Title: A Lattice Non-Perturbative Definition of Chiral Fermnion/Gauge Theory from Topological Order

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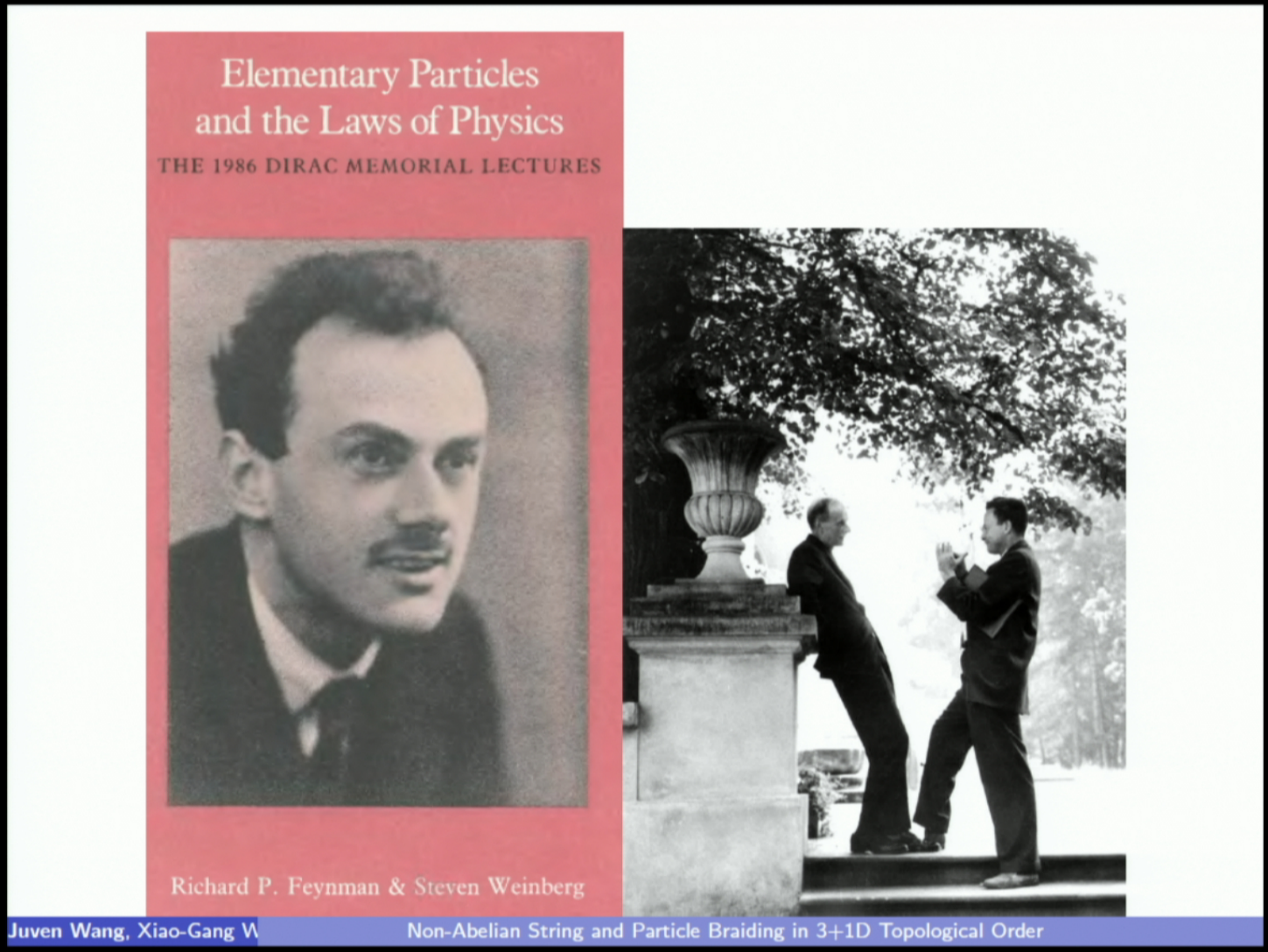
Abstract: A non-perturbative definition of anomaly-free chiral fermions and bosons in 1+1D spacetime as finite quantum systems on 1D lattice is proposed. In particular, any 1+1D anomaly-free chiral matter theory can be defined as finite quantum systems on 1D lattice with on-site symmetry, if we include strong interactions between matter fields. Our approach provides another way, apart from Ginsparg-Wilson fermions approach, to avoid the fermion-doubling challenge. In general, using the defining connection between gauge anomalies and the symmetry-protected topological orders, we propose that any truly anomaly-free chiral gauge theory can be non-perturbatively defined by putting it on a lattice in the same dimension. As an additional remark, we conjecture/prove the equivalence relation between 't Hooft anomaly-matching conditions and the boundary fullygapping rules.

