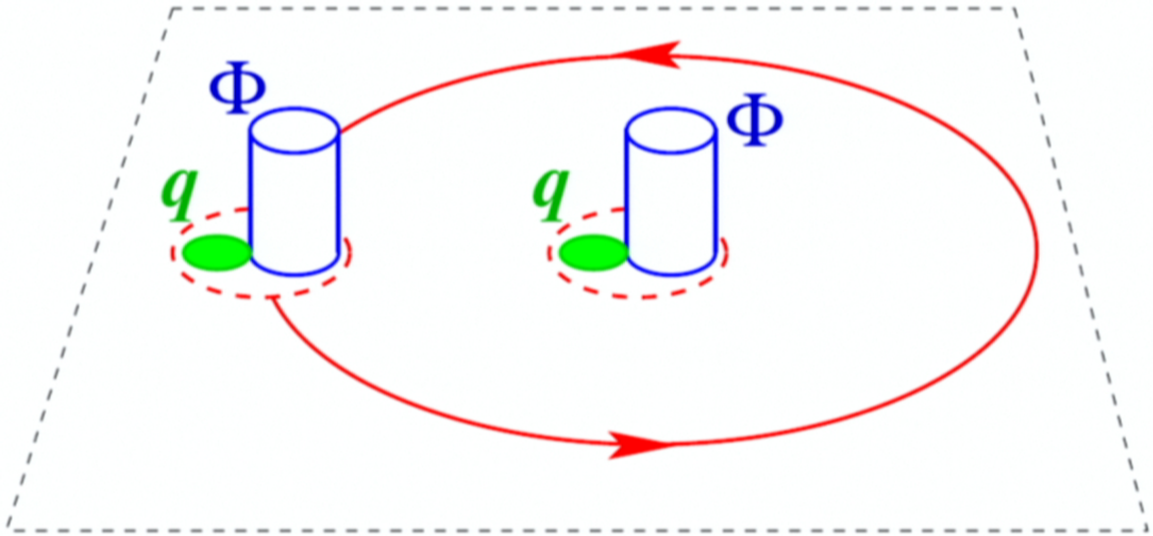

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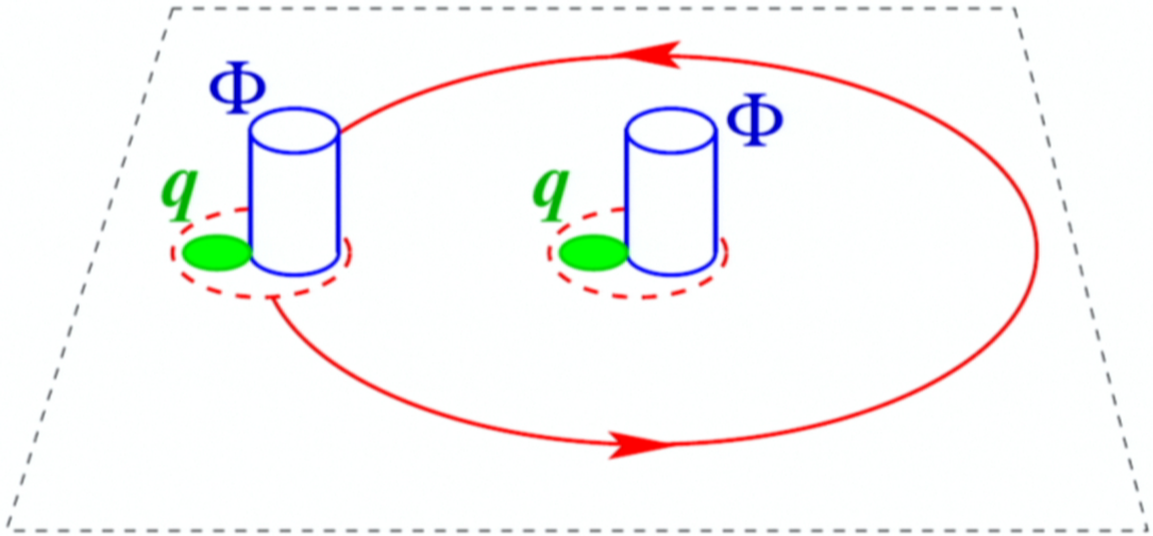

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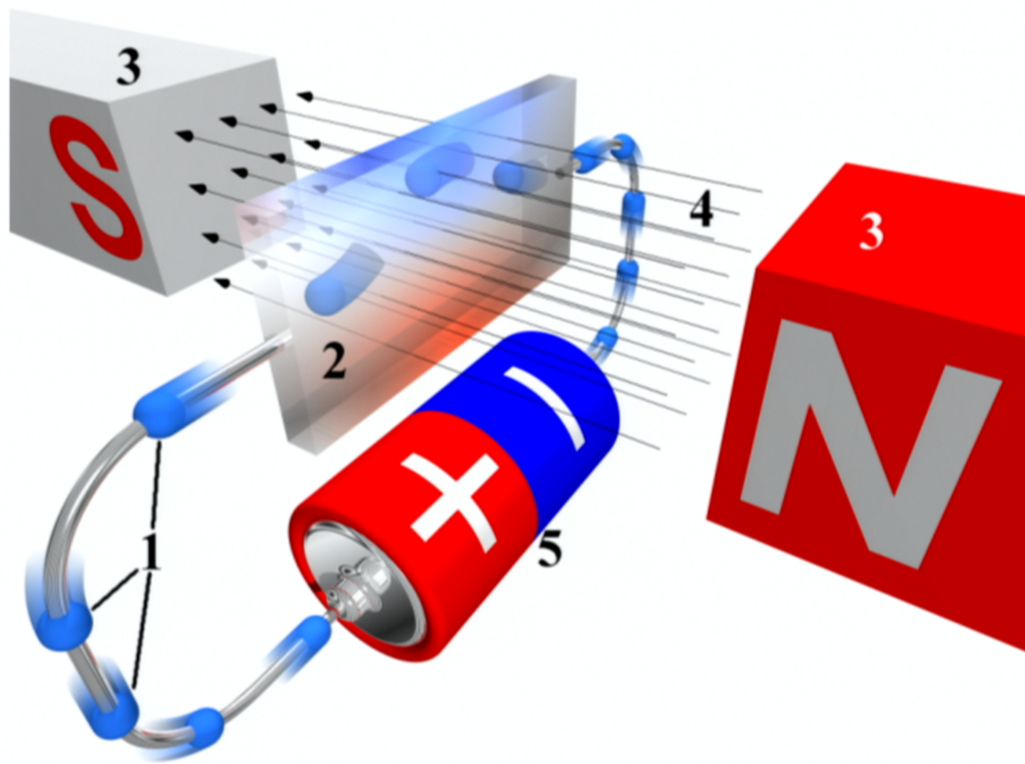
Hall Effect



$$E_y = v_x B$$

$$\rho_{\mathrm{H}} = \frac{E_y}{j_x} = \frac{B}{ne}$$

Fractional Quantum Hall Effect

Expect:

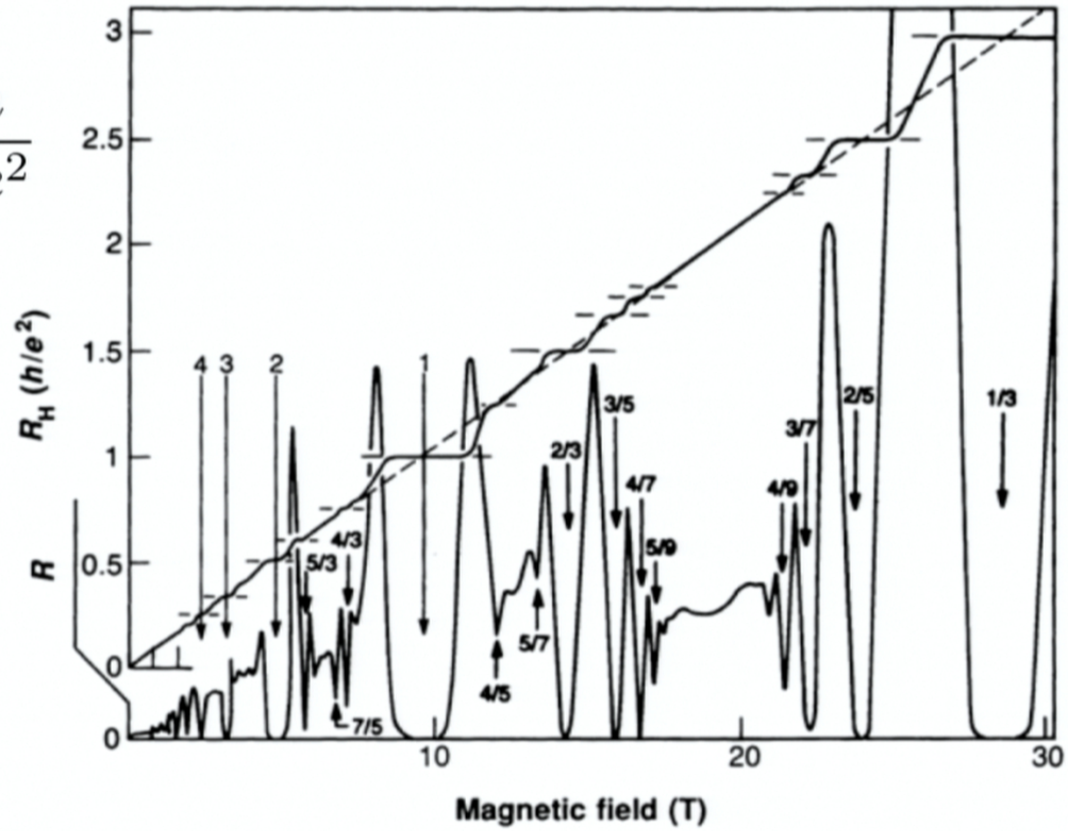
$$\rho_{\rm H} = \frac{B}{ne} = \frac{h}{\nu e^2}$$

where $\nu = \dfrac{N_e}{N_\Phi}$

$$N_\Phi = \frac{BA}{\Phi_0}$$

Find:

$$\nu = \frac{p}{q}$$



Eisenstein, Stormer, Science 248, 1990

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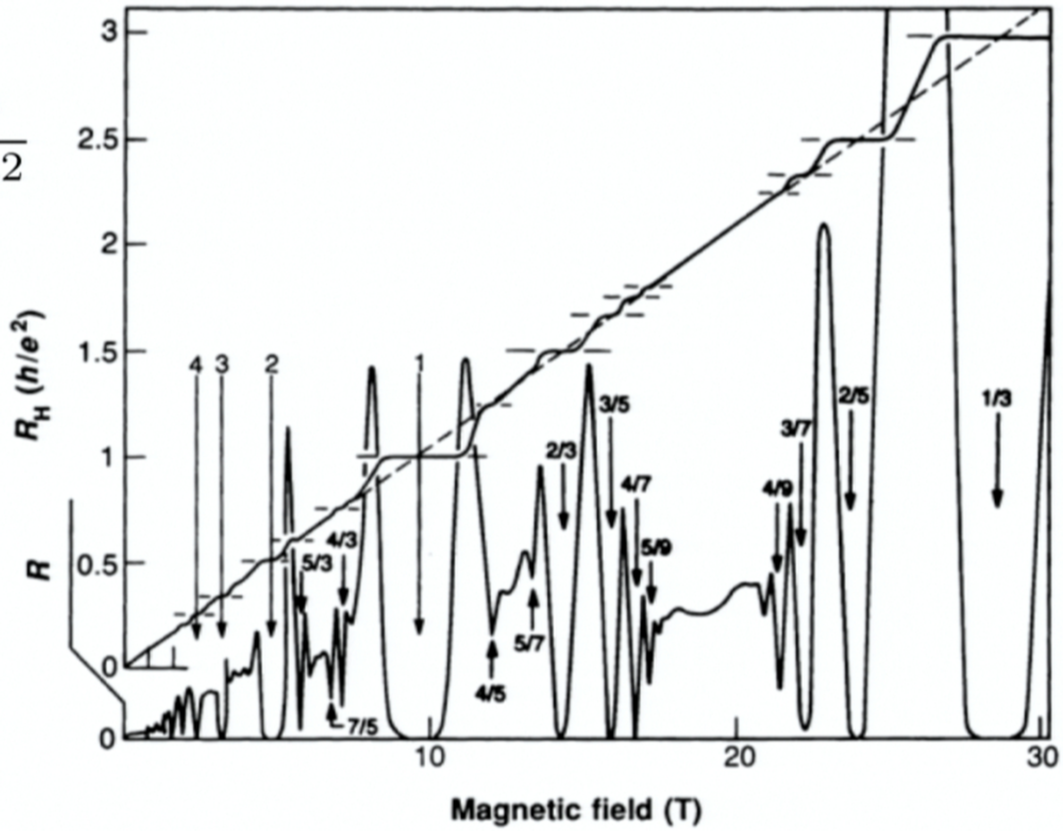

