Title: The quest for self-correcting quantum memory

Date: May 12, 2014 04:00 PM

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Abstract: <span>A self-correcting quantum memory is a physical system whose quantum state can be preserved over a long period of time without the need for any external intervention. The most promising candidates are topological quantum systems which would protect information encoded in their degenerate groundspace while interacting with a thermal environment. Many models have been suggested but several approaches have been shown to fail due to no-go results of increasingly general scope. In a nutshell, 2D topological models and many 3D topological models have point-like excitations which propagate freely and change the groundstate at any non-zero temperature. A recent suggestion is to introduce effective long-range interactions between those point-like excitations. In this presentation, I will first explain the desiderata for self-correction, review the recent advances and no-go results, and describe the current endeavours to define a self-correcting system in 2D and 3D. Time permitting, I will briefly present our recent work on the thermal instability of models which aim to introduce effective long-range interactions between point-like excitations (joint work with Beni Yoshida, John Preskill and David Poulin).</span>

# The Quest for Self-Correcting Quantum Memory


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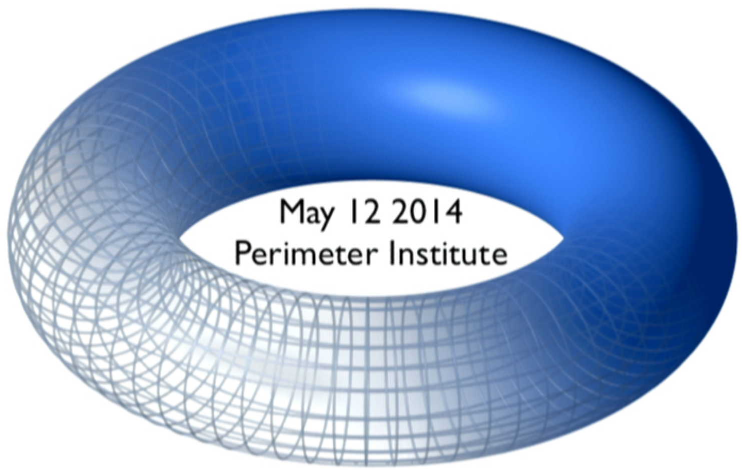
California Institute of Technology (Caltech)


INSTITUTE FOR QUANTUM INFORMATION AND MATTER

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# Self-correcting memory


**Quest for self-correcting memory**

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Self-correction
- Desiderata
- Canonical ex.
- Thermal stability

No-go results
- Stabilizer codes
- LCPCs

Alternatives
- Active QEC
- 4D toric code
- 2D + long-range
- 3D spin liquids
- Entropy protected

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Self-correcting memory = physical system which encode (quantum) information


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- reliably
- for a macroscopic period of time


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Encoding $|\psi_i\rangle \xrightarrow{\text{Noise}} |\psi_i'\rangle \xrightarrow{\text{Decoding}} |\psi_f\rangle$

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Noise

Encoding $|\psi_i\rangle$ → $|\psi'_i\rangle$ $\xrightarrow{\text{Decoding}}$ $|\psi_f\rangle$

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Typically, the degenerate groundspace of a local Hamiltonian
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Quantum  hard drive?

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# Self-correcting classical memories

## 2D ferromagnetic Ising model

$$H_{\text{Ising2D}} = -\sum_{\langle i,j \rangle} \sigma_z^i \otimes \sigma_z^j$$

↑↑↑↑↑↑↑↑↑
↑↑↑↑↑↑↑↑↑
↑↑↑↑↑↑↑↑↑ 0
↑↑↑↑↑↑↑↑↑

↓↓↓↓↓↓↓↓↓
↓↓↓↓↓↓↓↓↓
↓↓↓↓↓↓↓↓↓ 1
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➡ (small) magnetic field breaks degeneracy
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➡ with degenerate groundspace?
➡ with no local order parameter?
➡ stable spectrum?

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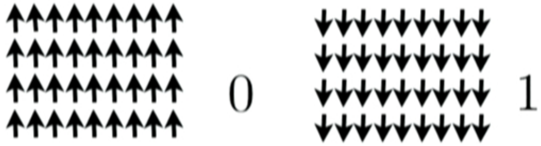


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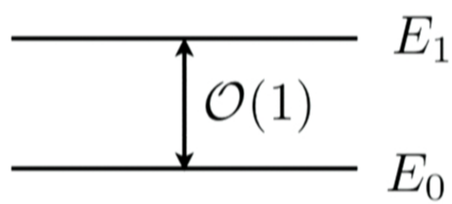


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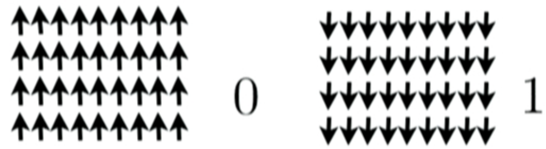


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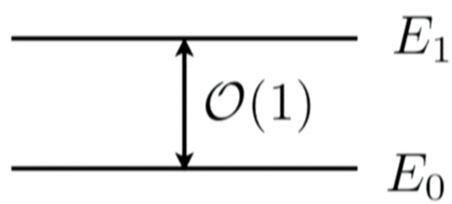


**Topologically ordered systems!**

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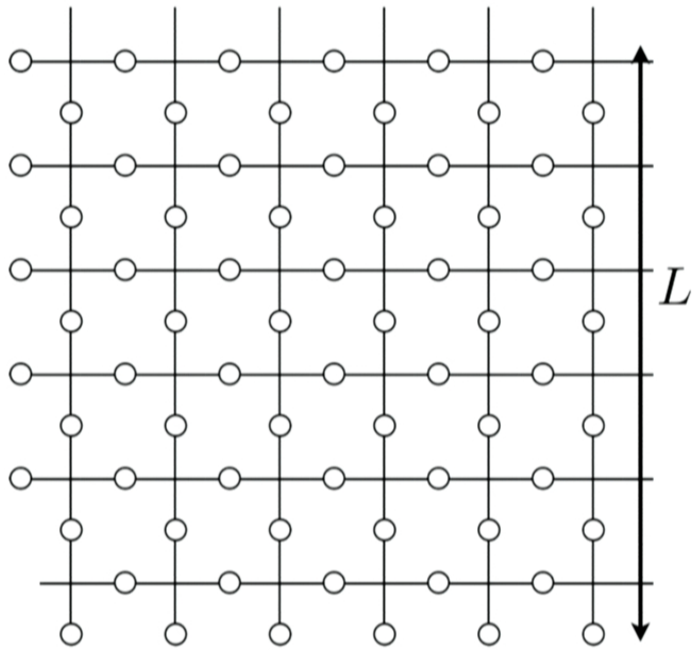


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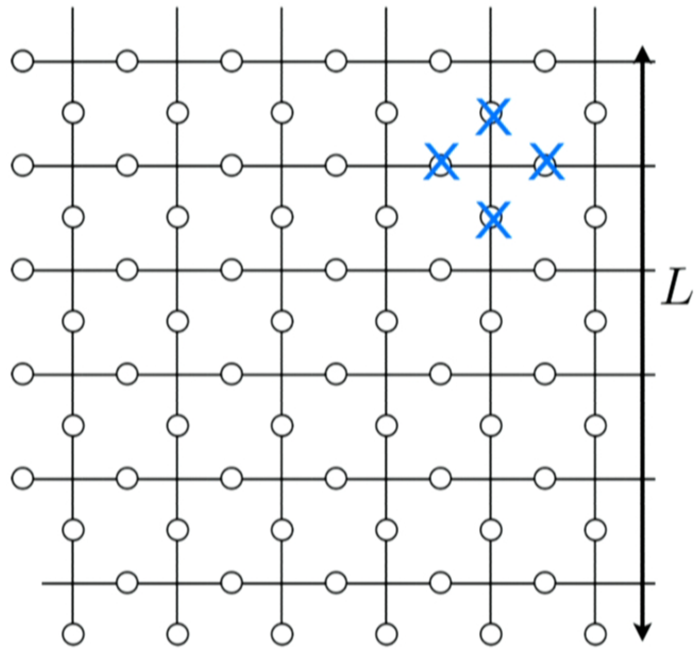


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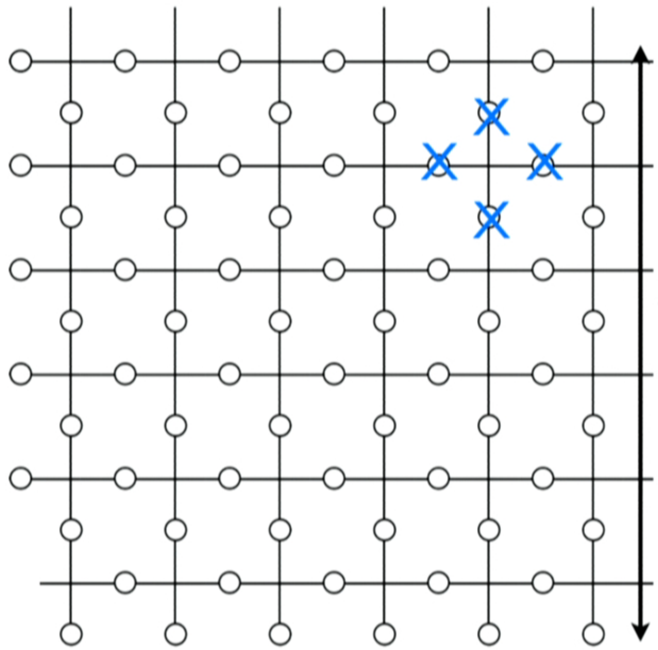


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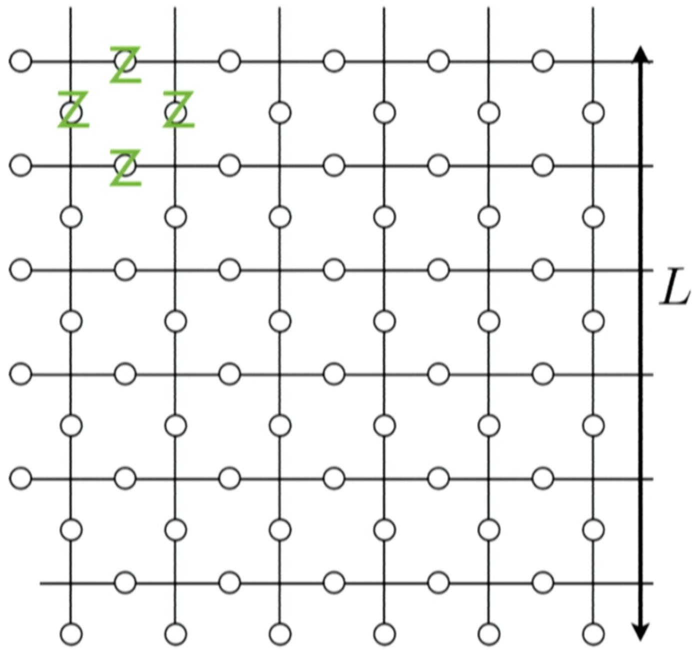


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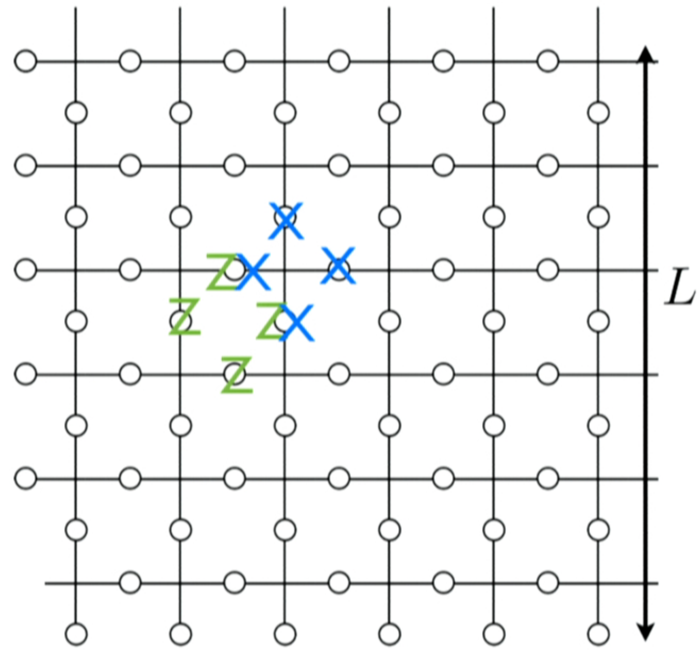
- Commuting local terms

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- Commuting local terms
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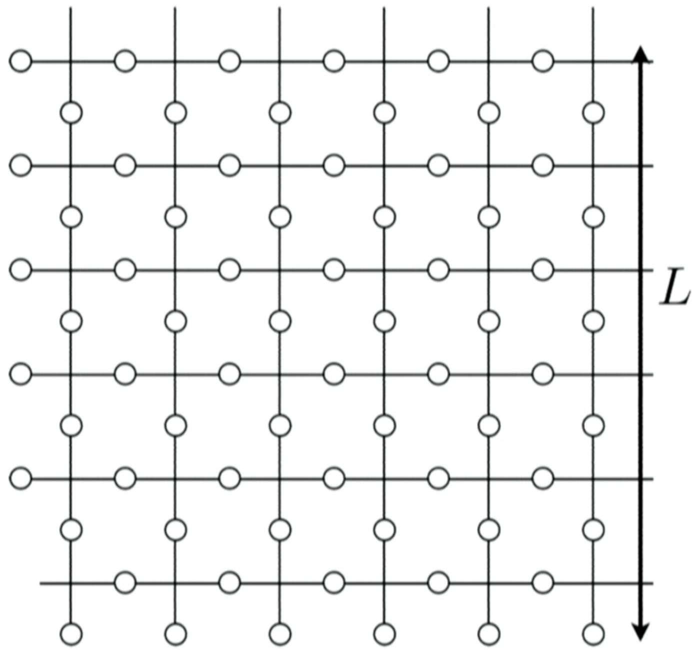