Non-Abelian String and Particle Braiding in 3+1D Topological Order

Juven Wang, Xiao-Gang Wen
(arXiv:1404.7854)

MIT/Perimeter Inst.

4K ONTARIO CMP SYMPOSIUM 2014


Elementary Particles
and the Laws of Physics
THE 1986 DIRAC MEMORIAL LECTURES
Richard P. Feynman & Steven Weinberg

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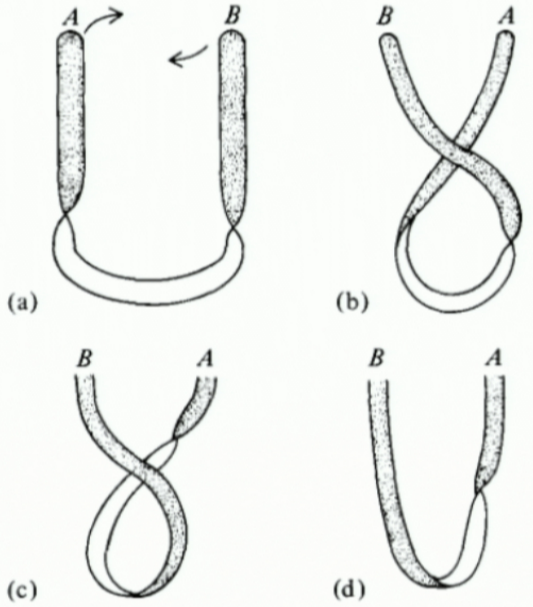

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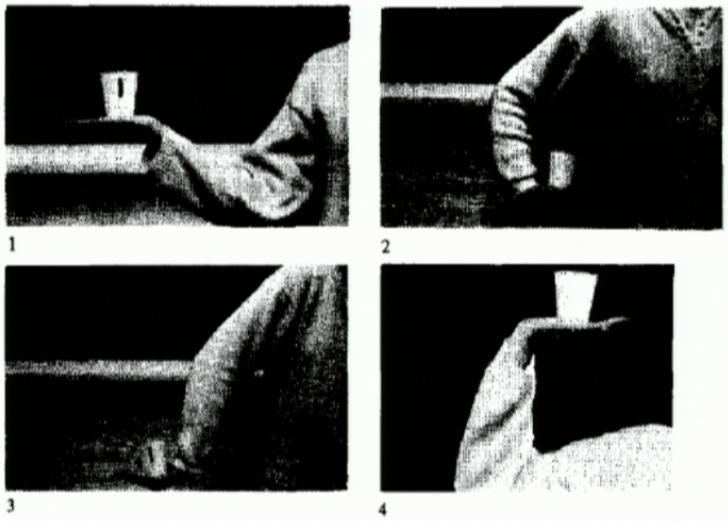


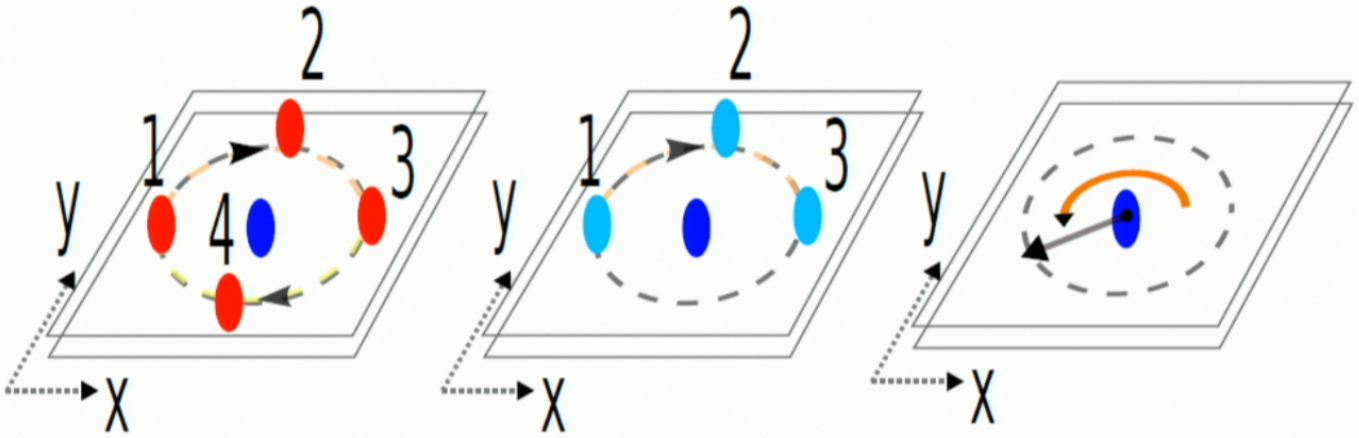
Outline: Three Key Questions

- **Q1**: What are the **braiding statistics** of excitations in 3+1D?
- **Q2**: How to understand a **3D topological order** from a 2D viewpoint?
- **Q3**: 2D topological order data: the topology-dept. **ground state degeneracy (GSD)** and SL(2,$\mathbb{Z}$) modular data (nonAbelian gauge/Berry structure of ground state). How to characterize the 3D topological order?

