Title: Entanglement, Ergodicity, and Many-Body Localization

Date: May 01, 2014 10:45 AM

URL: http://pirsa.org/14050020

Abstract: We are used to describing systems of many particles by statistical mechanics. However, the basic postulate of statistical mechanics â€“ ergodicity â€“ breaks down in so-called many-body localized systems, where disorder prevents particle transport and thermalization. In this talk, I will present a theory of the many-body localized (MBL) phase, based on new insights from quantum entanglement. I will argue that, in contrast to ergodic systems, MBL eigenstates are not highly entangled. I will use this fact to show that MBL phase is characterized by an infinite number of emergent local conservation laws, in terms of which the Hamiltonian acquires a universal form. Turning to the experimental implications, I will describe the response of MBL systems to quenches: surprisingly, entanglement shows logarithmic in time growth, reminiscent of glasses, while local observables exhibit power-law approach to â€œequilibriumâ€• values. I will support the presented theory with results of numerical experiments. I will close by discussing other directions in exploring ergodicity and its breaking in quantum many-body systems.

Entanglement, ergodicity, and many-body localization

Dima Abanin

Perimeter Institute for
Theoretical Physics

Four Corners Condensed Matter Meeting
Perimeter Institute, May 1, 2014

International Worker's Day in different cities

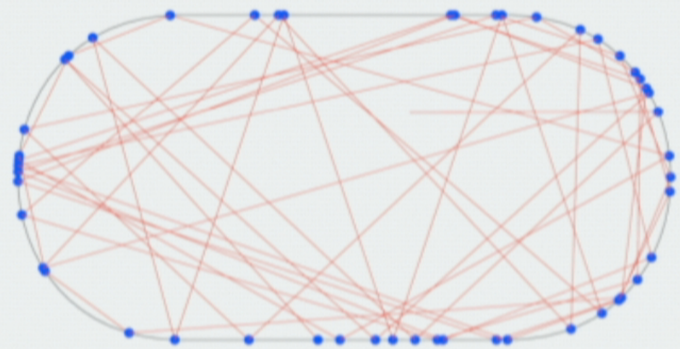
Stockholm

Montreal

Waterloo

Ergodicity and its breaking

System explores full phase space



Ergodicity and its breaking

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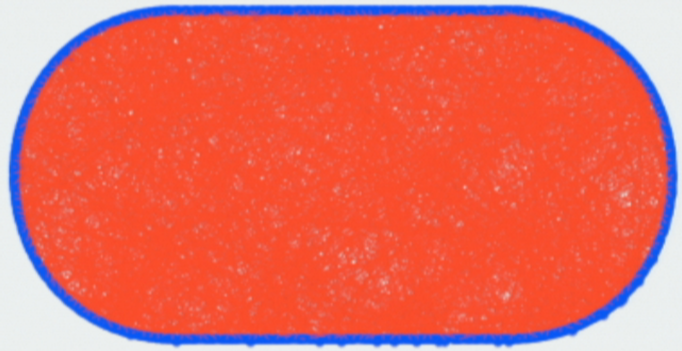
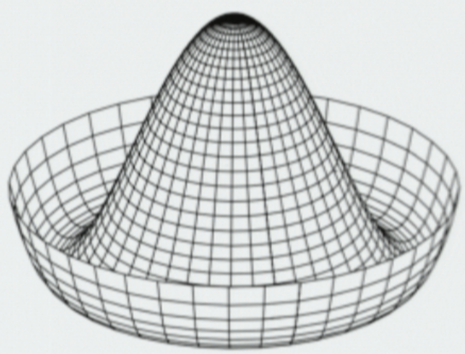
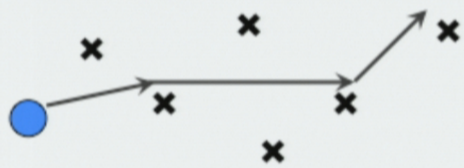


Ergodicity breaking

In phase transitions



Anderson localization: ergodicity breaking in quantum systems



Quantum memory → localization

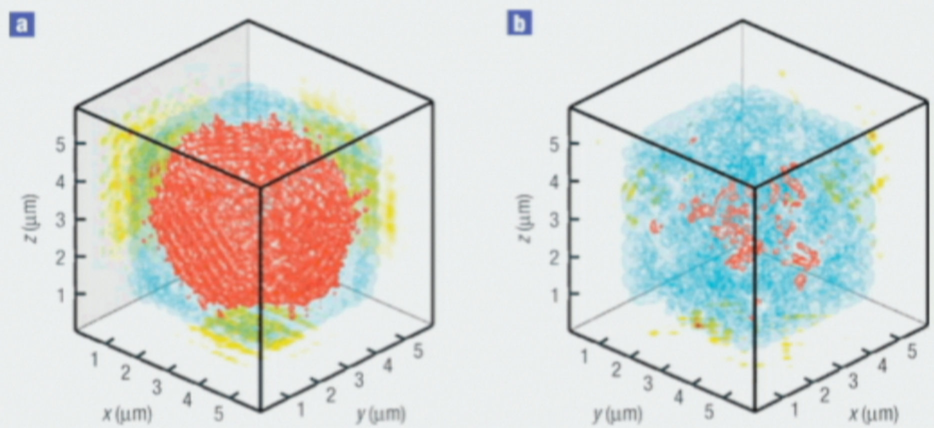
Anderson'58

Weak disorder

Extended states

Strong disorder

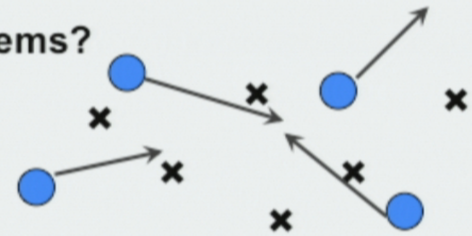
Localized



Many-body localization problem

When/how does ergodicity break in many-body systems?