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Groundstates

$$\forall s \; A_s|\psi\rangle = +|\psi\rangle$$

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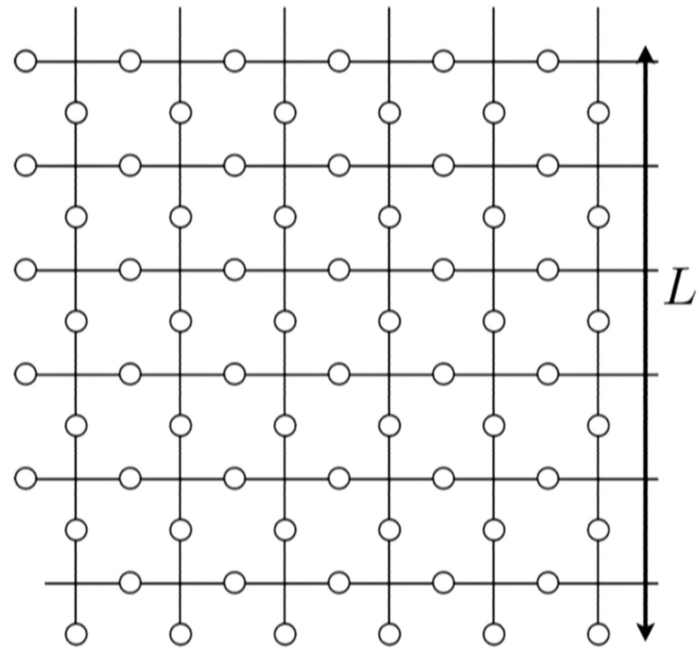


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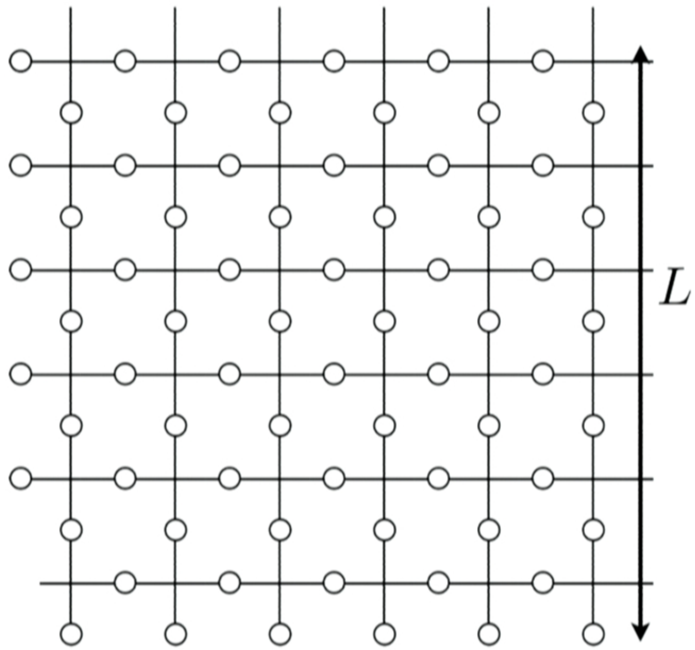


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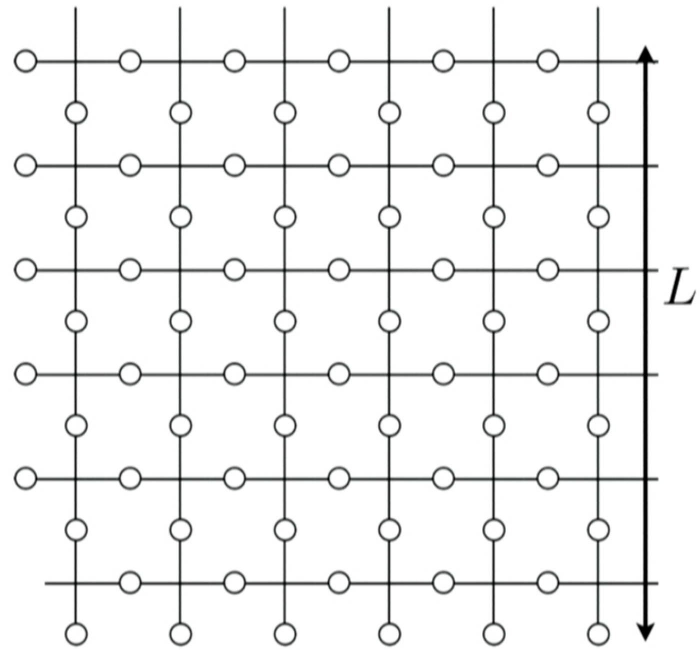


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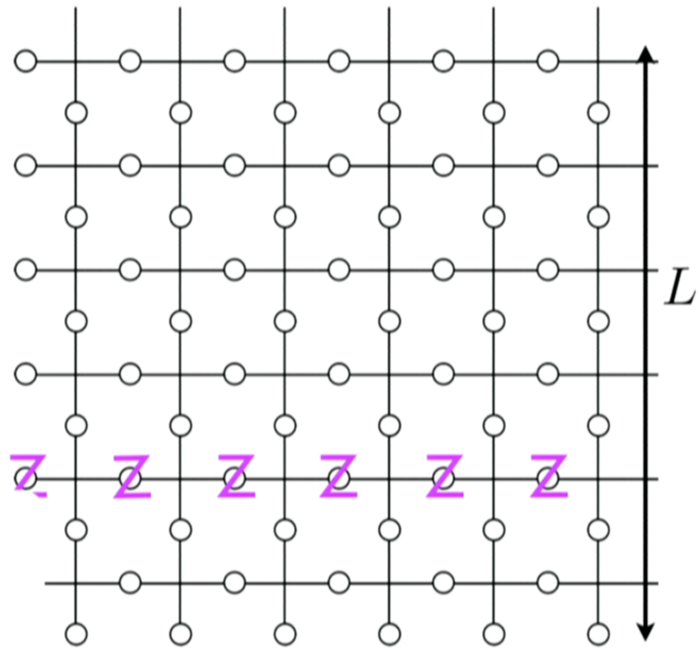
→ Toric code $K = Z^{\otimes L}$

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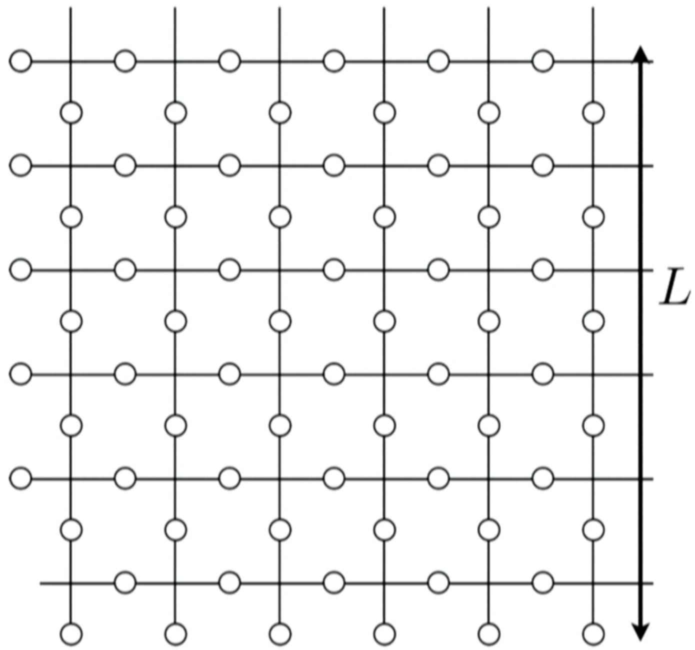


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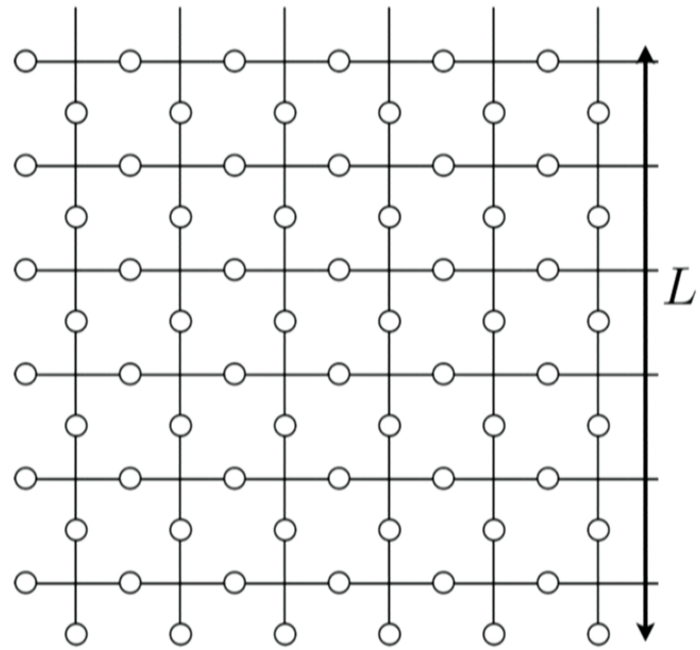


Energy of excitations

Energy meter

$\mathcal{O}(1)$

$\propto L$

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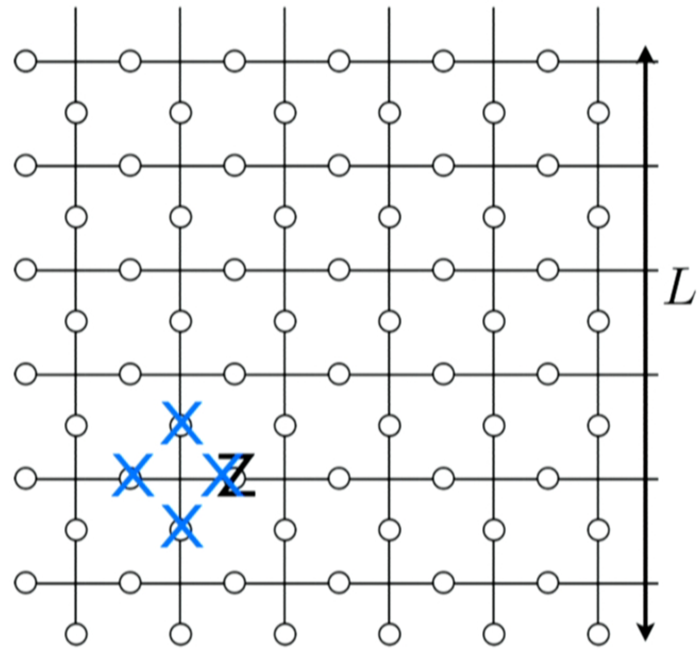


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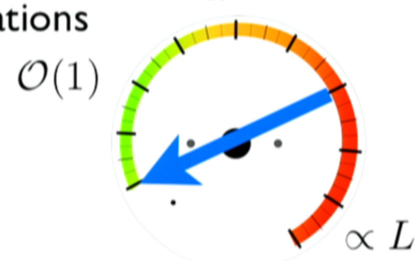


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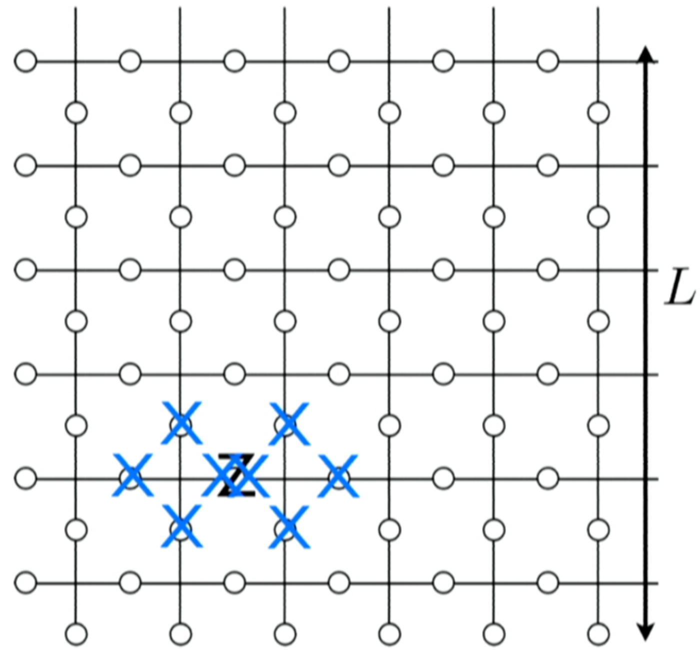
$$\forall p \; B_p|\psi\rangle = +|\psi\rangle$$

Logical operator: $[K,\, H]$

Toric code $K = Z^{\otimes L}$

Energy of excitations




Energy meter


**Quest for self-correcting memory**

Olivier Landon-Cardinal
olc@caltech.edu

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- Thermal stability

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$\mathcal{O}(1)$

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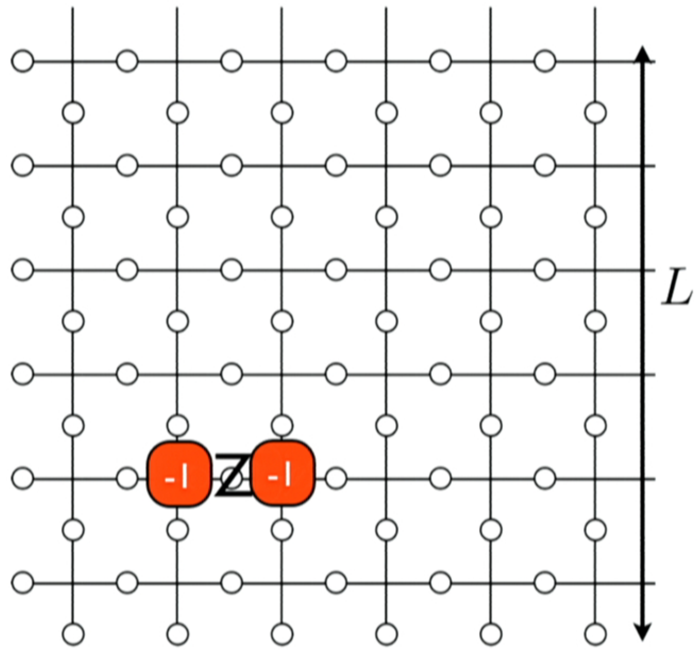


