Title: Renormalization Group Analysis of a Non-Fermi Liquid System

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Abstract: We devise a renormalization group analysis for quantum field theories with Fermi surface to study scaling behaviour of non- Fermi liquid states in a controlled approximation. The non-Fermi liquid fixed points are identified from a Fermi surface in (m+1) spatial dimensions, while the co-dimension of Fermi surface is also extended to a generic value. We also study superconducting instability in such systems as a function of dimension and co-dimension of the Fermi surface. The key point in this whole analysis is that unlike in relativistic QFT, the Fermi momentum kF enters as a dimensionful parameter, thus modifying the naive scaling arguments. The effective coupling constant is found to be a combination of the original coupling constant and kF.

RG Analysis of a Non-Fermi Liquid System

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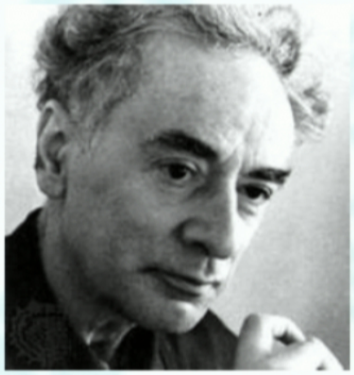
Plan of the talk

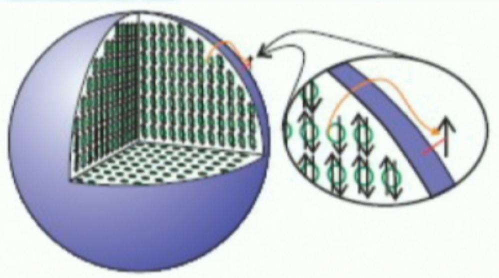
- Prologue → Landau Fermi-Liquid Theory
- Breakdown of FLT → Non-Fermi Liquids
- Ising-Nematic QCP
- Dimensional & Co-dimensional RG
- Critical Dimension
- Beta-Functions & Critical Exponents
- Stable NFL Fixed Point
- Superconducting Instability
- Epilogue

Landau Fermi-Liquid Theory



[***Landau, 50s***]: A finite density of interacting fermions doesn't depend on specific microscopic dynamics of individual systems :–

- **Ground state**: characterized by a sharp Fermi surface (FS) in momentum space
- **Low energy excitations**: weakly interacting quasi-particles around FS



