Title: Unitarity and crossing symmetry in the S-matirx of large N Chern-Simons theory with fundamental matter

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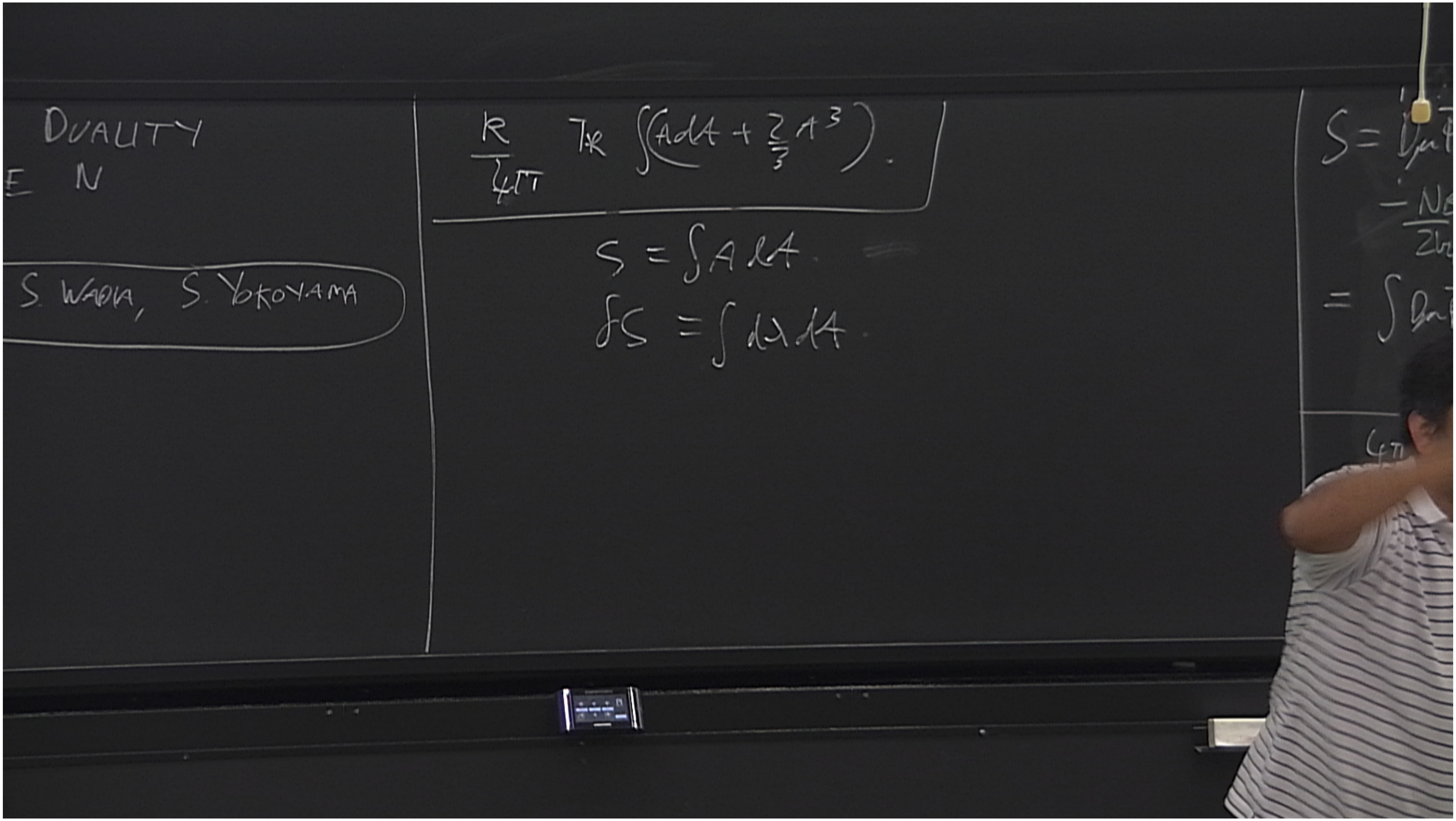
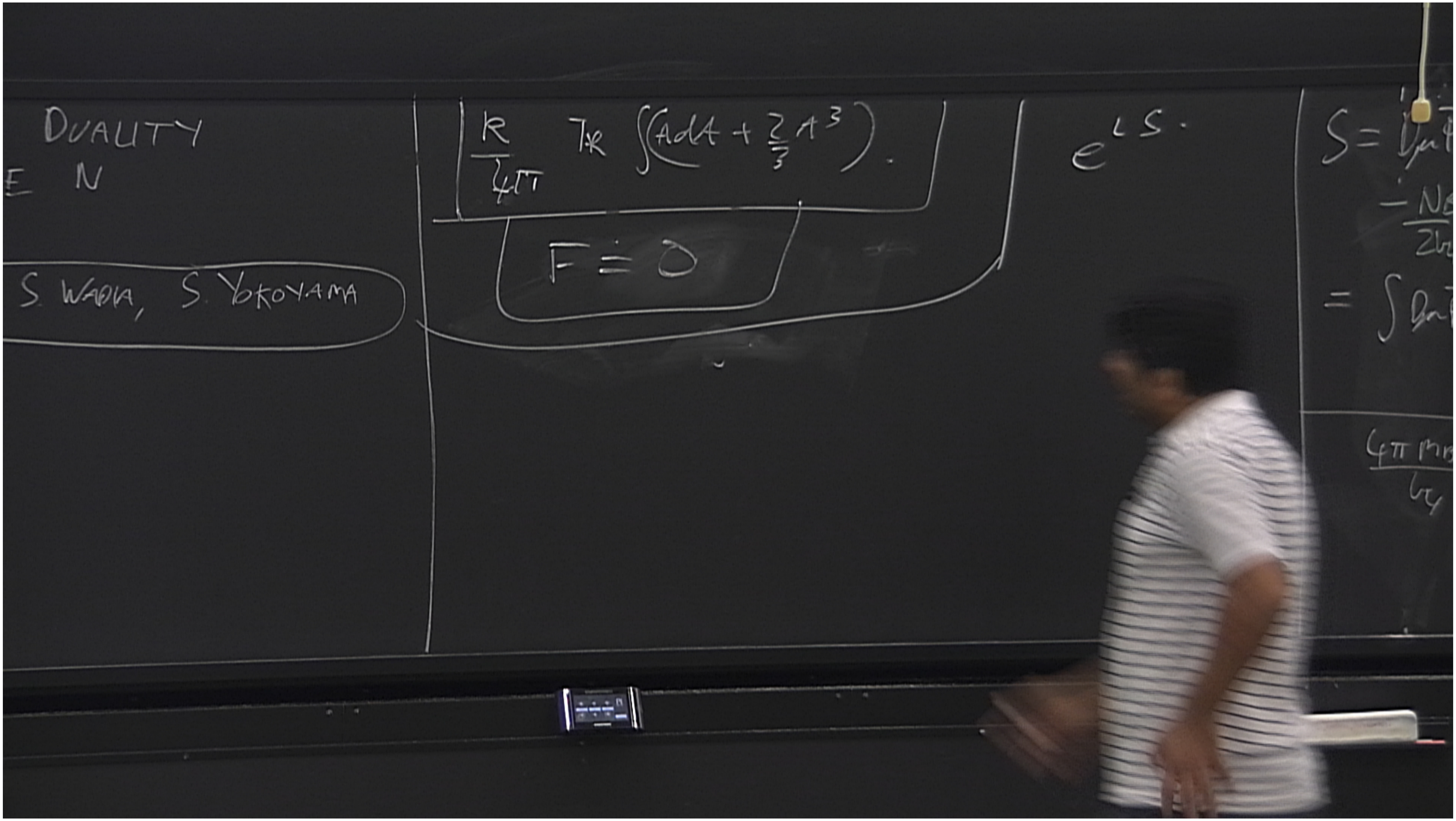
Abstract: We present explicit computations and conjectures for 2 â†’ 2 scattering matrices in large N U(N) Chern-Simons theories coupled to fundamental bosonic or fermionic matter to all orders in the â€™t Hooft coupling expansion. The bosonic and fermionic S-matrices map to each other under the recently conjectured Bose-Fermi duality after a level-rank transposition. The S-matrices presented in this paper may be regarded as relativistic generalization of Aharonov-Bohm scattering. They have unusual structural features: they include a non analytic piece localized on forward scattering, and obey modified crossing symmetry rules. We conjecture that these unusual features are properties of S-matrices in all Chern-Simons matter theories. The S-matrix in one of the exchange channels in our paper has an anyonic character; the parameter map of the conjectured Bose-Fermi duality may be derived by equating the anyonic phase in the bosonic and fermionic theories.

UNITARITY, CROSSING SYMMETRY AND DUALITY OF THE S MATRIX IN LARGE N CHERN SIMONS THEORIES

S. JAIN, M. MANDLIK, SM, T. TAKIMI, S. WADIA, S. YOKOYAMA

1. INTRO + BG
2. NON RELATIVISTIC LIMIT
3. CALCULATION
4. CONJECTURE


DUALITY
E N
S. WADIA, S. YOKOYAMA
$\frac{k}{4\pi} \mathrm{Tr} \int \left(A dA + \frac{2}{3} A^3\right).$
$S = \int A dA$
$\delta S = \int d\lambda dA$


DUALITY
N
S. WADIA, S. YOKOYAMA
$\frac{k}{4\pi} \mathrm{Tr} \int (AdA + \frac{2}{3} A^3)$.
$F = 0$
e^{iS}.

DUALITY

E N

S. WADIA, S. YOKOYAMA

$$\boxed{\frac{k}{4\pi} \mathrm{Tr} \int \left(A\,dA + \frac{2}{3} A^3\right).}$$

