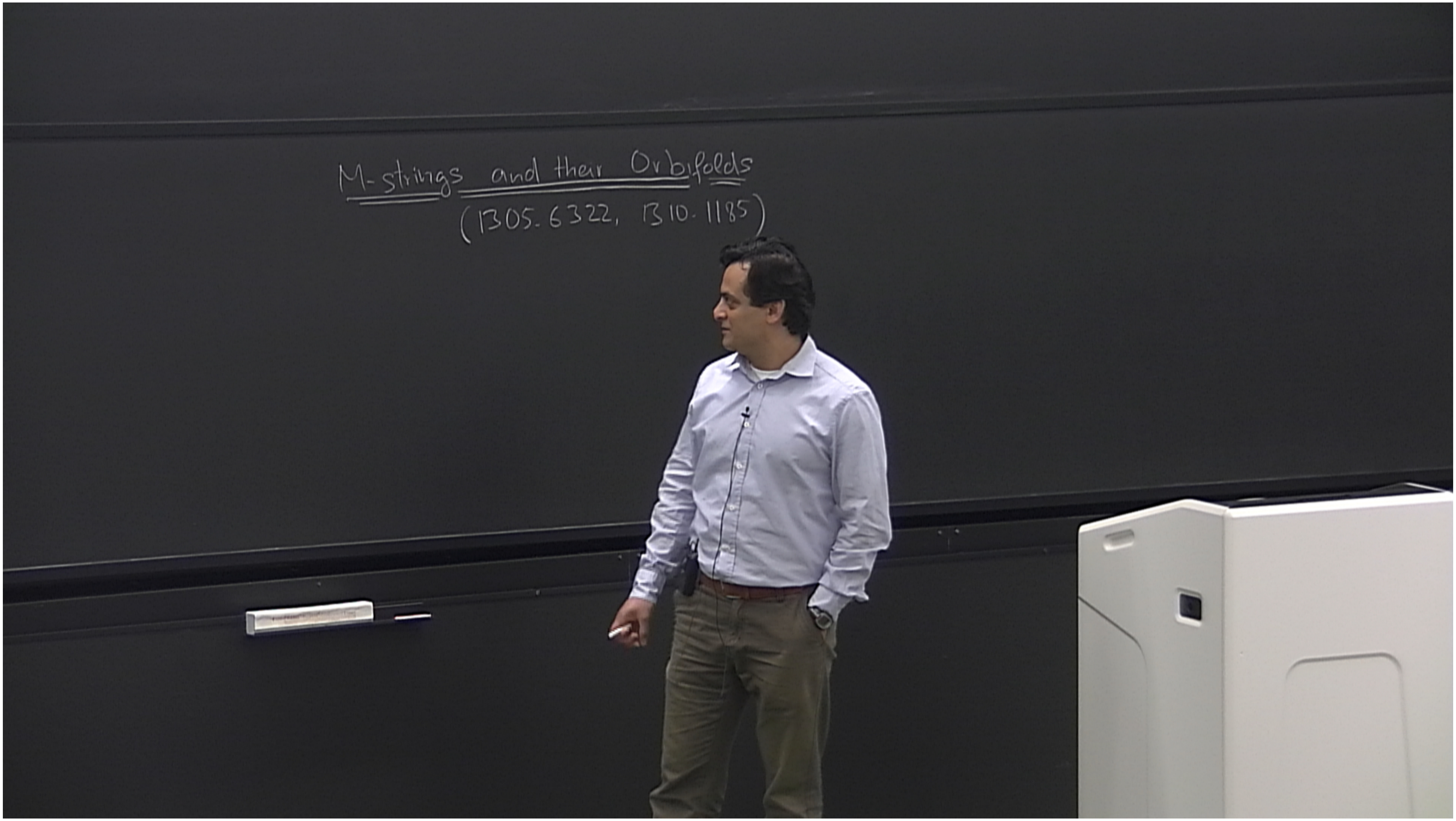
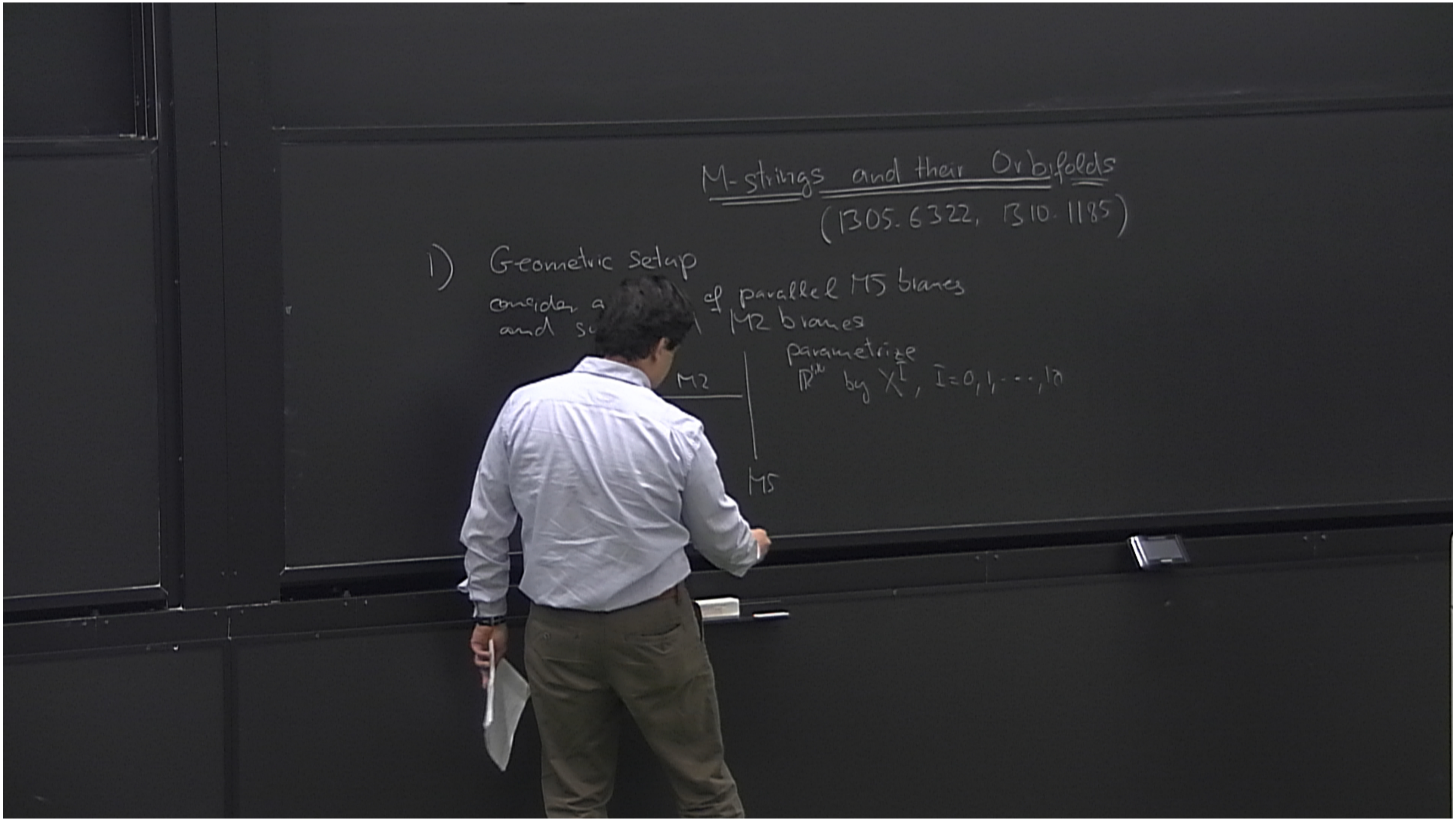
Title: M-strings and their orbifolds

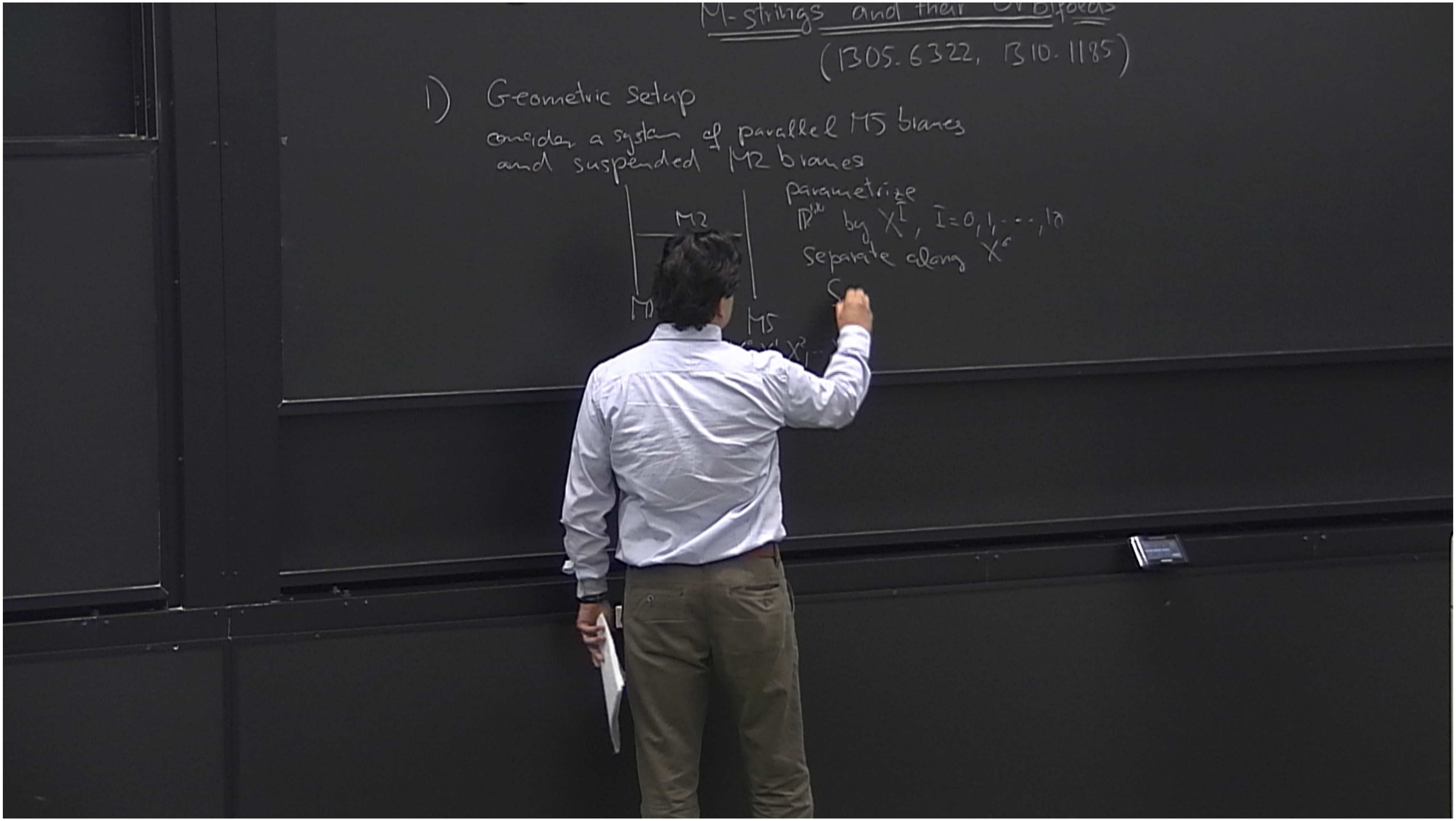
Date: May 20, 2014 02:00 PM

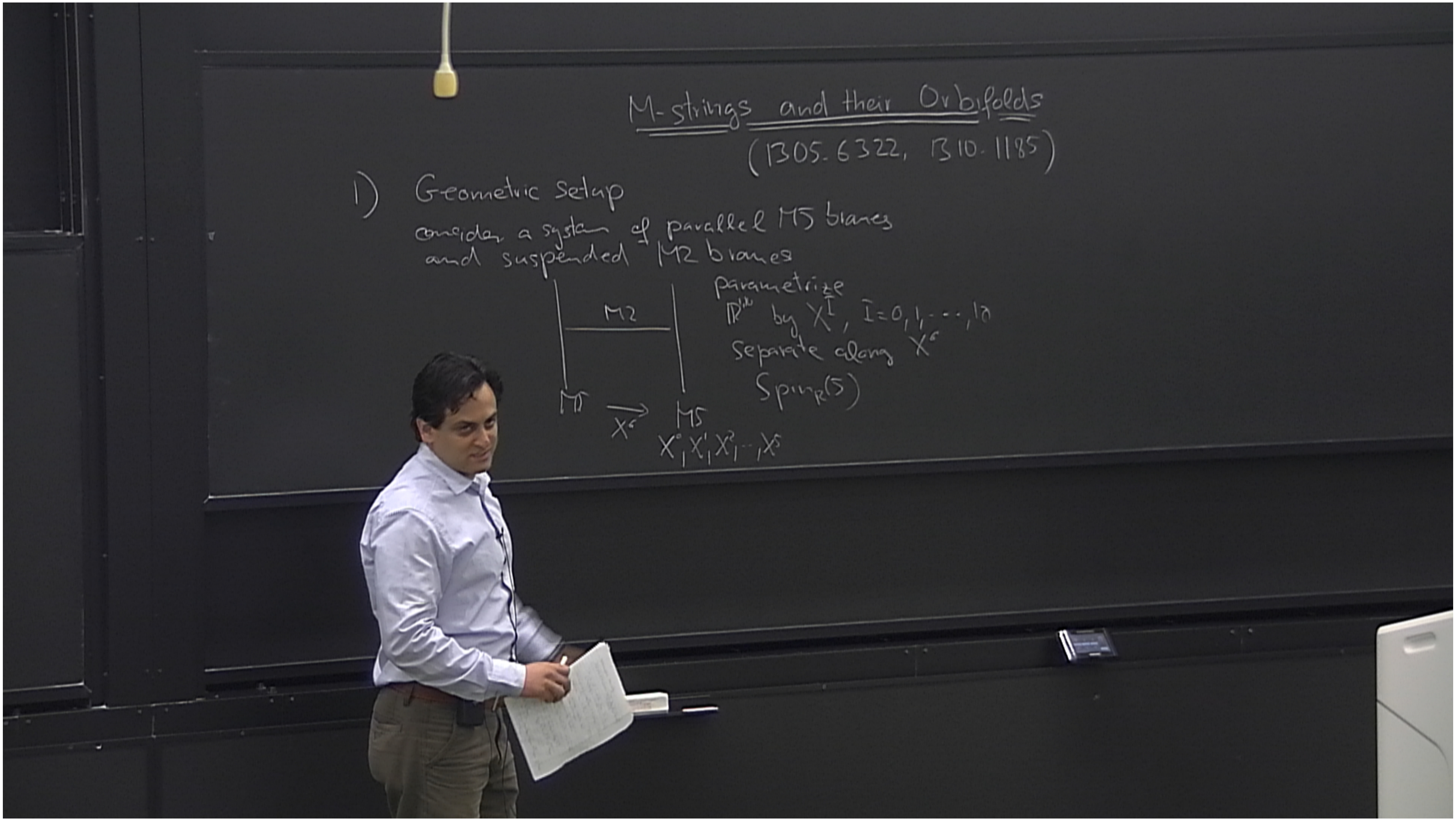
URL: http://pirsa.org/14050005

Abstract: We consider M-theory in the presence of M parallel M5-branes probing a transverse A_{N-1} singularity. This leads to a superconformal theory with (1,0) supersymmetry in six dimensions. We compute the supersymmetric partition function of this theory on a two-torus, with arbitrary supersymmetry preserving twists, using the topological vertex formalism. Alternatively, we show that this can also be obtained by computing the elliptic genus of an orbifold of recently studied M-strings. The resulting 2d theory is a (4,0) supersymmetric quiver gauge theory whose Higgs branch corresponds to strings propagating on the moduli space of SU(N)^(M-1) instantons on R^4 where the right-moving fermions are coupled to a particular bundle.


M-strings and their Orbifolds
(1305.6322, 1310.1185)


M-strings and their Orbifolds
(1305.6322, 1310.1185)
1) Geometric Setup
of parallel M5 branes
M2 branes
parametrize
$\mathbb{R}^{11}$ by X^I, $I=0,1,\dots,10$
M2
M5


M-strings and their orbifolds
(1305.6322, 1310.1185)
1) Geometric Setup
consider a system of parallel M5 branes
and suspended M2 branes
parametrize
R^{1,10} by X^I, I = 0, 1, ..., 10
separate along X^6
M2
M5


M-strings and their Orbifolds
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consider a system of parallel M5 branes
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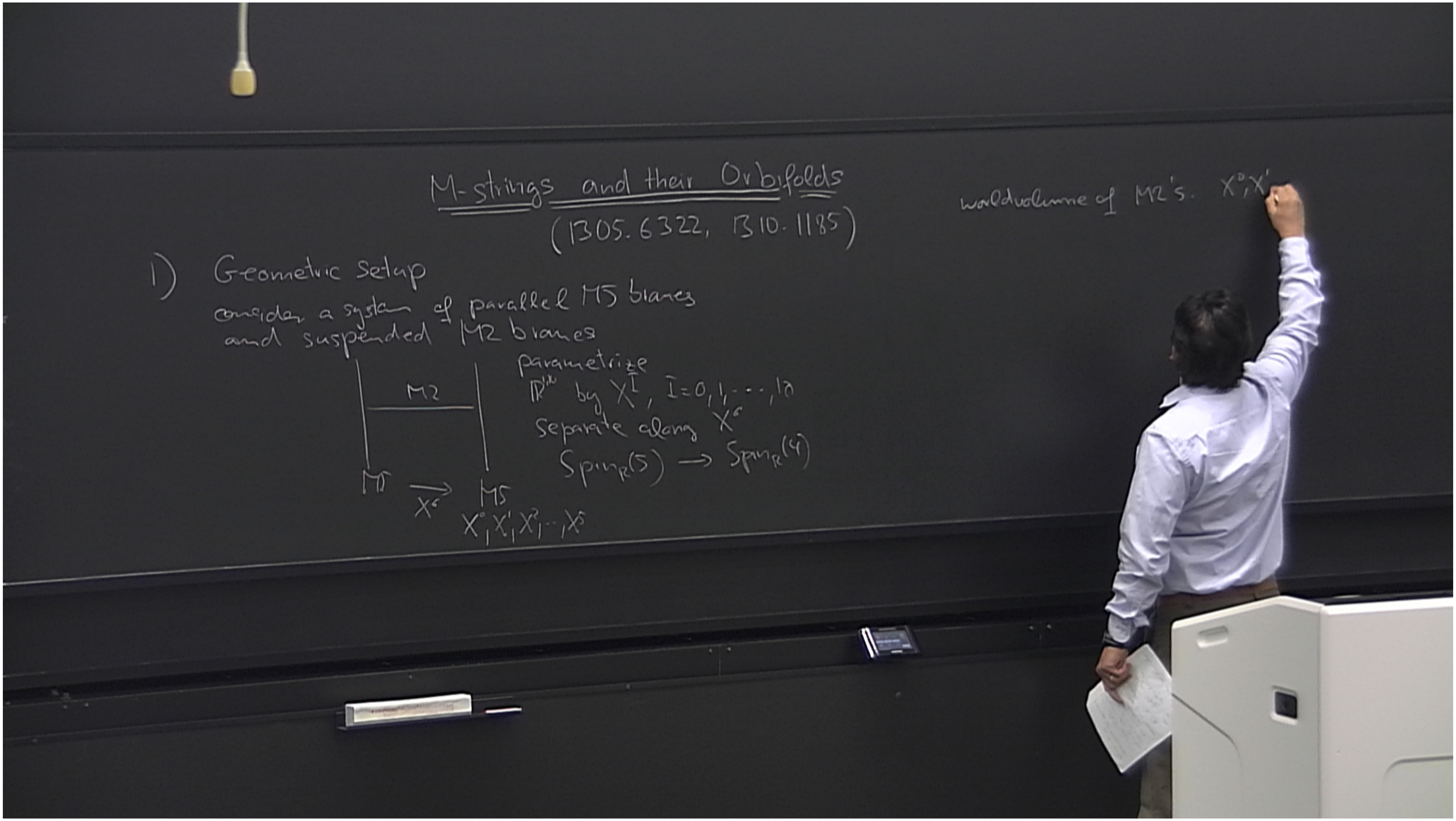
parametrize
$\mathbb{R}^{1,10}$ by X^I, $I = 0, 1, \ldots, 10$
separate along X^6
$Spin_R(5)$
M2
M5
X^6
M5
$X^0, X^1, X^2, \ldots, X^5$