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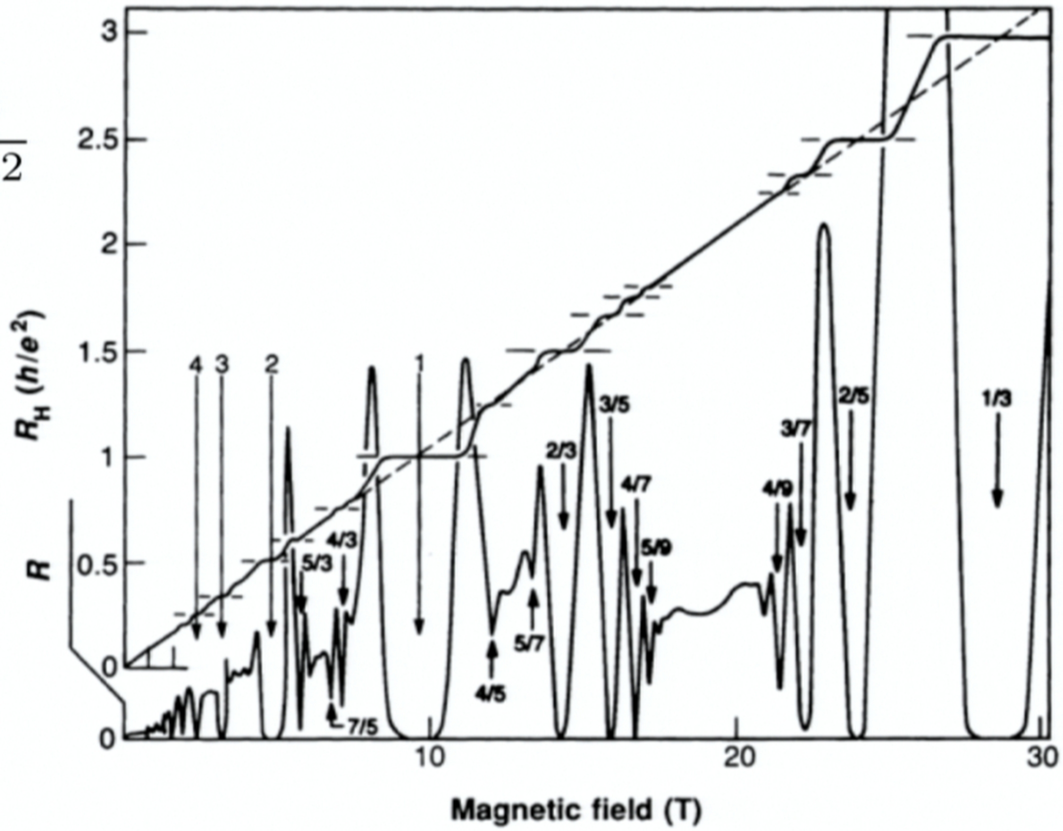

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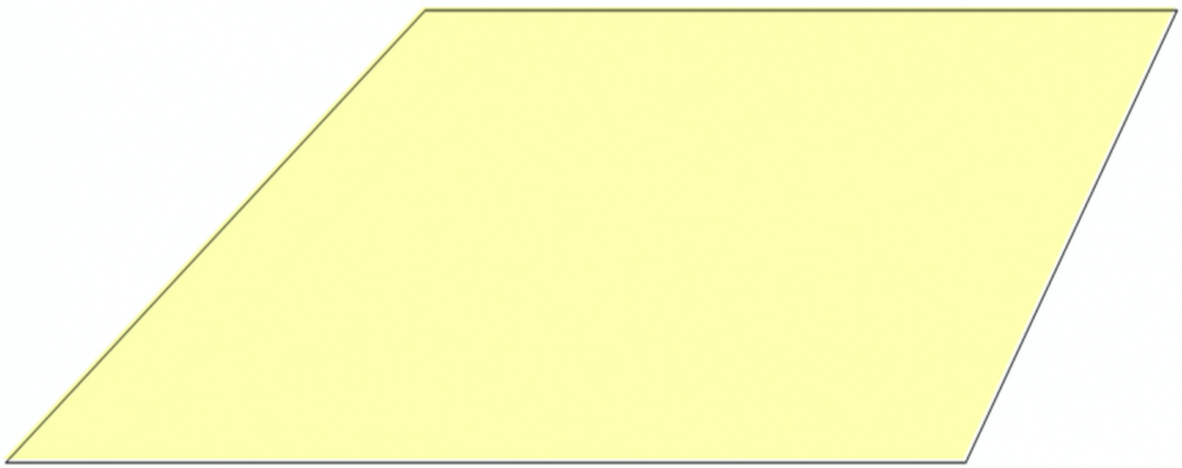
Fractional Charge

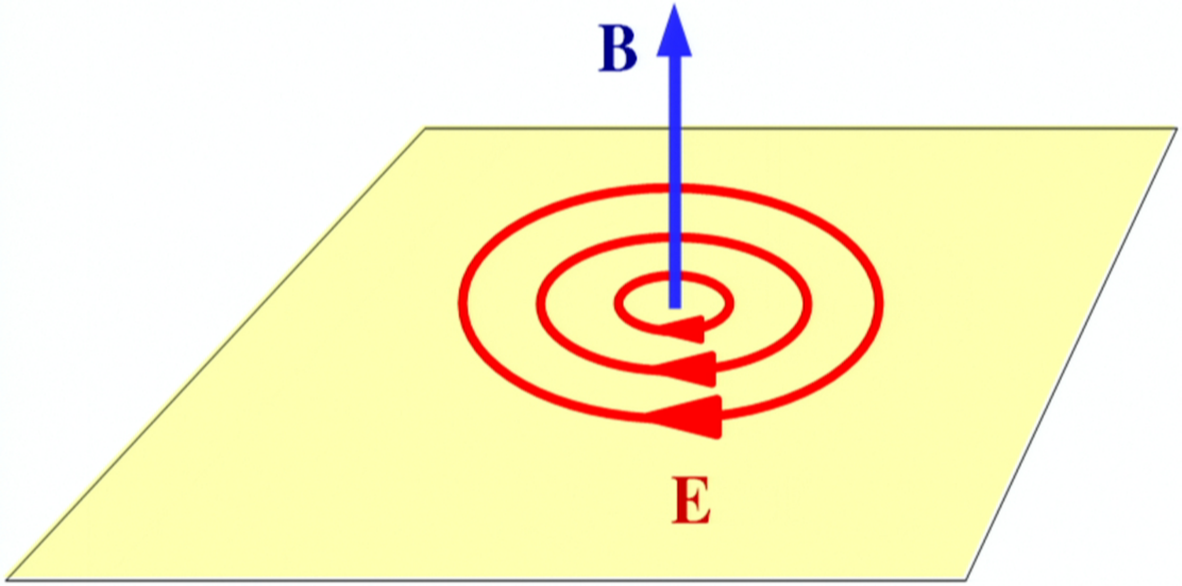
Adiabatically insert one flux quantum Φ_0



Fractional Charge

Adiabatically insert one flux quantum Φ_0

Maxwell: $\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$



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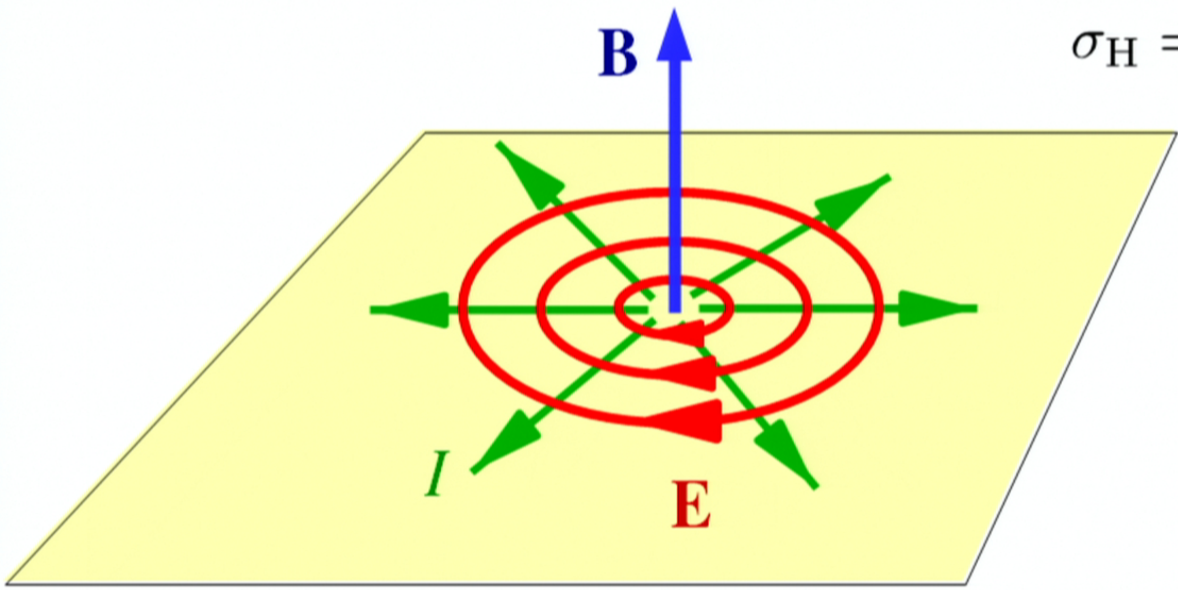
$$\sigma_{\rm H} = \frac{1}{\rho_{\rm H}} = \frac{\nu e^2}{h}$$



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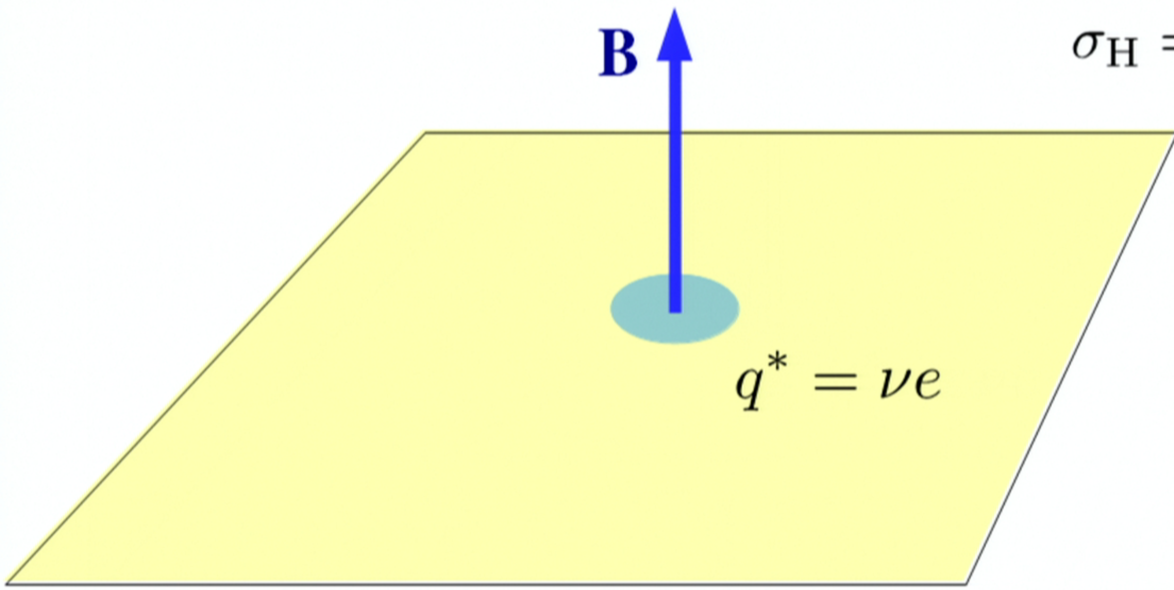
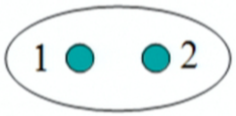


Non-Abelian Anyons

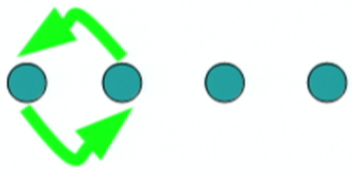
For fixed particle positions, we can have more than one wavefunction describing their combined state: $\Psi_1(\mathbf{x}_1, \mathbf{x}_2), \Psi_2(\mathbf{x}_1, \mathbf{x}_2) \ldots$



Exchanging particles positions may mix these wavefunctions:

$$\Psi_a(\mathbf{x}_2, \mathbf{x}_1) = \sum_b R_{ab} \Psi_b(\mathbf{x}_1, \mathbf{x}_2)$$

Exchanging 1 and 2:



$$\vec{\Psi} \to \mathbf{R}^{12} \vec{\Psi}$$

Exchanging 2 and 3:

$$\vec{\Psi} \to \mathbf{R}^{23} \vec{\Psi}$$

- Matrices $\mathbf{R}^{12}$ and $\mathbf{R}^{23}$ need not commute, hence ***Non-Abelian*** statistics
- For fixed particle positions, we have a non-trivial multi-dimensional Hilbert space where we can store quantum information

Majorana zero modes

Ettore Majorana (1906—1938 ?)

1937: Majorana fermion

- a fermion which is its own antiparticle

Majorana fermions in condensed matter setting:

$$\gamma = \gamma^\dagger, \quad \gamma^2 = 1$$

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$$\gamma = c + c^\dagger \qquad c = (\gamma_1 + i\gamma_2)/\sqrt{2}$$

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nature
International weekly journal of science

Nature News, February 2012:
"Quest for quirky quantum particles may have struck gold free"

Majorana wires

1D spinless p-wave superconductor(Kitaev 2001):

$$H = \mu \sum_{x=1}^{N} c_x^\dagger c_x - \sum_{x=1}^{N-1} (t c_x^\dagger c_{x+1} + |\Delta| e^{i\phi} c_x c_{x+1} + h.c.)$$

$$\mu = 0$$
$$t = |\Delta|$$

$$c_x = \frac{1}{2} e^{-i\frac{\phi}{2}} (\gamma_{B,x} + i\gamma_{A,x})$$

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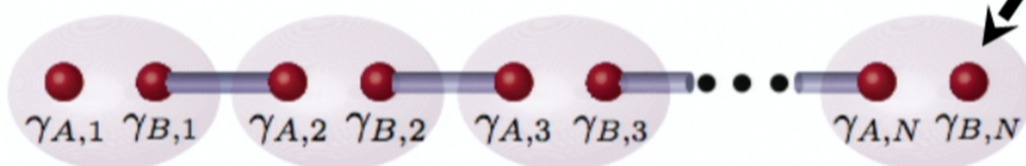
$$\Rightarrow \quad H = -it \sum_{x=1}^{N-1} \gamma_{B,x} \gamma_{A,x+1}$$

Unpaired Majorana fermions at the ends!



