Title: Random models of inflation

Date: May 13, 2014 11:00 AM

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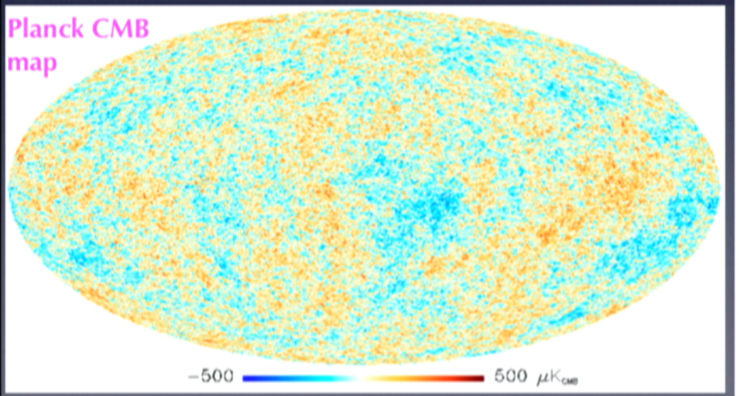
Abstract: Rather than writing down specific functional forms, one can generate inflation models via stochastic processes in order to explore generic properties of inflation models. I describe our explorations of the phenomenology of randomly-generated multi-field inflation models, both for canonical fields and for a braneworld-motivated scenario. Implications of some recent observational results, including BICEP2, will be discussed.

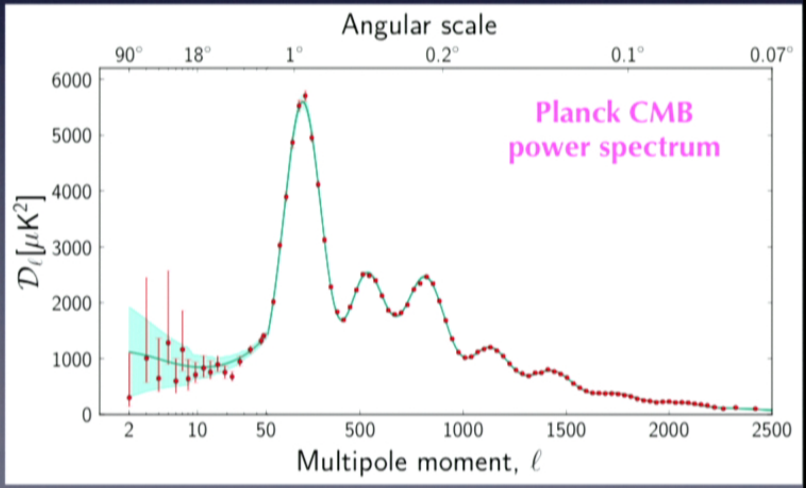
Random models of inflation

Andrew Liddle
May 2014

Inflation is ...

- A prolonged period of accelerated expansion in the very early Universe,
- which explains why the Universe is approximately homogeneous and flat,
- may explain the absence of relic particles predicted by fundamental physics,
- and is the leading paradigm for explaining the observed inhomogeneities in the Universe.





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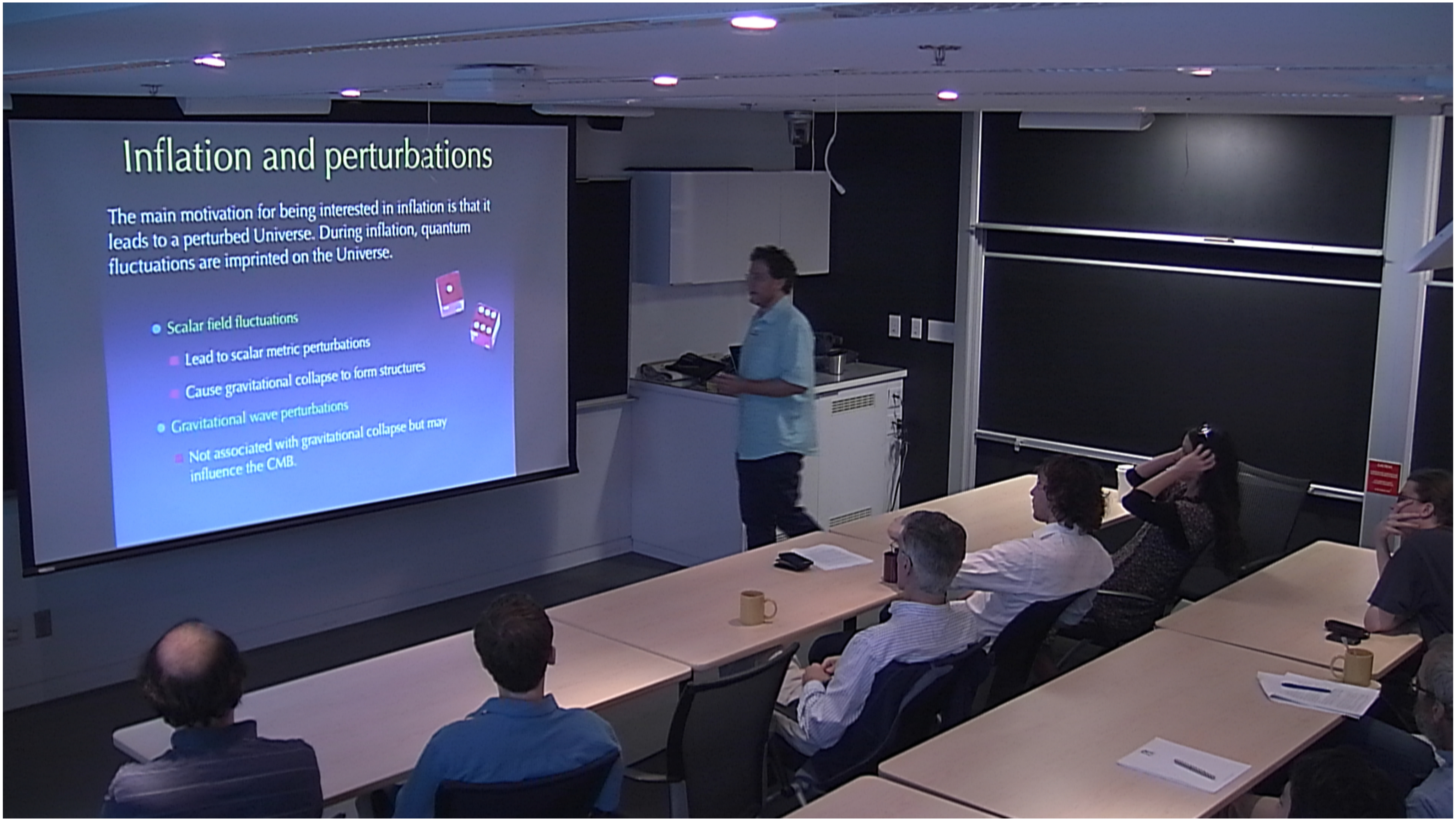
- Scalar field fluctuations
  - Lead to scalar metric perturbations
  - Cause gravitational collapse to form structures
- Gravitational wave perturbations
  - Not associated with gravitational collapse but may influence the CMB.



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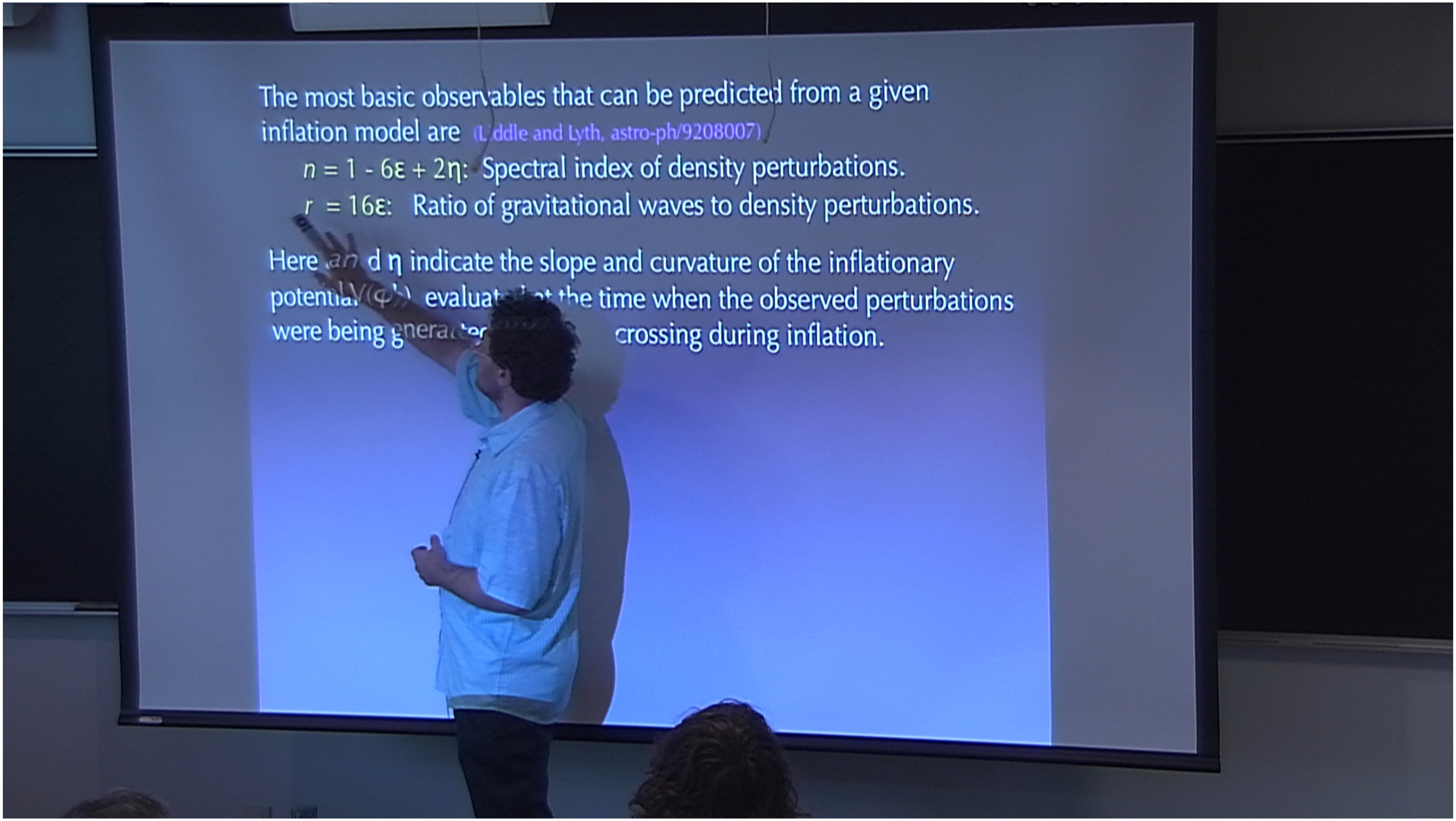
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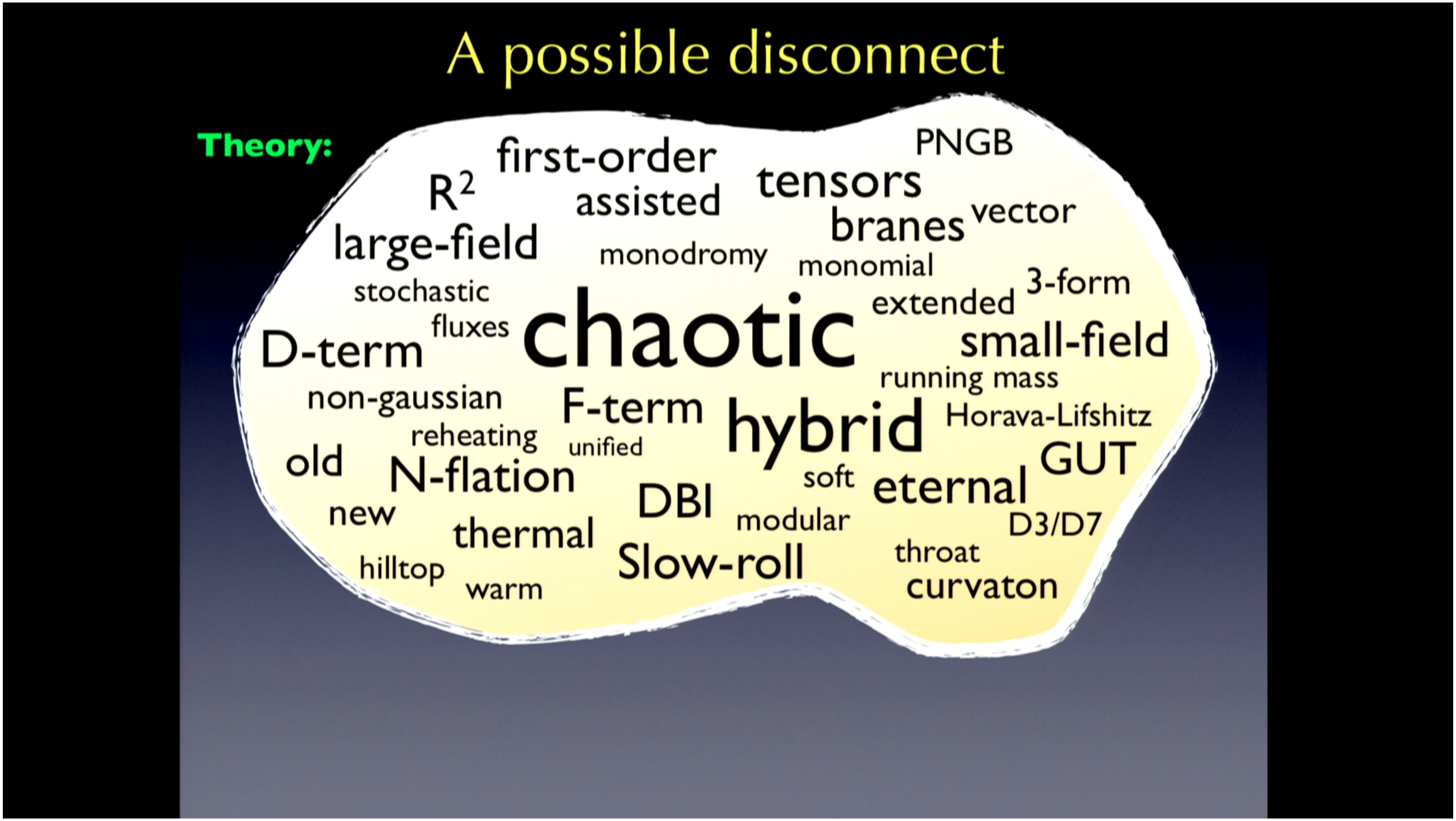
Additionally