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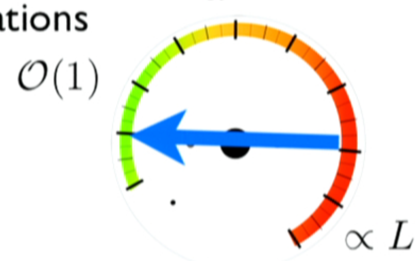


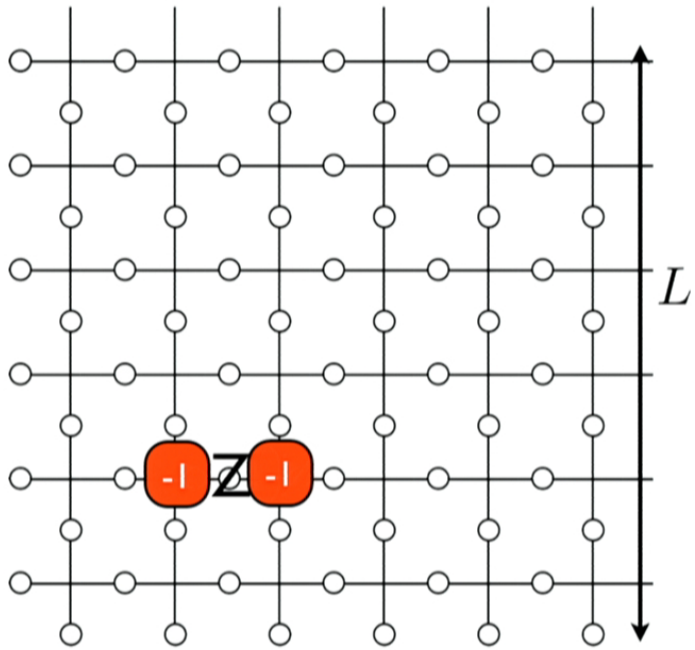


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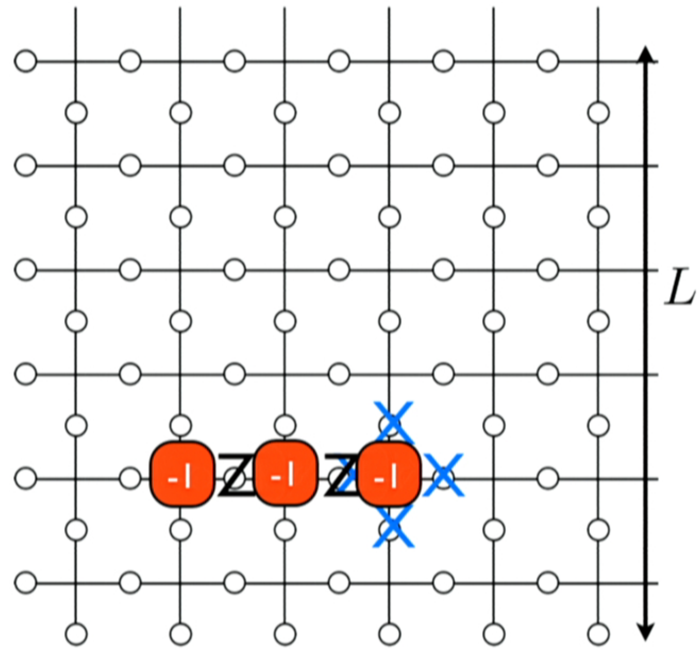


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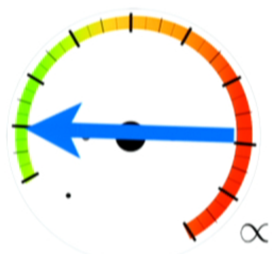

$\propto L$

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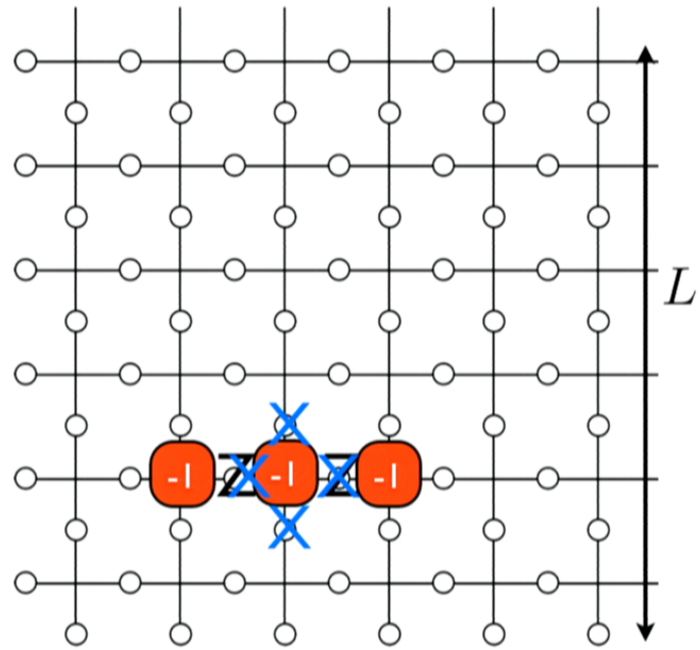


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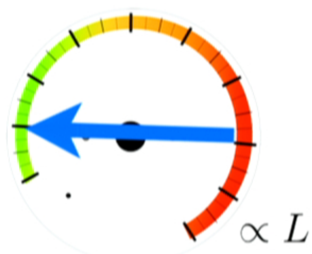


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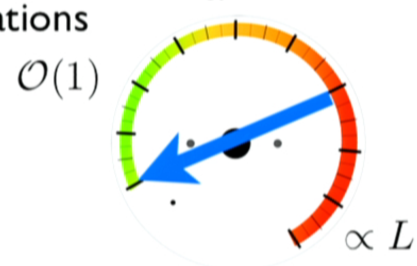


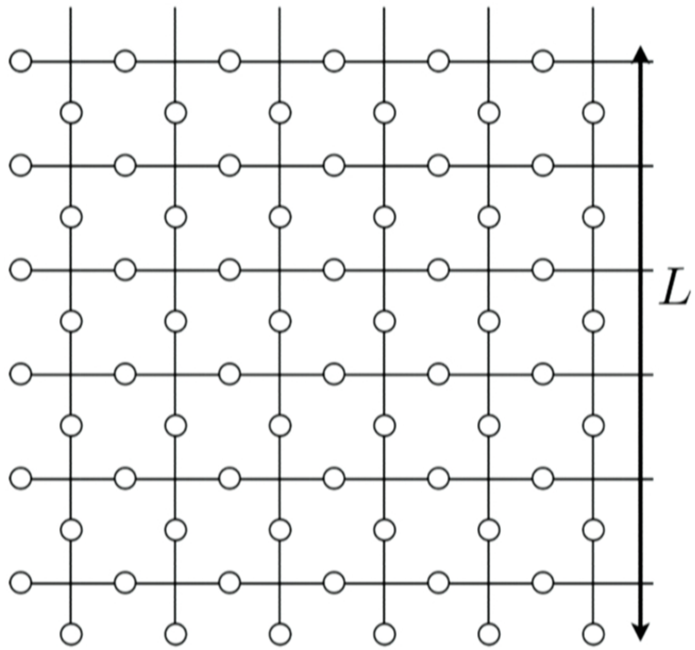


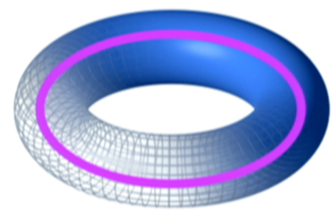

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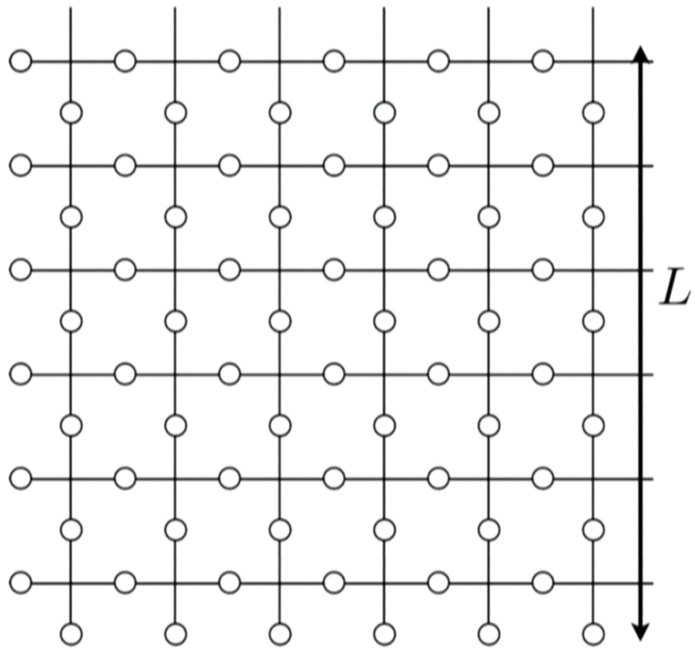


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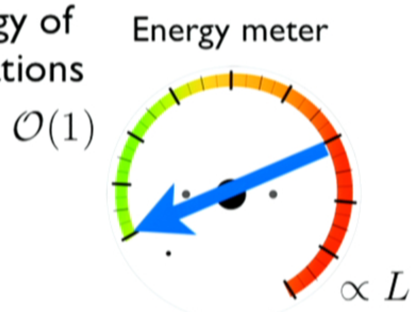


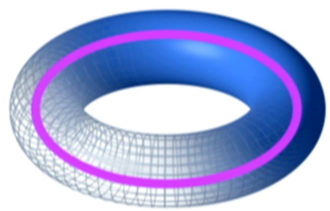

Thermal fluctuations can accumulate and corrupt the information.

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# Formal (possible) definition of self-correction

Thermalization requires detailed knowledge of system dynamics.

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Simplified model for thermalization

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Encoding $|\psi_i\rangle$ ——Noise——

$\longrightarrow |\psi_i'\rangle \xrightarrow{\text{Decoding}} |\psi_f\rangle$

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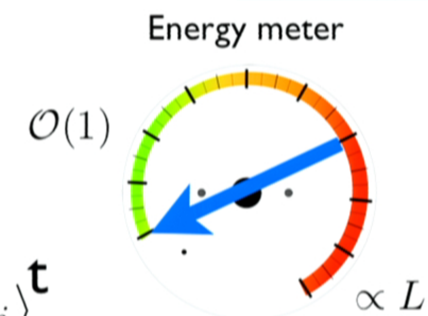


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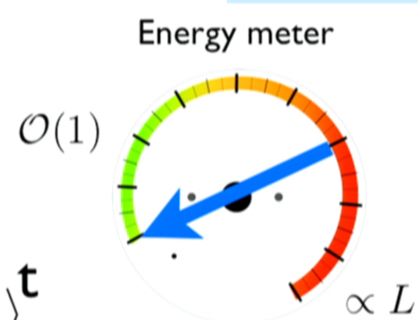


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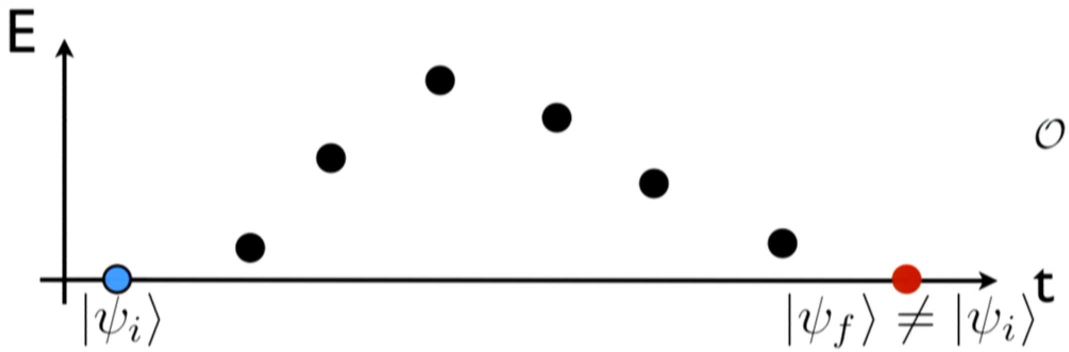


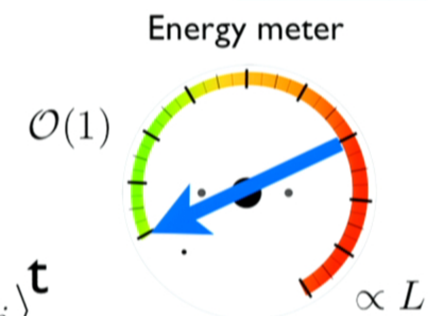


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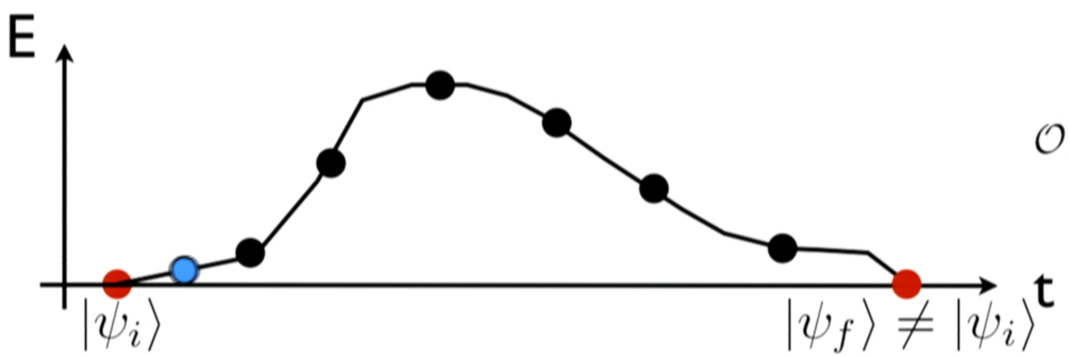