M-strings and their Orbifolds
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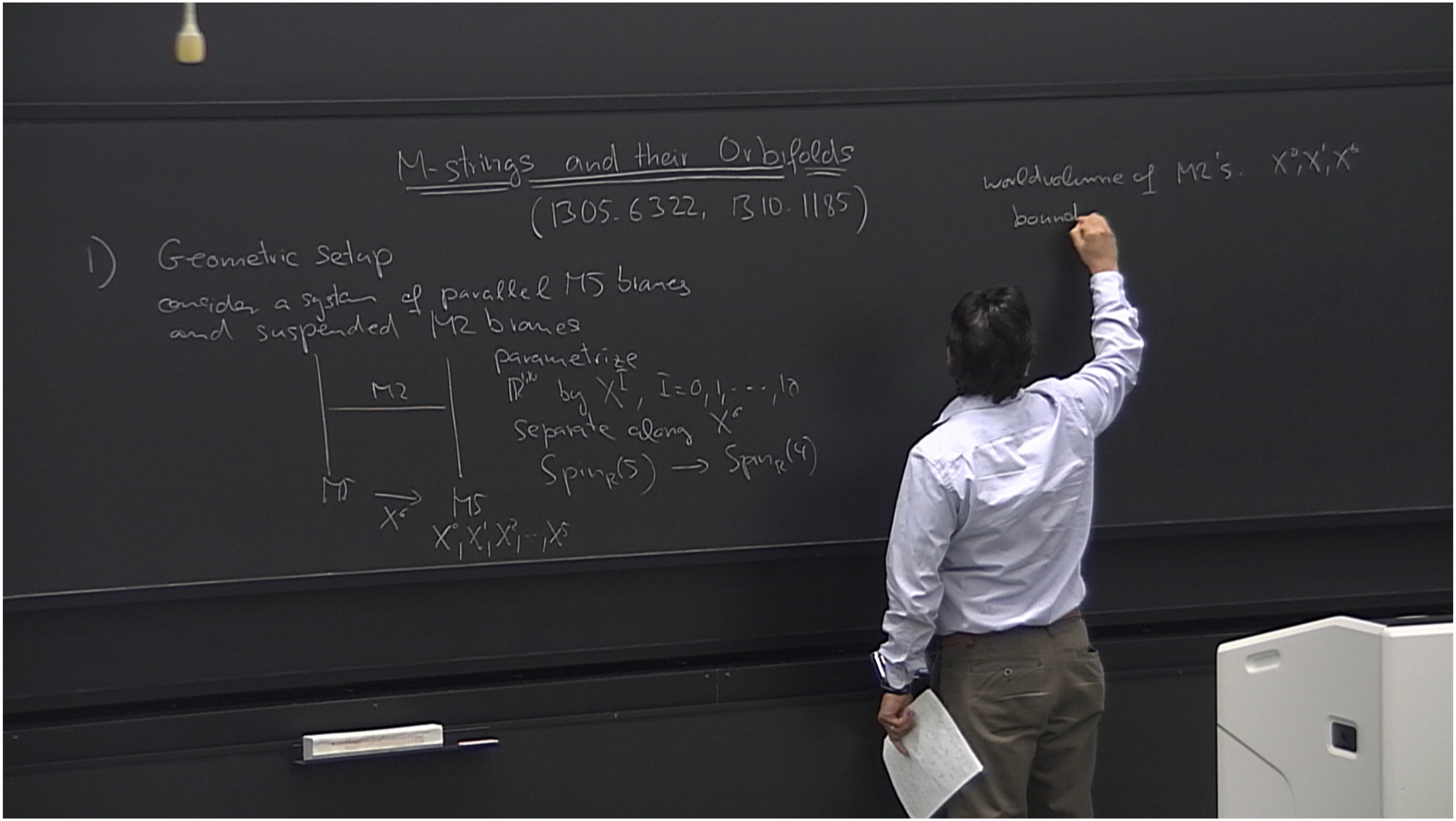
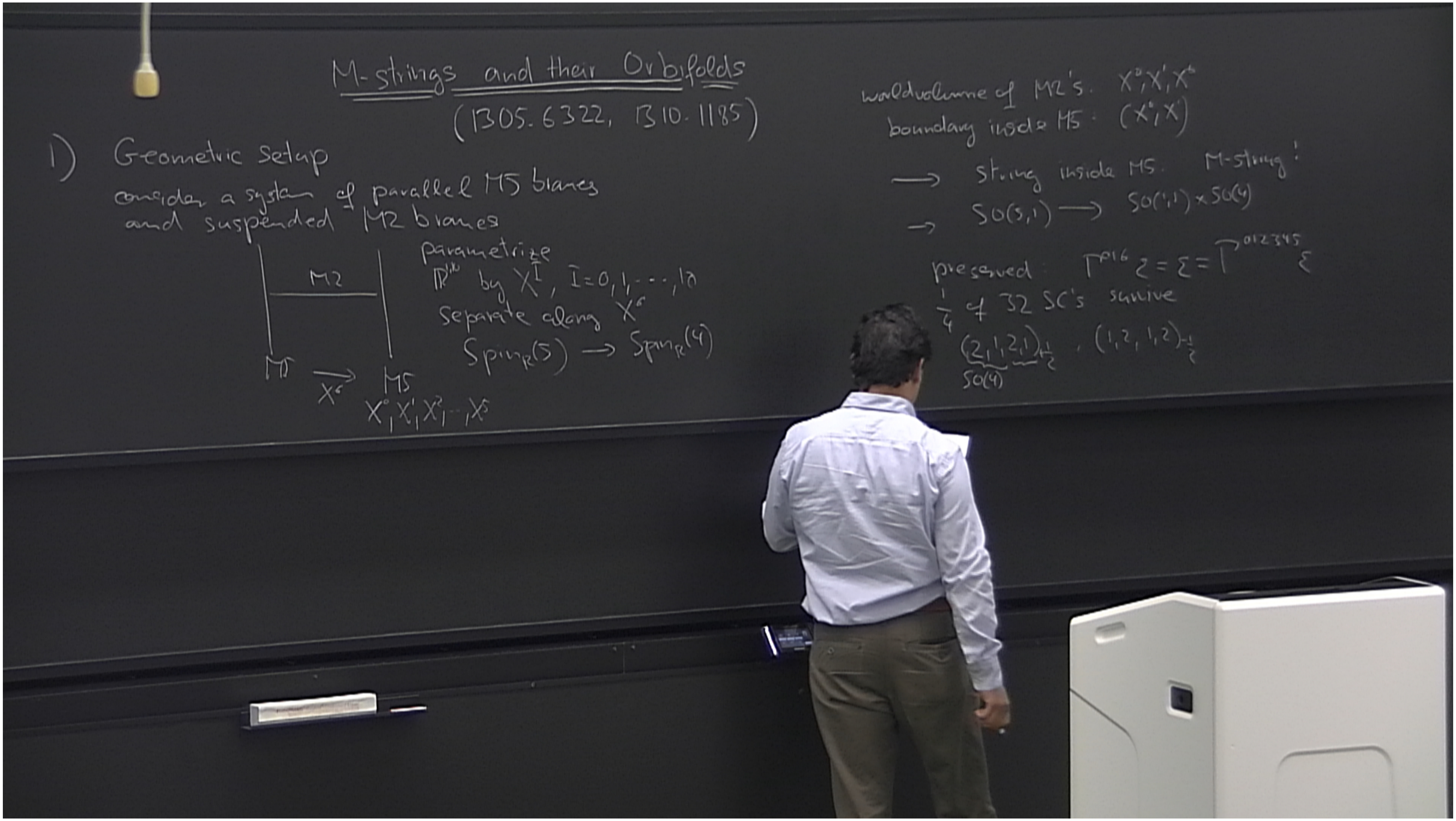
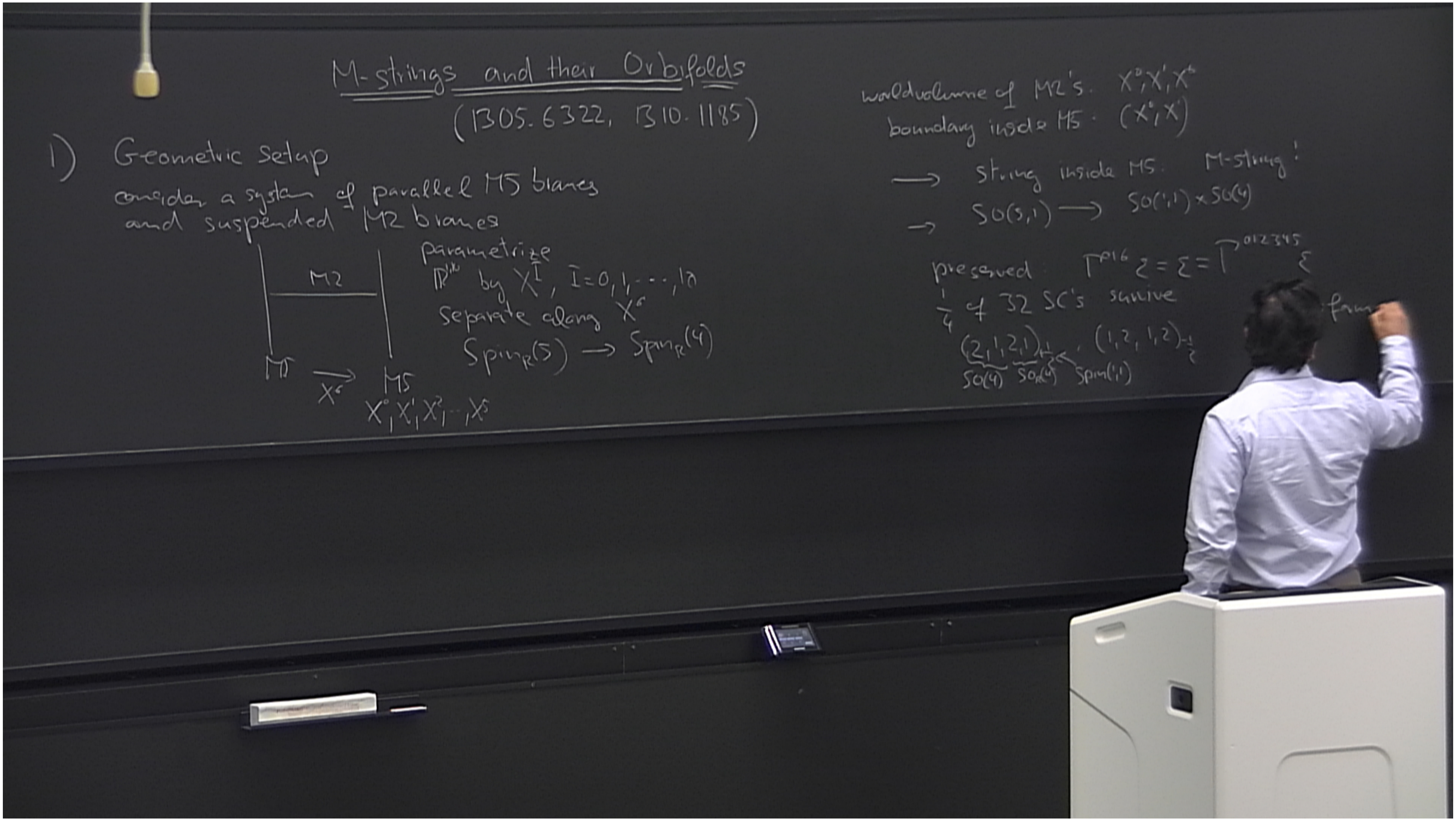
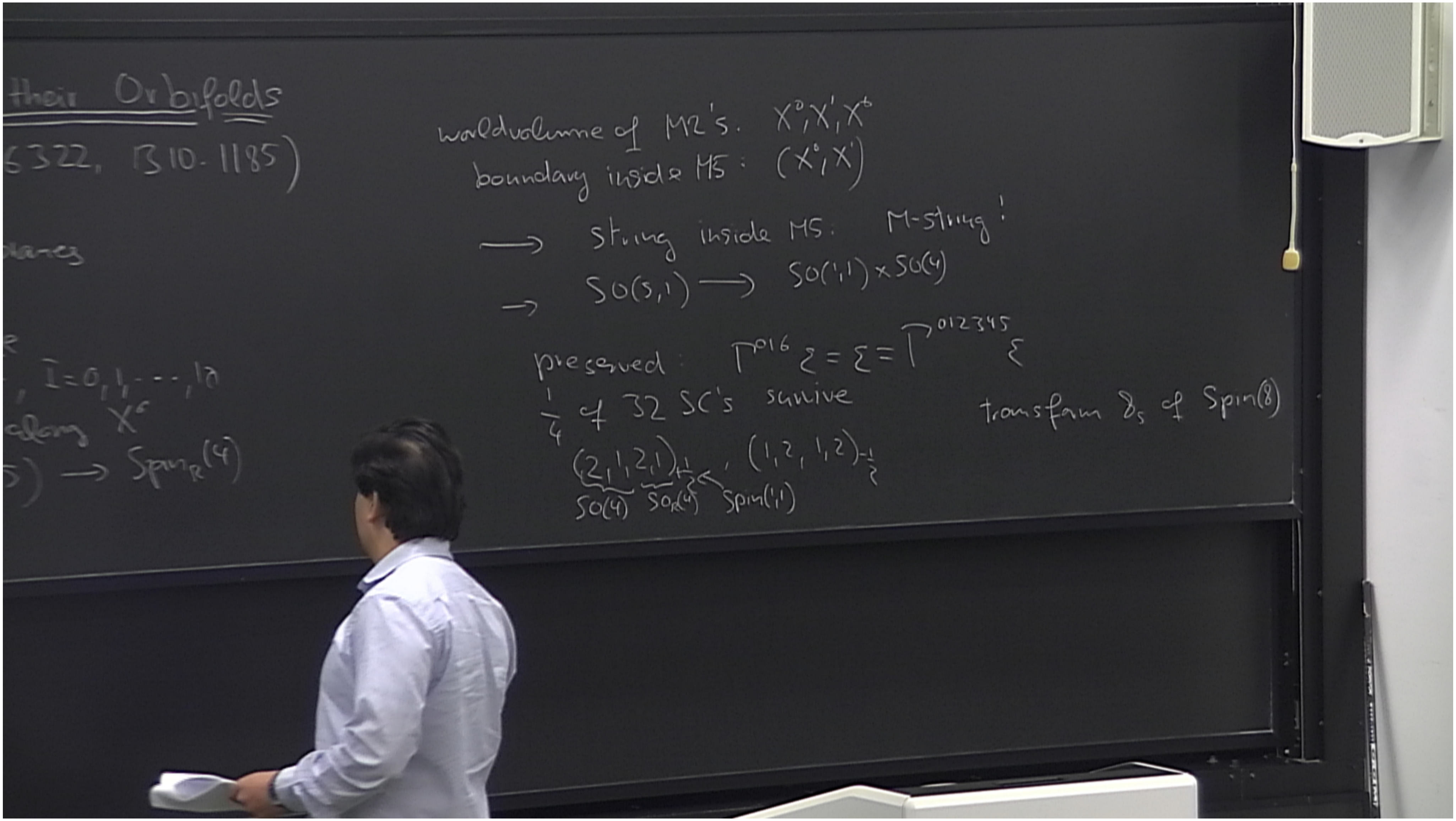
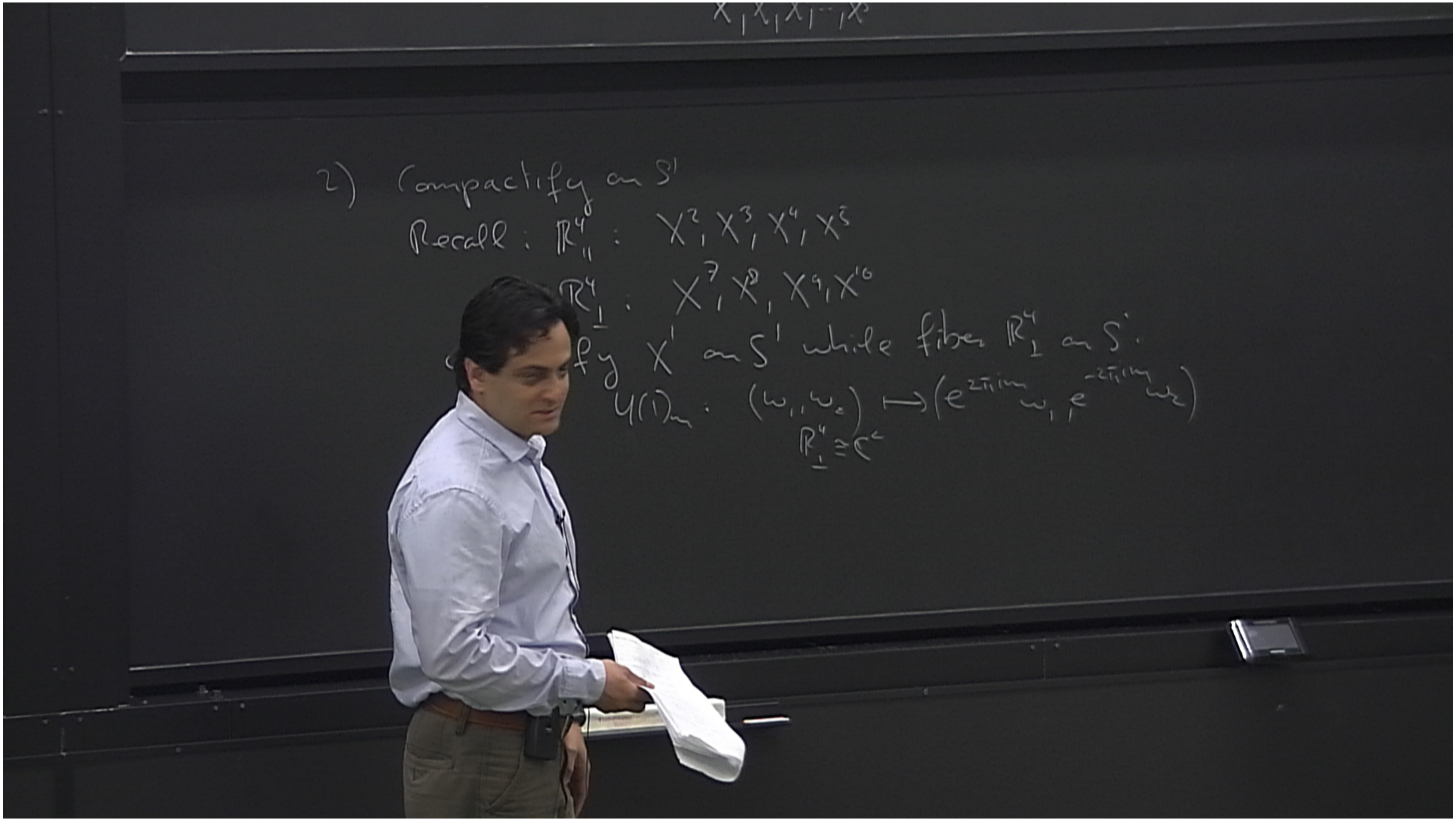
1) Geometric Setup
consider a system of parallel M5 branes
and suspended M2 branes
M2
M5
M5
X^6
X^0, X^1, X^2, ..., X^5
parametrize
R^{1,10} by X^I, I = 0, 1, ..., 10
Separate along X^6
Spin_R(5) → Spin_R(4)
worldvolume of M2's. X^0, X^1

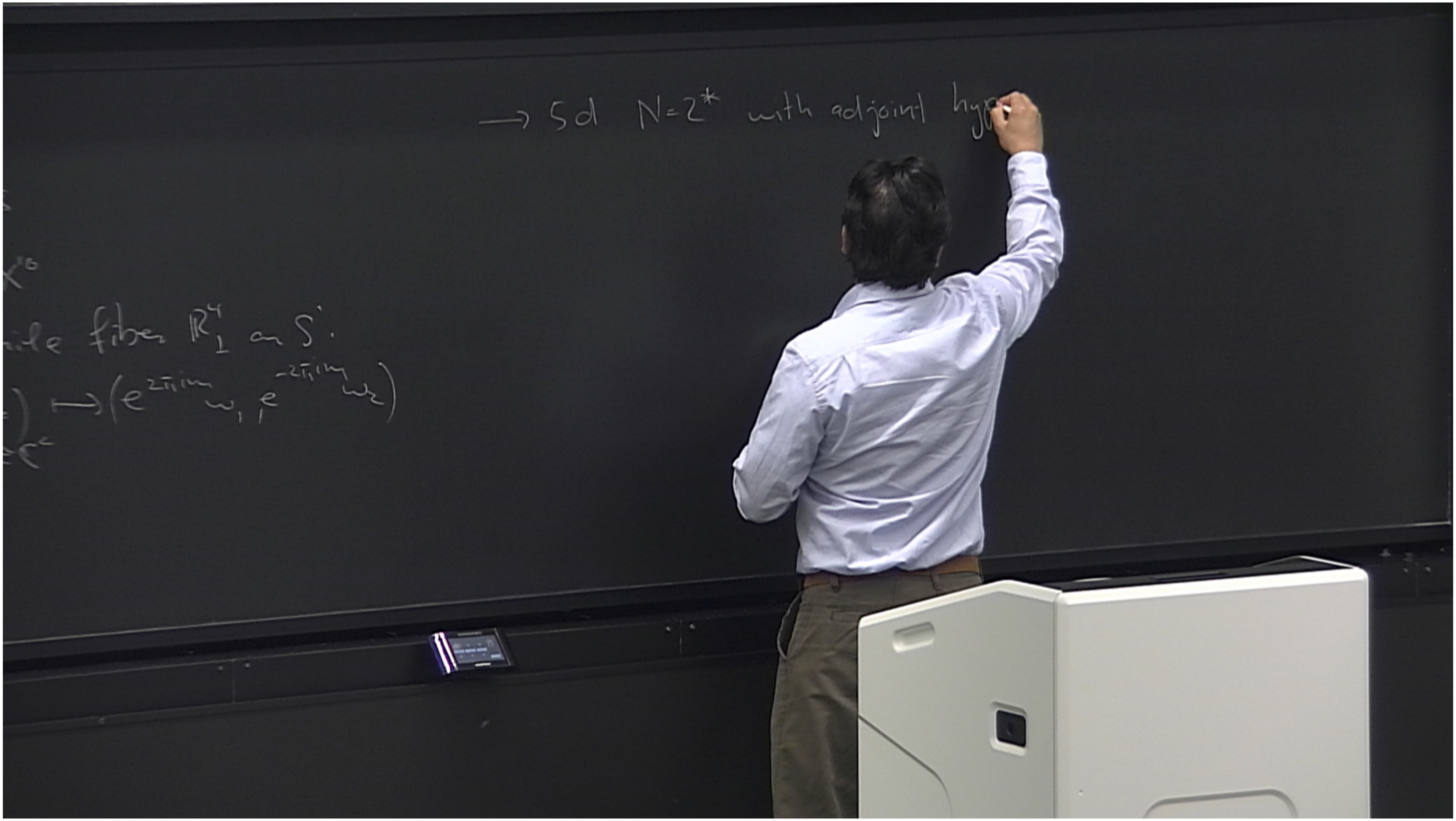

M-strings and their Orbifolds
(1305.6322, 1310.1185)
1) Geometric Setup
consider a system of parallel M5 branes
and suspended M2 branes
M2
M5
M5
X^6
$X^0, X^1, X^2, \ldots, X^5$
parametrize
$\mathbb{R}^{1,10}$ by X^I, $I = 0, 1, \ldots, 10$
separate along X^6
$Spin_R(5) \longrightarrow Spin_R(4)$
worldvolume of M2's. X^0, X^1, X^6
bound