$n_g = -2\epsilon$: Spectral index of gravitational waves.

$n_g = -r/8$: Consistency equation


A possible disconnect
Theory:
first-order
PNGB
R^2
assisted
tensors
branes
vector
large-field
monodromy
monomial
stochastic
3-form
extended
fluxes
chaotic
D-term
small-field
running mass
non-gaussian
F-term
hybrid
Horava-Lifshitz
reheating
unified
GUT
old
N-flation
soft
eternal
DBI
new
modular
D3/D7
thermal
throat
hilltop
Slow-roll
warm
curvaton

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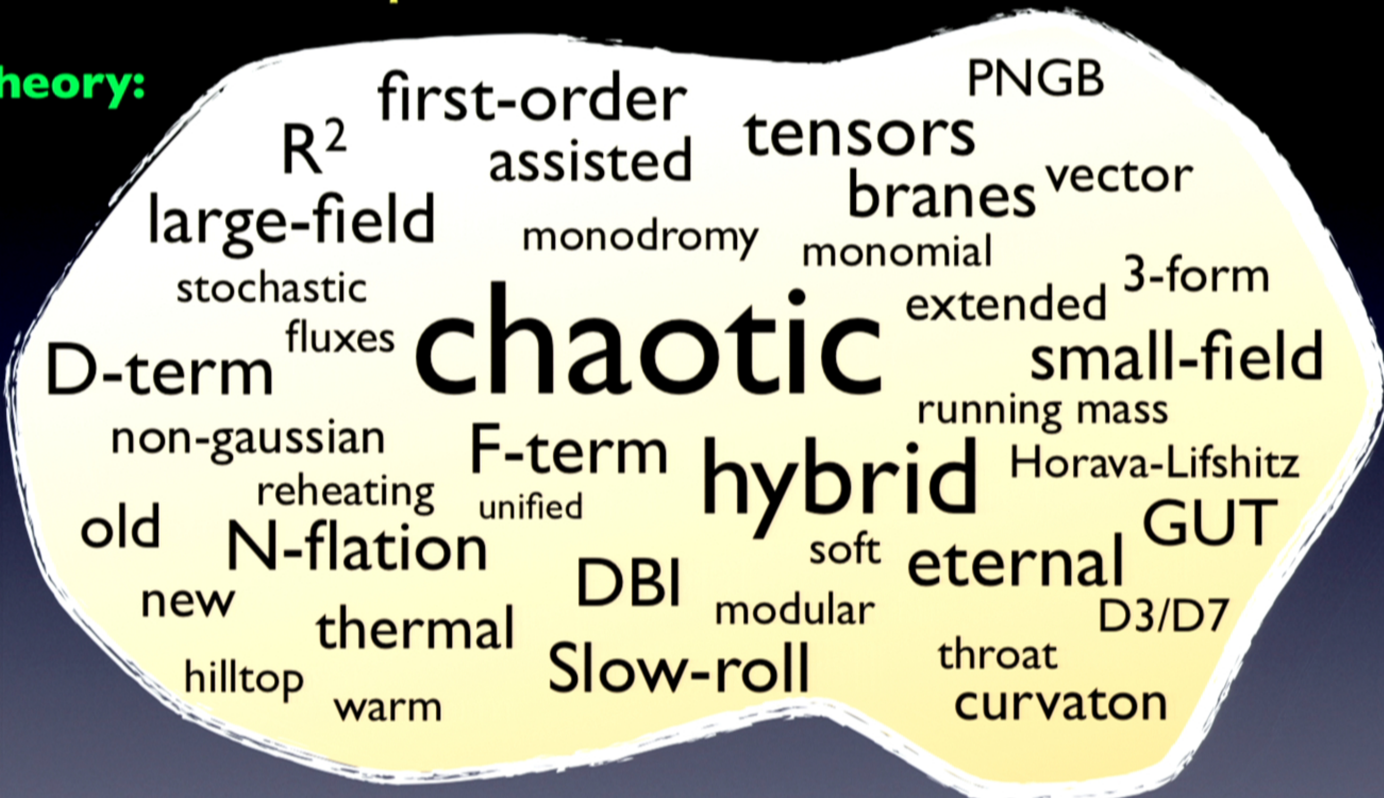

Present and future observations:

n already measured, *r* probably measured (?),

α may be measurable, f_{NL} perhaps, f_{iso} if we are lucky

Planck results: power spectrum

Main result: Exquisite measurement of the temperature power spectrum

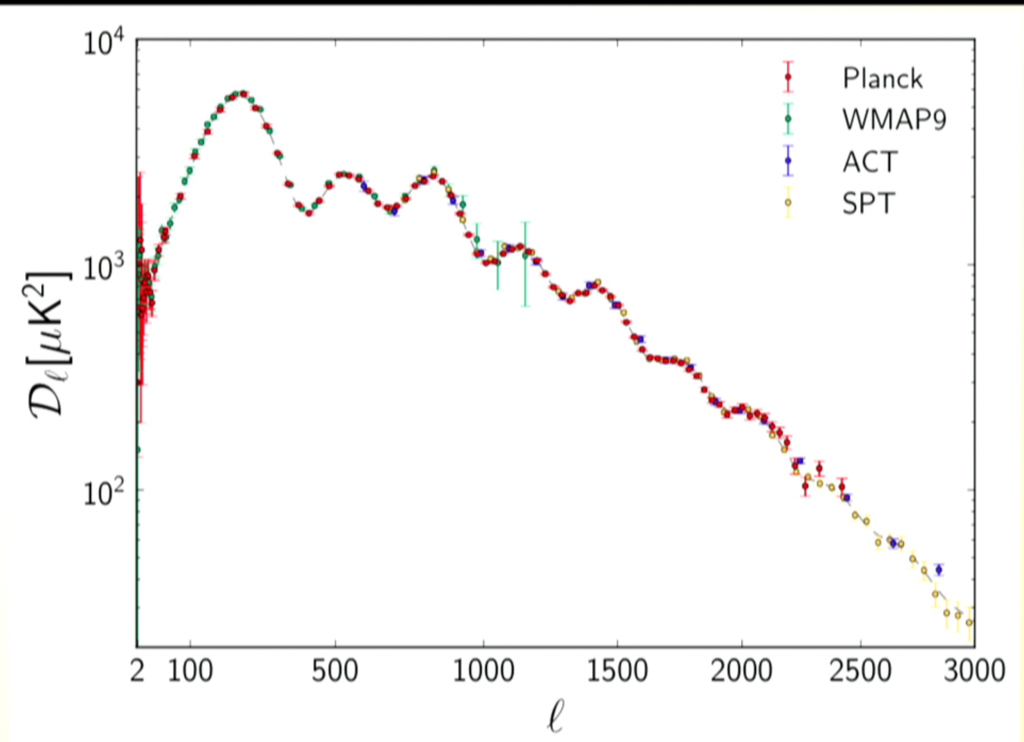

Fig. 25. Measured angular power spectra of *Planck*, WMAP9, ACT, and SPT. The model plotted is *Planck*'s best-fit model including *Planck* temperature, WMAP polarization, ACT, and SPT (the model is labelled [Planck+WP+HighL] in Planck Collaboration XVI (2013)). Error bars include cosmic variance. The horizontal axis is $\ell^{0.8}$.