$$\boxed{F = 0}$$

$$S_{CS} + \int \bar{\psi} \not{D} \psi.$$

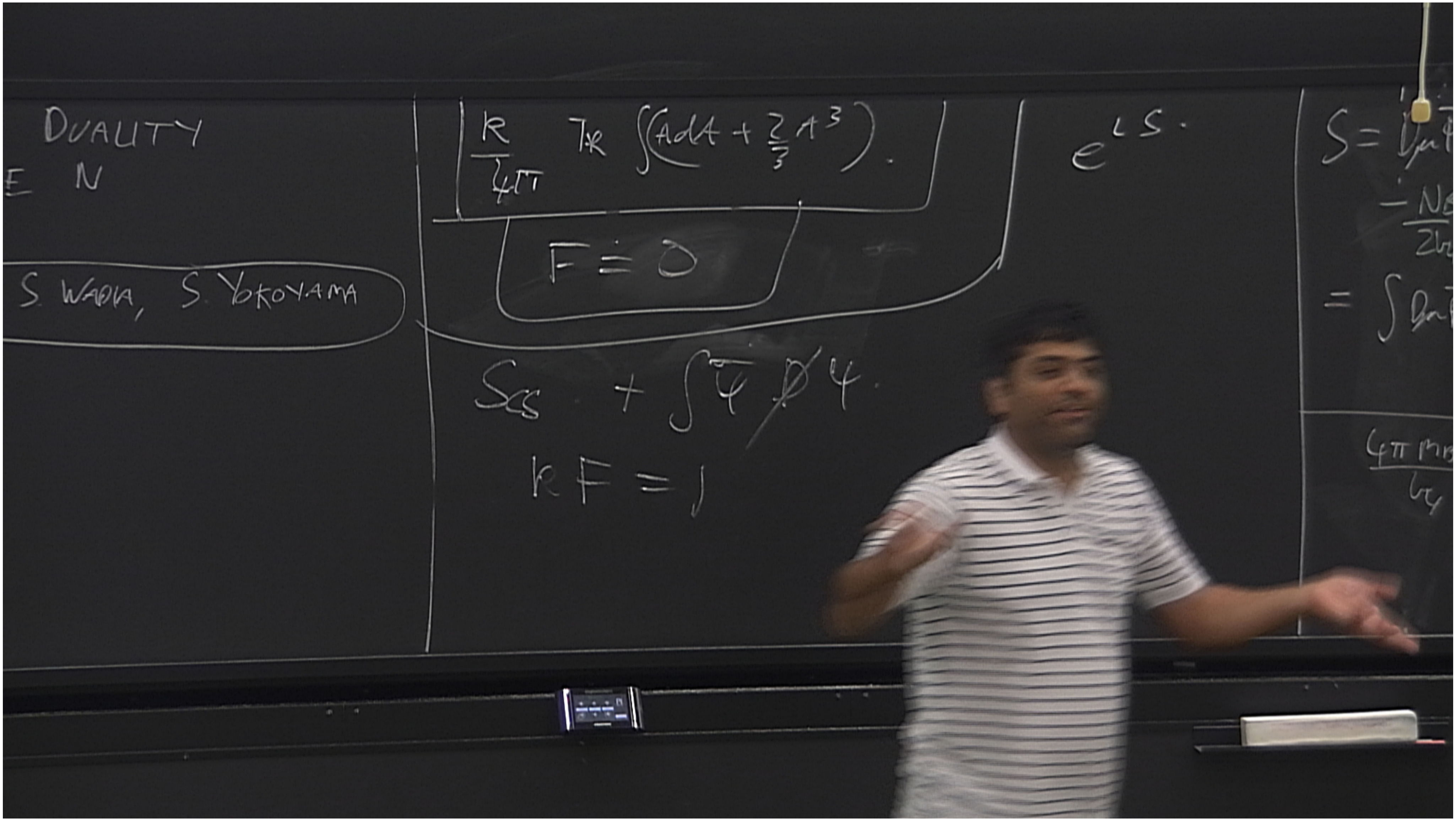
$$e^{iS}.$$

$$S = D_\mu \ldots$$

$$-\frac{N\ldots}{2\ldots}$$

$$= \int D_\mu \ldots$$

$$\frac{4\pi\, m\ldots}{\ldots}$$


DUALITY
S. WADIA, S. YOKOYAMA
F = 0
kF = 1

DUALITY

E N

S. WADIA, S. YOKOYAMA

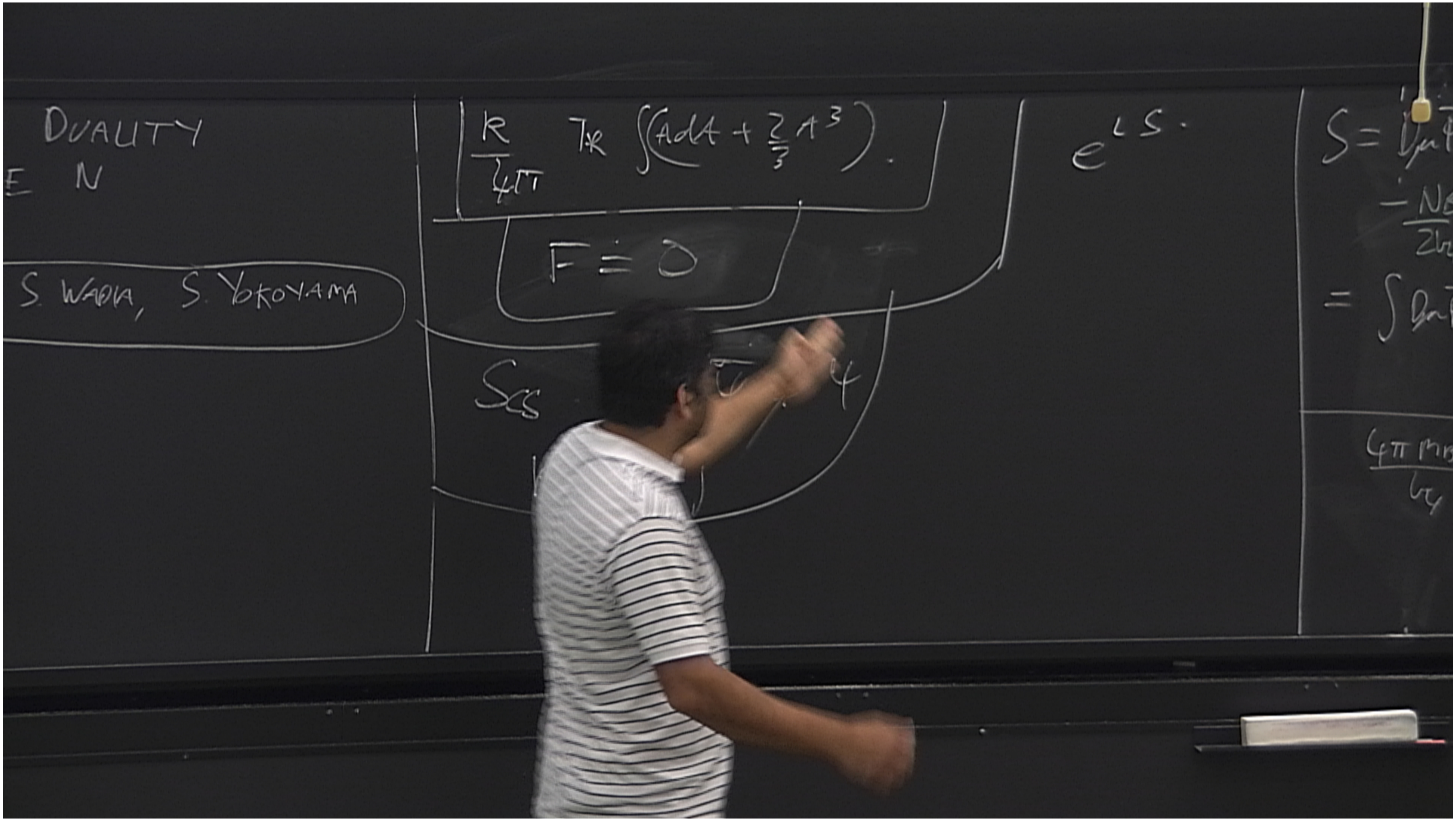
$$\boxed{\frac{k}{4\pi} \mathrm{Tr} \int \left(A\,dA + \frac{2}{3} A^3\right).}$$

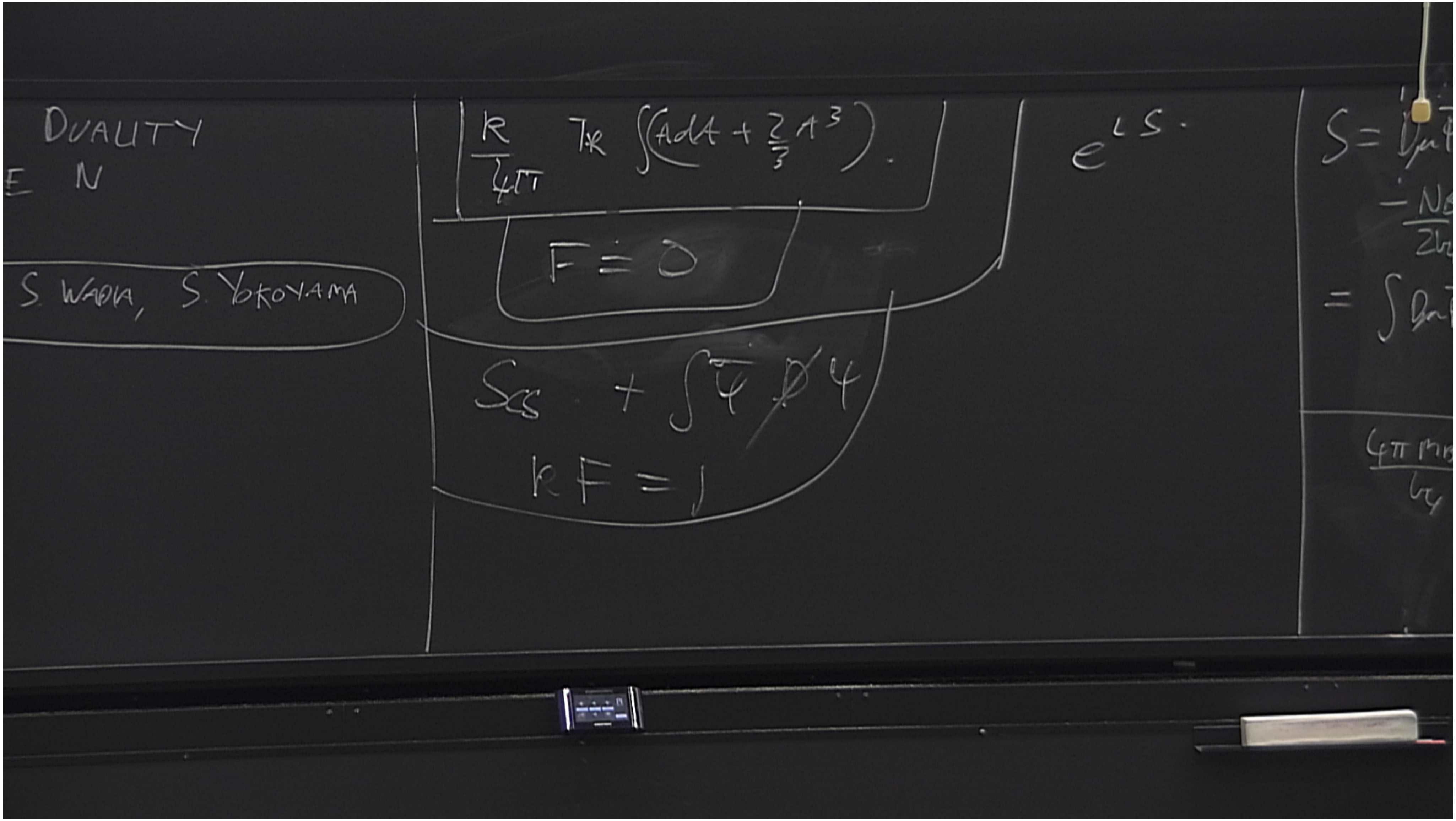
$$e^{iS}.$$

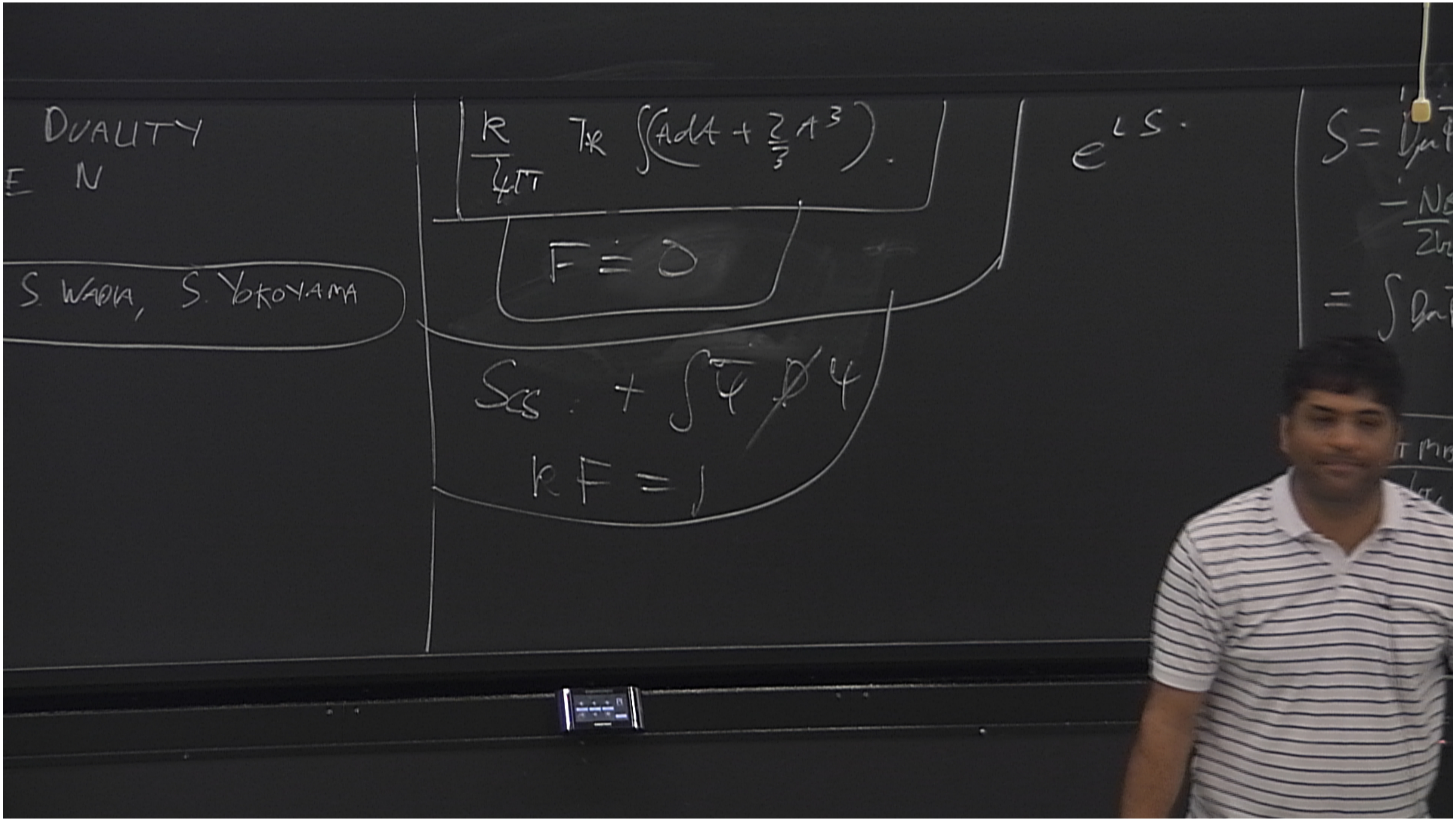
$$\boxed{F = 0}$$

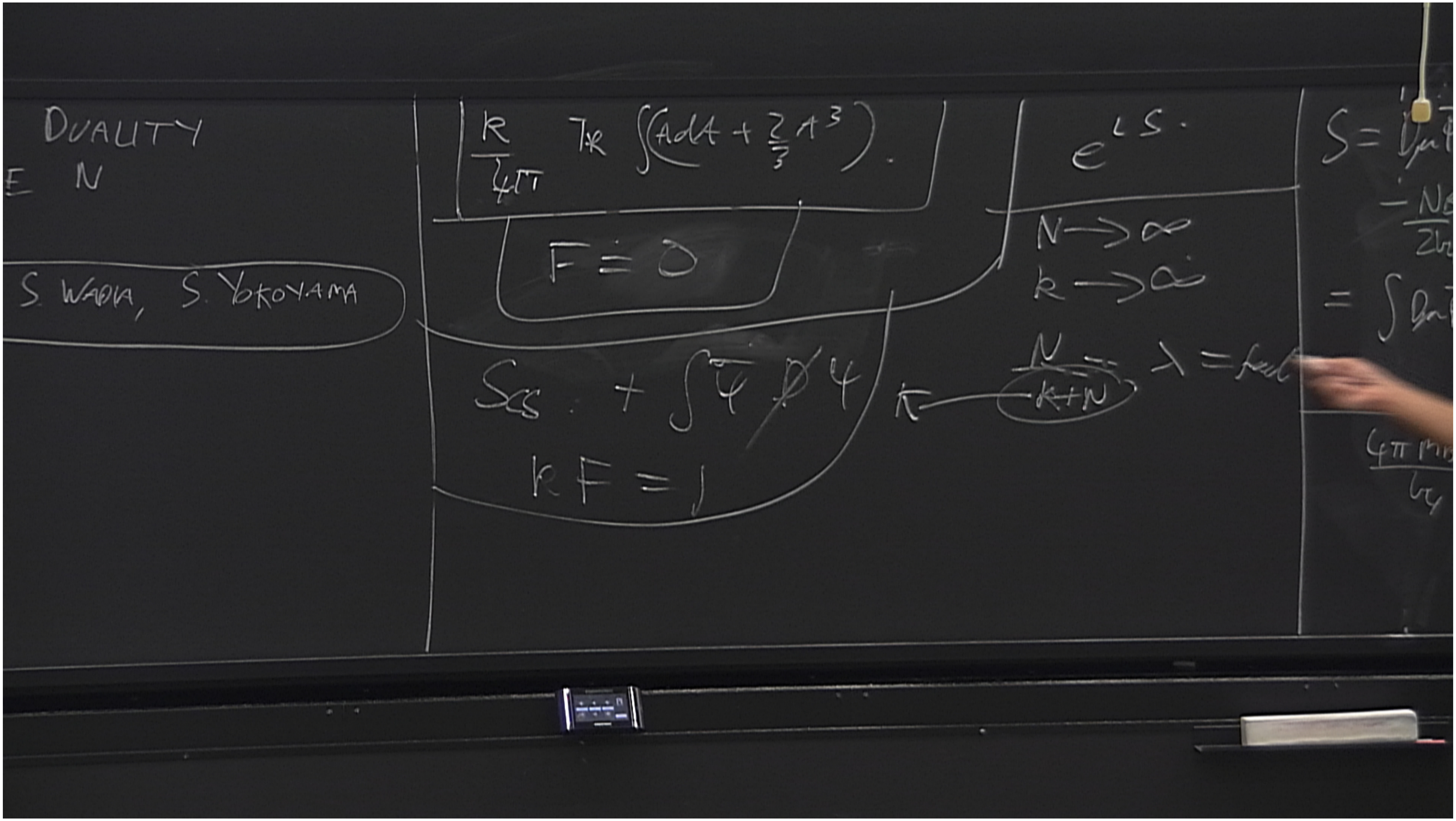
$$S_{CS} + \int \bar{\psi} \not{D} \psi.$$

$$kF = J$$


DUALITY
N
S. WADIA, S. YOKOYAMA
$\frac{k}{4\pi} \mathrm{Tr} \int (AdA + \frac{2}{3}A^3)$.
$F = 0$
e^{iS}.
S_{CS}


DUALITY
E N
S. WADIA, S. YOKOYAMA
$\frac{k}{4\pi} \mathrm{Tr} \int (A dA + \frac{2}{3} A^3)$.
$F = 0$
$S_{CS} + \int \bar{\psi} \not{D} \psi$
$kF = 1$
e^{iS}.


