Title: From locally covariant QFT to quantum gravity

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Abstract: Locally covariant quantum field theory (LCQFT) has proven to be a very successful framework for QFT on curved spacetimes. It is natural to ask, how far these ideas can be generalized and if one can learn something about quantum gravity, using LCQFT methods. In particular, one can use the relative Cauchy evolution to formulate the notion of background independence. Recently we have proven that background independence in this sense holds for effective quantum gravity, formulated as a perturbative QFT. Remarkably, the formalism of LCQFT can be extended to structures more general than spacetimes. The essential feature is the presence of the causal structure. An example application would be QFT on causal sets (work in progress).

From locally covariant QFT to quantum gravity

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PI, 15.05.2014

[1] Based on the joint work with Klaus Fredenhagen and Romeo Brunetti

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Difficulties in quantum gravity

- In contrast to QFT on curved spacetimes, in QG the spacetime structure is dynamical. Need for "background independance".

Ways around some of the problems

- **Non-renormalizability**: use Epstein-Glaser renormalization to obtain finite results for any fixed energy scale. Think of the theory as an effective theory. Outlook: use the renormalization group flow equations to look for a UV fixed point (asymptotic safety program).

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- **Dynamical nature of spacetime**: make a tentative split of the metric into background and perturbation, quantize the perturbation as a quantum field on a curved background, show background independence at the end.

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- **Dynamical nature of spacetime**: make a tentative split of the metric into background and perturbation, quantize the perturbation as a quantum field on a curved background, show background independence at the end.
- **Diffeomorphism invariance**: use the BV formalism to do the gauge fixing. Possible difficulties: base manifold is Lorentzian and non-compact, symmetry group is infinite dimensional, so is the configuration space.

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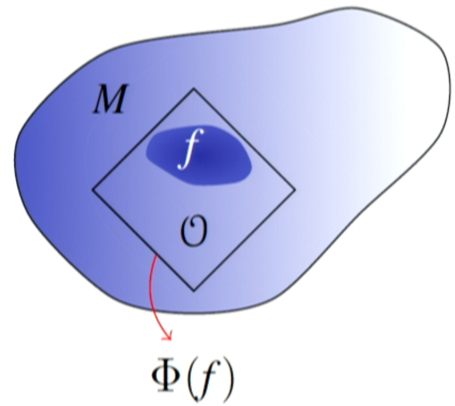
  - Some region $\mathcal{O}$ of spacetime where the measurement is performed,
  - An observable Φ, which we measure,
  - We don't measure the observable curvature at a point, but we have some smearing related to the experimantal uncertainty. This is modeled by smearing with a test function f. For example: $\Phi(f) = \int f(x)R(x)$.