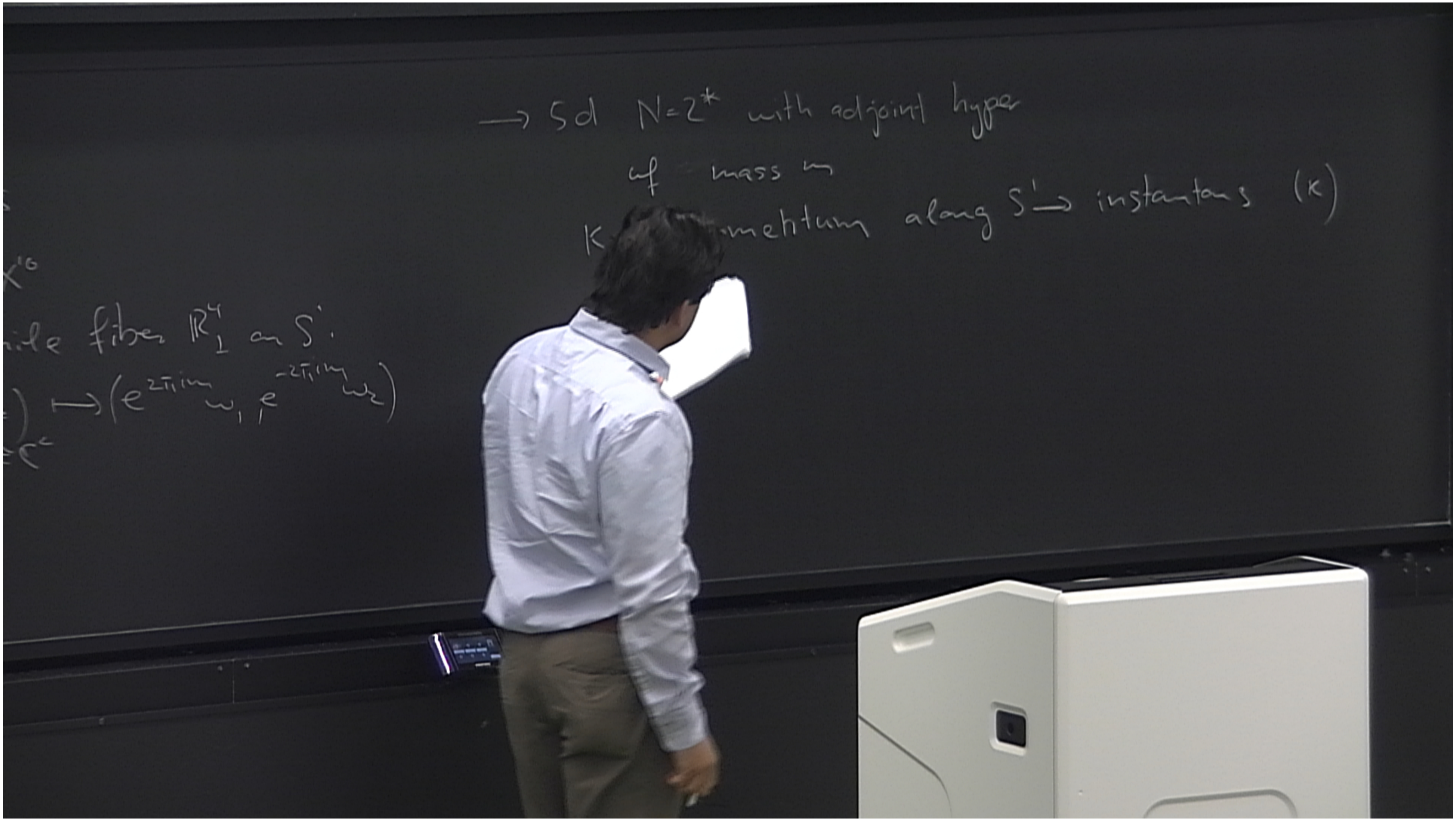

M-strings and their Orbifolds
(1305.6322, 1310.1185)
1) Geometric setup
consider a system of parallel M5 branes
and suspended M2 branes
M2
M5
M5
parametrize
by
separate along
worldvolume of M2's.
boundary inside M5 :
string inside M5. M-string!
preserved
of 32 SC's survive

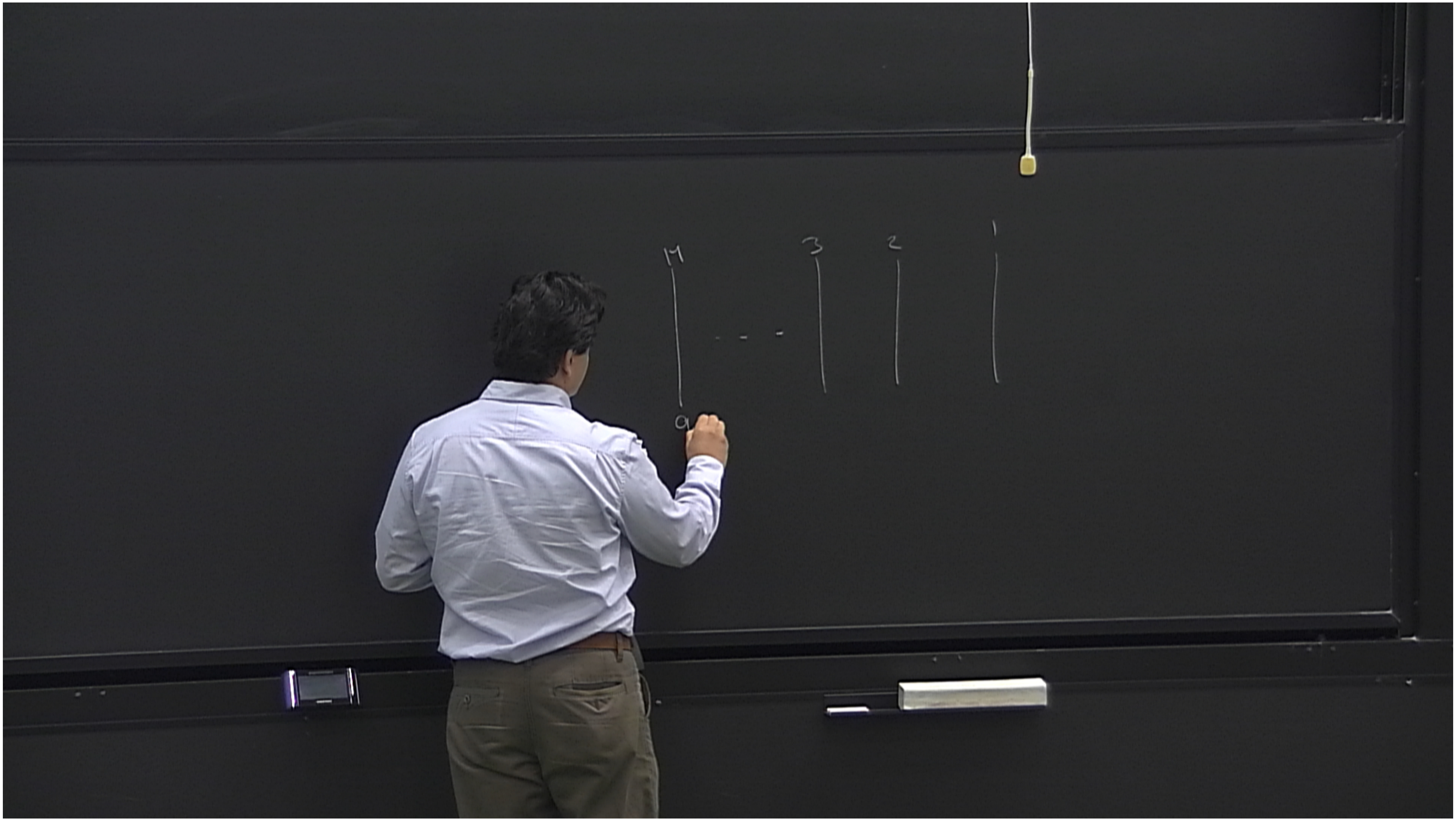

M-strings and their Orbifolds
(1305.6322, 1310.1185)
1) Geometric setup
consider a system of parallel M5 branes
and suspended M2 branes
M2
M5
M5
X^6
X^0, X^1, X^2, ..., X^5
parametrize
R^{1,10} by X^I, I = 0, 1, ..., 10
separate along X^6
Spin_R(5) → Spin_R(4)
worldvolume of M2's: X^0, X^1, X^6
boundary inside M5: (X^0, X^1)
→ string inside M5: M-string!
→ SO(5,1) → SO(1,1) × SO(4)
preserved Γ^{016} ε = ε = Γ^{012345} ε
1/4 of 32 SC's survive
(2,1,2,1)_{1/2} ⊕ (1,2,1,2)_{-1/2}
SO(4) SO_R(4) Spin(1,1)
from

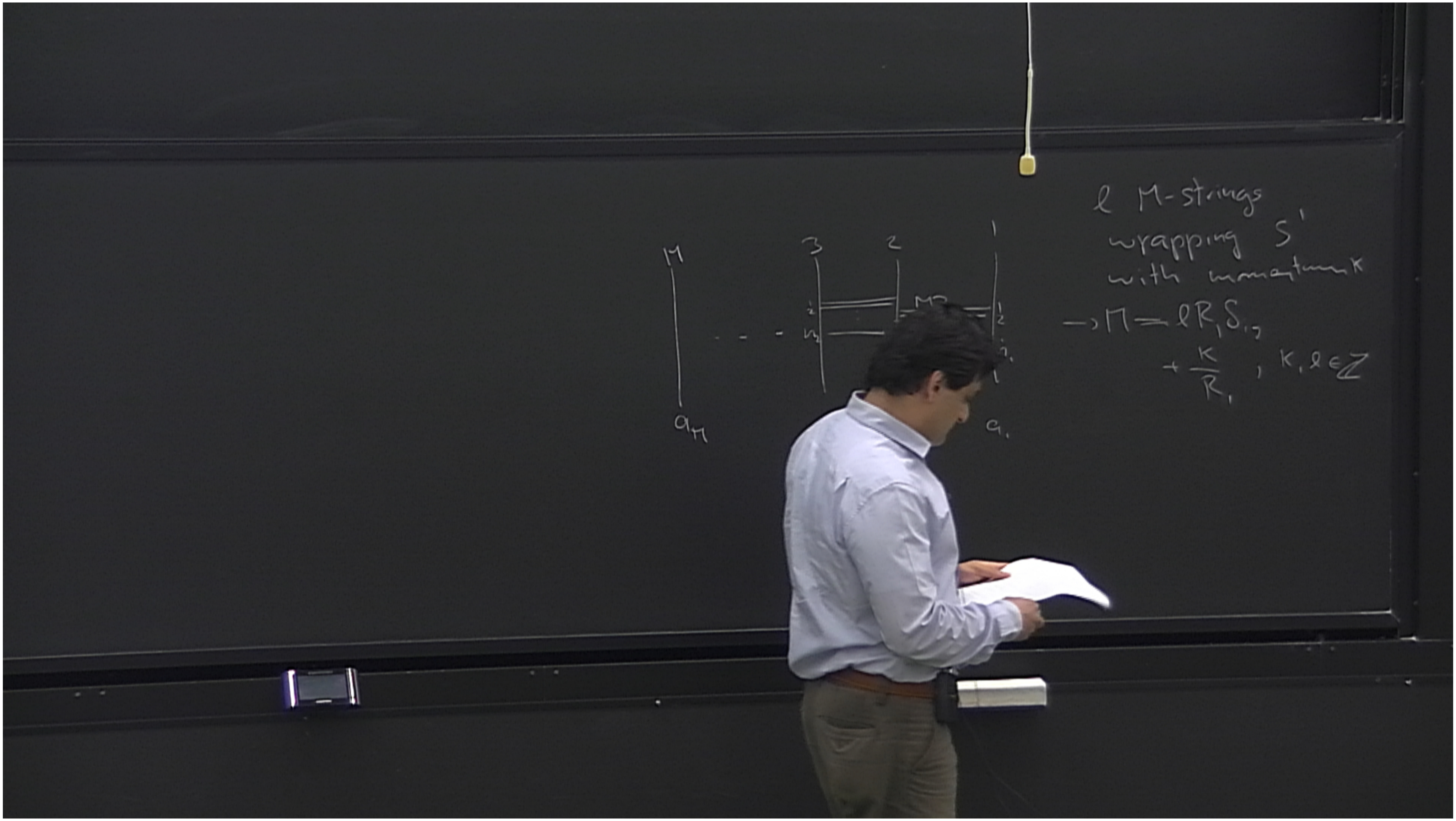

their Orbifolds
6322, 1310.1185)
worldvolume of M2's: X0, X1, X6
boundary inside M5: (X0, X1)
→ string inside M5: M-string!
SO(5,1) → SO(1,1) × SO(4)
preserved: Γ016 ε = ε = Γ012345 ε
1/4 of 32 SC's survive
transform 8s of Spin(8)
(2,1,2,1)1/2, (1,2,1,2)-1/2
SO(4) SOR(4) Spin(1,1)
I = 0, 1, ..., 10
along X6
→ SpinR(4)

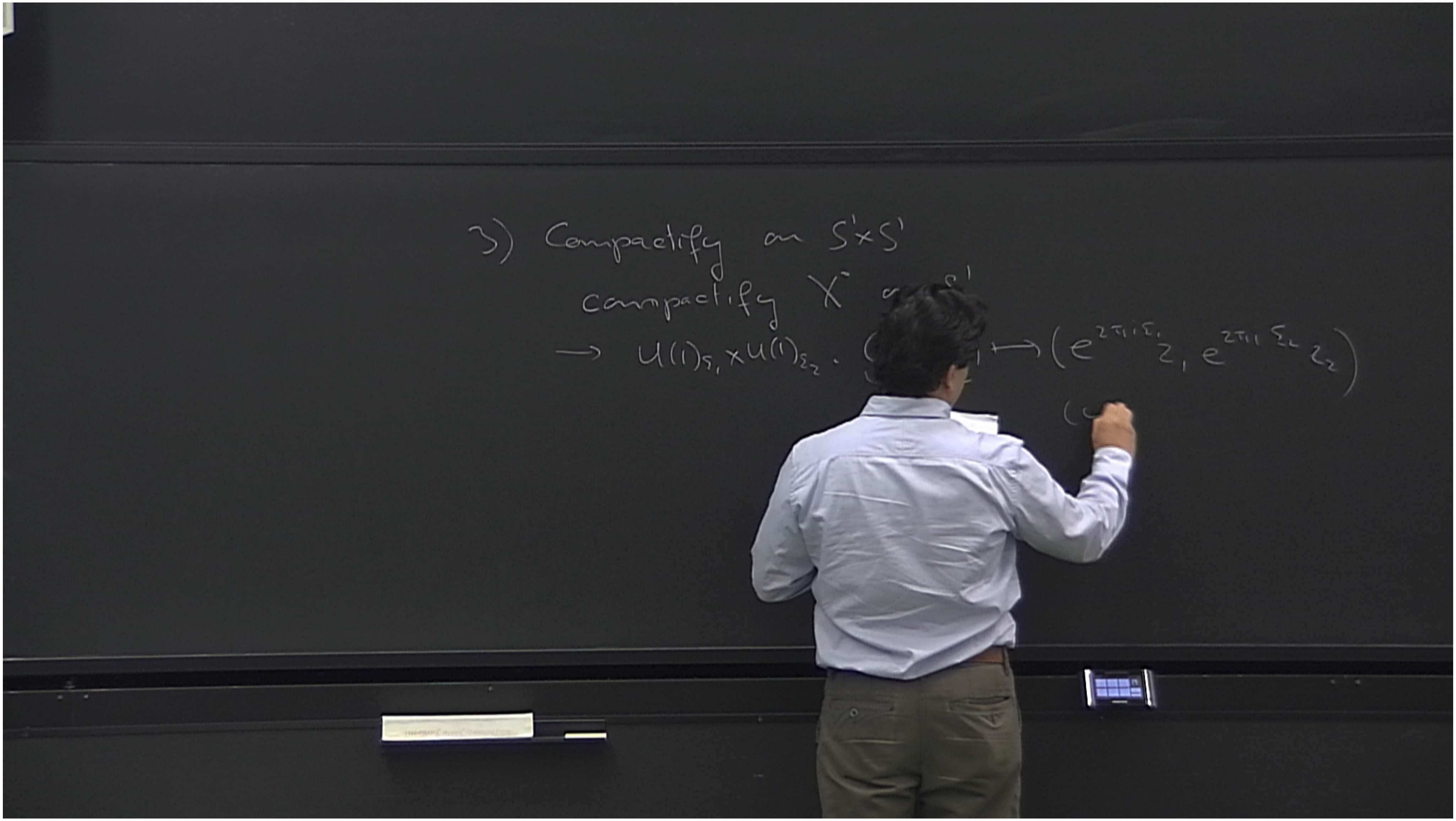

2) Compactify on S^1
Recall: $\mathbb{R}^4_{\parallel}$: X^2, X^3, X^4, X^5
$\mathbb{R}^4_{\perp}$: X^7, X^8, X^9, X^{10}
fy X^1 on S^1 while fiber $\mathbb{R}^4_{\perp}$ on S^1.
$U(1)_m$: $(w_1, w_2) \longmapsto (e^{2\pi i m} w_1, e^{-2\pi i m} w_2)$
$\mathbb{R}^4_{\perp} \cong \mathbb{C}^2$

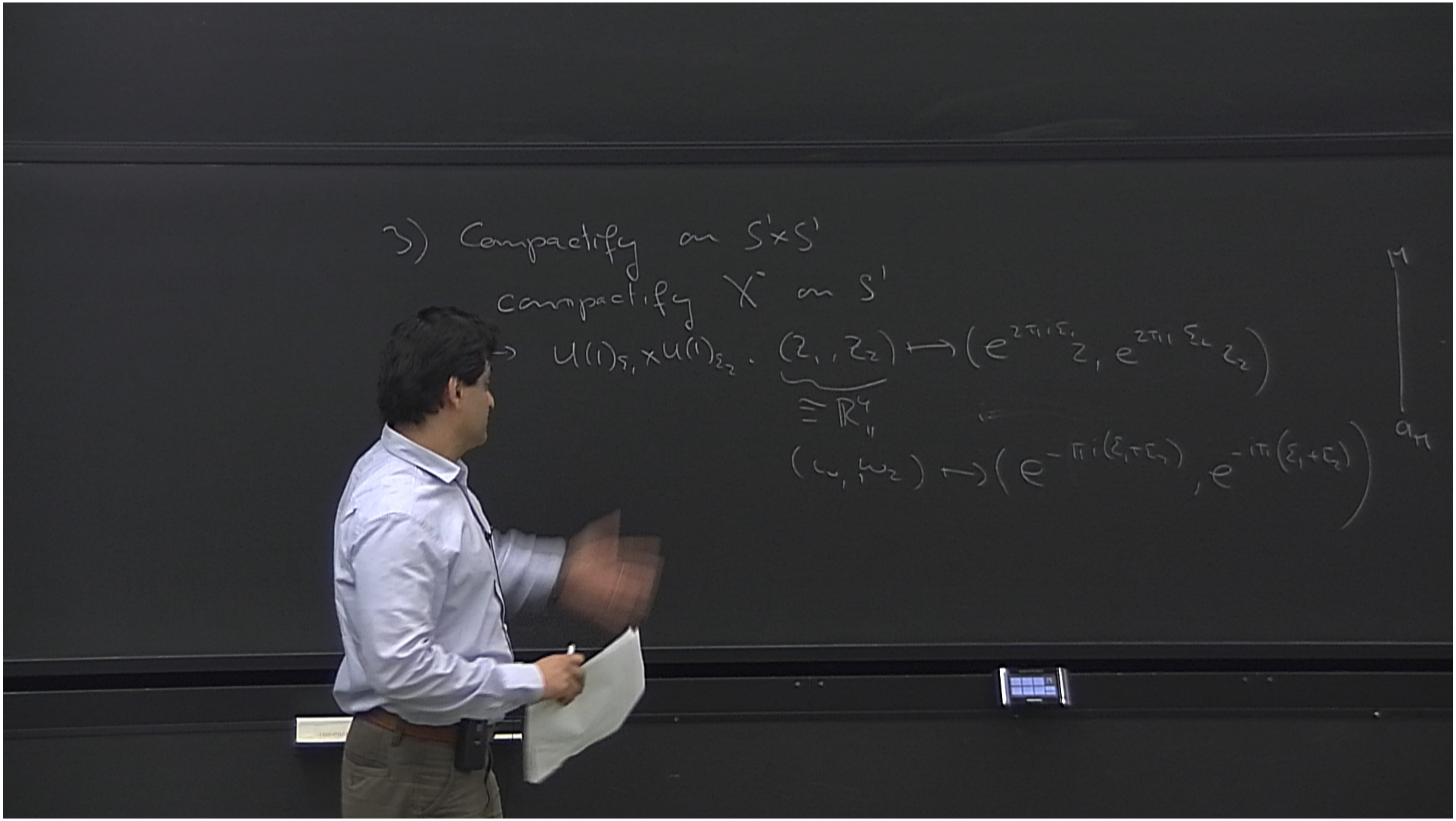

→ 5d N=2* with adjoint hyp
ile fiber R^4_1 on S^1:
) ↦ (e^{2πim} w_1, e^{-2πim} w_2)


→ 5d N=2* with adjoint hyper
of mass m
mentum along S¹ → instantons (k)
ile fiber R⁴ on S¹.


