

Title: Last gasp of a black hole: why unitary evaporation must be non-monotonic

Date: May 29, 2014 02:30 PM

URL: <http://pirsa.org/14050027>

Abstract: I will describe the relationship between radiated energy and entanglement entropy of massless fields at future null infinity (the "Page curve") in two-dimensional models of black hole evaporation. I will use this connection to derive a general feature of any unitary-preserving evaporation scenario: the Bondi mass of the hole must be non-monotonic. Time permitting, I will comment on time scales in such scenarios.

LAST GASP OF A BH

1404.0602

1405.5235

+ ---

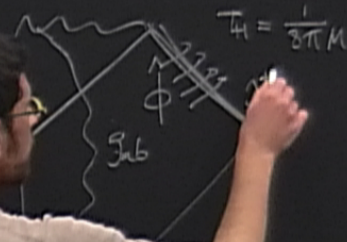
- 1) Entanglement in 2d.
- 2) Entropy and energy (at $\mathcal{I}^+$)
- 3) Time scale of evaporation.

LAST GASP OF A BH

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- 1) Entanglement entropy in 2d.
- 2) Entropy and energy (at t)
- 3) Time scales in BH evaporation



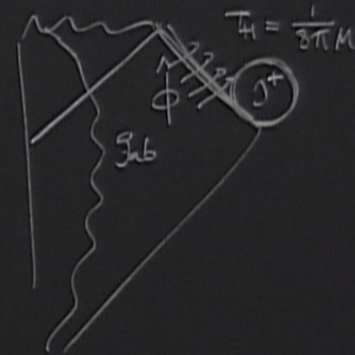
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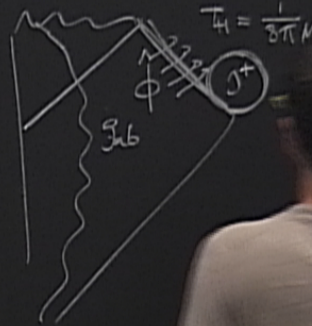
- 1) Entanglement entropy in 2d.
- 2) Entropy and energy (at $\mathcal{I}^+$)
- 3) Time scales in BH evaporation.



LAST GASP OF A BH

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- 1) Entanglement entropy in 2d.
- 2) Entropy and energy (at $\mathcal{I}^+$)
- 3) Time scales in BH evaporation

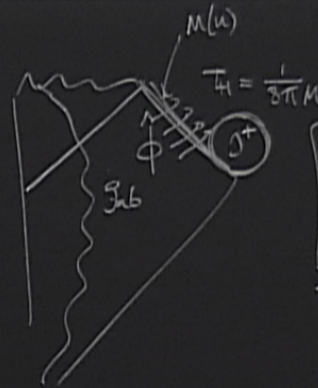


2: If evaporation unitary
then

LAST GASP OF A BH

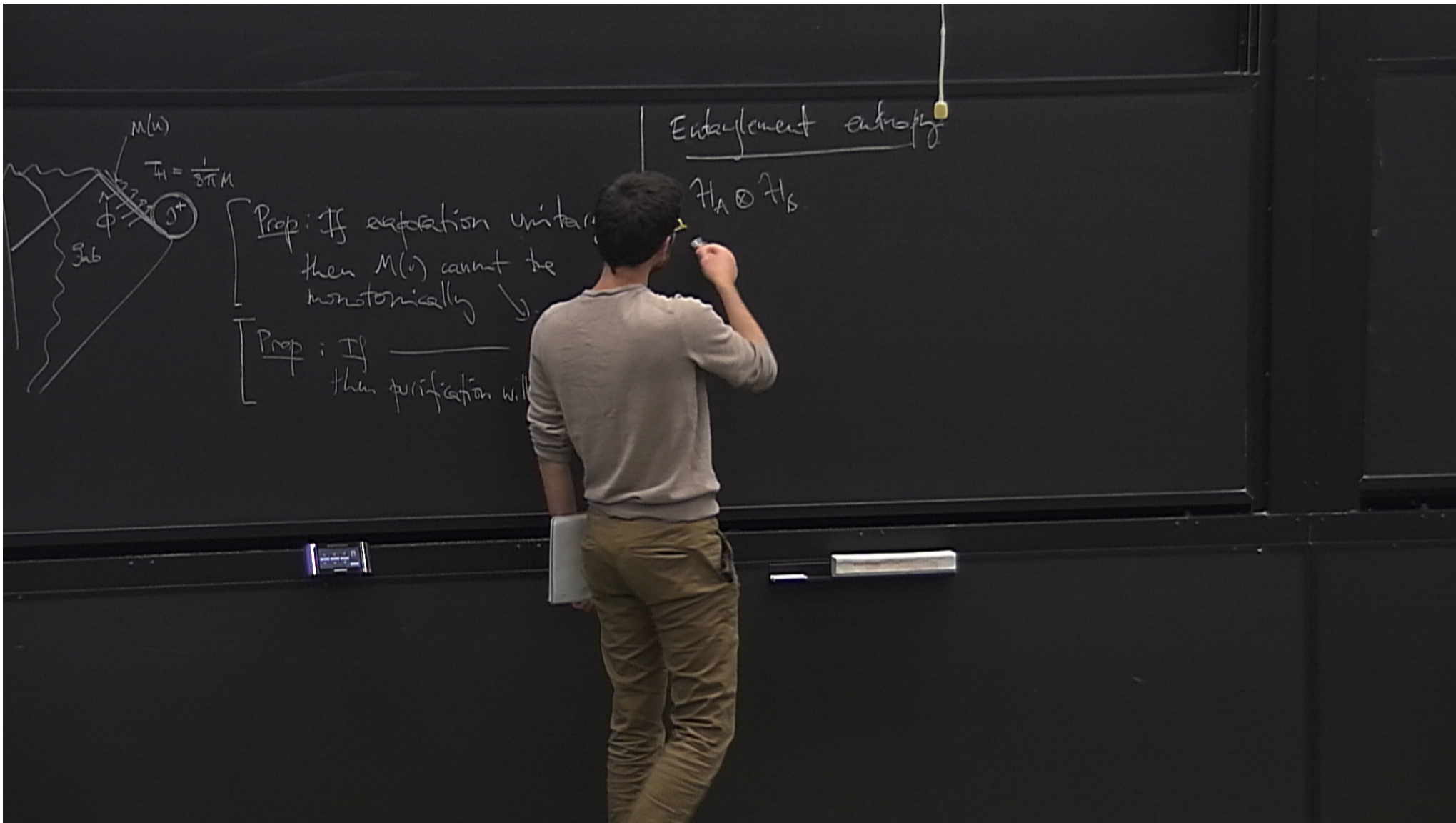
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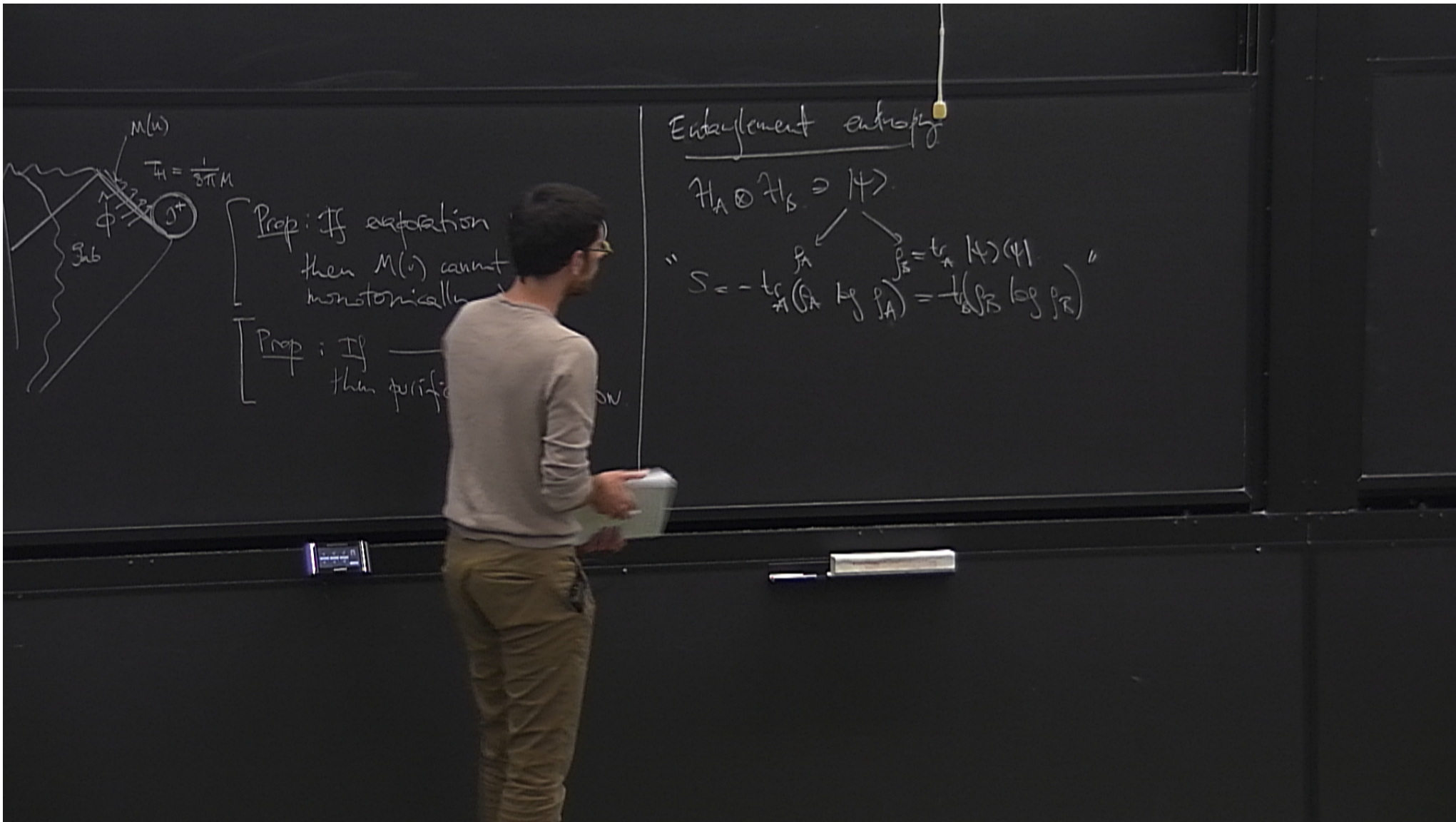
- 1) Entanglement entropy in
- 2) Entropy and energy
- 3) Time scales in BH formation

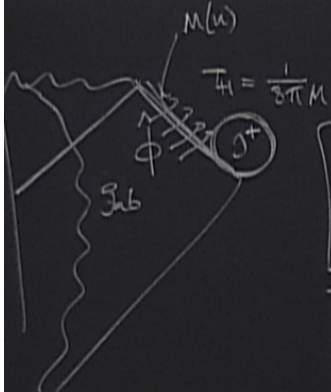


Prop: If evaporation unitary
then $M(u)$ cannot be
monotonically $\downarrow$.

Prop







$$T_H = \frac{1}{8\pi M}$$

Prop: If ρ is a stationary state then $M(u)$ is monotonic

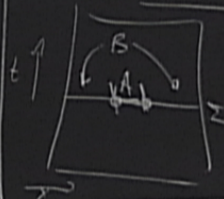
Prop: If ρ is a stationary state then $M(u)$ will be slow