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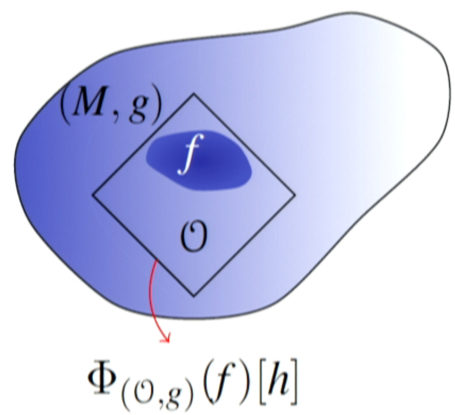
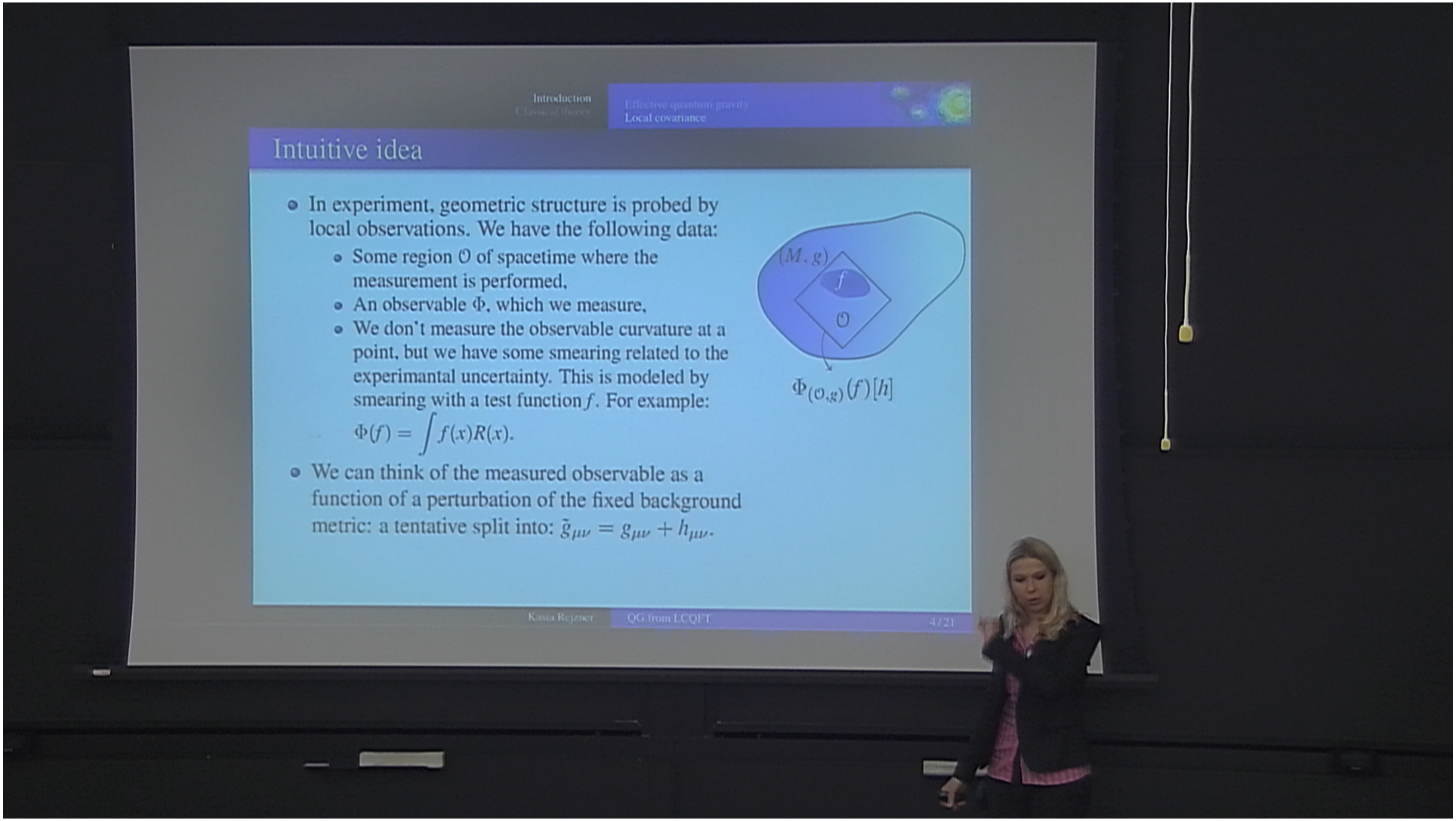
- We can think of the measured observable as a function of a perturbation of the fixed background metric: a tentative split into: $\tilde{g}_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$.