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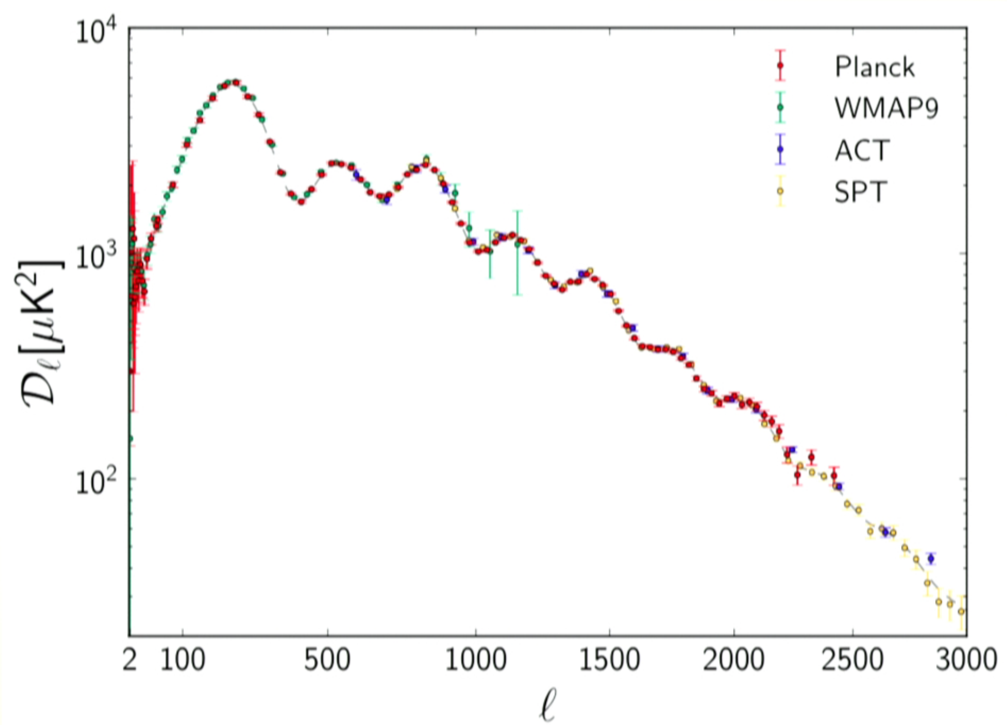

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Planck results: non-gaussianity

Main result: No detection of primordial non-gaussianity; limits strengthened. Detection of ISW-lensing bispectrum at 2 to 3 sigma.

- $f_{NL}^{local} = 2.7 \pm 5.8$
- $f_{NL}^{equil} = -42 \pm 75$
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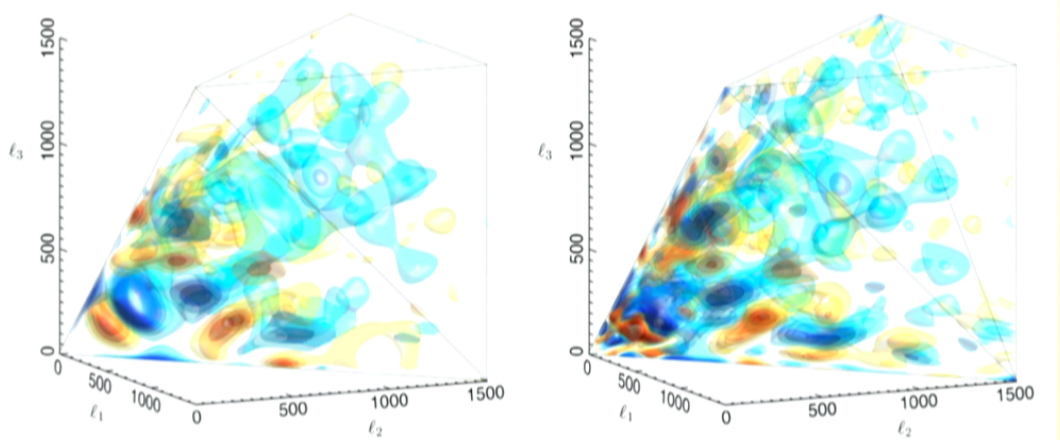
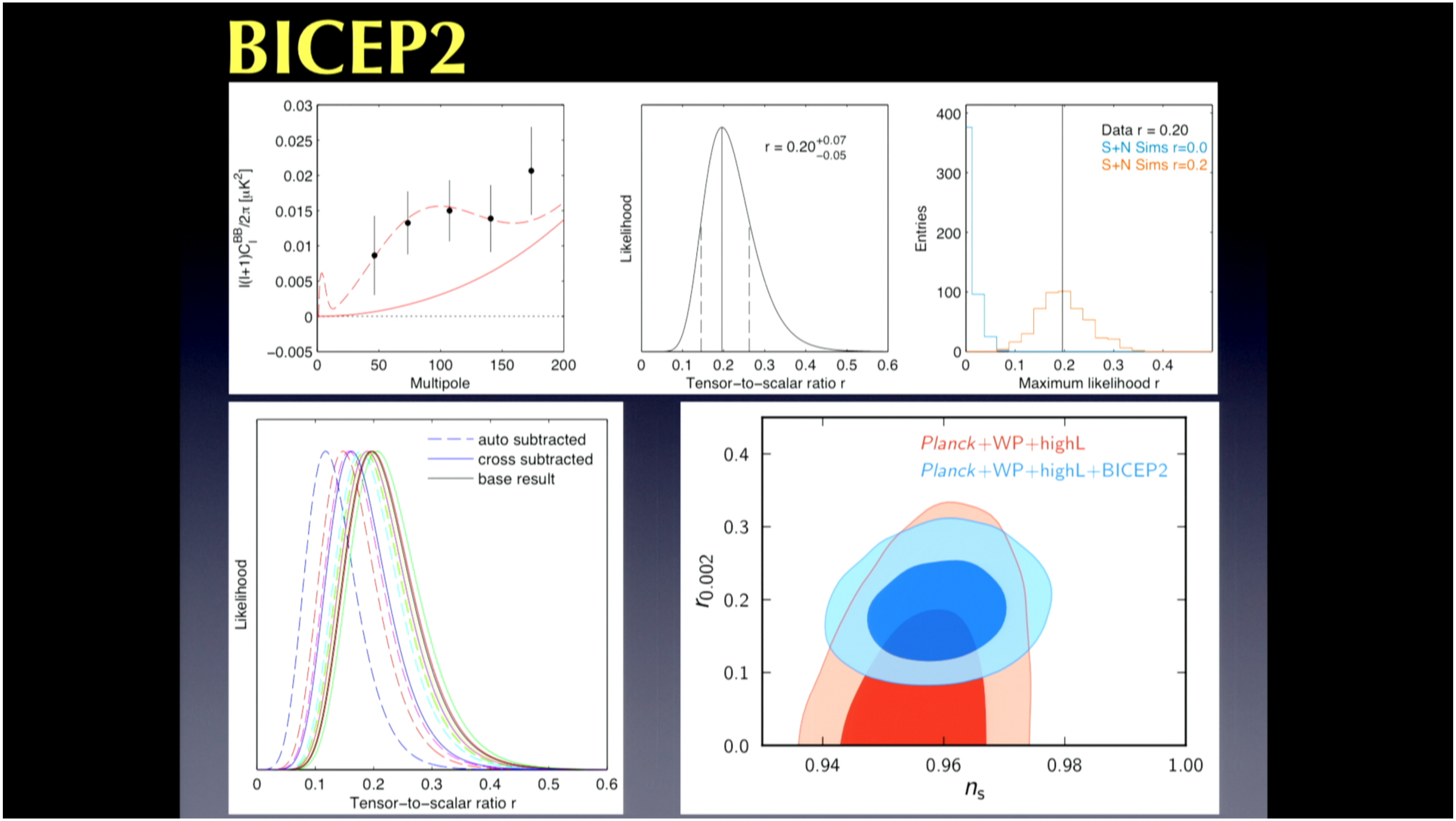
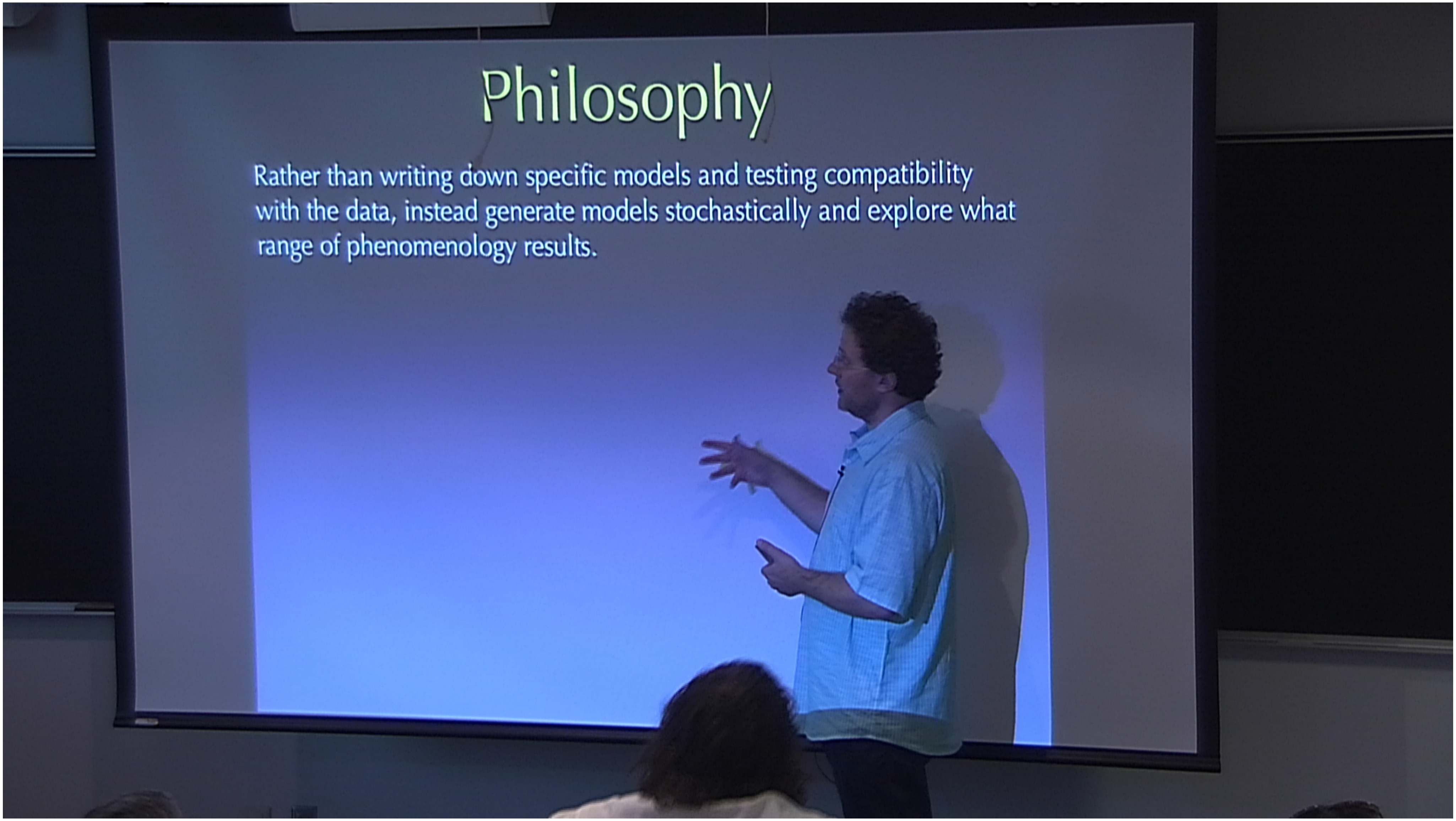


Fig. 7. *Planck* CMB bispectrum detail in the signal-dominated regime showing a comparison between full 3D reconstruction using hybrid Fourier modes (left) and hybrid polynomials (right). Note the consistency of the main bispectrum properties which include an apparently 'oscillatory' central feature for low-ℓ together with a flattened signal beyond to $\ell \lesssim 1400$. Note also the periodic CMB ISW-lensing signal in the squeezed limit along the edges of the tetrapyd.