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$$\mu = 0 \qquad t = |\Delta| \qquad c_x = \frac{1}{2} e^{-i\frac{\phi}{2}} (\gamma_{B,x} + i\gamma_{A,x})$$

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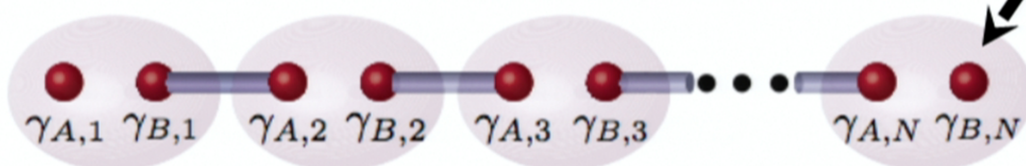
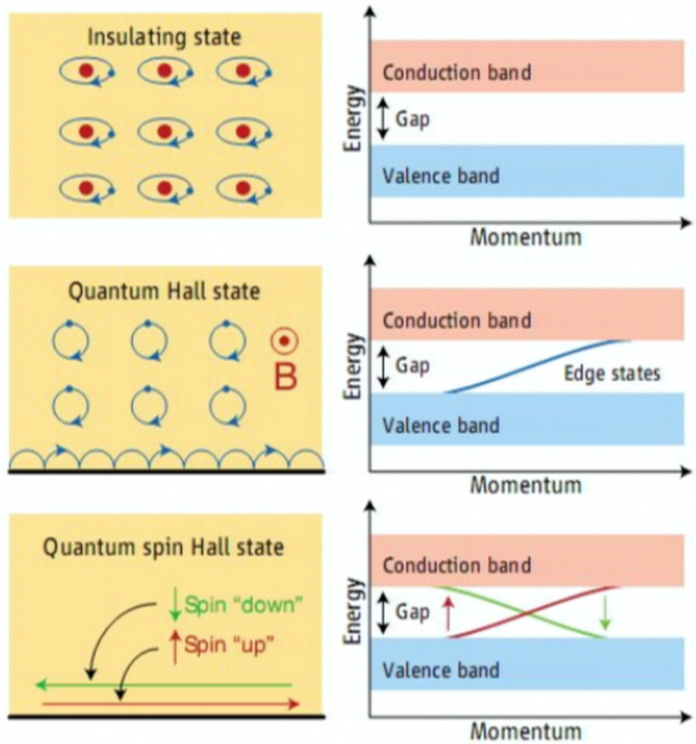


Realization in topological insulator edges



Kane & Mele, 2005; Bernevig, Hughes, Zhang, 2006; Fu & Kane, 2008

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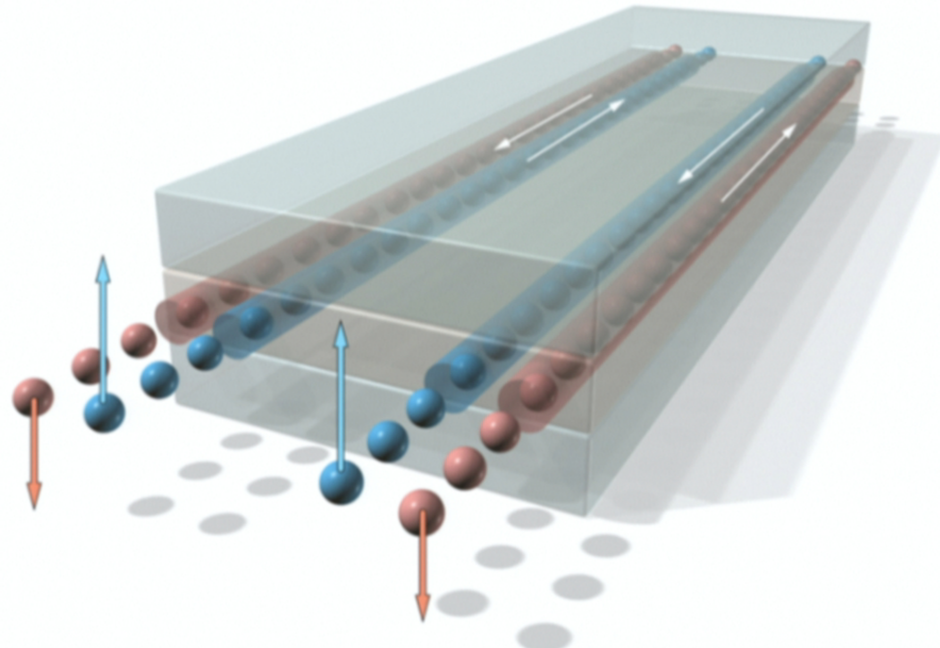

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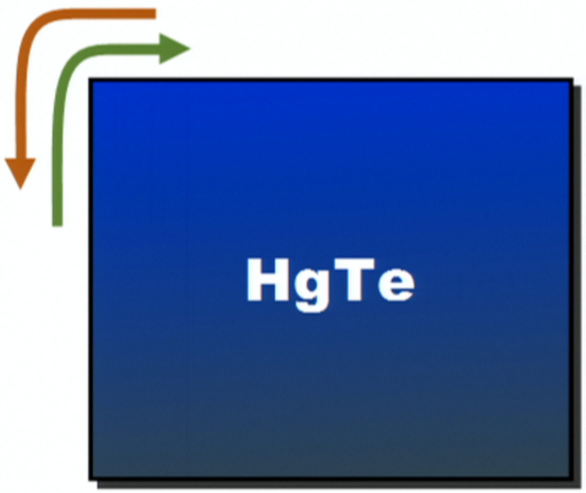
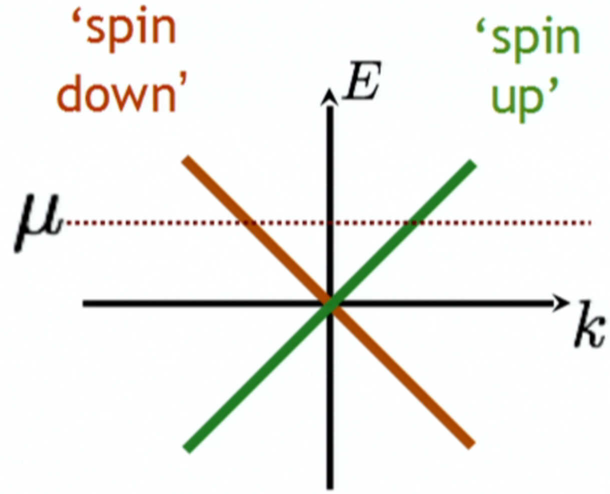


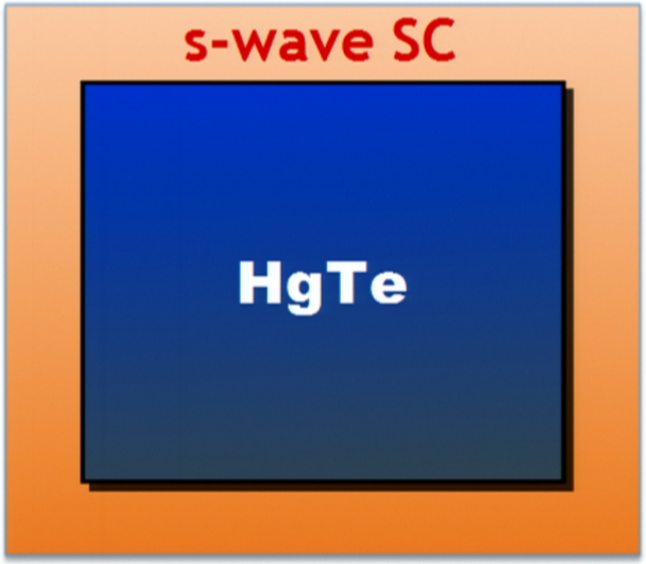


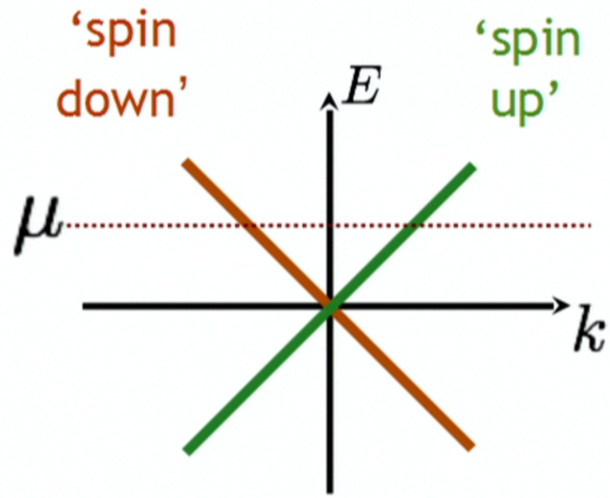
$$H_{\text{edge}} = \int dx[-\mu(\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L) - i\hbar v(\psi_R^\dagger \partial_x \psi_R - \psi_L^\dagger \partial_x \psi_L)]$$

1D and effectively 'spinless'! Just need superconductivity...

Fu & Kane, 2008

Realization in topological insulator edges





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$$+[\Delta\psi_R\psi_L + h.c.]$$

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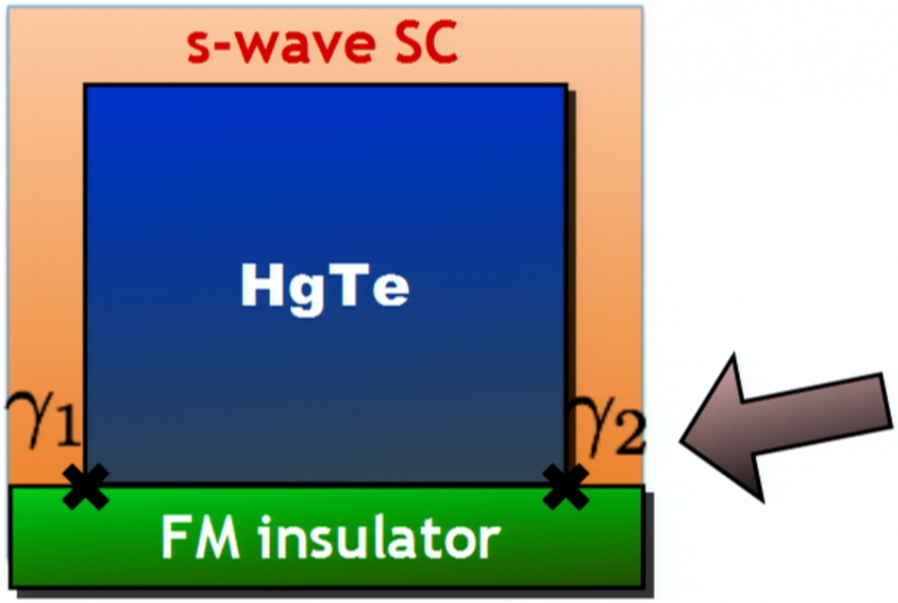
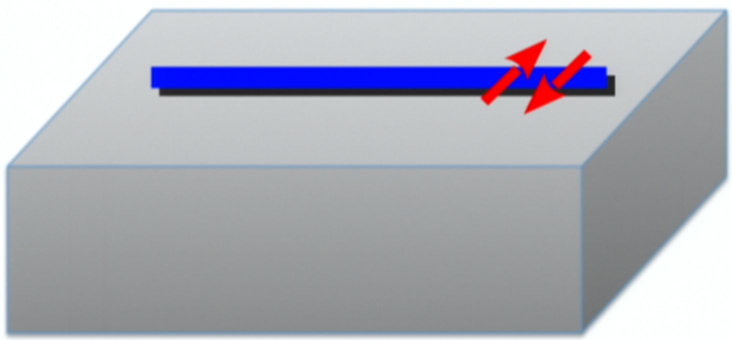


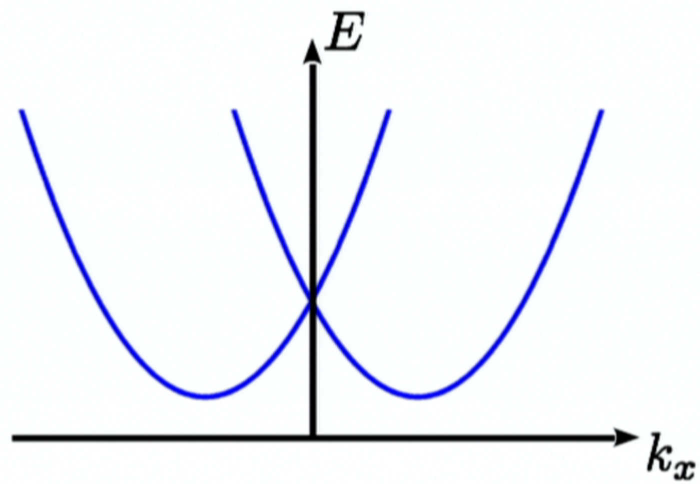
"Terminating" the SC wire by a magnetic gap: Majorana zero modes localised at the ends

Fu & Kane, 2008

Realization in 1D wires

1D spin-orbit-coupled
wire (e.g. InAs)