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Effective quantum gravity
Local covariance
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(M, g)
f
$\mathcal{O}$
$\Phi_{(\mathcal{O},g)}(f)[h]$
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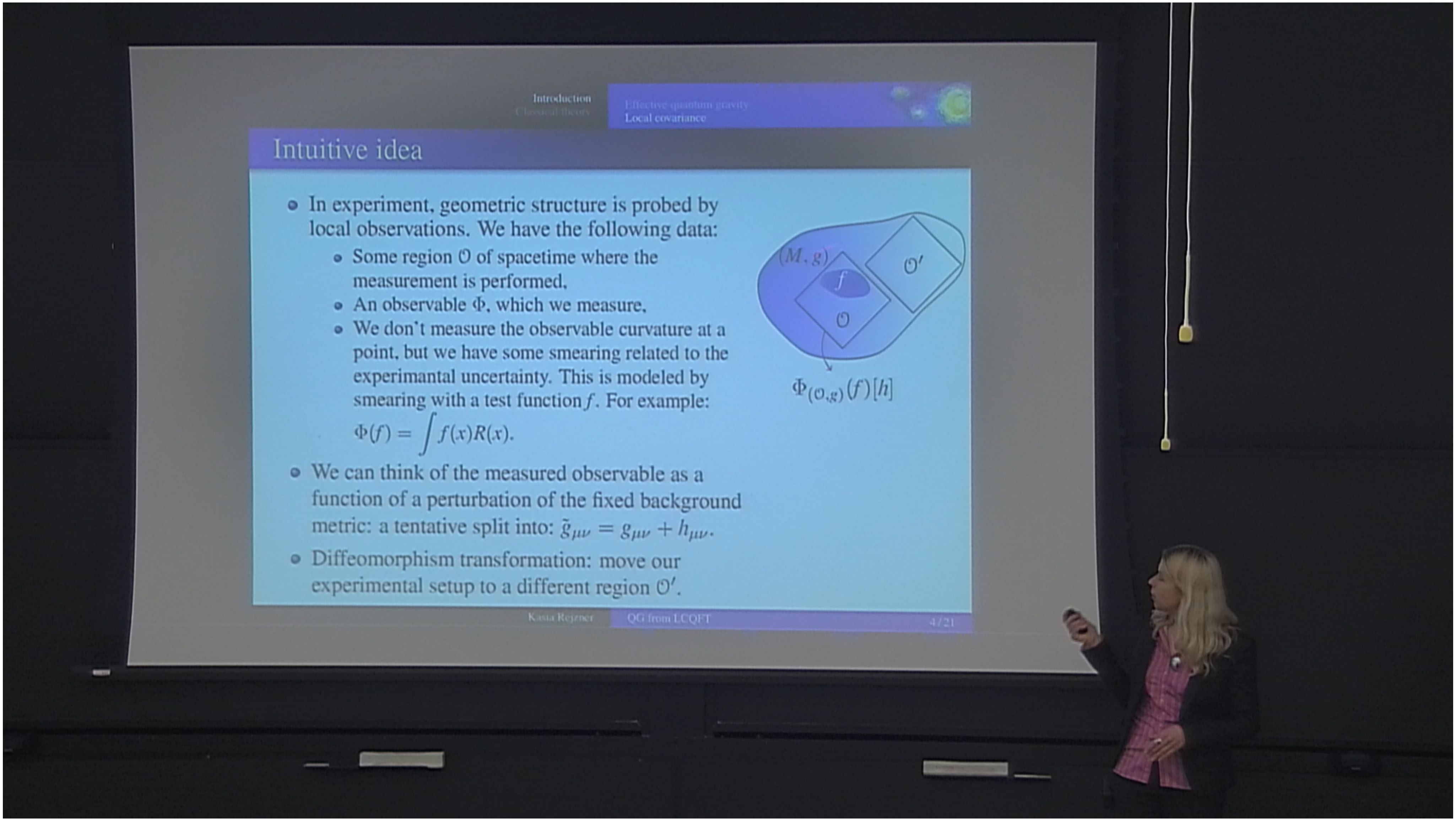
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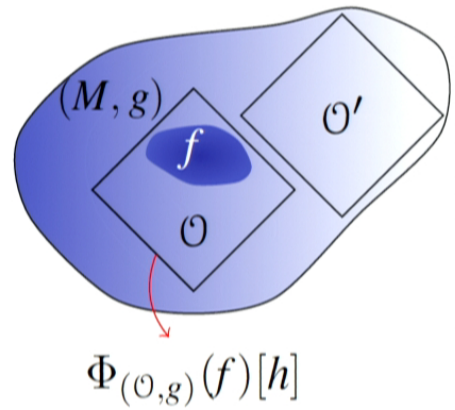

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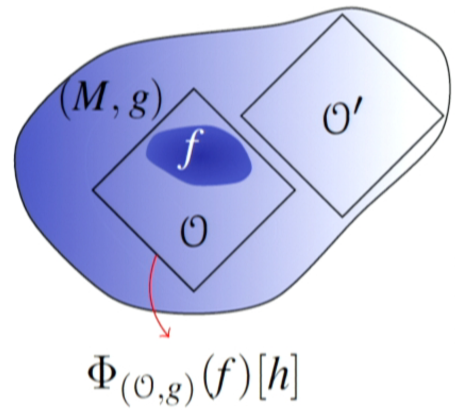