BICEP2
0.03
0.025
0.02
0.015
0.01
0.005
0
−0.005
0
50
100
150
200
Multipole
Likelihood
r = 0.20 +0.07 −0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
Tensor−to−scalar ratio r
400
300
200
100
0
Entries
Data r = 0.20
S+N Sims r=0.0
S+N Sims r=0.2
0
0.1
0.2
0.3
0.4
Maximum likelihood r
auto subtracted
cross subtracted
base result
Likelihood
0
0.1
0.2
0.3
0.4
0.5
0.6
Tensor−to−scalar ratio r
Planck+WP+highL
Planck+WP+highL+BICEP2
0.4
0.3
0.2
0.1
0.0
0.94
0.96
0.98
1.00


Philosophy
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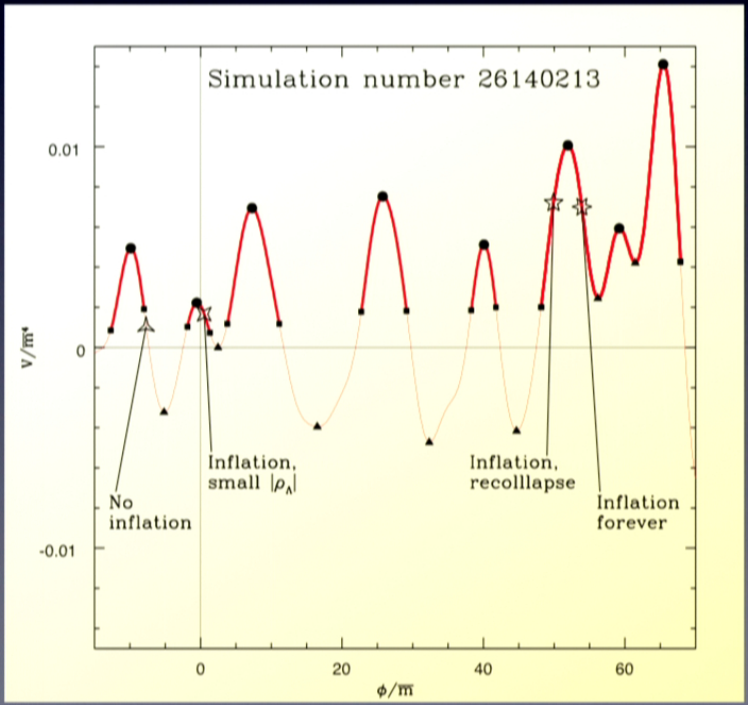
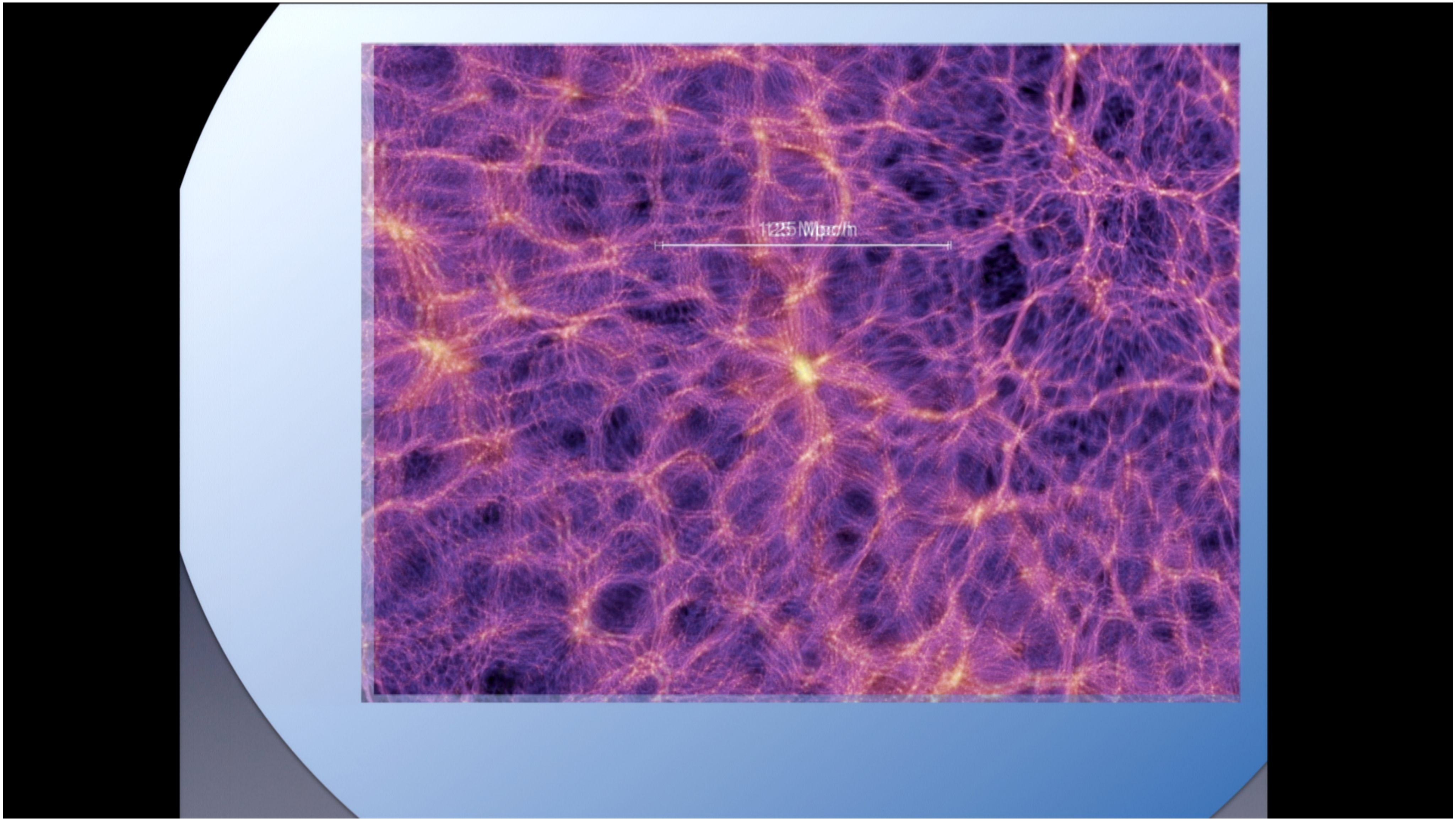
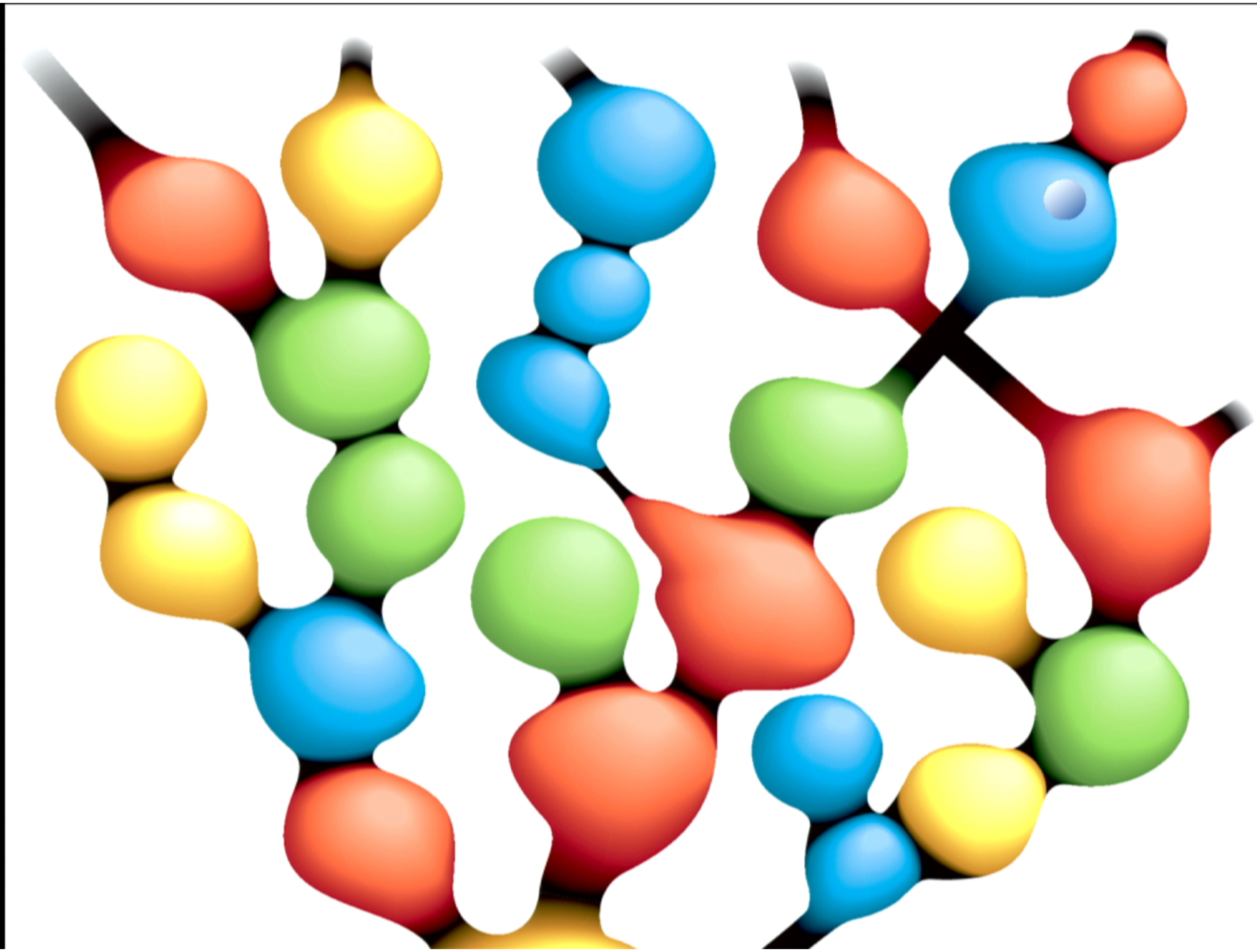