Entanglement entropy

$$\mathcal{H}_A \otimes \mathcal{H}_B \ni |\psi\rangle$$

$$S = -\text{tr}_A(\rho_A \log \rho_A) = -\text{tr}_B(\rho_B \log \rho_B)$$

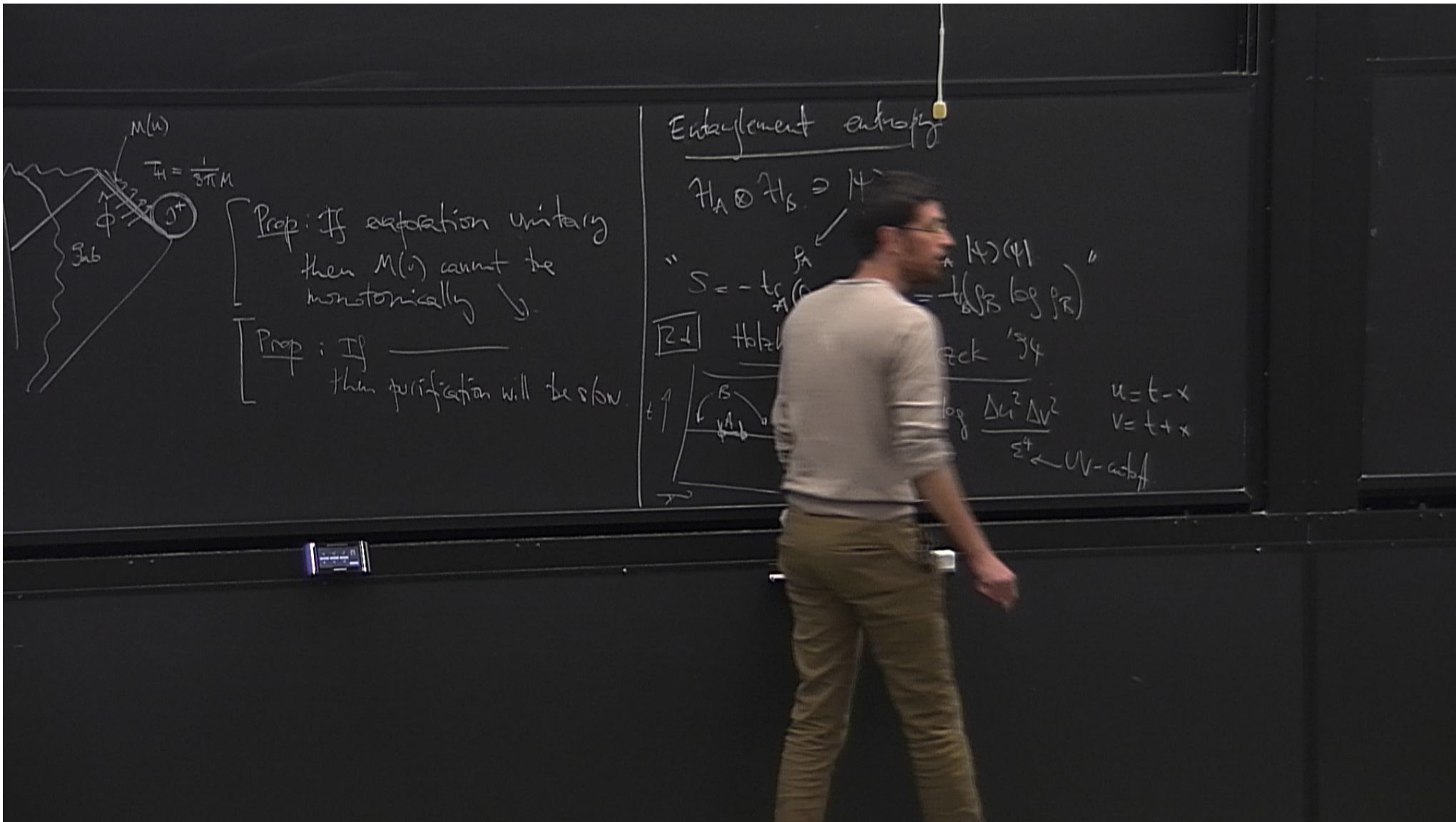
[24] Holography, Maldacena, Wilczek '94

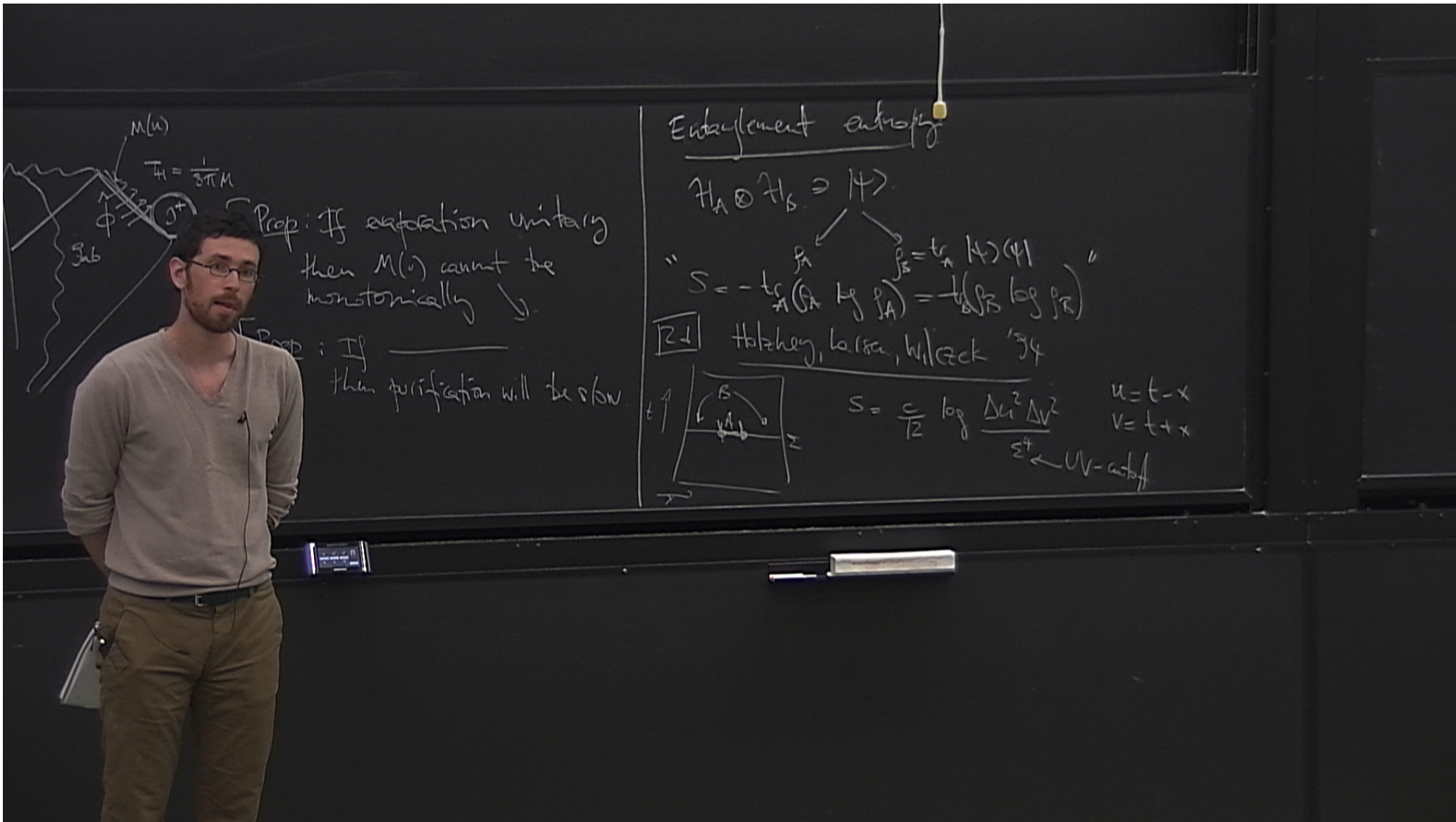


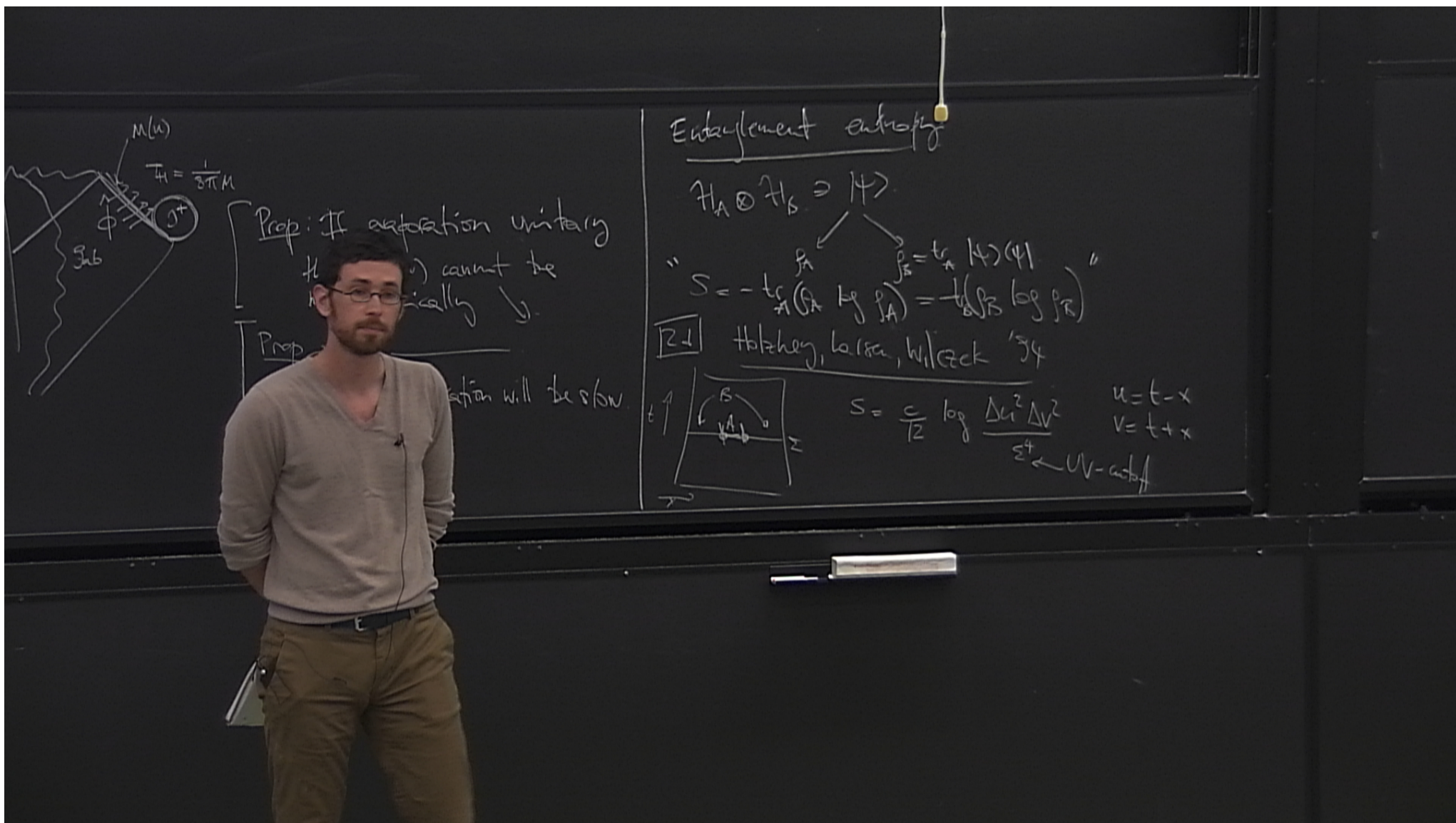
$$S = \frac{c}{12} \log \frac{\Delta u^2 \Delta v^2}{\epsilon^4}$$

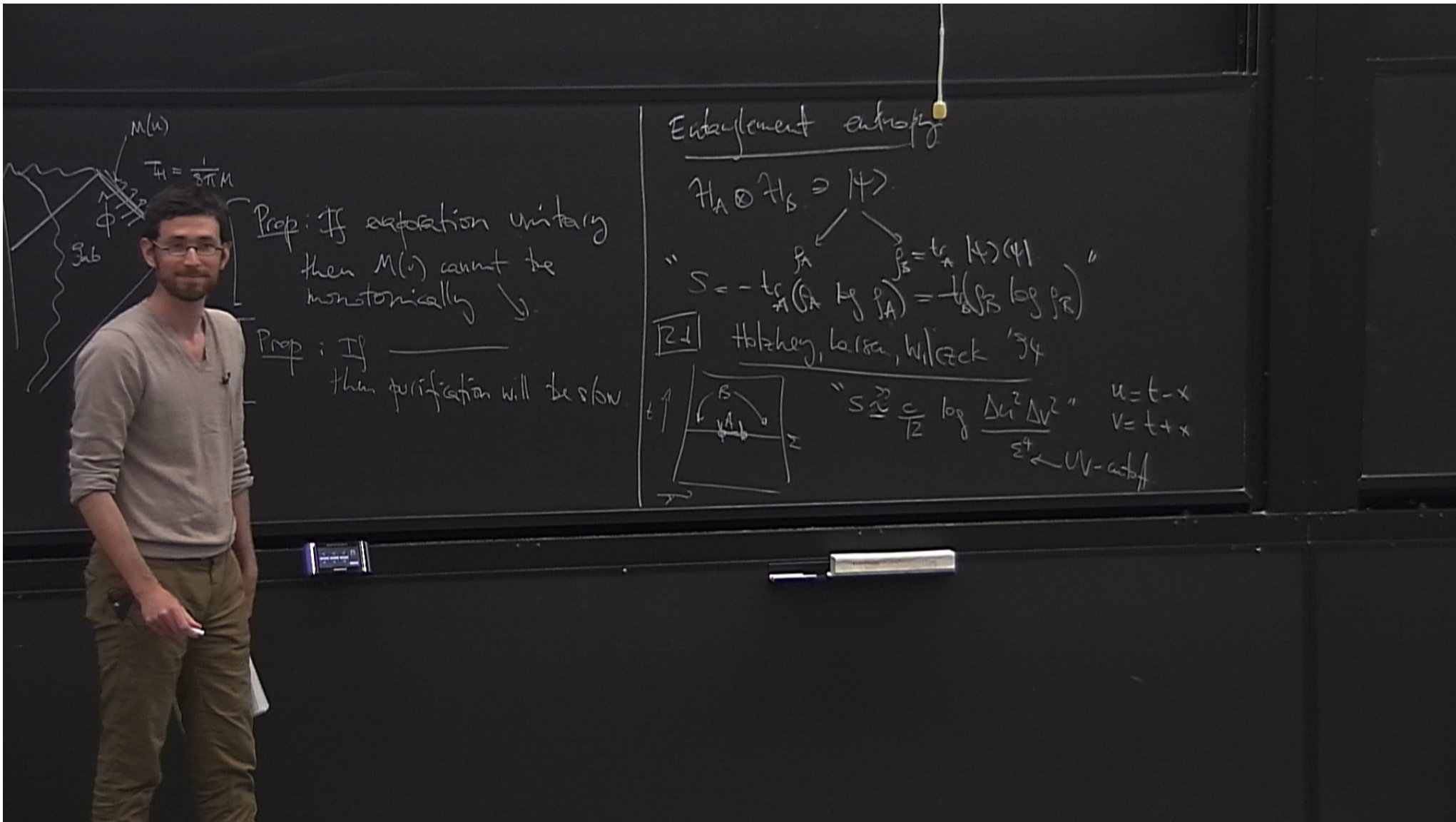
$$u = t - x$$

$$v = t + x$$











- 1) Express $S(D)$
- 2) Cutoff : Lorentz-invariant.

$\log p_R)$
 Σ^4
 ΔV^2
 $u = t - x$
 $v = t + x$
 $\Sigma^4 \leftarrow UV\text{-cutoff}$

operation unitary
 $M(\cdot)$ cannot be
 canonically $\downarrow$

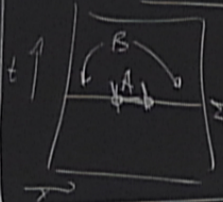
purification will be slow

Entanglement entropy

$$\mathcal{H}_A \otimes \mathcal{H}_B \ni |\Psi\rangle$$

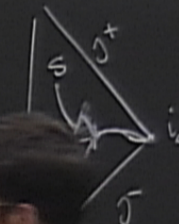
$$S = -\text{tr}_A(\rho_A \log \rho_A) = -\text{tr}_B(\rho_B \log \rho_B) = \frac{1}{2} I(A:\bar{A})$$

[21] Holzhey, Larsen, Wilczek '94

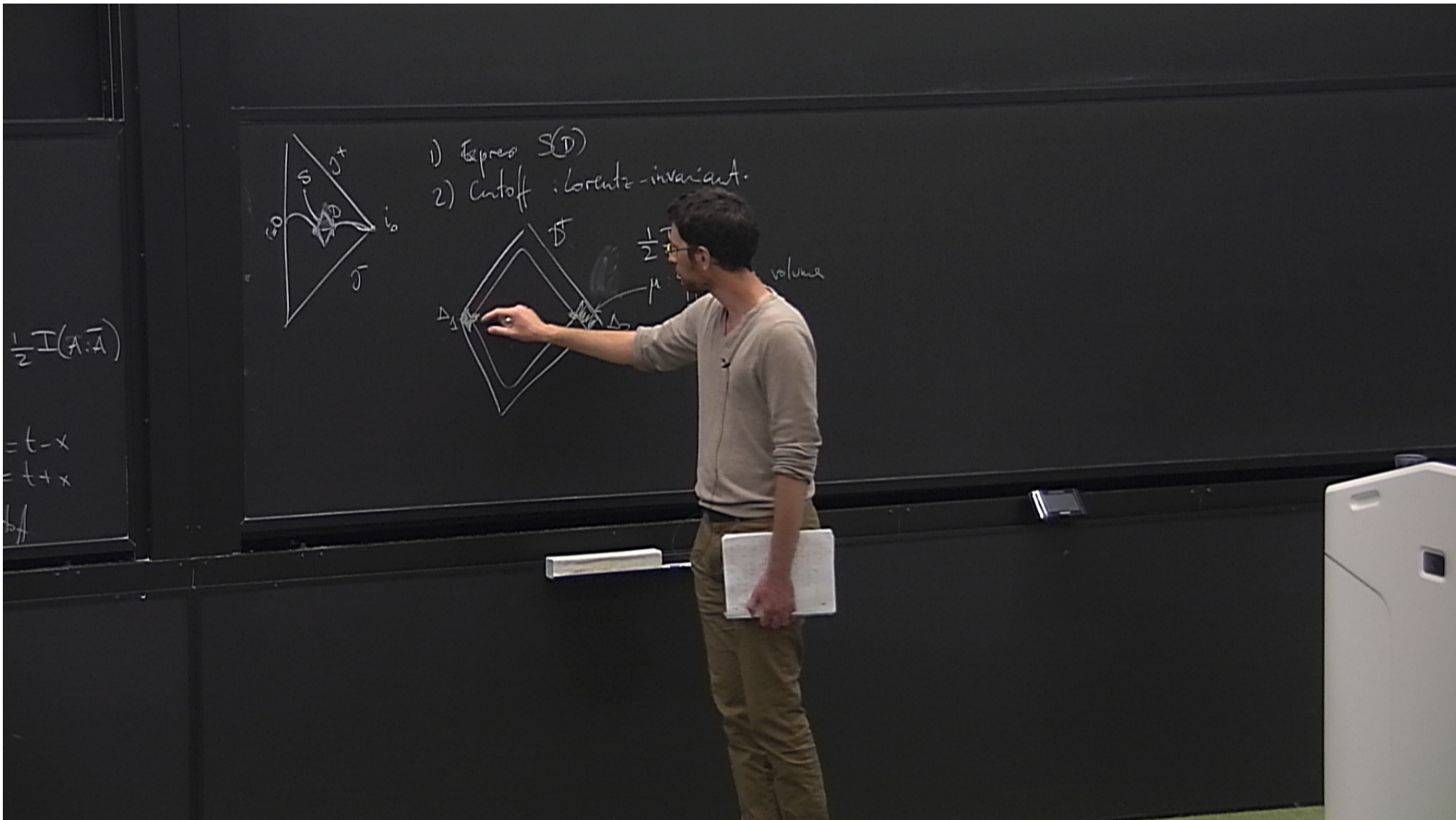


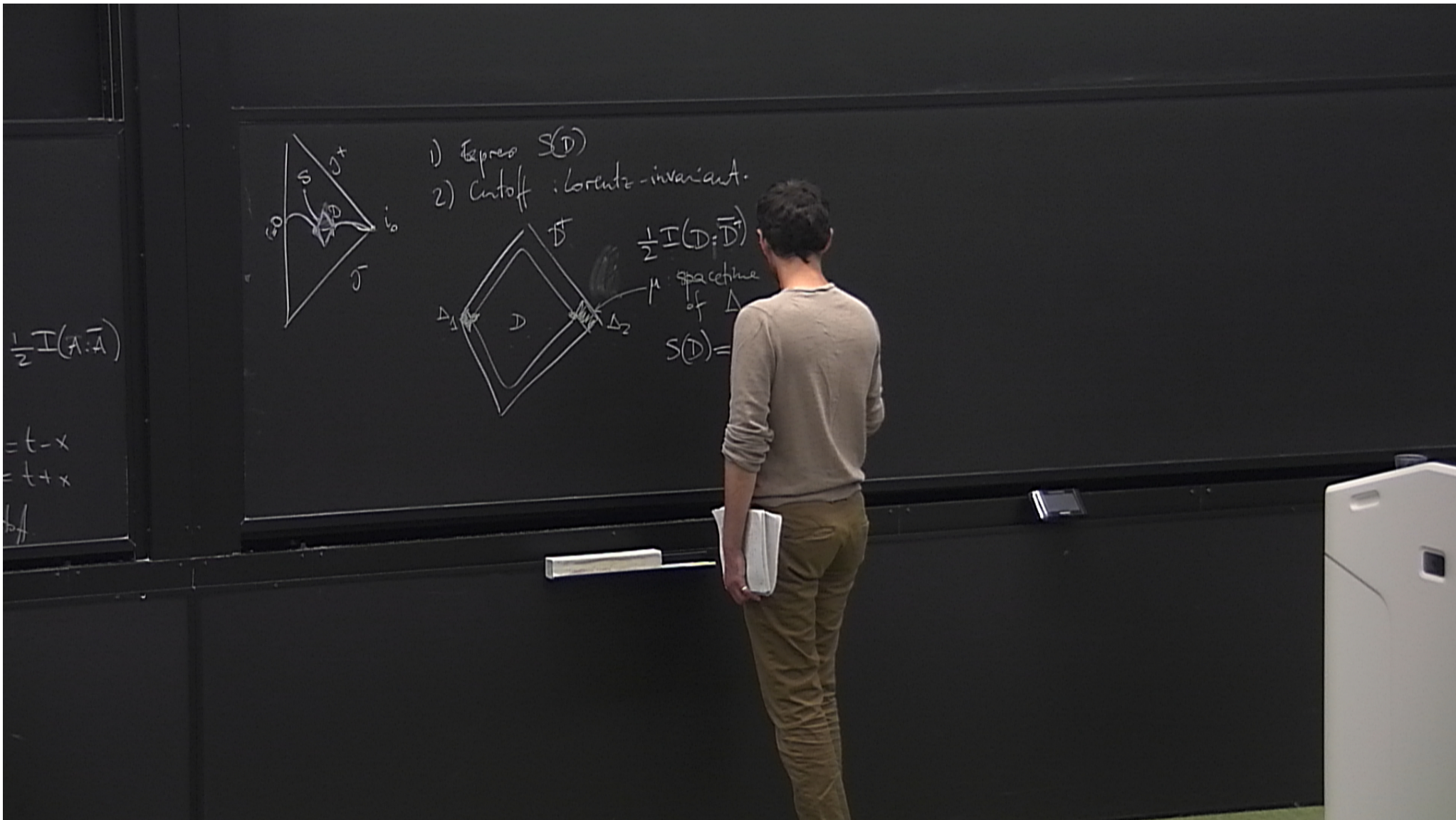
$$S \sim \frac{c}{12} \log \frac{\Delta u^2 \Delta v^2}{\epsilon^4}$$