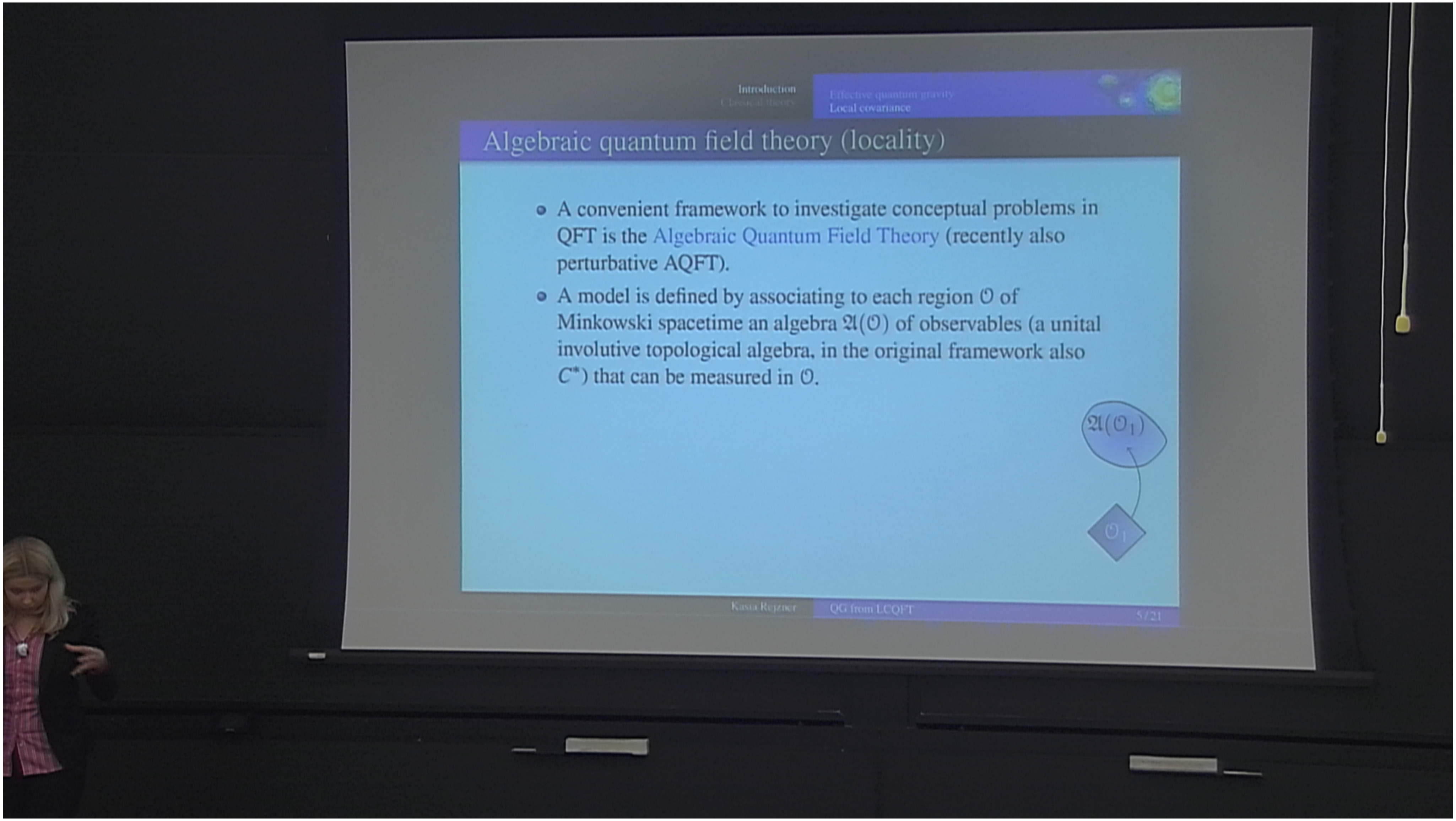

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A convenient framework to investigate conceptual problems in QFT is the Algebraic Quantum Field Theory (recently also perturbative AQFT).
A model is defined by associating to each region $\mathcal{O}$ of Minkowski spacetime an algebra $\mathfrak{A}(\mathcal{O})$ of observables (a unital involutive topological algebra, in the original framework also C^*) that can be measured in $\mathcal{O}$.
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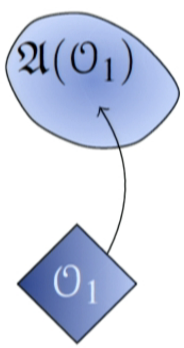

