Title: Gravity waves from Kerr/CFT

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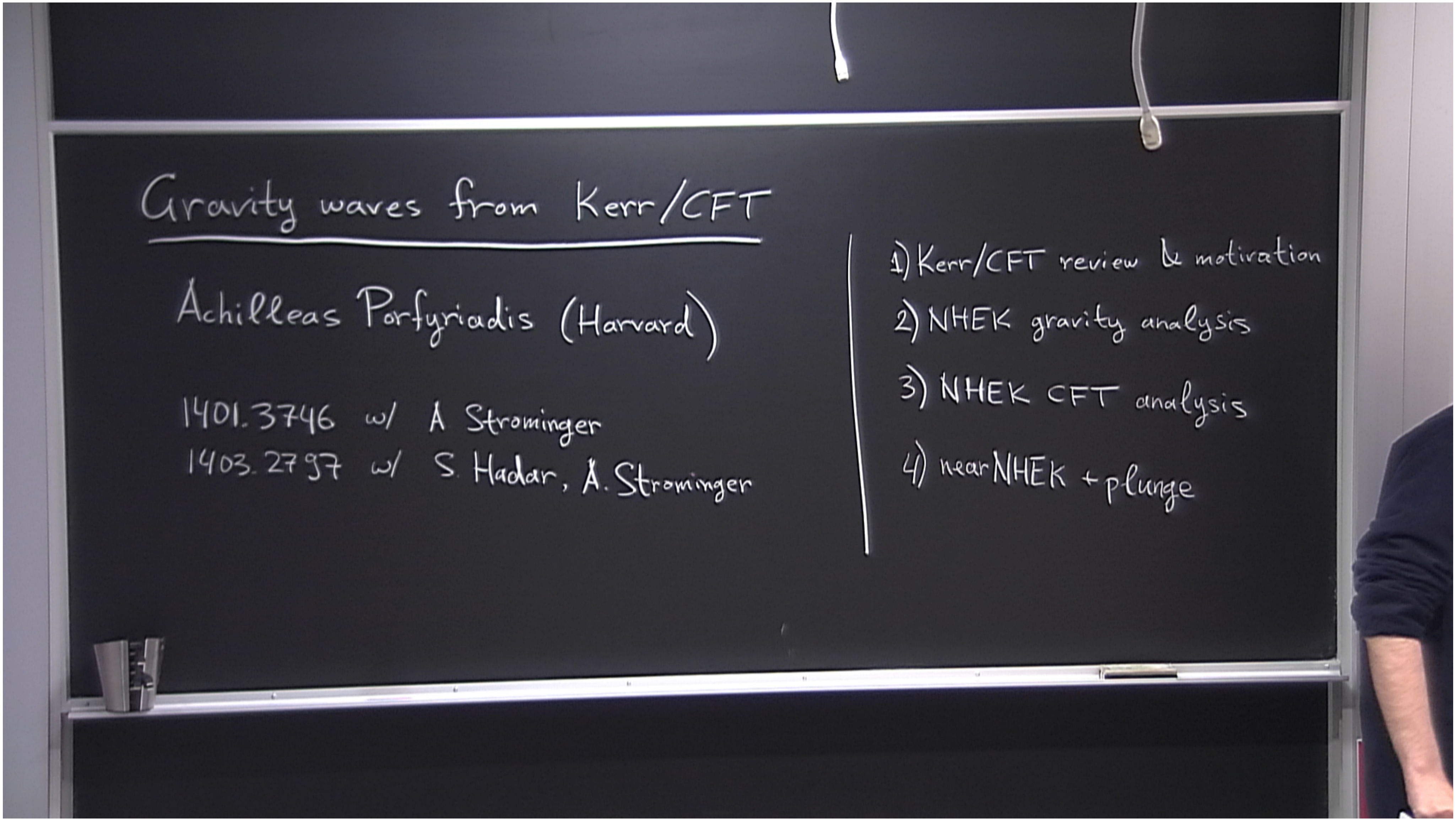
Abstract: Astronomical observation suggests the existence of near-extreme Kerr black holes whose horizons spin at nearly the speed of light. Properties of diffeomorphisms imply that the dynamics of the high-redshift near-horizon region of near-extreme Kerr, which includes the innermost-stable-circular-orbit (ISCO), is governed by an infinite-dimensional emergent conformal symmetry. This symmetry may be exploited to analytically, rather than numerically, compute a variety of potentially observable processes. In this talk I will show how we compute and study the conformal transformation properties of the gravitational radiation emitted by an orbiting massive object in the large-redshift near-horizon region. I will also use conformal symmetry of the near-horizon region to compute the gravitational radiation produced during the plunge phase following the object's crossing of the ISCO.