Do interactions destroy localization?



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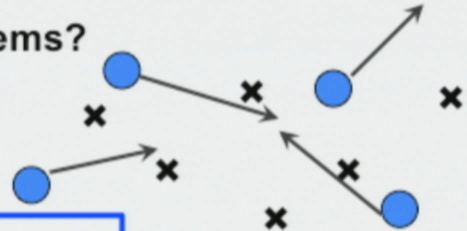


YES

NO

Many-body localization problem

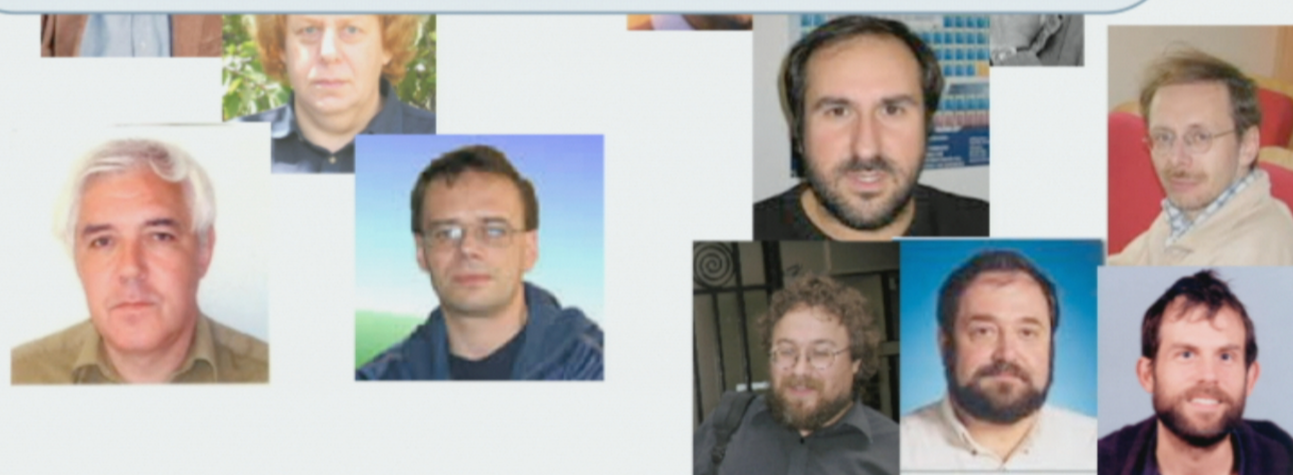
When/how does ergodicity break in many-body systems?

Do i

This talk:* *Universal dynamical properties

Local integrals of motion

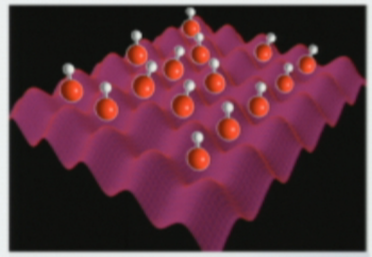
Many-body localization w/o disorder

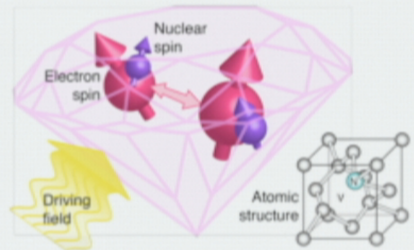


New experimental systems

Isolated & quantum-coherent. Tunable interactions and disorder

-Cold atoms, optical lattices





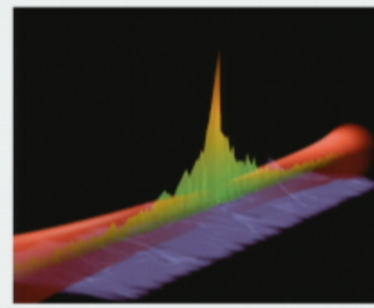
-Polar molecules

-Spin systems (NV-centers in diamond)

REPORTS

Three-Dimensional Anderson Localization of Ultracold Matter

S. S. Kondov, W. R. McGehee, J. J. Zirbel, B. DeMarco*

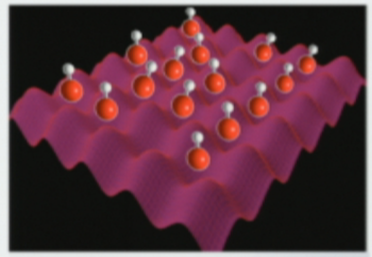


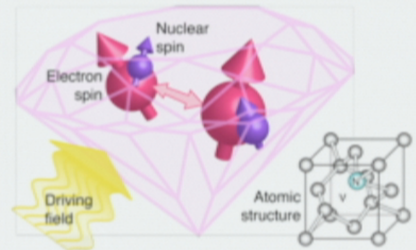
EXP: Ecole Normale, Florence, Urbana

New experimental systems

Isolated & quantum-coherent. Tunable interactions and disorder

-Cold atoms, optical lattices





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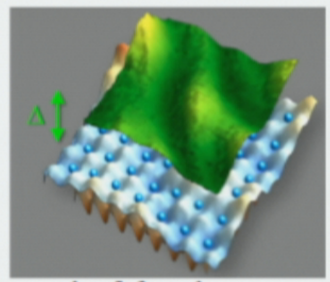
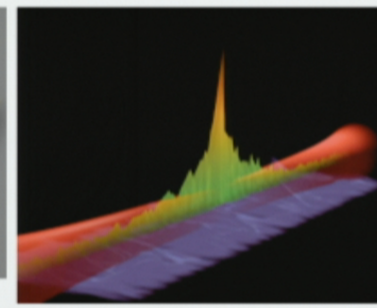