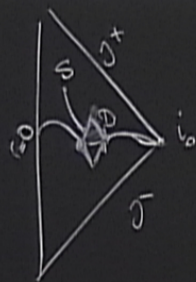
$u = t - x$
 $v = t + x$
 $\epsilon^4 \leftarrow UV\text{-cutoff}$



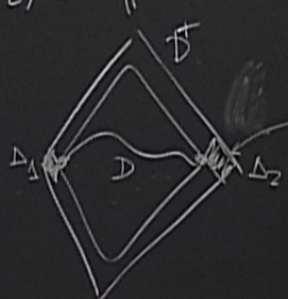
- 1) Express $S(D)$
- 2) Cutoff: Lorentz-invar







- 1) Express $S(D)$
- 2) Cutoff : Lorentz-invariant.



$$\frac{1}{2} I(D; \bar{D})$$

μ : spacetime volume of Δ

$$S(D) = \frac{1}{2} I(D; \bar{D})$$

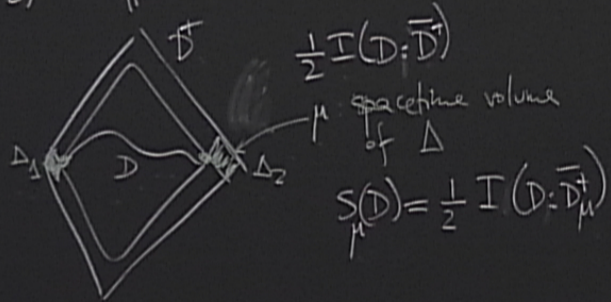
$$\frac{1}{2} I(A; \bar{A})$$

$$= t - x$$

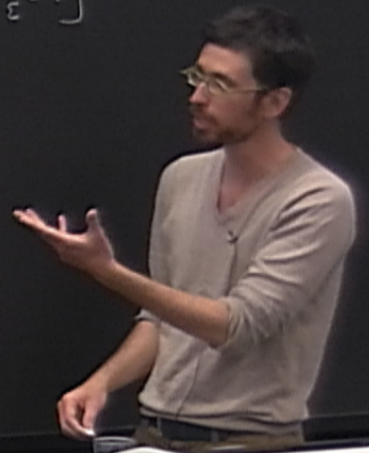
$$= t + x$$



- 1) Express $S(D)$
- 2) Cutoff : Lorentz-invariant.



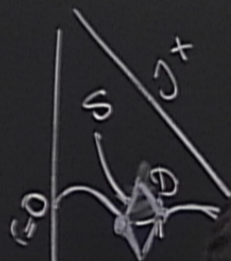
$$S_\mu(D) = \frac{1}{2} I(D; \bar{D}_\mu^+) = \frac{1}{2} \left[S_\varepsilon(D) + S_\varepsilon(\bar{D}_\mu^+) - S_\varepsilon(\Delta) \right]$$



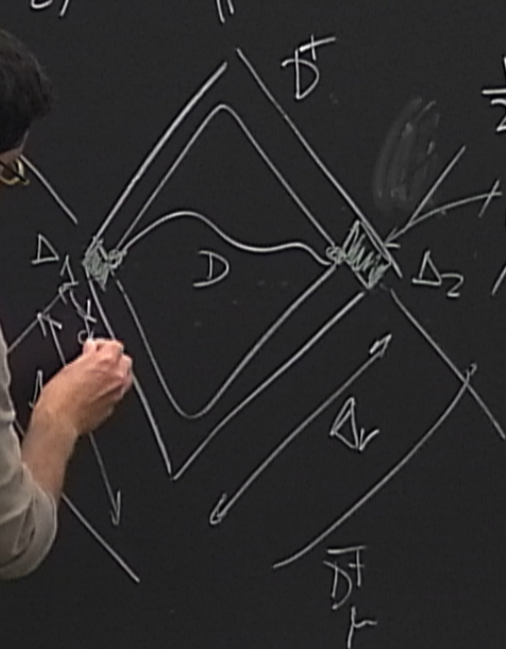


$$S_{\mu}(D) = \frac{1}{2} I(D; \overline{D}_{\mu}^+) = \frac{1}{2} \left[S_{\varepsilon}(D) + S_{\varepsilon}(D_{\mu}^+) - S_{\varepsilon}(\Delta) \right]$$

$\swarrow$ $\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} |_D$ $\downarrow$ $\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} |_{D_{\mu}^+}$ $\searrow$



- 1) Express $S(D)$
- 2) Cutoff : Lorentz-invariant.



$$\frac{1}{2} I(D; \bar{D}^+)$$

μ : spacetime volume of Δ

$$S(D) = \frac{1}{2} I(D; \bar{D}_\mu^+)$$

$$S_\mu(D) = \frac{1}{2} I(D; \bar{D}_\mu^+)$$

$$S_{\mu}(D) = -T(D; \overline{D}_{\mu}^{+}) = \frac{1}{2} \left[S_{\varepsilon}(D) + S_{\varepsilon}(D_{\mu}^{+}) - S_{\varepsilon}(\Delta) \right]$$

$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} |D| \quad \log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} |D_{\mu}^{+}| \quad \log \frac{\delta u^2 \delta v^2}{\varepsilon^4} |\Delta_1| + \log \frac{\delta u^2 \delta v^2}{\varepsilon^4} |\Delta_2|$$

$$S_{\mu}(D) = \frac{1}{2} I(D; \overline{D}_{\mu}^{+}) = \frac{1}{2} \left[S_{\varepsilon}(D) + S_{\varepsilon}(D_{\mu}^{+}) - S_{\varepsilon}(\Delta) \right]$$

$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} |_D$$

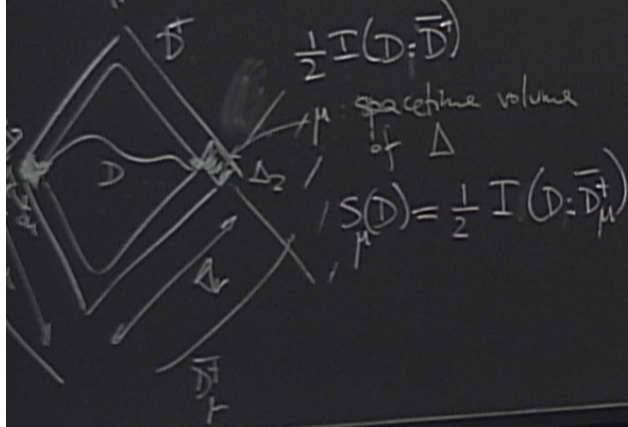
$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} |_{D_{\mu}^{+}}$$

$$- \frac{v^2}{\varepsilon^4} |_{\Delta_1} + \log \frac{\delta u^2 \delta v^2}{\varepsilon^4} |_{\Delta_2}$$

$$S_{\mu}(D) = \frac{c}{12} \log \frac{\Delta u^4 \Delta v^4}{\delta u_1^2 \delta v_1^2 \delta u_2^2 \delta v_2^2}$$

↑
μ small

Express $S(D)$
 Cutoff: Lorentz-invariant.



$$S_\mu(D) = \frac{1}{2} I(D; \bar{D}_\mu^+) = \frac{1}{2} \left[S_\varepsilon(D) + S_\varepsilon(D_\mu^+) - S_\varepsilon(\Delta) \right]$$

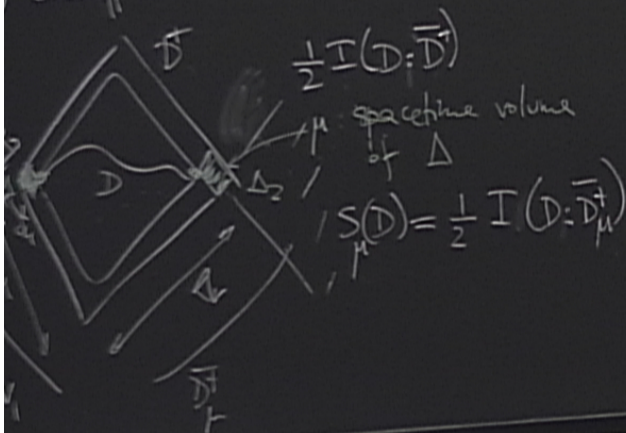
$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} \Big|_D$$

$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} \Big|_{\Delta_+}$$

$$\log \frac{\delta u^2 \delta v^2}{\varepsilon^4} \Big|_{\Delta_+} + \log \frac{\delta u^2 \delta v^2}{\varepsilon^4} \Big|_{\Delta_-}$$

$$S_\mu(D) = \frac{c}{h} \log \frac{\Delta u^4 \Delta v^4}{\delta u_+^2 \delta v_+^2 \delta u_-^2 \delta v_-^2}$$

Express $S(D)$
 Cutoff: Lorentz-invariant.



$$S_\mu(D) = \frac{1}{2} I(D; \bar{D}_\mu^+) = \frac{1}{2} \left[S_\varepsilon(D) + S_\varepsilon(D_\mu^+) - S_\varepsilon(\Delta) \right]$$

$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} \Big|_D$$

$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} \Big|_{D_\mu^+}$$

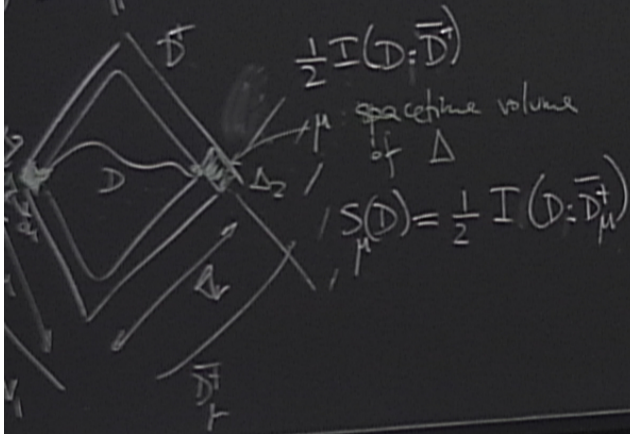
$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} \Big|_{\Delta_\perp}$$

$$+ \log \delta$$

$$S_\mu(D) = \frac{c}{h} \log \frac{\Delta u^4 \Delta v^4}{\delta u_1^2 \delta v_1^2 \delta u_2^2 \delta v_2^2}$$

μ axis

Express $S(D)$
 Cutoff: Lorentz-invariant.



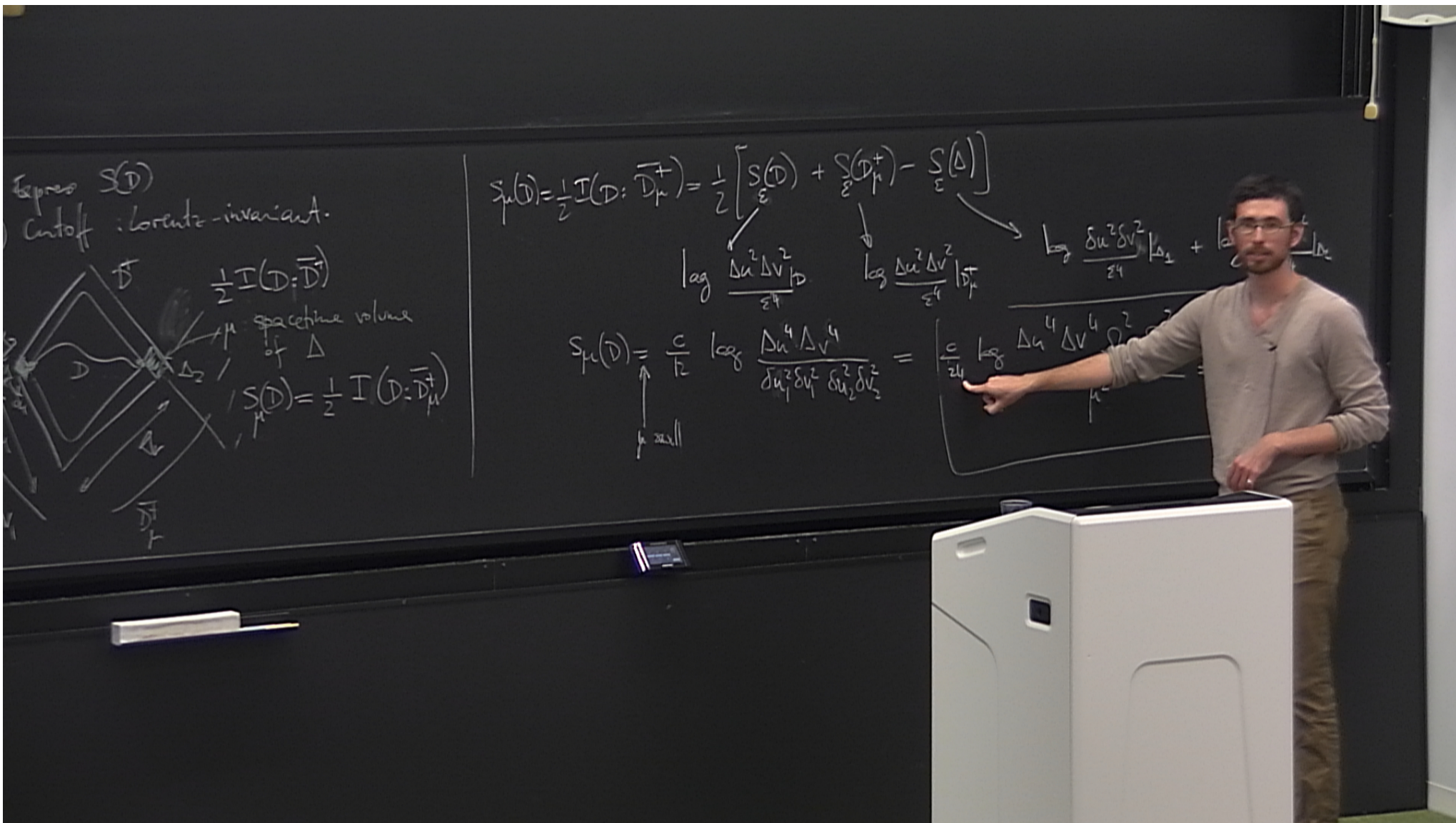
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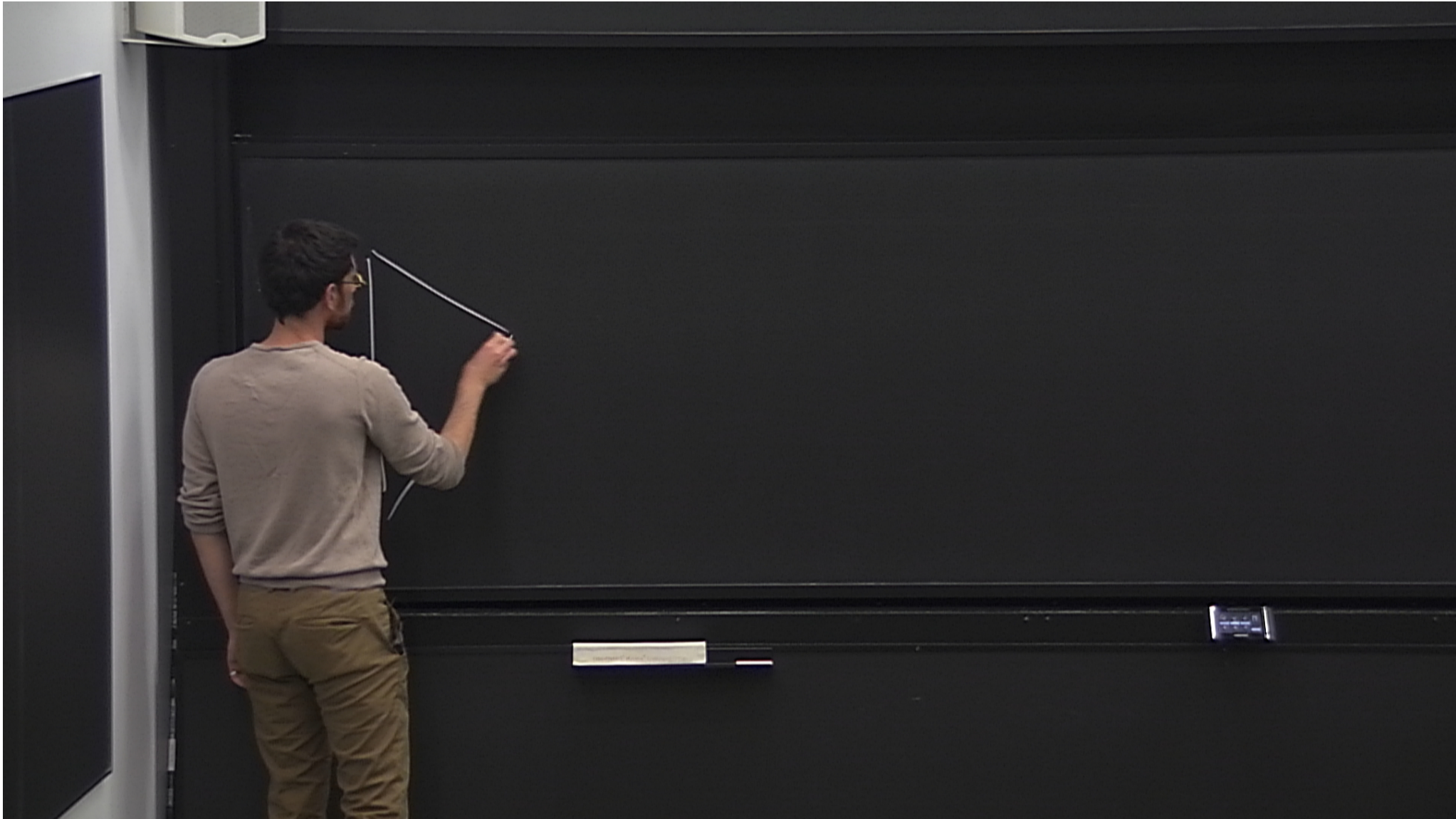
$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} \Big|_D$$