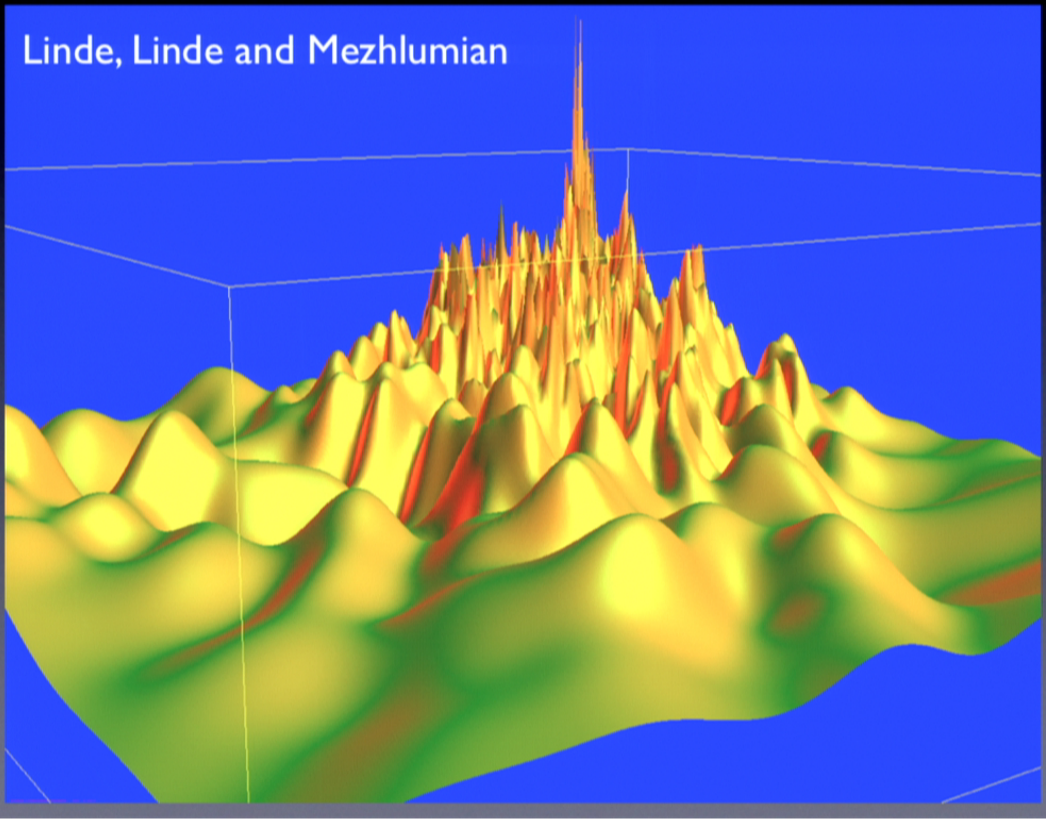
Early work of this kind was done by Tegmark (astro-ph/0410281). But he used only a single field, hence very limited phenomenology.


125 Mpc/h



The Multiverse ...

... is the notion that the properties of the Universe vary on extremely large scales, much bigger than the present observable Universe. This is predicted by simple inflationary models.



The Multiverse ...

... is worth considering as it provides so far the only plausible explanation for the observed cosmological constant:

- The multiverse contains untold numbers of regions with varying values of fundamental physics parameters including Λ.
- In those where Λ is negative, the Universe typically recollapses long before galaxies can form.
- In those where Λ is positive, the Universe typically goes into accelerated expansion, ceasing structure formation, long before galaxies can form.
- The exception is if Λ is extremely close to zero (in Planck units), as is observed.

Hence the multiverse is envisaged to be a vast arena almost entirely devoid of galaxies. In places where galaxies happen to exist, Λ is extremely small. If Λ is measured by lifeforms who inhabit galaxies, they inevitably find Λ to be small, even though this is not a fair representation of the greater multiverse.

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- The argument hinges on there being a huge number of possible vacuum states to the Universe, as the observed value of Λ is around $10^{-120}\ m_{Pl}^4$. String theory has been shown to provide at least 10^{500} possible vacuum states of different energies, and hence can provide a realization of Weinberg's argument.

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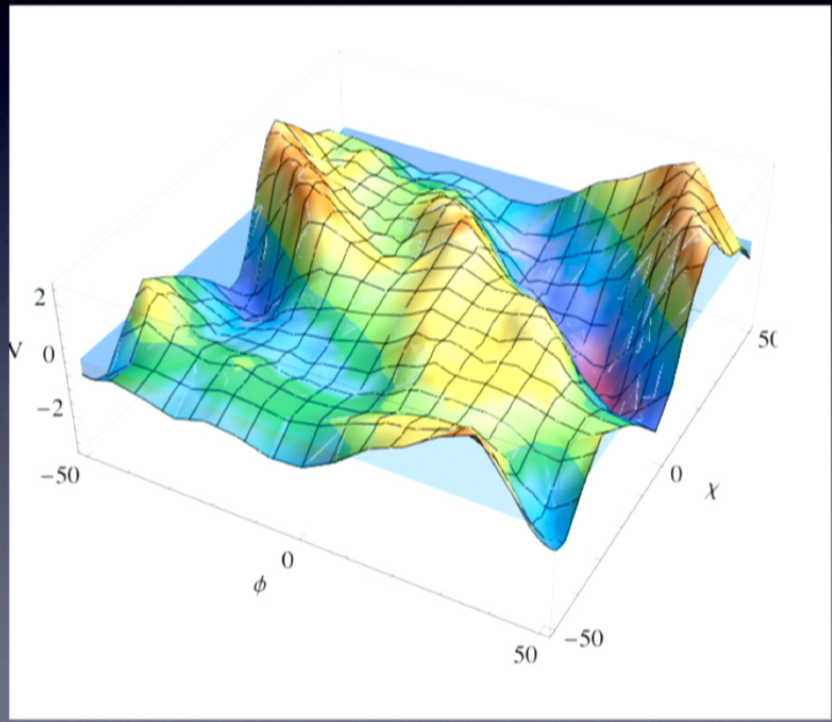
String landscape inflation

Frazer and Liddle, JCAP, arXiv:1101.1619

Our aim: to consider consequences of the string landscape picture for inflation by considering inflationary trajectories in a (toy-model) randomly-generated landscape.

We initially considered a two-dimensional landscape generated via a Fourier series approach.



New phenomenology:

- Isocurvature perturbations.
- Non-gaussianity.

Procedure

1. Generate a random potential $V(\phi)$ and start at $\phi = (0, 0)$.
2. If $V(0, 0) < 0$ then reject model, otherwise evolve to find the field trajectory.
3. If model gets stuck in eternal inflation, reject.
4. Once the model stops inflating, if the number of e-folds of inflation $N < 60$ we reject as insufficient inflation occurred. Otherwise find the local minimum to calculate ρ_{vac}, and if $\rho_{\text{vac}} < 0$, reject. If $N \geq 60$ and $\rho_{\text{vac}} \geq 0$, calculate observables.
5. Repeat many times to obtain a statistical sample.
6. (Change some assumptions and do it all again.)

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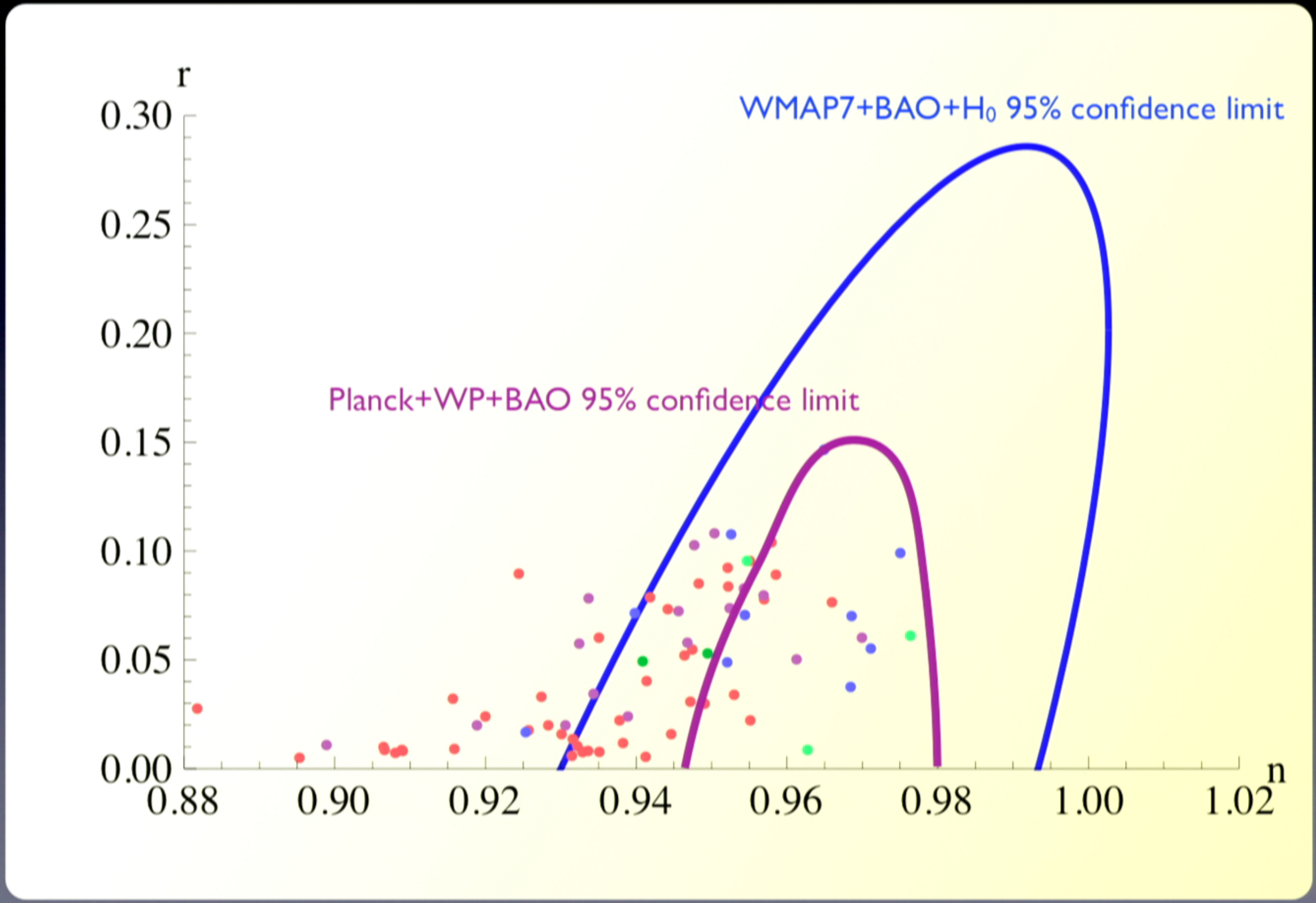
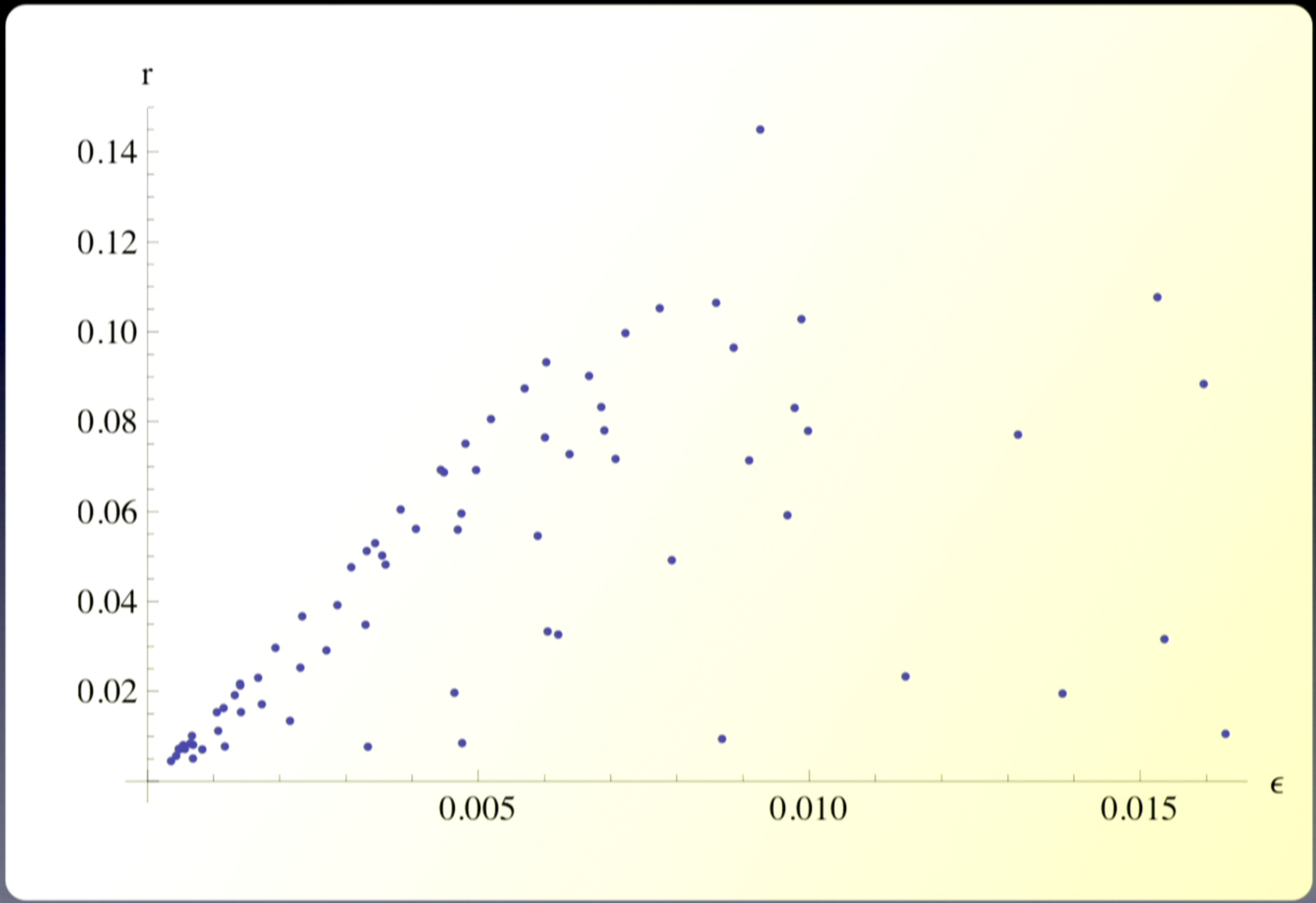
Successful runs (where inflation ends in a satisfactory vacuum state) are obtained only around one in 10^5 simulations. We analyzed approximately 5 x 10^6 cases thus generating around 100 model Universes. We can then explore their properties.