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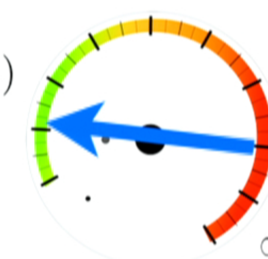


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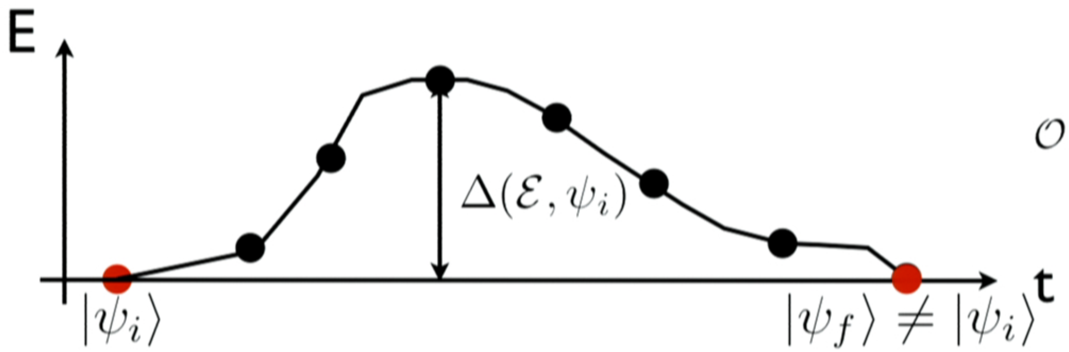
Sequence of local transformations changing the groundstate?

→ Maximum energy of intermediate states : energy barrier

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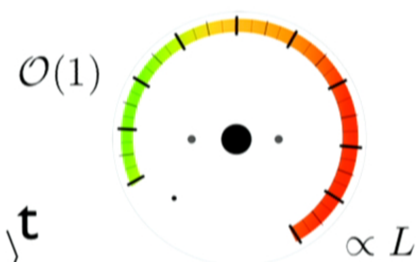


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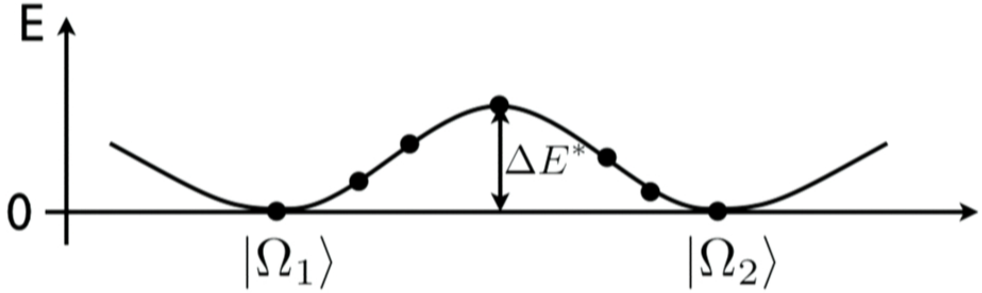
# Self-correction and energy barrier

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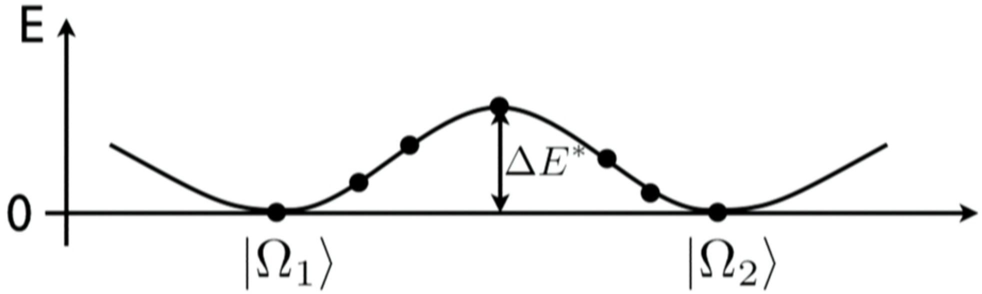
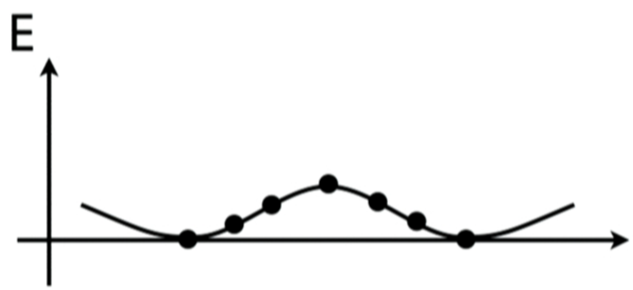



Thermally unstable system




$\Delta E^*$ independent of system size.

E.g., toric code

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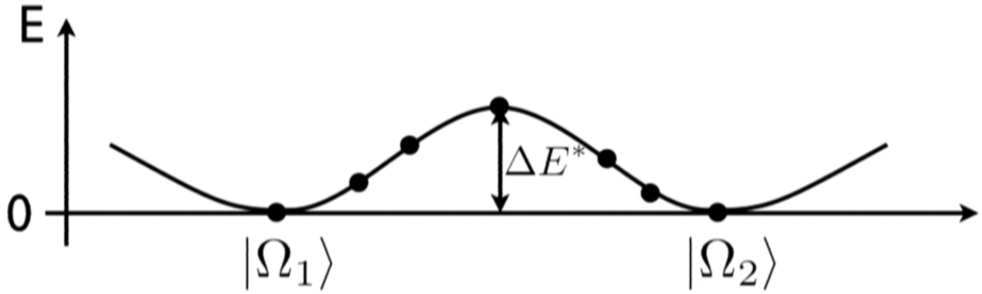


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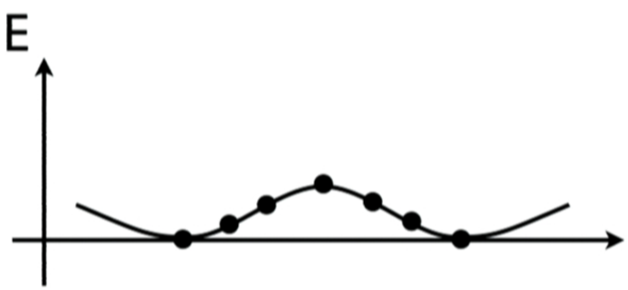


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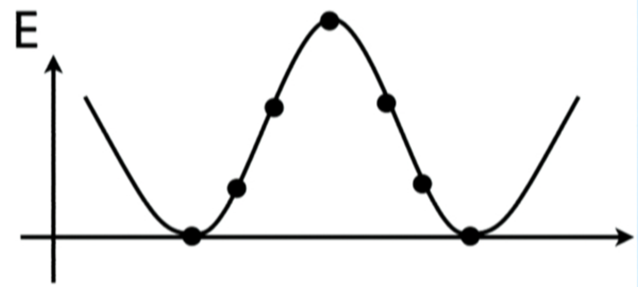
Self-correcting system




Macroscopic energy barrier.
$\Delta E^* \sim L^\alpha \quad \alpha > 0$

E.g., Ising

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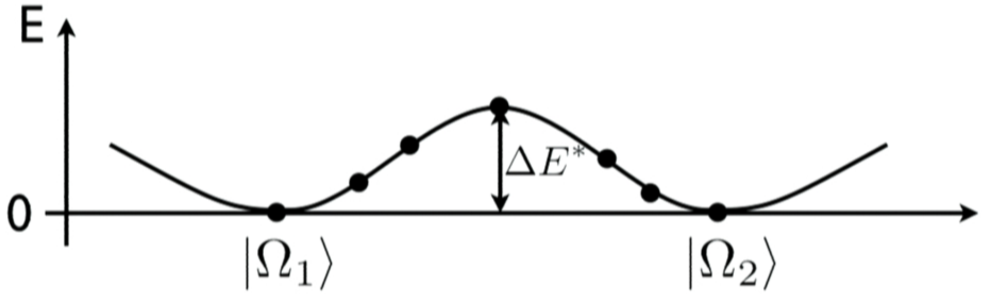


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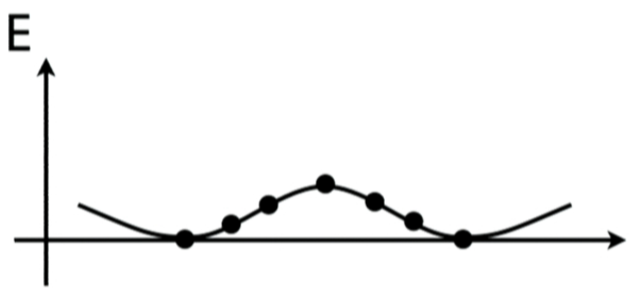


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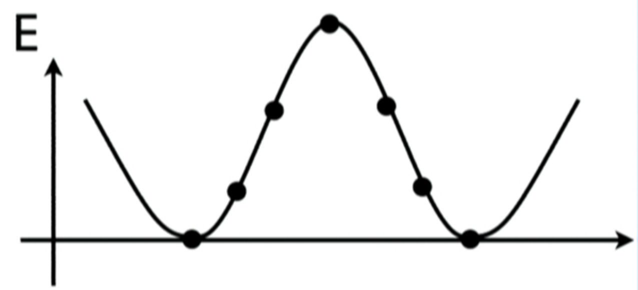


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E.g., Ising

Combining robustness to perturbations and thermal stability in 2D?

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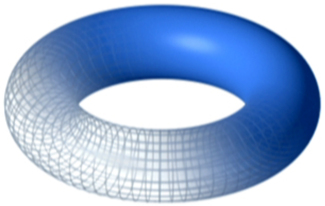
# No-go for stabilizer codes

Instability in Kitaev's toric code



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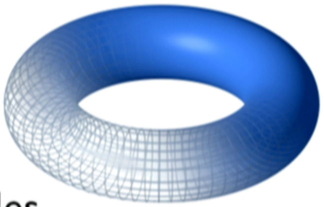

Instability in Kitaev's toric code

Key features
• logical operator is supported on a 1D string of particles

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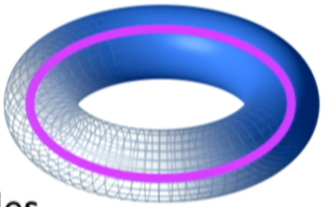

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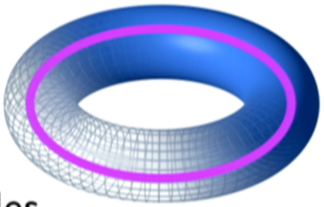

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Stabilizer codes: quantum codes defined by a group of commuting Pauli ops.

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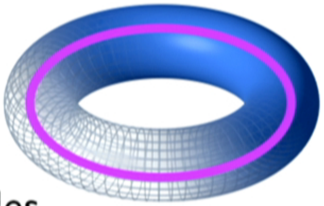
# No-go for stabilizer codes



Instability in Kitaev's toric code

Key features
- logical operator is supported on a 1D string of particles
- logical operator is a tensor product of single-body unitaries

Stabilizer codes: quantum codes defined by a group of commuting Pauli ops.
→ Shor code, Kitaev's toric code, Bombin's color codes, etc.