Interplay of disorder and interactions in an optical lattice

Hubbard model

S. S. Kondov, W. R. McGehee, B. DeMarco[1]

(Dated: May 28, 2013)

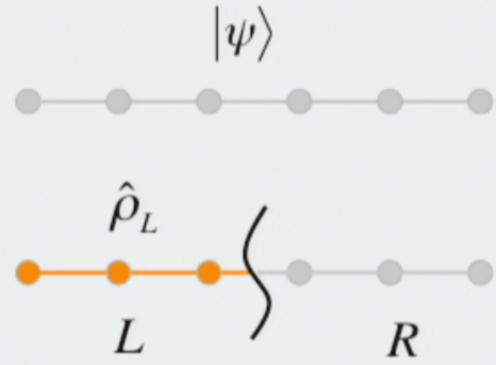
EXP: Ecole Normale, Florence, Urbana

Studying many-body localization experimentally now possible

Entanglement entropy

Pure many-body state $|\psi\rangle$

Reduced density matrix $\hat{\rho}_L = \mathrm{Tr}_R |\psi\rangle\langle\psi|$

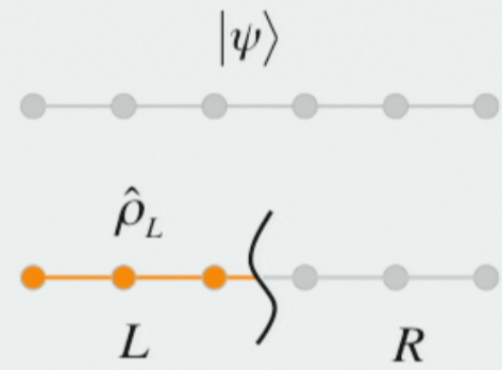


$$S_{ent} = -\mathrm{Tr}\rho_L \log \rho_L$$

A measure of "quantumness"

Entanglement entropy

Pure many-body state $|\psi\rangle$



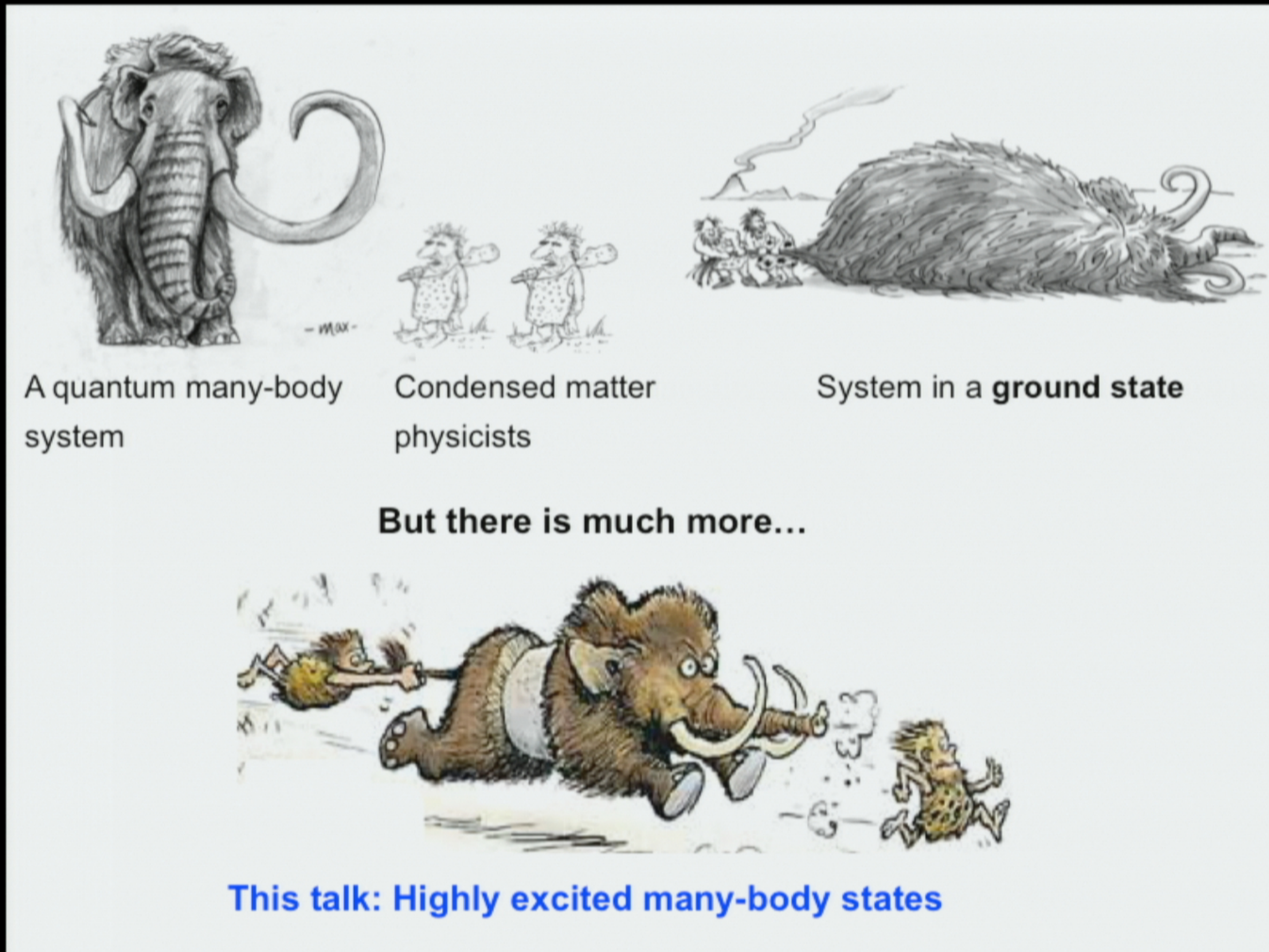
Reduced density matrix $\hat{\rho}_L = \mathrm{Tr}_R |\psi\rangle\langle\psi|$