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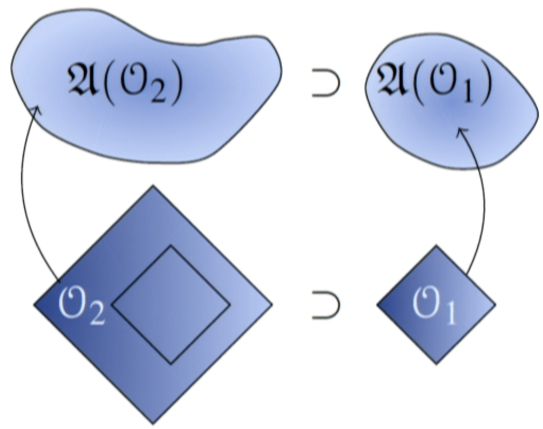

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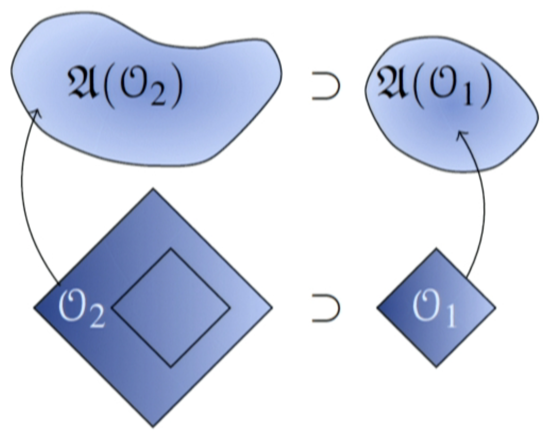