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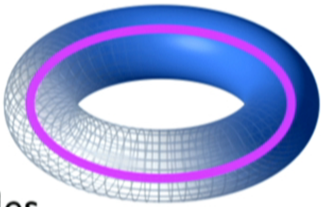

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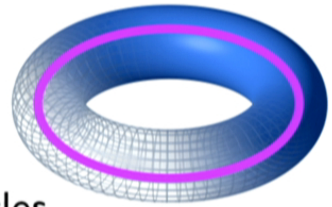

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$\Rightarrow \Omega$ g.s. of $H = -\sum_i g_i$

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- Canonical ex.
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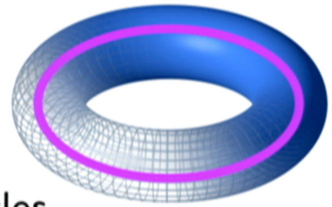

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General result for 2D topological stabilizer codes

Bravyi & Terhal.
New J. Phys. 11 (2009) 043029

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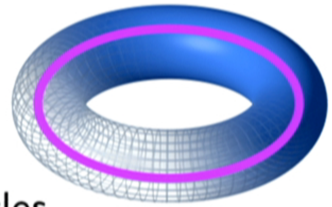

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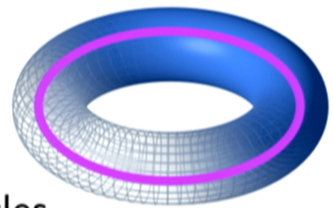

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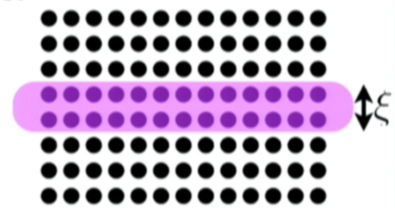


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# Local commuting projector codes (LCPCs)

N d-dim. spin particles (qudits) located on the vertices of a 2D lattice (V, E).

$$H = -\sum_{X \subset V} P_X$$

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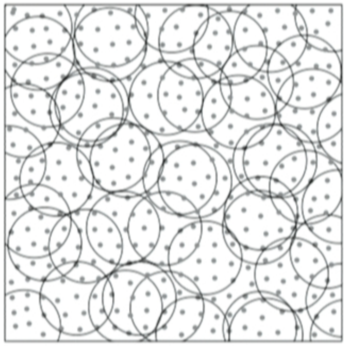

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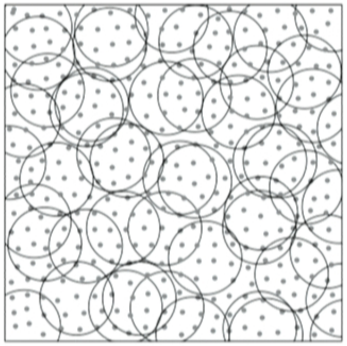

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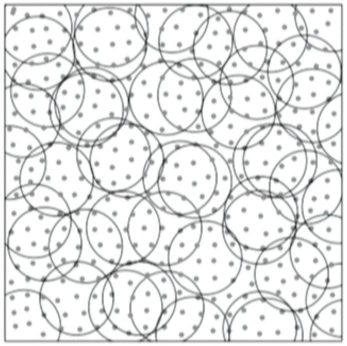

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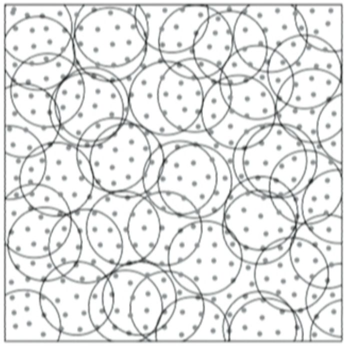

Include most topologically ordered models

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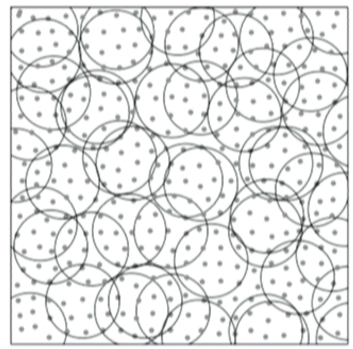

Include most topologically ordered models
- Kitaev toric code and quantum double models
- Topological color codes Bombin & Martin-Delgado. PRL **98**, 160502 (2007)
- Levin-Wen string net models Levin & Wen PRB **71** 045110 (2005)
- Turaev-Viro codes Koenig, Kuperberg, Reichardt. Ann Phys. **325**, 2707-2749 (2010)

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# Spectrum stability: local topological order

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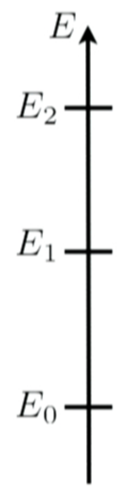


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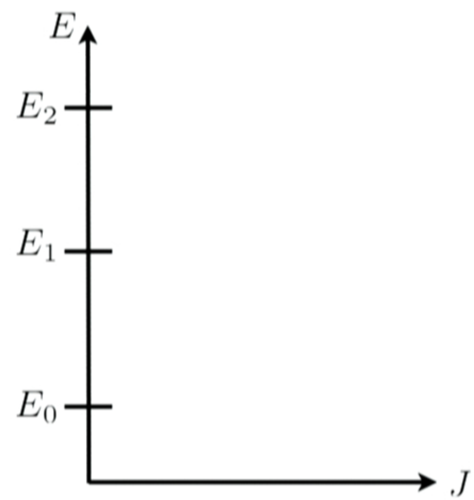


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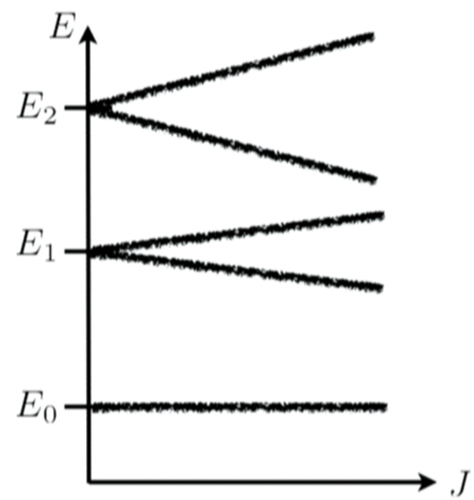
*Remark*: the op. norm of the perturbation grows with system size. $\|V\| = L^D \|V_X\|$

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2D LCPCs : logical operator supported on a 1D strip, but not a tensor product.

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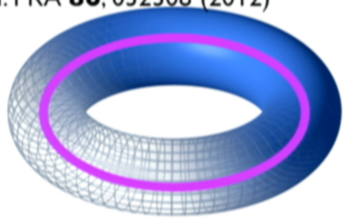

**Intuition**: excitations are point-like excitations which propagate without energy cost.

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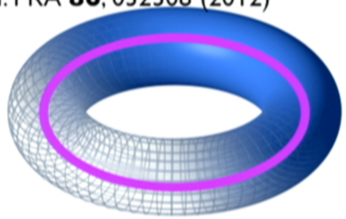

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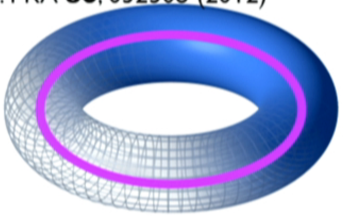

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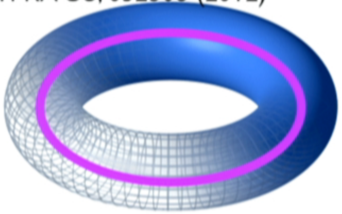

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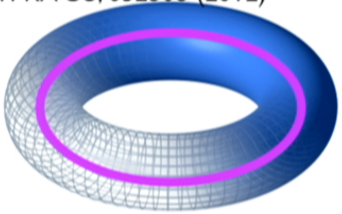

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Spectral stability $\cdots\cdots\rightarrow$ 2D topological Hamiltonian

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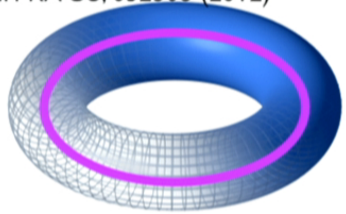

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Quasi-particles freely propagating $\longrightarrow$ Energy barrier independent of system size

Spectral stability $\cdots\cdots\rightarrow$ 2D topological Hamiltonian

**Quest for self-correcting memory**

Olivier Landon-Cardinal
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Self-correction
- Desiderata
- Canonical ex.
- Thermal stability

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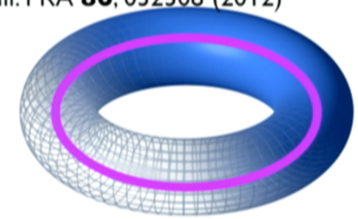

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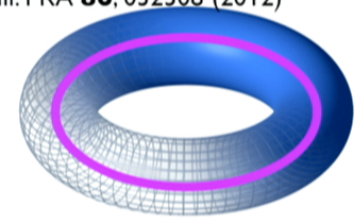

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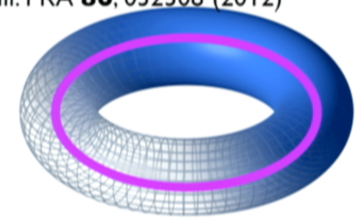

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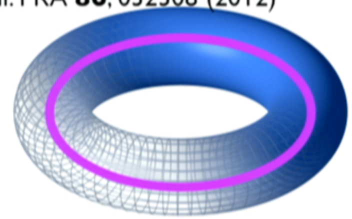

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Environment - applies random (unitary) tranformations
- checks energy locally

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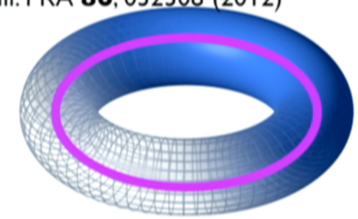

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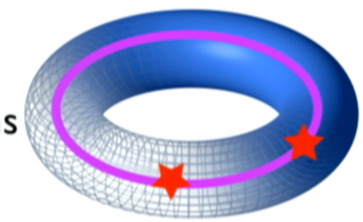

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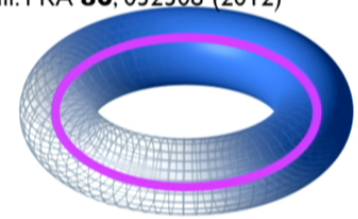

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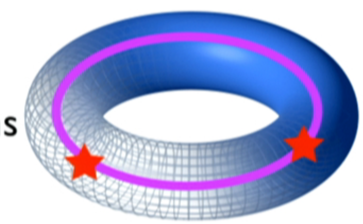

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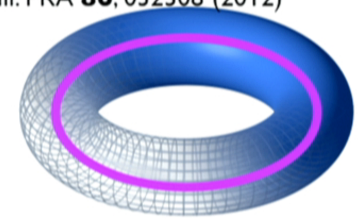

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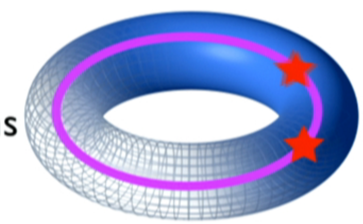

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2D *local topologically ordered* LCP code have an energy barrier independent of system size.

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Protection mechanisms other than energy barrier. → Entropy-protected 2D codes

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# *Active* error correction: quantum RAM

**Quest for self-correcting memory**

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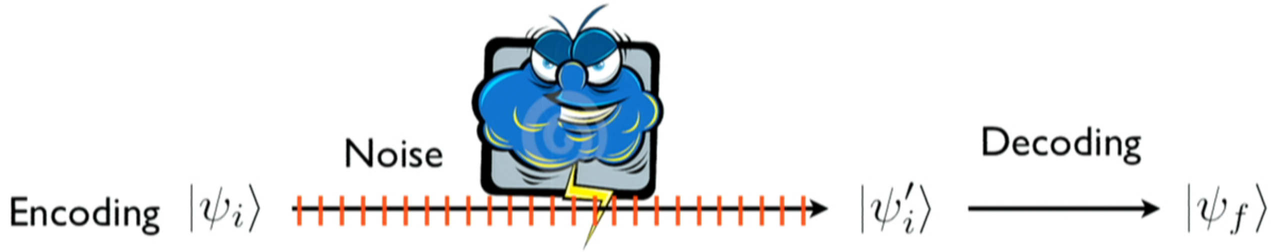


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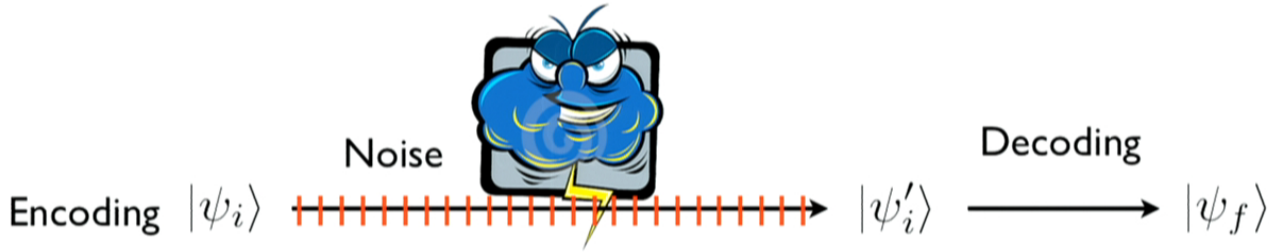


Existence of *threshold*
If noise level is low enough, perfect decoding for large system.



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# *Active* error correction: quantum RAM

Noise

Encoding $|\psi_i\rangle$ ⟶ $|\psi_i'\rangle$ $\xrightarrow{\text{Decoding}}$ $|\psi_f\rangle$

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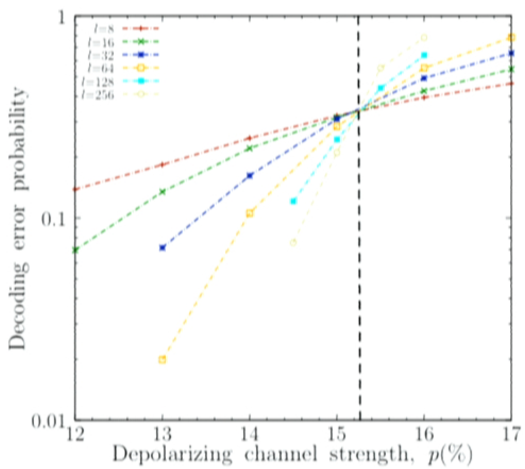


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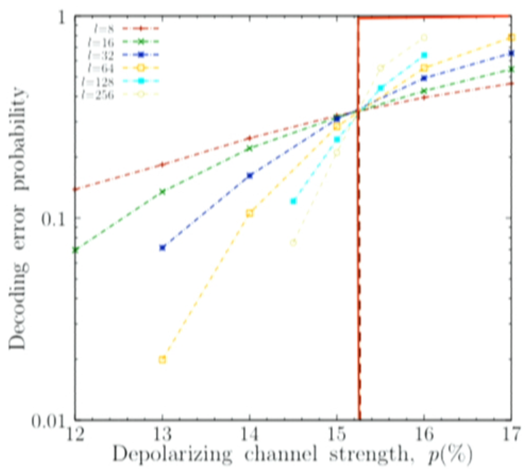


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Experimental friendly version of the toric code: surface codes

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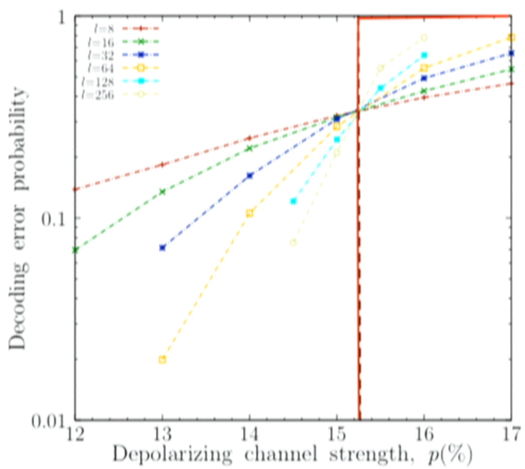