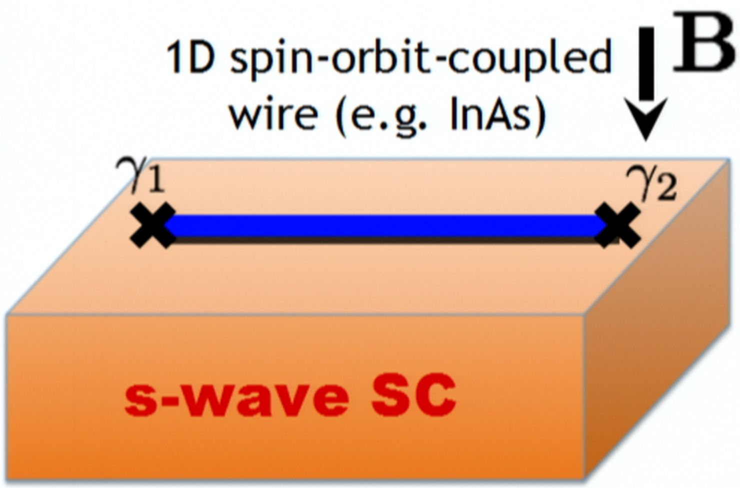


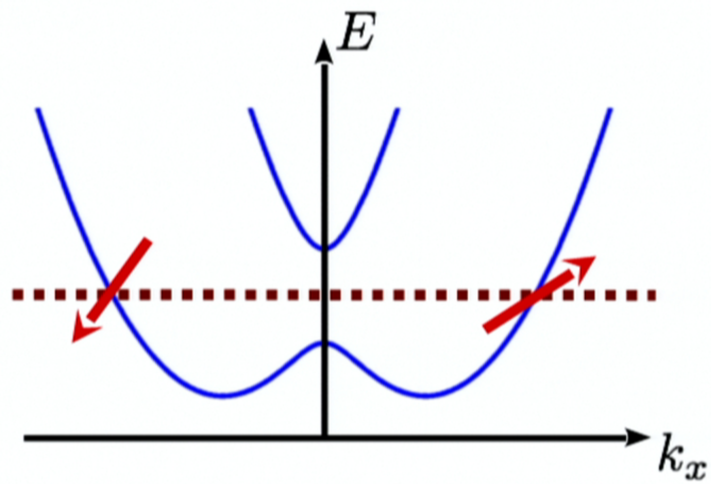


$$H = \int dx \psi^\dagger \left[-\frac{\partial_x^2}{2m} - \mu - i\hbar v \partial_x \sigma^y \right] \psi$$

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)

Realization in 1D wires





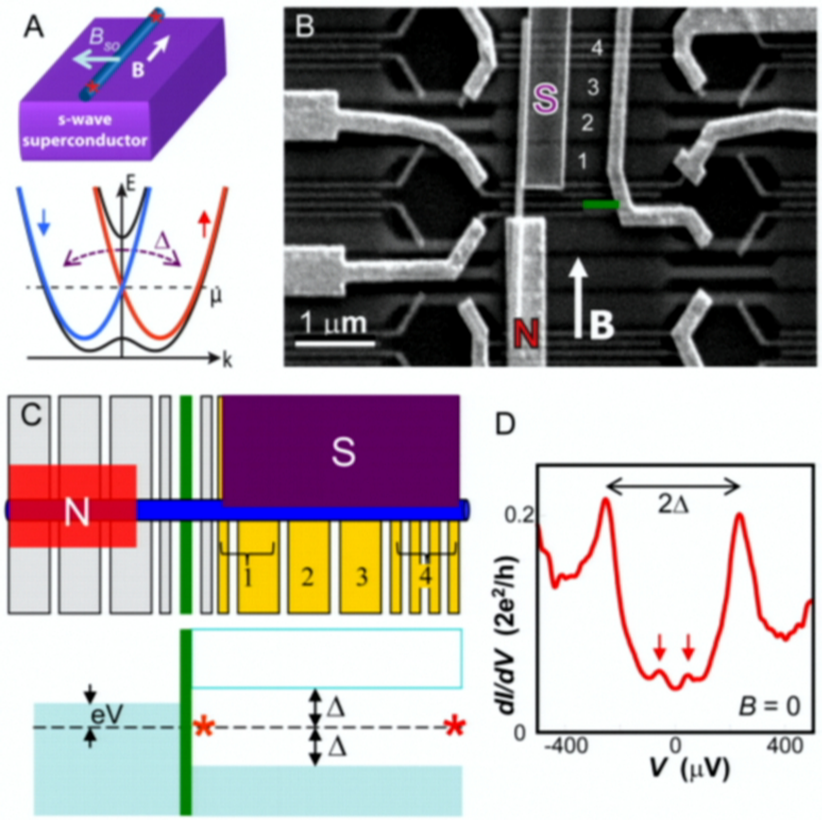
$$H = \int dx\, \psi^\dagger \left[-\frac{\partial_x^2}{2m} - \mu - i\hbar v \partial_x \sigma^y - \frac{g\mu_B B}{2}\sigma^z \right] \psi$$

$$+ (\Delta \psi_\uparrow \psi_\downarrow + h.c.)$$

Generates a **1D 'spinless' SC state with Majorana fermions!**

(Lutchyn, Sau, Das Sarma 2010; Oreg, Refael, von Oppen 2010)

First possible experimental realization



Mourik et al., Science 2012 (Kouwenhoven's group, Delft) following proposals by Lutchyn, Sau & Das Sarma, 2010;
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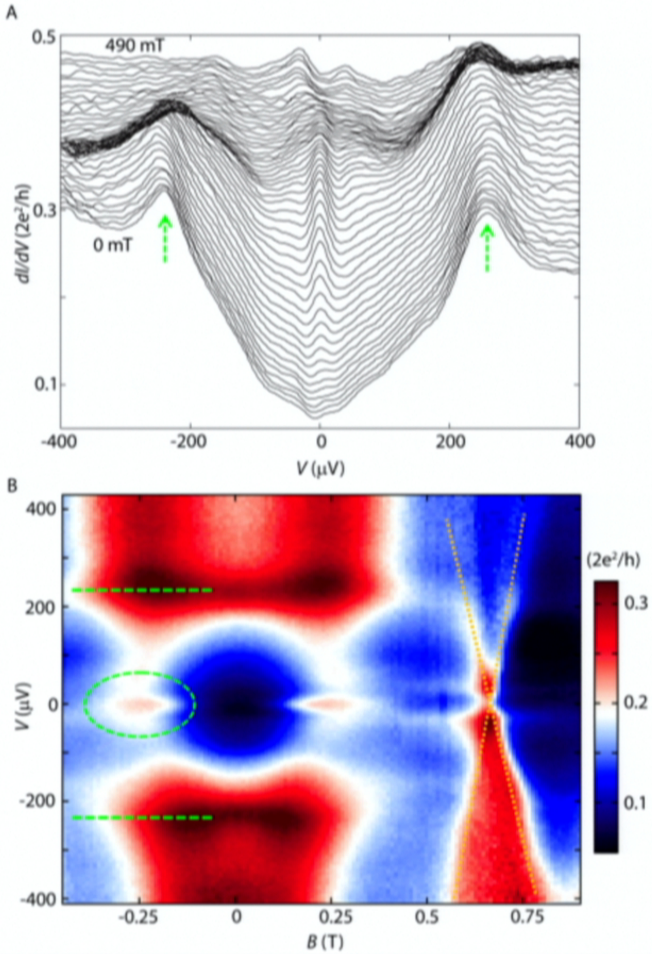

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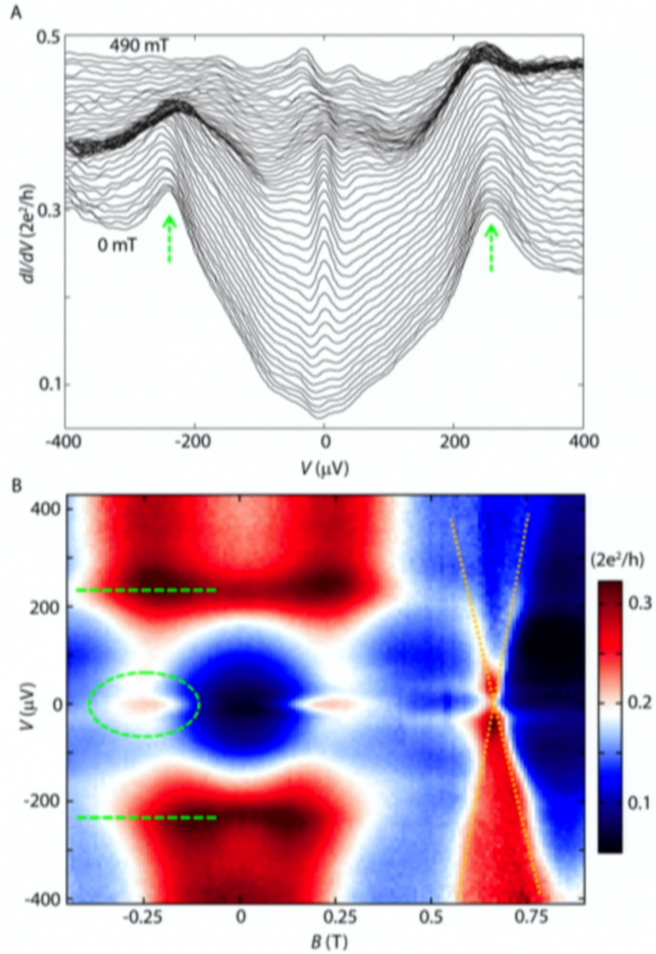

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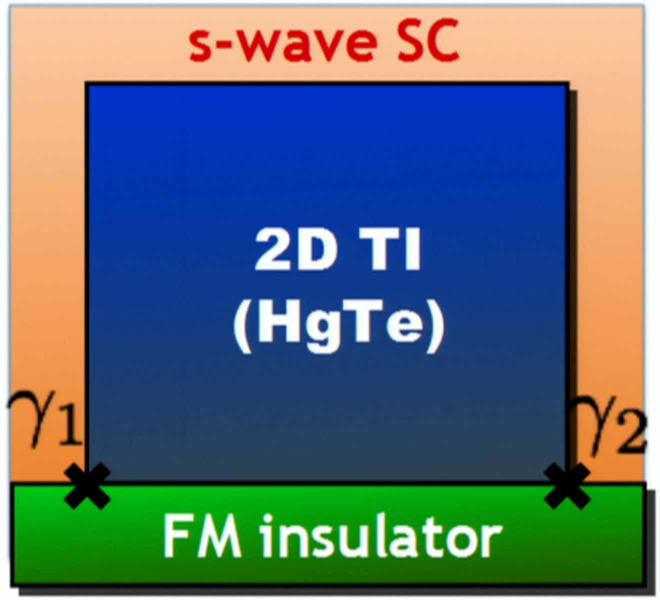
Great! Anything else?

Can this idea be generalised to other types of exotic (non-Abelian) zero modes?

At first glance, this is very doubtful:

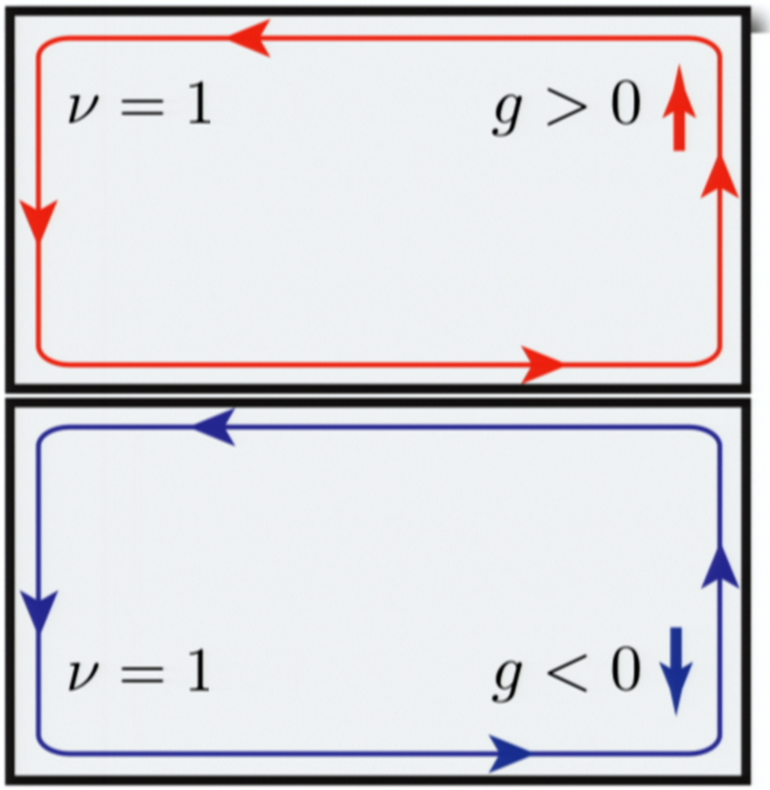
- $\mathbb{Z}_8$ classification of phases of interacting fermions in 1D (Fidkowski & Kitaev, 2009; Turner, Pollmann & Berg, 2010)

Recall topological insulator edges



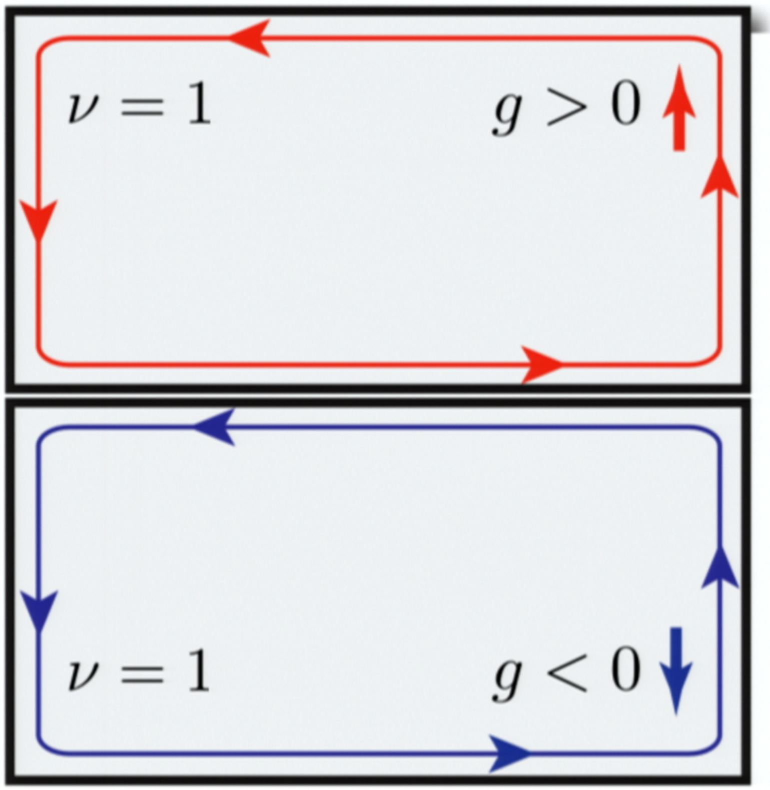
Fu & Kane, 2008

Realization in quantum Hall edges



Counter-propagating edge modes at the boundary between $g > 0$ and $g < 0$. The sign of g can be changed by stress.