Predicted perturbations: spectral index versus tensor-to-scalar ratio.



Consistency relation breakdown due to isocurvature perturbations.



Extension I: Canonical models

Frazer and Liddle, JCAP, arXiv:1111.6646

- Extension to *D* scalar fields instead of only 2
  - Done up to *D*=6, nothing much changes.
- Computation of cosmic non-gaussianity, using Transport Equations method of Mulryne-Seery-Wesley.

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  - Done up to *D*=6, nothing much changes.
- Computation of cosmic non-gaussianity, using Transport Equations method of Mulryne-Seery-Wesley.
  - It's always small.
- Fuller investigation of the effect of choice of the scale of features in the random potentials.
  - String theory suggests that the potential should have more structures than we used in our first paper. Success rate in generating models plummets as more features are included, but change does not seem radical.

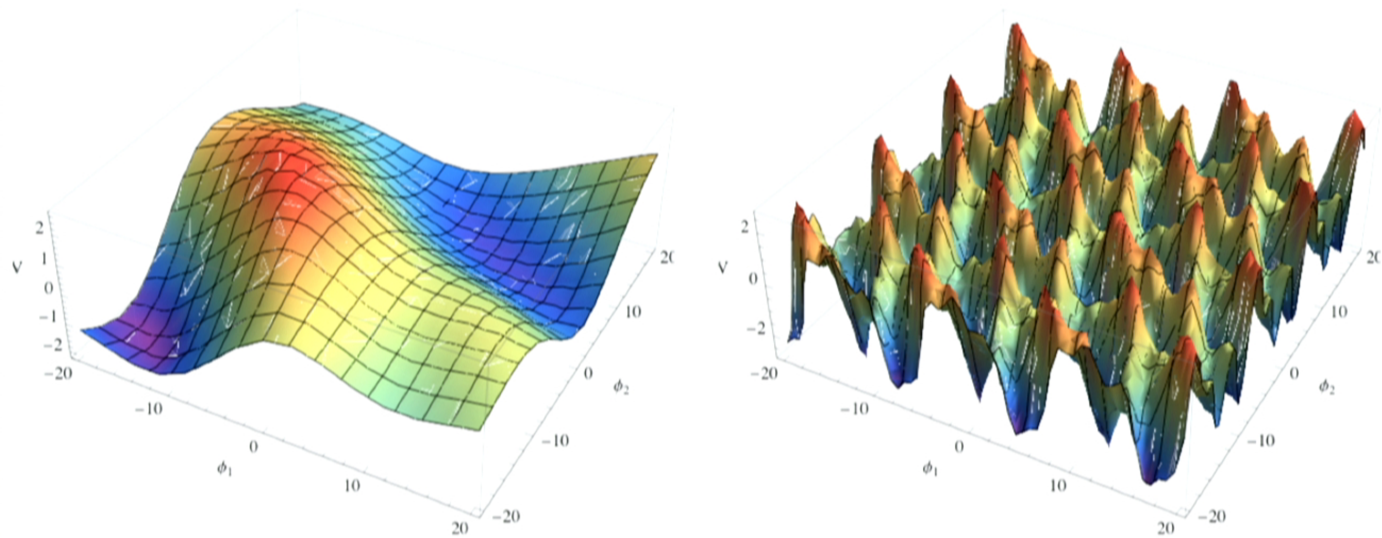
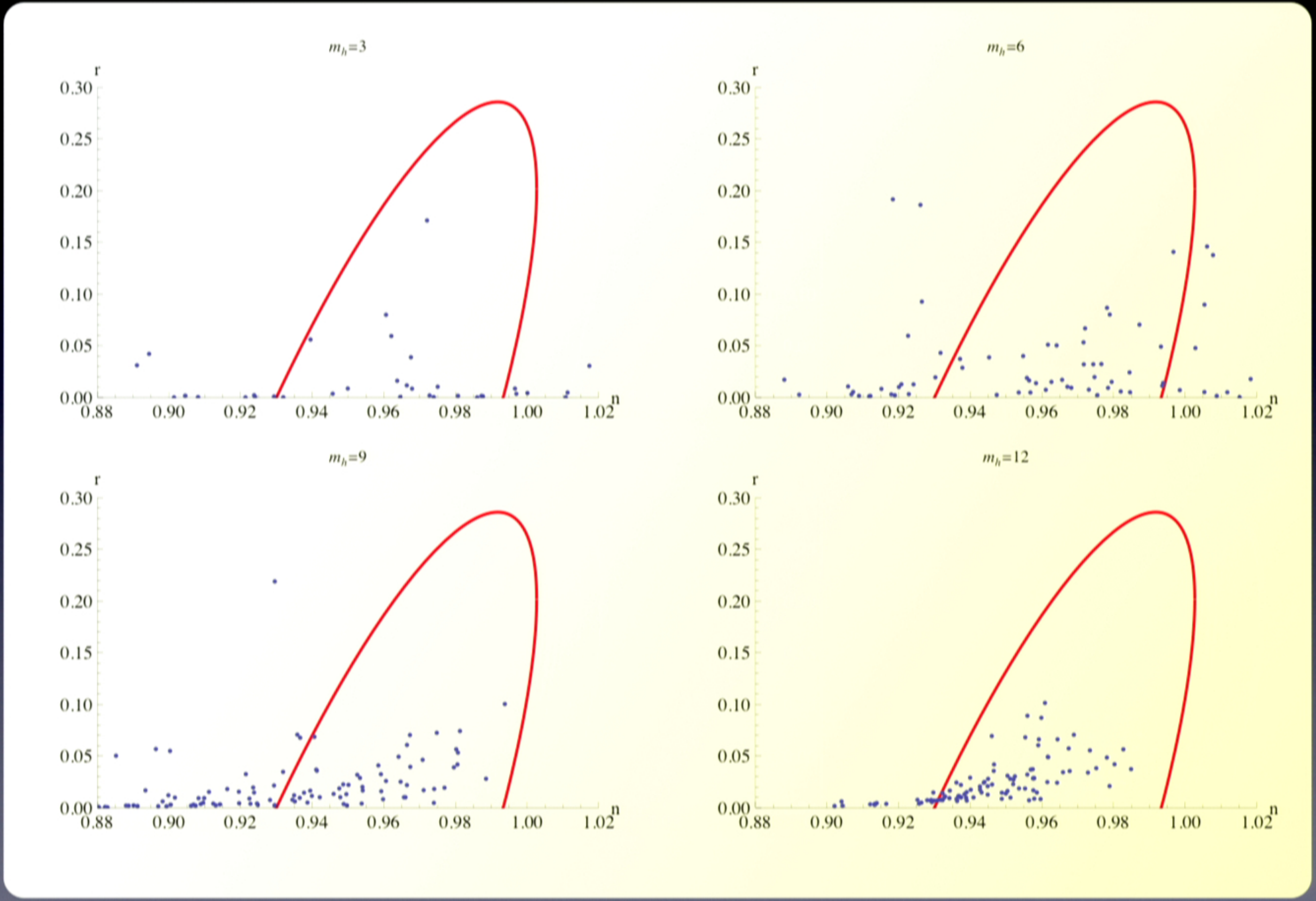
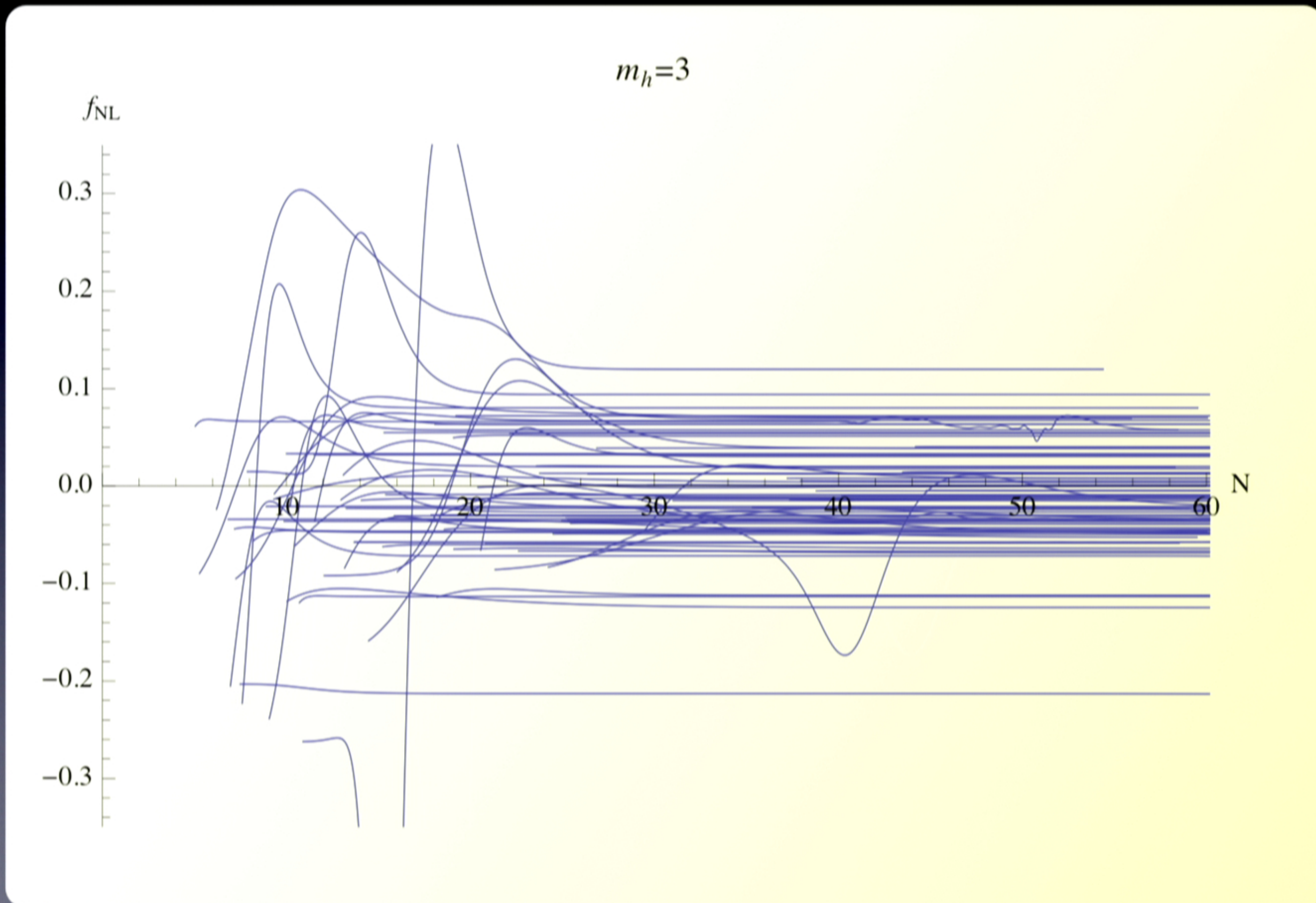