Braiding statistics in 2+1D

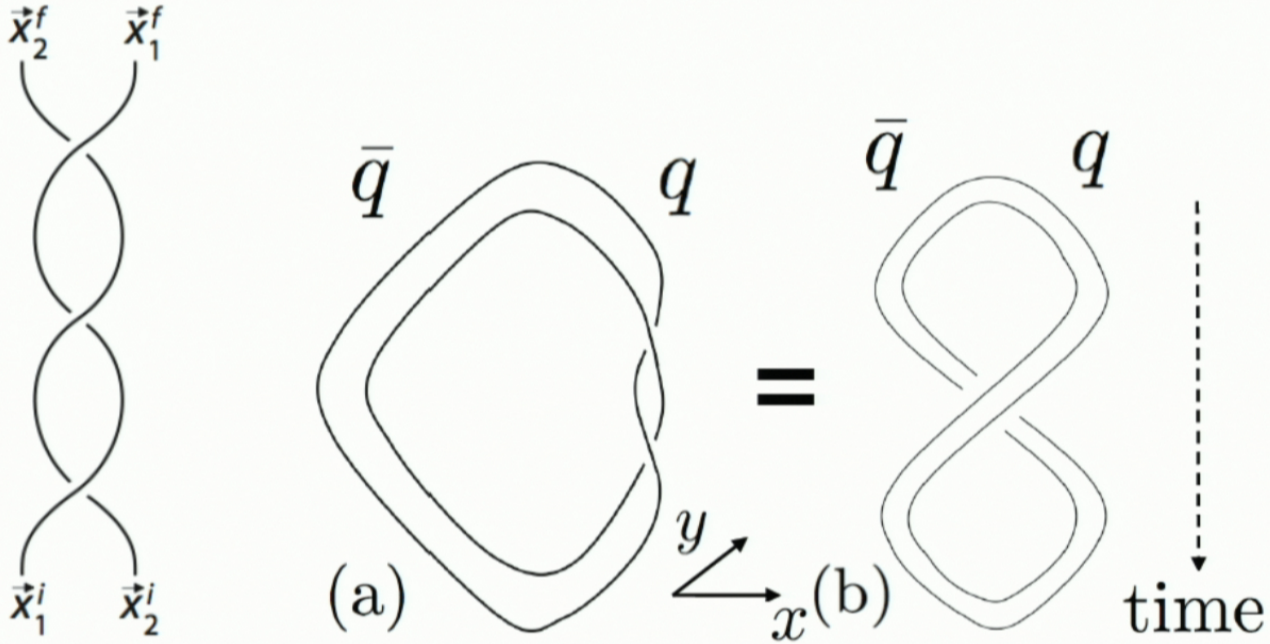
particle-particle braiding:

Mutual statistics, exchange statistics, spin



Braiding statistics in 2+1D

Mutual statistics and exchange statistics (**particle-particle braiding**):



spin statistics theorem

Braiding statistics in 3+1D

- **particle-particle braiding**:

fermionic or bosonic.

Exchange twice the overall phase gained of wavefunction is $e^{2i\theta} = 1$.
So $e^{i\theta} = \pm 1$.

- **particle(charge)-string(flux) braiding**:

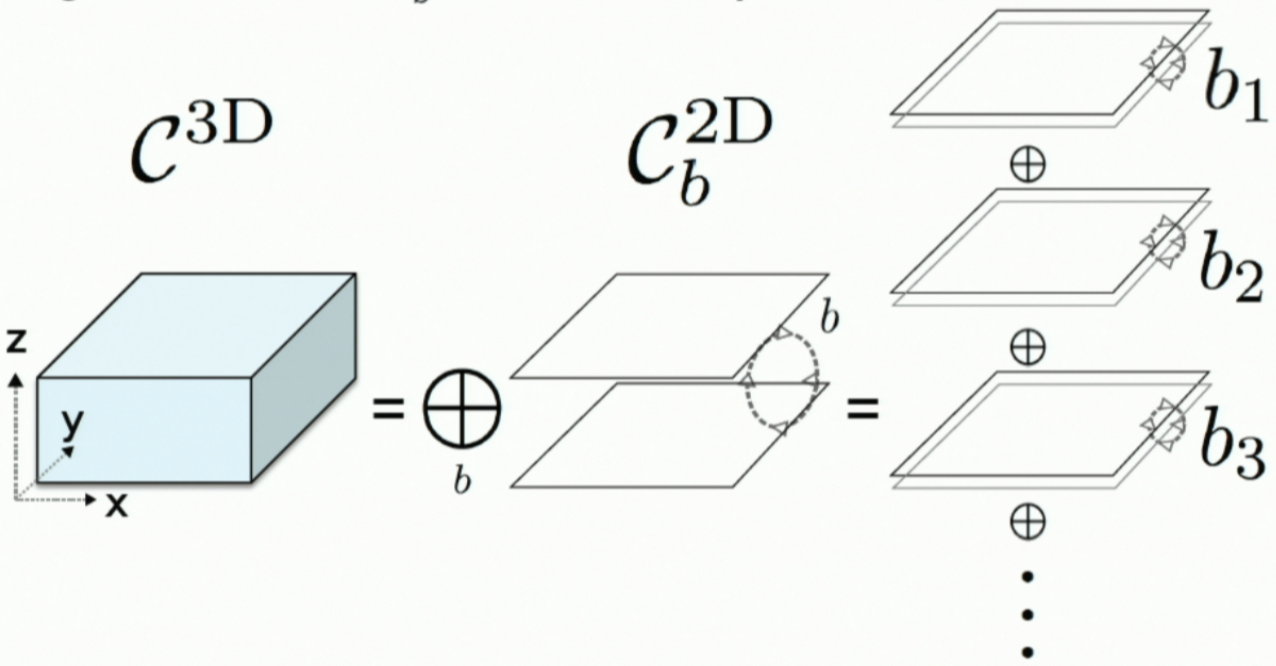
Aharonov-Bohm effect.

a discrete Z_N gauge theory (dynamical Z_2 superconductor, Z_2 toric code/spin liquids, mutual Chern-Simons, BF, ...)

- **string-string braiding?**

Where are strings? From 3D $\longrightarrow$ 2D

- The **Hilbert space of ground states** of 3D topological order $\mathcal{C}^{3D}$ as a **direct sum** of different b flux sectors of the Hilbert space of ground states of $\mathcal{C}_b^{2D}$. **NOT real space**.



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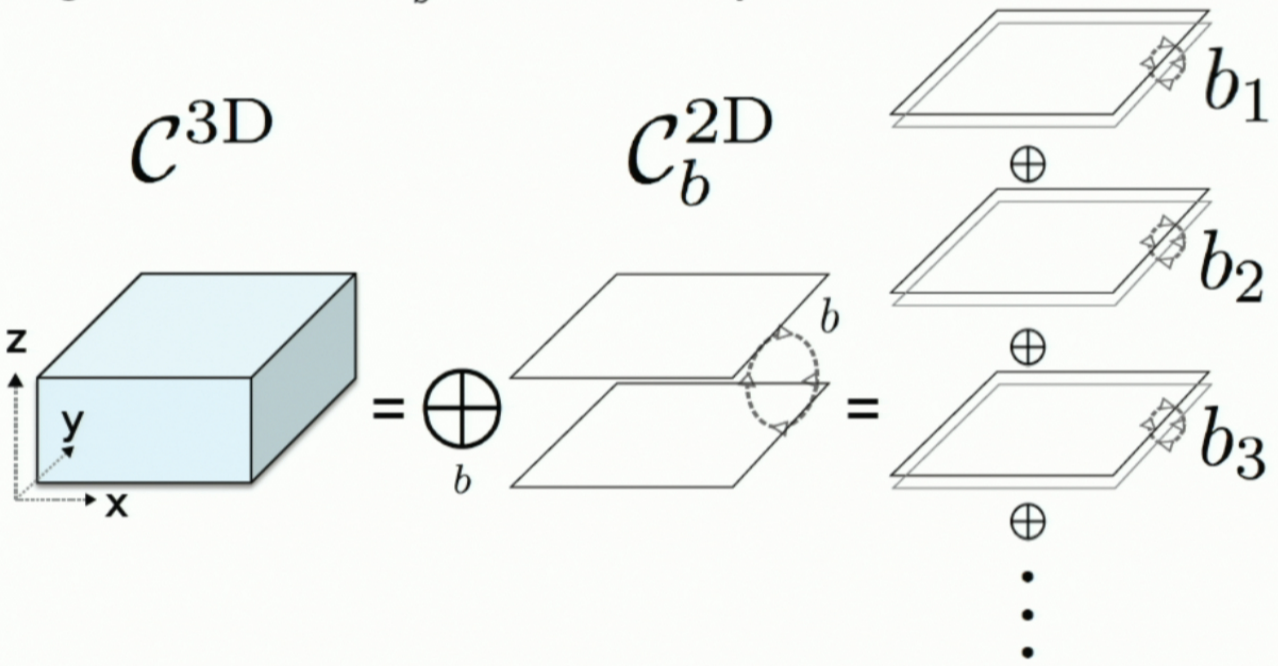