M
3
2
1


ℓ M-strings
wrapping S¹
with momentum k
→ M = ℓ R₁ S₁,
+ k/R₁ , k, ℓ ∈ Z


3) Compactify on $S^1 \times S^1$
compactify X^-
$\longrightarrow U(1)_{\xi_1} \times U(1)_{\xi_2}$.
$\longmapsto (e^{2\pi i \xi_1} z_1, e^{2\pi i \xi_2} z_2)$


3) Compactify on S¹×S¹
compactify X⁻ on S¹
→ U(1)ξ₁ × U(1)ξ₂ . (z₁, z₂) ↦ (e^{2πiξ₁} z₁, e^{2πiξ₂} z₂)
≅ ℝ⁴ℏ
(w₁, w₂) ↦ (e^{-πi(ξ₁+ξ₂)}, e^{-iπ(ξ₁+ξ₂)})
M
a_M

3) Compactify on $S^1 \times S^1$

compactify X^0 on S^1

$\longrightarrow U(1)_{\varepsilon_1} \times U(1)_{\varepsilon_2} \cdot \underbrace{(z_1, z_2)}_{\cong \mathbb{R}^4_{\parallel}} \longmapsto \left(e^{2\pi i \varepsilon_1} z_1, e^{2\pi i \varepsilon_2} z_2\right)$

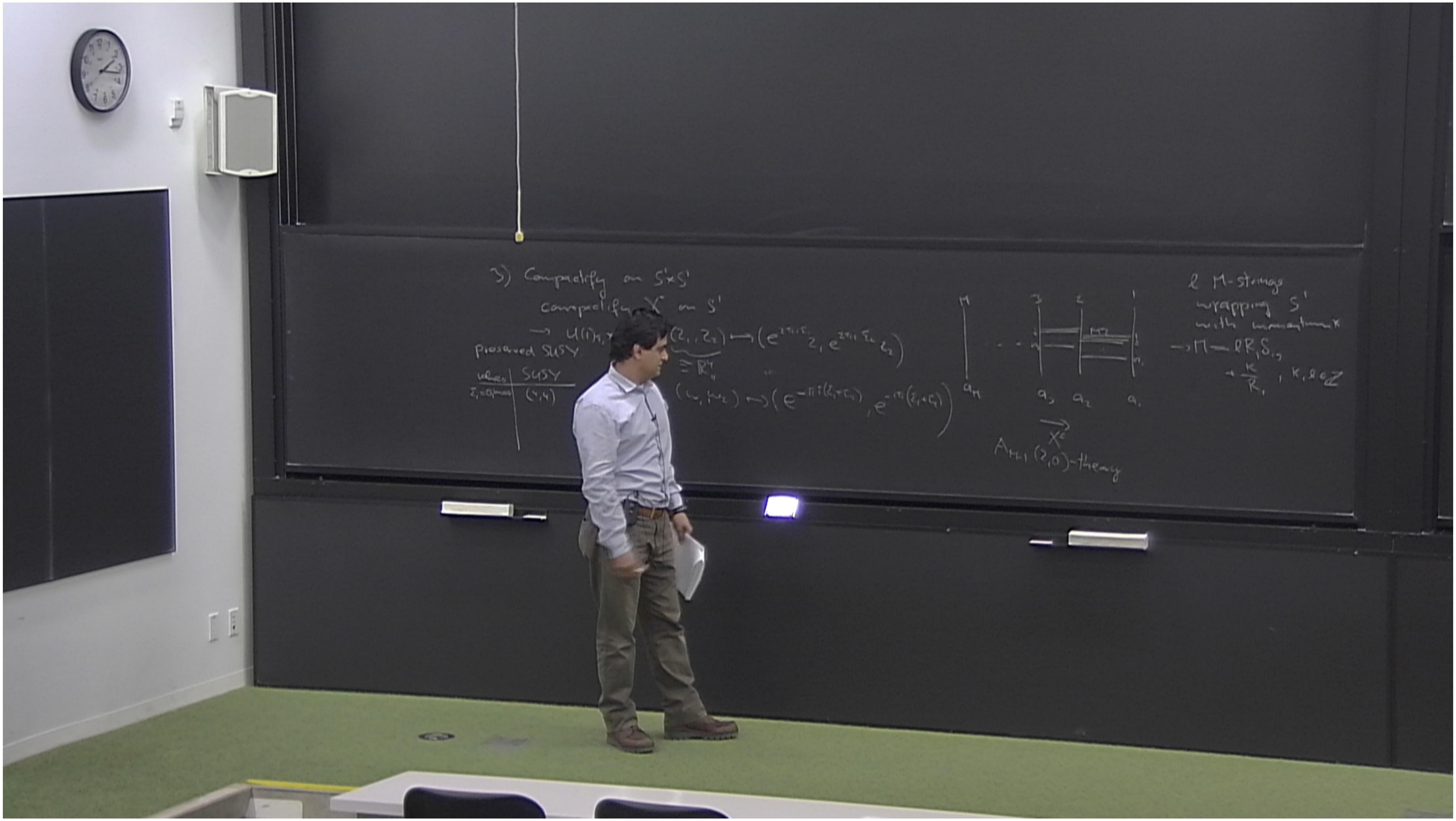
$(w_1, w_2) \longmapsto \left(e^{-\pi i(\varepsilon_1+\varepsilon_2)}, e^{-i\pi(\varepsilon_1+\varepsilon_2)}\right)$

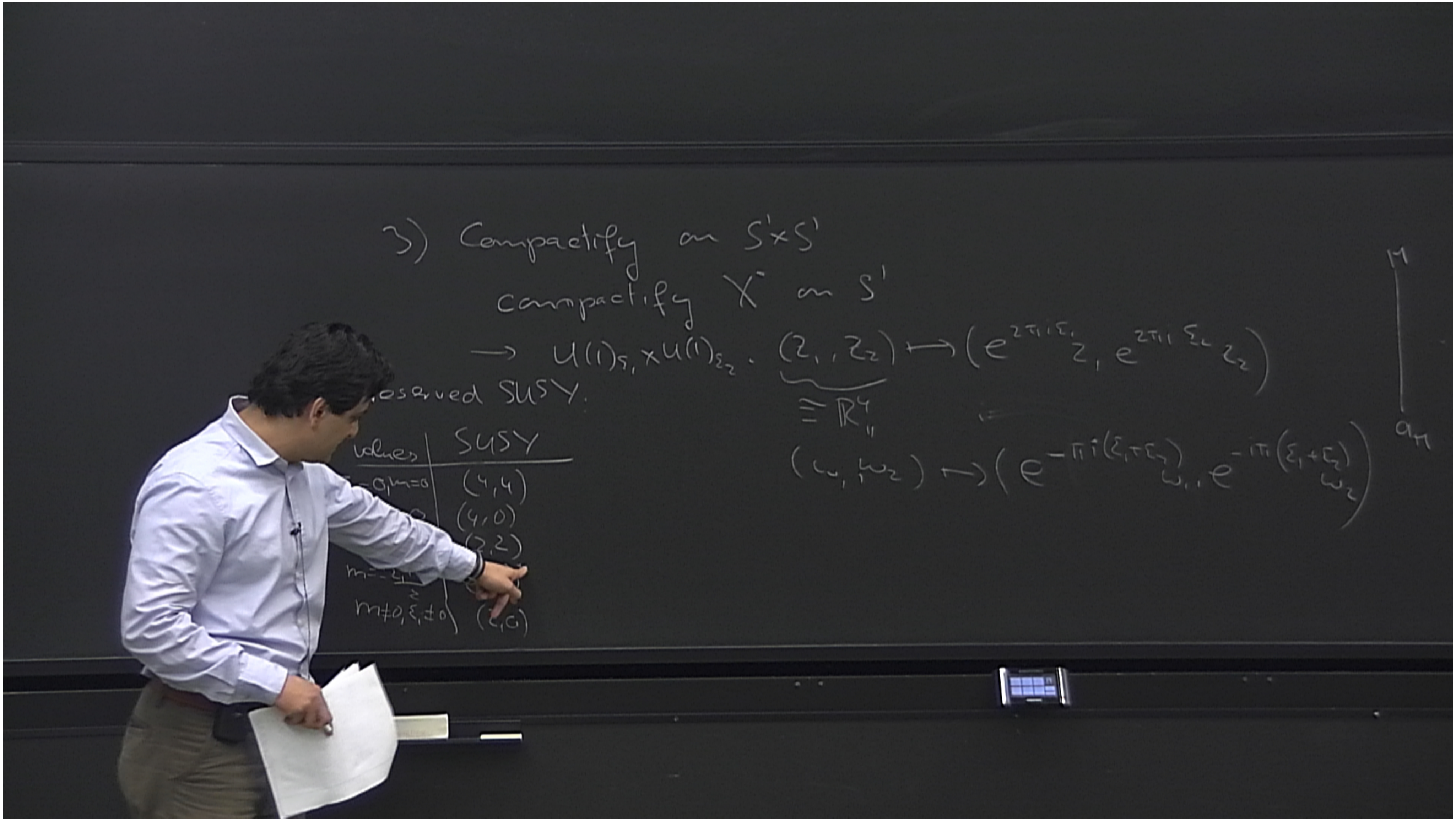
preserved SUSY:

values	SUSY
$\varepsilon_1 = 0, \mu = 0$	$(4,4)$

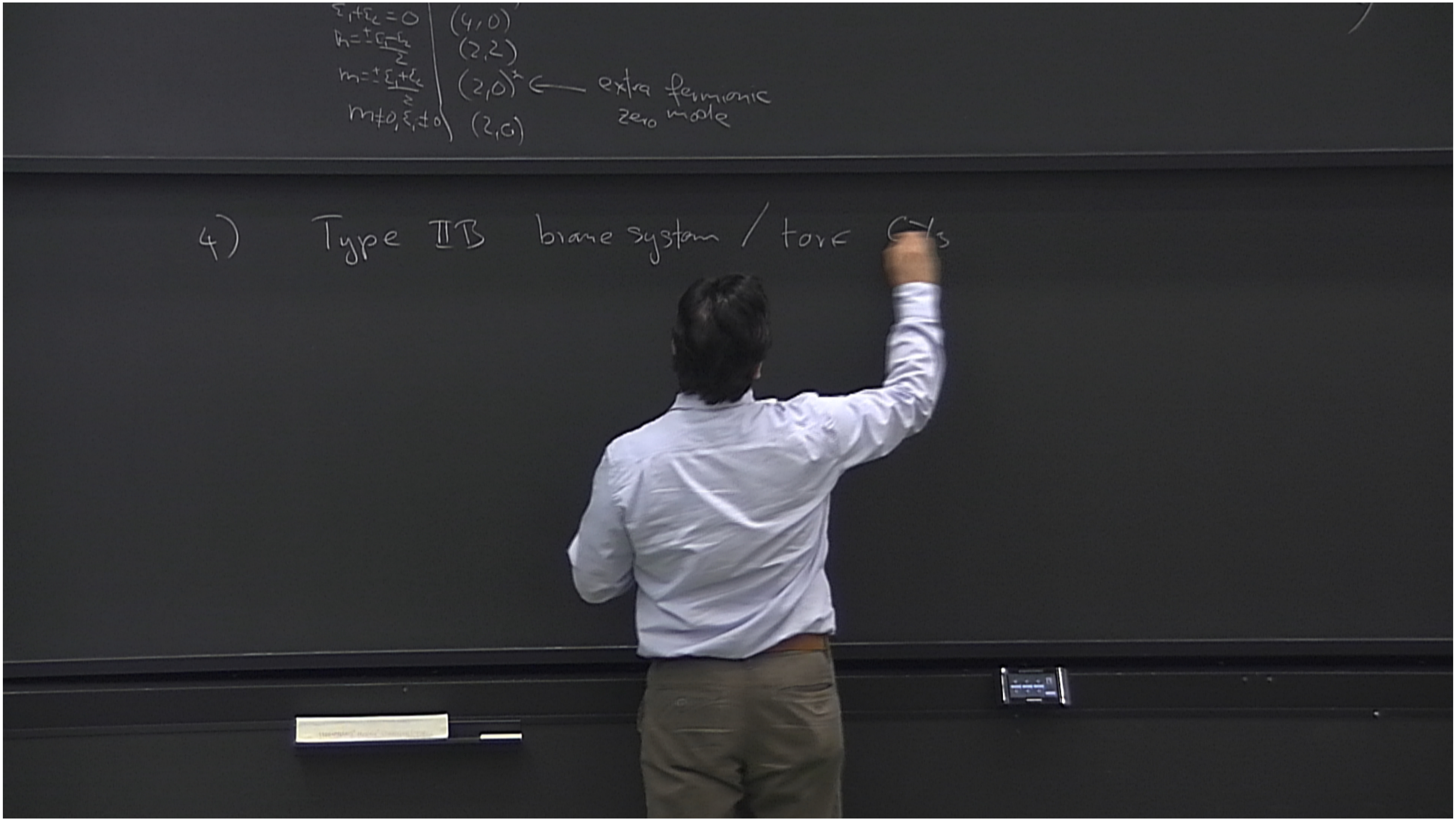
M

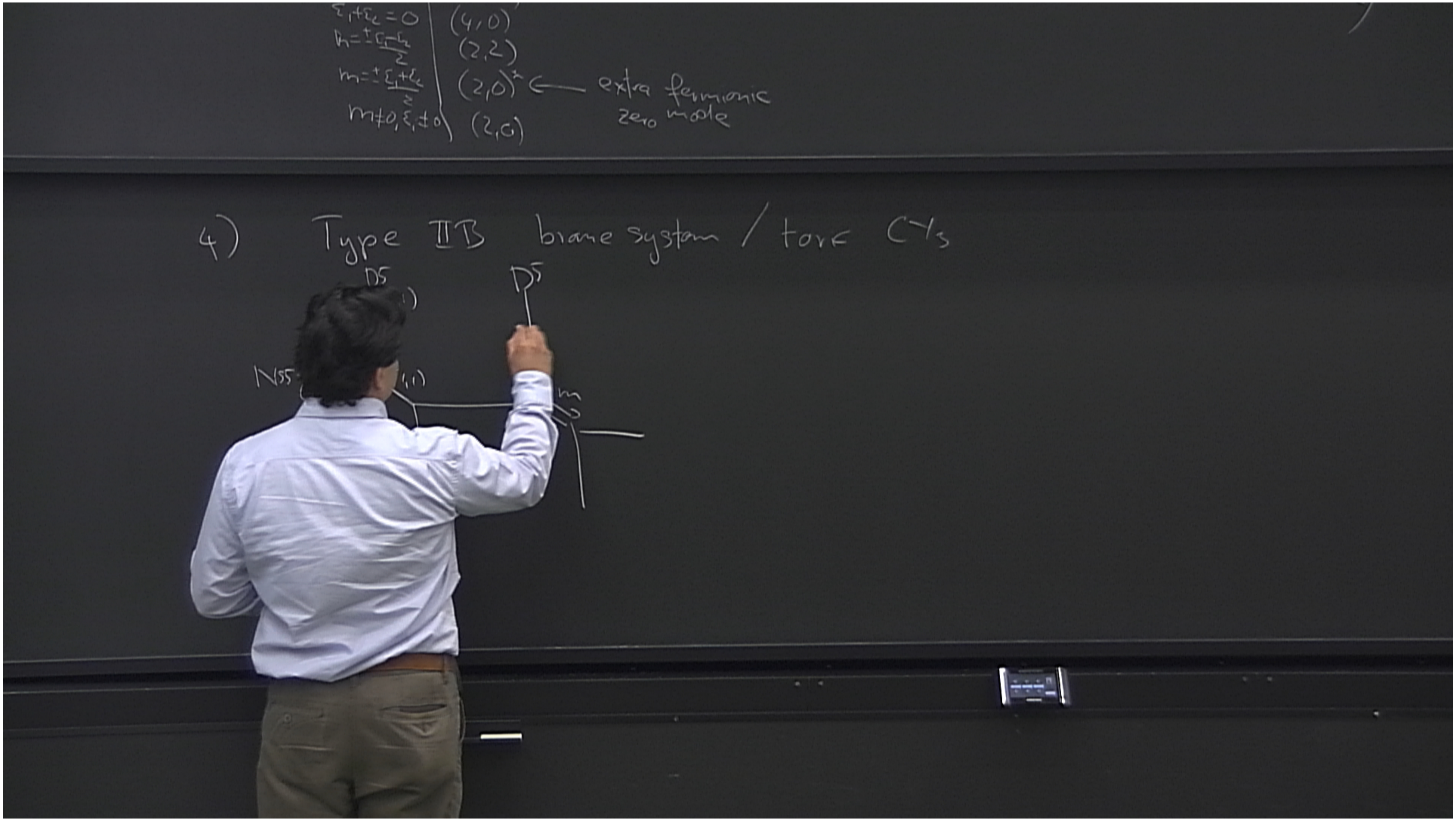
a_M

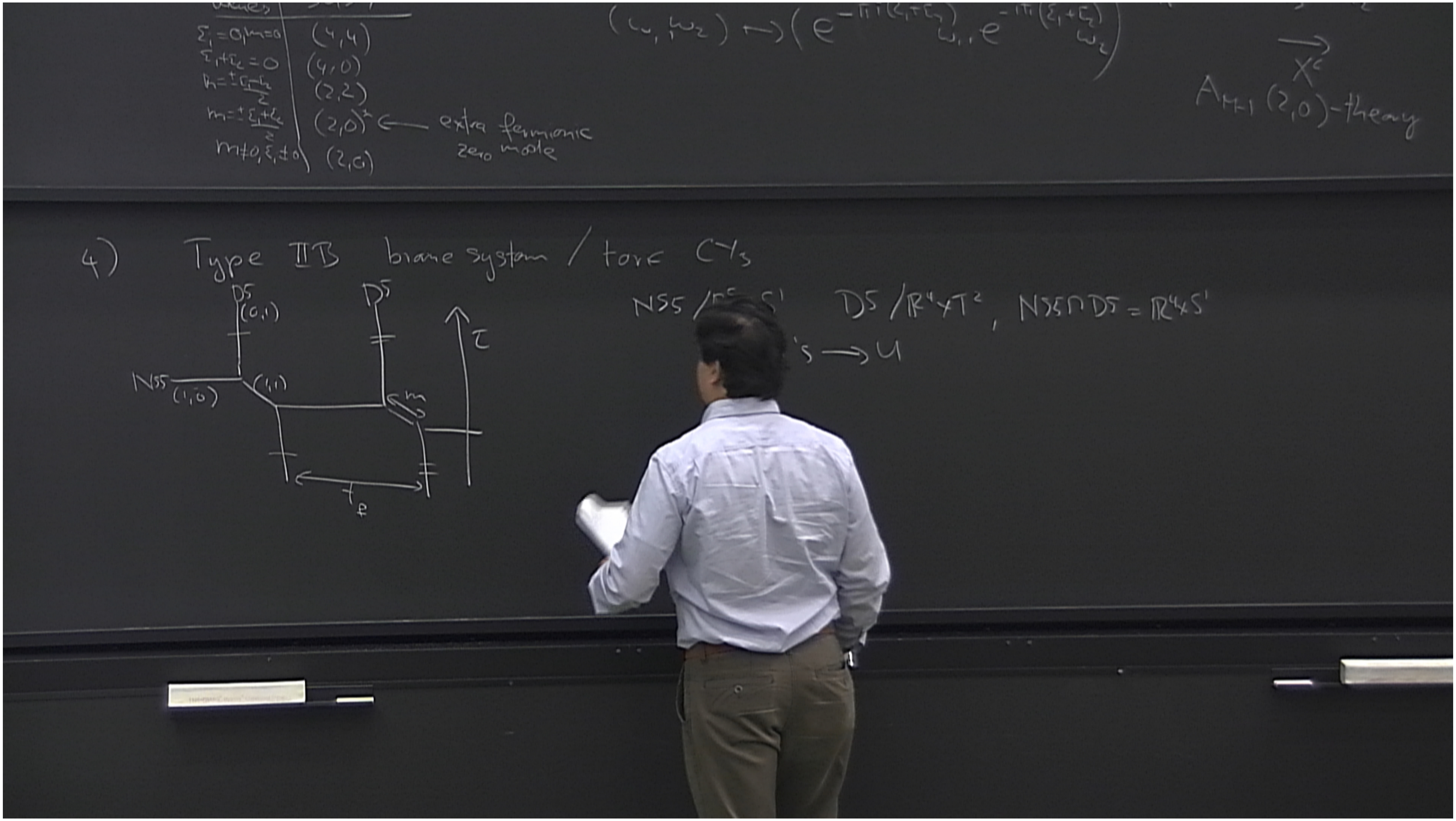

3) Compactify on S'xS'
compactify X on S'
Preserved SUSY
SUSY
(4,4)
l M-strings
wrapping S'
A_{M-1} (2,0)-theory

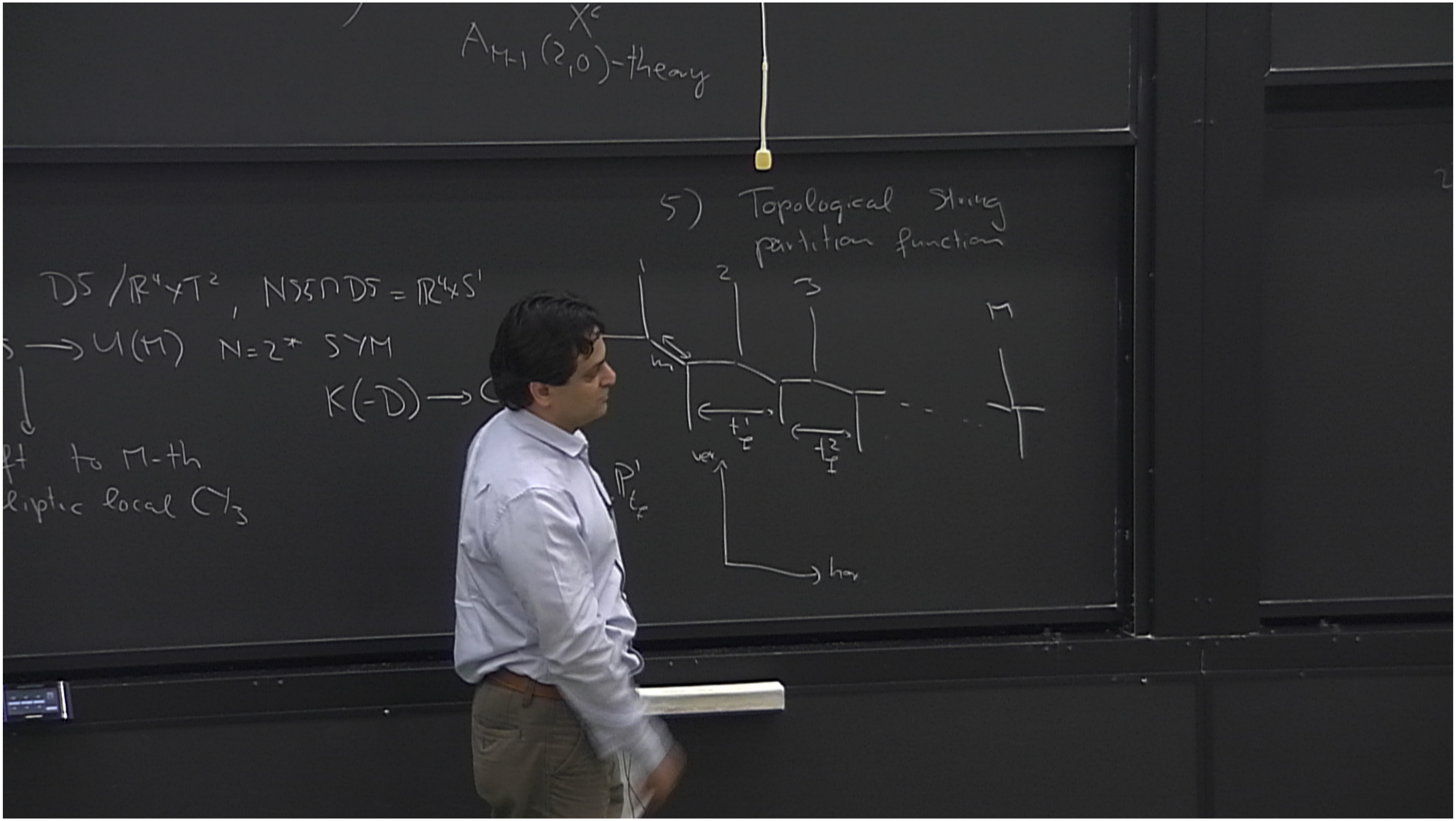


3) Compactify on S'×S'
compactify X on S'
→ U(1) × U(1)
≅ R
served SUSY
SUSY
(4,4)
(4,0)
(2,2)
(4,0)

