Title: Localization-Protected Quantum Order

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Abstract: <span>Systems that are many-body Anderson localized can exhibit symmetry-breaking long-range order or topological order in regimes where such order would be destroyed at equilibrium by thermal fluctuations. The ordering is dynamical: an ordered initial state stays ordered, being "protected" by the localization of all fluctuations. The simplest examples are quantum Ising models with static randomness. For the random quantum Ising chain this feature has been "known" but apparently not appreciated for close to half a century. <br><br>Ref.: Huse, et al., Phys. Rev. B 88, 014206 (2013).</span>

Localization Protected Quantum Order.

D.H., R. Nandkishore, V. Oganesyan, A. Pal, S. Sondhi
(CUNY) (Harvard)

P.R.B 88, 014206 (2013). also: D. Pekker, et al. ....
Vosk + Altman....

- Many-body localization allows a new world of ordered* phases and quantum phase transitions that is invisible to equilibrium stat. mech.

* topological and/or symmetry-breaking

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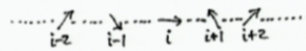
Example: transverse-field Ising spin chain:



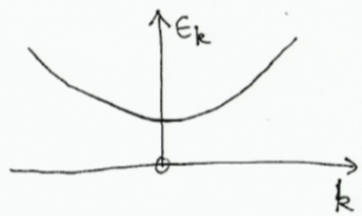

isolated system ↓

First: no disorder: $H = -\sum_i \left( J \sigma_{iz} \sigma_{(i+1)z} + h \sigma_{ix} \right) +$ nothing else

Jordan-Wigner + Boguliubov $\quad = \sum_k \epsilon_k c_k^\dagger c_k$ free fermions.

<u>Paramagnetic phase:</u> $h > J$ ground state is $\longrightarrow \quad \sigma_{ix} = +1$

a fermion is a flip of $\sigma_{ix}$: $\longleftarrow \quad \sigma_{ix} = -1$




Ground state is nondegenerate.

Spectrum is gapped. roughly:

$h$ = energy of stationary fermion

$J$ = kinetic energy of fermion

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$$H = -\sum_{i=1}^{L-1}\left(J\sigma_{iz}\sigma_{(i+1)z} + h\sigma_{ix}\right) - h\sigma_{Lx} = \sum_k \epsilon_k c_k^\dagger c_k$$

Ferromagnetic phase: Two "Schrödinger cat" ground states:
$\boxed{J > h}$

$$|0,\pm\rangle = \left(|\cdots\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\rangle \pm |\cdots\downarrow\downarrow\downarrow\downarrow\downarrow\cdots\rangle\right)/\sqrt{2}$$

Even / Odd under $\mathbb{Z}_2$ symmetry operation: $F = \prod_{i=1}^{L}\sigma_{ix}$

Energy splitting is $\epsilon_0 \sim \left(\frac{h}{J}\right)^L \to 0$ for $L \to \infty$.

So broken-symmetry state

$$|\cdots\uparrow\uparrow\uparrow\uparrow\uparrow\cdots\rangle = \left(|0,+\rangle + |0,-\rangle\right)/\sqrt{2}$$ is

dynamically stable for $L \to \infty$ ("protected" by an energy gap to making domain walls)

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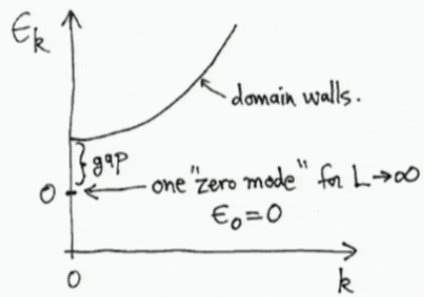
$$H = -J\sum_{i=1}^{L-1}\sigma_{iz}\sigma_{(i+1)z} - h\sum_{i=1}^{L}\sigma_{ix} = \sum_k \epsilon_k \underbrace{C_k^\dagger C_k}_{\text{fermions}}$$

Ferromagnetic phase, $J>h$: Fermions are <u>domain</u> <u>walls</u>:

$$\underset{\text{position}\rightarrow}{C_x^\dagger}|0,+\rangle = \big(|\cdots\uparrow\uparrow\uparrow\underset{x}{\downarrow}\downarrow\downarrow\cdots\rangle + |\cdots\downarrow\downarrow\downarrow\underset{x}{\uparrow}\uparrow\uparrow\cdots\rangle\big)/\sqrt{2}$$

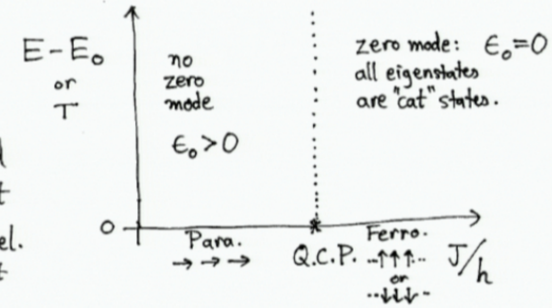
Without disorder these domain walls are mobile: $J$ = energy of wall. $h$ = moves the wall.