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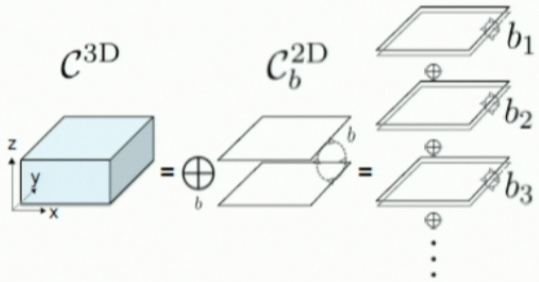
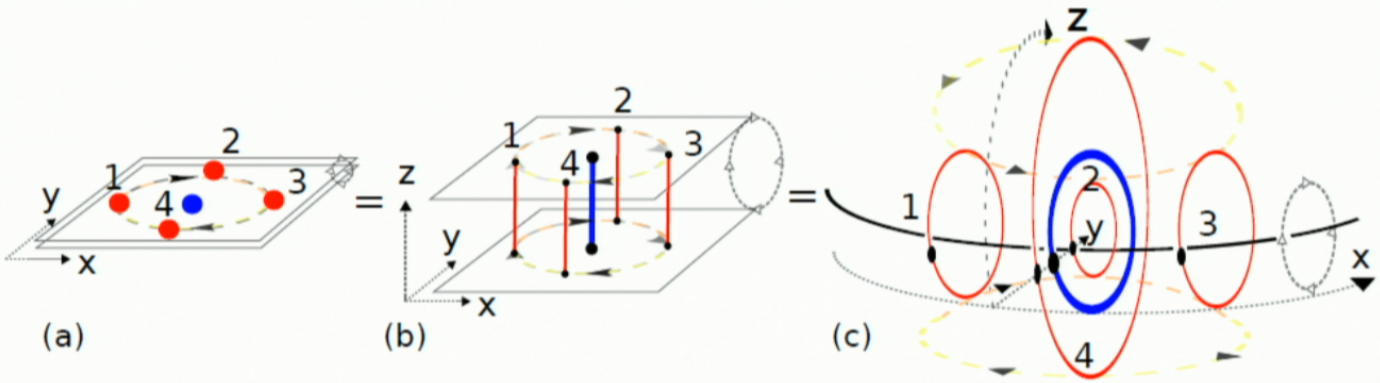
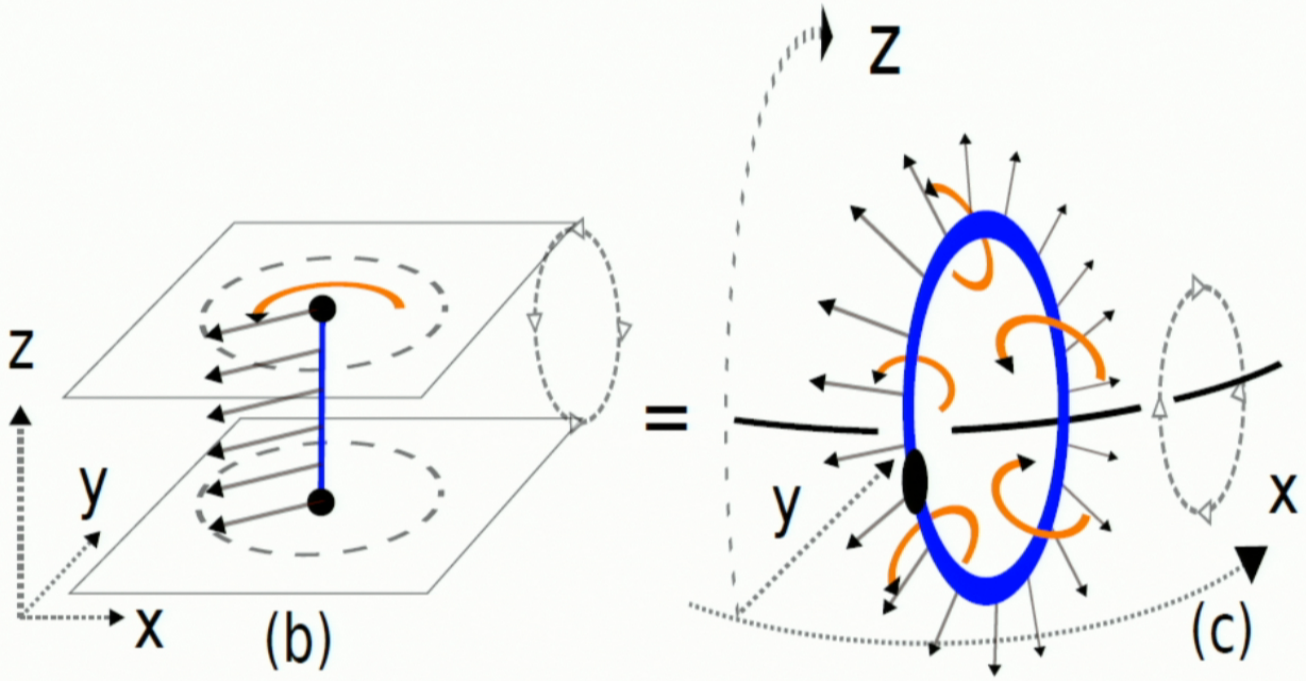