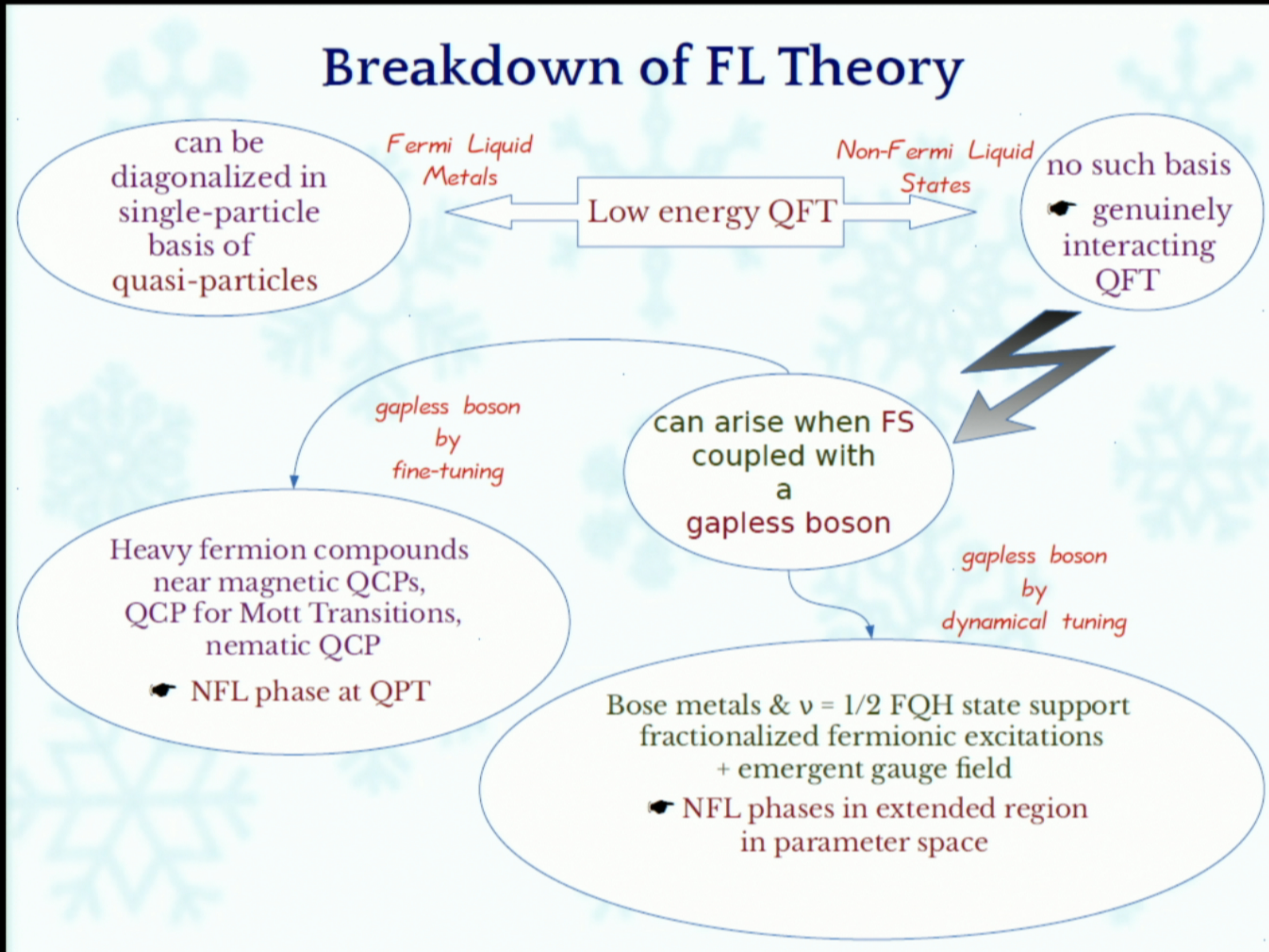
$$\left(\omega = 0\,, \quad k_\perp \equiv k - k_F = 0\right)$$

⇓

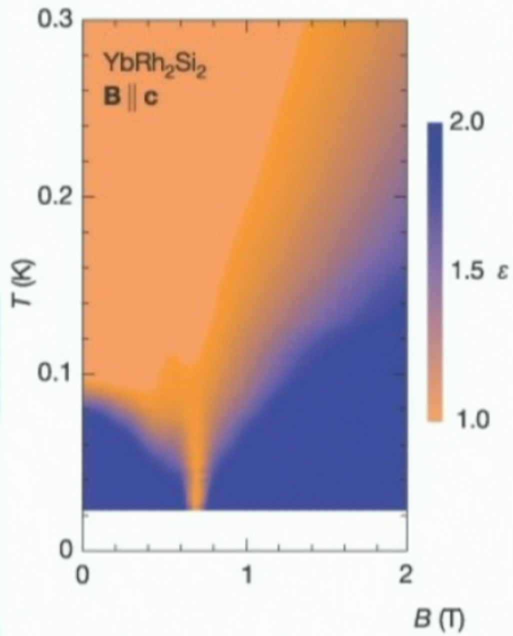
$$G_R(\omega, \vec{k}) = \frac{Z}{\omega - v_F\, k_\perp + i\,\Gamma}$$

❶ Quasi-particles have long life time ☛ Decay rate $\Gamma \sim \omega^2$

❷ Electron has a finite over-lap with quasi-particle adiabatically connected to non-interacting Fermi gas ☛ quasi-particle wt $Z > 0$