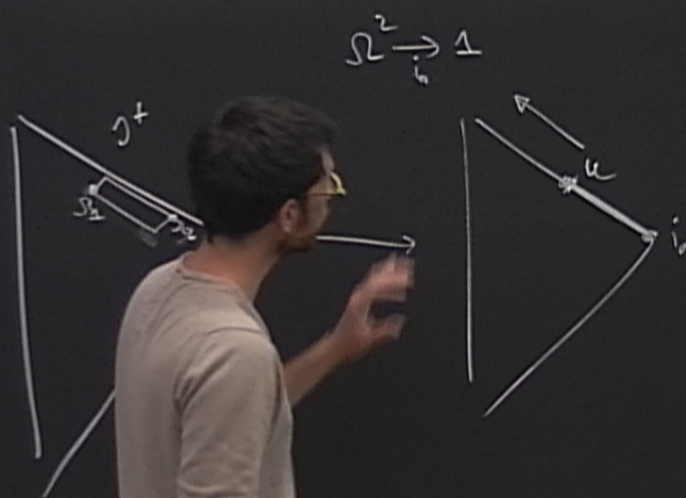
$$\log \frac{\Delta u^2 \Delta v^2}{\varepsilon^4} \Big|_{D_\mu^+}$$

$$\log \frac{\delta u^2 \delta v^2}{\varepsilon^4} \Big|_{\Delta_1} + \log \frac{\delta u^2 \delta v^2}{\varepsilon^4} \Big|_{\Delta_2}$$

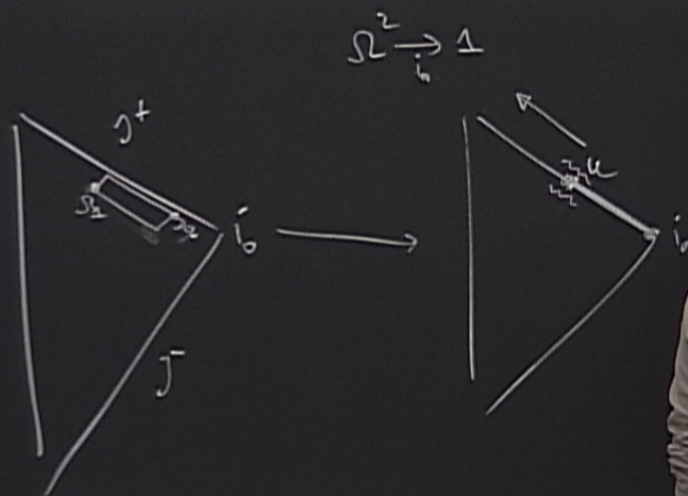
$$S_\mu(D) = \frac{c}{12} \log \frac{\Delta u^4 \Delta v^4}{\delta u_1^2 \delta v_1^2 \delta u_2^2 \delta v_2^2} = \left[\frac{c}{12} \log \frac{\Delta u^4 \Delta v^4 S_1^2 S_2^2}{\mu^2} \right] = S_\mu(D)$$