Gravity waves from Kerr/CFT
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1401.3746 w/ A. Strominger
1403.2797 w/ S. Hadar, A. Strominger
1) Kerr/CFT review & motivation
2) NHEK gravity analysis
3) NHEK CFT analysis
4) near NHEK + plunge

Kerr metric in BL coords ($G = c = \hbar = 1$):

$$ds^2 = -\frac{\Delta}{\rho^2}\left(d\hat{t} - a\sin^2\theta\, d\hat{\phi}\right)^2 + \frac{\sin^2\theta}{\rho^2}\left[(\hat{r}^2 + a^2)\,d\hat{\phi} - a\,d\hat{t}\right]^2 + \frac{\rho^2}{\Delta}\,d\hat{r}^2 + \rho^2\,d\theta^2$$

$$\Delta \equiv \hat{r}^2 - 2M\hat{r} + a^2\,, \qquad \rho^2 \equiv \hat{r}^2 + a^2\cos^2\theta$$

$$M\,,\ J = aM \leq M^2$$

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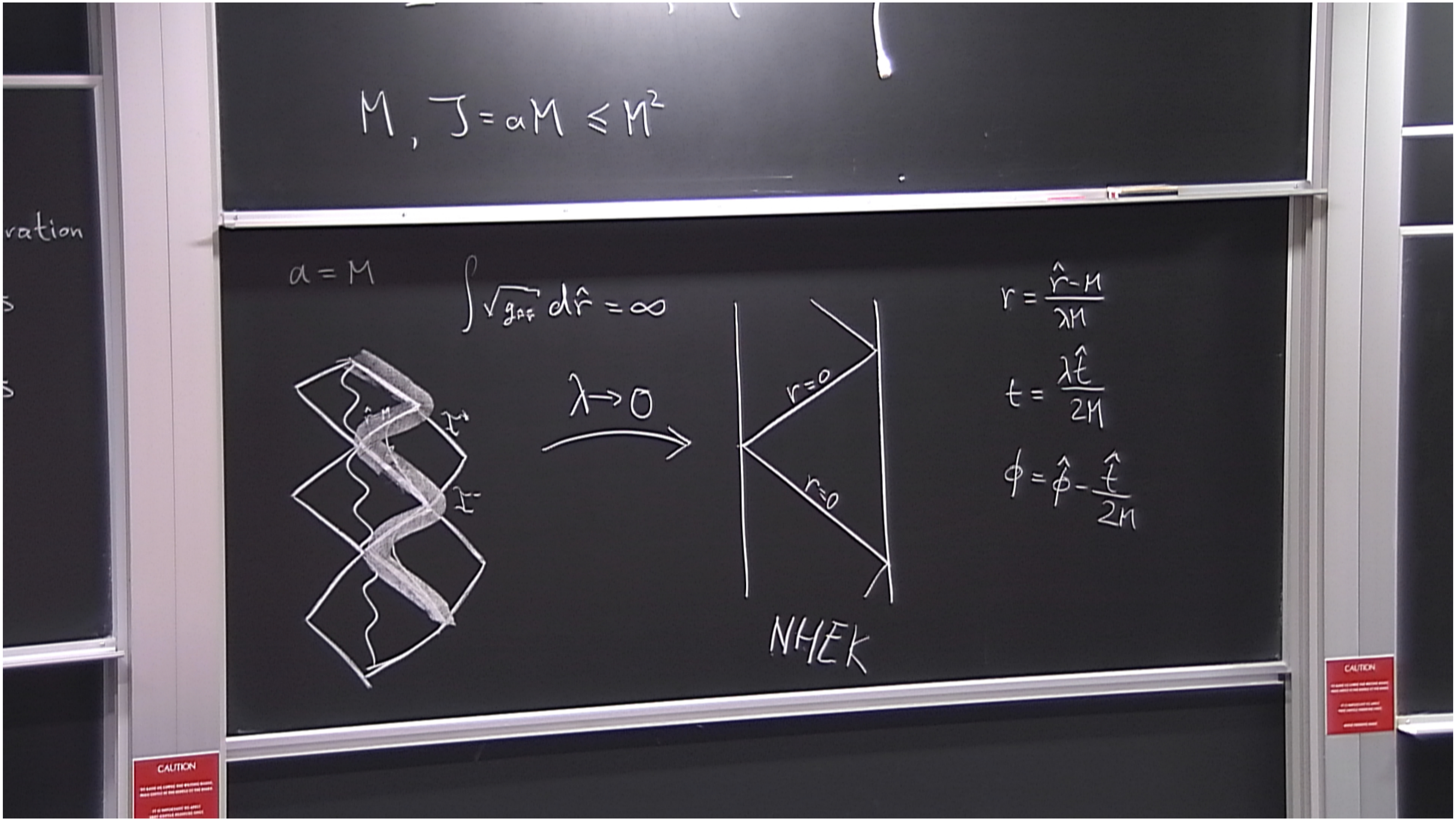
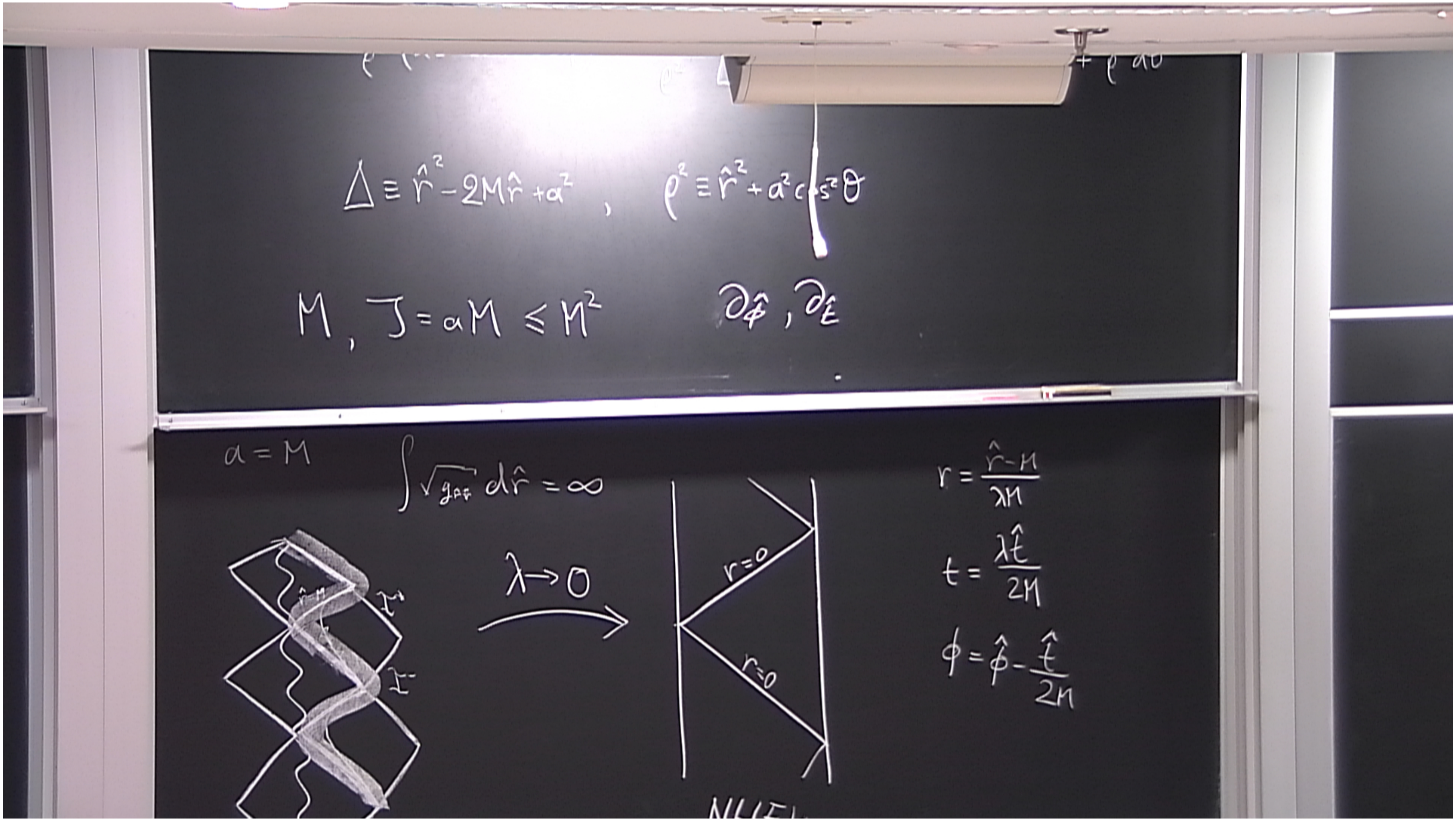