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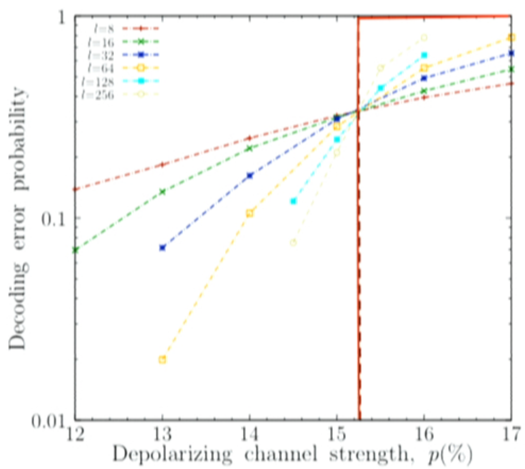


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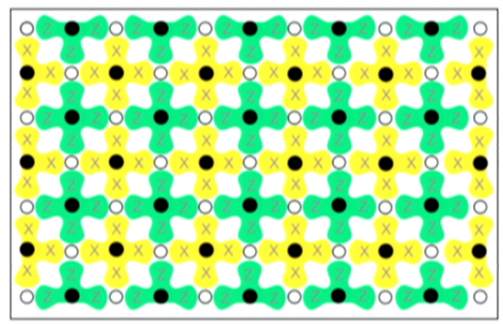

Fowler et al. PRA 86, 032324 (2012)

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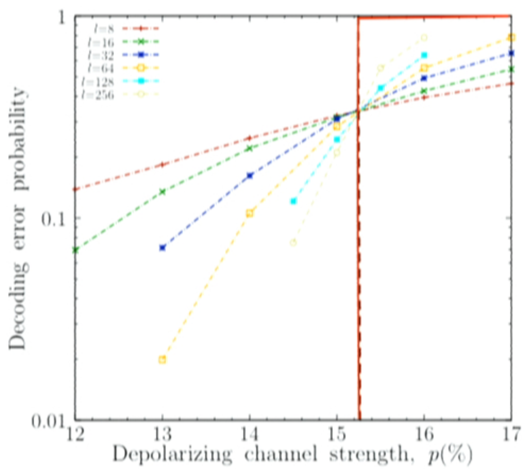


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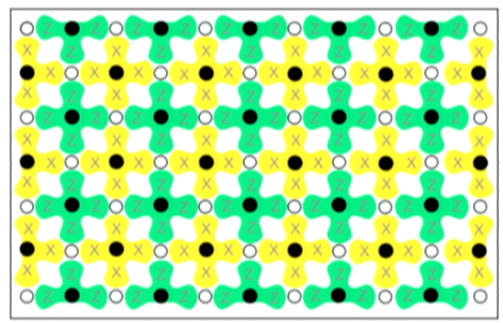

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**Not self-correction!**

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# 4D toric code: phase transition at finite T

Let's embed active error correction into the quantum system...

**Quest for self-correcting memory**

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**Pb**: classical post-processing is non-local.

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4D toric code

Dennis, Kitaev, Landahl, Preskill, J. Math. Phys., **43**, 4452 (2002)

1 qubit per plaquette

Edge operator $X^{\otimes 6}$

Cube operator $Z^{\otimes 6}$

Phase transition at non-zero T (strong criterion for self-correction)!

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4D local interactions $\longrightarrow$ long-range interactions in smaller D

$\mathcal{O}(1)$ $\qquad$ $L^{\frac{4-D}{D}}$

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# 3D topological systems: self-correction?

Structure of
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2D — point-like excitations, e.g., 2D toric code

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# 3D topological systems: self-correction?

Structure of excitations and logical operators

2D — point-like excitations, e.g., 2D toric code

3D — 3D toric code: point-like excitations

4D — no point-like excitations, e.g., 4D toric code

No-go for 3D   Beni Yoshida. Ann. Phys **326**, (2011) 15-95

**Quest for self-correcting memory**

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Self-correction
- Desiderata
- Canonical ex.
- Thermal stability

No-go results
- Stabilizer codes
- LCPCs

Alternatives
- Active QEC
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- 2D + long-range
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- local
- translation-invariant
- scale-invariant

stabilizer codes have point-like excitations.

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**Haah cubic code**    Haah. PRA **83**, 042330 (2011)

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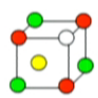
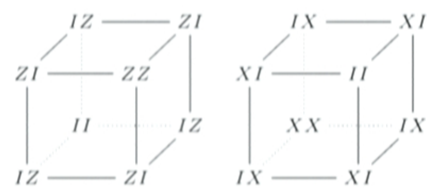
$H = -\frac{1}{2}\sum_{cubes}$  + 

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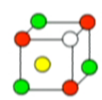
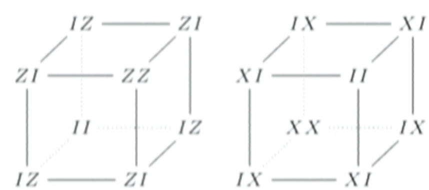
$H = -\frac{1}{2}\sum_{cubes}$  $+$ 




Self-correction?

Bravyi and Haah.
PRL **107**, 150504 (2011)

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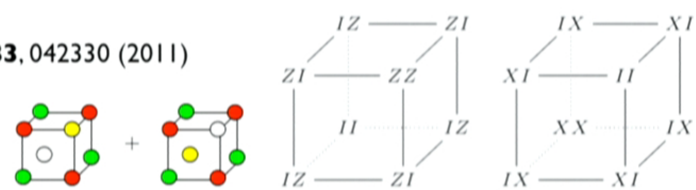
$H = -\frac{1}{2}\sum_{cubes}$ 


Self-correction? Energy barrier $\log L$

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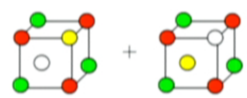
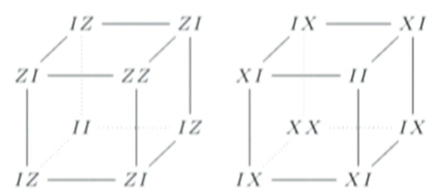
$H = -\frac{1}{2}\sum_{cubes}$  + 

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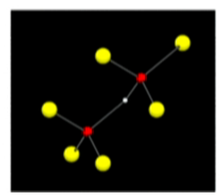
Fractal logical operator



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# 3D topological quantum spin glasses

## Numerical study

Bravyi and Haah.
PRL **111**, 200501 (2013)

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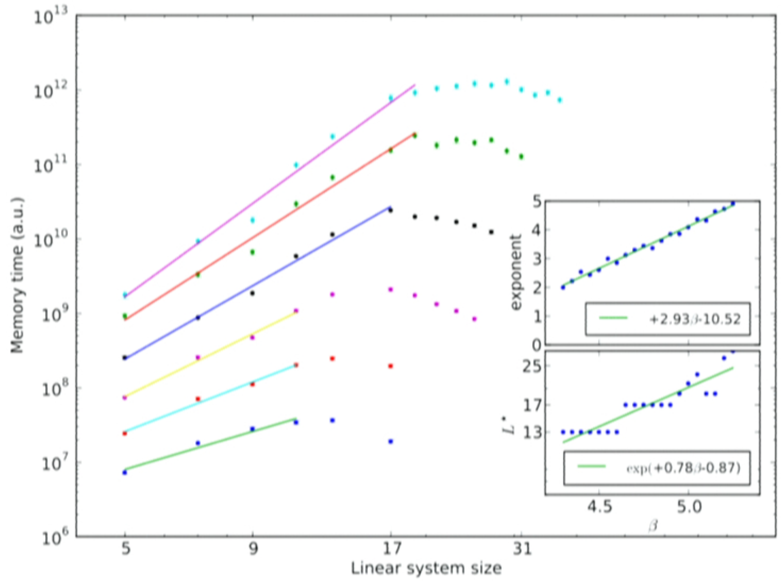


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for L below a critical size
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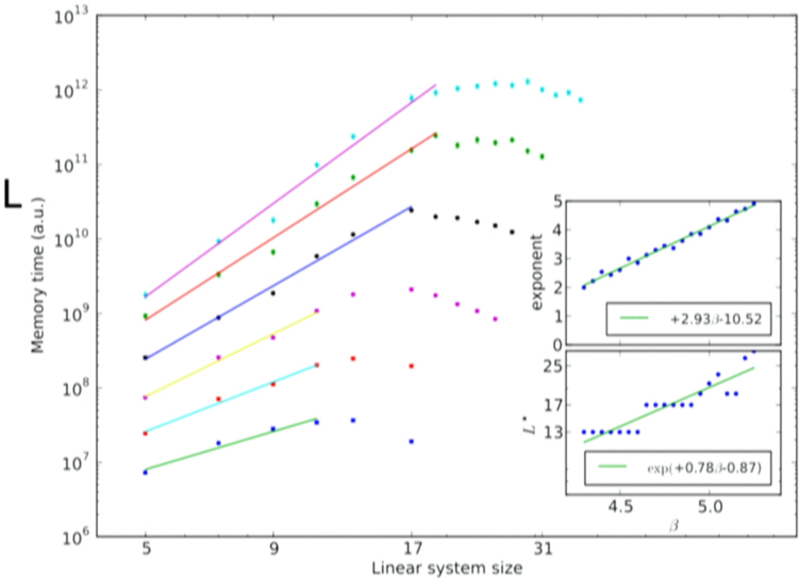


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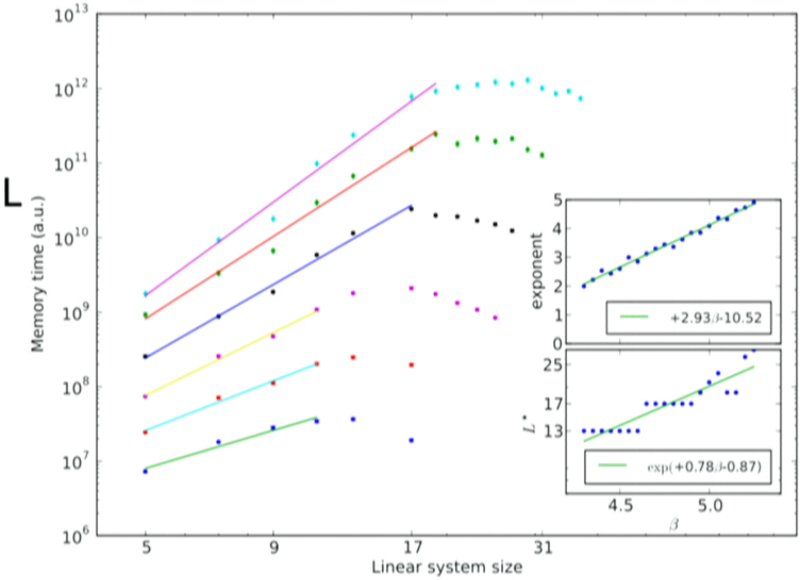


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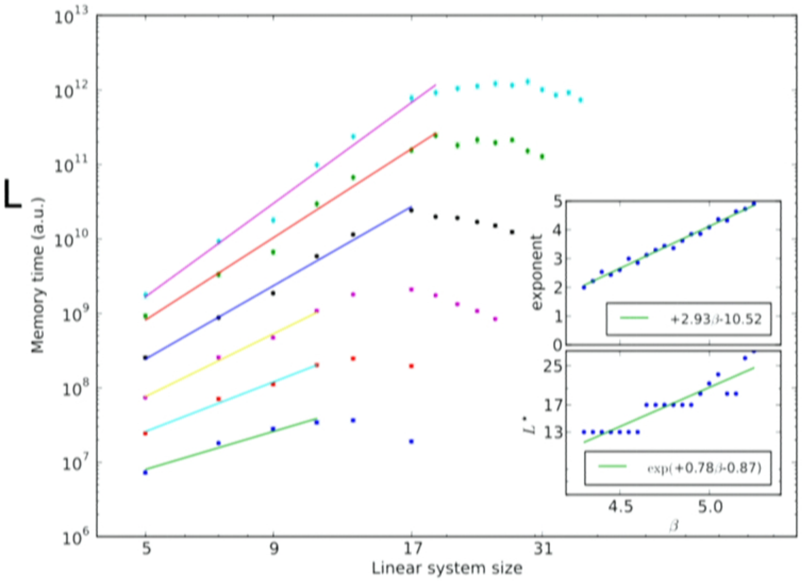


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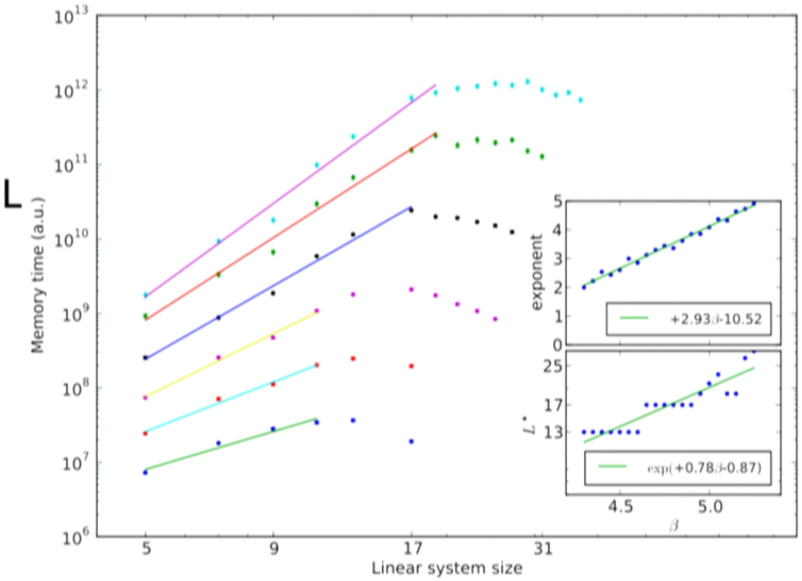
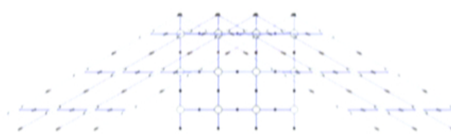