DUALITY
N
S. WADIA, S. YOKOYAMA
$\frac{k}{4\pi} \mathrm{Tr} \int (A dA + \frac{2}{3} A^3)$.
F = 0
$S_{CS} + \int \bar{\psi} \not{D} \psi$
kF = 1
e^{iS}.


DUALITY
N
S. WADIA, S. YOKOYAMA
F = 0
kF = 1
N → ∞
k → ∞

$Tr \int \left(AdA + \frac{2}{3}A^3\right).$

$e^{iS}.$

$F = 0$

$N \to \infty$

$k \to \infty$

$\frac{N}{k+N} = \lambda = \text{fixed}$

$+ \int \bar{\psi} \not{D} \psi$

$F = 1$

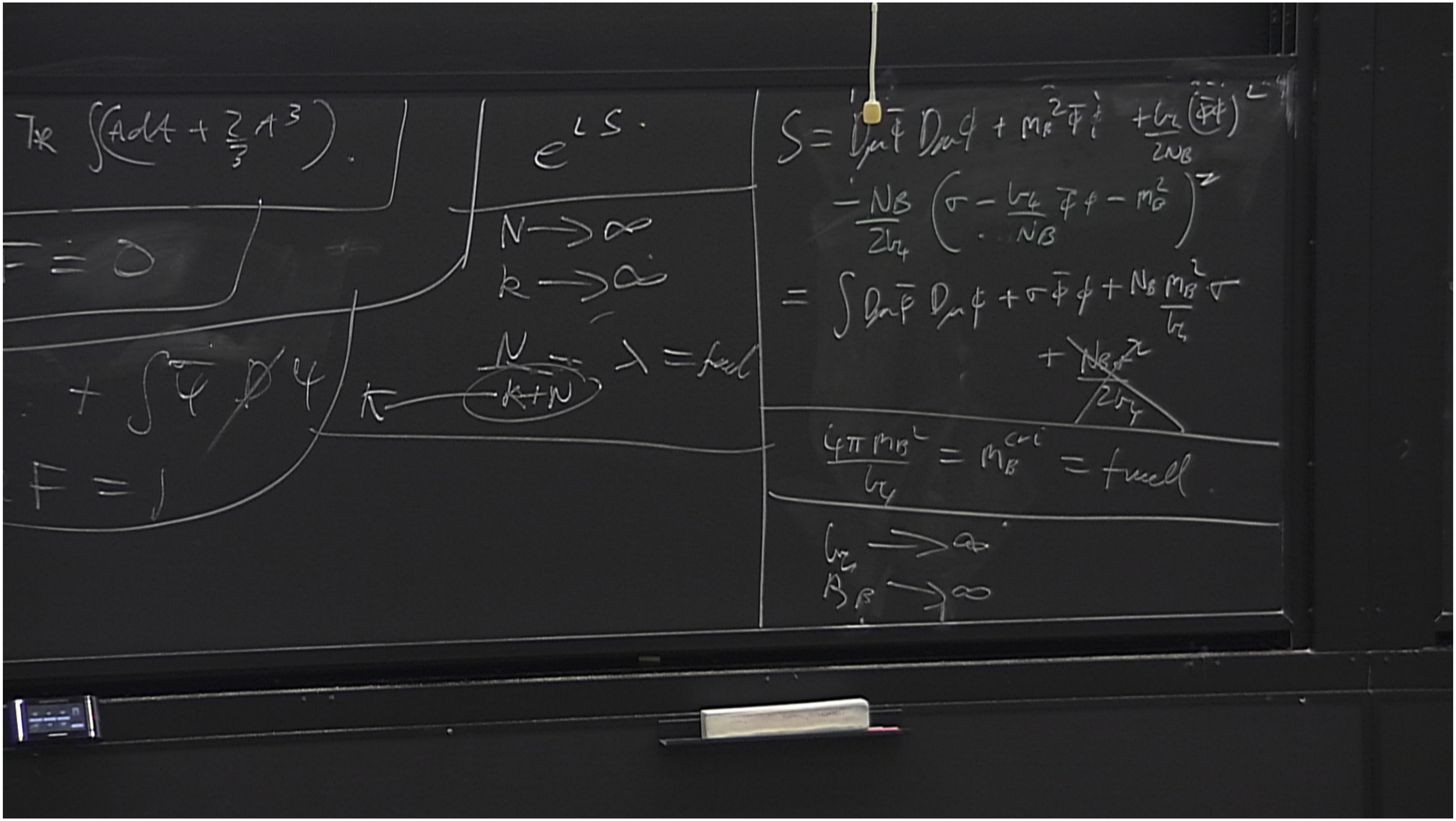
$S = D_\mu \bar{\phi} D_\mu \phi + m_B^2 \bar{\phi}\phi + \frac{b_4}{2N_B}(\bar{\phi}\phi)^2$

$- \frac{N_B}{2b_4}\left(\sigma - \frac{b_4}{N_B}\bar{\phi}\phi - m_B^2\right)$

$= \int D_\mu \bar{\phi} D_\mu \phi + \sigma \bar{\phi}\phi + N_B \frac{m_B^2}{b_4}\sigma$

$+ \frac{N_B \sigma^2}{2b_4}$

$\frac{4\pi m_B^2}{b_4} = m_B^{cri} = \text{fixed}.$


Tr ∫(AdA + 2/3 A³).
F = 0
+ ∫ Ψ̄ D̸ Ψ
F = 1
e^{iS}.
N → ∞
k → ∞
N/(k+N) = λ = fixed
S = D_μΦ̄ D_μΦ + m_B² Φ̄Φ + λ₄/(2N_B) (Φ̄Φ)²
− N_B/(2λ₄) (σ − λ₄/N_B Φ̄Φ − m_B²)²
= ∫ D_μΦ̄ D_μΦ + σΦ̄Φ + N_B m_B²/λ₄ σ
+ N_B σ²/(2λ₄)
4π m_B²/λ₄ = m_B^{cri} = fixed
λ₄ → ∞
B_B → ∞

$\mathrm{Tr}\int\left(A\,dA+\frac{2}{3}A^3\right)$.

$F=0$

$+\int\bar{\Psi}\,\cancel{D}\,\Psi$

$F=1$

e^{iS}.

$N\to\infty$

$k\to\infty$

$\frac{N}{k+N}=\lambda=\text{fixed}$

$S=D_\mu\bar{\Phi}D_\mu\Phi+m_B^2\bar{\Phi}\Phi+\frac{\lambda_4}{2N_B}(\bar{\Phi}\Phi)^2$

$-\frac{N_B}{2\lambda_4}\left(\sigma-\frac{\lambda_4}{N_B}\bar{\Phi}\phi-m_B^2\right)^2$

$=\int D_\mu\bar{\Phi}D_\mu\phi+\sigma\bar{\Phi}\cdot\phi+N_B\frac{m_B^2}{\lambda_4}\sigma+\cancel{\frac{N_B\sigma^2}{2\lambda_4}}$

$\frac{4\pi m_B^2}{\lambda_4}=m_B^{cri}=\text{fixed}.$

$\lambda_4\to\infty$

$\lambda_B\to\infty$

$Tr \int (A dA + \frac{2}{3} A^3)$.

$F = 0$

$+ \int \bar{\psi} \cancel{D} \psi$

$F = 1$

e^{iS}.

$N \to \infty$

$k \to \infty$

$\frac{N}{k+N} = \lambda = \text{fixed}$

$k \leftarrow k+N$

$S = D_\mu \bar{\Phi} D_\mu \phi + m_B^2 \bar{\Phi} \phi + \frac{b_4}{2N_B} (\bar{\Phi}\phi)^2$

$- \frac{N_B}{2b_4} \left(\sigma - \frac{b_4}{N_B} \bar{\Phi} \phi - m_B^2 \right)^2$

$= \int D_\mu \bar{\Phi} D_\mu \phi + \sigma \bar{\Phi} \cdot \phi + N_B \frac{m_B^2}{b_4} \sigma$

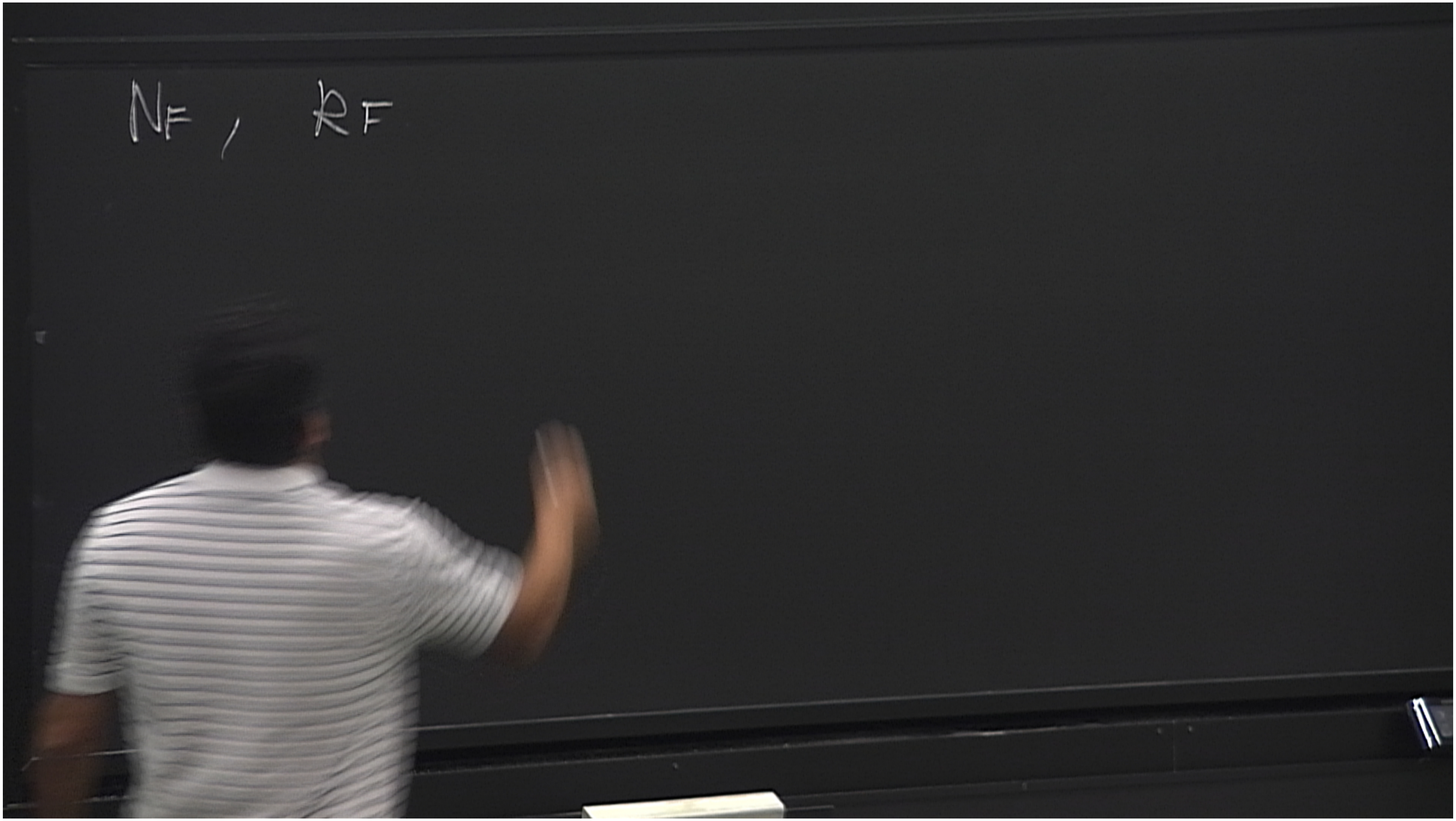
$+ \cancel{\frac{N_B \sigma^2}{2 b_4}}$

$\frac{4\pi m_B^2}{b_4} = m_B^{cri} = \text{fixed}$.