Except: "Domain walls" at end of chain cost no energy: these are "Majorana modes". The two ends together make the "$k=0$" fermion, with zero energy $\epsilon_0=0$.

$C_0, C_0^\dagger$ "flip" states odd $\leftrightarrow$ even.

"Eigenstate Phase diagram":




Critical point is only "easily" detected in ground state. But it is "there" at all $E$, $T$, for this model.
(gap closes, nature of exact eigenstates changes)

Next, add disorder: but preserve $\mathbb{Z}_2$ symmetry

$$H = -\sum_{i=1}^{L-1} J_i \sigma_{iz}\sigma_{(i+1)z} - \sum_{i=1}^{L} h_i \sigma_{ix}$$

now fermions move in a random potential in 1-D: they are all Anderson-localized:

$$H = \sum_{\alpha} \epsilon_\alpha C_\alpha^\dagger C_\alpha$$

$C_\alpha^\dagger$ creates a fermion in <u>localized</u> "orbital" $\alpha$.

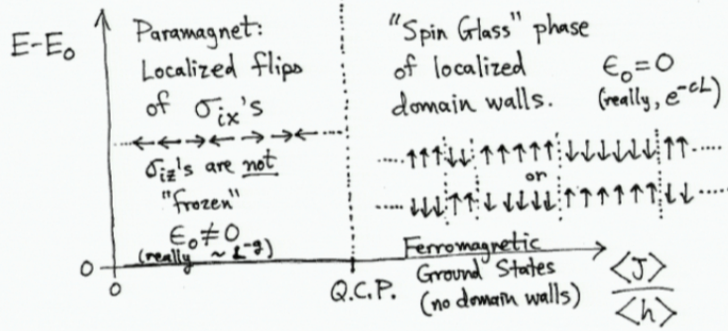
With disorder: $H = -\sum_{i=1}^{L-1} J_i \sigma_{iz}\sigma_{(i+1)z} - \sum_{i=1}^{L} h_i \sigma_{ix} = \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha$

<u>Dynamical</u> Phase Diagram:

(with disorder)

McCoy-Wu 1960's

DSFisher 1990's




Localized Paramagnet: Patterns of $\langle \sigma_{ix} \rangle$ are dynamically stable. localized states do <u>not</u> break $\mathbb{Z}_2$ symmetry

Localized "Spin Glass": Patterns of $\langle \sigma_{iz} \rangle$ that break $\mathbb{Z}_2$ symmetry are also dynamically stable. (protected by localization)

Quantum Phase Transition in highly-excited states.
Same universality class as for ground state. (Pekker, et al.)

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e.g.:

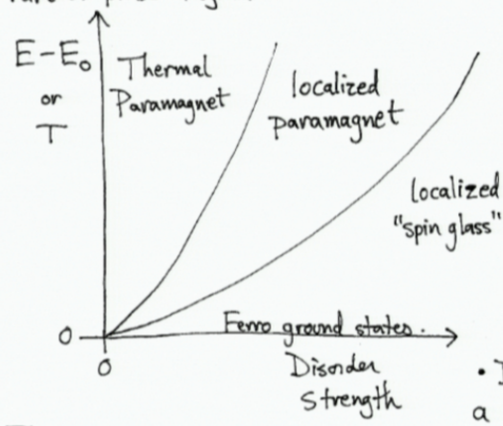
Then, add "interactions": $H_{\text{spin}} = -\sum_i \left( J_i \sigma_{iz}\sigma_{(i+1)z} + h_i \sigma_{ix} + J'\sigma_{iz}\sigma_{(i+2)z} \right)$

still preserve $\mathbb{Z}_2$ symmetry

Localized phases are stable to weak interactions. (Basko, et al. 2006)

Numerics from exact diagonalizations, Kjäll, et al. 2014 arXiv

Part of phase diagram:




Thermal phase: $H_{\text{spin}}$ is enough to thermalize system.

Localized phases are frozen under dynamics of $H_{\text{spin}}$, but will thermalize (and lose order) under (slower?) dynamics of spin-phonon interactions.

• Does many-body localization play a role in any "spin-glassy" materials? •

Transitions are only dynamical, are invisible to equilibrium thermodynamics.

Spin-glass-like dynamics, but no divergence of nonlinear susceptibility.