$M, J = aM \leq M^2$
$a = M$
$\int \sqrt{g_{\hat{r}\hat{r}}}\, d\hat{r} = \infty$
$\lambda \to 0$
$r=0$
$r=0$
NHEK
$r = \frac{\hat{r} - M}{\lambda M}$
$t = \frac{\lambda \hat{t}}{2M}$
$\phi = \hat{\phi} - \frac{\hat{t}}{2M}$
CAUTION
CAUTION

Hadar, A. Strominger

4) near NHEK + plunge

$$ds^2 = 2M^2\,\Gamma(\theta)\left[-r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2(\theta)\,(d\phi + r\,dt)^2\right] \quad \text{NHEK}$$

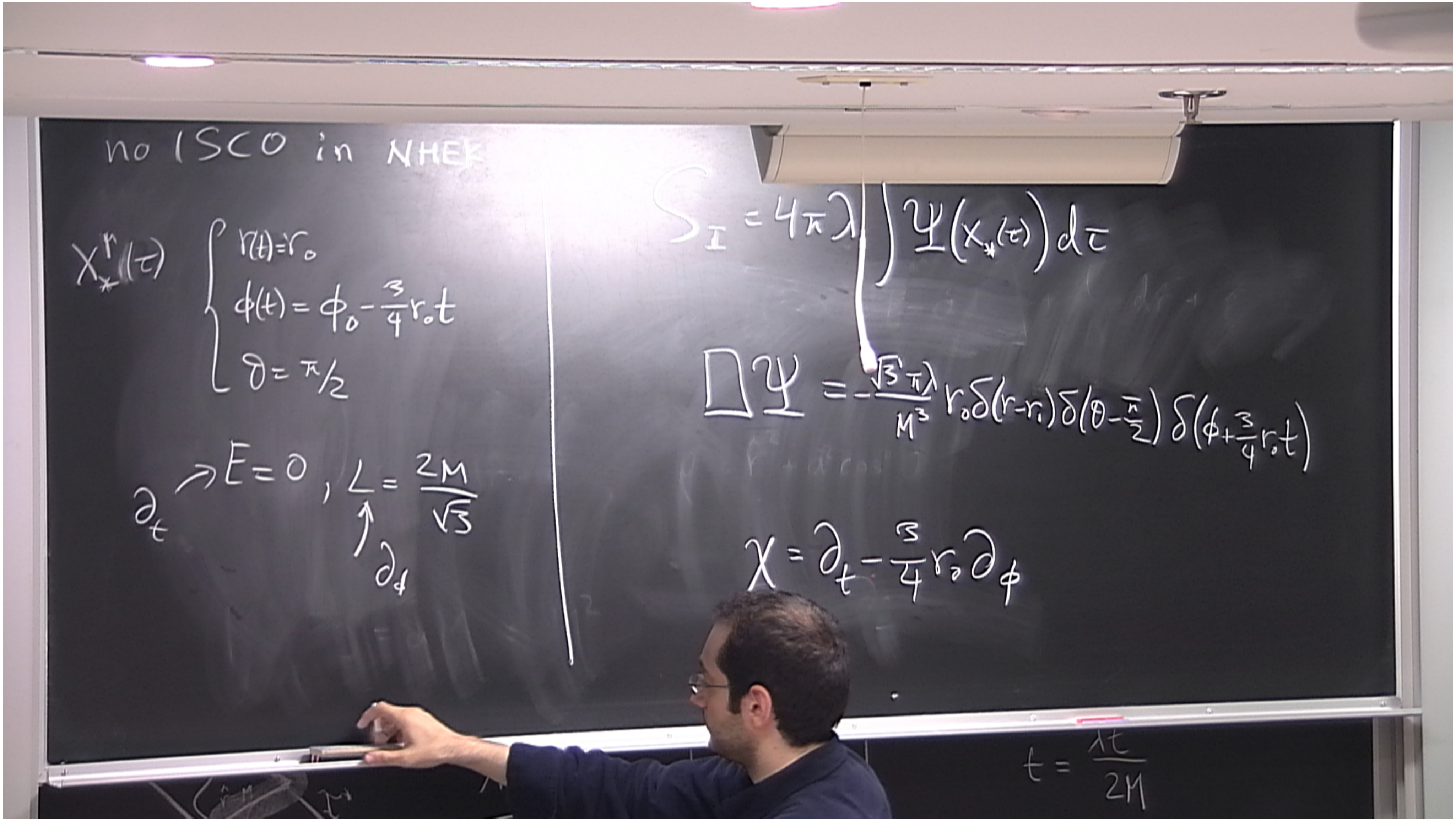


$\Delta \equiv \hat{r}^2 - 2M\hat{r} + a^2 \,, \quad \rho^2 \equiv \hat{r}^2 + a^2\cos^2\theta$
$M \,,\; J = aM \leq M^2$
$\partial_{\hat{\phi}} \,, \partial_{\hat{t}}$
$a = M$
$\int \sqrt{g_{\phi\phi}}\, d\hat{r} = \infty$
$\lambda \to 0$
$r=0$
$r=0$
$r = \frac{\hat{r} - M}{\lambda M}$
$t = \frac{\lambda \hat{t}}{2M}$
$\phi = \hat{\phi} - \frac{\hat{t}}{2M}$

$$\partial_t \longrightarrow SL(2,R)_R \times U(1)_L \longleftarrow \partial_\phi$$

$$\Big\downarrow \text{ASG} \quad \text{s.t. } J = M^2$$

$$\mathrm{Vir}_L \qquad c_L = 12J \,,\; T_L = \frac{1}{2\pi} \Rightarrow S_{\text{Cardy}} = \frac{\pi^2}{3} c_L T_L = S_{BH}$$


no ISCO in NHEK
$X_*^r(\tau)$ $\begin{cases} r(t) = r_0 \\ \phi(t) = \phi_0 - \frac{3}{4} r_0 t \\ \theta = \pi/2 \end{cases}$
$\partial_t \to E = 0, \quad L = \frac{2M}{\sqrt{3}} \leftarrow \partial_\phi$
$S_I = 4\pi\lambda \int \Psi(x_*(\tau))\, d\tau$
$\Box \Psi = -\frac{\sqrt{3}\pi\lambda}{M^3} r_0 \delta(r - r_0)\, \delta(\theta - \frac{\pi}{2})\, \delta(\phi + \frac{3}{4} r_0 t)$
$\chi = \partial_t - \frac{3}{4} r_0 \partial_\phi$
$t = \frac{\lambda t}{2M}$

$$\Psi = \sum_{\ell,m} e^{im(\phi + \frac{3}{4} r_0 t)} S_\ell(\theta) R_{\ell m}(r)$$

$$D_\theta S_\ell^m(\theta) = -K_\ell^m S_\ell^m(\theta)$$

$[0, \pi]$

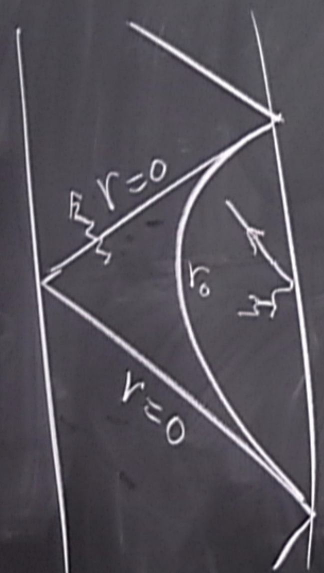
$[0,\pi]$

$$\omega \equiv -\frac{3}{4} r_0 m$$

$$\partial_r\left(r^2 \partial_r R_{\ell m}\right) + \left(2m^2 - K_\ell + \frac{2\omega m}{r} + \frac{\omega^2}{r^2}\right) R_{\ell m} = -\frac{\sqrt{3}\,\lambda}{2M} S_\ell\!\left(\frac{\pi}{2}\right) r_0\, \delta(r - r_0)$$

$$\omega \equiv -\frac{3}{4} r_0 m$$

$$\partial_r\left(r^2 \partial_r R_{\ell m}\right) + \left(2m^2 - K_\ell + \frac{2\omega m}{r} + \frac{\omega^2}{r^2}\right) R_{\ell m} = -\frac{\sqrt{3}\lambda}{2M} S_\ell\left(\frac{\pi}{2}\right) r_0 \delta(r-r_0)$$