$$S_{ent} = -\mathrm{Tr}\rho_L \log \rho_L$$

A measure of "quantumness"

Characterize/classify quantum order in ground states
Beyond conventional order parameters

Vidal, Kitaev, Preskill'02, Vidal'03-'14, Verstraete, Cirac'04-14, Levin, Wen'06, Kitaev, Preskill'06, Turner et al'11 Wen et al'11-14


A quantum many-body system
Condensed matter physicists
System in a **ground state**
But there is much more…
This talk: Highly excited many-body states

A model of many-body localization

Jordan-Wigner

Spinless interacting 1D fermions ≈ Random-field XXZ spin-1/2 chain

$$H = \sum_i E_i c_i^+ c_i + t\sum_i c_i^+ c_{i+1} + h.c. + V\sum_i n_i n_{i+1}$$

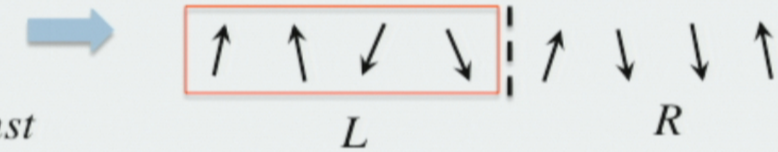
$$H = \sum_i h_i S_i^z + J_\perp \sum_i (S_i^+ S_{i+1}^- + h.c) + J_z \sum_i S_i^z S_{i+1}^z$$

Many-body localization at strong disorder (numerics)

Oganesyan, Huse'07, Prosen et al'08, Pal, Huse'10, Monthus, Garel'10,…

Numerics: entanglement propagation in localized systems

Hamiltonian evolution



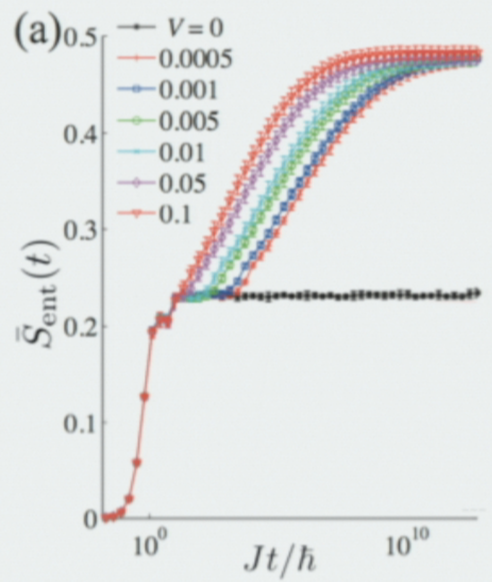
-Anderson-localized: $S_{ent}(t) \leq const$