Realization in quantum Hall edges



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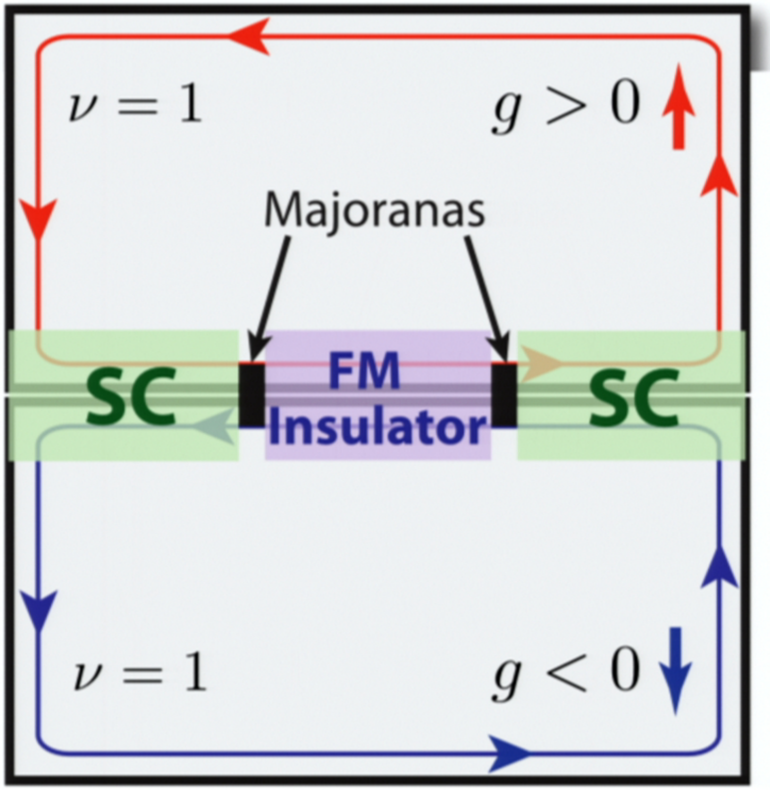
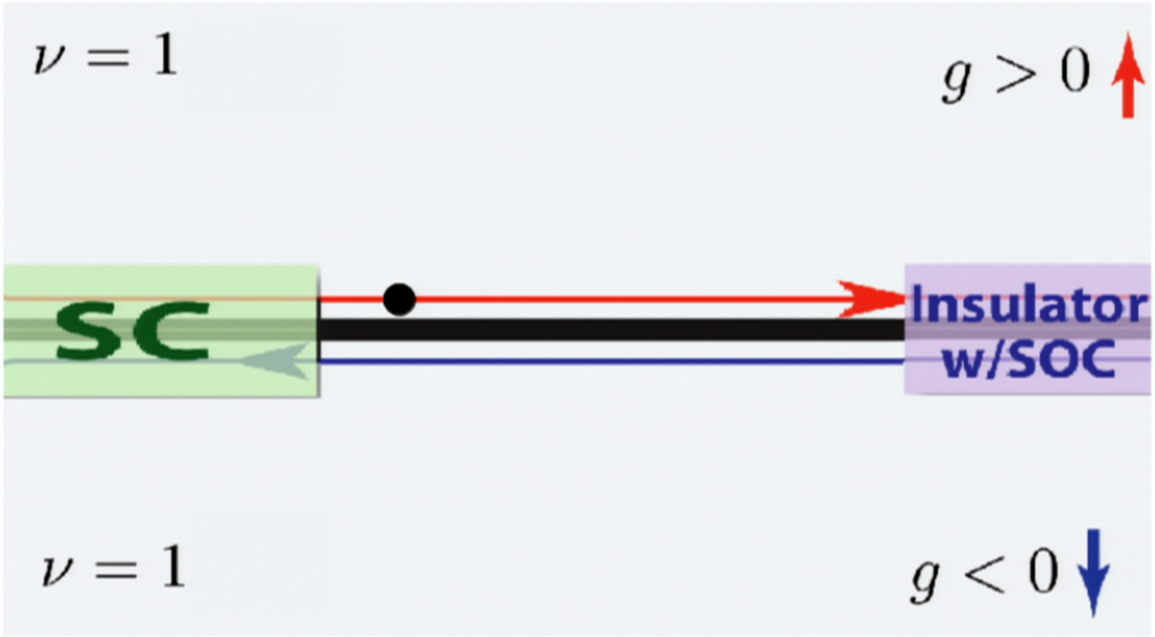
Realization in quantum Hall edges

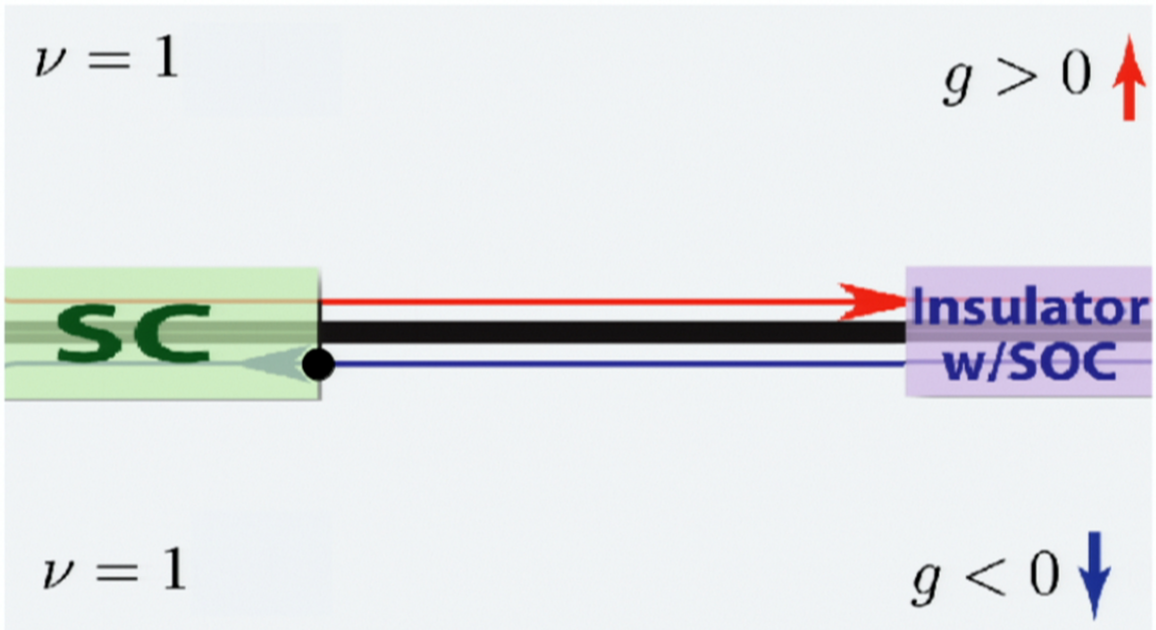


Counter-propagating edge modes at the boundary between $g > 0$ and $g < 0$. The sign of g can be changed by stress.

Majorana zero mode

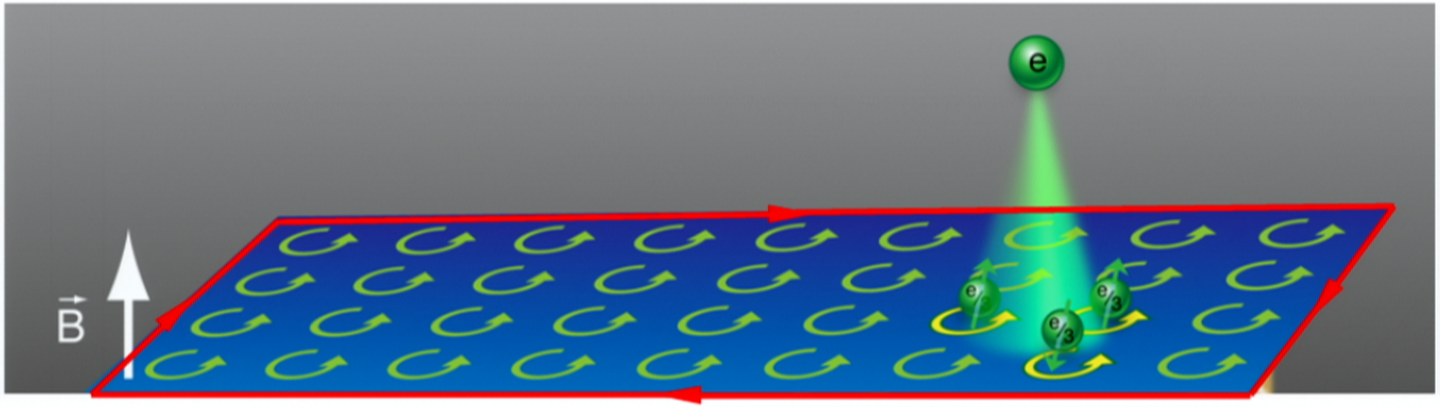


Majorana zero mode



Fractional Quantum Hall Effect

$\nu = 1/3$ Laughlin state:



chiral edge state

Quasiparticles propagating along the edge are fractionally charged: $e^* = e/3$

Parafermions vs Majoranas

<u>*Upshot:*</u>

Majorana Fermions: $\gamma^2 = 1$

$$\gamma_y \gamma_x = -\gamma_x \gamma_y$$

Parafermions: $\alpha^N = 1$

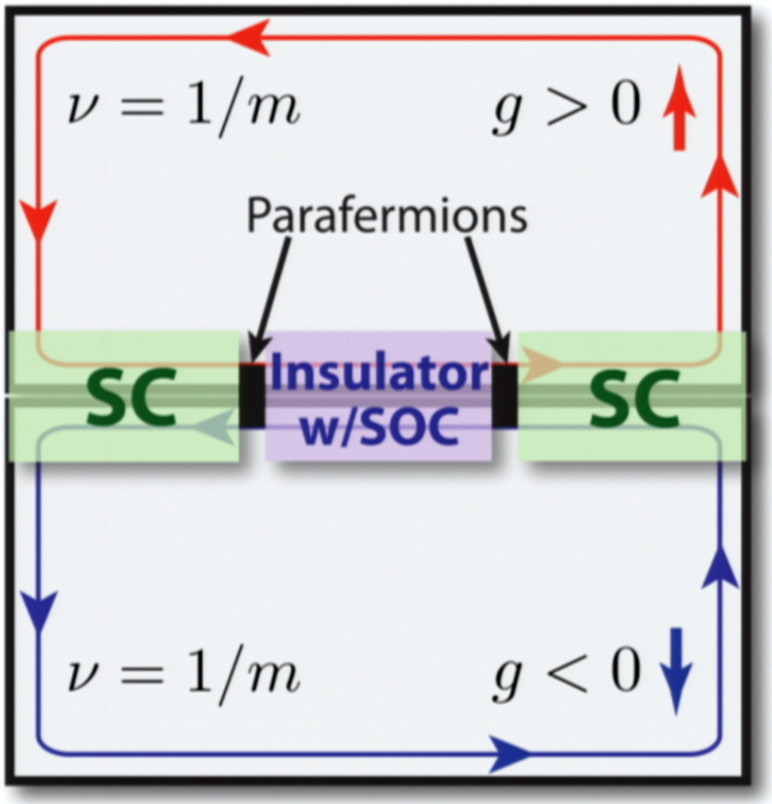
$$\alpha_y \alpha_x = \alpha_x \alpha_y e^{\frac{2\pi i}{N} \mathrm{sgn}(x-y)}$$

Majoranas <--> 1D quantum Ising model

Parafermions <--> 1D quantum Clock/Potts model

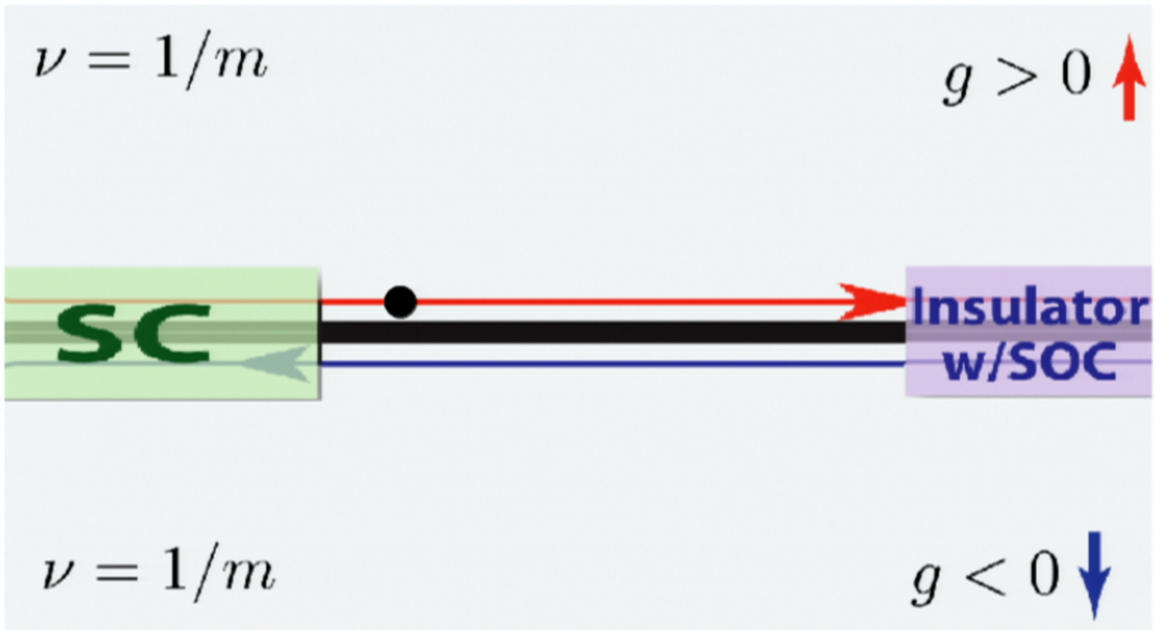
Paul Fendley, arXiv:1209.0472

What about fractional quantum Hall edges?