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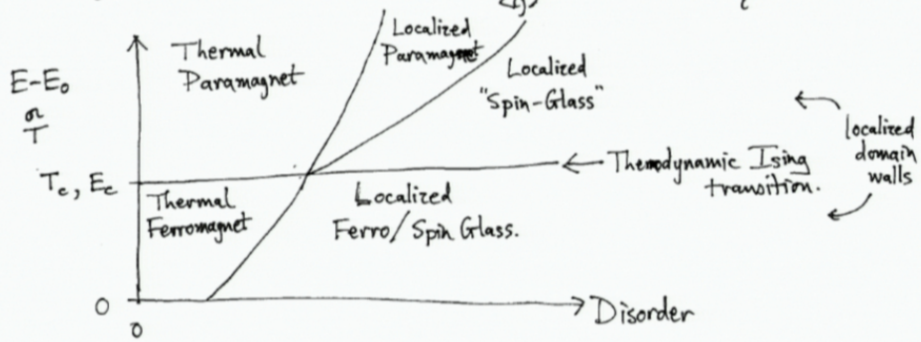
Higher dimensions: $H_{spin} = -\sum_{\langle ij \rangle} J_{ij}\,\sigma_{iz}\sigma_{jz} - \sum_i h_i \sigma_{ix} + \text{nothing else}$




Thermal Ferromagnet: System <u>is</u> a "heat bath" of volume $L^d$ <u>for itself</u>.

Barrier to flip $\cdots\uparrow\uparrow\uparrow\uparrow\cdots \leftrightarrow \cdots\downarrow\downarrow\downarrow\downarrow\cdots$ is $\sim L^{d-1}$. System (for $T>0$) thermally activates itself <u>over</u> the barrier via its own unitary quantum dynamics (no "external" bath needed).

Localized Ferromagnet and/or Spin Glass: Isolated system $H_{spin}$ is <u>not</u> a bath. Must quantum tunnel <u>through</u> barrier to do global flip.

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Can also have localization-protected topological order in systems with quenched disorder + discretely topologically-ordered ground states (S.P.T.s, etc.)

Bahri, et al. ; Chandran et al., 2013 arxiv.

highly-excited states of lattice gauge theories, Kitaev-type models, ....

Dynamically stable topological order* in isolated, disordered systems.

*of a "spin-glass" type.

Systems are "protected" against thermal fluctuations by localizing all such possible fluctuations.

A "taxonomy" of "Schrödinger cat" states: e.g. $(|\text{"up"}\rangle \pm |\text{"down"}\rangle)/\sqrt{2}$
in closed quantum systems

1) Localized cats: $(|A\rangle + |D\rangle)/\sqrt{2}$

$|A\rangle$, $|D\rangle$ "macroscopically" different, each localized.

If symmetry-related, rate of $|A\rangle \longleftrightarrow |D\rangle \sim e^{-cV}$

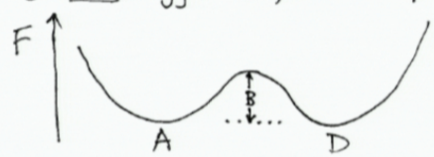
under unitary time evolution. $V \sim$ volume of system.

2) Thermal cats:

a) Barrier low enough: system is a big enough bath for itself to thermally activate itself over the barrier

rate of $|A\rangle \longleftrightarrow |D\rangle \sim e^{-B/T}$ (or faster if $B=0$)

$B =$ free energy barrier, $T =$ temperature.



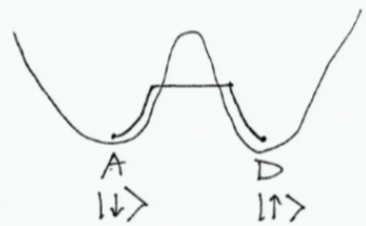
Thermal cats (cont'd):

b) Barrier too high. Happens for infinite-range Ising model: (but not for finite d.)

$$H = -\underbrace{\sum_{i,j}^{N} J_{ij}\,\sigma_{iz}\sigma_{jz}}_{\text{Ferromagnetic}} + \sum_{i=1}^{N} h_i \sigma_{ix}$$

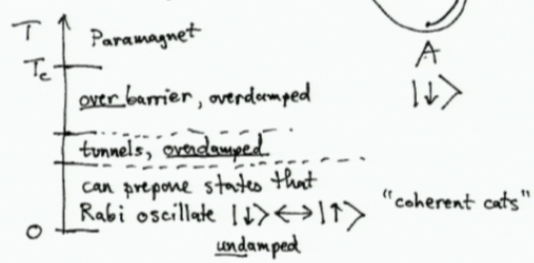
Zhao, Kerridge, H. 2013 arXiv

Saddle point for $|A\rangle \longleftrightarrow |D\rangle$ is partly thermally activated, partly tunneling (bath is not "strong" enough to get over barrier):




Ferro phase contains three dynamical phases:

in $N \to \infty$ limit.



SUMMARY

Localization Protected Quantum Order

- localize all thermal fluctuations.
- showed simplest case of Ising model:
  localize all domain walls to freeze a given magnetization pattern.
- new type of quantum phase transition that can occur in low dimensions (e.g. $d=1$) and in highly-excited states.

"Schrödinger cat" states in Ising ferromagnets come in a variety of "species".