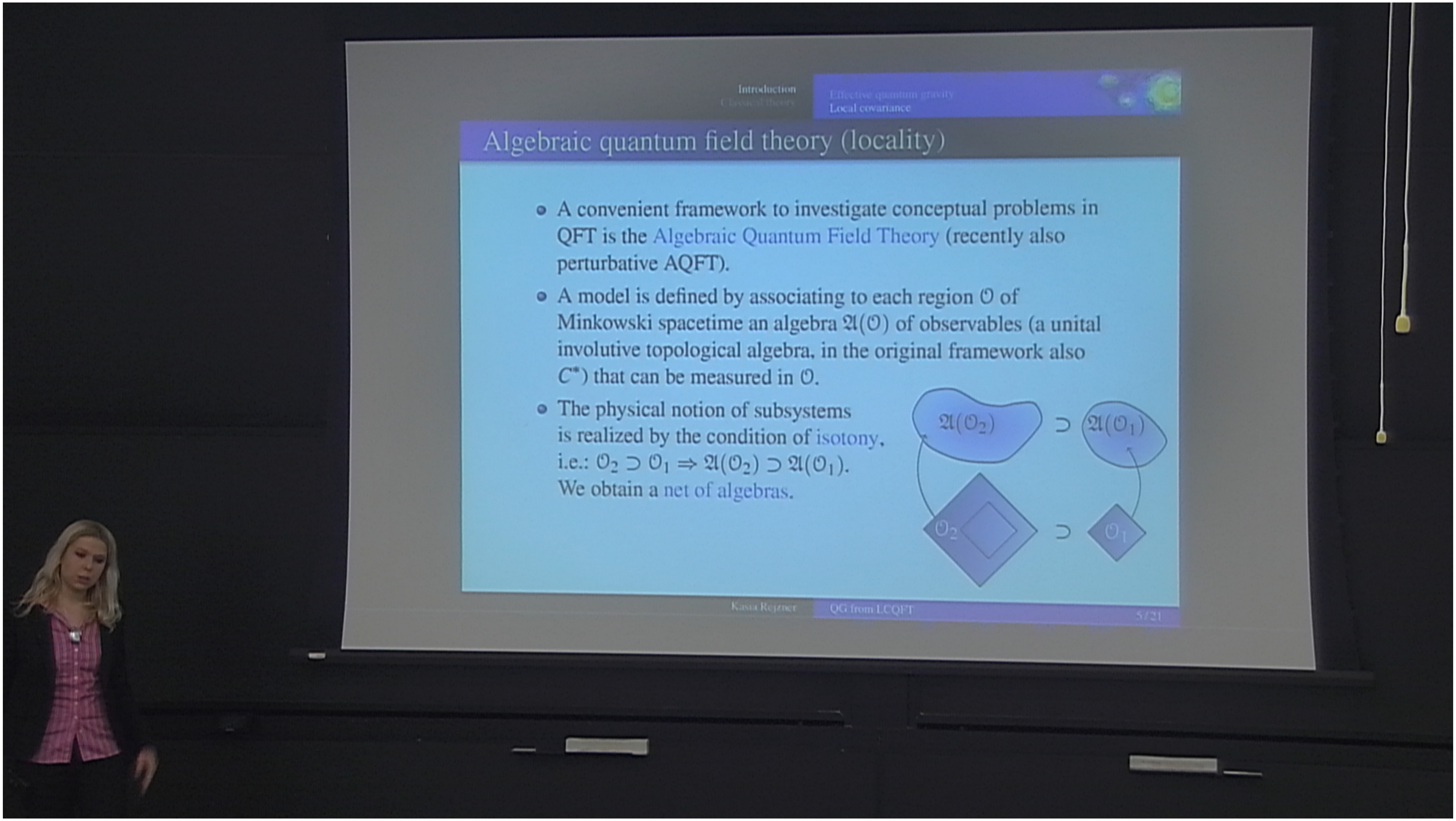
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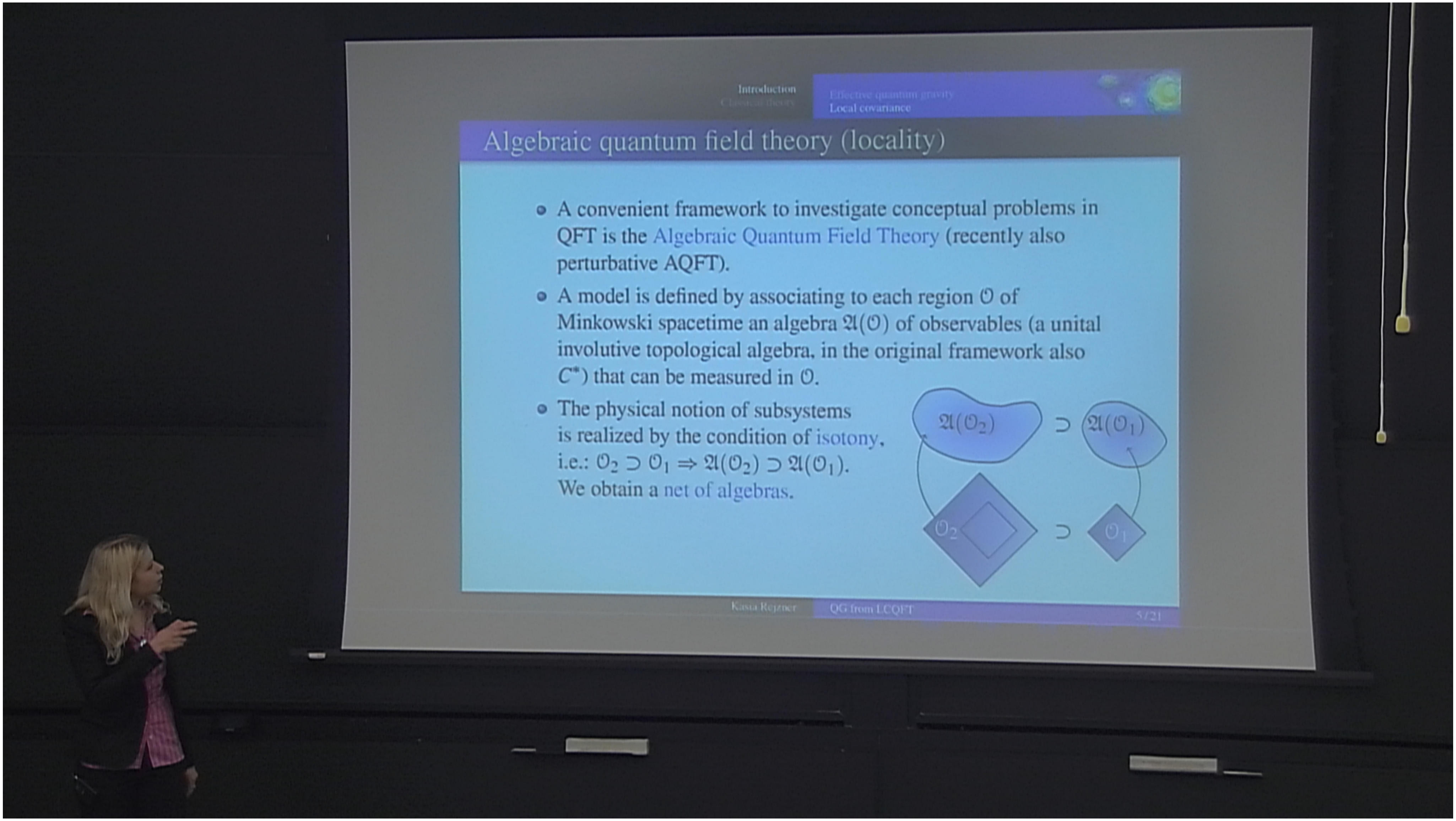