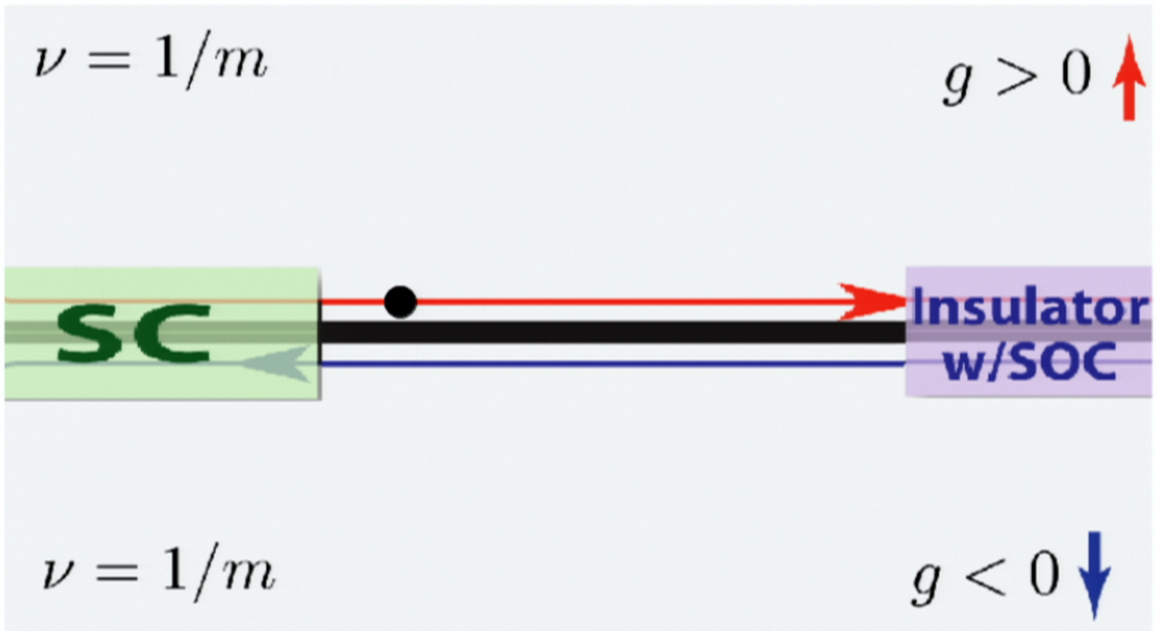


Counter-propagating ***fractionalized*** edge modes at the boundary between $g > 0$ and $g < 0$. The sign of g can be changed by stress.

Parafermionic zero mode

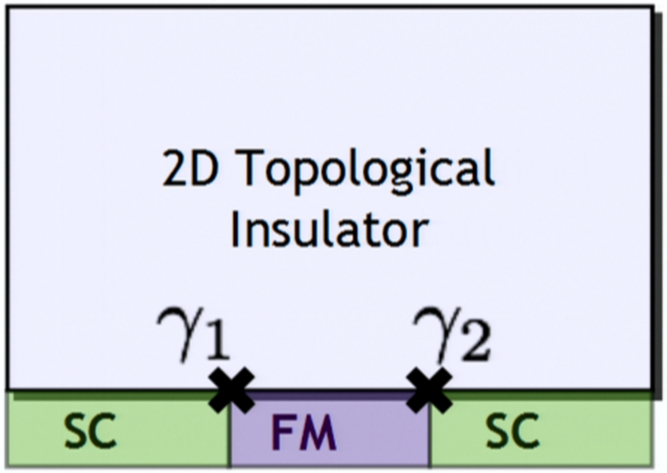


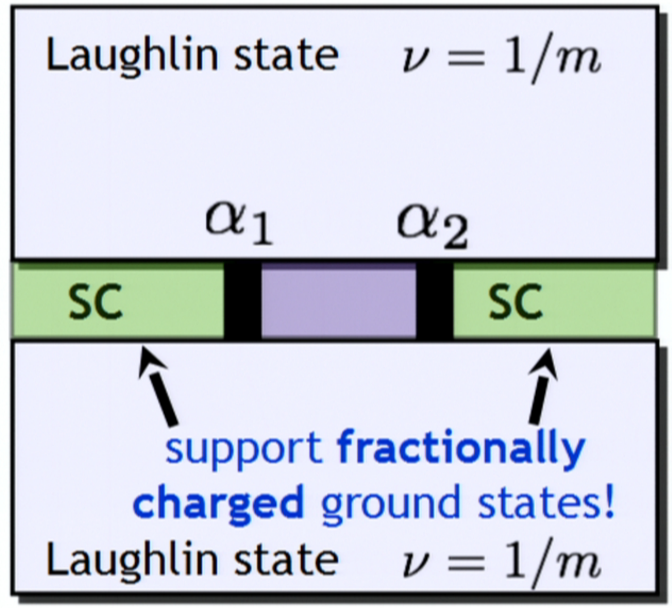
Parafermionic zero mode



Majorana vs. parafermion zero-modes

Fu & Kane (2009)





$$\gamma_i^2 = 1 \qquad \gamma_1\gamma_2 = -\gamma_2\gamma_1$$

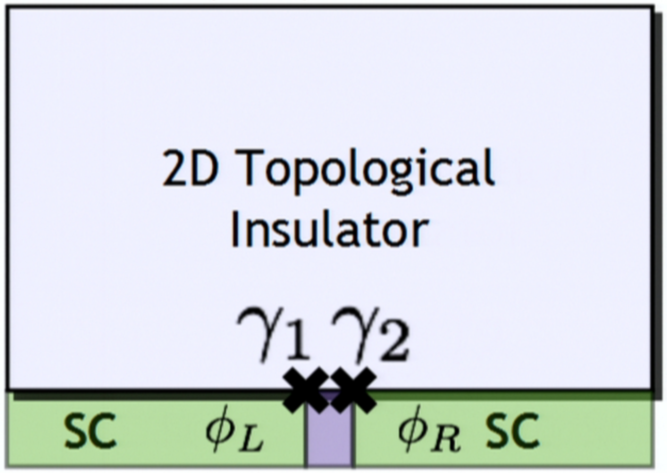
$$\alpha_i^{2m} = 1 \qquad \alpha_1\alpha_2 = e^{i\frac{\pi}{m}}\alpha_2\alpha_1$$

$$|\text{even}\rangle \equiv |0\rangle \overset{\gamma_i}{\longleftrightarrow} |\text{odd}\rangle \equiv |1\rangle$$

$$|0\rangle \overset{\alpha_i^\dagger}{\rightarrow} |1/m\rangle \rightarrow |2/m\rangle \rightarrow \cdots \rightarrow |2 - 1/m\rangle$$

Majorana vs. parafermion zero-modes

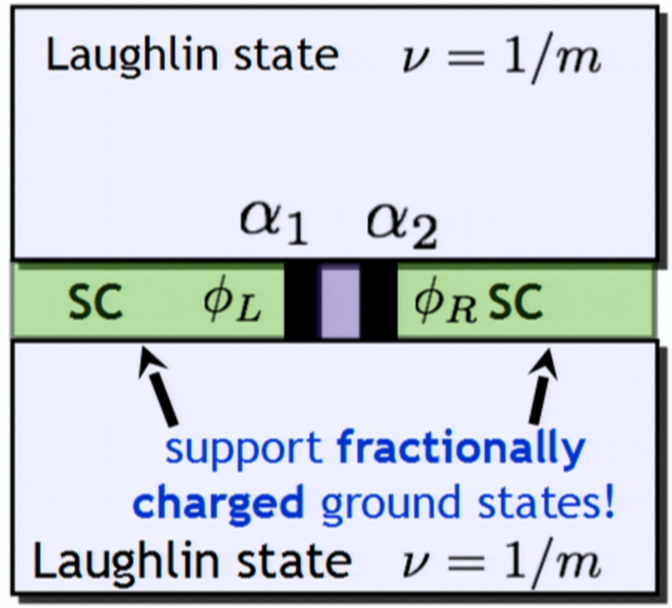
Fu & Kane (2009)



Fractional Josephson effect

$$I \propto \sin\left(\frac{\Delta\phi}{2}\right)$$

Kitaev (2001)

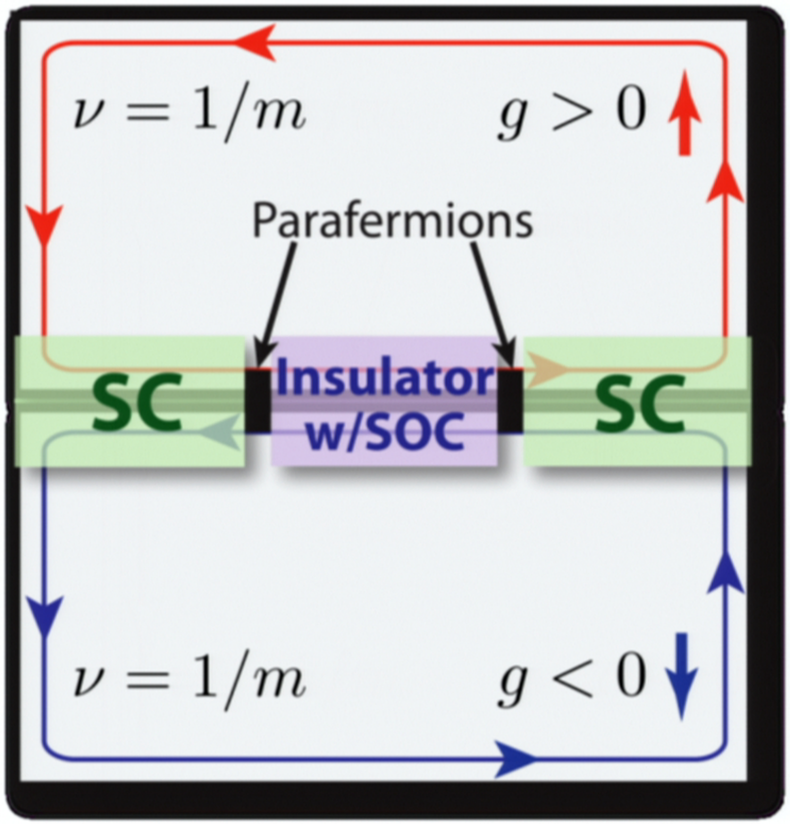


$$I \propto \sin\left(\frac{\Delta\phi}{2m}\right)$$

Effect persists even with **electronic** relaxation processes

Experimental realizations?

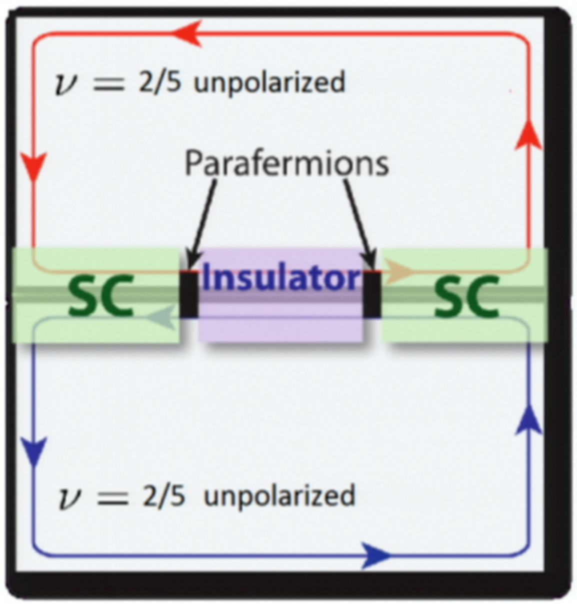
Is this feasible?