-Many-body localized: slow growth of entanglement

$$S_{ent}(t) \propto \log t$$

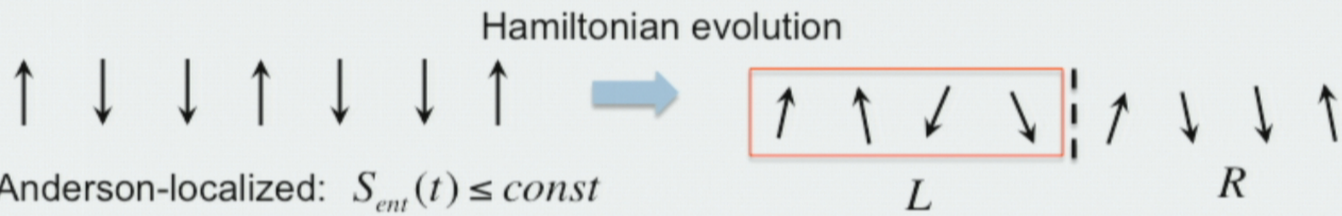
-"Glassy" spread of entanglement

-Very long time scales



Bardarson, Pollmann, Moore'12

Numerics: entanglement propagation in localized systems



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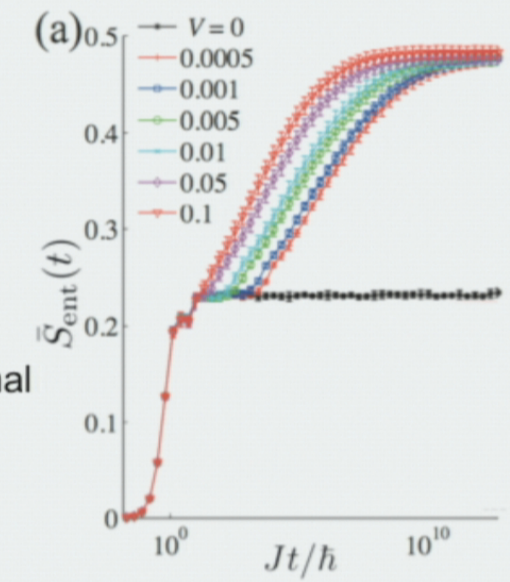
-Very long time scales

-Entanglement <u>extensive in system size,</u> non-thermal

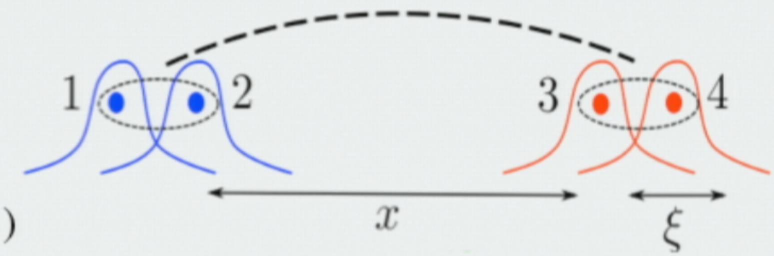
Very slow equilibration? Slow particle transport??



Bardarson, Pollmann, Moore'12

The mechanism of entanglement growth: Toy model

$$|\psi_0\rangle = \frac{1}{2}(c_1^+ + c_2^+)(c_3^+ + c_4^+)|0\rangle$$



Assume weak interactions

Eigenstate $|\alpha\beta\rangle = c_\alpha^+ c_\beta^+ |0\rangle + O(e^{-x/\xi})$

Energy: $E_{\alpha\beta} = E_\alpha + E_\beta + \boxed{C_{\alpha\beta} V e^{-x/\xi}}$

Reduced density matrix

$$\rho(t) = \frac{1}{2}\begin{bmatrix} 1 & \cos\omega t \\ \cos\omega t & 1 \end{bmatrix}$$

$$\omega \sim \frac{V}{\hbar} e^{x/\xi} \qquad \boxed{t_{deph} \sim \frac{2\pi}{\omega} \sim \frac{\hbar}{V} e^{x/\xi}}$$

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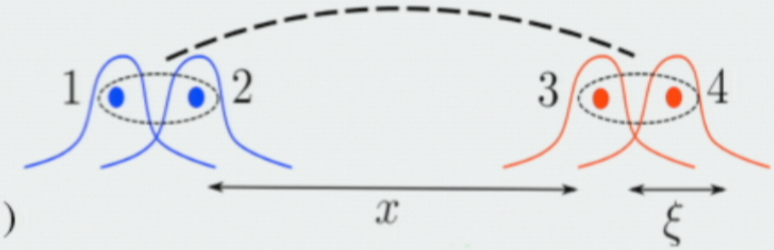

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Interaction-induced dephasing → entanglement generation

Serbyn, Papic, Abanin PRL '13

Case of many particles

Intuition: Eigenstates at **small *V*** are "close" to non-interacting eigenstates

Non-interacting: occupation numbers

$$n_\alpha = \left\langle c_\alpha^+ c_\alpha \right\rangle = 0,1$$

Interacting: obtain by small local deformations

$$n_\alpha = \left\langle c_\alpha^+ c_\alpha \right\rangle \approx 0,1$$

Energy: perturbation theory in *V*

$$E = \sum E_\alpha n_\alpha + V \sum C_{\alpha\beta} n_\alpha n_\beta e^{-\frac{|R_\alpha - R_\beta|}{\xi}} + V^2 \sum C_{\alpha\beta\gamma} n_\alpha n_\beta n_\gamma e^{-\frac{|R_\alpha - R_\beta| + |R_\gamma - R_\beta|}{\xi}} \ldots$$