$r=0$: $\boxed{r^{-im} e^{+i\omega/r}}$ $\quad r^{im} e^{-i\omega/r}$

$r=\infty$ $\quad \underset{D}{r^{h-1}}$ $\quad \underset{N}{\boxed{r^{-h}}}$

$$h \equiv \frac{1}{2} + \sqrt{\frac{1}{4} + K_\ell - 2m^2}$$

$$R_{\ell m}(r) = X\,\Theta(r_0 - r)\,W(r) + Z\,\Theta(r - r_0)\,M(r)$$

$$j^{\mu}_{KG} \sim \Psi^{*} \overleftrightarrow{\nabla}^{\mu} \Psi$$

$$\boxed{F^{KG}_{\ell m} = \frac{\lambda^2}{4} r_0 m^{-1} e^{-\pi m} S_{\ell}\left(\frac{\pi}{2}\right) \frac{|\Gamma(h+im)|^2}{\Gamma(2h)^2} \left|M_{im,h-\frac{1}{2}}\left(\frac{3im}{2}\right)\right|^2}$$

$r < r_0$

$$S = S_{CFT} + \sum_{\ell} \int dt^+ dt^- J_\ell(t^+, t^-) \mathcal{O}_\ell(t^+, t^-)$$

$$t^+ = \phi, \; t^- = t$$

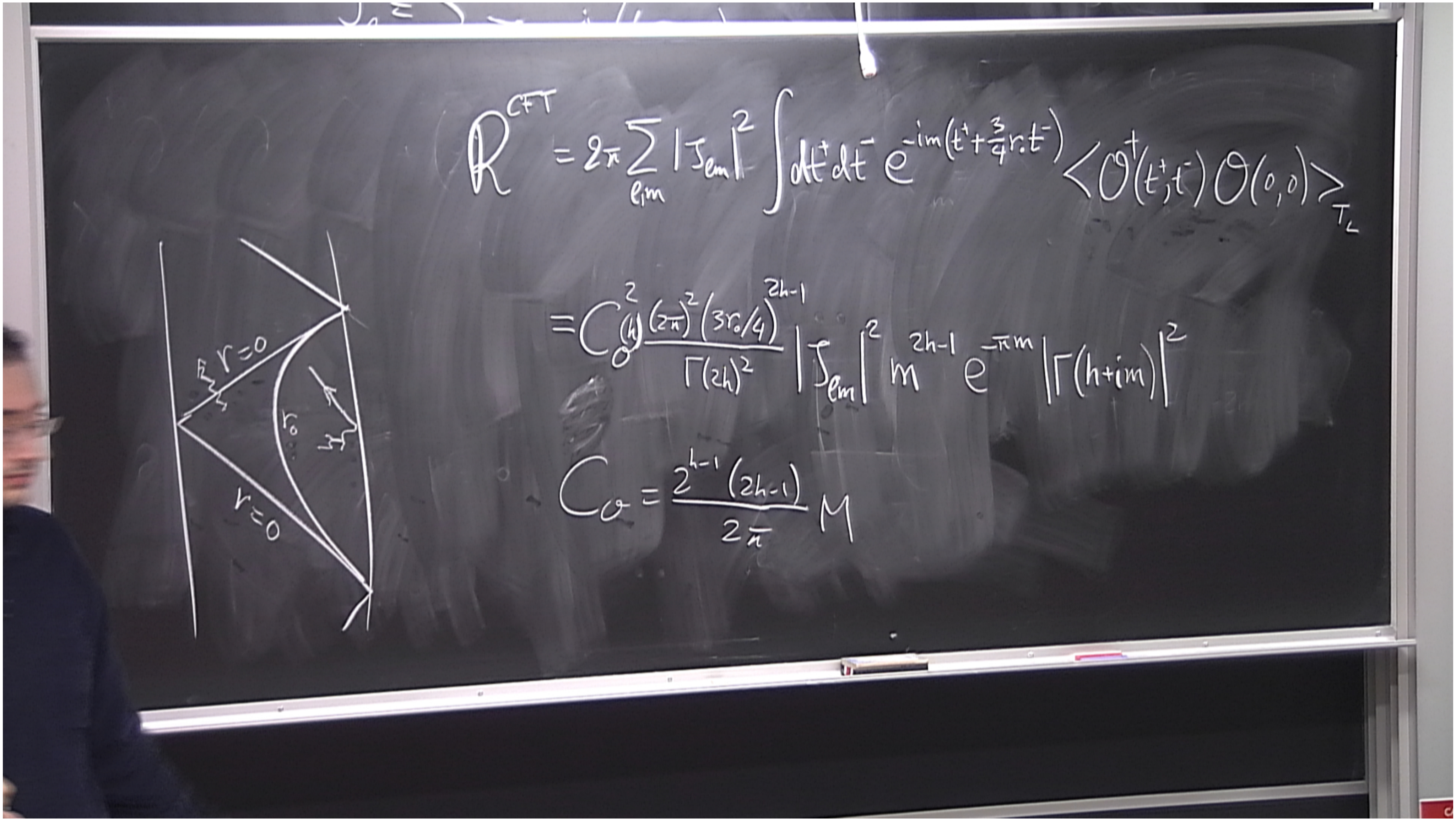
$r < r_0$

$$S = S_{CFT} + \sum_{\ell} \int dt^+ dt^- \, J_\ell(t^+, t^-) \, \mathcal{O}_\ell(t^+, t^-)$$