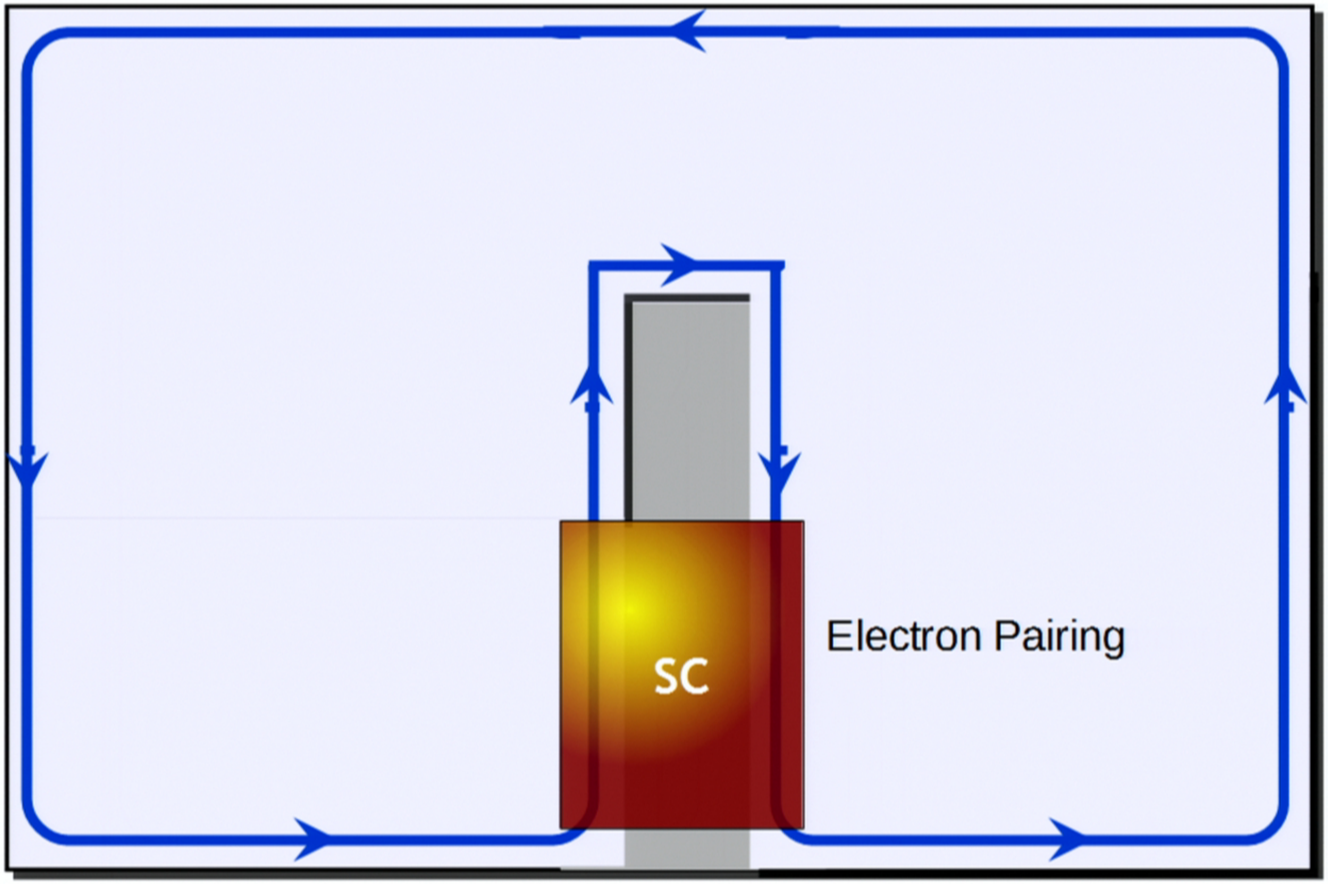
Which Parts are Really Necessary?

So do we really need the insulator at all?

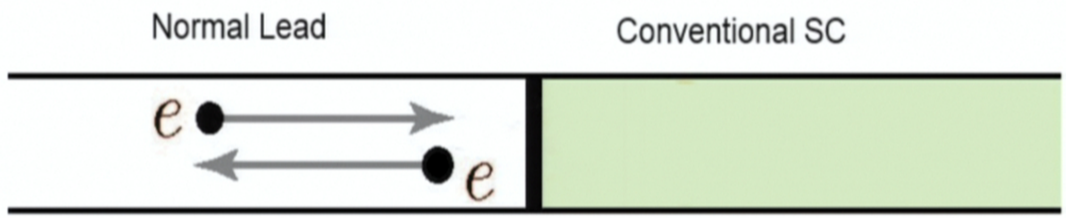
D. Clarke, J. Alicea, KS, arXiv:1312.6123



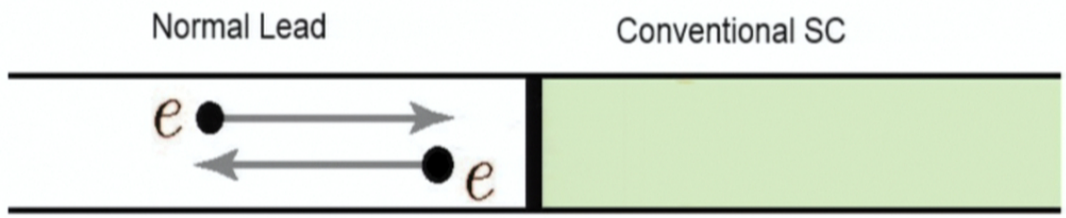
Realization in quantum Hall edges



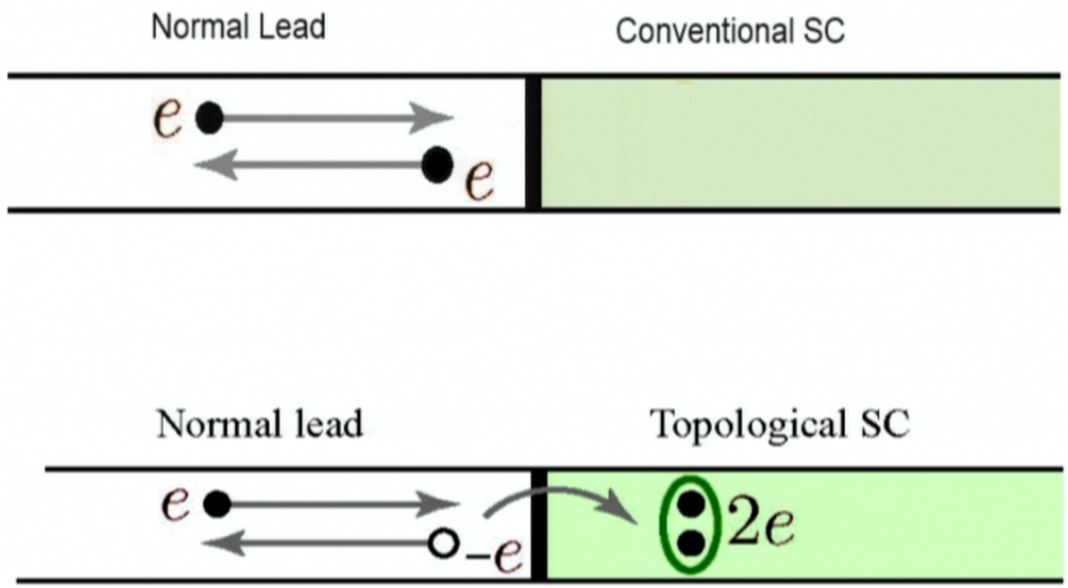
Normal vs. Andreev reflection



Normal vs. Andreev reflection

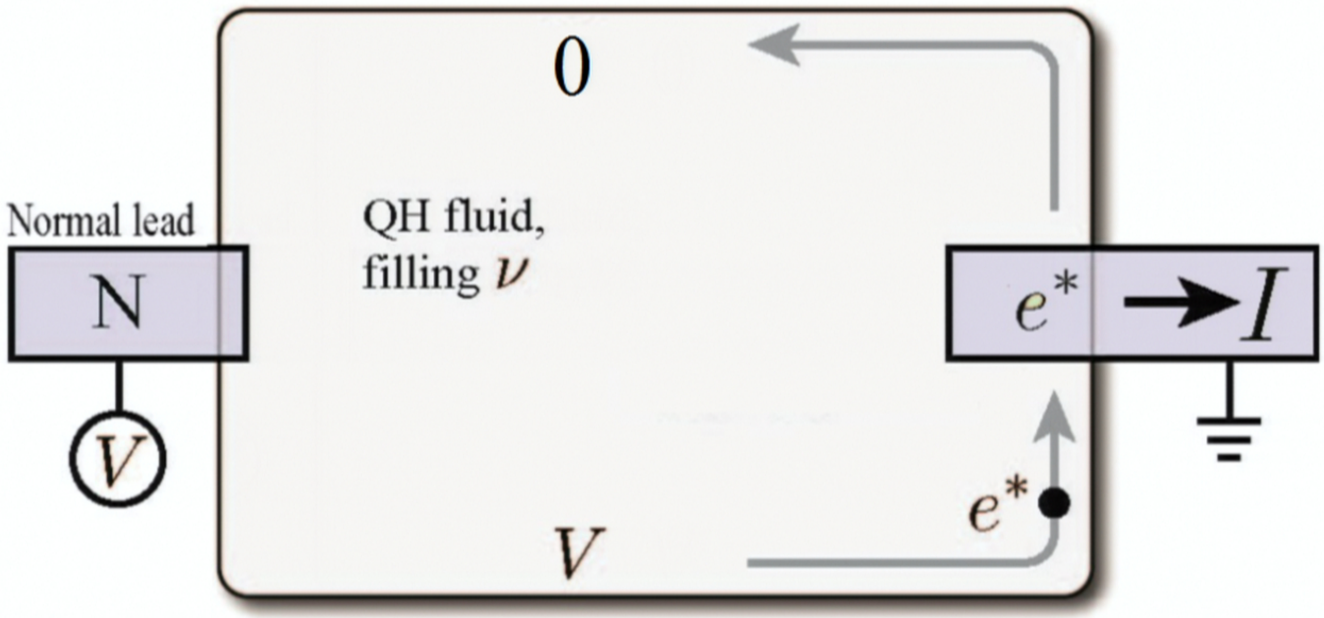


Normal vs. Andreev reflection



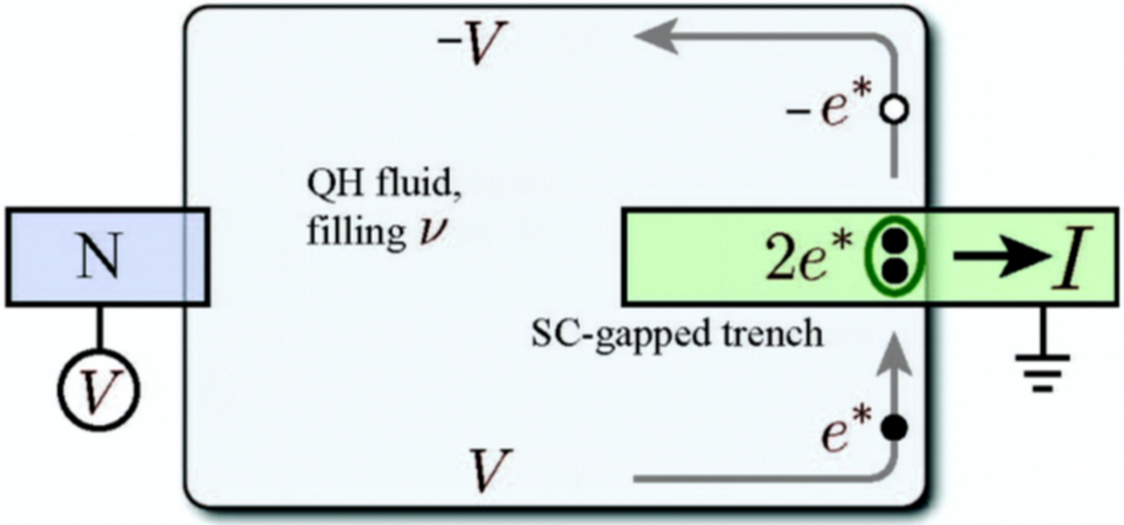
Conductance is *doubled* relative to a single channel wire.