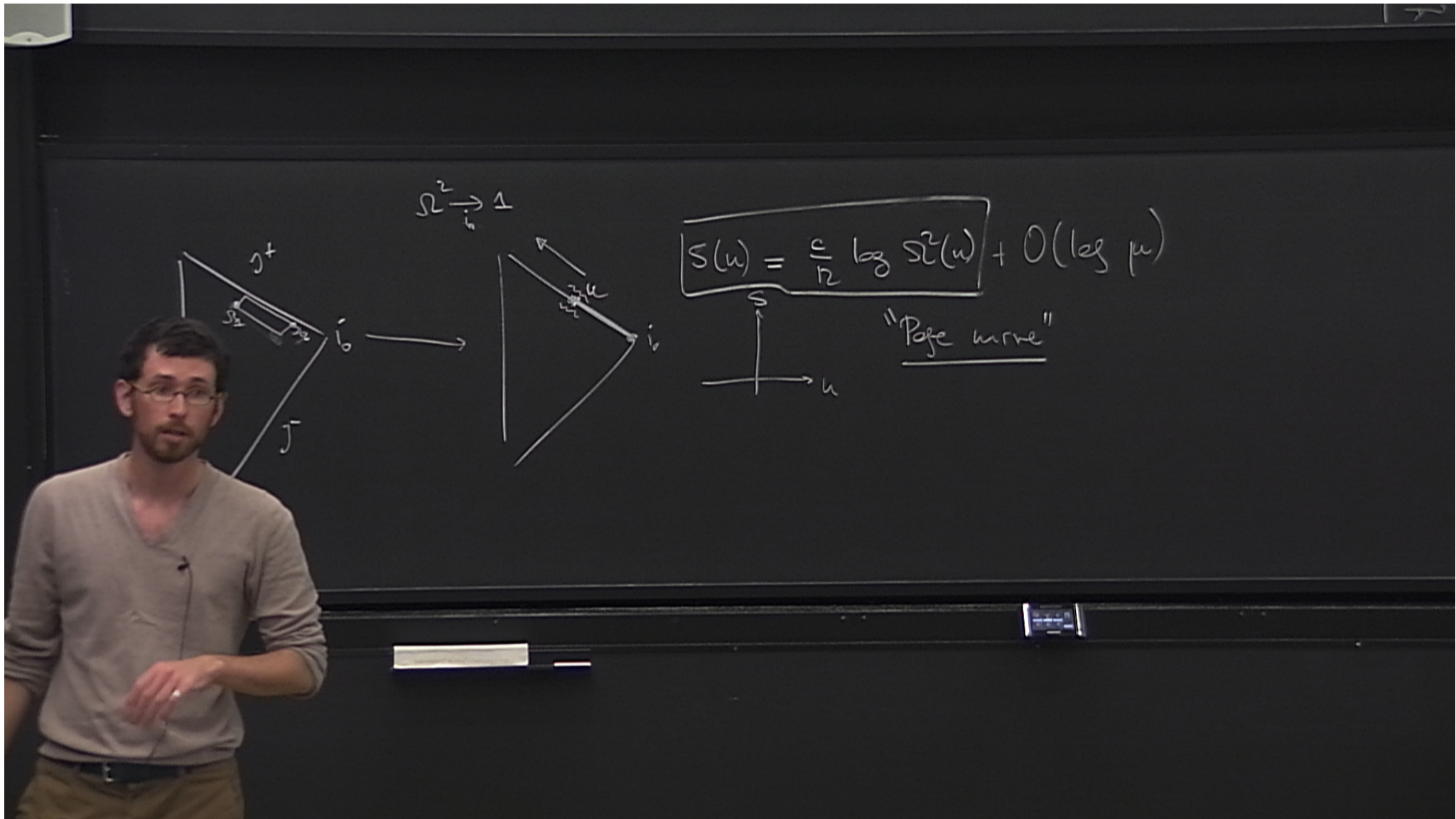


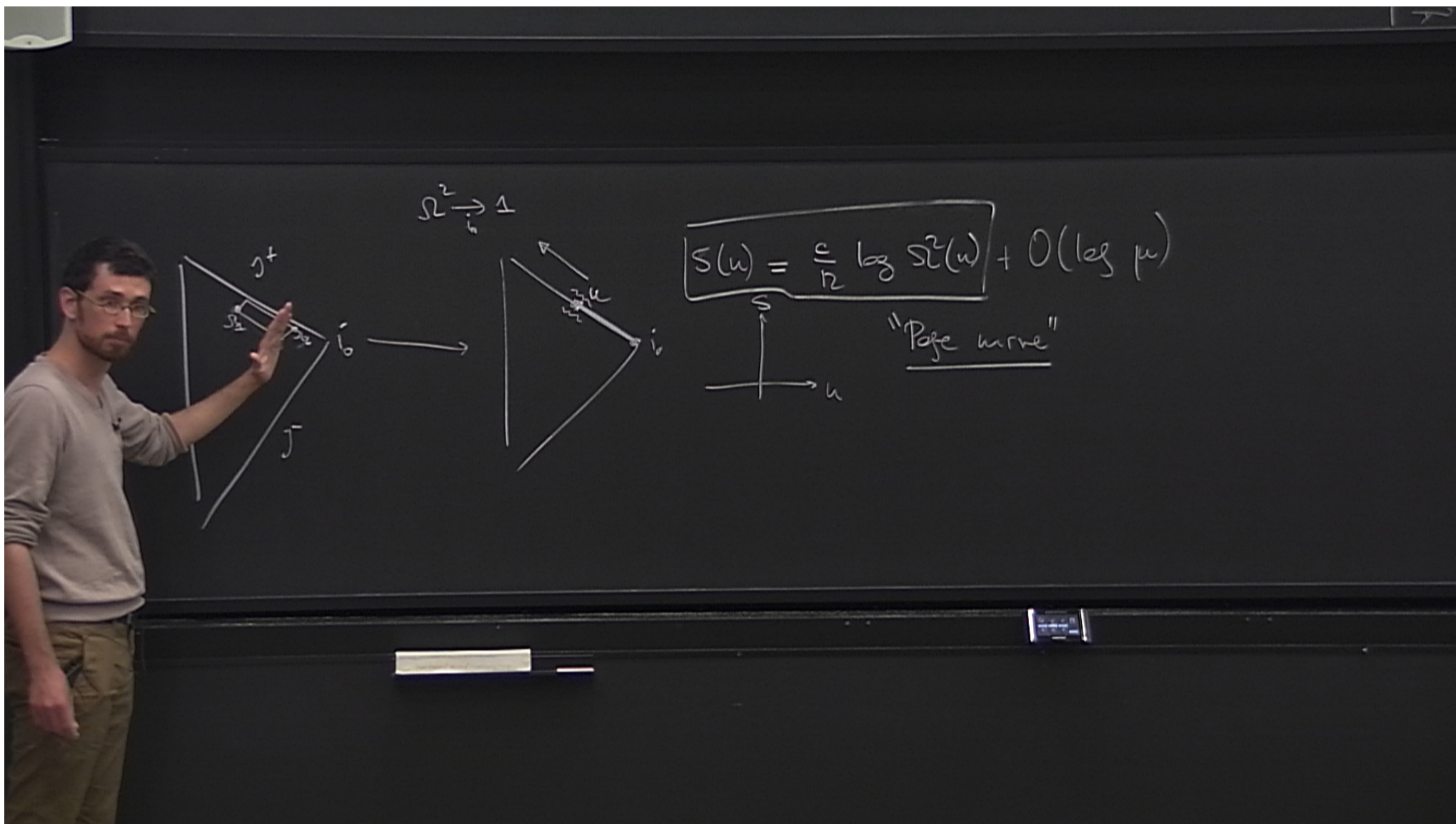


$$S(u) = \frac{c}{2} \log S^2(u) + O(\log \mu)$$



$$S \leq \frac{c}{12} \log S^2(u) + O(\log \mu)$$

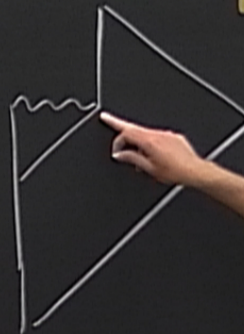


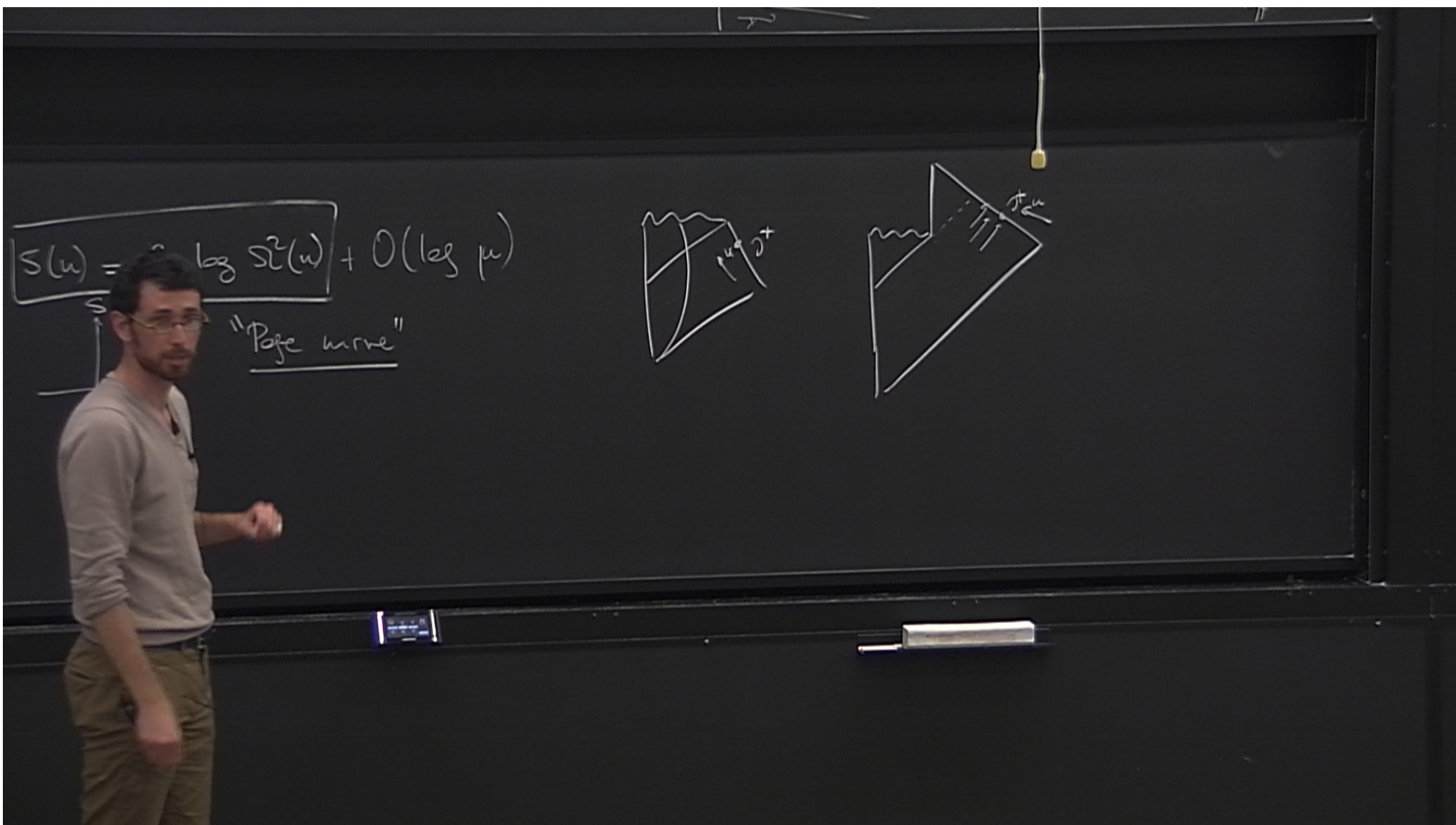


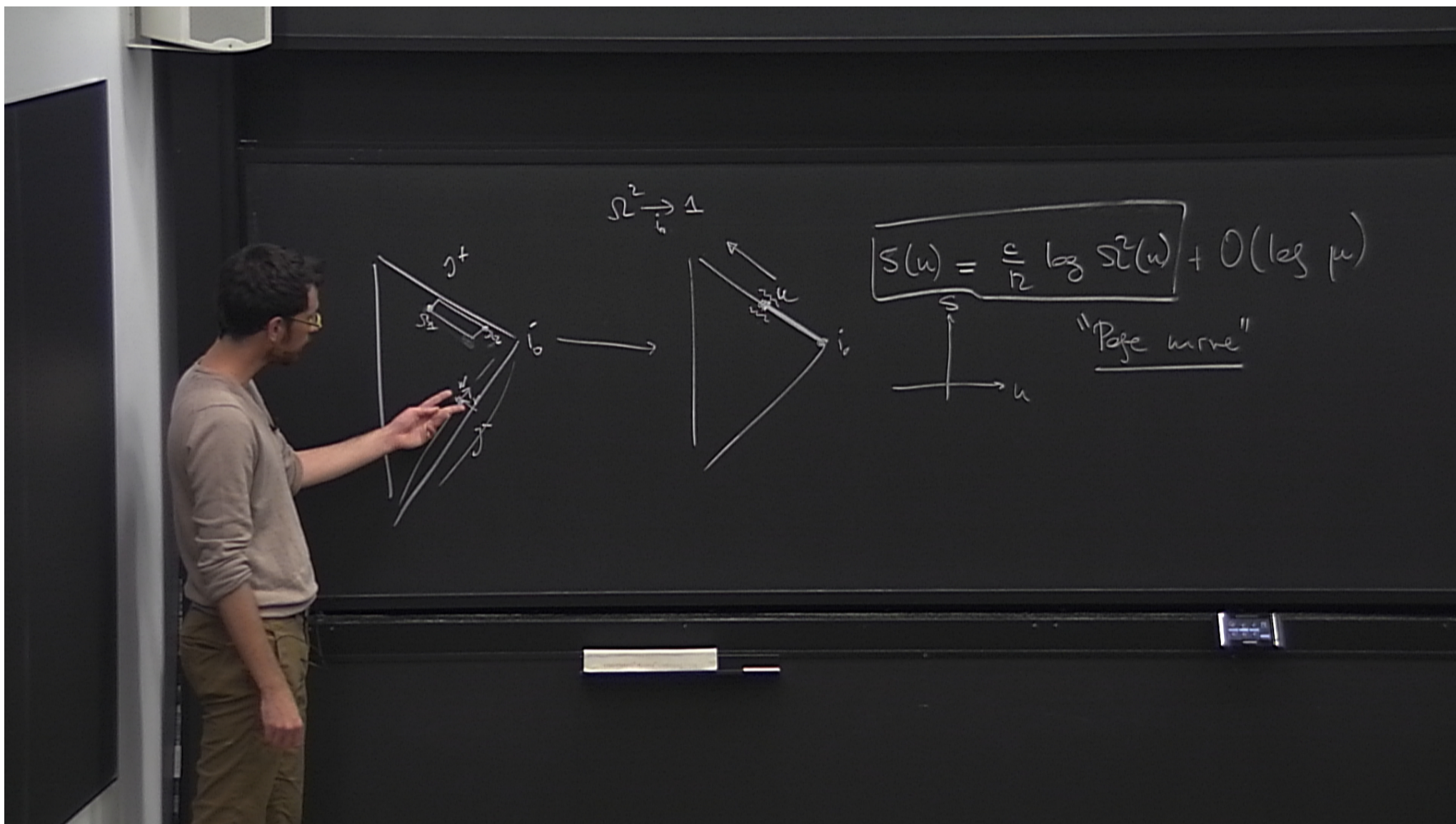
$$S(u) = \frac{c}{12} \log S^2(u) + O(\log u)$$

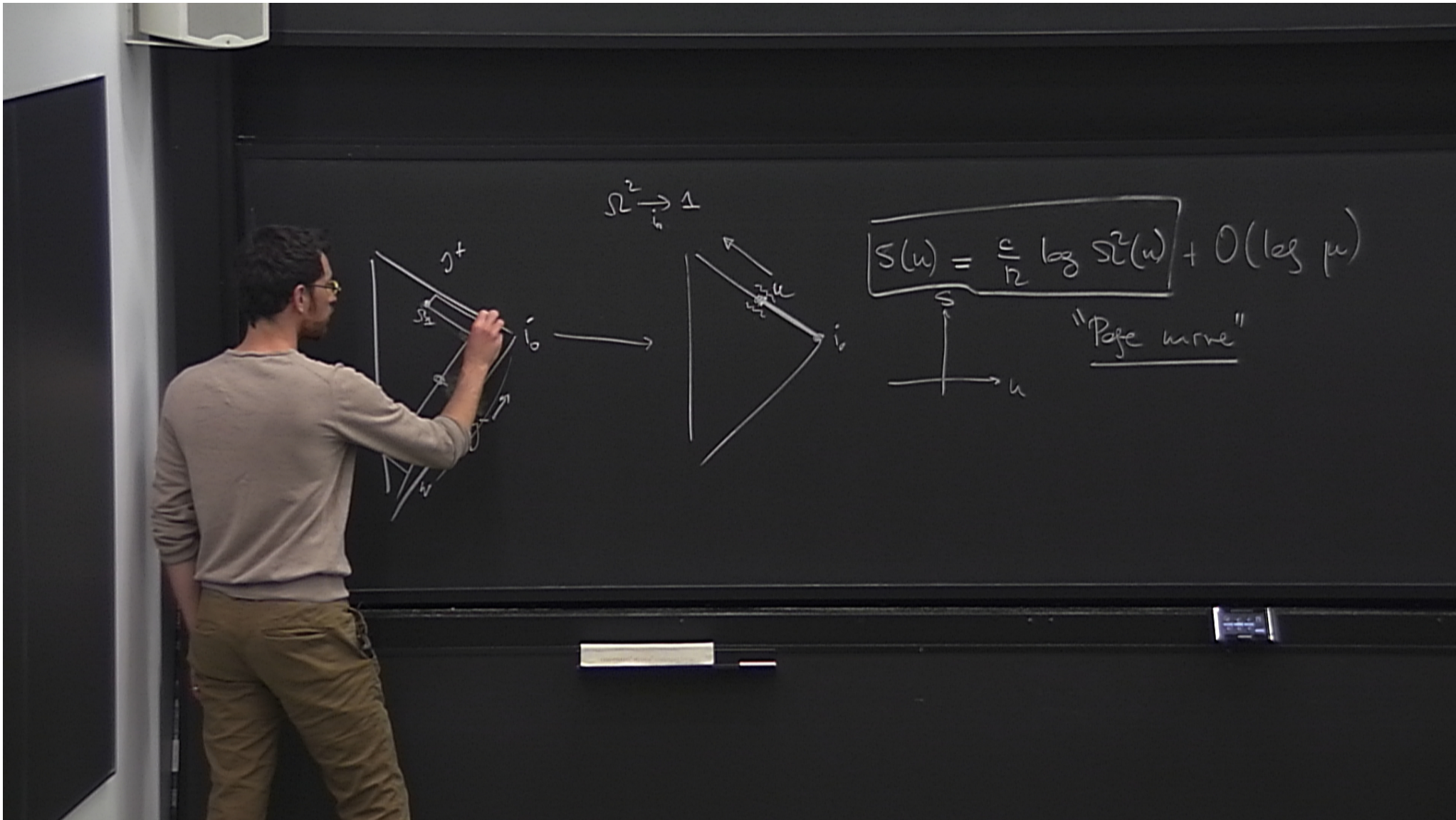


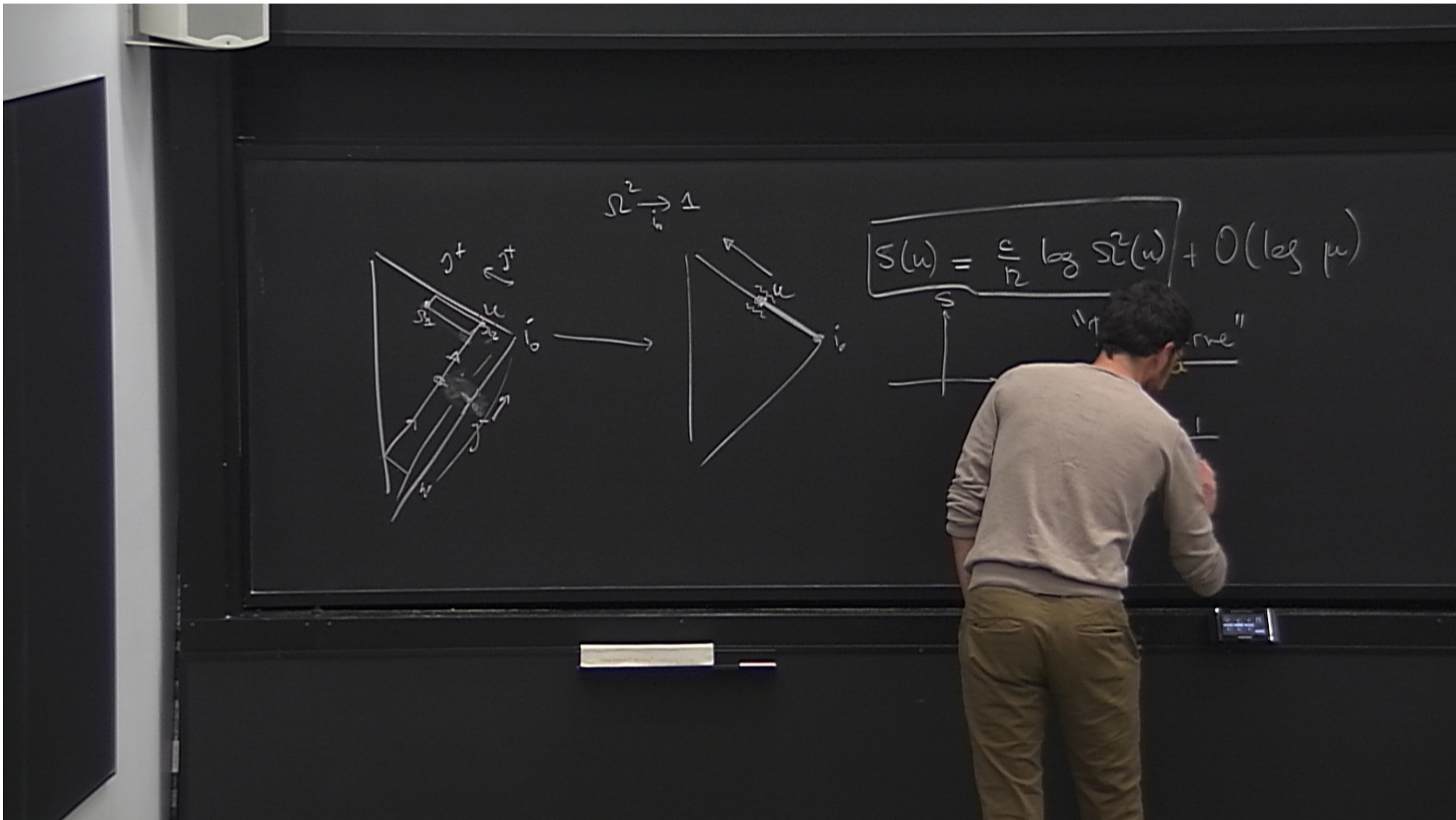
"Page curve"

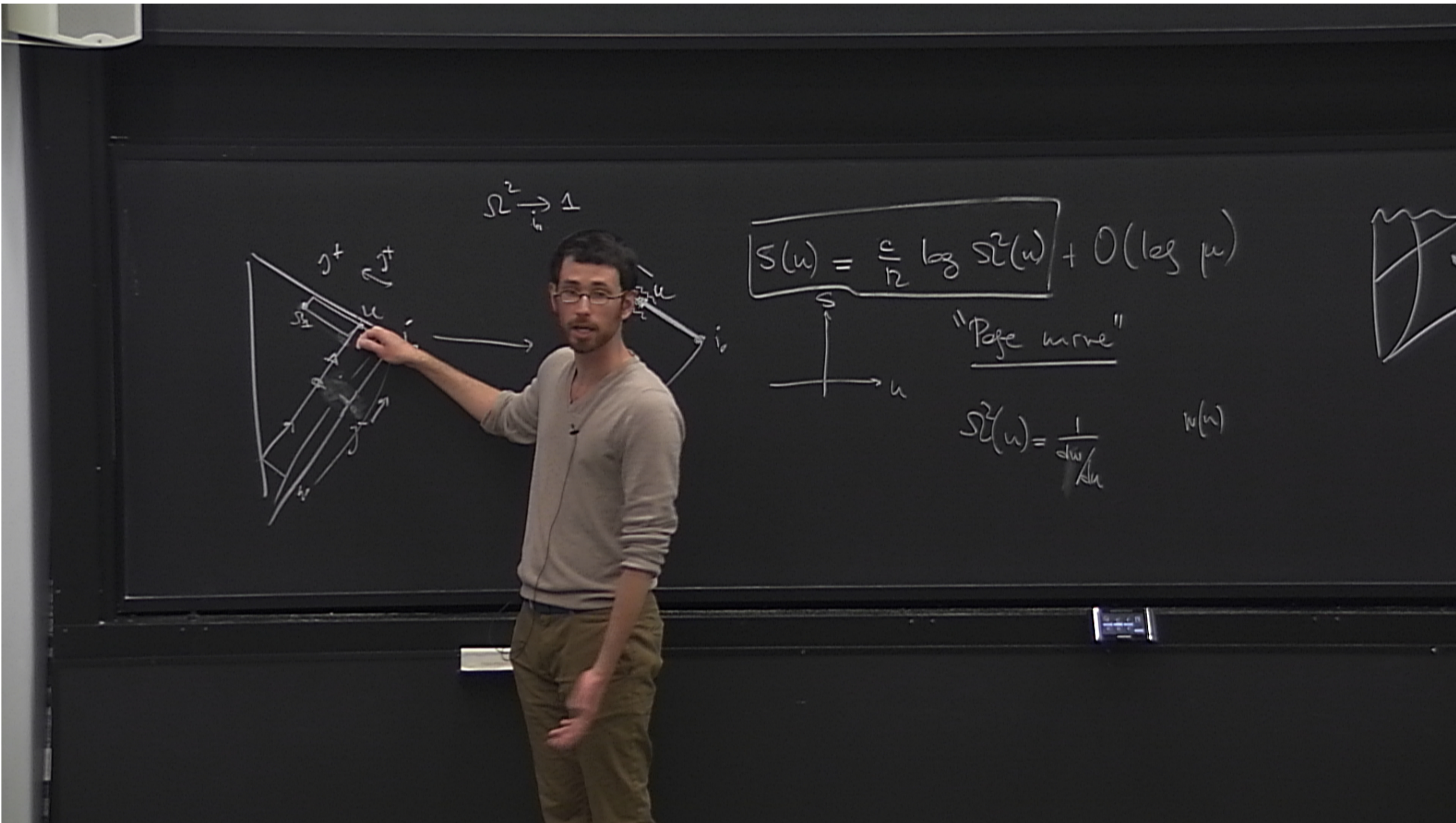






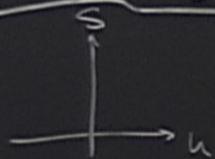




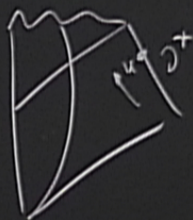


$$S(u) = \frac{c}{12} \log S^2(u) + O(\log \mu)$$

"Page curve"