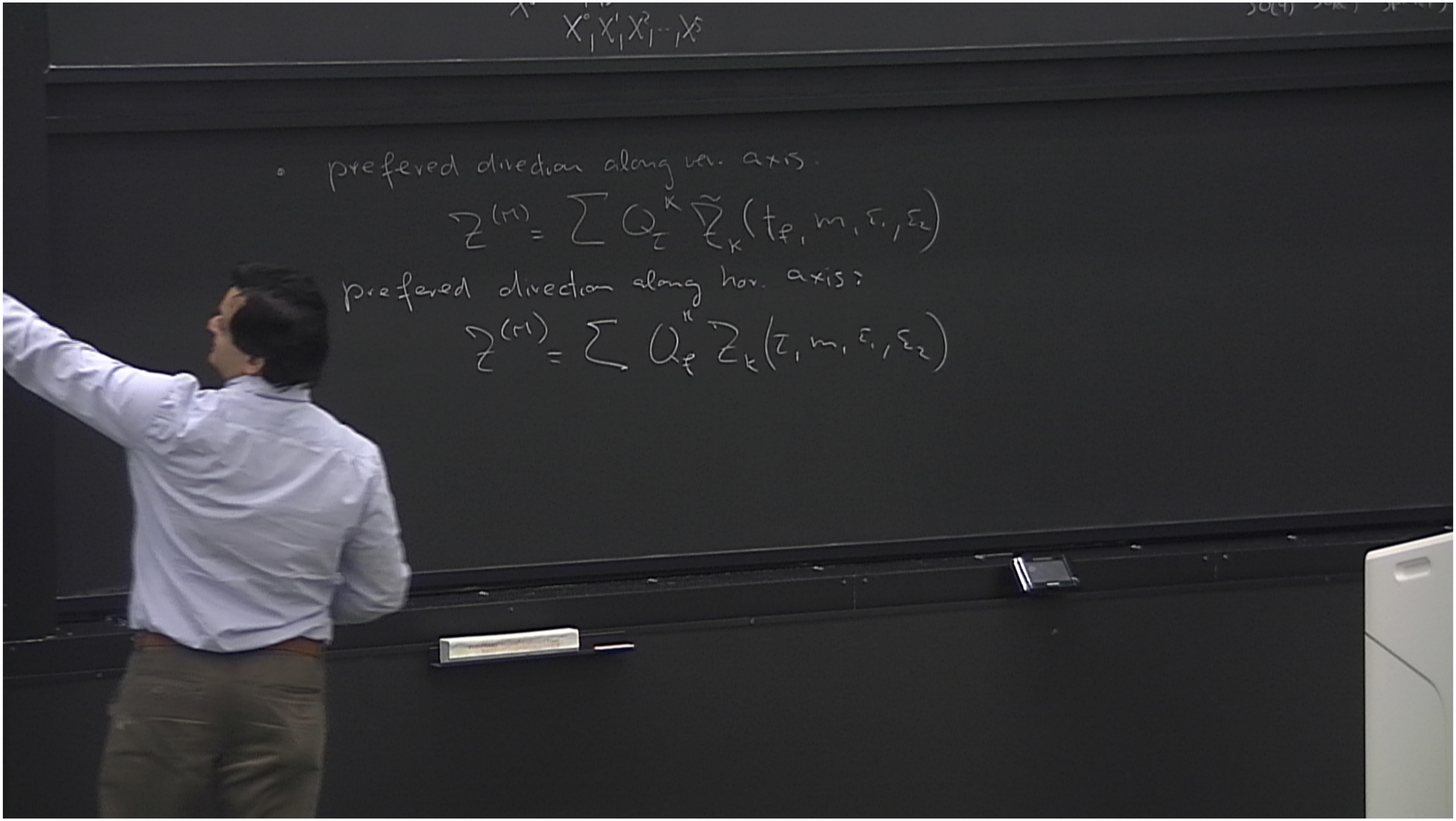

(4,0)
(2,2)
(2,0) ← extra fermionic zero mode
(2,0)
4) Type IIB brane system / toric


$\epsilon_1+\epsilon_2=0$ (4,0)
$m=\pm\frac{\epsilon_1-\epsilon_2}{2}$ (2,2)
$m=\pm\frac{\epsilon_1+\epsilon_2}{2}$ $(2,0)^2$ ← extra fermionic zero mode
$m\neq 0, \epsilon_1\neq 0$ (2,0)
4) Type IIB brane system / toric CY3s
D5
D5
NS5
(1,1)
m

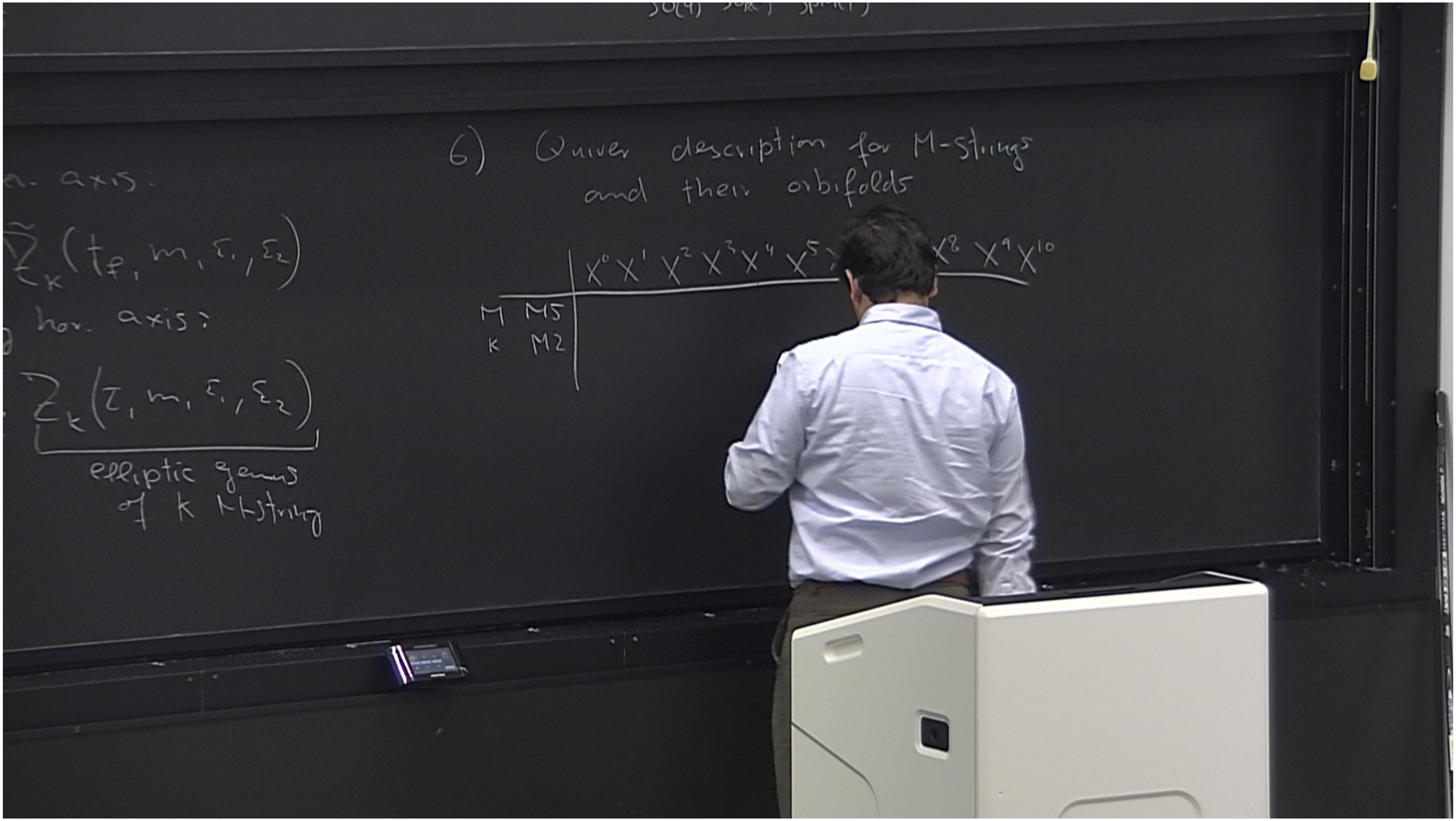

extra fermionic zero mode
A_{N-1} (2,0)-theory
4) Type IIB brane system / toric CY_3
D5
(0,1)
NS5
(1,0)
(1,1)
m
τ
t_F
$D5/\mathbb{R}^4 \times T^2$, $NS5 \cap D5 = \mathbb{R}^4 \times S^1$

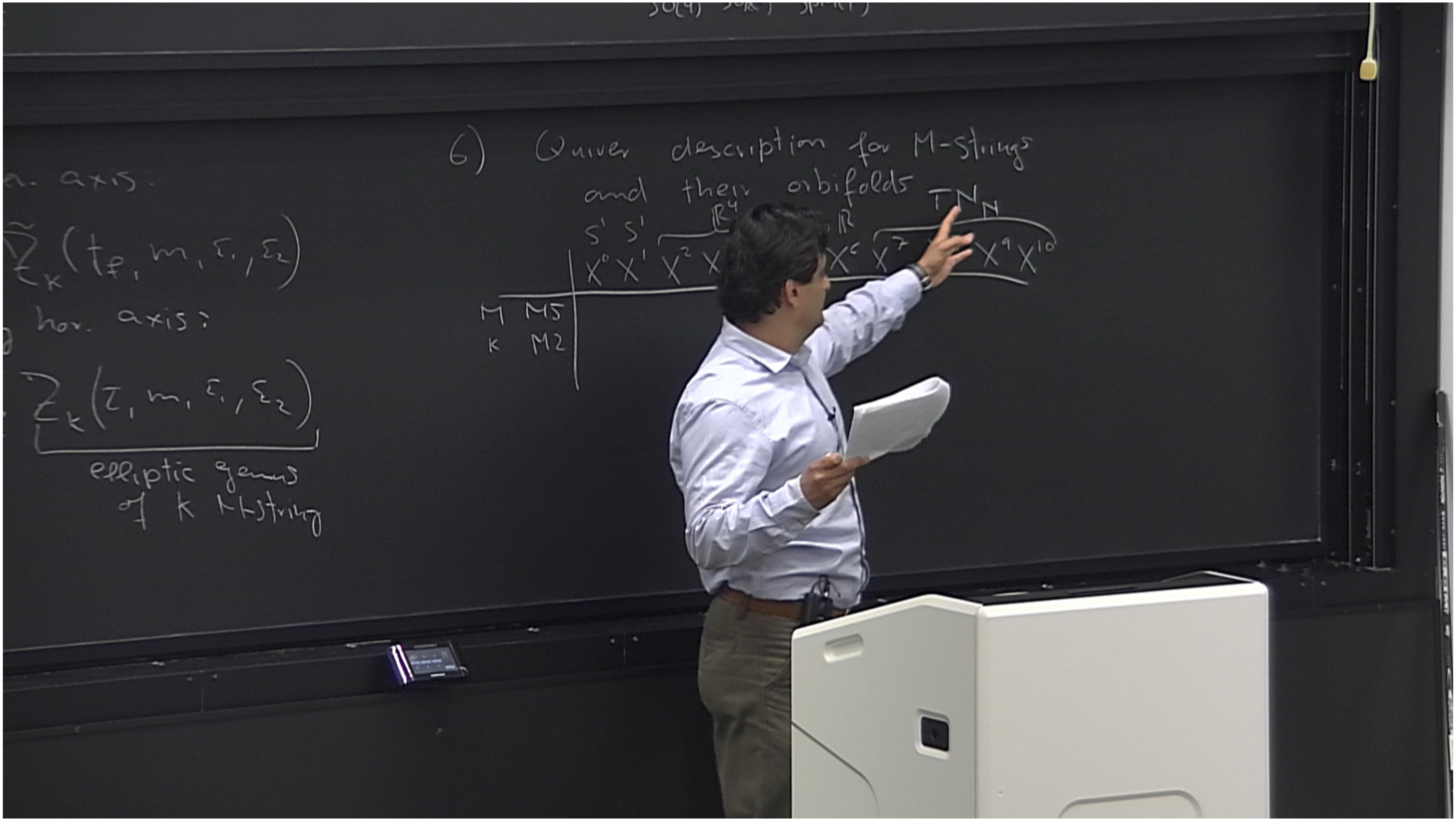


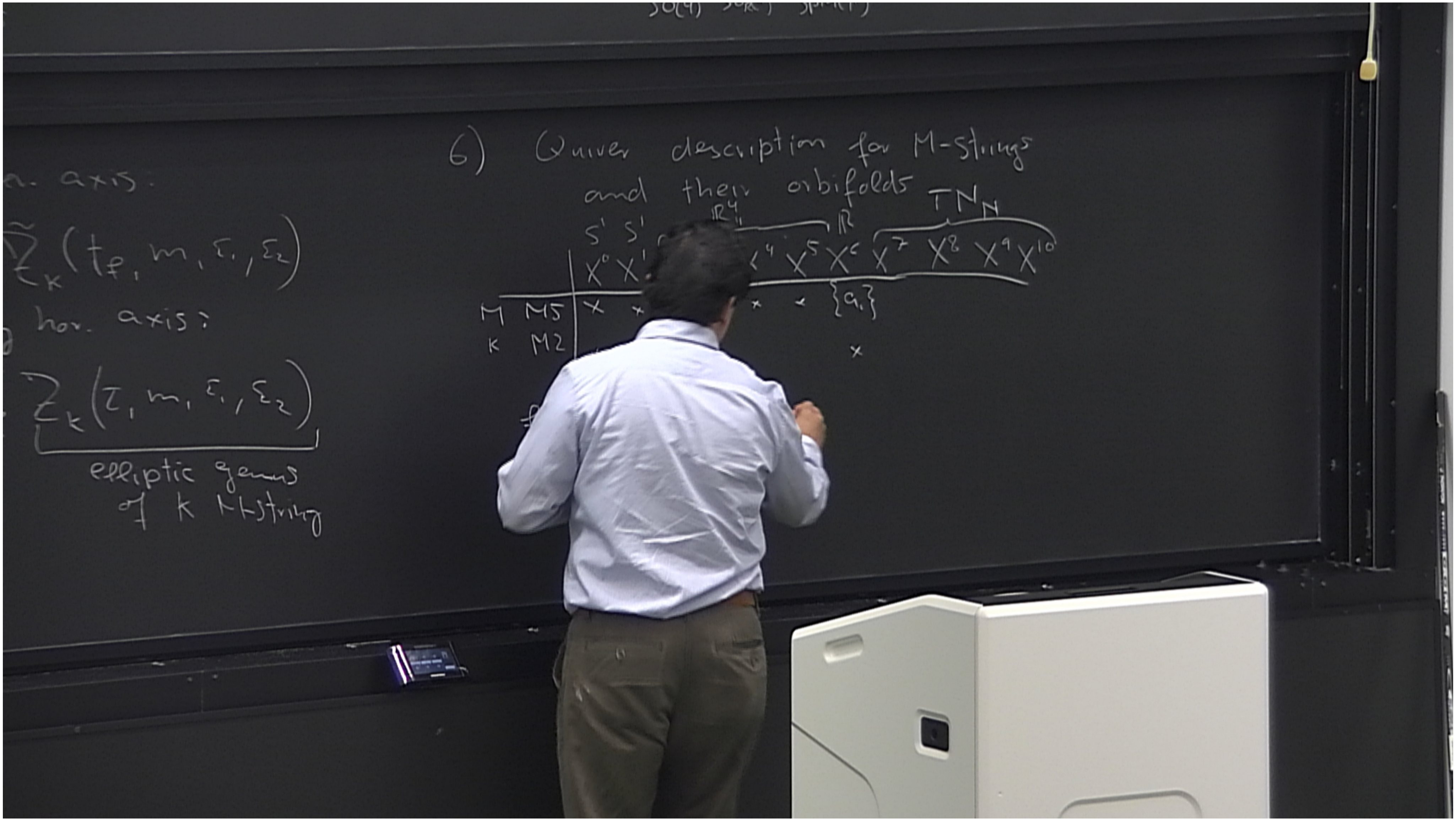
A_{M-1} (2,0)-theory
5) Topological String partition function
D5 / $\mathbb{R}^4 \times T^2$, NS5 ∩ D5 = $\mathbb{R}^4 \times S^1$
→ U(M) N=2* SYM
K(-D) →
to M-th
elliptic local CY3
ver
hor



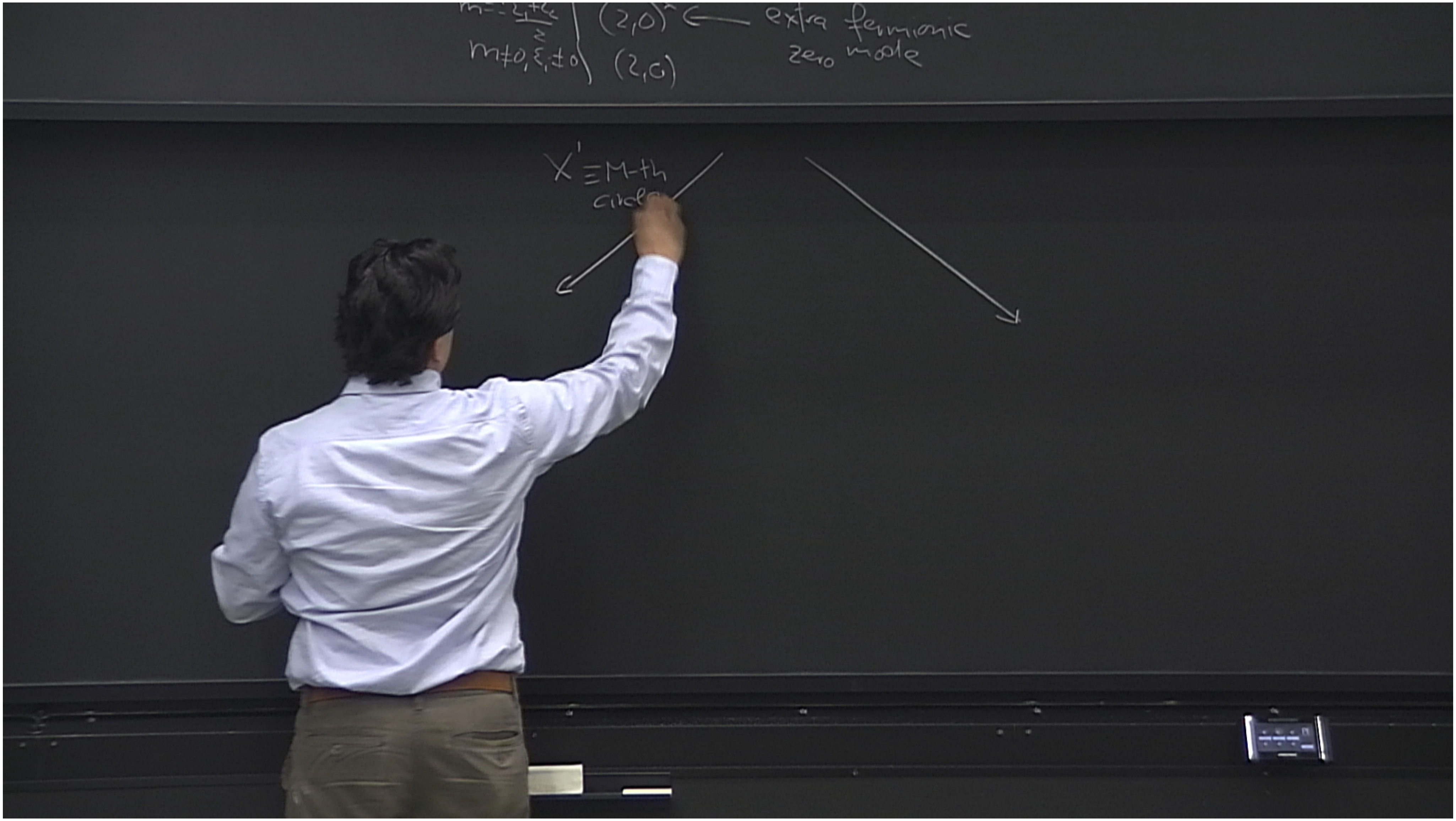
$X^0, X^1, X^2, \dots, X^5$
• prefered direction along ver. axis.
$Z^{(M)} = \sum Q_\tau^k \tilde{Z}_k(t_f, m, \epsilon_1, \epsilon_2)$
prefered direction along hor. axis:
$Z^{(M)} = \sum Q_f^k Z_k(\tau, m, \epsilon_1, \epsilon_2)$

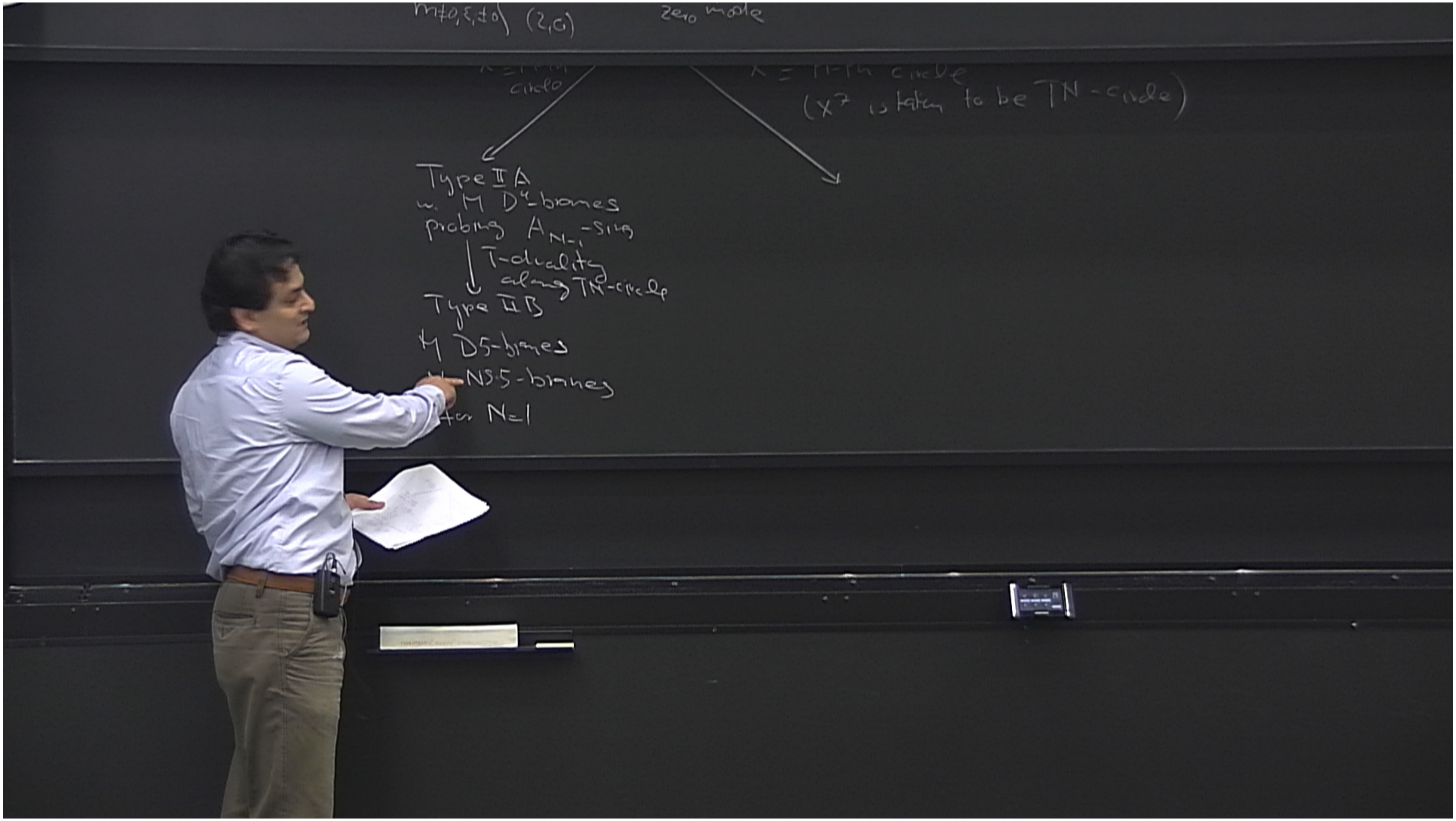

6) Quiver description for M-strings
and their orbifolds
a. axis:
hor. axis:
elliptic genus
of k M-string
M M5
k M2