$$h_L = h_R = h$$

$$t^+ = \phi, \quad t^- = t$$

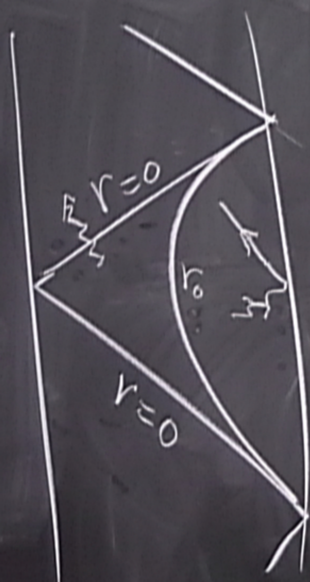
$$J_\ell = \sum_{l,m} J_{lm} \, e^{im\left(\phi + \frac{3}{4} r_0 t\right)}$$



$R^{CFT} = 2\pi \sum_{l,m} |J_{lm}|^2 \int dt^+ dt^- \, e^{-im(t^+ + \frac{3}{4} r_0 t^-)} \langle \mathcal{O}^\dagger(t^+,t^-) \mathcal{O}(0,0) \rangle_{T_L}$
$= C_{\mathcal{O}}^2(h) \frac{(2\pi)^2 (3r_0/4)^{2h-1}}{\Gamma(2h)^2} |J_{lm}|^2 m^{2h-1} e^{-\pi m} |\Gamma(h+im)|^2$
$C_{\mathcal{O}} = \frac{2^{h-1}(2h-1)}{2\pi} M$
$r=0$
$r=0$
r_0

$$J_\ell = \sum_{\ell,m} J_{\ell m} e^{im(\phi + \frac{3}{4} r_0 t)}$$

$$R^{CFT} = 2\pi \sum_{\ell,m} |J_{\ell m}|^2 \int dt^+ dt^- \, e^{-im(t^+ + \frac{3}{4} r_0 t^-)} \langle O^\dagger(t^+,t^-)\, O(0,0)\rangle_{T_L}$$



$$= C_{O}^{(h)\,2} \frac{(2\pi)^2 (3r_0/4)^{2h-1}}{\Gamma(2h)^2} |J_{\ell m}|^2 m^{2h-1} e^{-\pi m} |\Gamma(h+im)|^2$$

$$C_O = \frac{2^{h-1}(2h-1)}{2\pi} M$$

$$\boxed{R^{CFT}_{\ell m} = F^{Ka}_{\ell m}}$$

$$O_\ell(t^+,t^-) = \sum_{\ell,m} O_{\ell m} e^{-imt^+}$$

$$\kappa \equiv \sqrt{1-\frac{a^2}{M^2}} \ll 1$$

near NHEK

$$T_L = \frac{1}{2\pi}\ , \quad T_R = \frac{\kappa}{2\pi}$$

$$ds^2 = 2M^2\Gamma(\theta)\left[-\tilde{r}(\tilde{r}+2\kappa)dt^2 + \frac{d\tilde{r}^2}{\tilde{r}(\tilde{r}+2\kappa)} + d\theta^2 + \Lambda^2(\theta)\left(d\phi + (\tilde{r}+\kappa)dt\right)^2\right]$$

$$\mathcal{O}_\ell(t,t) = \sum_{\ell,m} \mathcal{O}_{\ell m}\, e^{-imt}$$

ISCO in near NHEK is $\tilde{r} = \infty$

plunge

$$\tilde{E} = 0, \quad \tilde{L} = \frac{2M}{\sqrt{3}}$$

$$\begin{cases} \tilde{t}(\tilde{r}) = \dfrac{1}{2\kappa} \ln \dfrac{1}{\tilde{r}(\tilde{r}+2\kappa)} + \tilde{t}_0 \\[2ex] \tilde{\phi}(\tilde{r}) = \dfrac{3\tilde{r}}{4\kappa} + \dfrac{1}{2} \ln \dfrac{\tilde{r}}{\tilde{r}+2\kappa} + \tilde{\phi}_0 \end{cases}$$