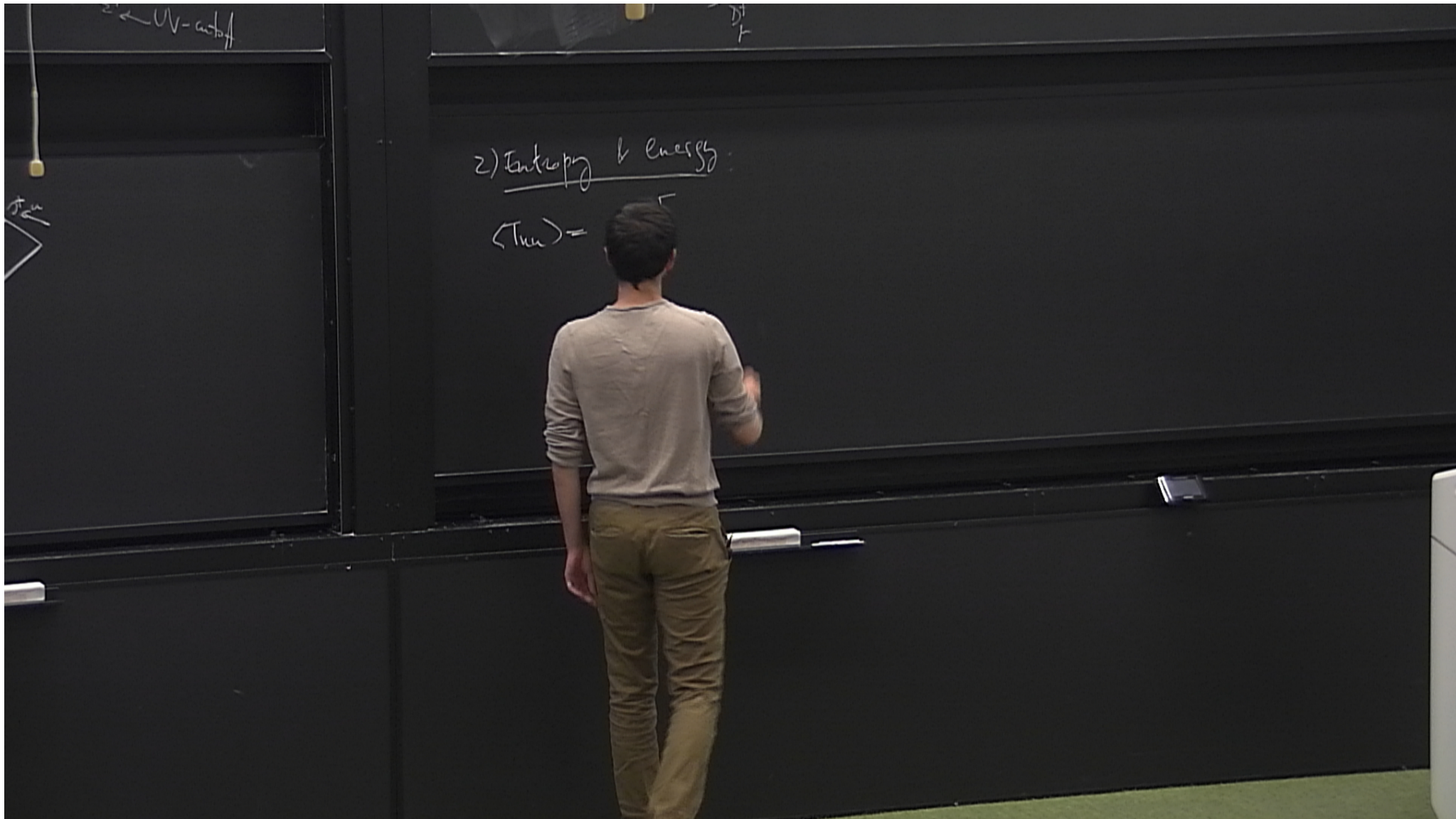


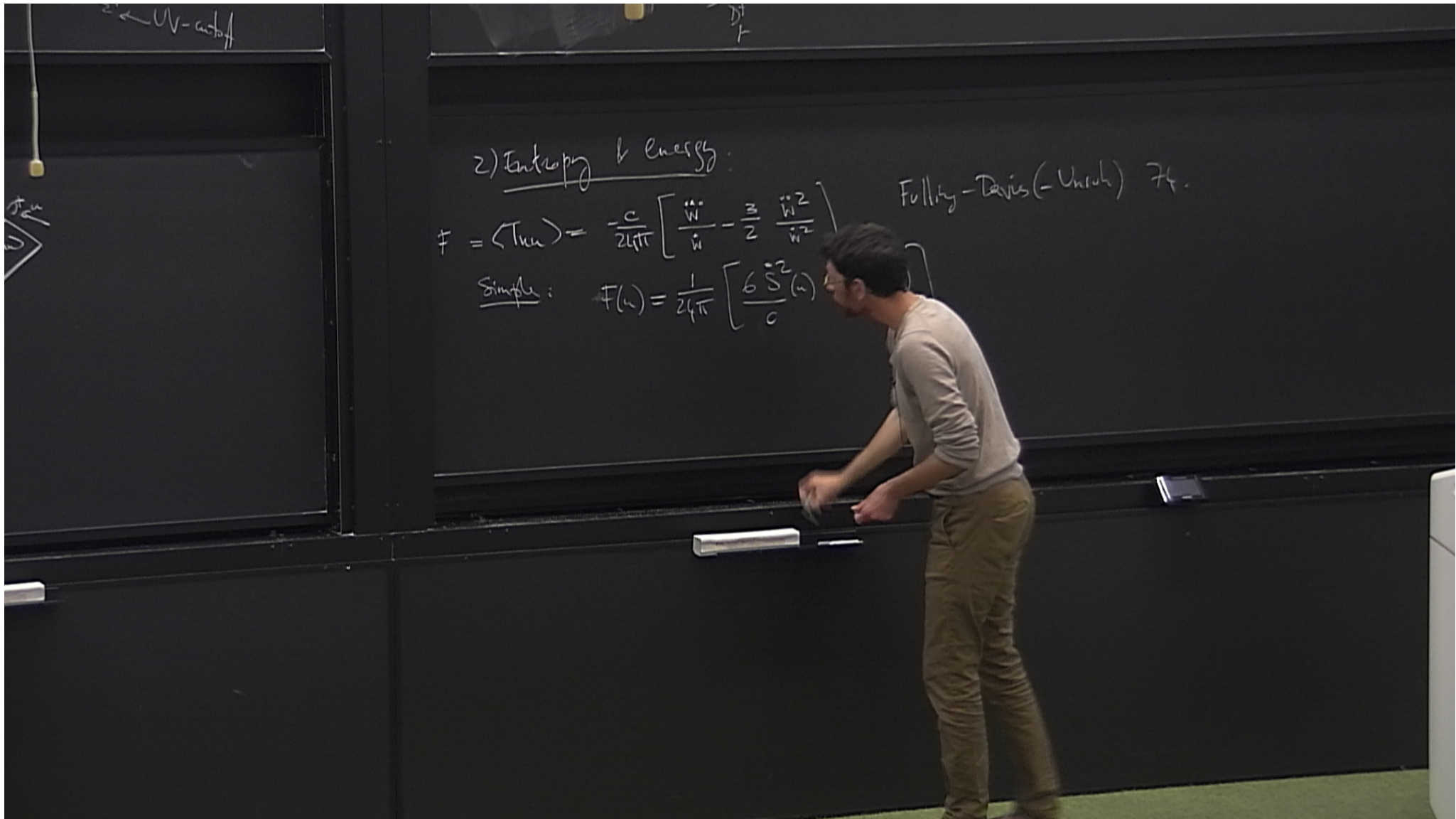
$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{\dot{w}}$$



$$S(u) = -\frac{c}{12} \log \dot{w}(u)$$







2) Entropy & Energy:

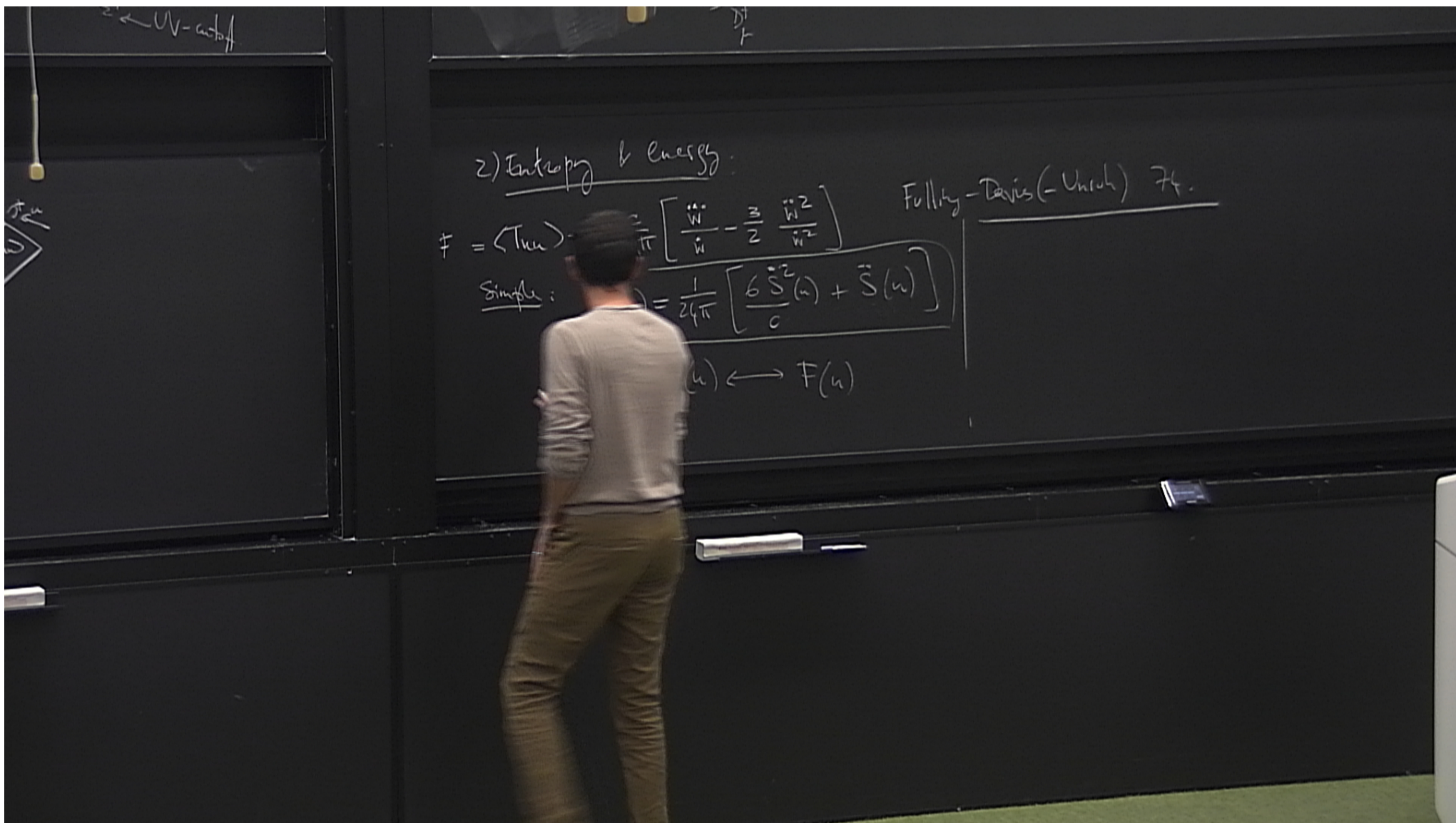
$$F = \langle T_{nn} \rangle = \frac{c}{24\pi} \left[\frac{\ddot{w}}{\dot{w}} - \frac{3}{2} \frac{\ddot{w}^2}{\dot{w}^2} \right]$$

Fully-Derived (Unruh) 74.

Simple:

$$F(u) = \frac{1}{24\pi} \left[\frac{6 \ddot{S}^2(u)}{c} + \ddot{S}(u) \right]$$

$$S(u) \longleftrightarrow F(u)$$



$$k = \frac{1}{4m}$$

2) Entropy & Energy:

$$\mathcal{F} = \langle T_{uu} \rangle = \frac{-c}{24\pi} \left[\frac{\ddot{w}}{\dot{w}} - \frac{3}{2} \frac{\dot{w}^2}{\dot{w}^2} \right]$$

Full: -Davis (-Unruh) 74.

Simple: $\mathcal{F}(u) = \frac{1}{24\pi} \left[\frac{6 \ddot{S}^2(u)}{c} + \dots(u) \right]$

$$\frac{dE}{du} = \alpha T_H \frac{dS}{du} + \quad S(u) \longleftrightarrow \mathcal{F}(u)$$

2) Entropy & Energy:

$$F = \langle T_{nn} \rangle = -\frac{c}{24\pi} \left[\frac{\ddot{w}}{\dot{w}} - \frac{3}{2} \frac{\ddot{w}^2}{\dot{w}^2} \right]$$

Simple:

$$F(u) = \frac{1}{24\pi} \left[\frac{6 \ddot{S}^2(u)}{c} + \ddot{S}(u) \right]$$

$$\frac{dE}{du} = \alpha T_H \frac{dS}{du} + S(u) \longleftrightarrow F(u)$$

Fulling-Davis(-Unruh) 74.

$$\psi(u) = e^{6S(u)/c}$$

$$V(u) = \frac{12\pi}{c} F(u)$$

$$-\ddot{\psi}(u) + V(u) \psi(u) = 0$$

2) Entropy & Energy:

$$F = \langle T_{nn} \rangle = \frac{-c}{24\pi} \left[\frac{\ddot{w}}{\dot{w}} - \frac{3}{2} \frac{\dot{w}^2}{w^2} \right]$$

Simple:

$$F(u) = \frac{1}{24\pi} \left[\frac{6 \ddot{S}^2(u)}{c} + \ddot{S}(u) \right]$$

$$\frac{dF}{du} = \alpha T_H \frac{dS}{du} + S(u) \longleftrightarrow F(u)$$

Fully-Derived (-Unruh) 74.

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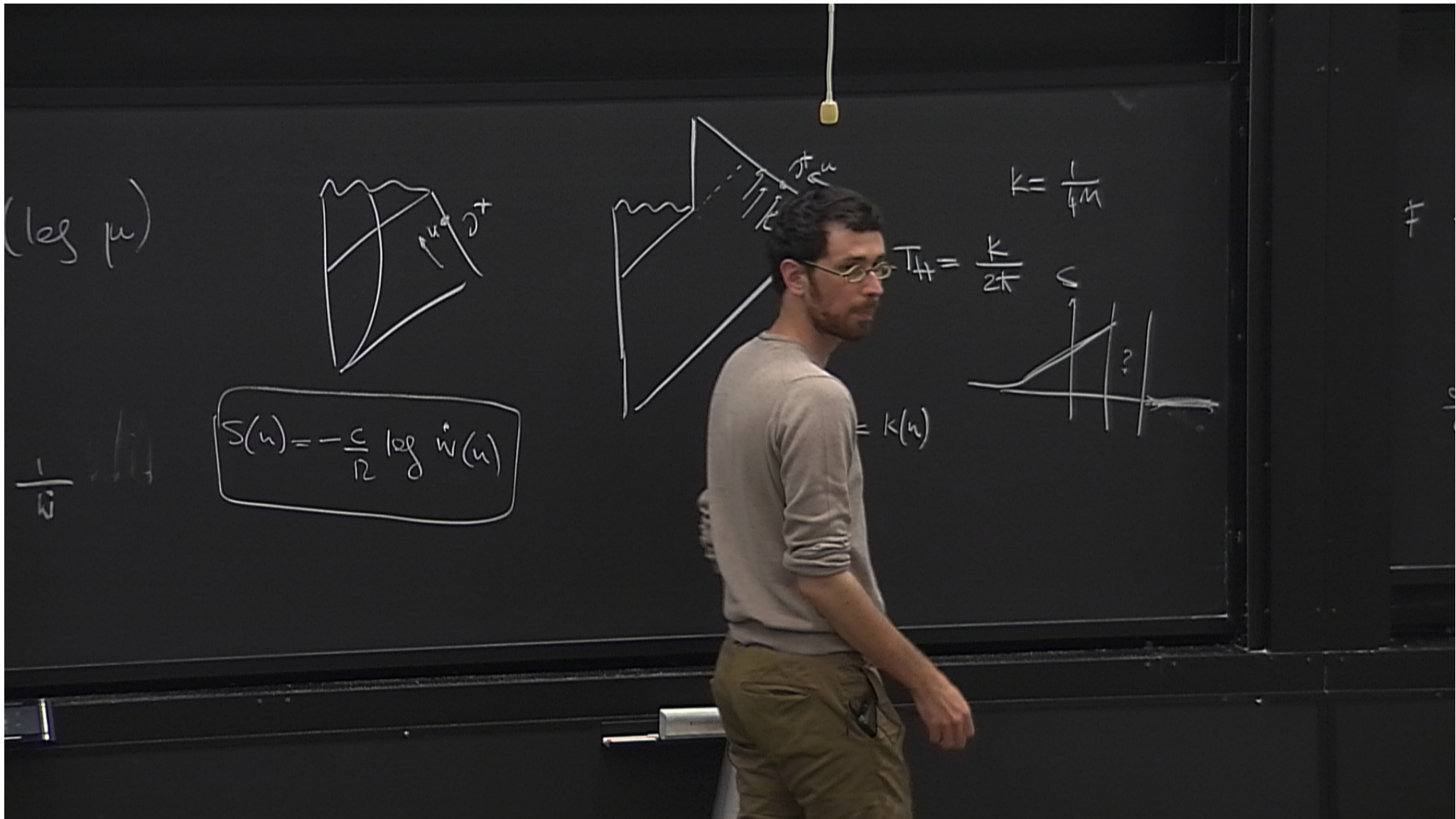
Davis (- Unruh) 74.

$$\psi(u) = e^{\frac{6S(u)}{c}}$$

$$V(u) = \frac{12\pi}{c} \ddot{\phi}(u)$$

$$-\ddot{\psi}(u) + V(u)\psi(u) = 0$$

provides intuition



$\psi(x)$ 74.