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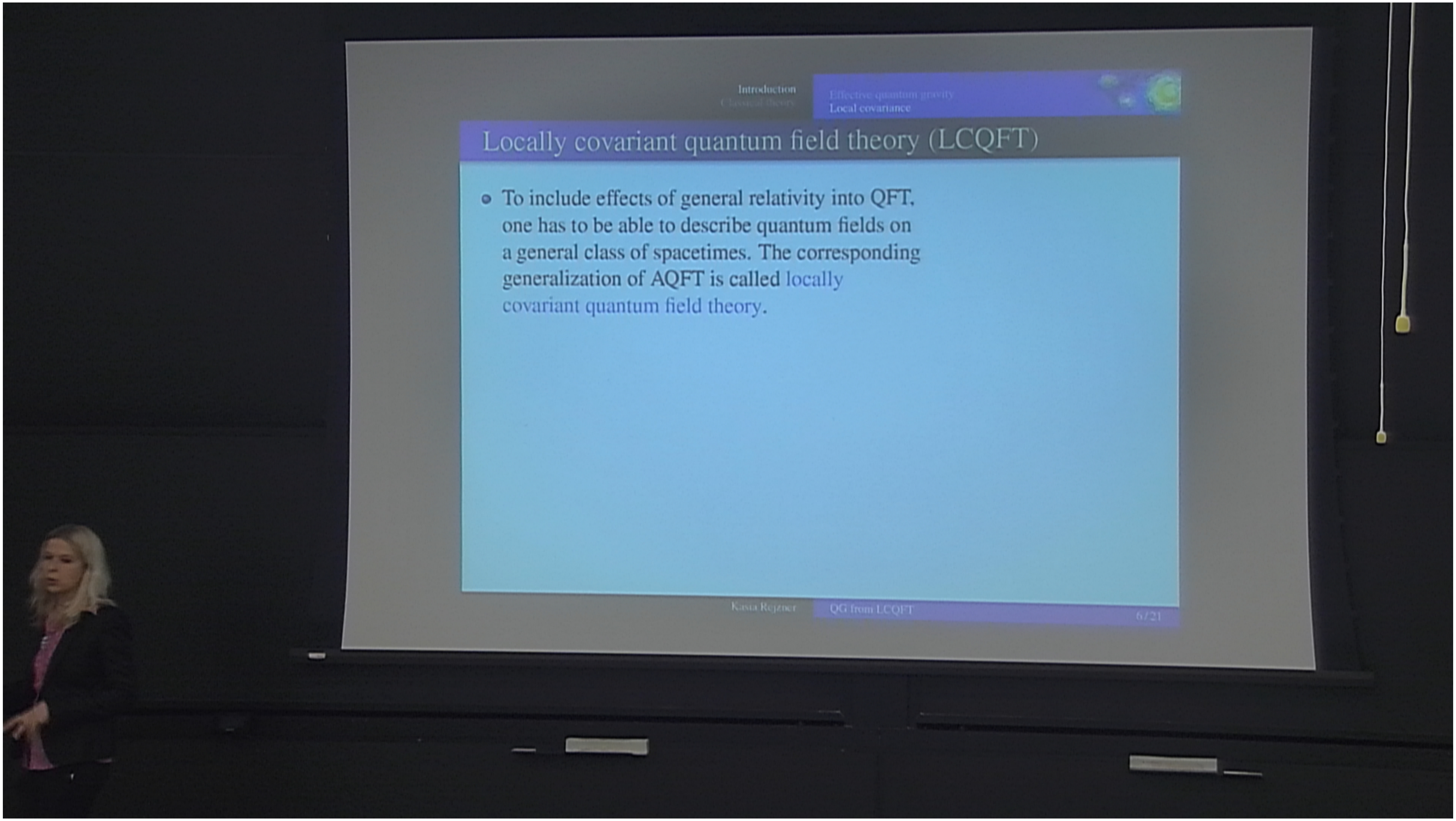
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Locally covariant quantum field theory (LCQFT)