$$\theta = \frac{\pi}{2}$$

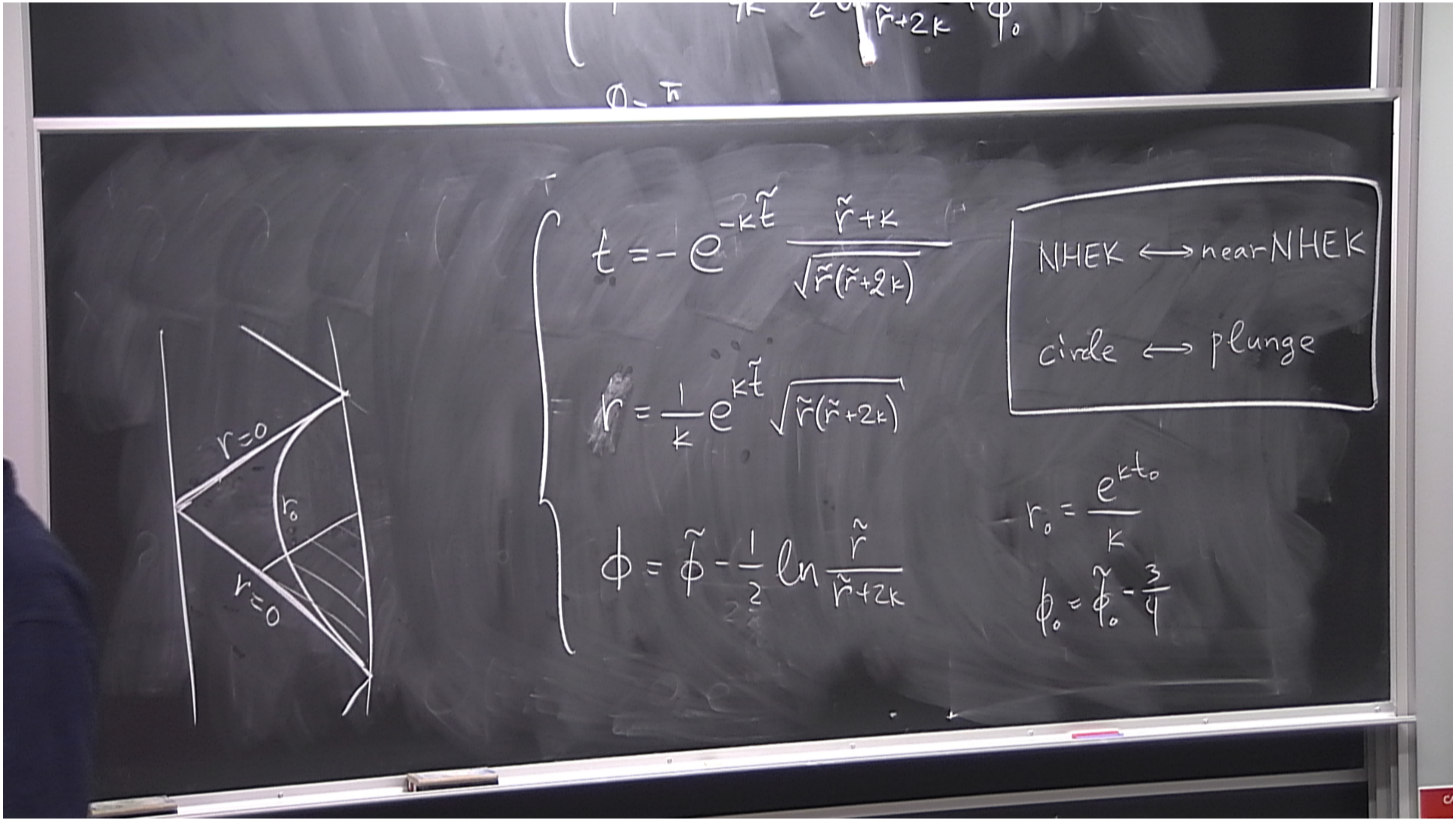
$$\kappa \equiv \sqrt{1-\frac{a^2}{M^2}} \ll 1$$

near NHEK

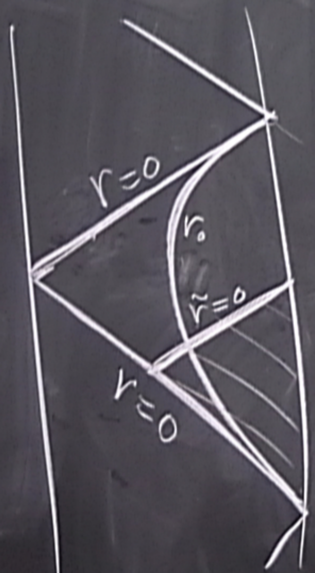
$$T_L = \frac{1}{2\pi}, \quad T_R = \frac{\kappa}{2\pi}$$

$$ds^2_{\text{near NHEK}} = 2M^2\,\Gamma(\theta)\left[-\tilde{r}(\tilde{r}+2\kappa)\,dt^2 + \frac{d\tilde{r}^2}{\tilde{r}(\tilde{r}+2\kappa)} + d\theta^2 + \Lambda^2(\theta)\left(d\tilde{\phi} + (\tilde{r}+\kappa)\,d\tilde{t}\right)^2\right]$$

$$ds^2_{\text{NHEK}} = \quad -r^2 dt^2 + \frac{dr^2}{r^2} \qquad (d\phi + r\,dt)^2$$