Where are strings? From 3D $\longrightarrow$ 2D

Self Flipping (Spin? Topological Spin) by 2π



NonAbelian statistics from 3+1D Abelian twisted discrete gauge theory

Dimensional Reduction in the Hilbert space

$$\begin{aligned}
&\mathcal{C}^{3D}_{(Z_2)^4,\omega_{4,IV}} = \oplus_b \mathcal{C}^{2D}_b \\
&= \mathcal{C}^{2D}_{(Z_2)^4} \oplus 10\, \mathcal{C}^{2D}_{(Z_2)\times(Z_2)^3_{(ijl)},\omega^{(ijl)}_{3,III}} \oplus 5\mathcal{C}^{2D}_{(Z_2)^4,\omega_{3,III}\times\omega_{3,III}\times\ldots} \\
&= \mathcal{C}^{2D}_{(Z_2)^4} \oplus 10\, \mathcal{C}^{2D}_{(Z_2)\times(D_4)} \oplus 5\mathcal{C}^{2D}_{(Z_2)^4,\omega_{3,III}\times\omega_{3,III}\times\ldots}
\end{aligned} \tag{1}$$

D_4 is nonAbelian Dihedral group,

so $\mathcal{C}^{2D}_{(Z_2)\times(D_4)}$ has nonAbelian sts.

also $\mathcal{C}^{2D}_{(Z_2)^4,\omega_{3,III}\times\omega_{3,III}\times\ldots}$ has nonAbelian sts.

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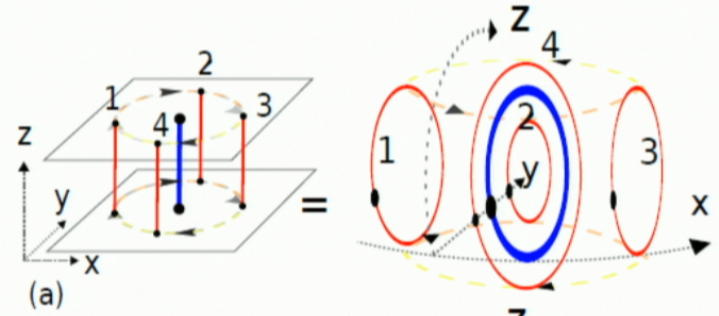

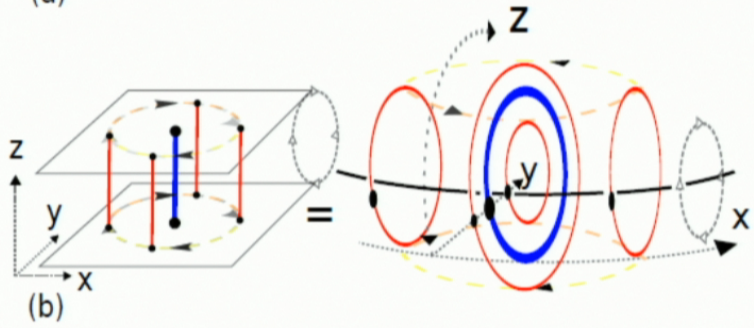

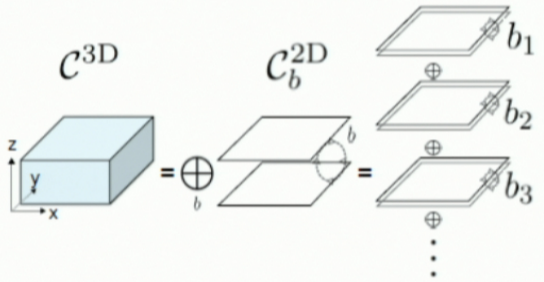

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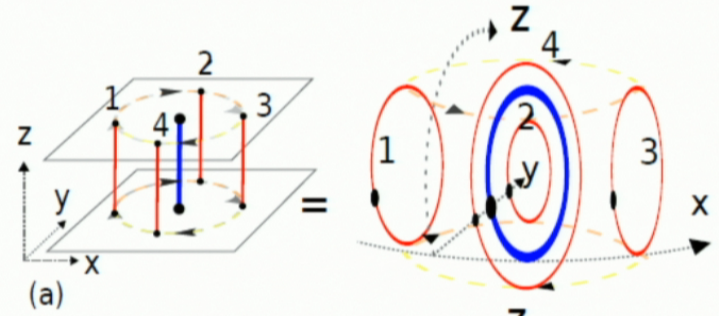