Other 3D proposals

Welded codes

K. Michnicki, arXiv:1208.3496.



$\Delta E^* \sim L^{2/3}$

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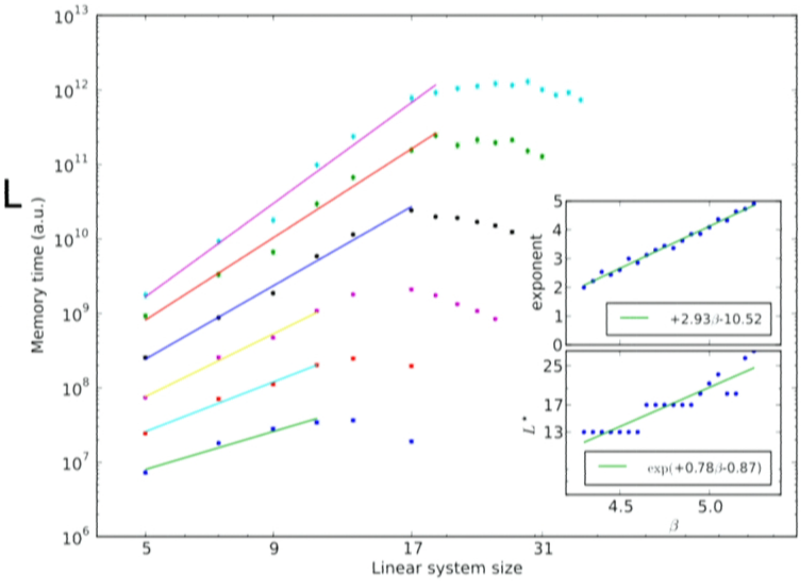


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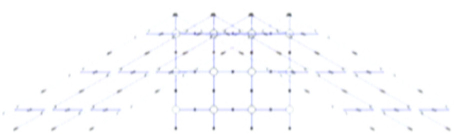

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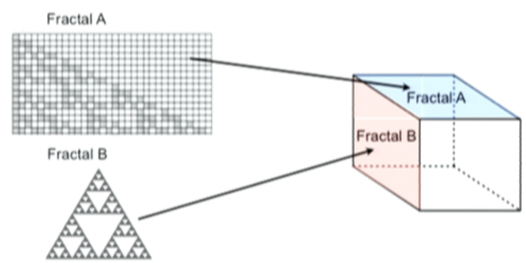
### Fractal spin liquids

Yoshida PRB **88**, 125122 (2013)
Annals of Physics **338**, 134 (2013)

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# Entropy protected codes

Fractal logical operators in 2D?

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# Entropy protected codes

Fractal logical operators in 2D?

Quantum double + defect lines

Brown, Al-Shimary, Pachos. arXiv:1307.6222

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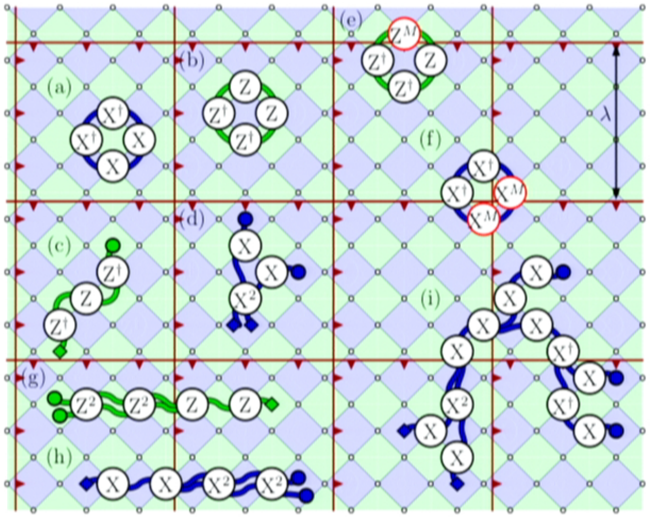

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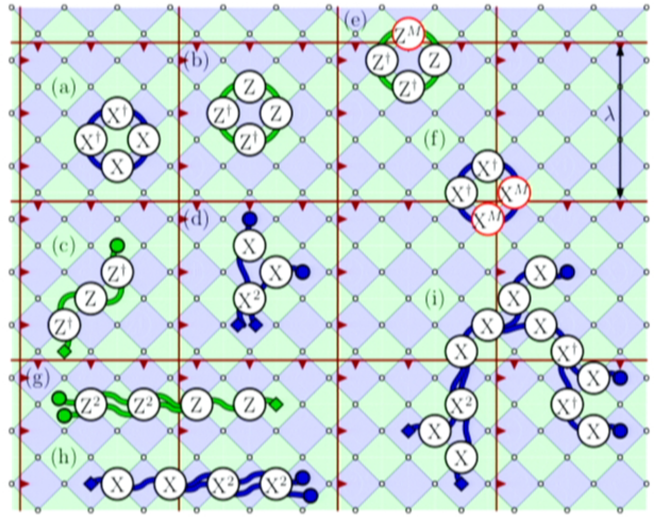


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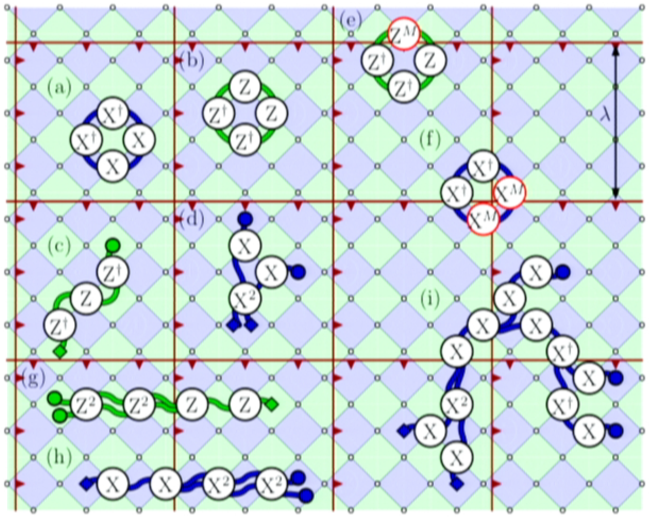


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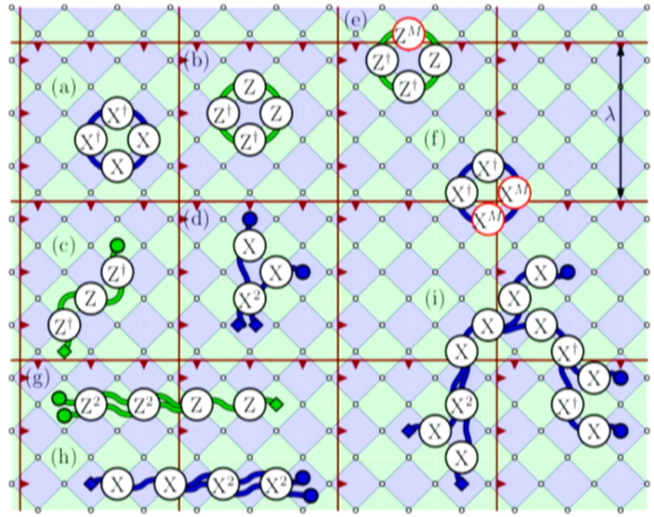
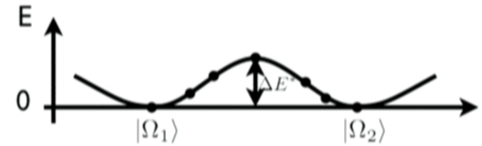



Criterion for self-correction: energy barrier

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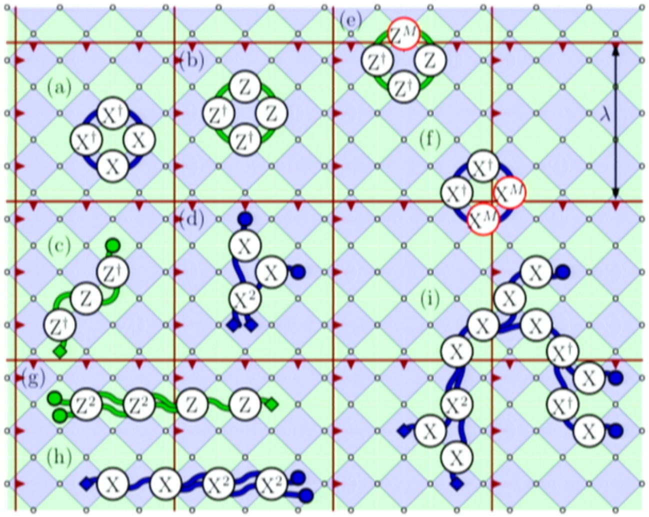


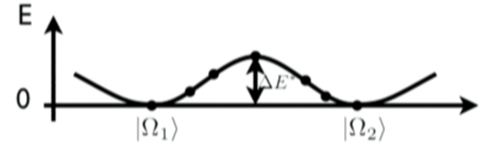


Criterion for self-correction: energy barrier

Non-zero T: minimization of free energy $F = E - TS$

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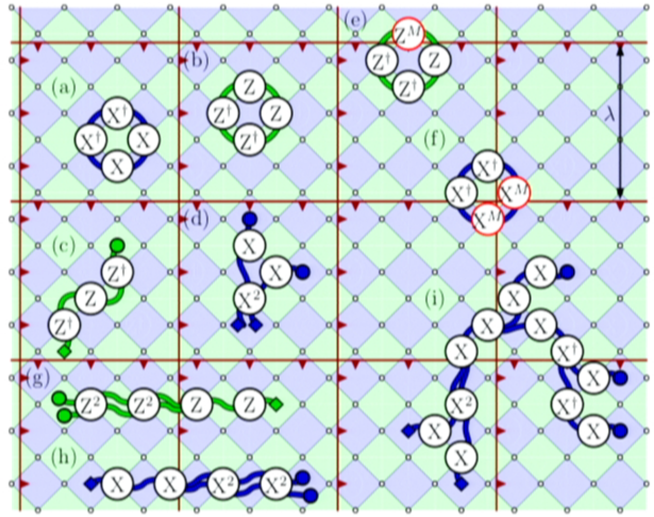

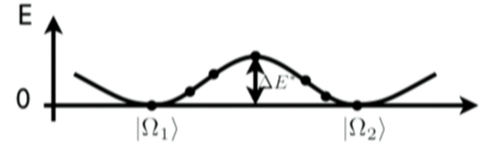


Criterion for self-correction: energy barrier

→ Non-zero T: minimization of free energy $F = E - TS$

→ Phase transition in Ising model

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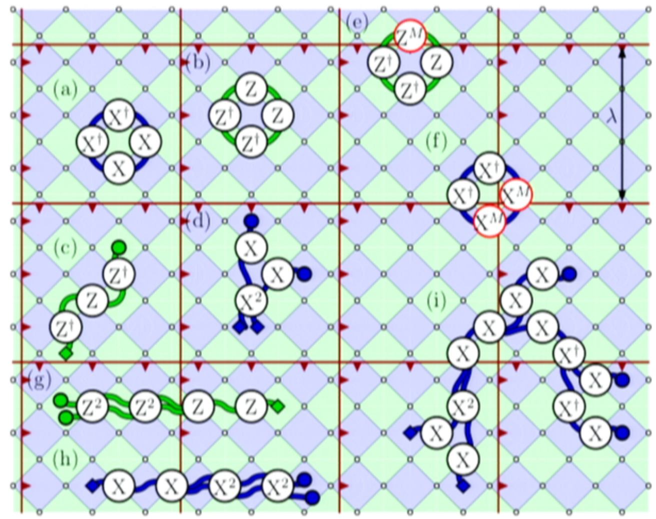

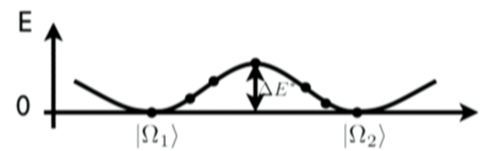


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Energy barrier $\propto L$
Available energy $\propto L^2$

Entropy protected topological codes?

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2D thermal instability: pt-like excitations propagating at no energy cost.

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# 2D models with long-range interactions

2D thermal instability: pt-like excitations propagating at no energy cost.

↪ Let's introduce long-range interaction between quasi-particles.

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2D thermal instability: pt-like excitations propagating at no energy cost.

↪ Let's introduce long-range interaction between quasi-particles.