Breakdown of FL Theory
can be diagonalized in single-particle basis of quasi-particles
Fermi Liquid Metals
Low energy QFT
Non-Fermi Liquid States
no such basis
☛ genuinely interacting QFT
can arise when FS coupled with a gapless boson
gapless boson by fine-tuning
Heavy fermion compounds near magnetic QCPs, QCP for Mott Transitions, nematic QCP
☛ NFL phase at QPT
gapless boson by dynamical tuning
Bose metals & ν = 1/2 FQH state support fractionalized fermionic excitations + emergent gauge field
☛ NFL phases in extended region in parameter space

Unusual Scaling Phenomenology



[Custers et al, Nature (2003)]

$[\rho(T) - \rho_0] \propto T^{\varepsilon}$

ε =1 for NFL (yellow)

ε =2 for FL (blue)

❶ Calculational framework that replaces FL theory needed.

❷ QFT of metals ☛ low symmetry + extensive gapless modes need to be kept in low energy theories ☛ less well understood compared to relativistic QFTs.

Dimension as A Tuning Parameter

- Couplings irrelevant for d > upper critical dim d_C.

- For d < d_C couplings become slightly relevant & theories flow into a perturbative fixed point.

- Modify spacetime dim continuously to gain a controlled access to NFL states

  ☛ Extend

  ❶ FS dim [Chakravarty, Norton and Syljuasen (1995) ☛ issues with k_F]

  or

  ❷ FS co-dim [Senthil and Shankar (2009); Dalidovich and Lee (2013)]

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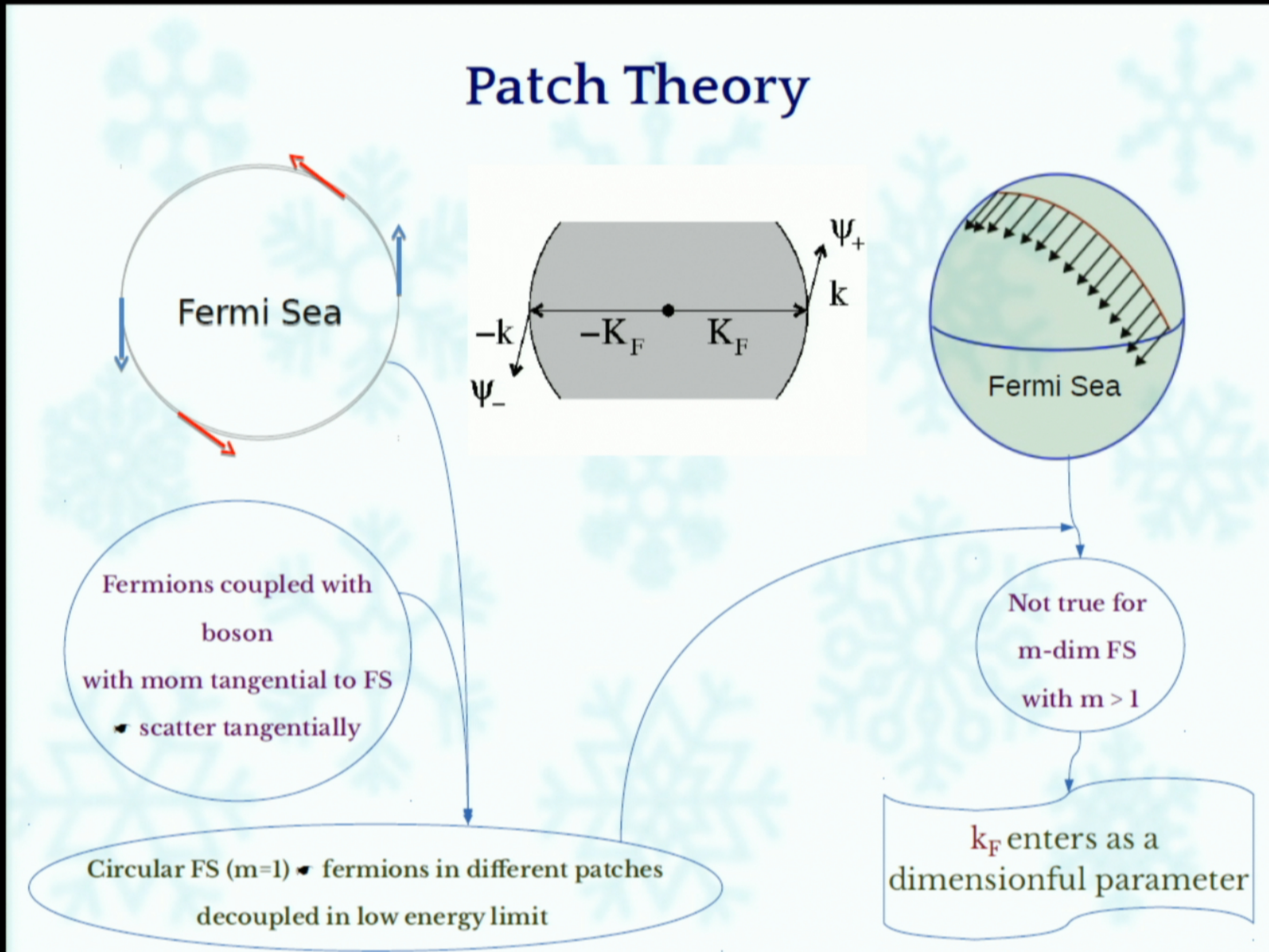