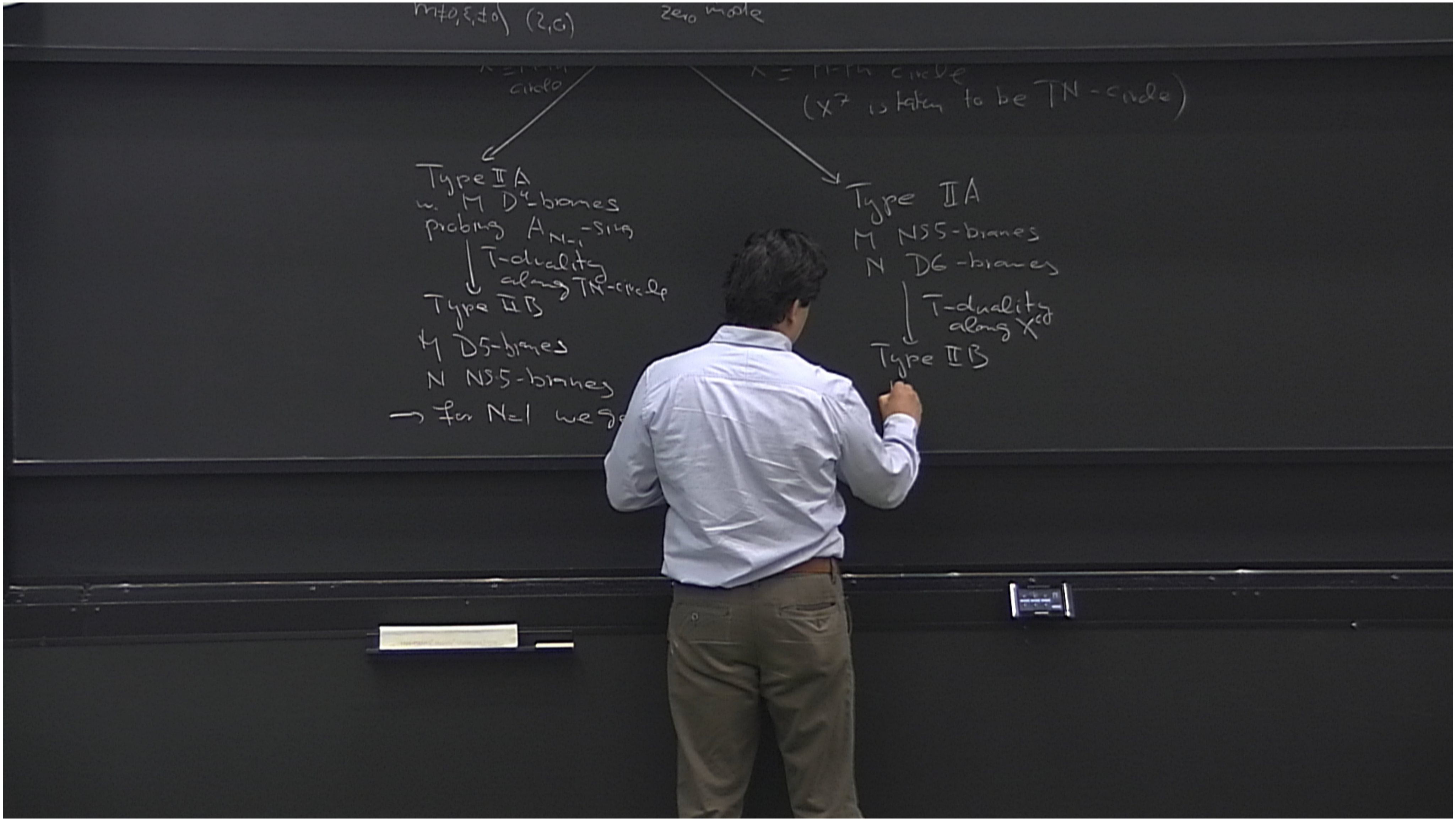

6) Quiver description for M-strings
and their orbifolds
hor. axis:
M M5
k M2
elliptic genus
of k M-string

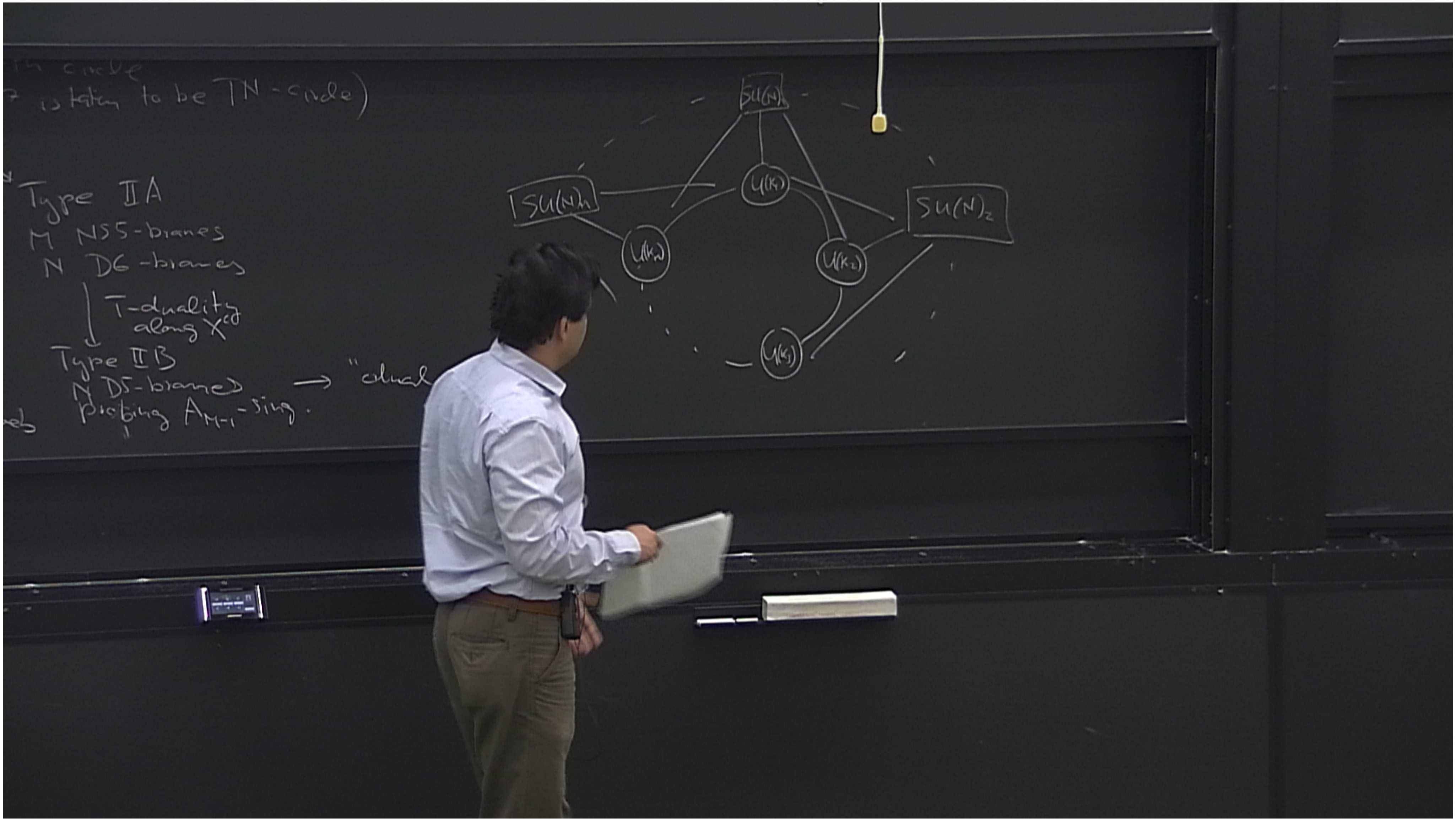


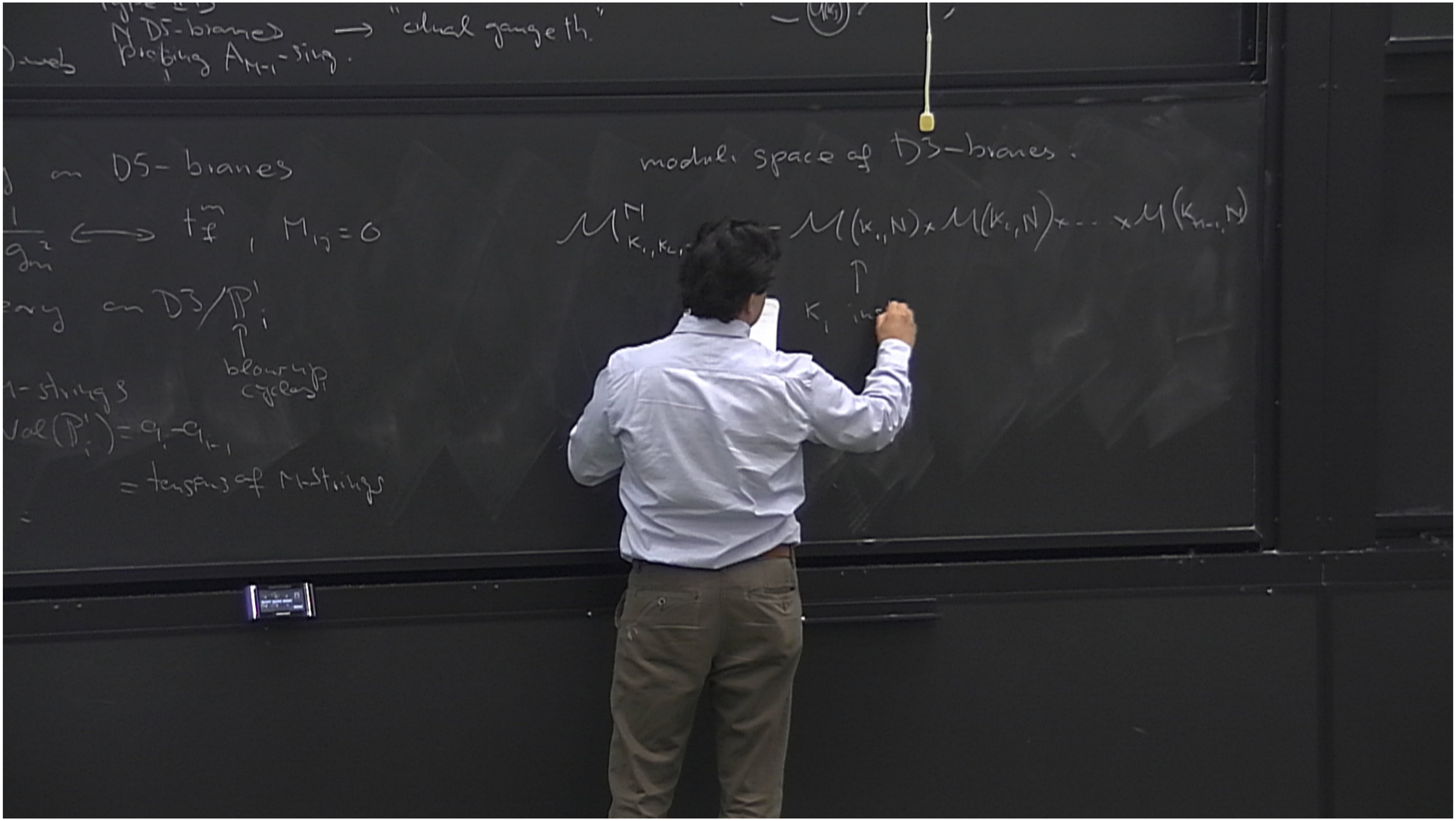
6) Quiver description for M-strings
and their orbifolds
elliptic genus
of k M-string

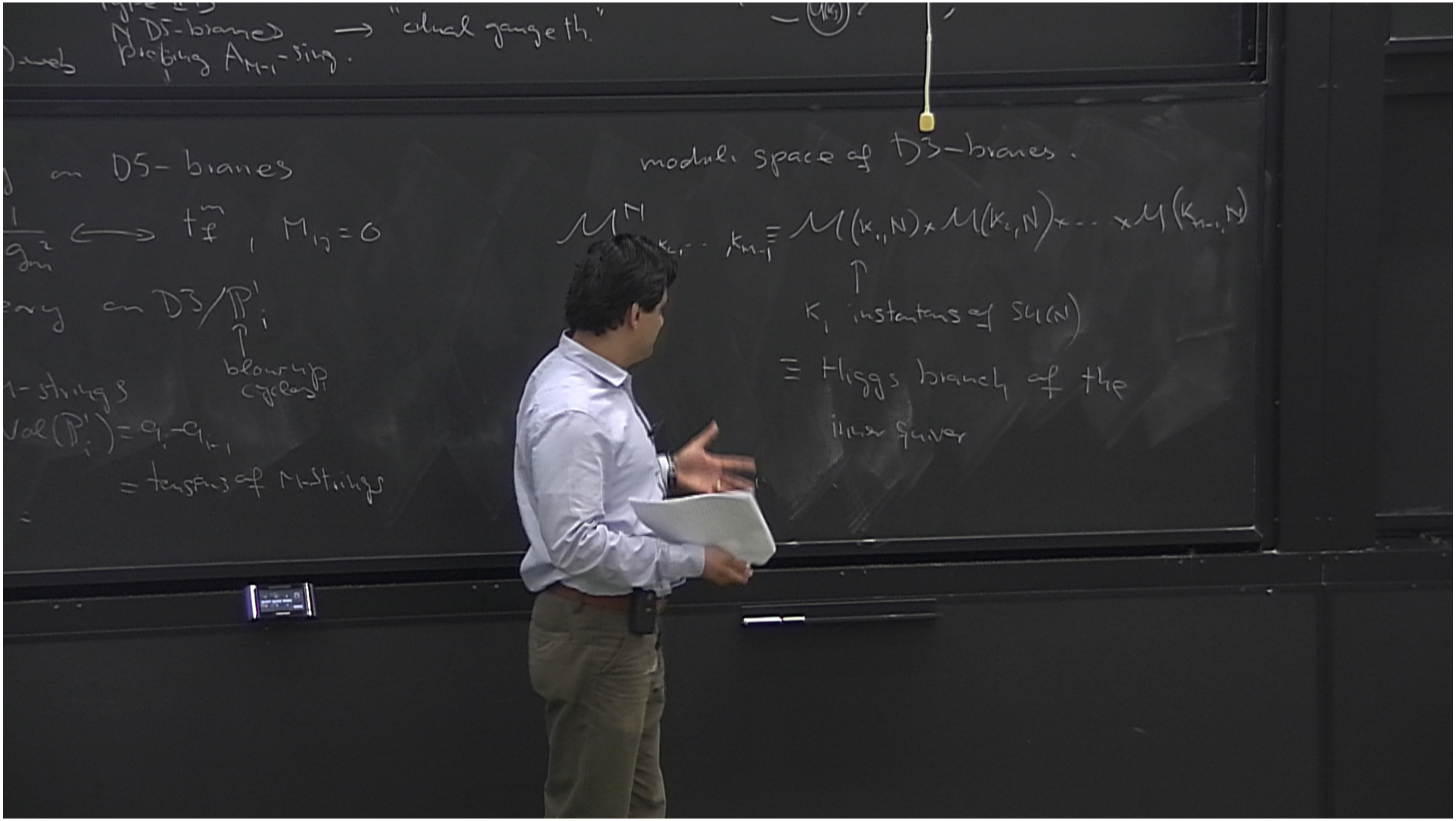

(2,0)^2 ← extra fermionic zero mode
m≠0, ξ≠0 (2,0)
X' ≡ M-th

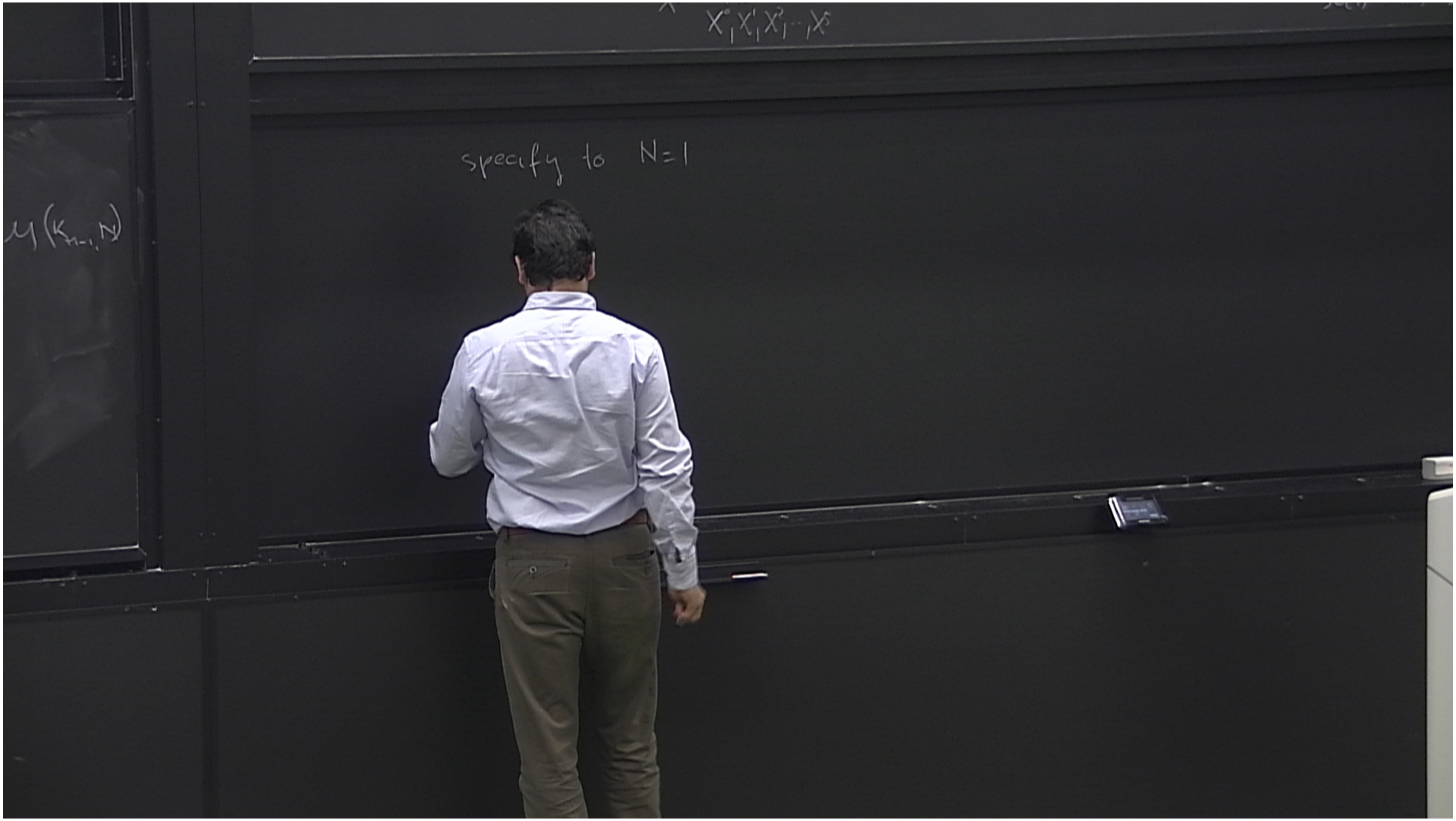

zero mode
circle
(x^7 is taken to be TN-circle)
Type IIA
w. M D4-branes
probing A_{N-1}-sing
T-duality
along TN-circle
Type IIB
M D5-branes
NS5-branes
for N=1

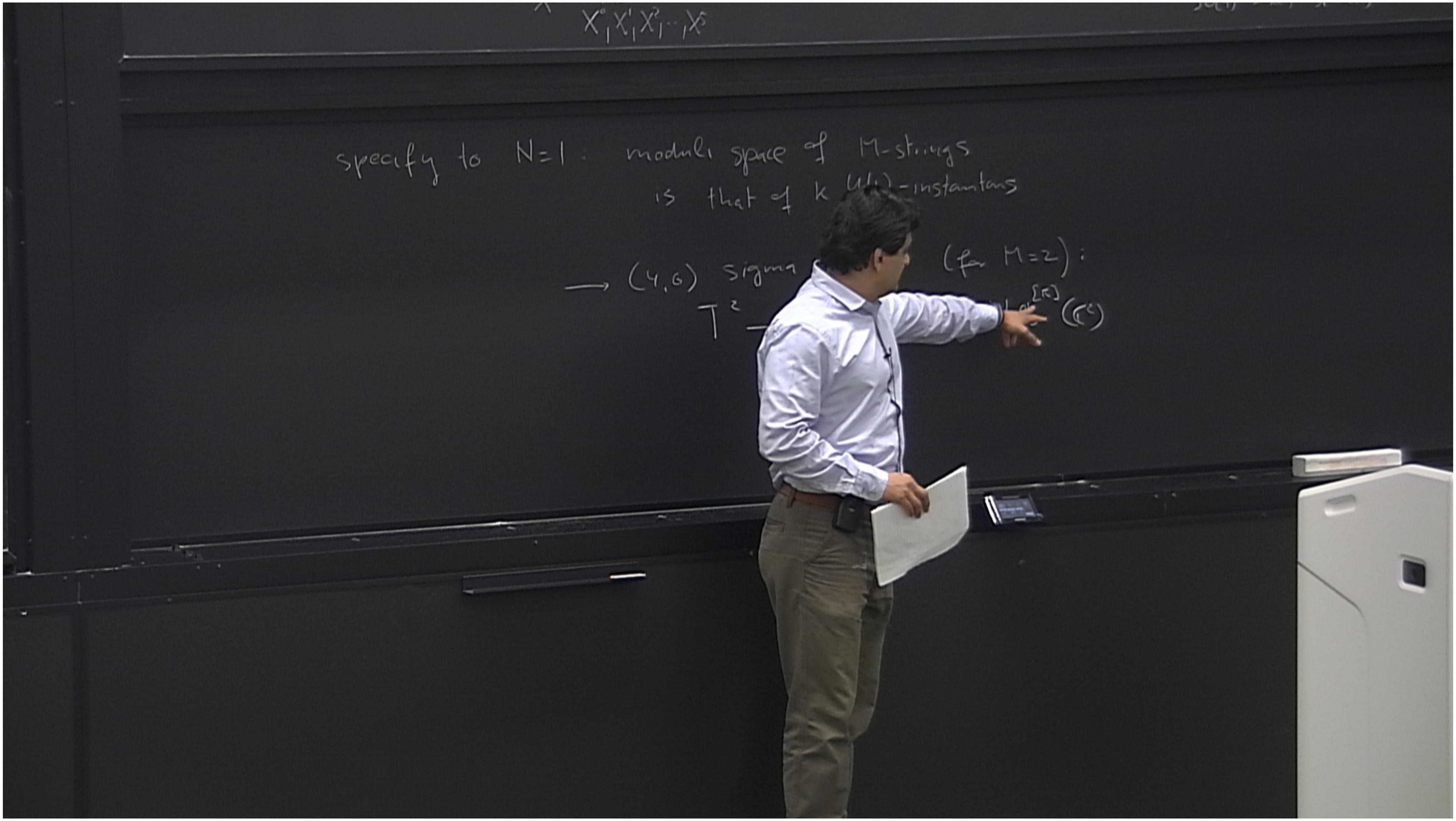

zero mode
(x^7 is taken to be TN-circle)
Type IIA
w. M D4-branes
probing A_{N-1}-sing
T-duality
along TN-circle
Type IIB
M D5-branes
N NS5-branes
→ for N=1 we
Type IIA
M NS5-branes
N D6-branes
T-duality
along x^6
Type IIB