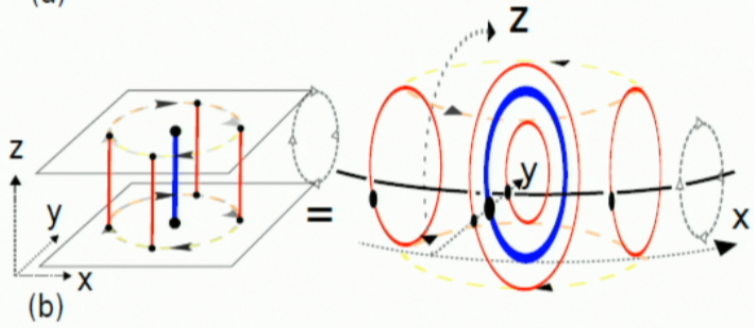

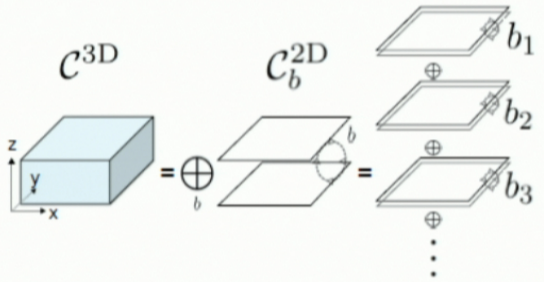

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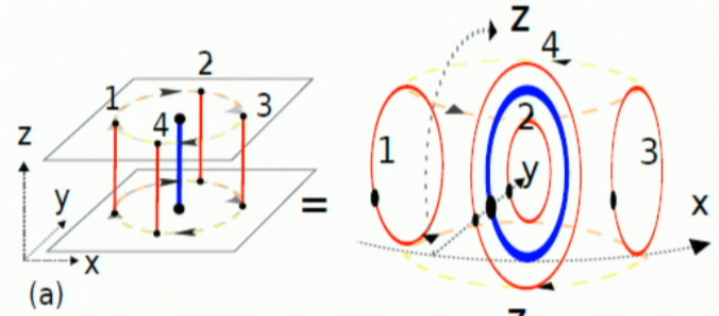
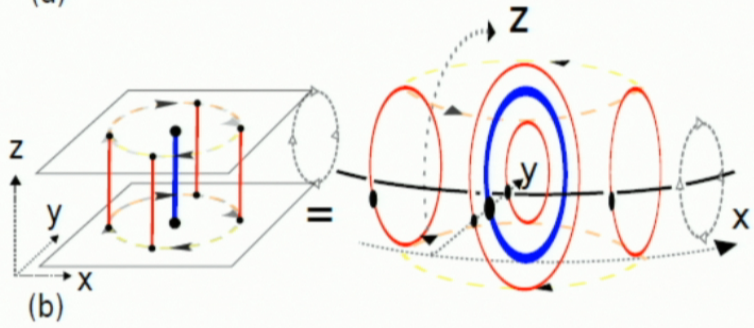
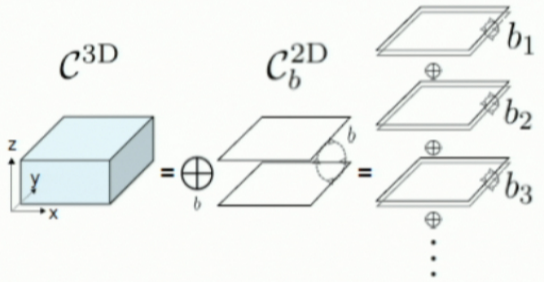
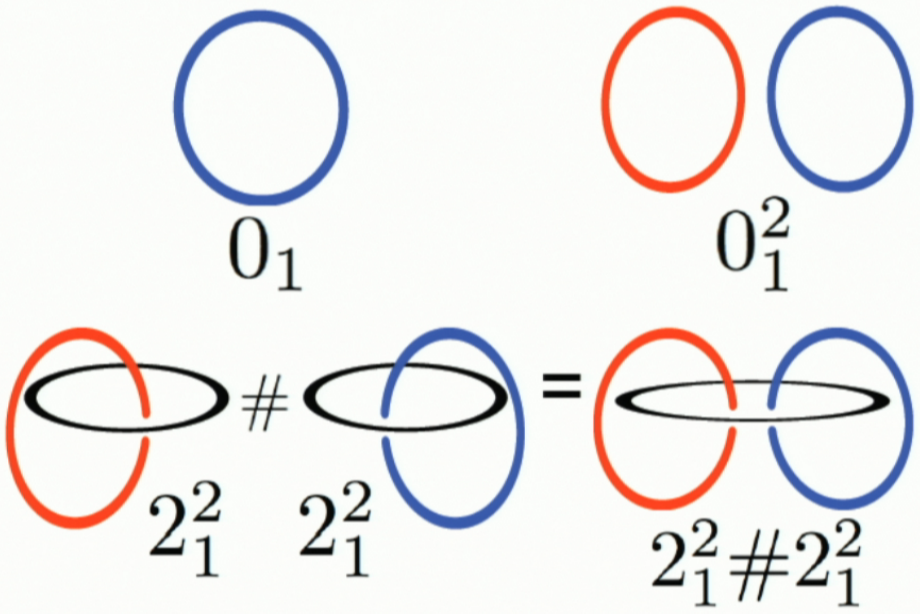