$b_4 \to \infty$

$B_B \to \infty$

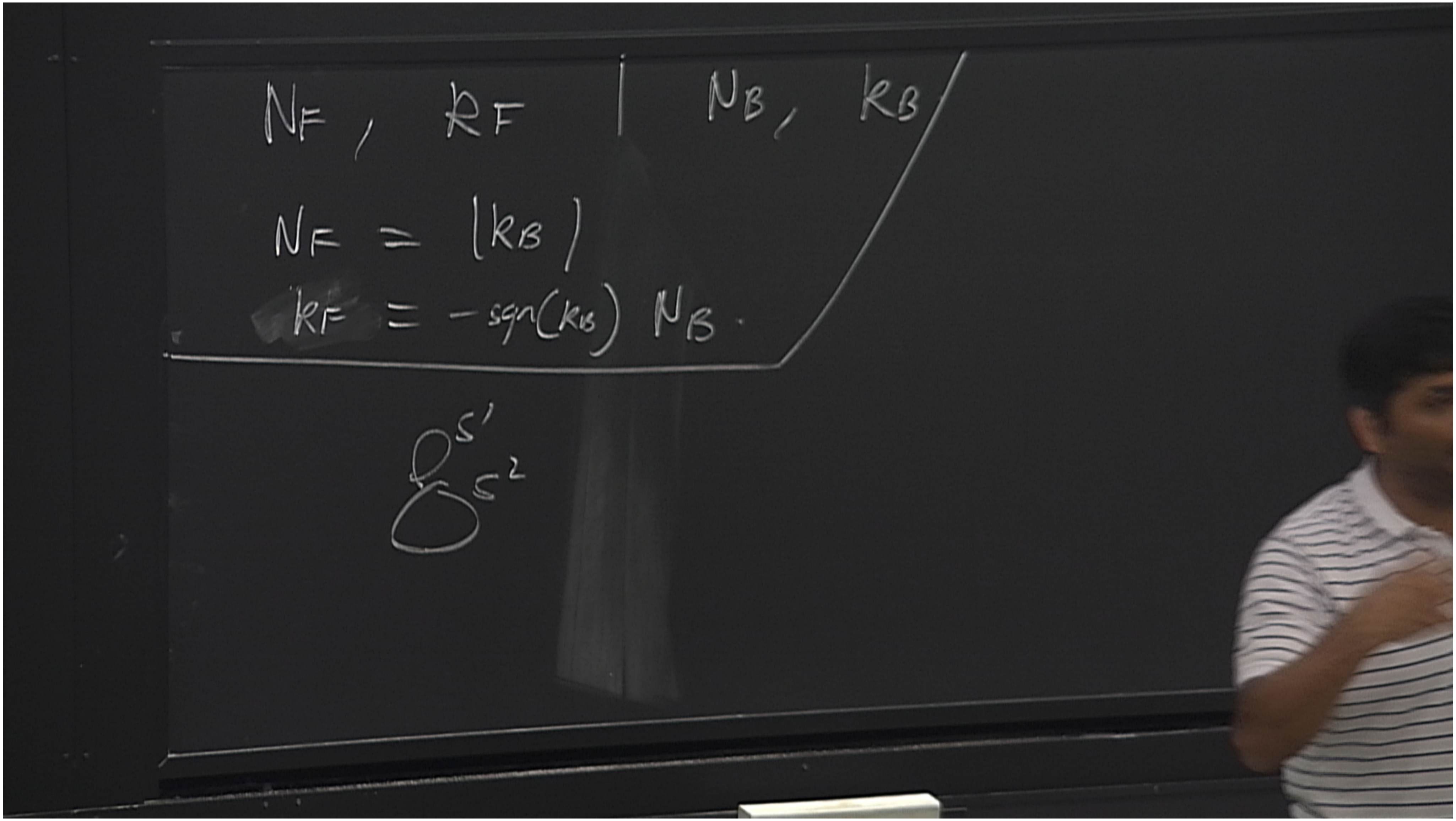
N_F , R_F

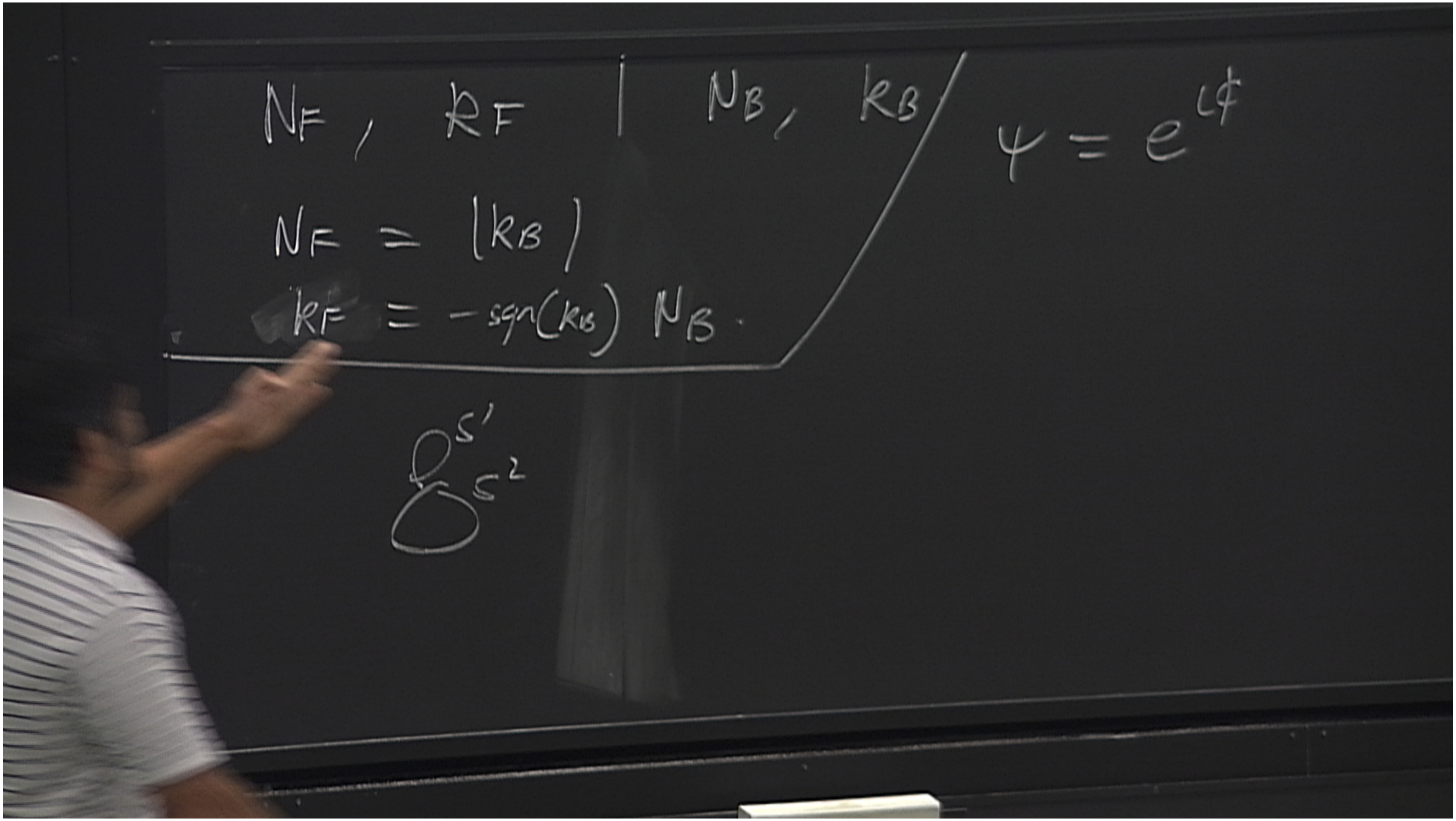

NF , kF

$N_F, \; k_F \;\;|\;\; N_B, \; k_B.$

$N_F = |k_B|$

$k_F = -\mathrm{sgn}(k_B)\, N_B.$


N_F , k_F | N_B , k_B
$N_F = |k_B|$
$k_F = -\mathrm{sgn}(k_B)\, N_B$.
S^1
S^2

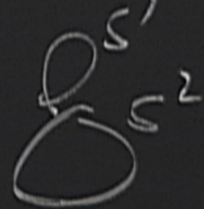

$N_F , k_F \mid N_B , k_B$
$\psi = e^{i\phi}$
$N_F = |k_B|$
$k_F = -\mathrm{sgn}(k_B) N_B$
S^1
S^2

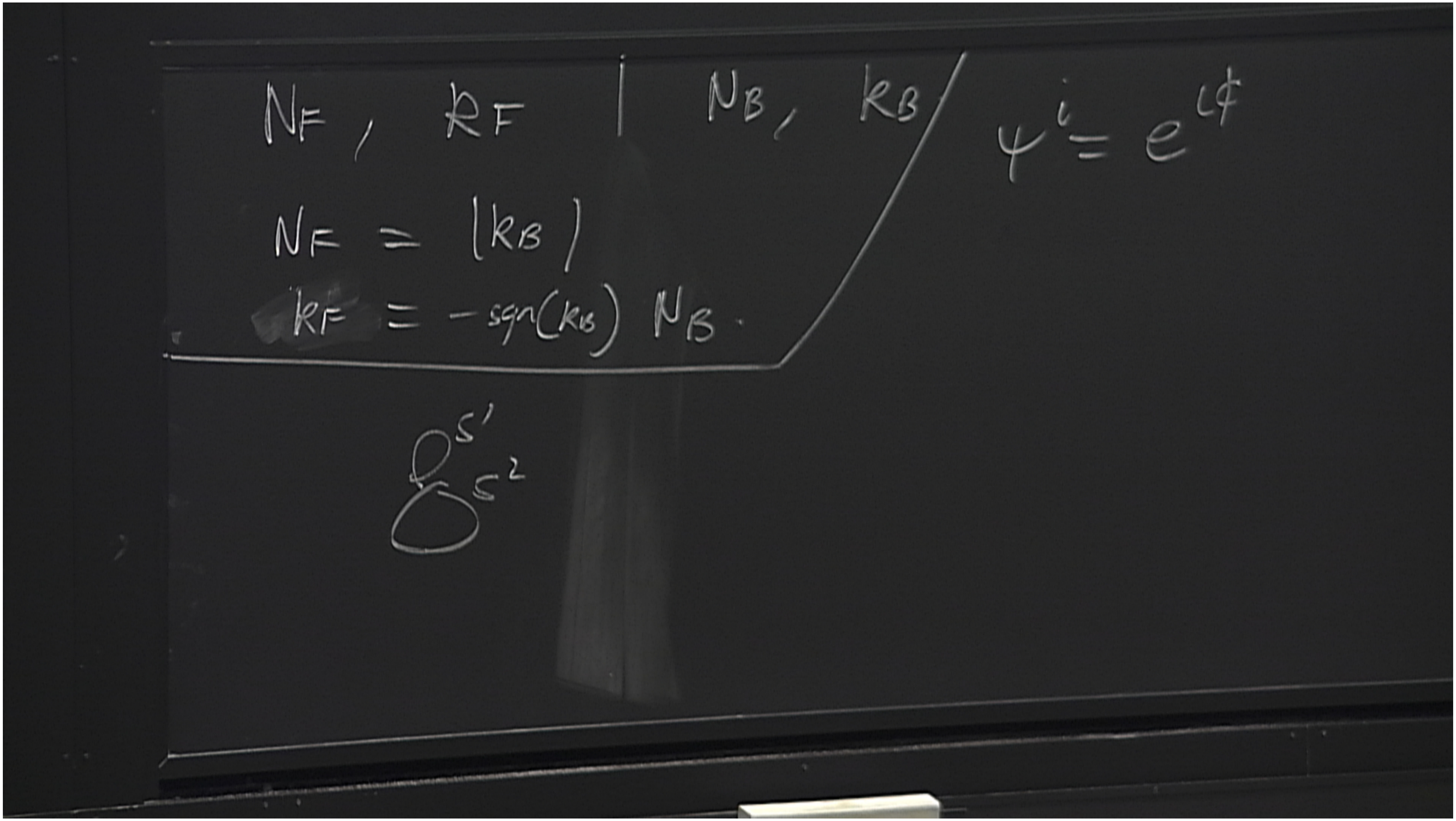
$$N_F, \quad k_F \quad \Big| \quad N_B, \quad k_B$$