$$e^{6S(x)/c}$$

$$\frac{12\pi}{c} \psi(x)$$

$$-\ddot{\psi}(x) + V(x)\psi(x) = 0$$

provides intuition

If $\dot{\psi}(\pm\infty) = 0$ (hence $\psi(\pm\infty) = 0$)

$$\frac{dE}{du} = \alpha T_H \frac{dS}{du} + S(u) \longleftrightarrow F(u)$$

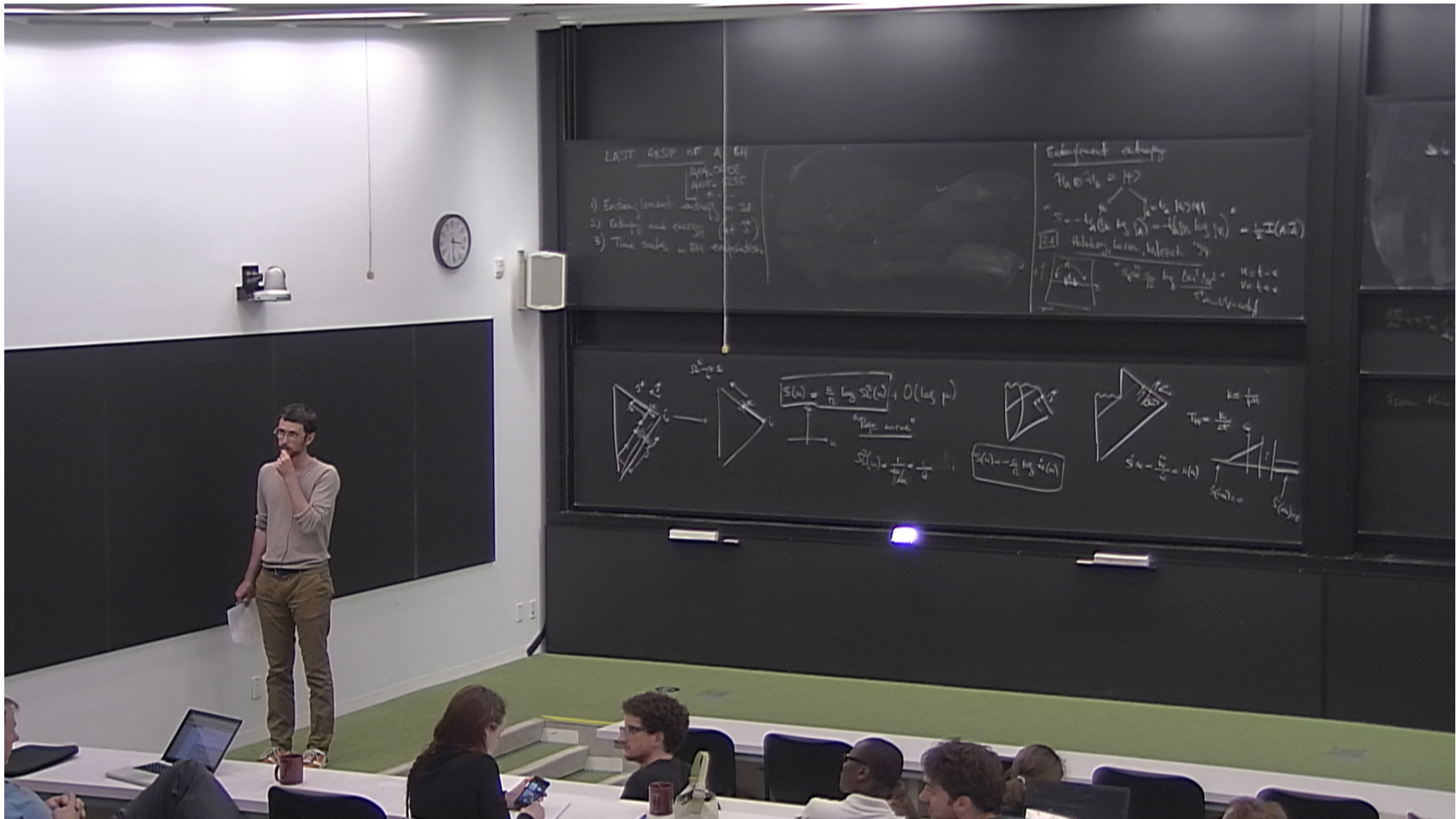
From this: $\exists (u_1, u_2) \dots < 0$

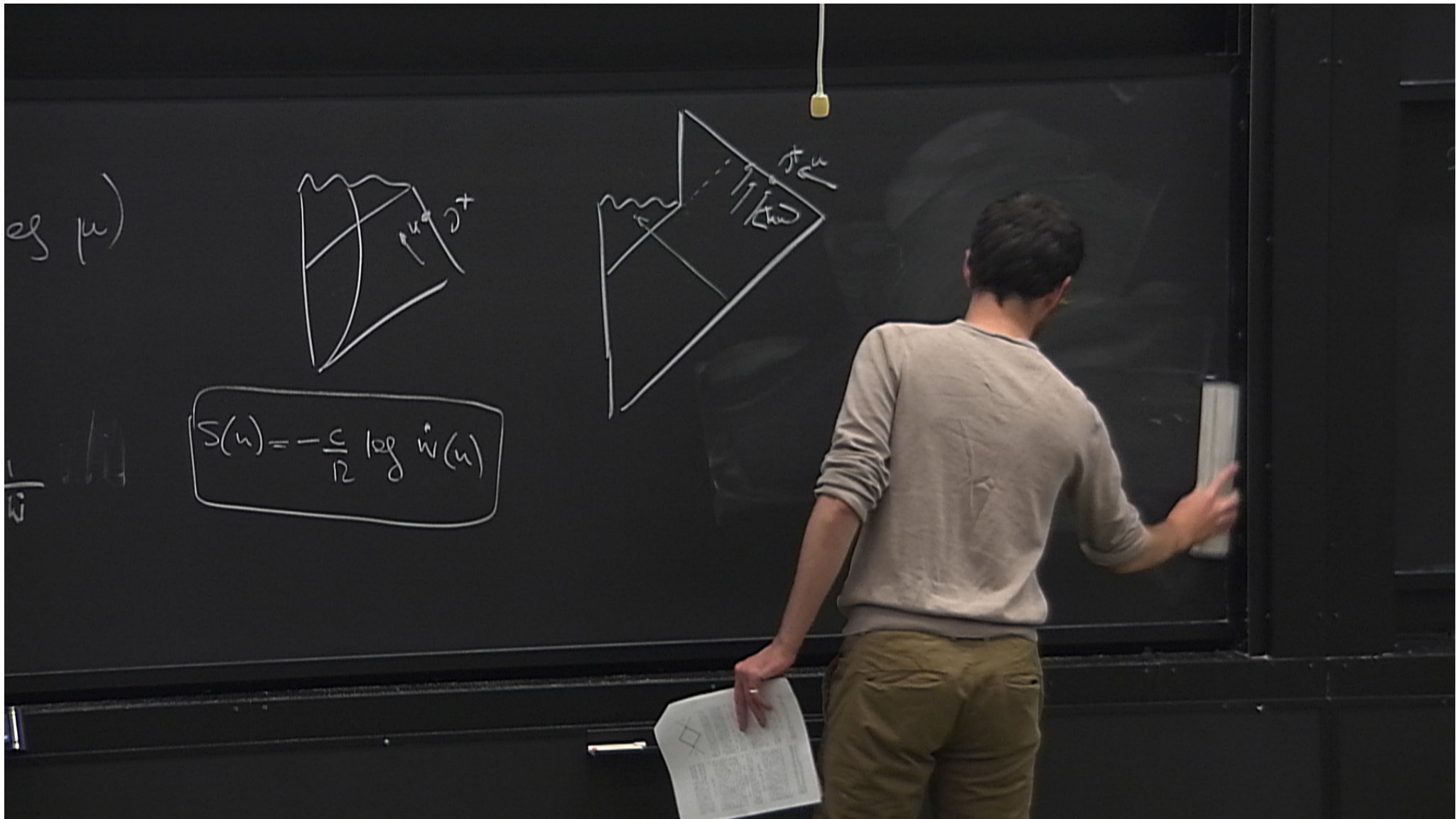
$$\frac{dE}{du} = \alpha T_H \frac{dS}{du} + S(u) \longleftrightarrow F(u)$$

From this: $\exists (u_1, u_2) \quad F(u) < 0 \quad \text{for } u \in (u_1, u_2)$
 $\rightarrow$ "last resp" of a BH.

From this: $\exists (u_1, u_2) \quad F(u) < 0 \quad \text{for } u \in (u_1, u_2)$

$\rightarrow$ "lost resp" of a BH.

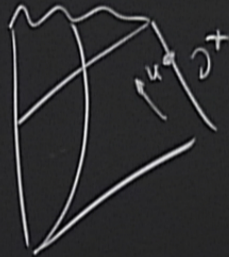




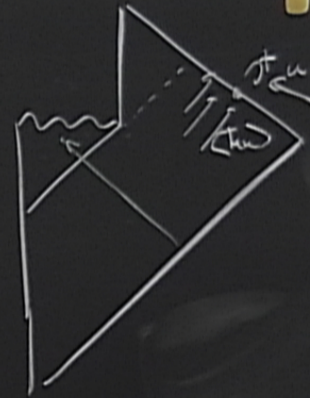
$$\log S^2(u) + O(\log u)$$

"Page curve"

$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{w}$$



$$S(u) = -\frac{c}{12} \log \dot{w}(u)$$

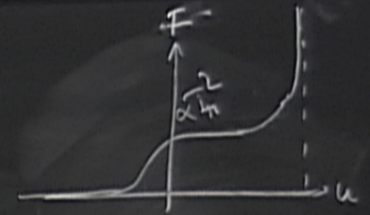
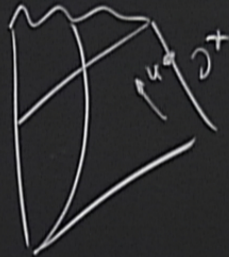


$$S^2(u) + O(\log u)$$

"Page curve"

$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{w}$$

$$S(u) = -\frac{c}{12} \log \hat{w}(u)$$



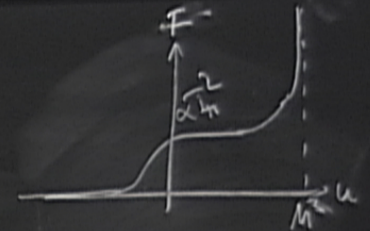
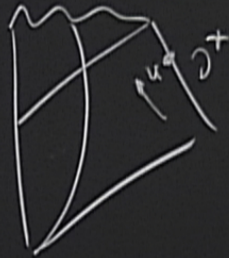
$$F(u) = \frac{c}{12 \log(u)}$$

$$S^2(u) + O(\log u)$$

"Page curve"

$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{w}$$

$$S(u) = -\frac{c}{12} \log \dot{w}(u)$$



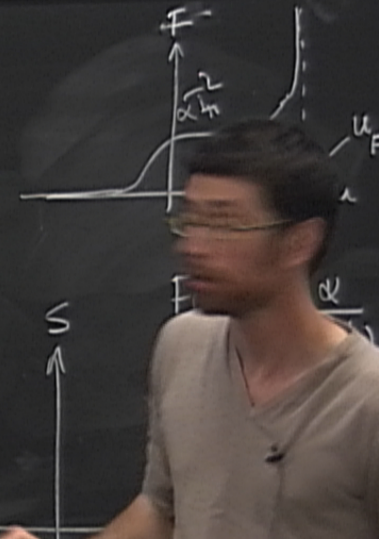
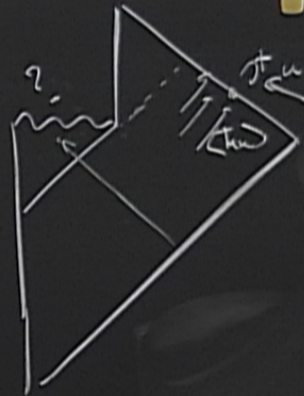
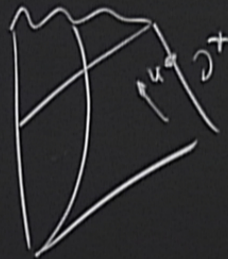
$$F(u) = \frac{\alpha}{h^2(u)}$$

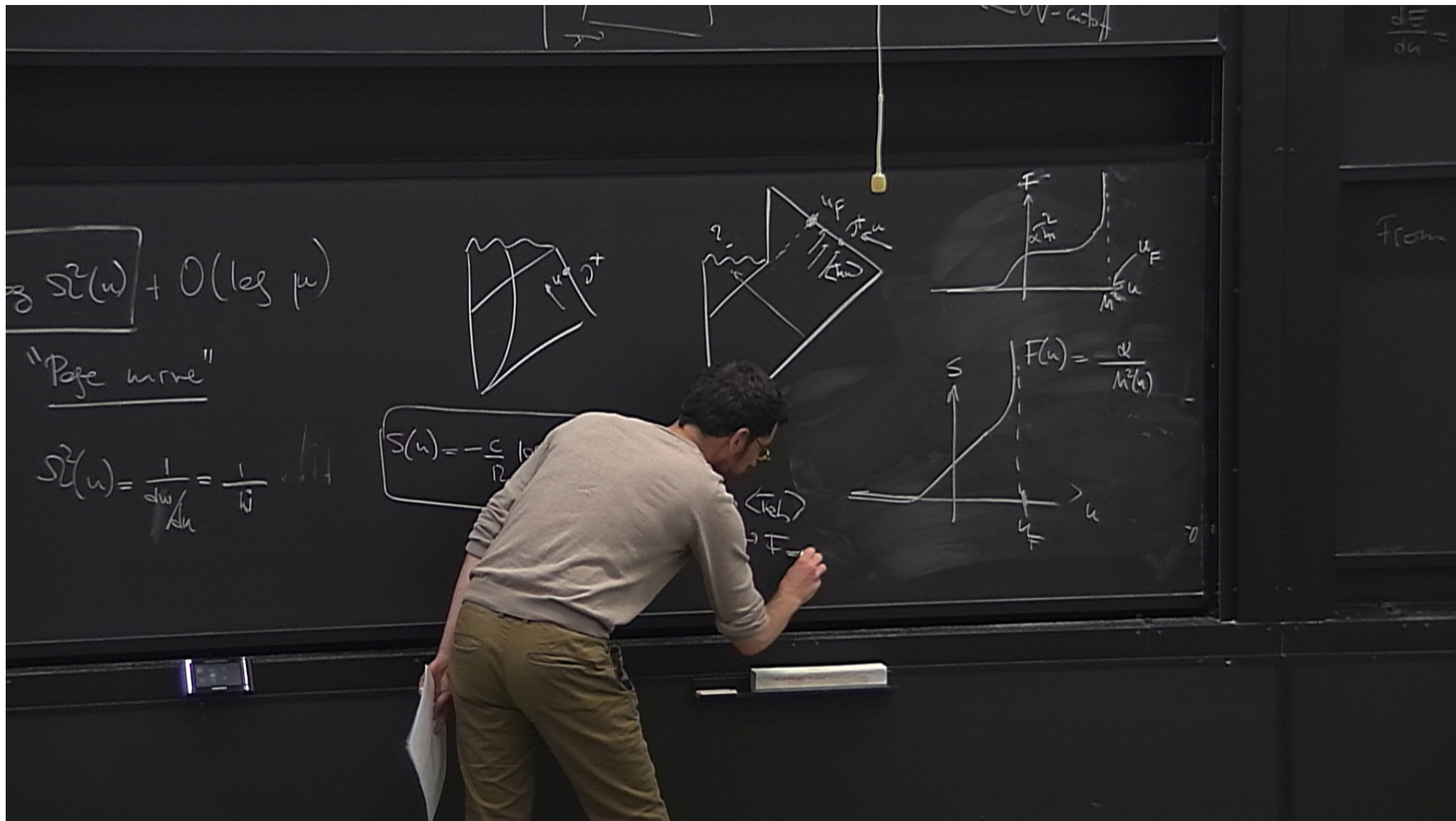
$$(\mu) + O(\log \mu)$$

"curve"

$$\mu = \frac{1}{\frac{dw}{d\mu}} = \frac{1}{\dot{w}}$$

$$S(\mu) = -\frac{c}{12} \log \dot{w}(\mu)$$

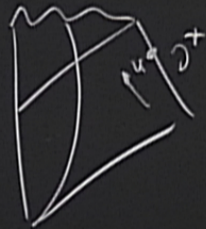




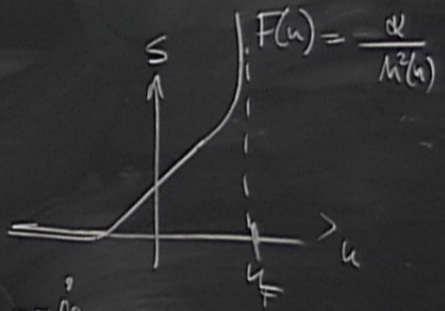
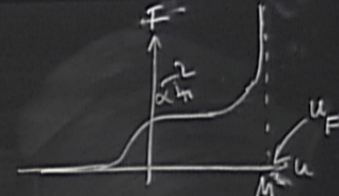
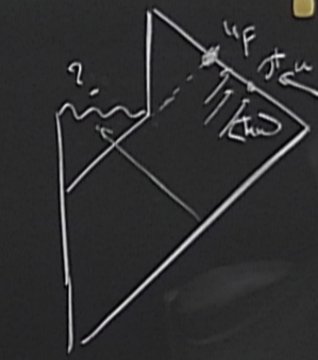
$$S^2(u) + O(\log u)$$

"Page curve"

$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{w}$$



$$S(u) = -\frac{c}{12} \log \dot{w}(u)$$



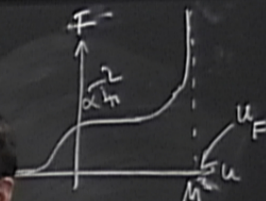
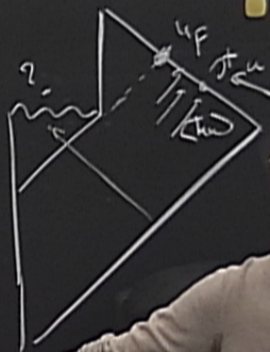
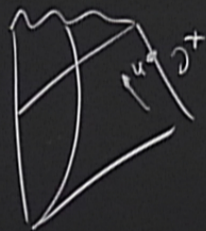
$$\rightarrow F = \frac{c}{12} = -\frac{1}{12}$$

$$S^2(u) + O(\log u)$$

"Page curve"

$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{w}$$

$$S(u) = -\frac{c}{12} \log \dot{w}(u)$$



$$F(u) = \frac{u}{M^2(u)}$$

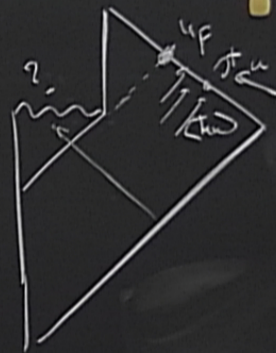
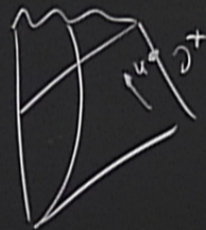
$$\rightarrow F = \frac{u}{M^2} = -$$

$$S^2(u) + O(\log u)$$

"Page curve"