Figure 2. Example of two-field potentials with $m_{\mathrm{h}} = 15.8\, M_{\mathrm{Pl}}$ and $m_{\mathrm{h}} = 2.0\, M_{\mathrm{Pl}}$ respectively.



m_h=3
m_h=6
m_h=9
m_h=12
r
n
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.88
0.90
0.92
0.94
0.96
0.98
1.00
1.02

Evolution of non-gaussianity



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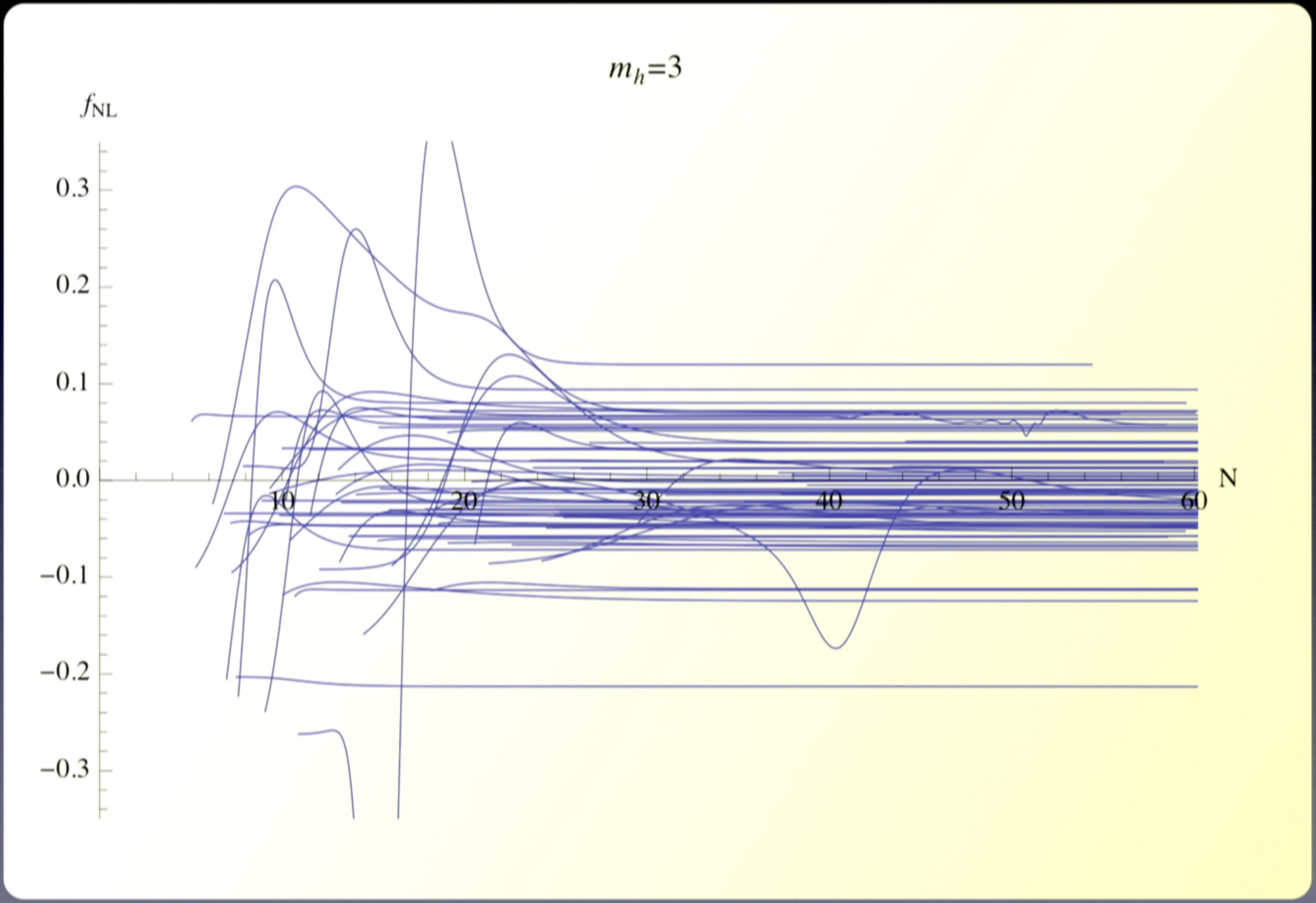
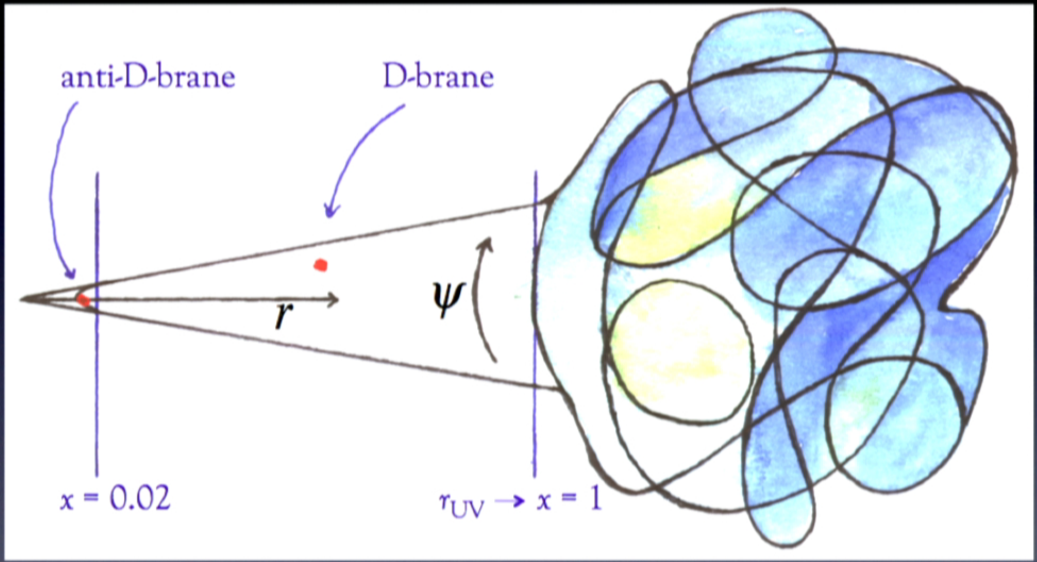


Extension II: D-brane model

Dias-Frazer-Liddle, JCAP, arXiv:1203.3792 (plus March 2013 erratum)



Many unknown Wilson coefficients that need to be sampled randomly, representing flux/ branes/etc

See also N. Agarwal, R. Bean, L. McAllister and G. Xu, Universality in D-brane Inflation, JCAP 1109 (2011) 002, arXiv:1103.2775
L. McAllister, S. Renaux-Petel and G. Xu, A Statistical Approach to Multifield Inflation: Many-field Perturbations Beyond Slow Roll, astro-ph/1207.0317.