1-body energy

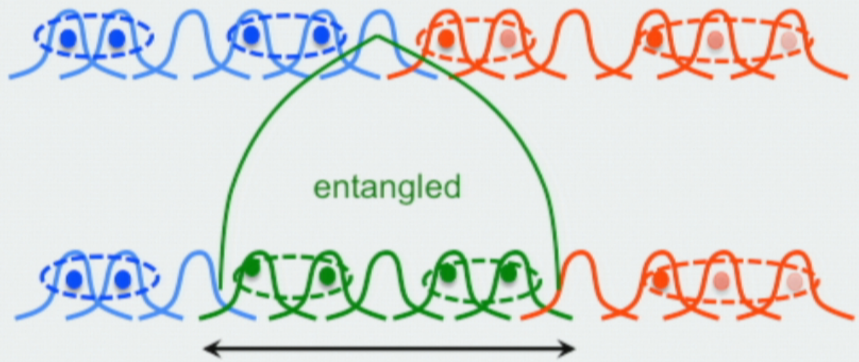
2-body interactions

3-body interactions

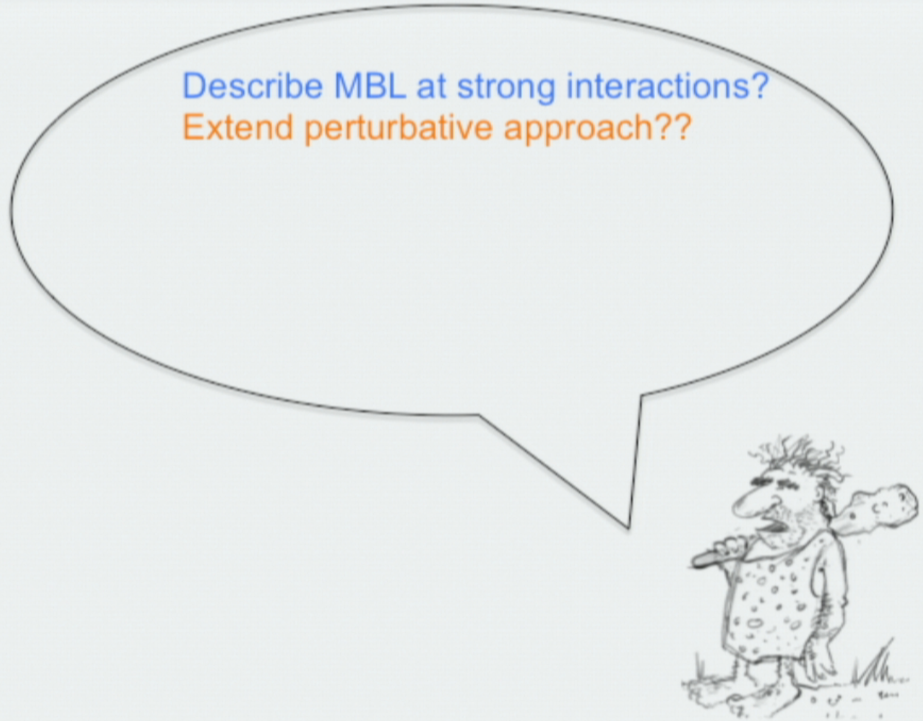
The laws of entanglement growth

Initial product state is a superposition of many eigenstates

$$t(x) \sim \frac{\hbar}{V} e^{x/\xi}$$



$$x(t) \sim \log \frac{Vt}{\hbar}$$


Describe MBL at strong interactions?
Extend perturbative approach??

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Extend perturbative approach??

KEY: ENTANGLEMENT STRUCTURE

LOCAL INTEGRALS OF MOTION

Hint: at weak interactions, we assumed

$$\left[c_\alpha^+ c_\alpha, H\right] \approx 0$$

Constructing local integrals of motion

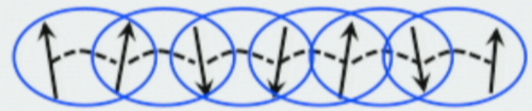
$$H_0 = \sum_i h_i s_i^z + J_z s_i^z s_{i+1}^z \qquad \uparrow\ \uparrow\ \downarrow\ \downarrow\ \uparrow\ \downarrow\ \uparrow \qquad s_i^z = \pm 1$$

Constructing local integrals of motion

$$H_0 = \sum_i h_i s_i^z + J_z s_i^z s_{i+1}^z$$

$$s_i^z = \pm 1$$

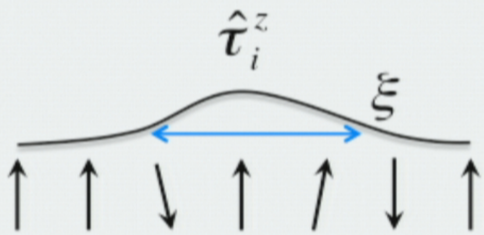
$$H = H_0 + \sum_i J_x s_i^+ s_{i+1}^- + h.c.$$



Local unitary

Hamiltonian diagonalized by sequence of local unitary transformations

$$U^+ H U = H_{diag}$$



Local integral of motion $\hat{\tau}_i^z = U \hat{s}_i^z U^+$

$$[\hat{\tau}_z^i, H] = 0$$

"Effective spins", form a complete set

Serbyn, Papic, Abanin, PRL'13
Huse, Oganesyan, arXiv'13

Universal Hamiltonian of many-body localized phase

$[\hat{\tau}_z^i, H] = 0$ → Hamiltonian depends only on $\hat{\tau}_z^i$'s

$$H = \sum_i H_i \tau_z^i + \sum_{ij} H_{ij} \tau_z^i \tau_z^j + \sum_{ijk} H_{ijk} \tau_z^i \tau_z^j \tau_z^k + ..$$

$H_{ij} \propto \exp(-|i-j|a/\xi)$, random

Quantum bits which cannot relax

Exponentially decaying (random) interactions → dephasing

Serbyn, Papic, Abanin, PRL'13
Huse, Oganesyan, arXiv'13

Structure of localized eigenstates

Conjecture: MBL eigenstates are obtained from product states by nearly local unitary transformations (recently proven by Imbrie'14)

Implication 1: Integrals of motion

$$H = \sum_i H_i \tau_z^i + \sum_{ij} H_{ij} \tau_z^i \tau_z^j + \sum_{ijk} H_{ijk} \tau_z^i \tau_z^j \tau_z^k + ..$$