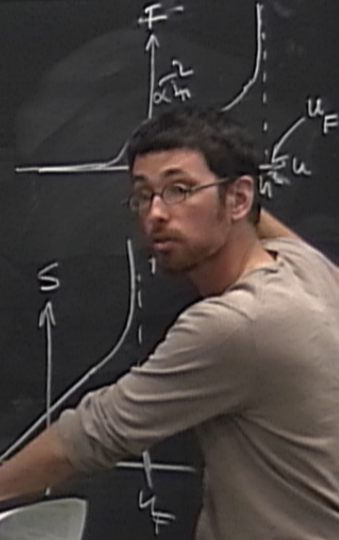
$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{w}$$

$$S(u) = -\frac{c}{12} \log \dot{w}(u)$$



$\frac{dw}{du} \rightarrow \langle T \rangle$

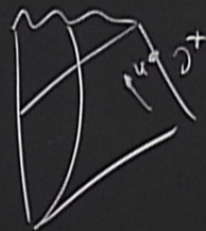
$$\rightarrow T = \frac{c}{12}$$



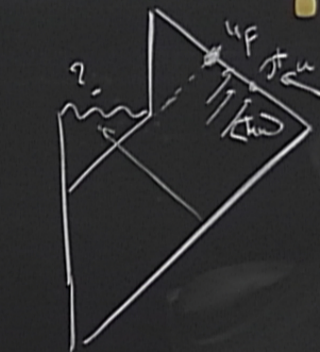
$$S^2(u) + O(\log u)$$

"Page curve"

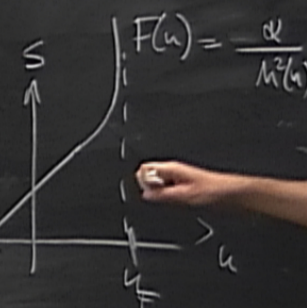
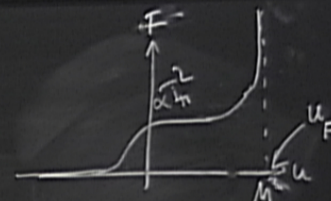
$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{w}$$



$$S(u) = -\frac{c}{12} \log \dot{w}(u)$$



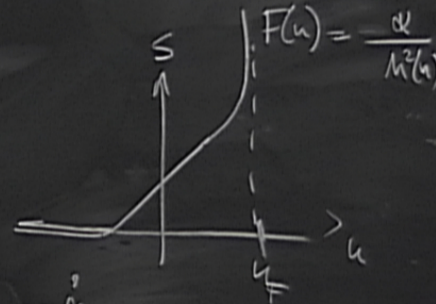
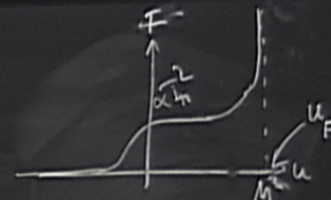
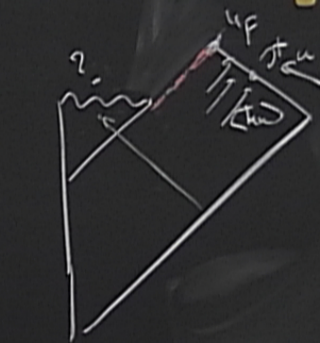
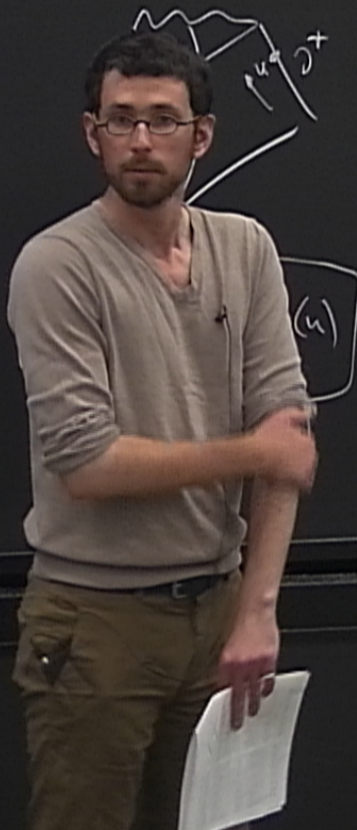
$$\rightarrow F = \frac{c}{12} = -\frac{1}{12}$$



$$S^2(u) + O(\log u)$$

"Page curve"

$$S^2(u) = \frac{1}{\frac{dw}{du}} = \frac{1}{w}$$



$$\rightarrow F = \frac{\alpha}{M^2} = -\frac{1}{M}$$



LAST GRASP OF A BH

- 1) Entanglement entropy in 2d
- 2) Entropy and energy (at T)
- 3) Time scales in BH evaporation

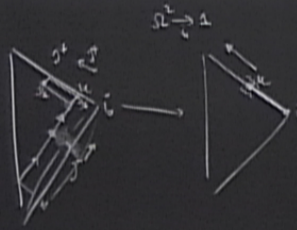
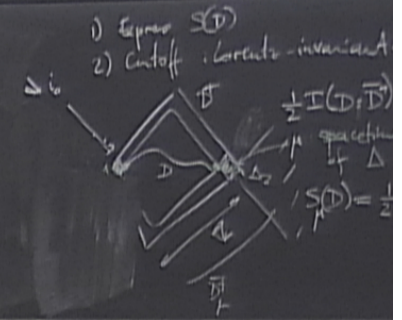
Entanglement entropy

$$H_A \otimes H_B = H$$

$$S = -\log_2 \text{Tr}(\rho_A) = -\log_2 \text{Tr}(\rho_B) = -\frac{1}{2} I(A:B)$$

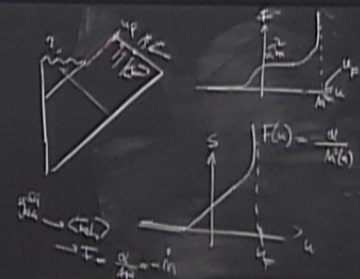
2d Holography, Lorentz, W_{QFT} '94

$$S \sim \frac{c}{12} \log \frac{\Delta_1^2 \Delta_2^2}{z^2} \quad u = t-x, \quad v = t+x$$



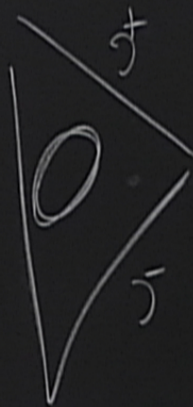
$$S(u) = \frac{c}{12} \log S^2(u) \sim O(\log \mu)$$

$$S(u) = -\frac{c}{12} \log w(u)$$



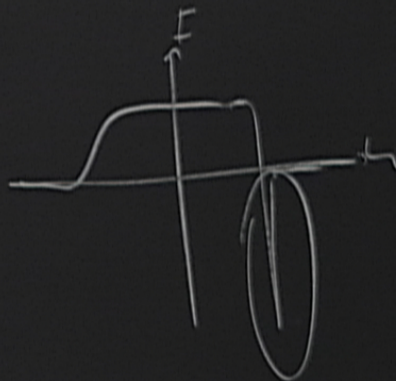
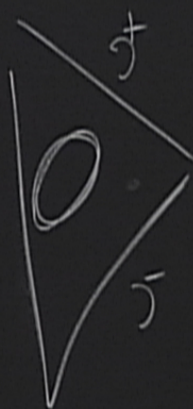
From this: $\exists (u_1, u_2) \quad E(u) < 0 \quad \text{for } u \in (u_1, u_2)$

$\rightarrow$ "lost gest" a BH.



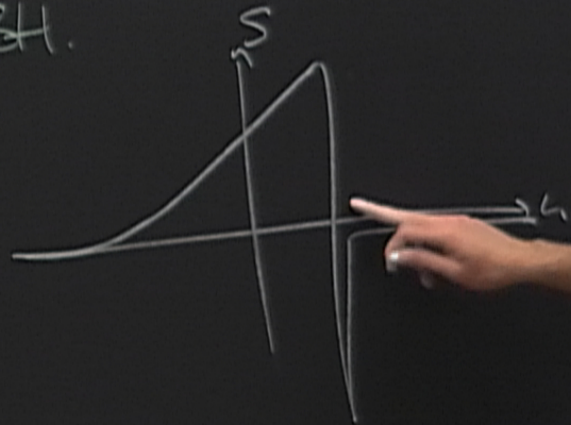
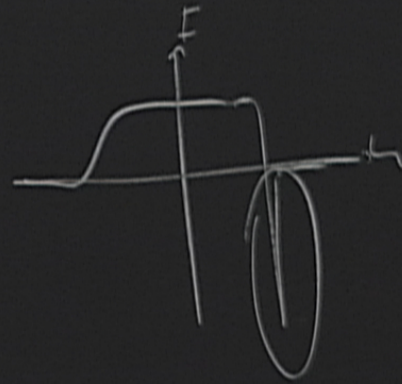
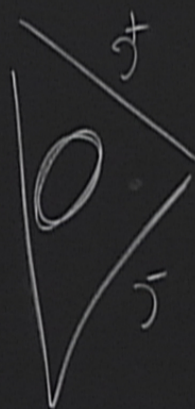
From this: $\exists (u_1, u_2) \quad F(u) < 0$ for $u \in (u_1, u_2)$

$\rightarrow$ "lost resp" of a BH.



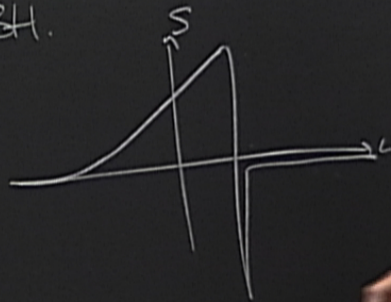
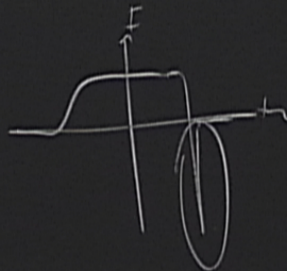
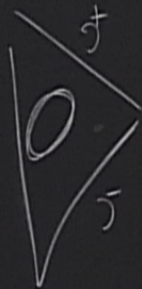
From this: $\exists (u_1, u_2) \quad F(u) < 0 \quad \text{for } u \in (u_1, u_2)$

$\rightarrow$ "lost resp" of a BH.

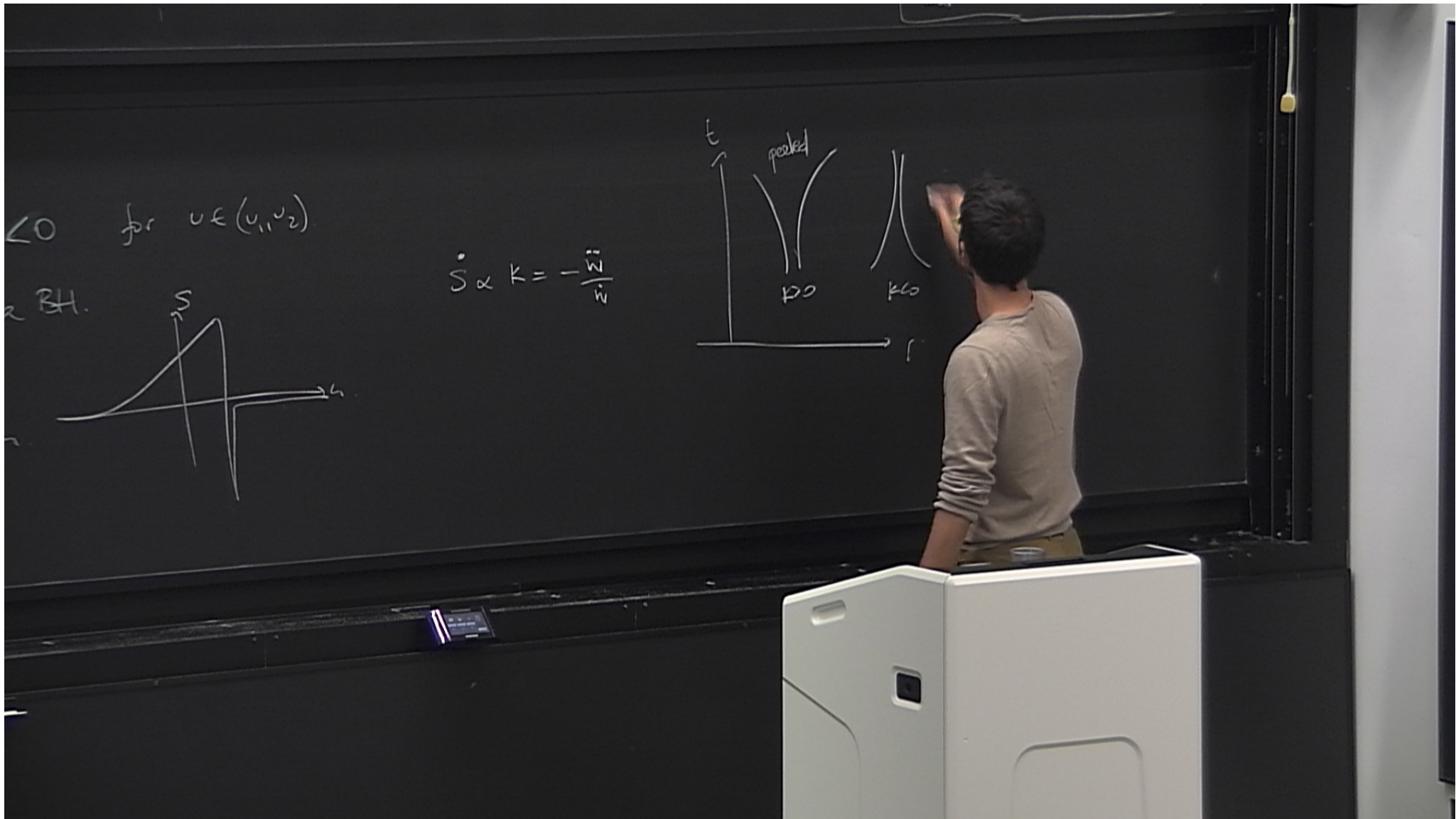


From this: $\exists (u_1, u_2) \quad F(u) < 0 \quad \text{for } u \in (u_1, u_2)$

$\rightarrow$ "last resp" of a BH.



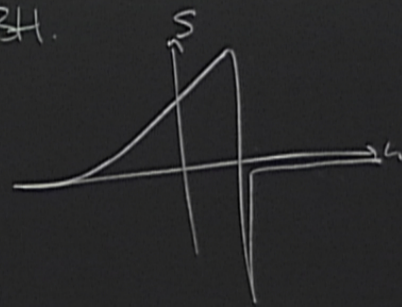
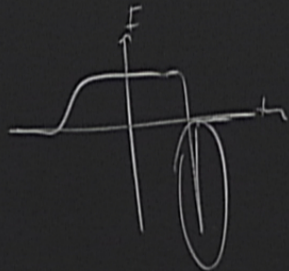
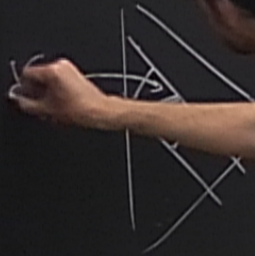
$$\dot{u} = -\frac{M}{u}$$



From this: $\exists (u_1, u_2) \quad F(u) < 0 \quad \text{for } u \in (u_1, u_2)$

$\rightarrow$ "last resp" of a BH.

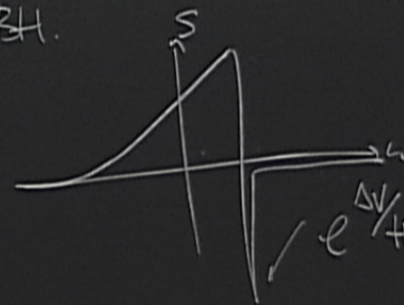
$$\dot{S} \propto K = -\frac{\dot{M}}{M}$$



From this: $\exists (u_1, u_2) \quad F(u) < 0$ for $u \in (u_1, u_2)$

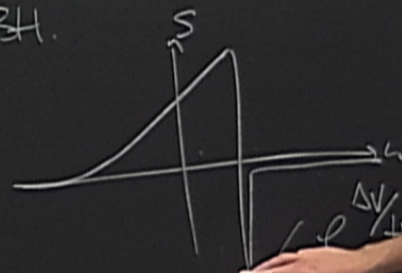
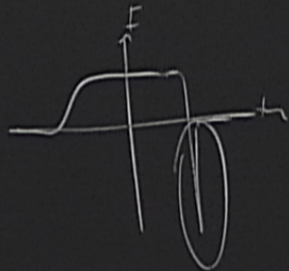
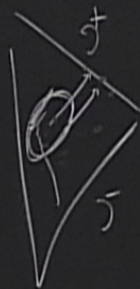
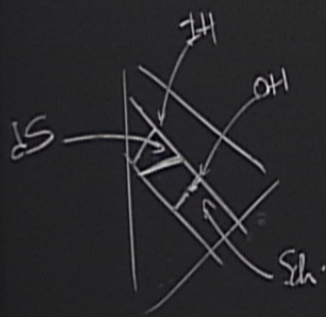
$\rightarrow$ "loss" of a BH.

$$\dot{S} \propto K = -\frac{2}{3} \frac{\dot{M}}{M}$$



From this: $\exists (u_1, u_2) \quad F(u) < 0 \quad \text{for } u \in (u_1, u_2)$

$\rightarrow$ "last resp" of a BH.



$\dot{S} \propto K$

From this: $\exists (u_1, u_2) \quad F(u) < 0 \quad \text{for } u \in (u_1, u_2)$

$$\dot{S} \propto K = -\frac{1}{2} \frac{\dot{S}}{S}$$