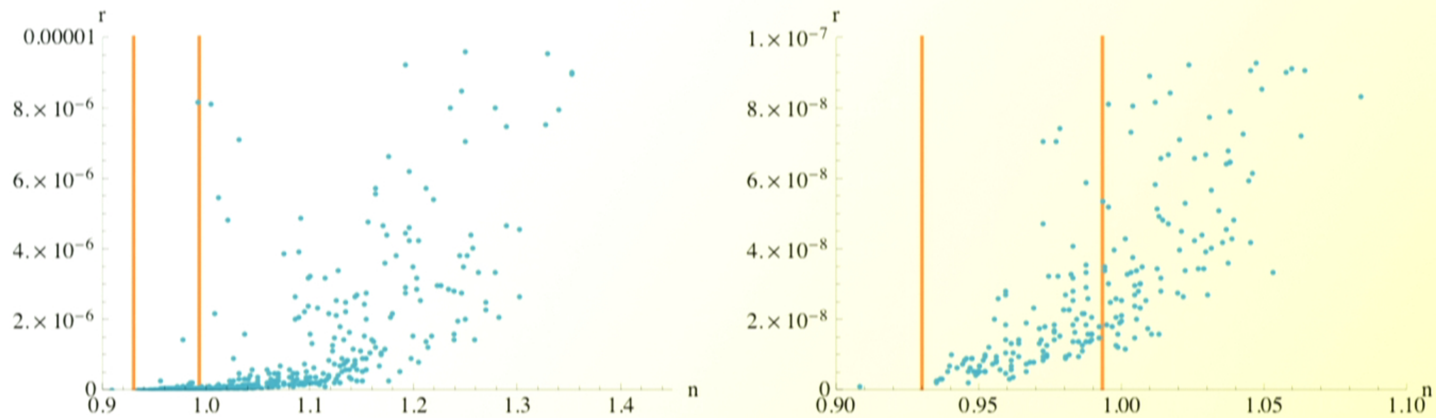
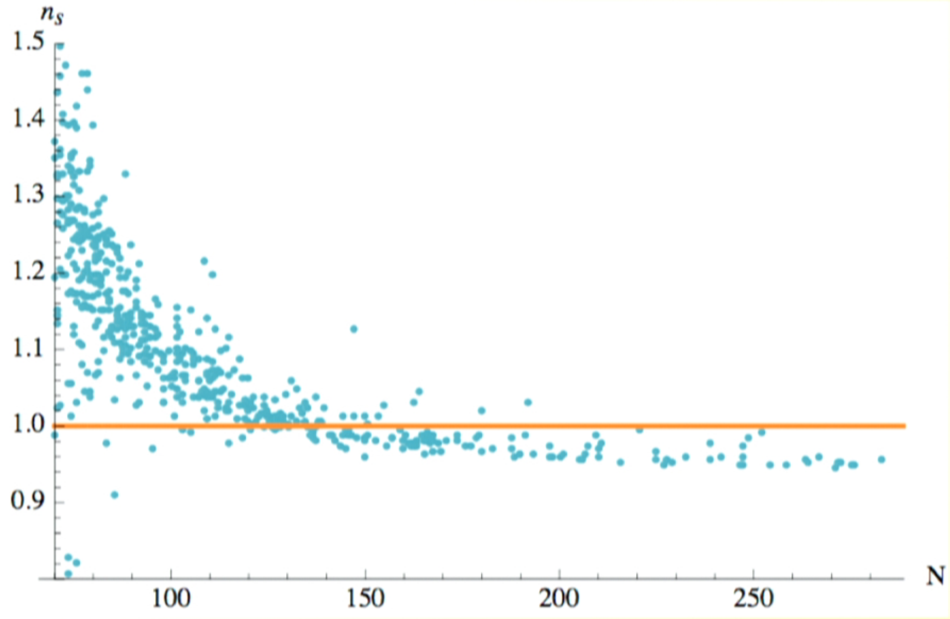


Figure 17. Plot of the values of n_s versus r. The right panel greatly expands the vertical scale. The orange lines indicate the 95% confidence limits using WMAP data.



Successful trajectories are inflection point trajectories. However, if horizon exit occurs before the inflection point is reached, the spectrum is blue. Models with red spectral indices tend to have >120 e-foldings in total.

Conclusions

- Inflation remains an excellent explanation of the origin of structure.
- There is no evidence yet of any complexity in inflationary dynamics.
- Landscape models may indicate limits to the predictiveness of inflationary models. They also tend to give rather boring observational predictions.
- In string-motivated incarnations, the large *r* value found by BICEP2 poses serious observational difficulties.

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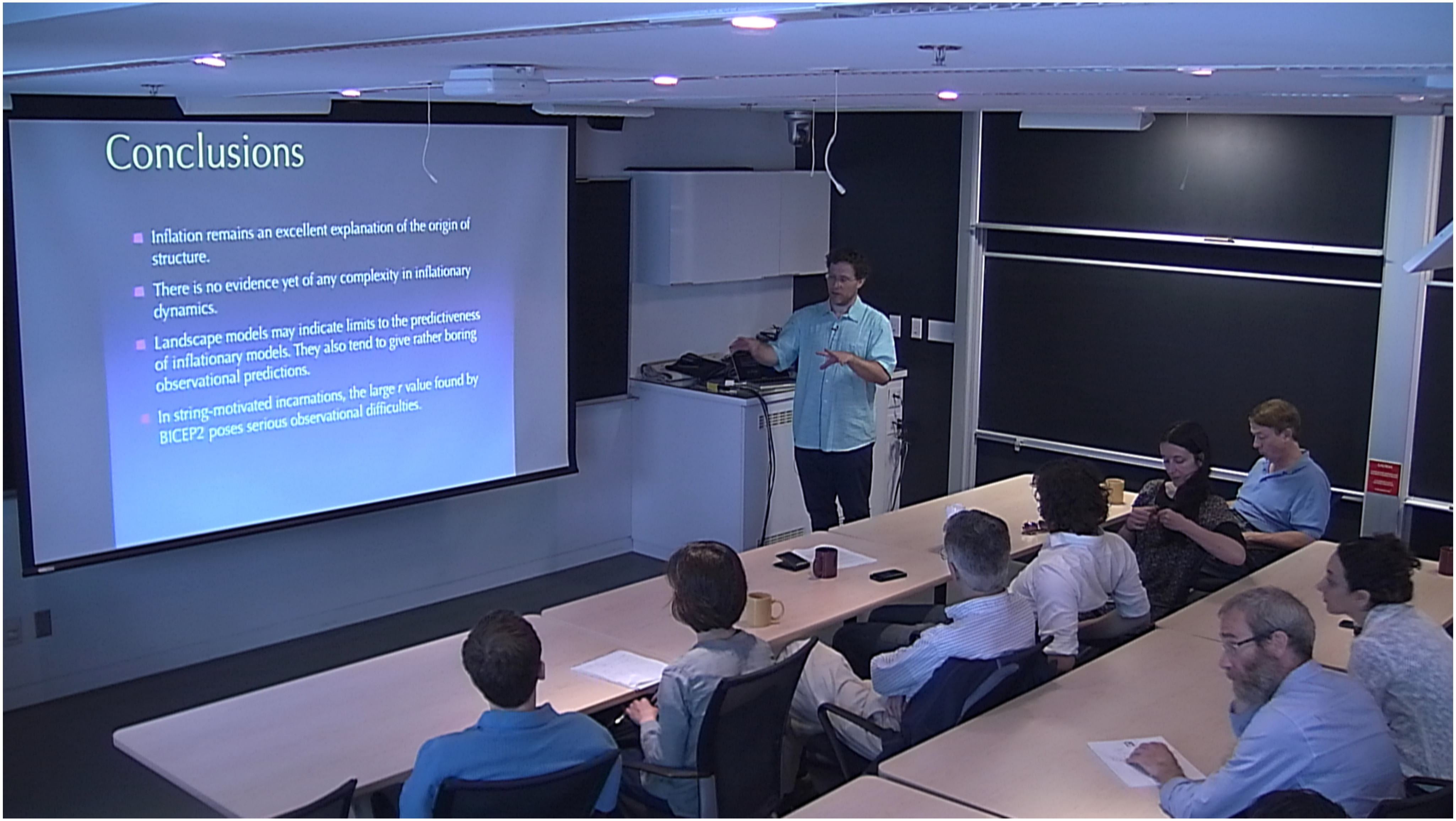
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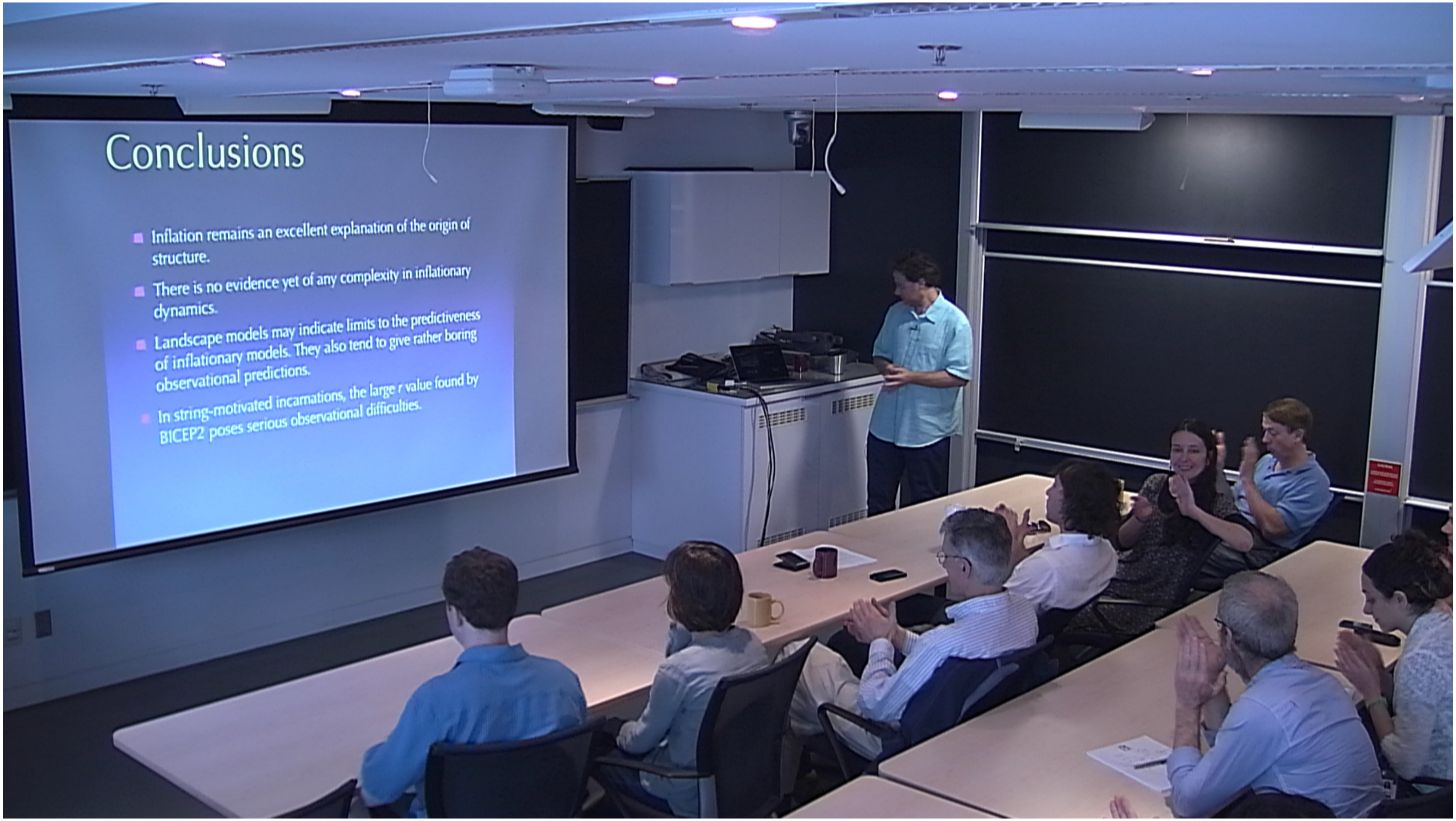
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