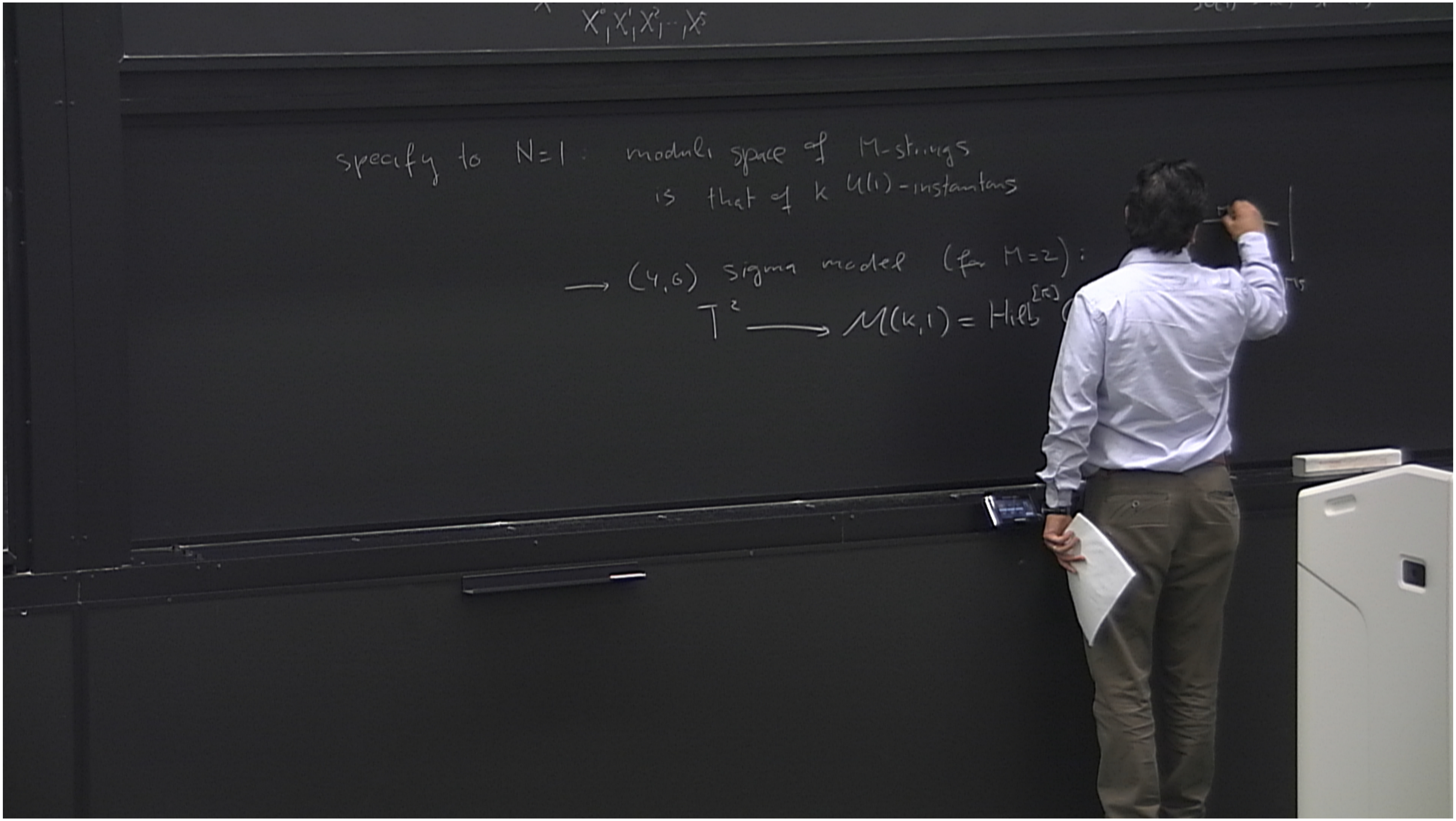

is taken to be TN-circle)
Type IIA
M NS5-branes
N D6-branes
T-duality along X^6
Type IIB
N D5-branes
probing A_{M-1}-sing.
"dual
$SU(N)$
$SU(N)_1$
$SU(N)_2$
$U(k)$
$U(k_1)$
$U(k_2)$
$U(k_3)$

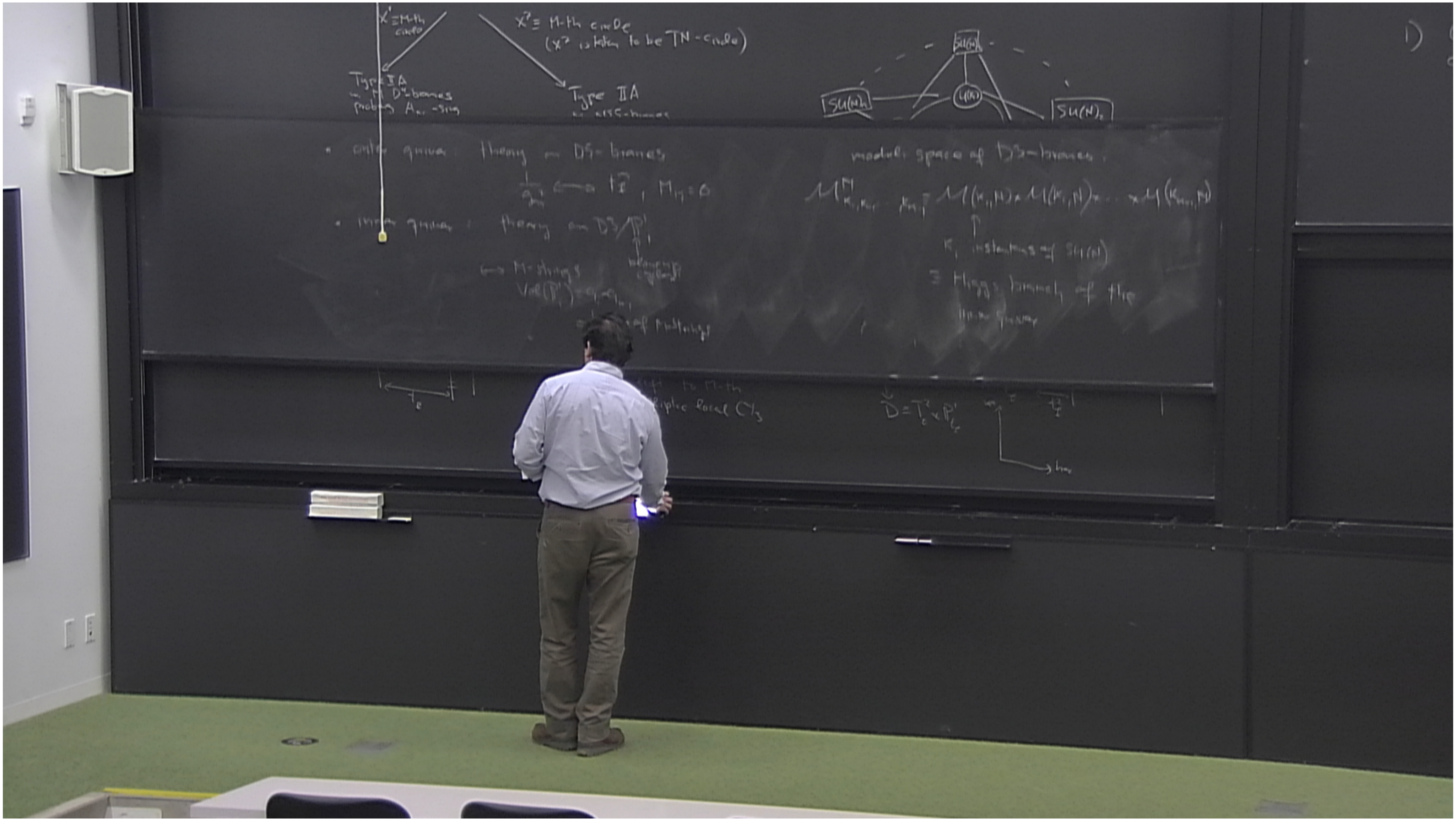

N D5-branes → "dual gauge th."
on D5-branes
moduli space of D3-branes .
blowup cycles
= tensions of M-strings

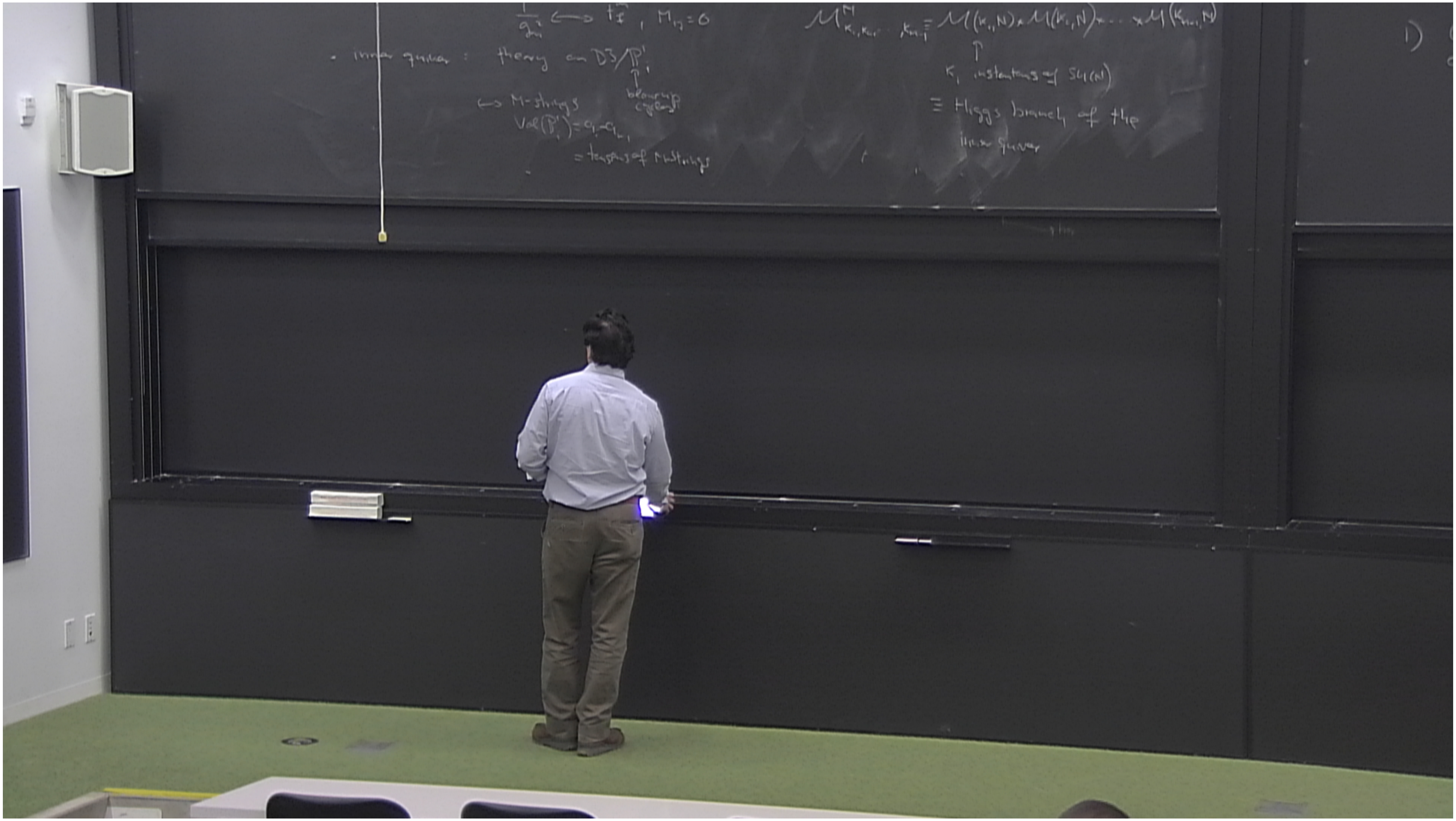

N D5-branes → "dual gauge th."
probing A_{M-1}-sing.
on D5-branes
$\frac{1}{g_m^2} \longleftrightarrow t_f^m$, $M_{ij} = 0$
on D3/$\mathbb{P}^1_i$
blowup cycles $\mathbb{P}^1_i$
M-strings
$Vol(\mathbb{P}^1_i) = a_i - a_{i-1}$
= tensions of M-strings
moduli space of D3-branes:
$\mathcal{M}^M_{k_1, \ldots, k_{M-1}} \equiv \mathcal{M}(k_1, N) \times \mathcal{M}(k_2, N) \times \cdots \times \mathcal{M}(k_{M-1}, N)$
k_1 instantons of SU(N)
≡ Higgs branch of the
inner quiver

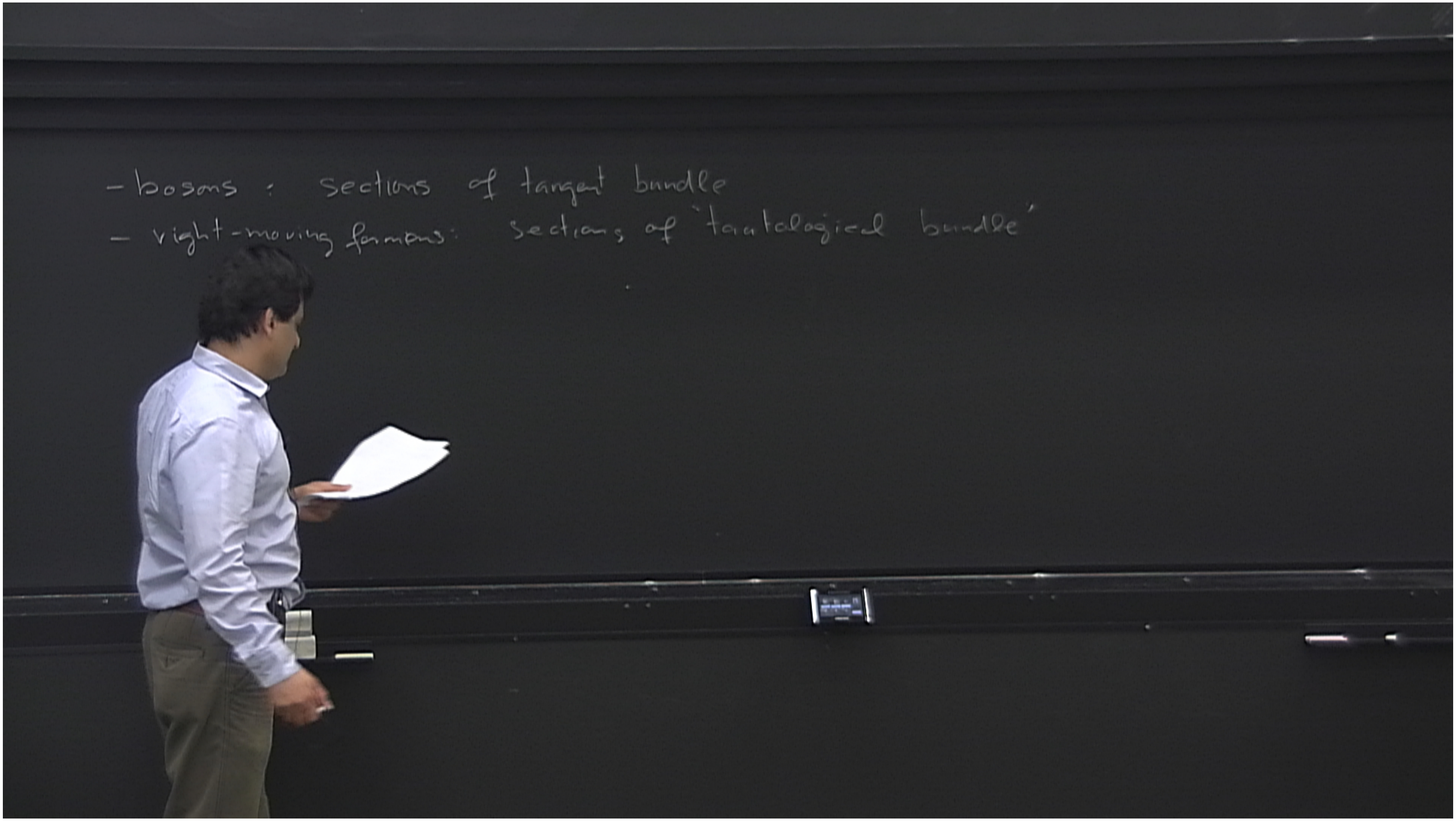

specify to N=1

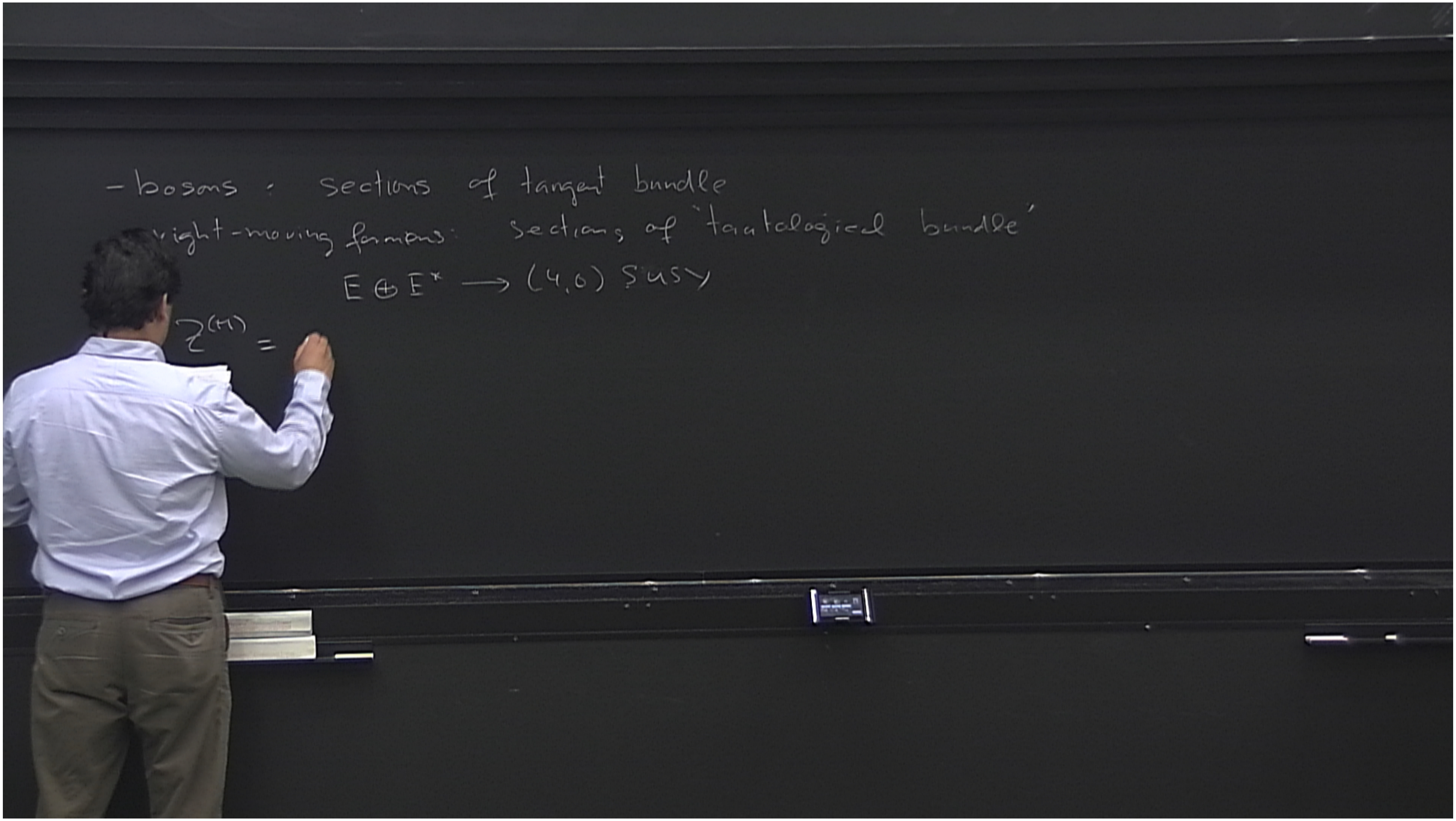

X⁰, X¹, X², …, X⁵
specify to N=1: moduli space of M-strings
is that of k U(1)-instantons
→ (4,6) sigma
(for M=2):
T²

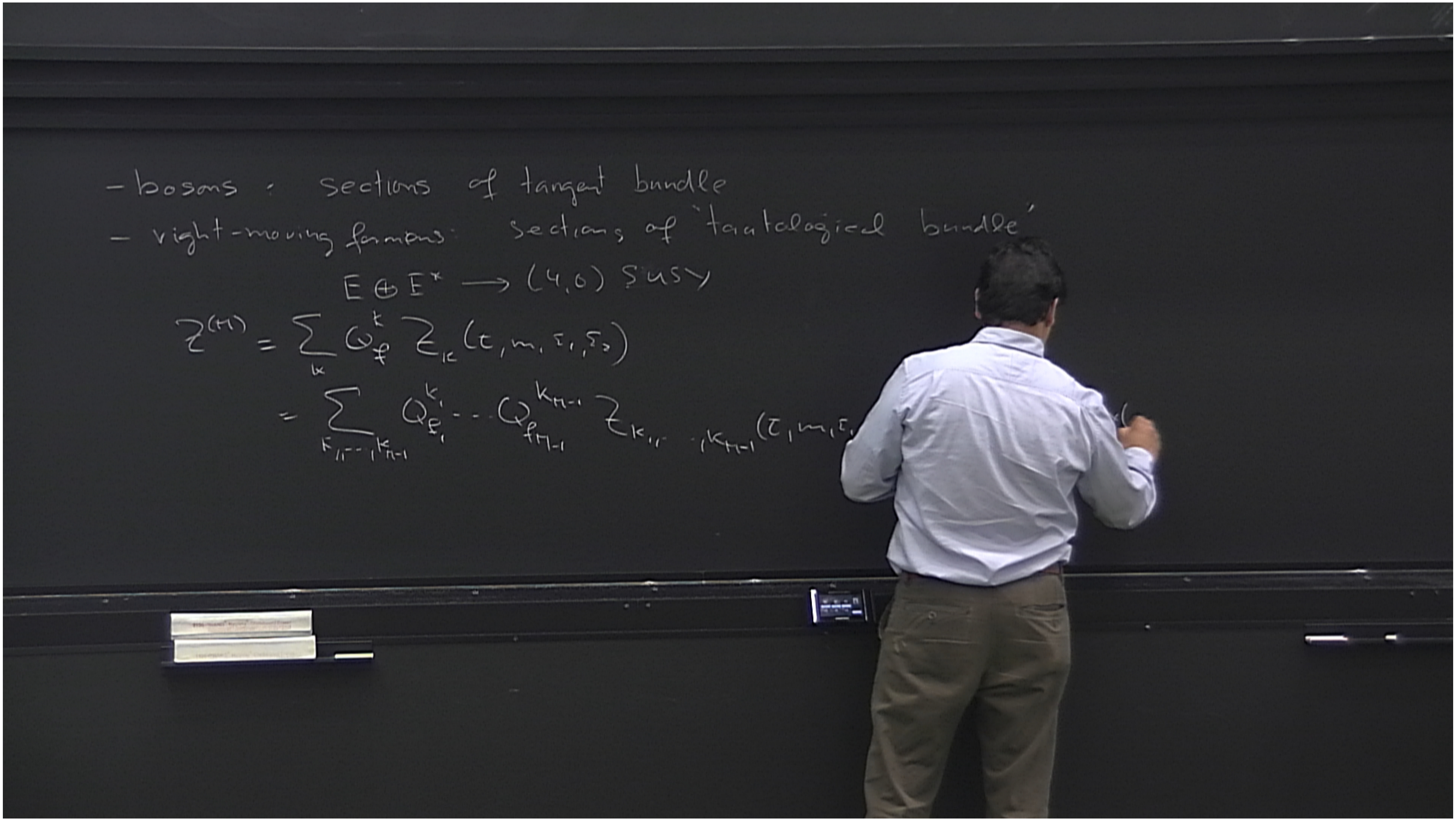

$X^0, X^1, X^2, \ldots, X^5$
specify to N=1 : moduli space of M-strings
is that of k U(1)-instantons
→ (4,6) sigma model (for M=2):
$T^2 \longrightarrow \mathcal{M}(k,1) = \mathrm{Hilb}^{[k]}$