Implication 2: Eigenstates have low entanglement entropy, "area-law"

$$S_{ent}(L) \leq Const$$

Contrast: "volume-law" of excited states in ergodic systems

$$S_{ent}(L) \sim L$$

Efficient classical simulation of MBL states

Experimental signatures

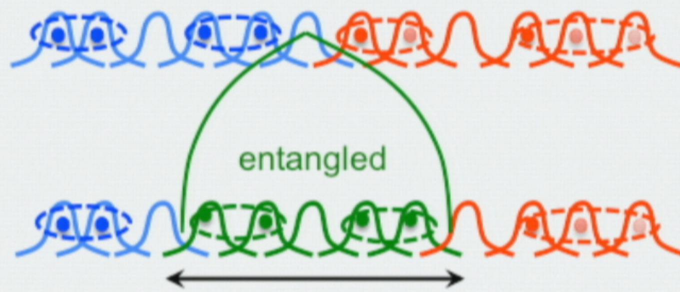
Quench experiment

-Steady non-thermal state
"Local diagonal ensemble"

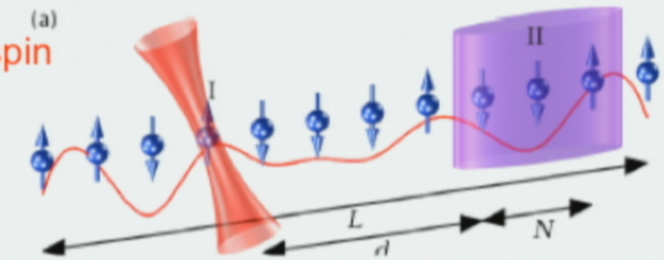
$$\left\langle \tau_z^i(t) \right\rangle = Const$$

-Universal growth of entanglement

$$S_{ent}(t) \sim \log(t)$$

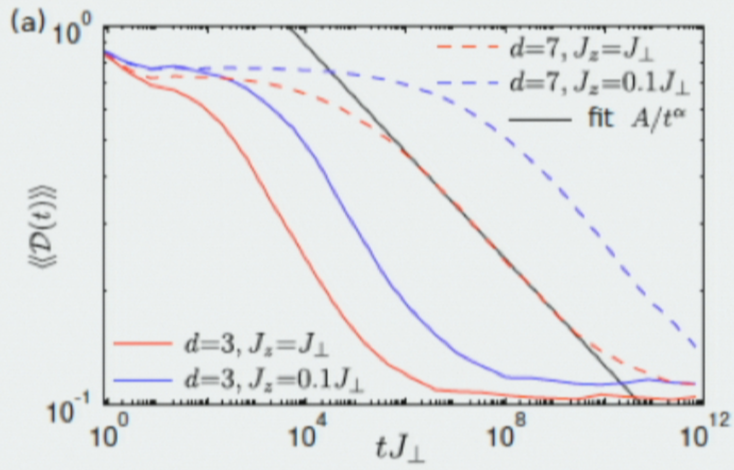


Probing many-body localization with spin-echo



Dephasing → power-law spin-echo decay $\bar{\mathcal{D}}(t) \sim t^{-\xi \ln 2}$

Directly probe MBL in spin systems, cold atoms



Serbyn et al, arXiv'14

Relating effective spins to physical operators

Effective spins are long-lived qubits. How to relate them to physical operators?

Relating effective spins to physical operators

Effective spins are long-lived qubits. How to relate them to physical operators?

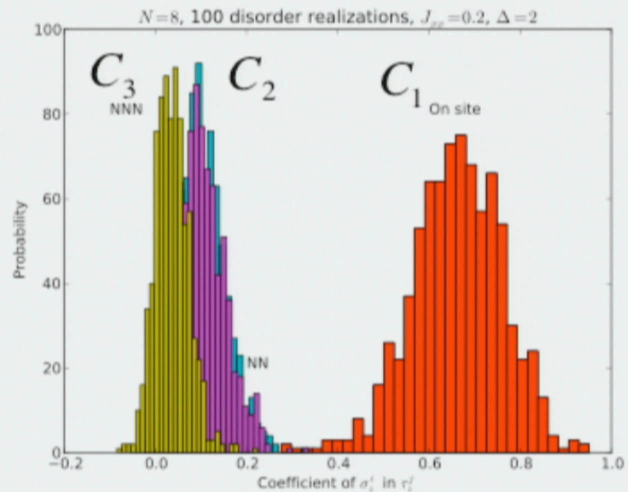
Approach 1: Find local unitary which diagonalizes a MBL Hamiltonian

$$\tau_z^i = U s_z^i U^+$$

Express τ_z^i via physical operators

$$\tau_z^i = \boxed{C_1 s_z^i + C_2 s_z^{i+1} + C_3 s_z^{i+2}} + D_1 s_z^i s_x^{i+1} +$$

Approach 2: Variational, works for large systems (in progress)

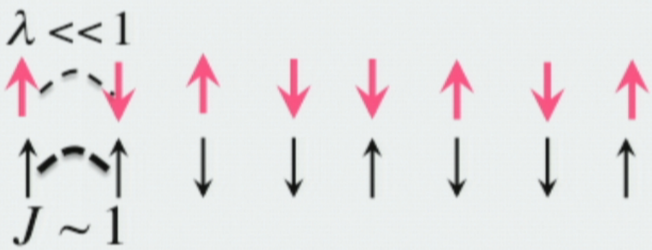


Chandran, Kim, Vidal, DA '14

Localization without quenched disorder

(Schiulaz, Muller'13, Huveneers, de Roeck'13, Grover, Fisher'13..)