Two Terminal Hall Conductance



$$I = \sigma_H V \qquad \sigma_H = e^2\nu / h$$

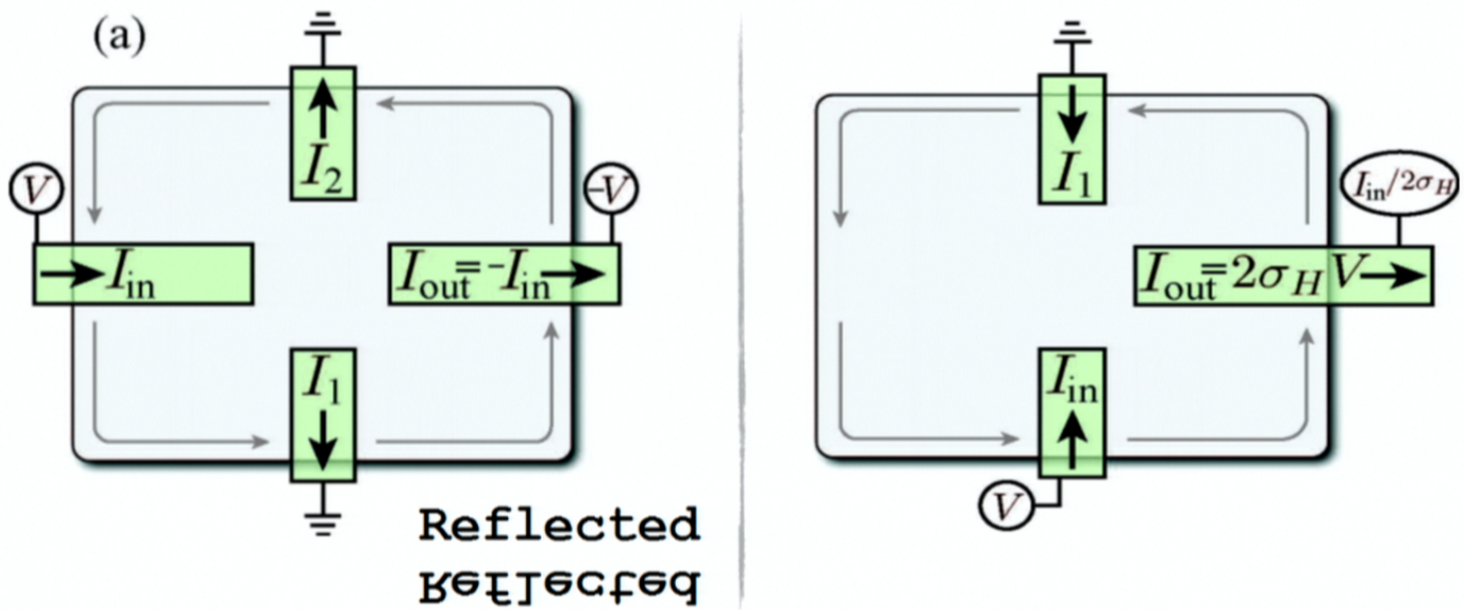
Doubled Hall Voltage



Equilibration and energy dissipation occur only at the normal lead! (in the ideal case)

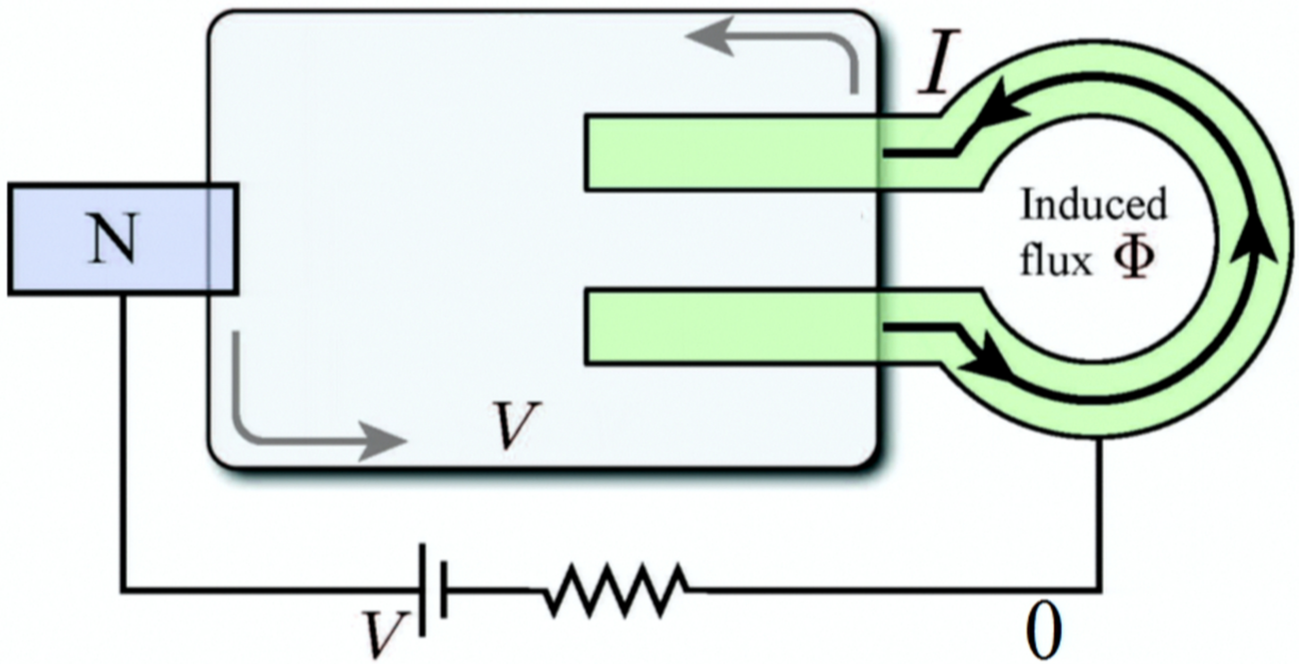
$$V_{Out} - V_{SC} = V_{SC} - V_{in}$$

Current and Voltage Manipulation

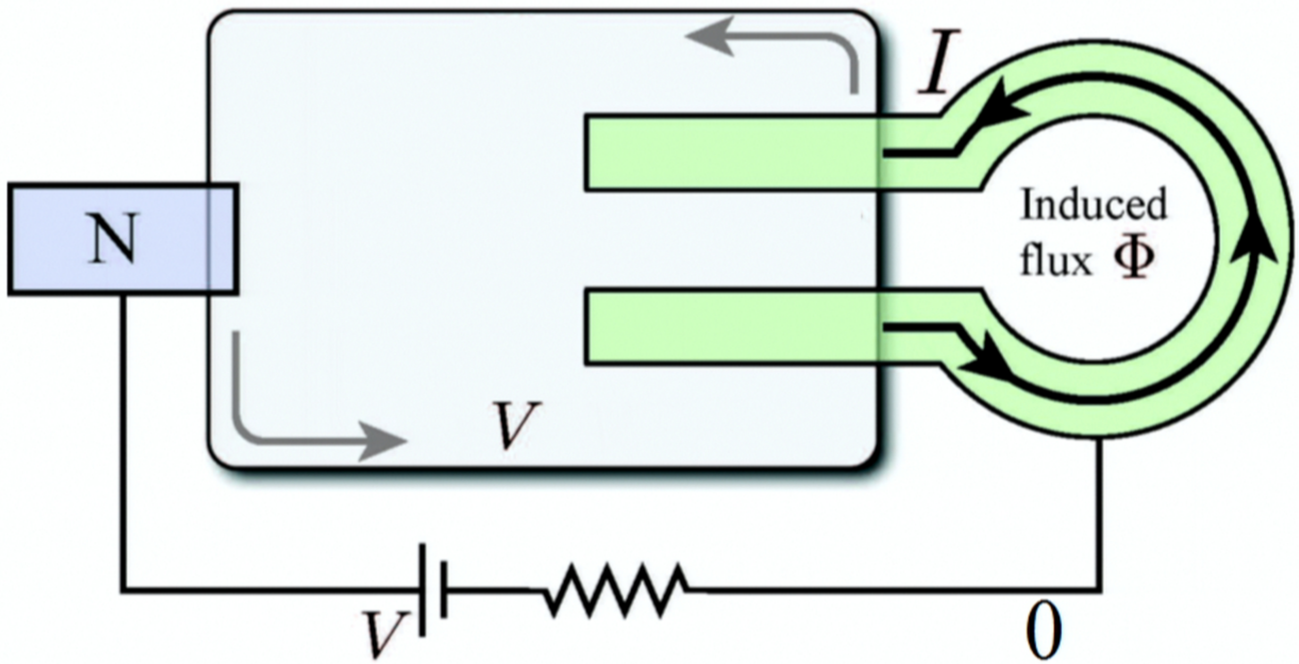


Unusual circuit elements follow from Kirchoff's rules!

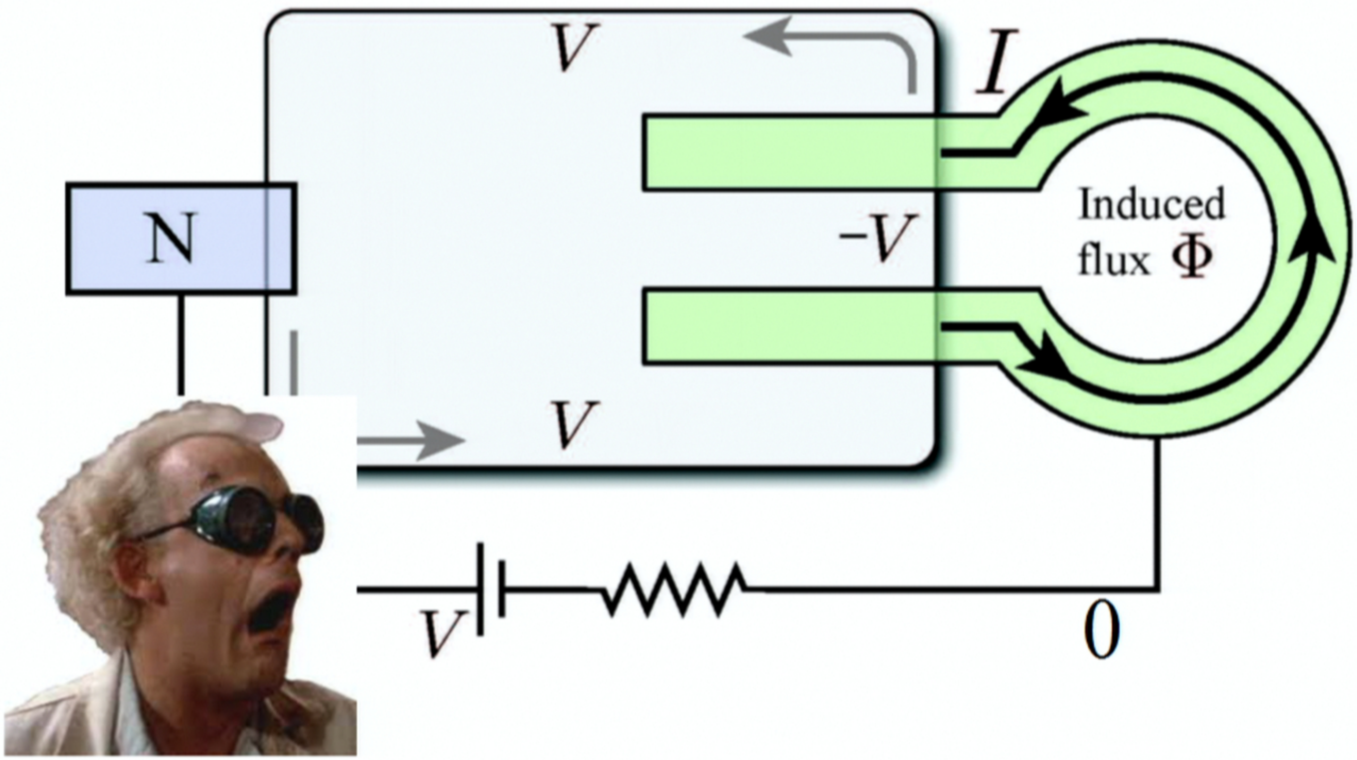
Capacitive Flux Storage



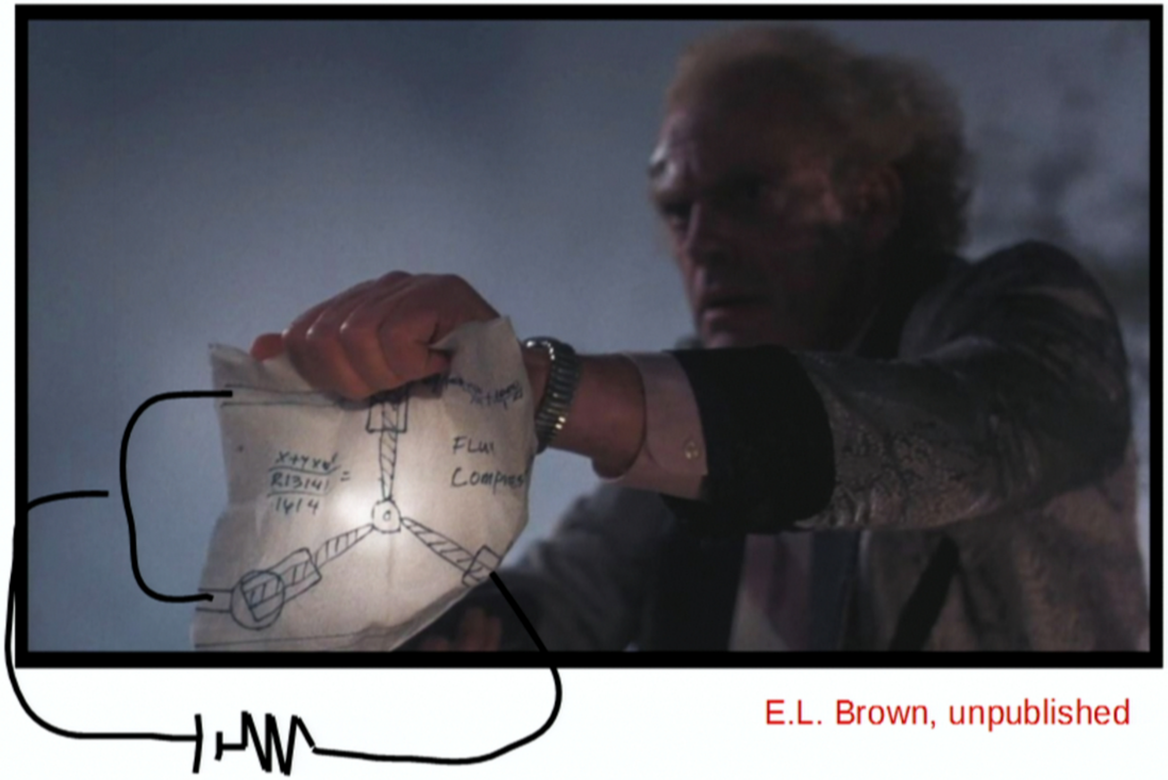
Capacitive Flux Storage



Capacitive Flux Storage



Flux Capacitor!



E.L. Brown, unpublished

(Possible) Applications of new circuit elements

- Low Temperature and Low Power Amplifiers, Transistors, Logic Gates, etc.
- "On Chip" control elements for anyons
- Detection and utilisation of local magnetic fields
- New transport signatures of non-Abelian zero modes in hybrid FQH/SC systems
- A *new use* for non-Abelian anyons!