$t = -e^{-\kappa \tilde{t}} \frac{\tilde{r}+\kappa}{\sqrt{\tilde{r}(\tilde{r}+2\kappa)}}$
$r = \frac{1}{\kappa} e^{\kappa \tilde{t}} \sqrt{\tilde{r}(\tilde{r}+2\kappa)}$
$\phi = \tilde{\phi} - \frac{1}{2} \ln \frac{\tilde{r}}{\tilde{r}+2\kappa}$
NHEK ⟷ near NHEK
circle ⟷ plunge
$r_0 = \frac{e^{\kappa t_0}}{\kappa}$
$\phi_0 = \tilde{\phi}_0 - \frac{3}{4}$
r=0
r_0
r=0

$$\tilde{r}+2\kappa \qquad \phi_0$$



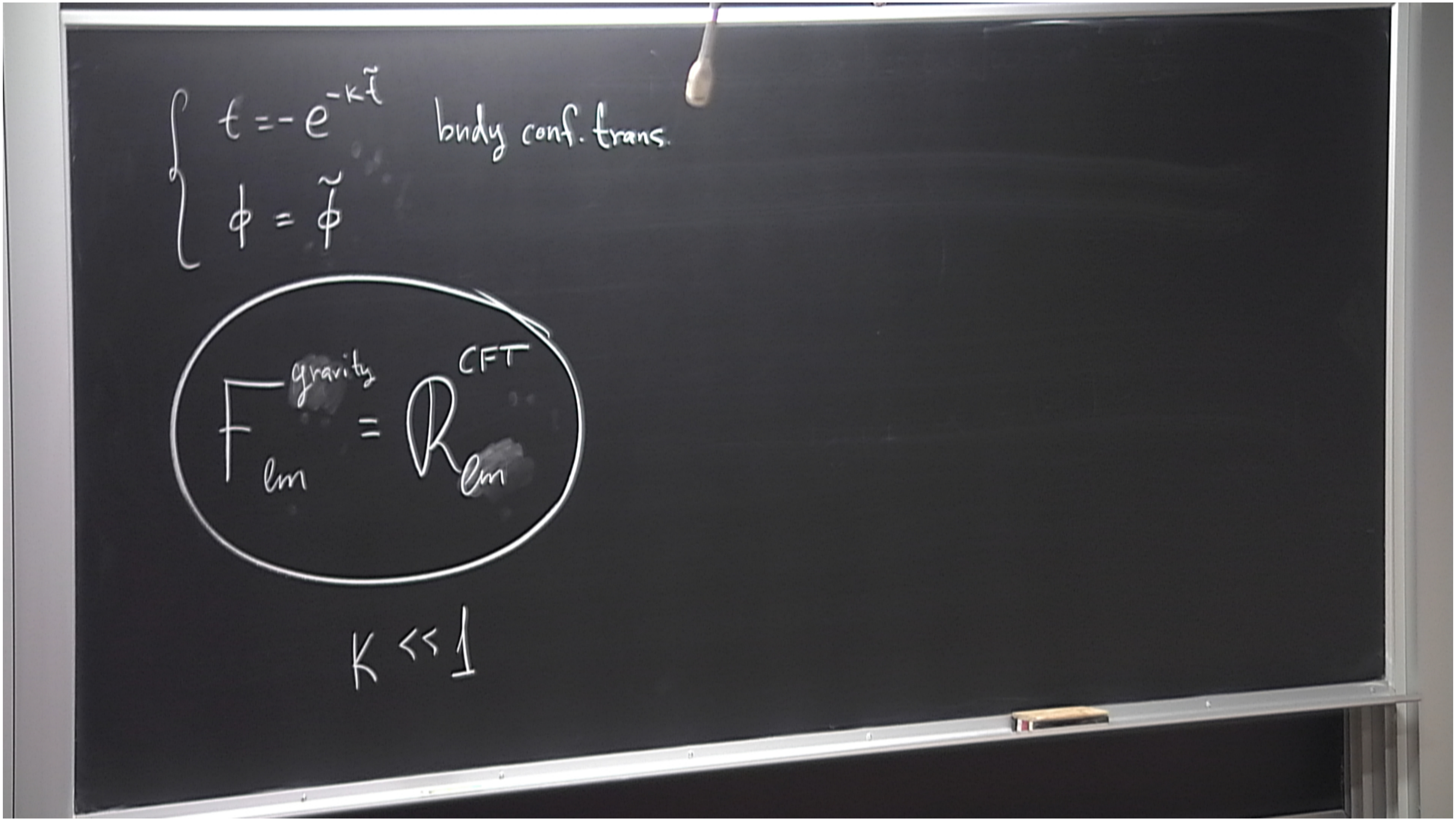
$$\begin{cases} t = -e^{-\kappa \tilde{t}} \dfrac{\tilde{r}+\kappa}{\sqrt{\tilde{r}(\tilde{r}+2\kappa)}} \\[2ex] r = \dfrac{1}{\kappa} e^{\kappa \tilde{t}} \sqrt{\tilde{r}(\tilde{r}+2\kappa)} \\[2ex] \phi = \tilde{\phi} - \dfrac{1}{2} \ln \dfrac{\tilde{r}}{\tilde{r}+2\kappa} \end{cases}$$

NHEK $\longleftrightarrow$ near NHEK

circle $\longleftrightarrow$ plunge

$$r_0 = \frac{e^{\kappa t_0}}{\kappa}$$

$$\phi_0 = \tilde{\phi}_0 - \frac{3}{4}$$


$t = -e^{-\kappa \tilde{t}}$
bndy conf. trans.
$\phi = \tilde{\phi}$
$F^{gravity}_{\ell m} = R^{CFT}_{\ell m}$
$\kappa \ll 1$