Patch Theory
Fermi Sea
−k
ψ_-
$-K_F$
K_F
ψ_+
k
Fermi Sea
Fermions coupled with
boson
with mom tangential to FS
➨ scatter tangentially
Not true for
m-dim FS
with m > 1
Circular FS (m=1) ➨ fermions in different patches
decoupled in low energy limit
k_F enters as a
dimensionful parameter

Role of "k_F"

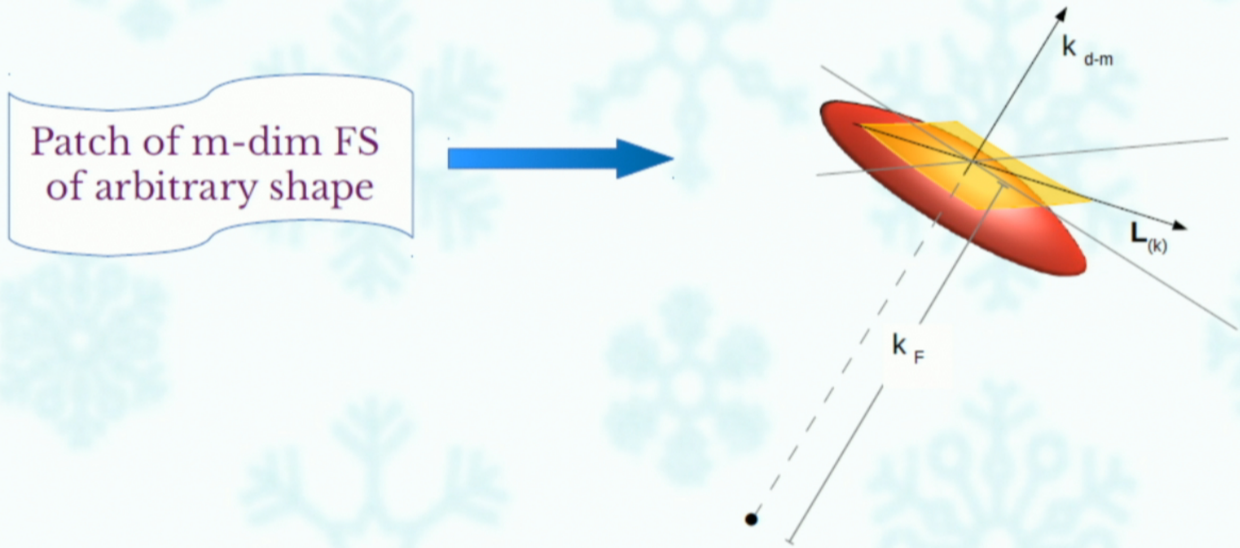
- Dalidovich & Lee [Phys. Rev. B 88, 245106 (2013)]
  ☛ DR for circular FS extending co-dim to a generic value with m=1 fixed ☛ (2+1)-d ISN QCP described by a stable NFL state slightly below $d_c = 5/2$.