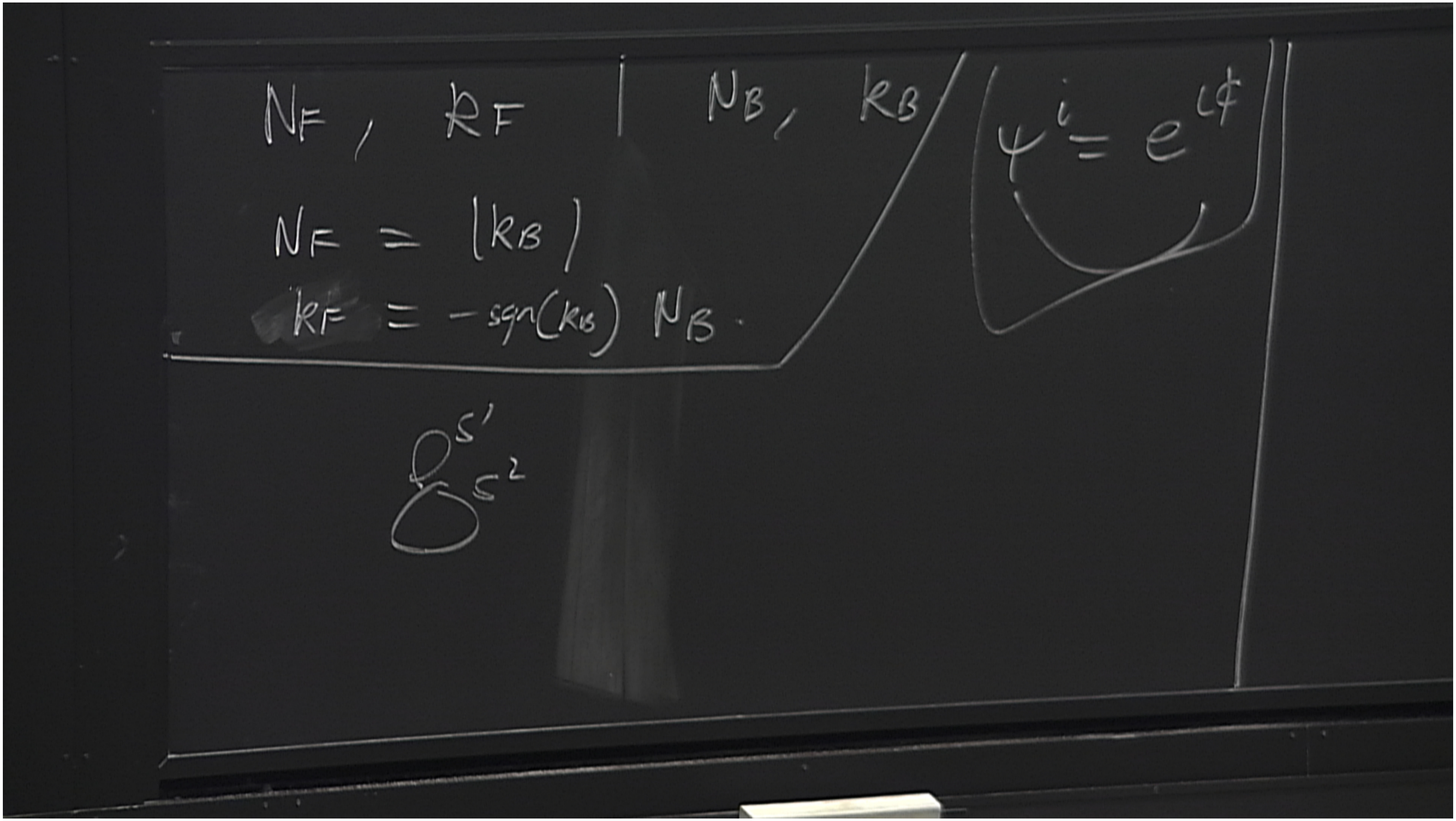
$$\psi = e^{i\phi}$$

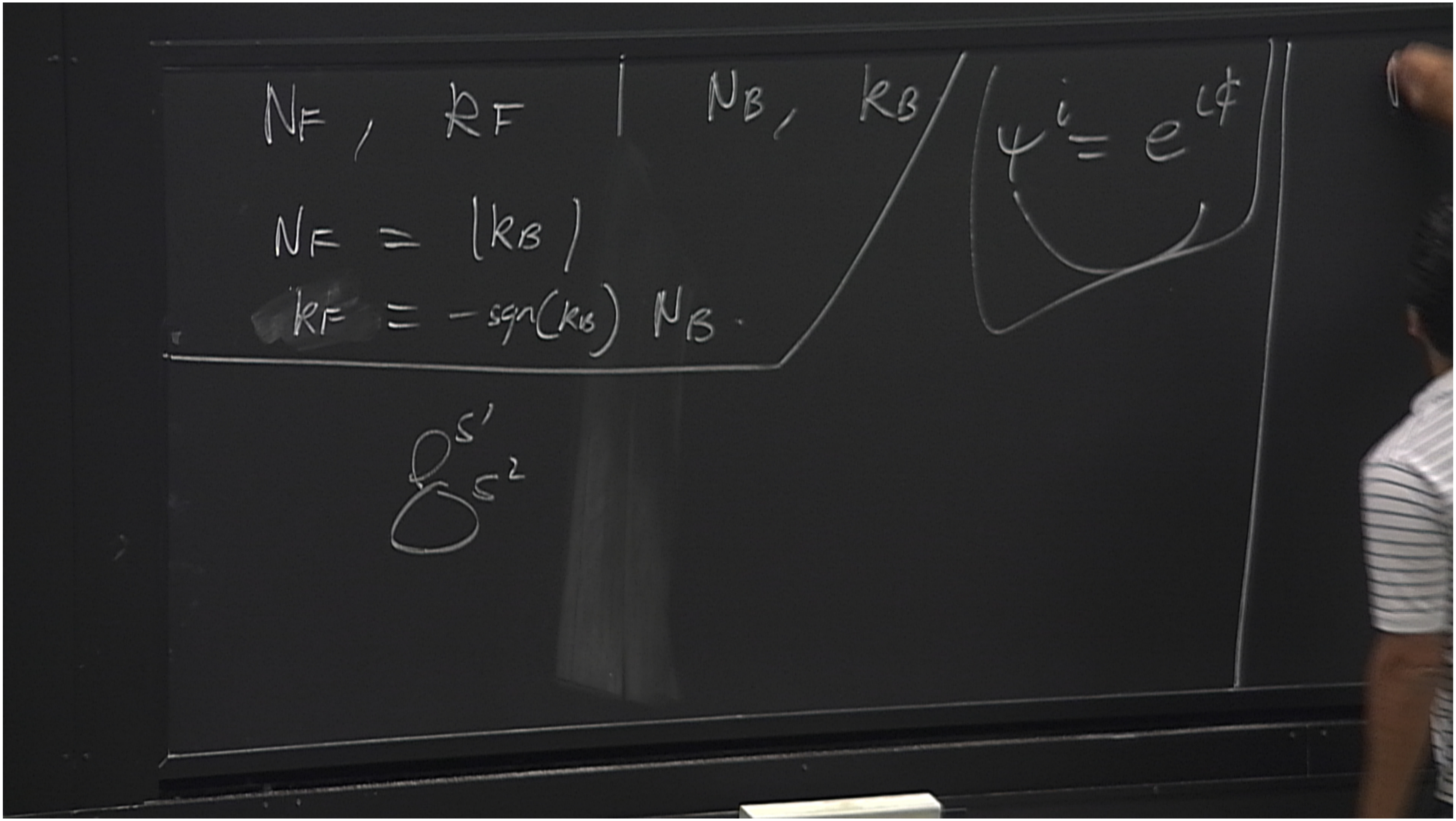
$$N_F = |k_B|$$

$$k_F = -\mathrm{sgn}(k_B)\, N_B .$$

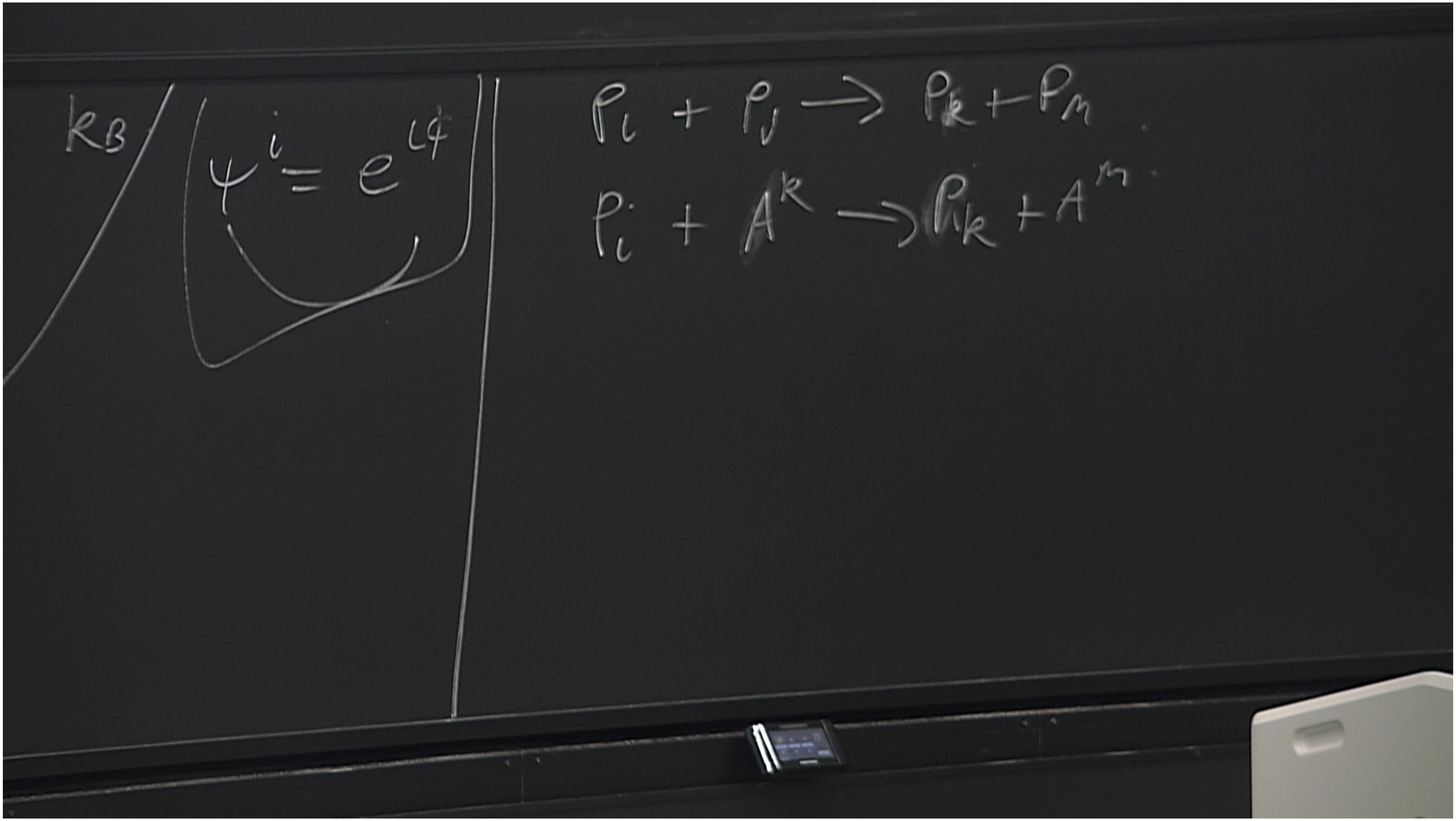


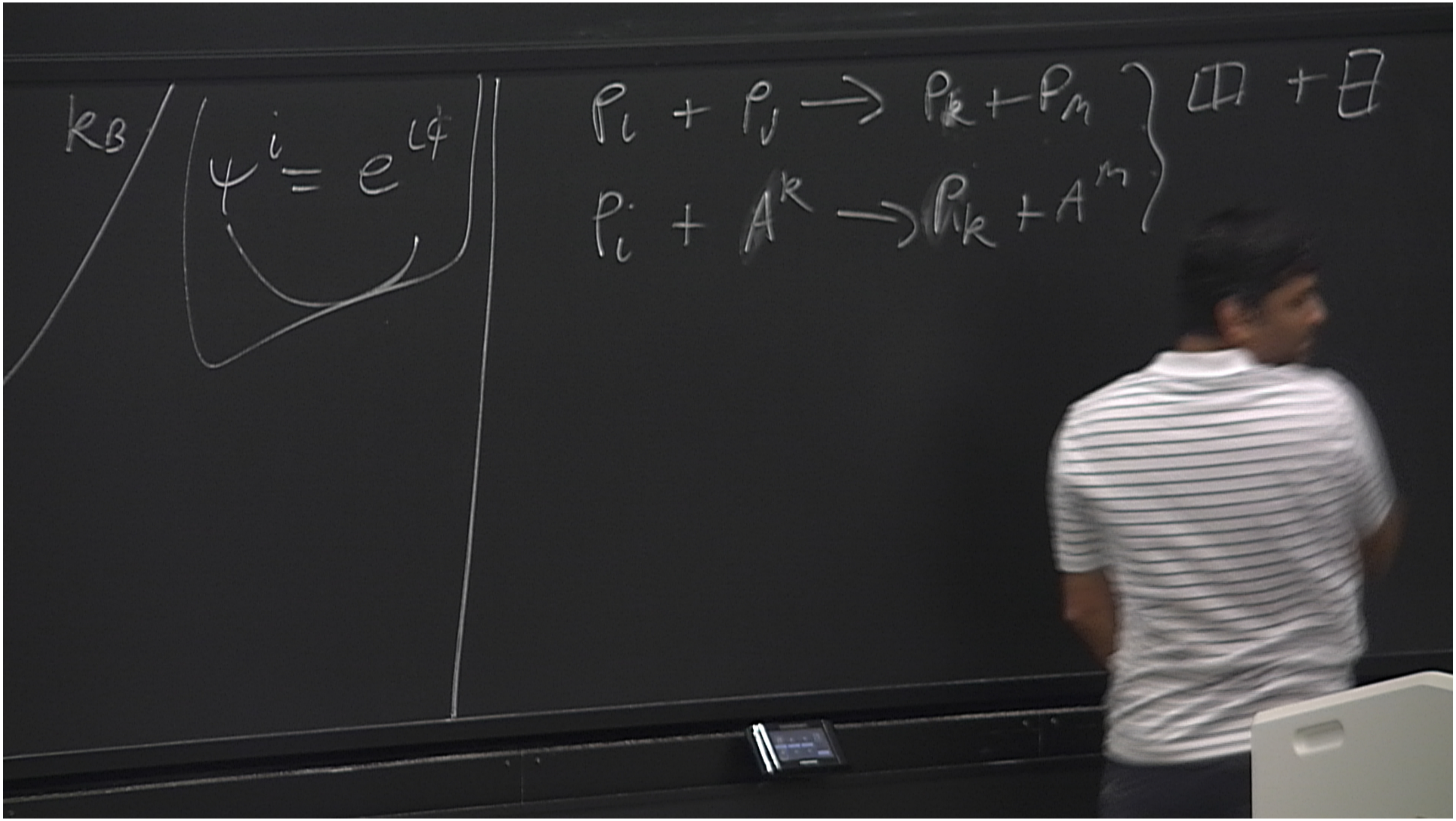

N_F, k_F | N_B, k_B
$\psi^i = e^{i\phi}$
$N_F = |k_B|$
$k_F = -\mathrm{sgn}(k_B)\, N_B$.
S^1
S^2

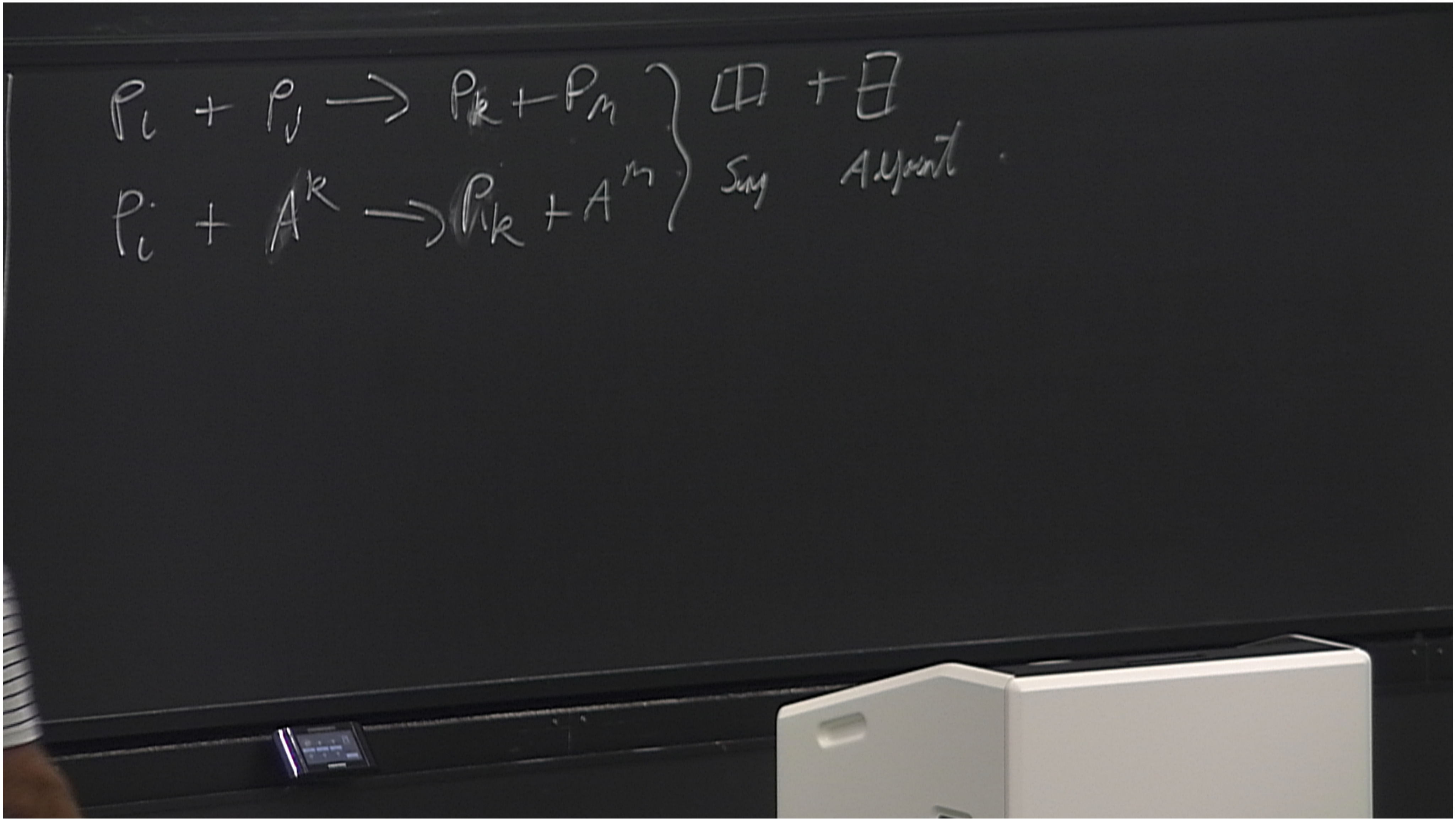

$N_F, \; k_F \quad | \quad N_B, \; k_B$
$N_F = |k_B|$
$k_F = -\mathrm{sgn}(k_B)\, N_B .$
$\psi^i = e^{i\phi}$
S^1
S^2



$N_F, k_F \mid N_B, k_B$
$N_F = |k_B|$
$k_F = -\mathrm{sgn}(k_B)\, N_B$.
$\psi^i = e^{i\phi}$
S^1
S^2