Type IIA
Type IIA
moduli space of D3-branes
instantons of SU(N)


theory on D3/$\mathbb{P}^1$
M-strings
$\equiv$ Higgs branch of the


- bosons : sections of tangent bundle
- right-moving fermions: sections of 'tautological bundle'


- bosons : sections of tangent bundle
right-moving fermions: sections of 'tautological bundle'
E ⊕ E* ⟶ (4,0) susy
Z^(H) =


- bosons : sections of tangent bundle
- right-moving fermions: sections of 'tautological bundle'
$E \oplus E^* \longrightarrow (4,0)$ SUSY
$Z^{(M)} = \sum_k Q_f^k Z_k(\tau, m, \epsilon_1, \epsilon_2)$
$= \sum_{k_1, \dots, k_{M-1}} Q_{f_1}^{k_1} \cdots Q_{f_{M-1}}^{k_{M-1}} Z_{k_1, \dots, k_{M-1}}(\tau, m, \epsilon,$

- bosons : sections of tangent bundle
- right-moving fermions: sections of 'tautological bundle'

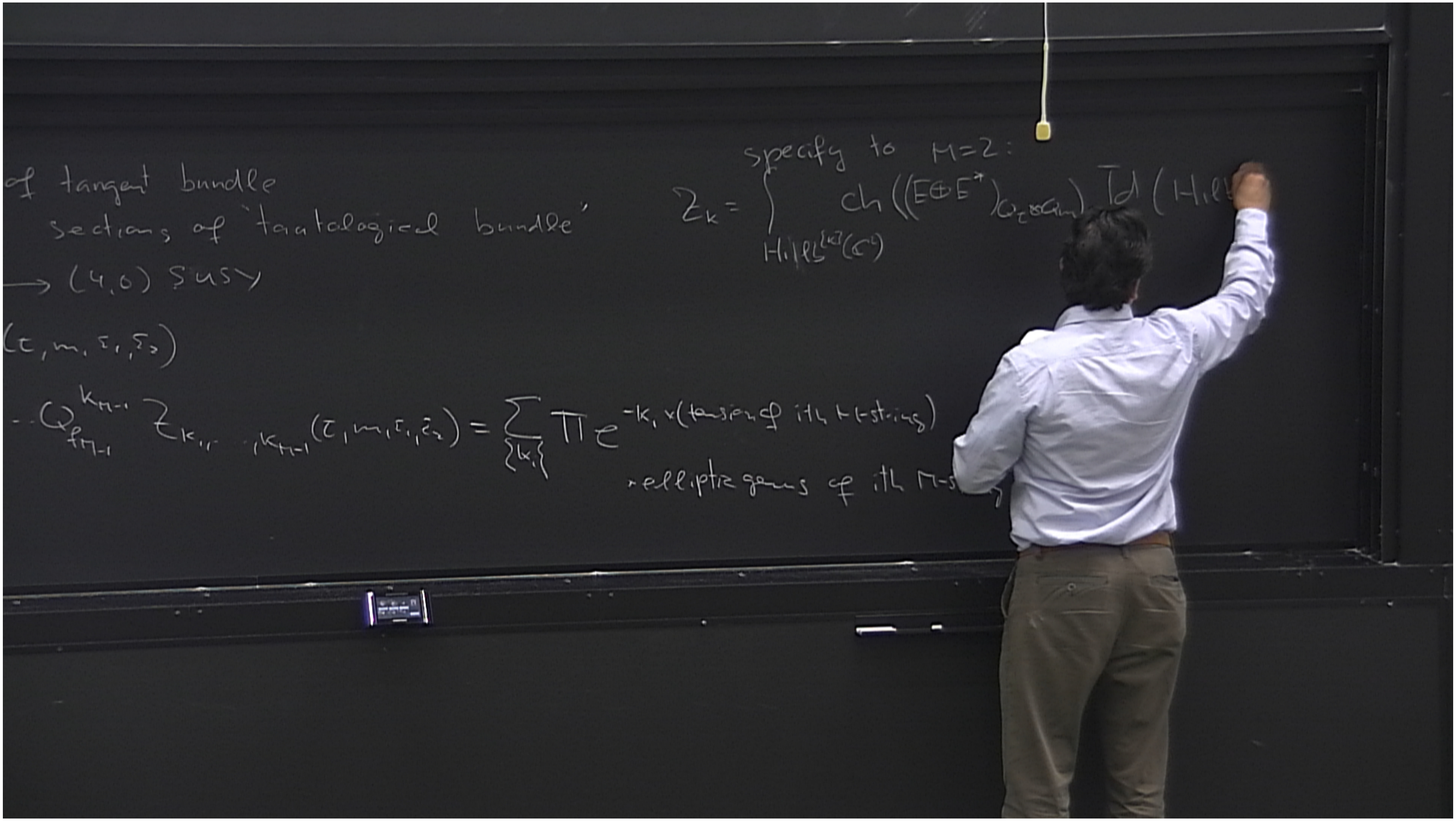
$$E \oplus E^* \longrightarrow (4,0) \text{ susy}$$

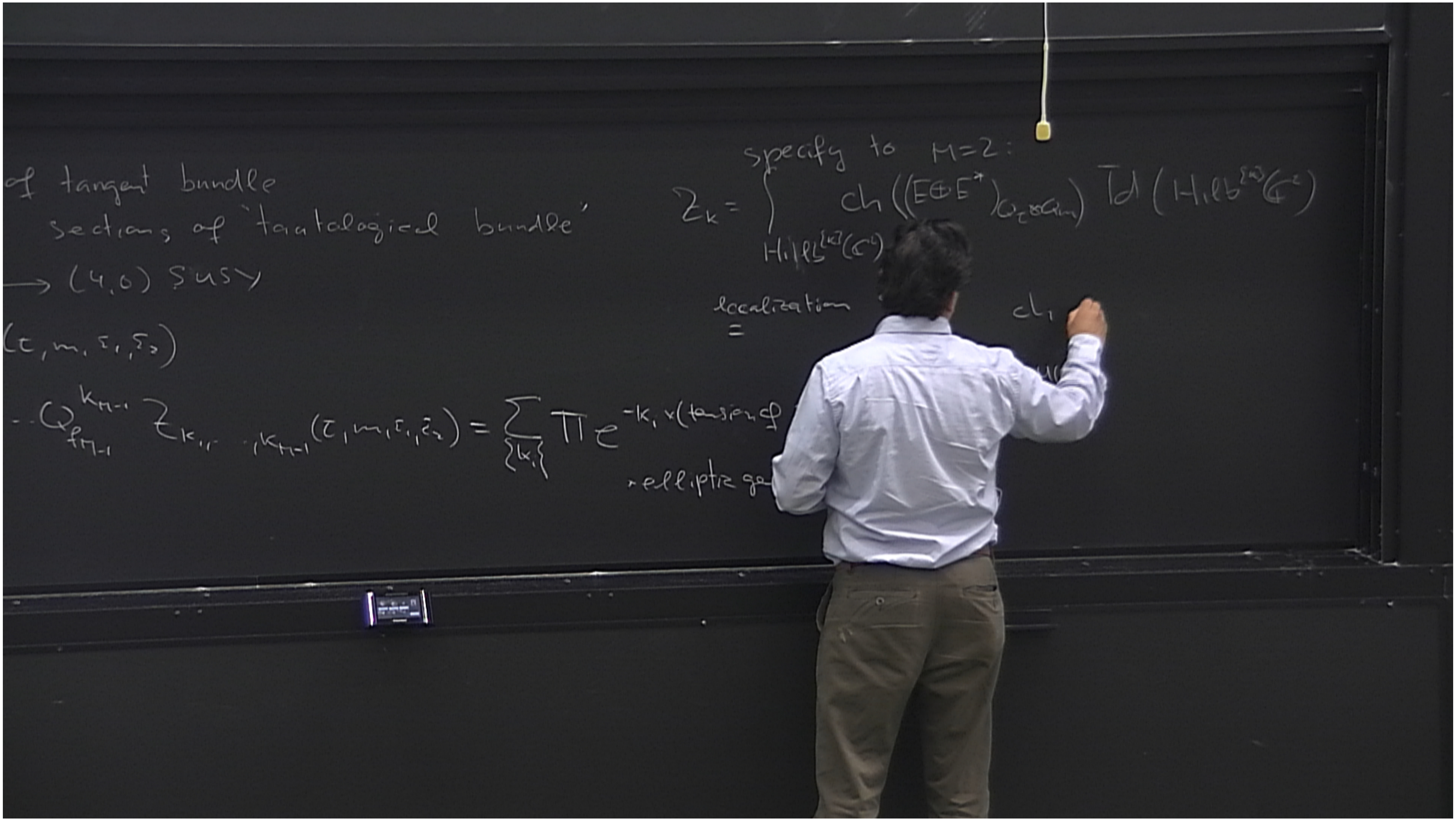
$$Z^{(M)} = \sum_k Q_f^k \, Z_k(\tau, m, \epsilon_1, \epsilon_2)$$

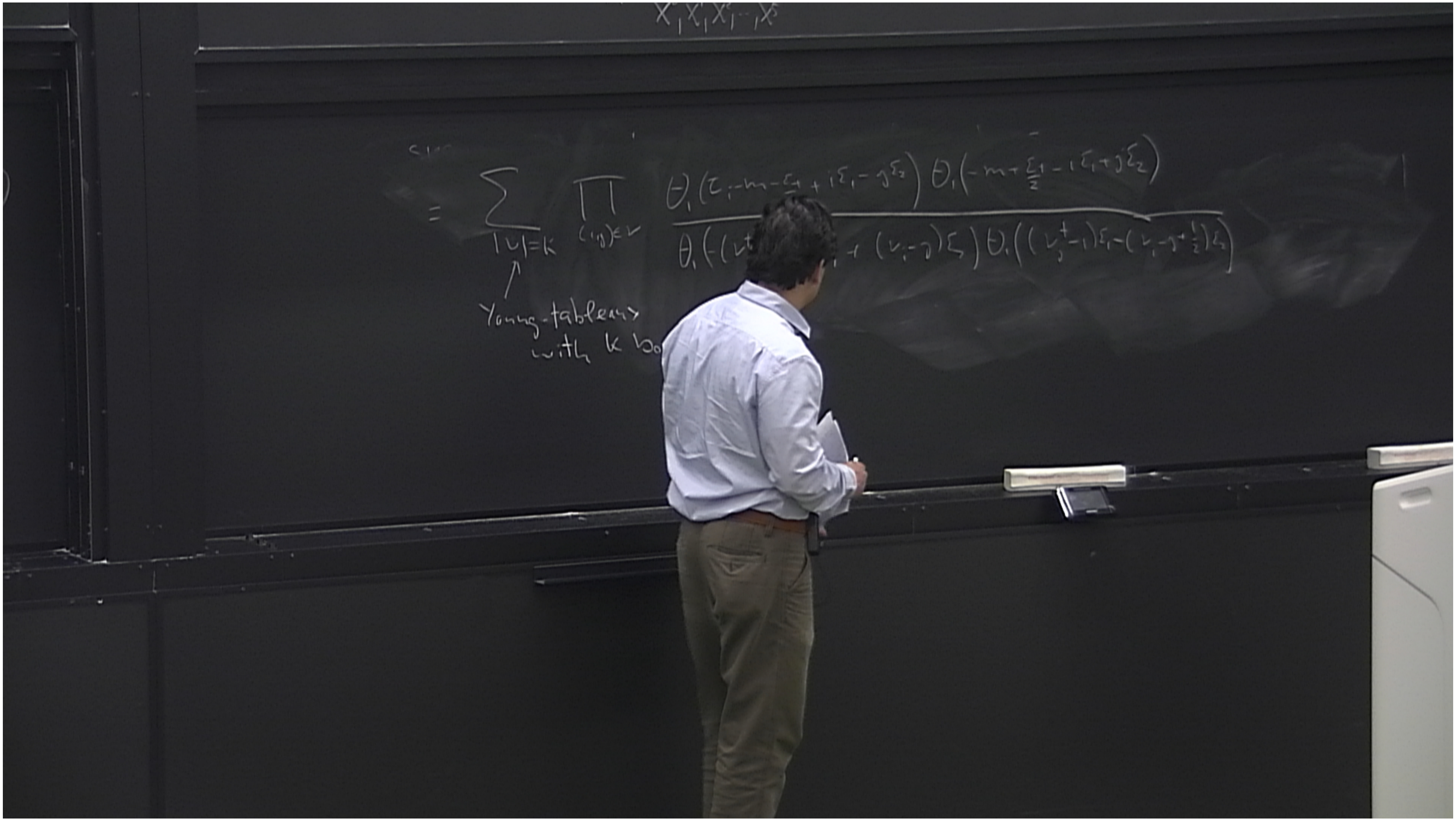
$$= \sum_{k_1,\dots,k_{M-1}} Q_{f_1}^{k_1} \cdots Q_{f_{M-1}}^{k_{M-1}} \, Z_{k_1,\dots,k_{M-1}}(\tau, m, \epsilon_1, \epsilon_2) = \sum_{\{k_i\}} \prod e \dots$$

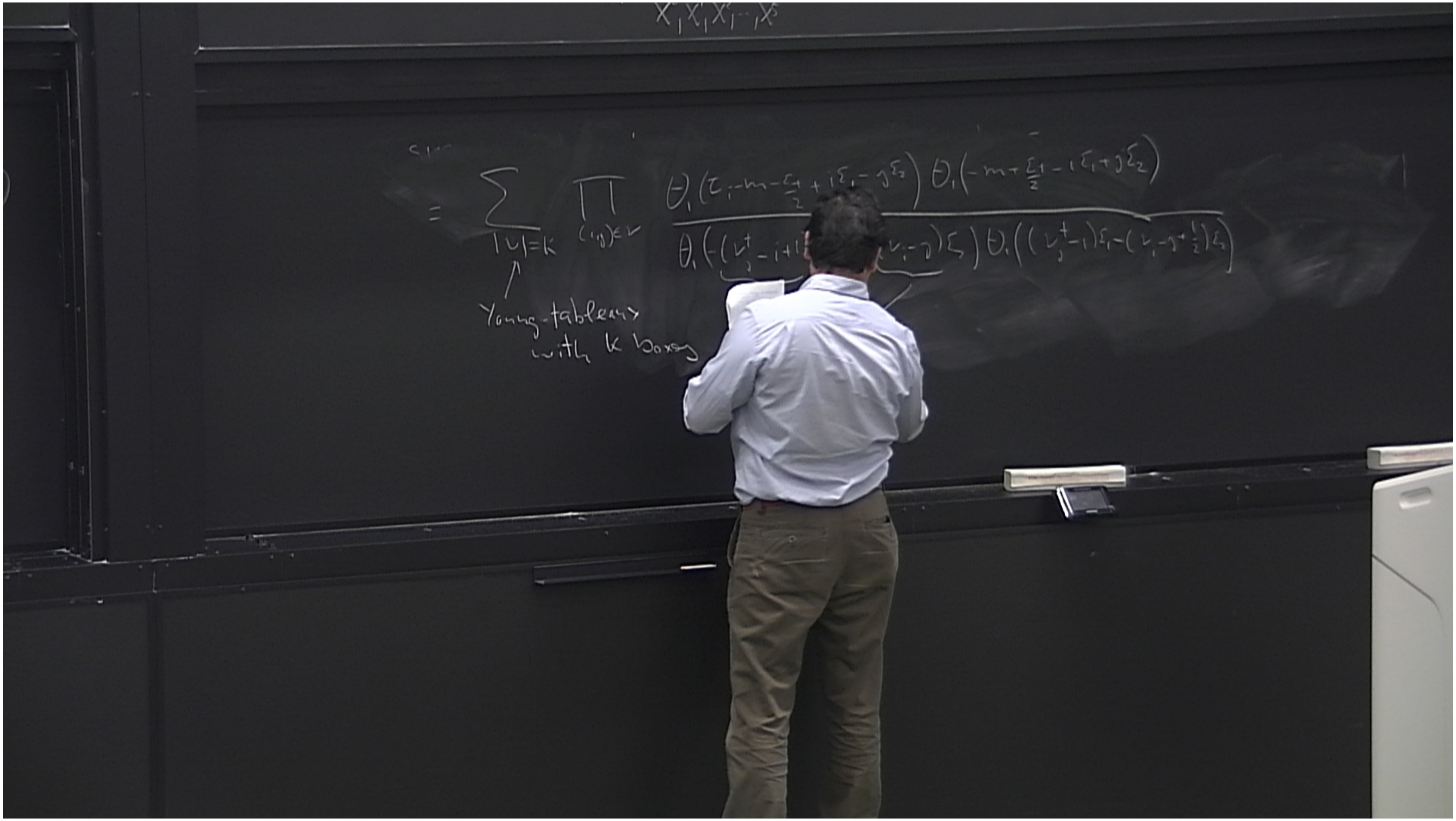
-strings)

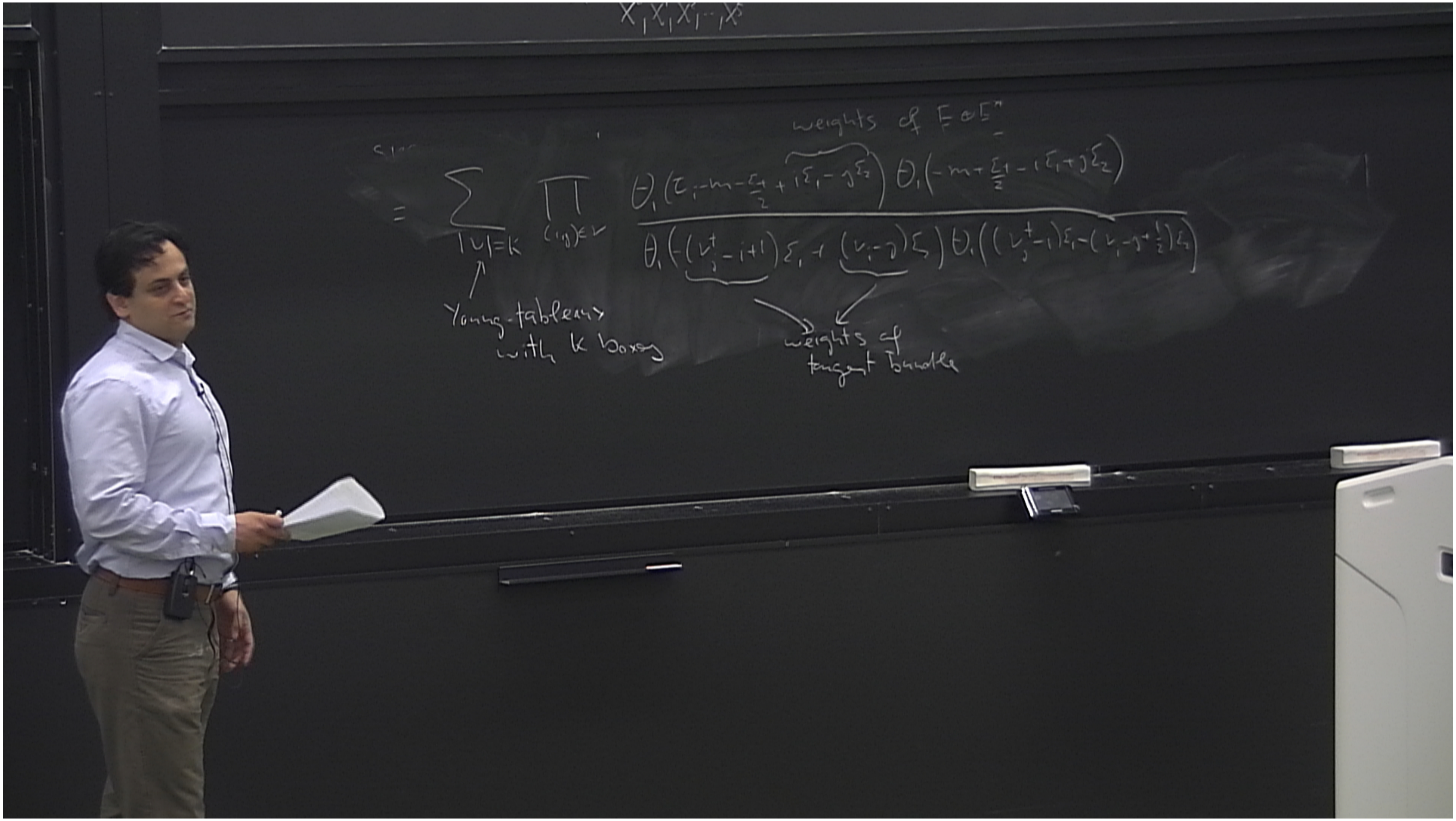
ith M-strin


of tangent bundle
sections of 'tautological bundle'
→ (4,0) susy
(τ, m, ε1, ε2)
specify to M=2:
Zk = ∫ ch((E⊕E*)
Hilb[k](C2)
= Σ Π e−ki × (tension of ith M-string)
{ki}
× elliptic genus of ith M-s


of tangent bundle
sections of 'tautological bundle'
(4,0) SUSY
specify to M=2:
localization


Young-tableaux
with k


Young-tableaux
with k boxes


weights of E ⊕ E*
Young-tableaux
with k boxes
weights of
tangent bundle