- We devise DR extending both **dim** & **co-dim** ☛ FS with m>1 included naturally.

- k_F enters as a dimensionful parameter modifying naïve scaling arguments for m>1.

- The global FS should be considered. But patch coords still allow to extract leading order effect of the global couplings of different patches through k_F.

Generic Fermi Surface



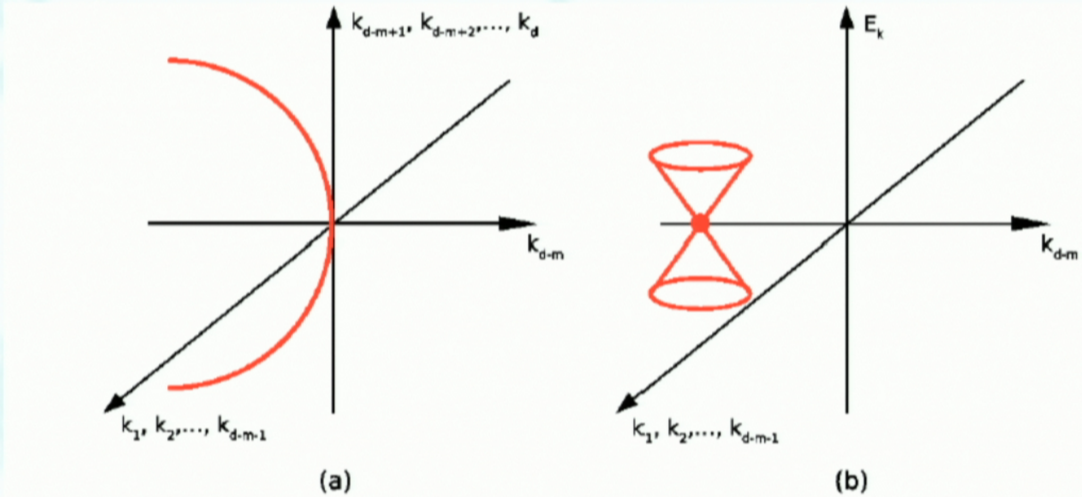
- At a chosen point on FS : $k_{d\text{-}m} \perp$ local S^m ☛ its magnitude measures deviation from k_F .
- $L_{(k)} = (k_{d-m+1} , k_{d-m+2} ,..., k_d)$ ☛ tangential along the local S^m.

Action

2 patches of m-dim FS
coupled with one critical boson
in (m+1)-space & one time dim:

$$
\begin{aligned}
S \;=\;& \sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^{m+2}k}{(2\pi)^{m+2}}\psi^{\dagger}_{s,j}(k)\left[ik_0 + s k_{d-m} + \vec{L}^2_{(k)}\right]\psi_{s,j}(k) \\
&+\; \frac{1}{2}\int \frac{d^{m+2}k}{(2\pi)^{m+2}}\left[k_0^2 + k_{d-m}^2 + \vec{L}^2_{(k)}\right]\phi(-k)\,\phi(k) \\
&+\; \frac{e}{\sqrt{N}}\sum_{s=\pm}\sum_{j=1}^{N}\int \frac{d^{m+2}k\,d^{m+2}q}{(2\pi)^{2m+4}}\;\phi(q)\;\psi^{\dagger}_{s,j}(k+q)\,\psi_{s,j}(k)
\end{aligned}
$$