KB
$\psi^i = e^{i\phi}$
$\rho_i + \rho_j \to \rho_k + \rho_m$
$\rho_i + A^k \to \rho_k + A^m$

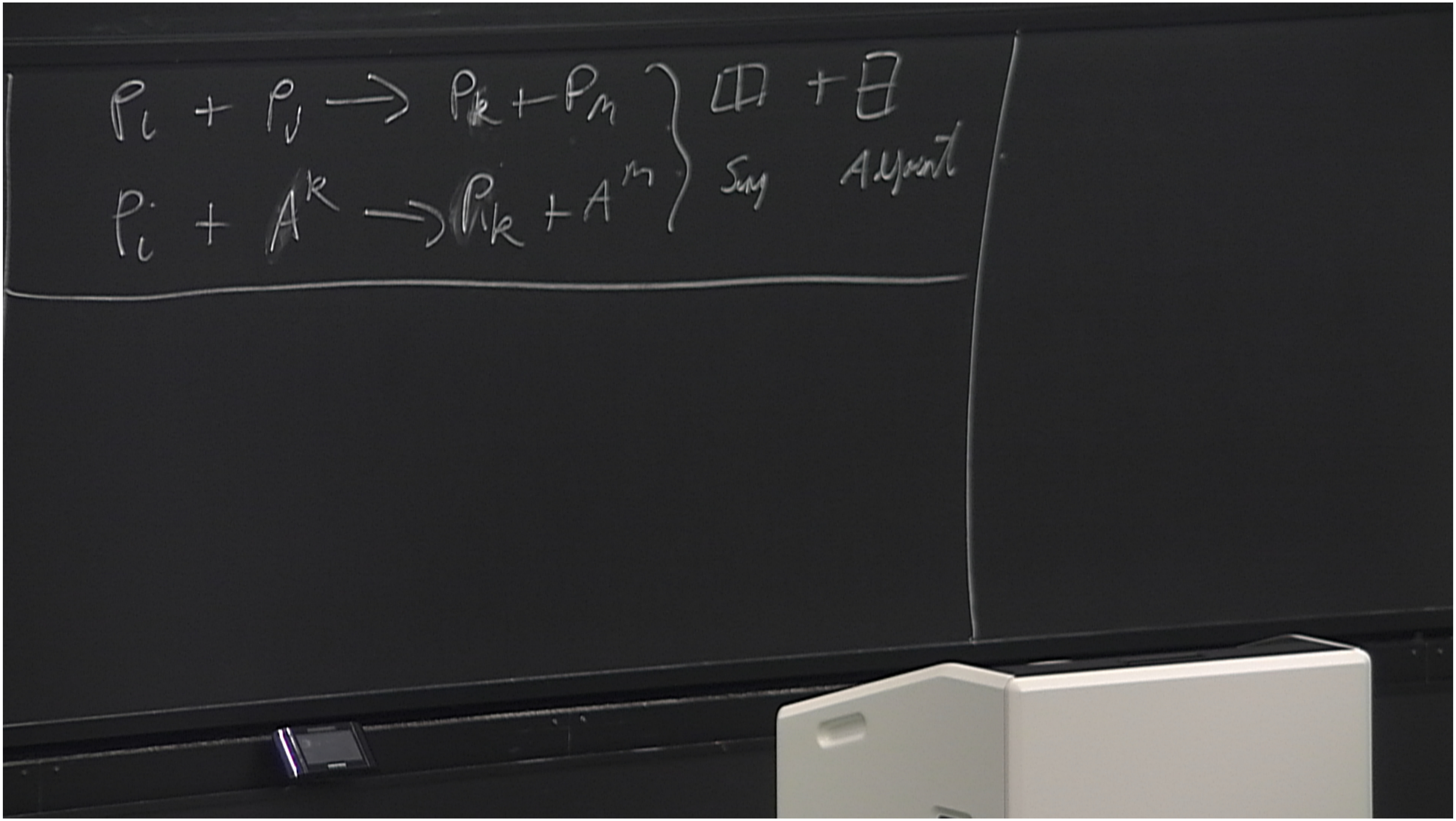

KB
ψ^i = e^{iφ}
ρ_i + ρ_j → ρ_k + ρ_m
ρ_i + A^k → ρ_k + A^m

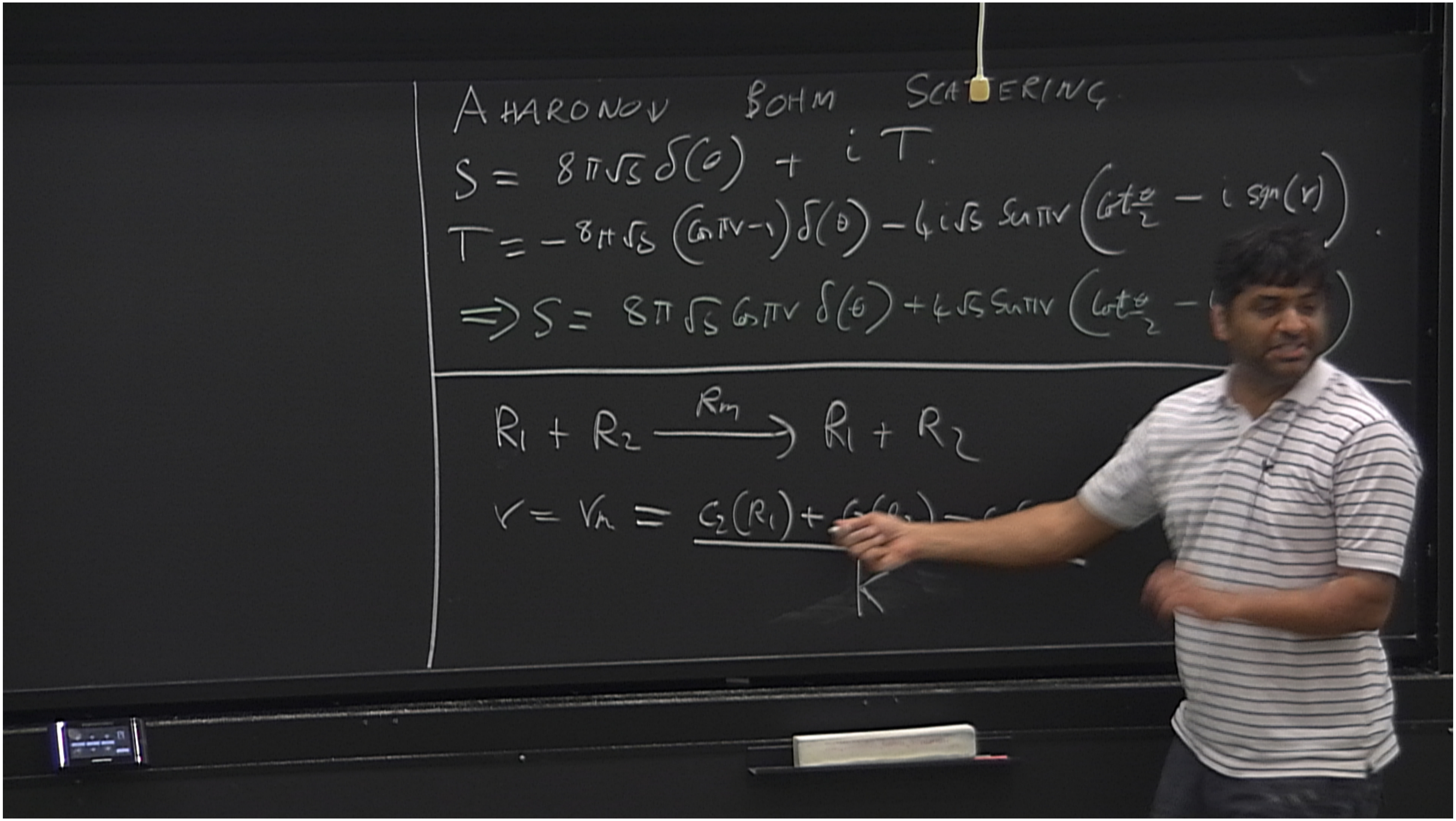

$P_i + P_j \longrightarrow P_k + P_m$
$P_i + A^k \longrightarrow P_{ik} + A^m$
Sym
Adjoint

$$P_i + P_j \longrightarrow P_k + P_m$$

$$P_i + A^k \longrightarrow P_{ik} + A^m$$

Sym   Adjoint


$P_i + P_j \longrightarrow P_k + P_m$
$P_i + A^k \longrightarrow P_{ik} + A^m$
Sym
Adjoint


AHARONOV BOHM SCATTERING
$S = 8\pi\sqrt{s}\,\delta(\theta) + iT.$
$T = -8\pi\sqrt{s}\,(\cos\pi\nu - 1)\,\delta(\theta) - 4i\sqrt{s}\,\sin\pi\nu\left(\cot\frac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right).$
$\Rightarrow S = 8\pi\sqrt{s}\,\cos\pi\nu\,\delta(\theta) + 4\sqrt{s}\,\sin\pi\nu\,(\cot\frac{\theta}{2} -$
$R_1 + R_2 \xrightarrow{R_m} R_1 + R_2$

AHARONOV BOHM SCATTERING

$$S = 8\pi\sqrt{s}\,\delta(\theta) + iT.$$

$$T = -8\pi\sqrt{s}\,(\cos\pi\nu - 1)\,\delta(\theta) - 4i\sqrt{s}\,\sin\pi\nu\left(\cot\tfrac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right).$$

$$\Rightarrow S = 8\pi\sqrt{s}\,\cos\pi\nu\,\delta(\theta) + 4\sqrt{s}\,\sin\pi\nu\left(\cot\tfrac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right)$$

$$R_1 + R_2 \xrightarrow{R_m} R_1 + R_2$$

$$\nu = \nu_m = \frac{C_2(R_1) + C_2(R_2) - C_2(R_m)}{K}$$

AHARONOV BOHM SCATTERING.

$$S = 8\pi\sqrt{s}\,\delta(\theta) + iT.$$

$$T = -8\pi\sqrt{s}\,(\cos\pi\nu - 1)\,\delta(\theta) - 4i\sqrt{s}\,\sin\pi\nu\left(\cot\tfrac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right).$$

$$\Rightarrow S = 8\pi\sqrt{s}\,\cos\pi\nu\,\delta(\theta) + 4\sqrt{s}\,\sin\pi\nu\left(\cot\tfrac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right)$$

$$R_1 + R_2 \xrightarrow{R_m} R_1 + R_2$$

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AHARONOV BOHM SCATTERING.

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$$\Rightarrow S = 8\pi\sqrt{s}\,\cos\pi\nu\,\delta(\theta) + 4\sqrt{s}\,\sin\pi\nu\left(\cot\tfrac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right)$$