Knots and Links pattern in 3D

Unknot, Unlink, Hopf links, a connected sum of two Hopf links



String Braiding

$\Leftrightarrow$ **nonAbelian statistic** by threading a third string into two strings (unlinks $0_1^2 \rightarrow$ Hopf links $2_1^2 \# 2_1^2$). (for certain Abelian twisted gauge theory)

Braiding statistics - (G, ω_4) of $\mathcal{C}^{3D}_{G,\omega_4} = \oplus_b \mathcal{C}^{2D}_b$	$b = 0$ braiding 2-strings 0_1^2	$b \neq 0$ braiding 3-strings $2_1^2 \# 2_1^2$
$(G_{Abel}, 1)$	Abelian st.	Abelian st.
$(G_{Abel}$, without $\omega_{4,\mathrm{IV}})$	Abelian st.	Abelian st.
$(G_{Abel}$, with $\omega_{4,\mathrm{IV}})$	Abelian st.	nonAbelian st.
$(G_{nAbel}, 1)$	nonAbelian st.	(non)Abelian st.
(G_{nAbel}, ω)	nonAbelian st.	nonAbelian st.

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(G_{nAbel}, ω)	nonAbelian st.	nonAbelian st.

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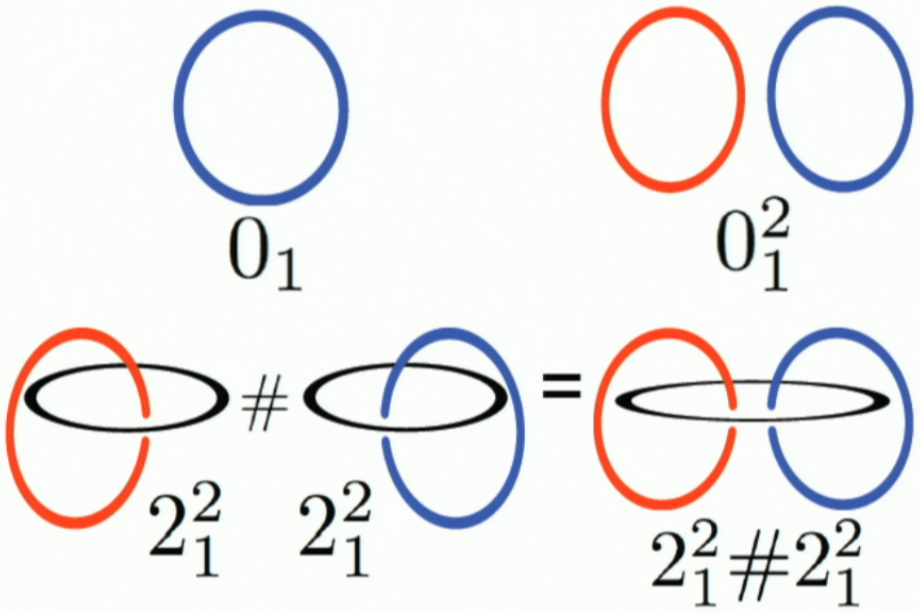

Outline: Three Key Messages

- **Q1**: **braiding statistics** of excitations in 3+1D? particle, string, 3(multi-)strings.

- **Q2**: How to understand a **3D topological order** from a 2D viewpoint?

$\mathcal{C}^{3D} = \oplus_b \mathcal{C}_b^{2D}$

- **Q3**:How to characterize the 3D topological order?

topological order data: the topology-dept. **GSD** and SL(3,$\mathbb{Z}$) modular data (gauge/Berry structure of ground state) and Mapping Class Group.