Two coupled XY spin chains, "fast" σ_i^z and "slow" s_i^z



$$H_{\text{int}} = W \sum_i \sigma_i^z s_i^z$$

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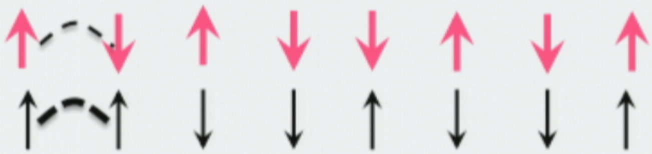
(Schiulaz, Muller'13, Huveneers, de Roeck'13, Grover, Fisher'13..)

Two coupled XY spin chains, "fast" σ_i^z and "slow" s_i^z

$\lambda << 1$



$J \sim 1$

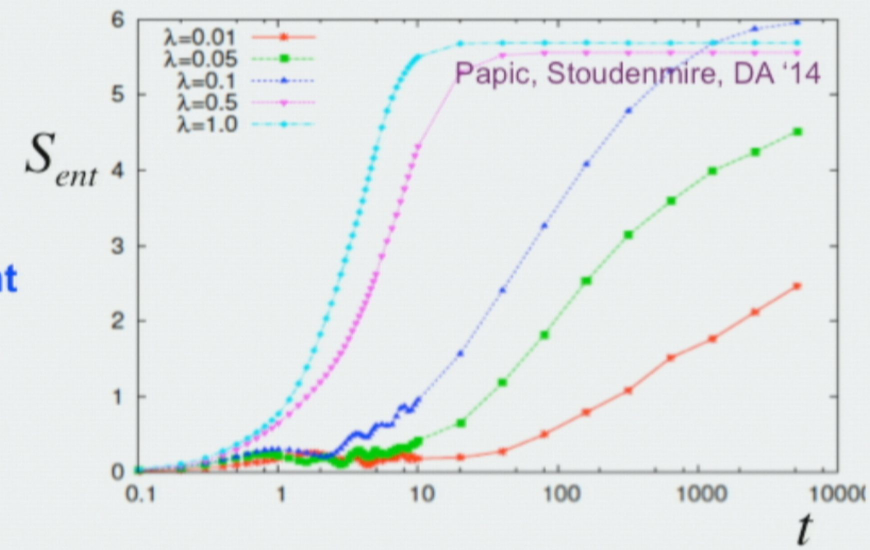
$$H_{\text{int}} = W \sum_i \sigma_i^z s_i^z$$

Entanglement growth

Localization at $\lambda << 1$?

Preliminary:

Log-growth of entanglement

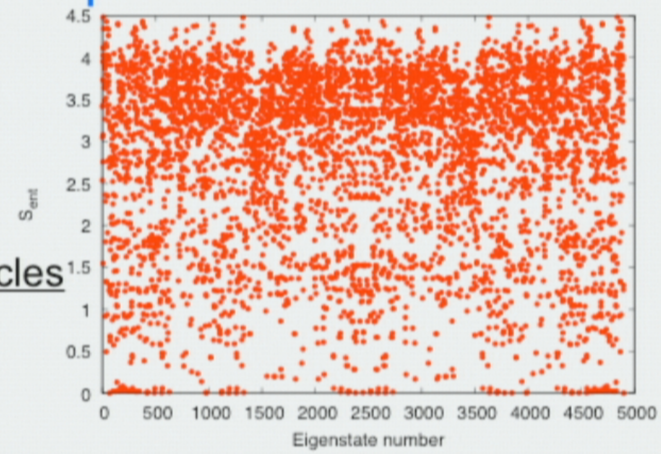


Localization without quenched disorder

Broad distribution of saturated S_{ent}

High and low-entropy states at the same energy

5+5 particles

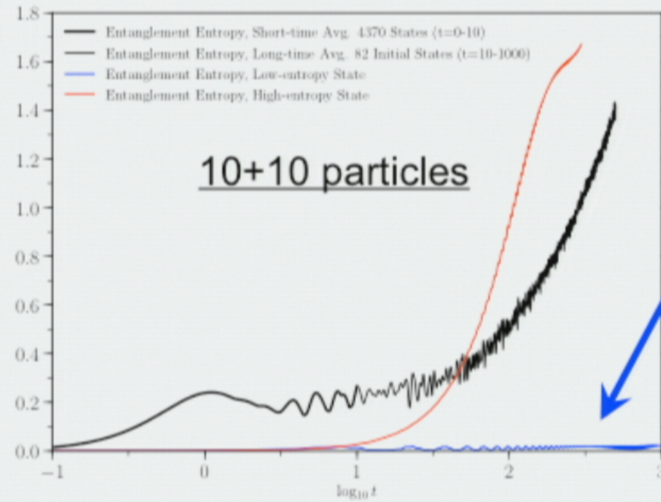


Are there many low-entropy states in thermodynamic limit?

YES!

But more work needed

Papic, Stoudenmire, DA '14



low-entropy state

Summary

-Local integrals of motion → ergodicity breaking

-Universal dynamics in MBL phase

-Many-body localization without quenched disorder

<u>MESSAGE:</u> ENTANGLEMENT GIVES INSIGHTS INTO ERGODICITY AND ITS BREAKING