↪ effective interaction

### Toric-boson model

Hamma, Castelnovo, Chamon.
PRB **79**, 245122 (2009)

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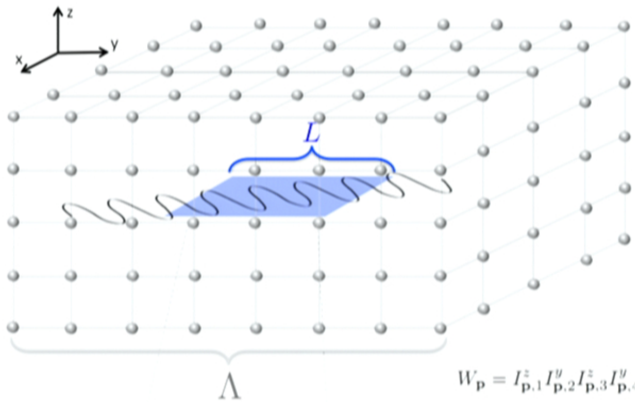
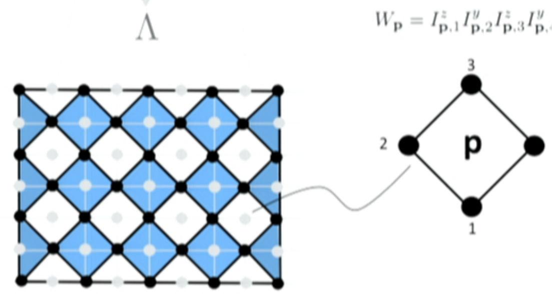
## Repulsive interaction

Chesi, Röthlisberger, Loss. PRA **82**, 022305 (2010)

## Coupling to free bosons

Pedrocchi, Hutter, Wootton, Loss.
PRA **88**, 062313 (2013)

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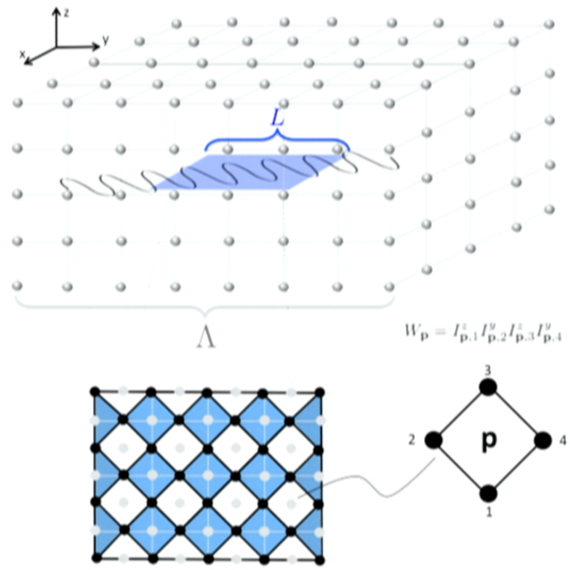


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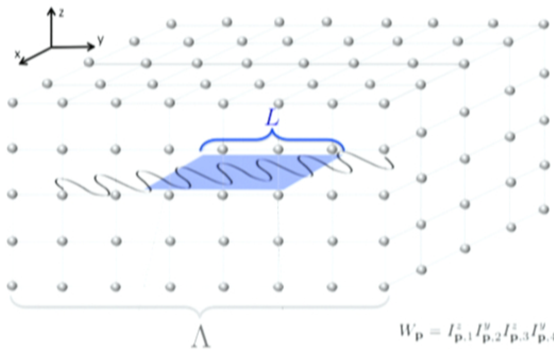


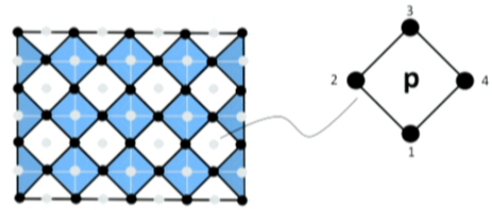


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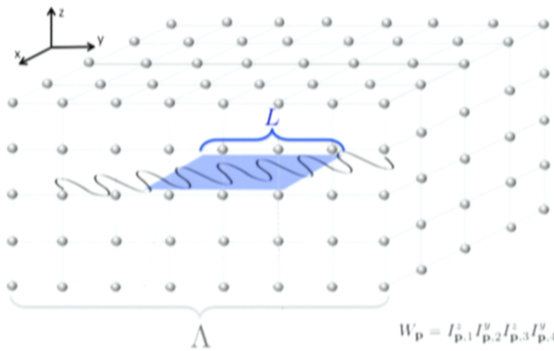


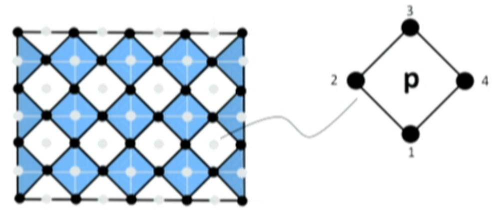


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$$H_W = - \sum_{p \neq p'} \frac{A^2}{4\pi t |R_p - R_{p'}|}$$

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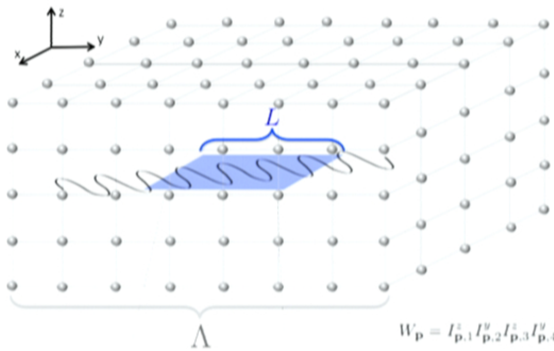


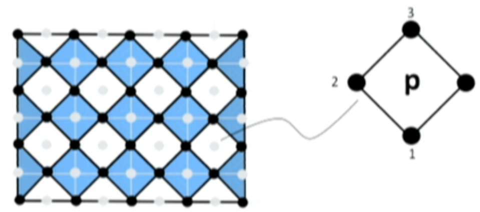


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Energy cost to add an anyon to the vaccum $\langle a_p + a_p^\dagger \rangle \sim \frac{A^2}{t} L$

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Field eq. = Poisson eq. with source term $\nabla^2 \phi = -w(x)$

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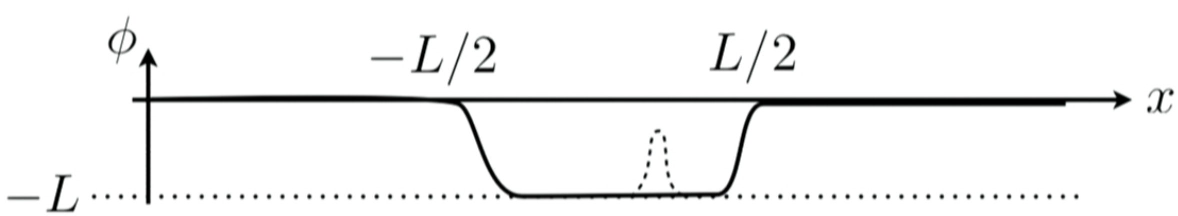


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$$Q[\theta] = \theta^2 \qquad \frac{d\langle \theta \rangle_\varepsilon}{d\varepsilon} = -2\langle \theta \rangle_\varepsilon \left( \langle \theta^2 \rangle_\varepsilon - \langle \theta \rangle_\varepsilon^2 \right)$$

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+Gaussian approx.

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**Quest for self-correcting memory**

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- Desiderata
- Canonical ex.
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# Perturbation gives mass to particles

Lattice model

$$H_\varepsilon = \epsilon_0 \sum_i a_i^\dagger a_i - t \sum_{\langle i,j\rangle} a_i^\dagger a_j \boxed{+\varepsilon \sum_i a_i^\dagger a_i}$$

Field theoretic model

$$H_\varepsilon = \int d^D x \left( \frac{1}{2}(\nabla\phi)^2 - w(x)\phi(x) + \varepsilon\phi^2 \right)$$

High occupation of bosonic modes is energetically penalized
➡Effective cutoff

Bosons become massive.
➡Interaction becomes short-ranged

Proof for Pedrocchi model $\theta_p \equiv a_p + a_p^\dagger$ $\langle \theta_p \rangle = \mathrm{Tr}\left[e^{-H}\theta_p\right]/Z \sim L$

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$Q[\theta] = \theta^4$

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$Q[\theta] = \theta^4$ $\quad \langle \theta \rangle_\varepsilon \simeq \frac{\langle \theta \rangle_0}{\sqrt{1 + 8\epsilon V \langle \theta \rangle_0^2}}$

Energy barrier is unstable under perturbation.

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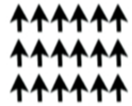

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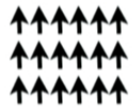

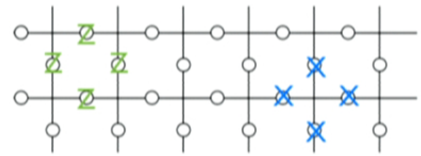

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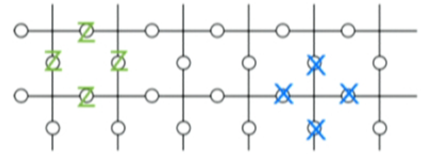

No-go results in 2D for

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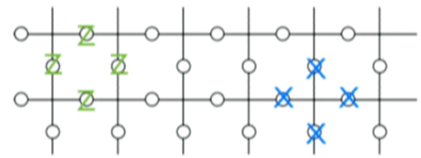

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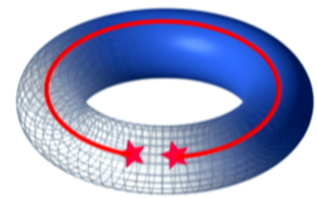

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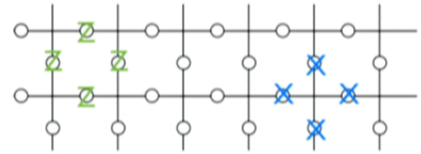

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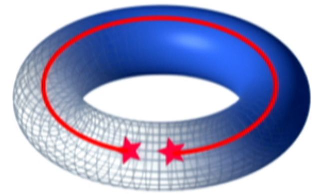

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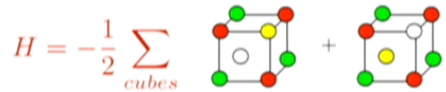


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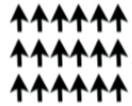

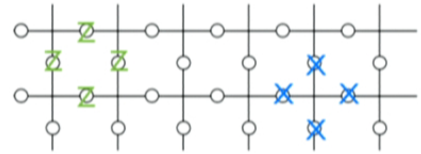

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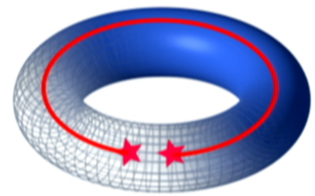

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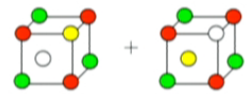
$$H = -\frac{1}{2} \sum_{cubes}$$



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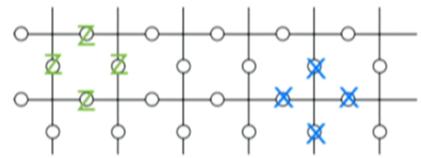

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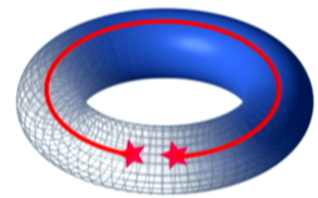

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New insight into emergence of anyons from microscopic Hamiltonian.

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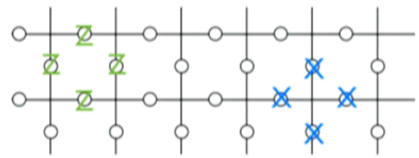

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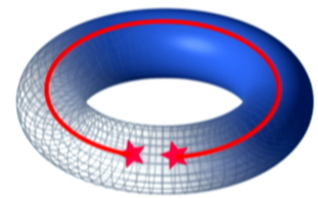

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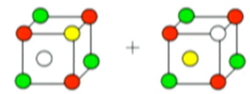
$H = -\frac{1}{2}\sum_{cubes}$ 

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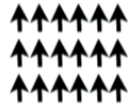

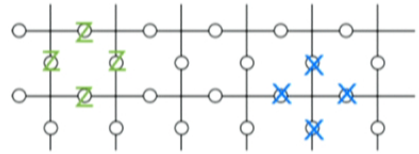

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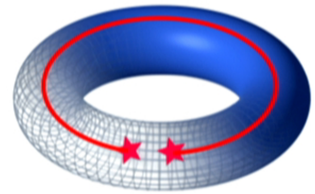

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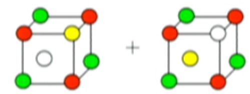

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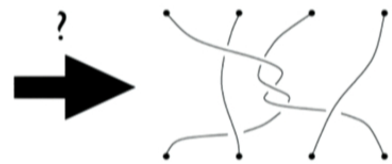
Anyon model

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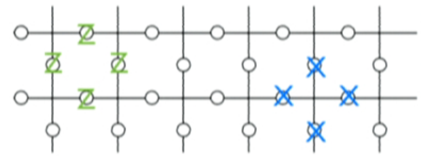

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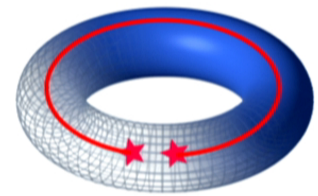

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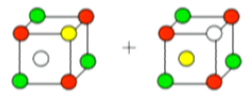
$$H = -\frac{1}{2}\sum_{cubes}$$ 

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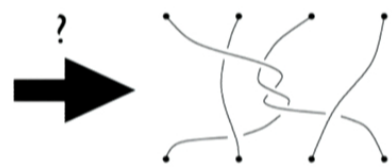

Contributions of quantum information to theoretical physics...

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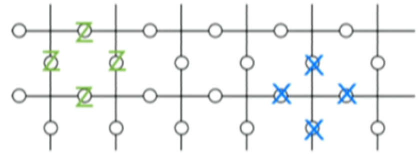

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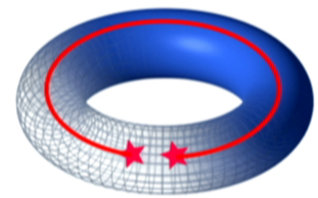

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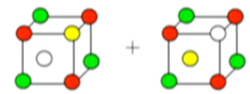

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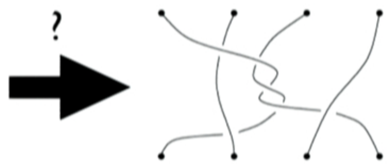

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