$$R_1 + R_2 \xrightarrow{R_m} R_1 + R_2$$

$$\nu = \nu_n = \frac{C_2(R_1) + C_2(R_2) - C_2(R_m)}{k}$$

AHARONOV BOHM SCATTERING

$$S = 8\pi\sqrt{s}\,\delta(\theta) + iT.$$

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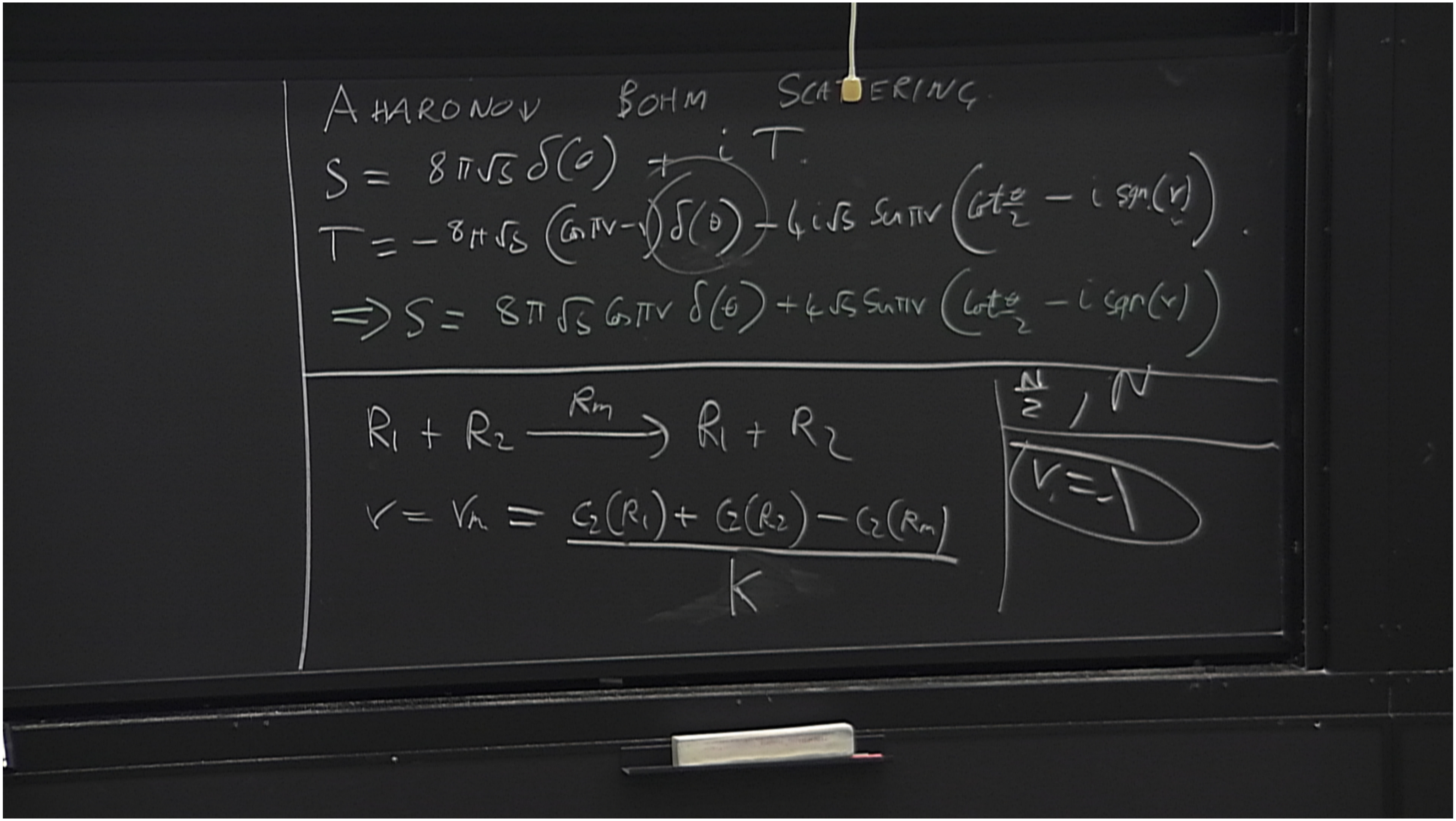
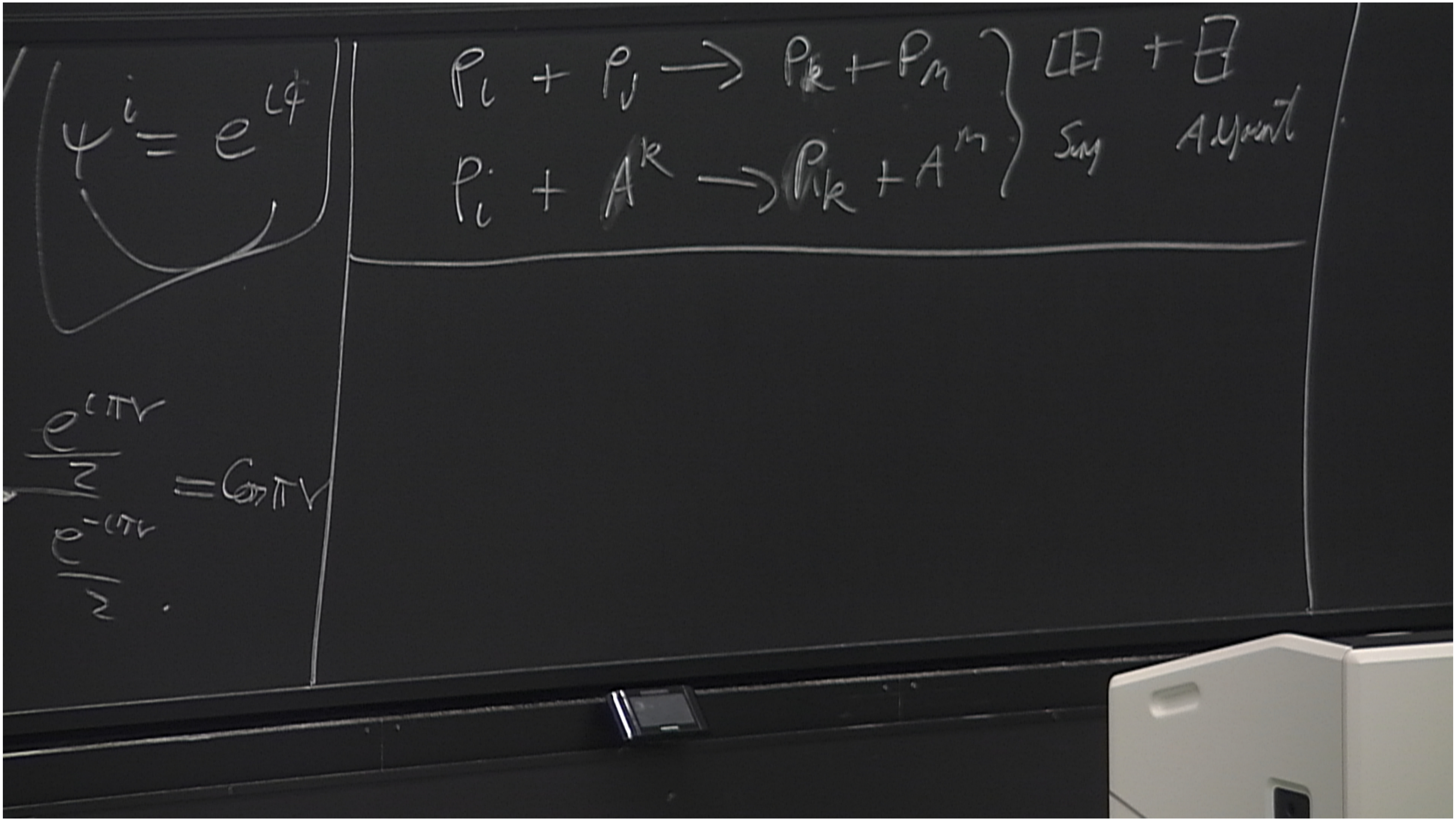
$$\Rightarrow S = 8\pi\sqrt{s}\,\cos\pi\nu\,\delta(\theta) + 4\sqrt{s}\,\sin\pi\nu\left(\cot\tfrac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right)$$

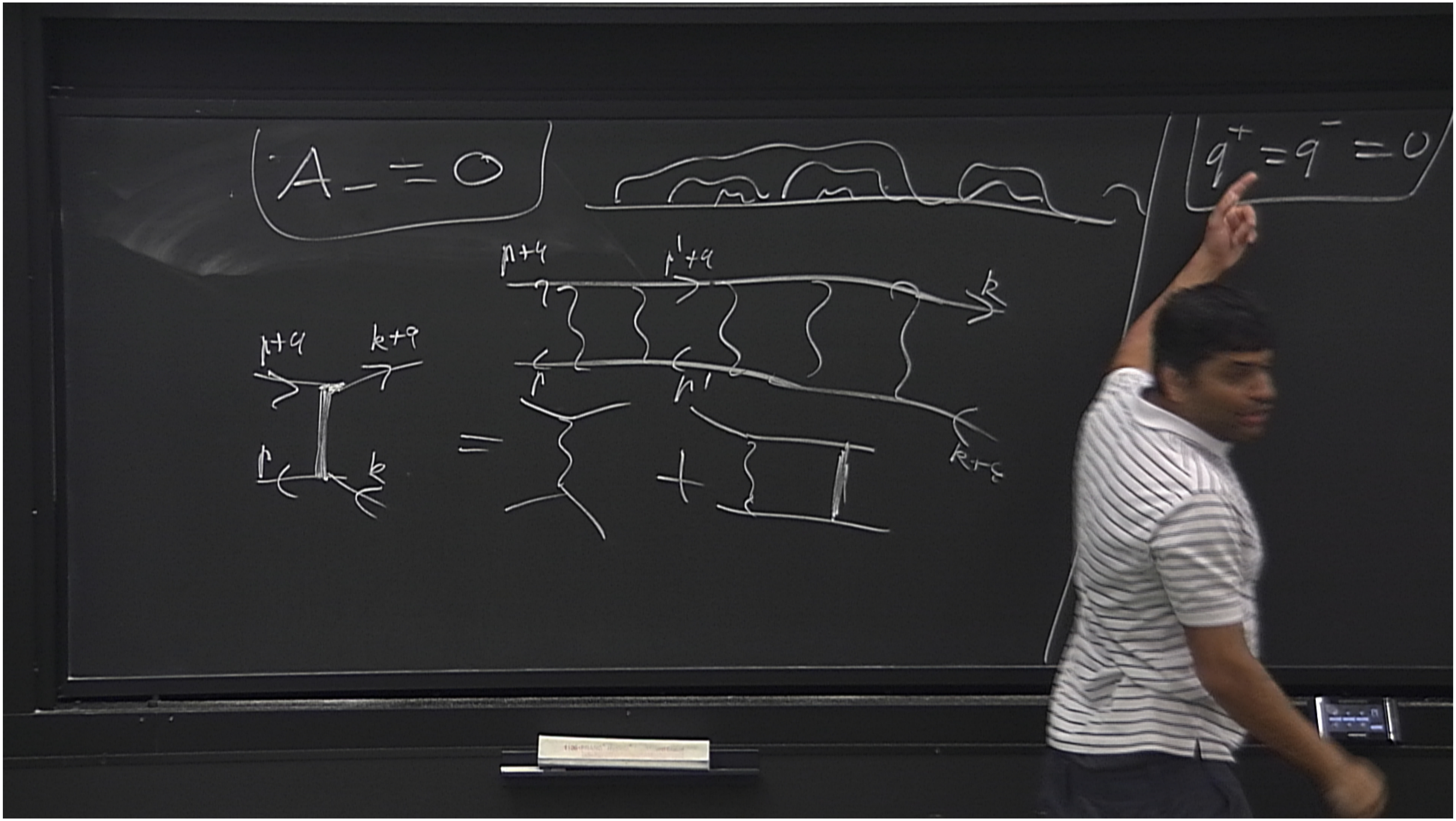
$$R_1 + R_2 \xrightarrow{R_m} R_1 + R_2$$

$$\nu = \nu_n = \frac{C_2(R_1) + C_2(R_2) - C_2(R_m)}{K}$$

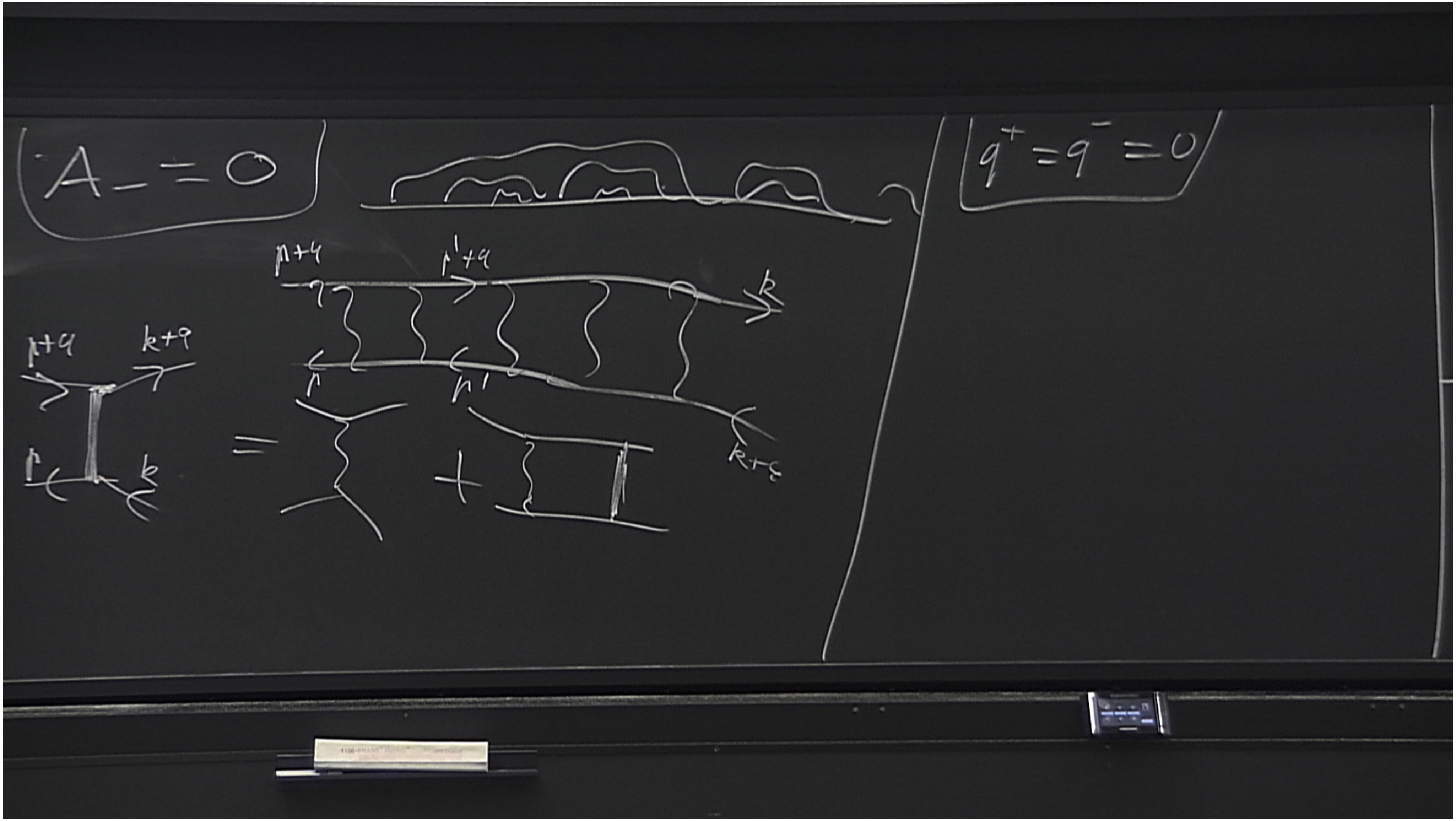
$\frac{N}{2}$ /

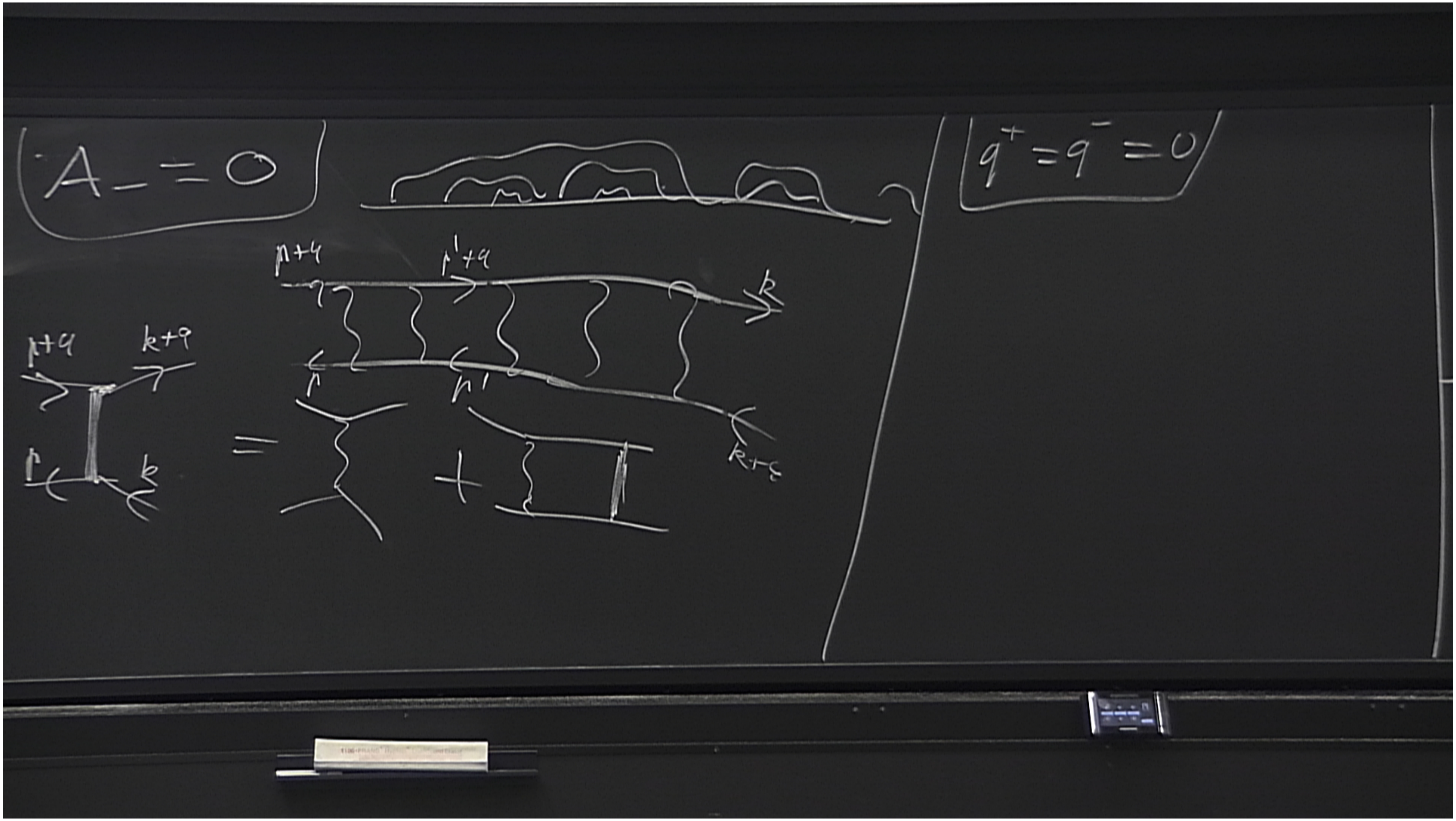

AHARONOV BOHM SCATTERING
$S = 8\pi\sqrt{s}\,\delta(\theta) + iT.$
$T = -8\pi\sqrt{s}\,(\cos\pi\nu - 1)\,\delta(\theta) - 4i\sqrt{s}\,\sin\pi\nu\left(\cot\frac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right).$
$\Rightarrow S = 8\pi\sqrt{s}\cos\pi\nu\,\delta(\theta) + 4\sqrt{s}\,\sin\pi\nu\left(\cot\frac{\theta}{2} - i\,\mathrm{sgn}(\nu)\right)$
$R_1 + R_2 \xrightarrow{R_m} R_1 + R_2$
$\nu = \nu_m = \frac{C_2(R_1) + C_2(R_2) - C_2(R_m)}{k}$
$\frac{N}{2}, N$
$\nu_1 = 1$