Line of Dirac Points

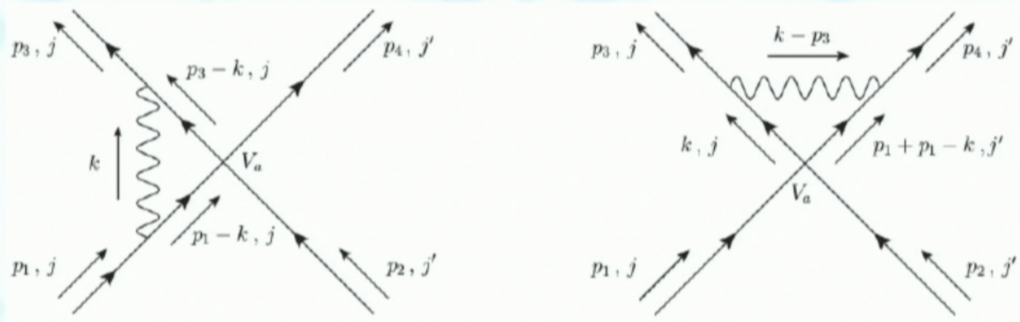


(a) m-dim FS embedded in d-dim mom space.

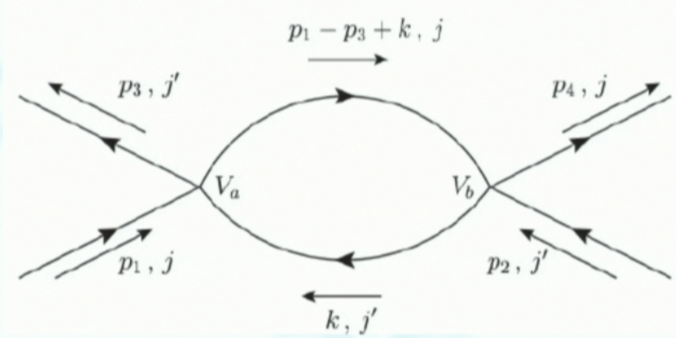
(b) Spinor has 2 bands: $E_k = E_F \pm \sqrt{\sum_{i=1}^{(d-m-1)} k_i^2 + \delta_k^2}$

For each $L_{(k)}$ ☛ Dirac point $\equiv \left(k_1=0, k_2=0, \ldots, k_{d-m} = -(L_{(k)})^2 \right)$ around which energy disperses linearly like a Dirac fermion in the (d-m)-dim subspace.

One-Loop Diagrams



(i) (ii)



Beta-Fns for V_a's

- One-loop divergent terms proportional to

$$\frac{e_{eff}\ \mu^{d_v}}{\epsilon} V_b \qquad \text{for} \qquad d = d_c - \epsilon\,;$$

$$\frac{k_F^{\frac{m}{2}}\,\mu^{2d_v}}{\epsilon} V_b\,V_c \qquad \text{for} \qquad d - m = 1 - \epsilon\,.$$

- Both valid only for d=3 - ε, m=2. Since d=3, m=2 correspond to real systems, we assume both conditions hold.

- Effective coupling constants:

$$\tilde{V}_a = \tilde{k}_F\, V_a$$

Epilogue

- RG analysis for QFTs with FS ☛ scaling behaviour of NFL states in a controlled approx.

- m-dim FS with its co-dim extended to a generic value ☛ stable NFL fixed points identified using $\epsilon = d_c - d$ as perturbative parameter.

- SC instability in such systems as a fn of dim & co-dim of FS.

- Key point ☛ $\mathbf{k_F}$ enters as a dimensionful parameter unlike in relativistic QFT ☛ modify naive scaling arguments.

- Effective coupling constants ☛ combinations of original coupling constants & $\mathbf{k_F}$.