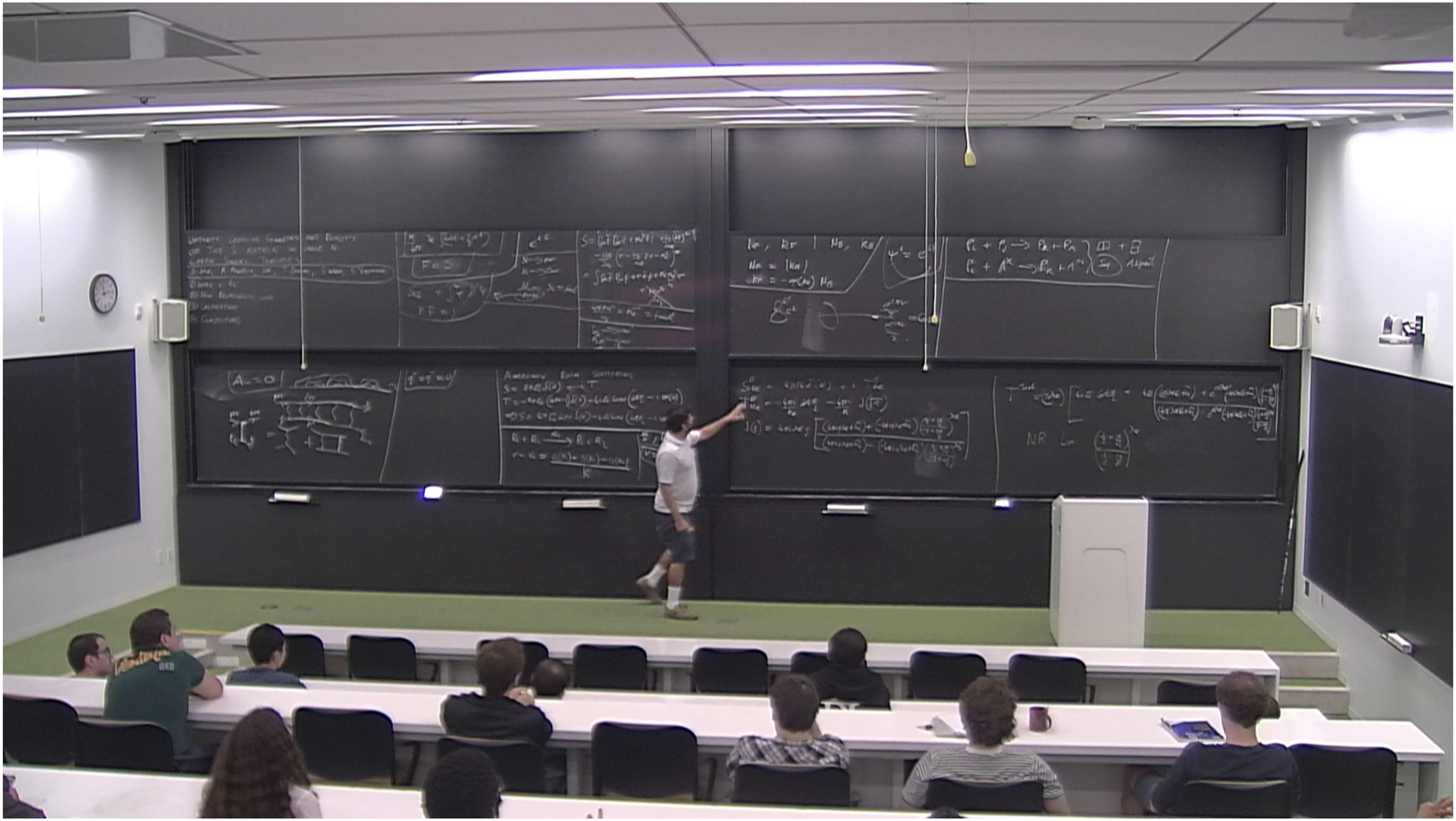

ρi + ρj → ρk + ρm
ρi + A^k → ρk + A^m
Sym
Adjoint



A_ = 0
q⁺ = q⁻ = 0
p+q
p'+q
k
p
p'
k+q
p+q
k+q
p
k
=
+


A_ = 0
q^+ = q^- = 0
p+q
p'+q
k
p
p'
k+q

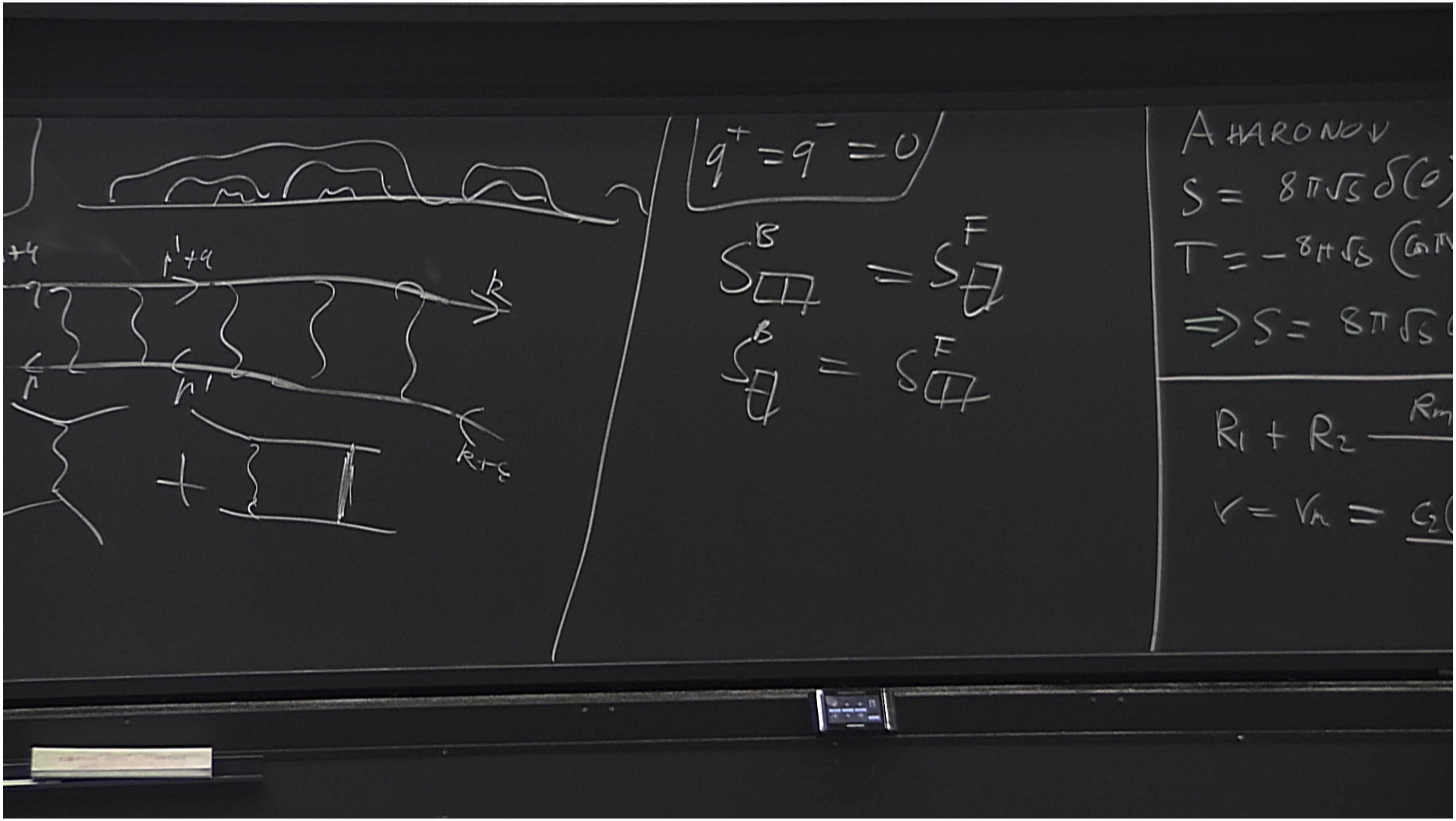

A_ = 0
q+ = q- = 0
p+q
p'+q
k
p
p'
k+q
p+q
k+q
p
k



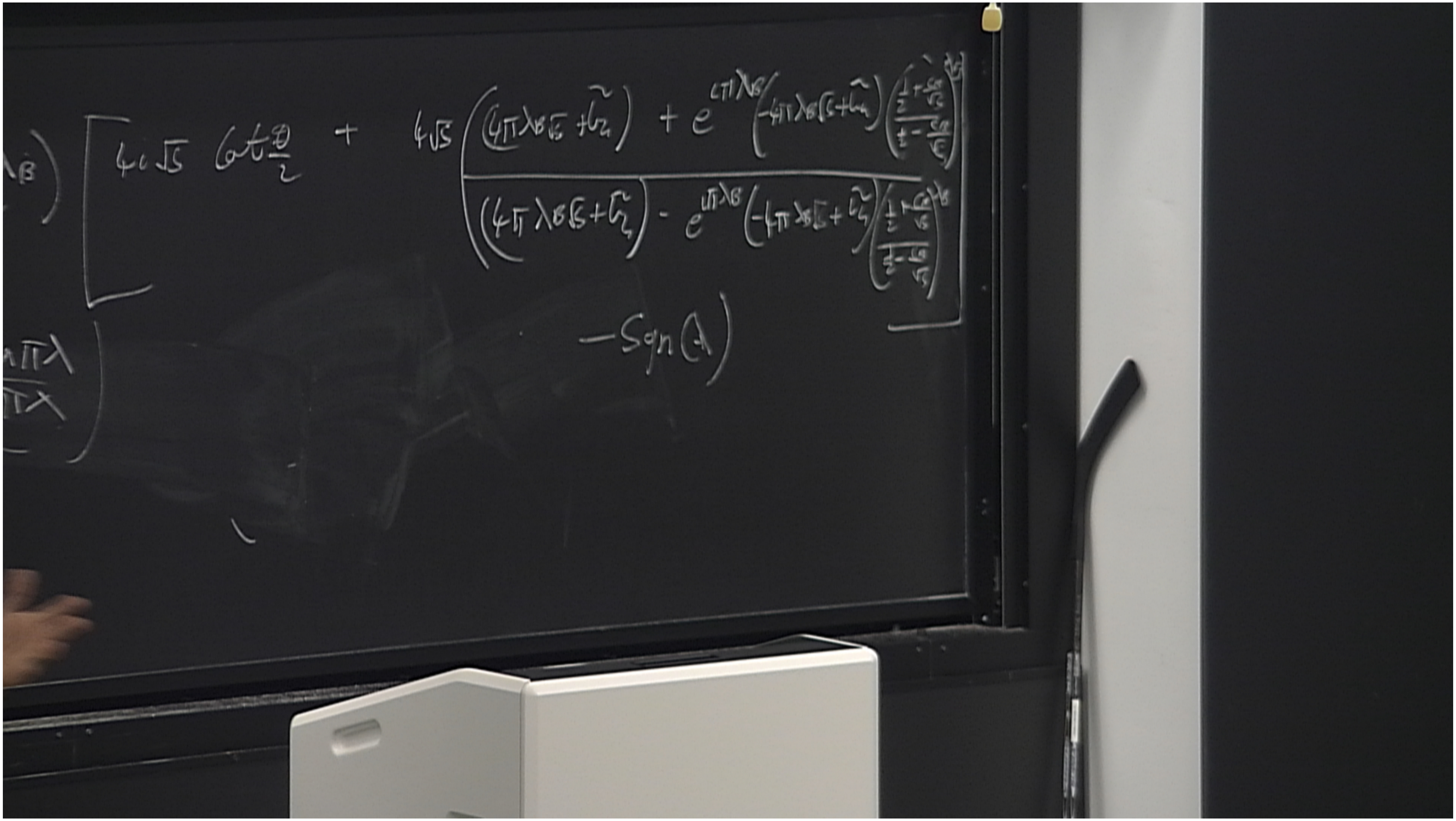
$$S^{B}_{\nu e} = 8\pi\sqrt{s}\,\delta(\theta) + i\,T^{B}_{\nu e}$$

$$T^{B}_{\nu e} = -\frac{4\pi i}{k_B}\cot\frac{\theta}{2} - \frac{4\pi i}{k}\,J\left(\sqrt{-t}\right)$$

$$J(q) = 4\pi i \lambda_B\, q \left[\frac{(4\pi i q \lambda_B + \tilde{b}_4) + (-4\pi i q \lambda_B + \tilde{b}_4)\left(\frac{\frac{1}{2} + \frac{c_B}{q}}{-\frac{1}{2} + \frac{c_B}{q}}\right)^{\lambda_B}}{(4\pi i q \lambda_B + \tilde{b}_4) - (-4\pi i q \lambda_B + \tilde{b}_4)\left(\frac{\frac{1}{2} + \frac{c_B}{iq}}{-\frac{1}{2} + \frac{c_B}{iq}}\right)^{\lambda_B}}\right]$$



p'+q
k
k+ε
q+ = q- = 0
S B = S F
S B = S F
AHARONOV
S = 8π√s δ(
T = -8π√s
⇒ S = 8π√s
R1 + R2
r = rh =


−Sgn(λ)