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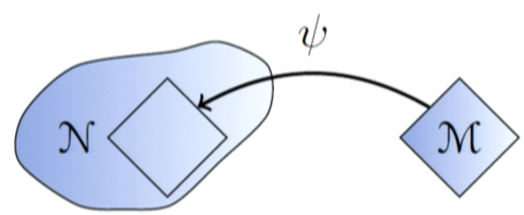
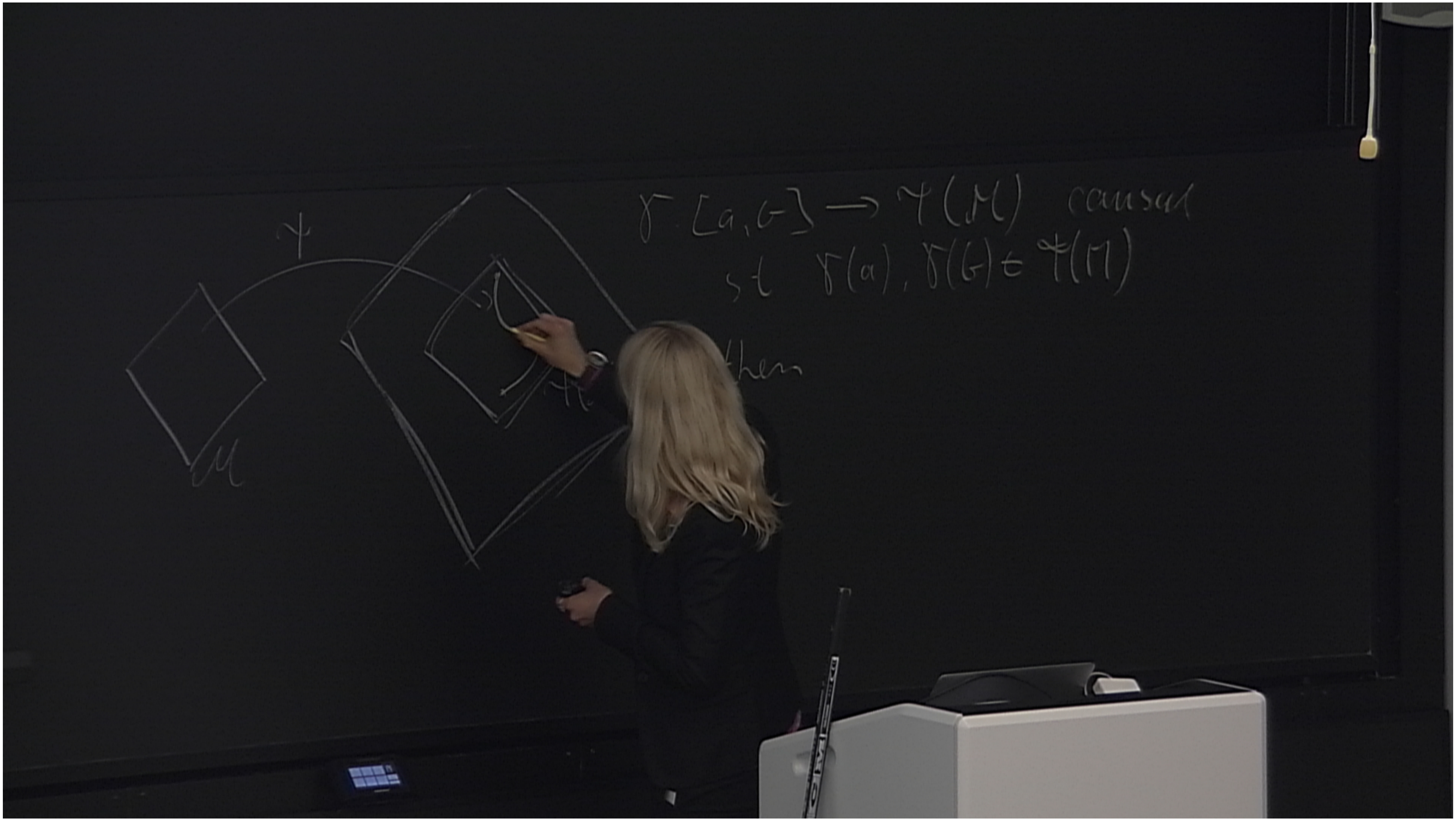
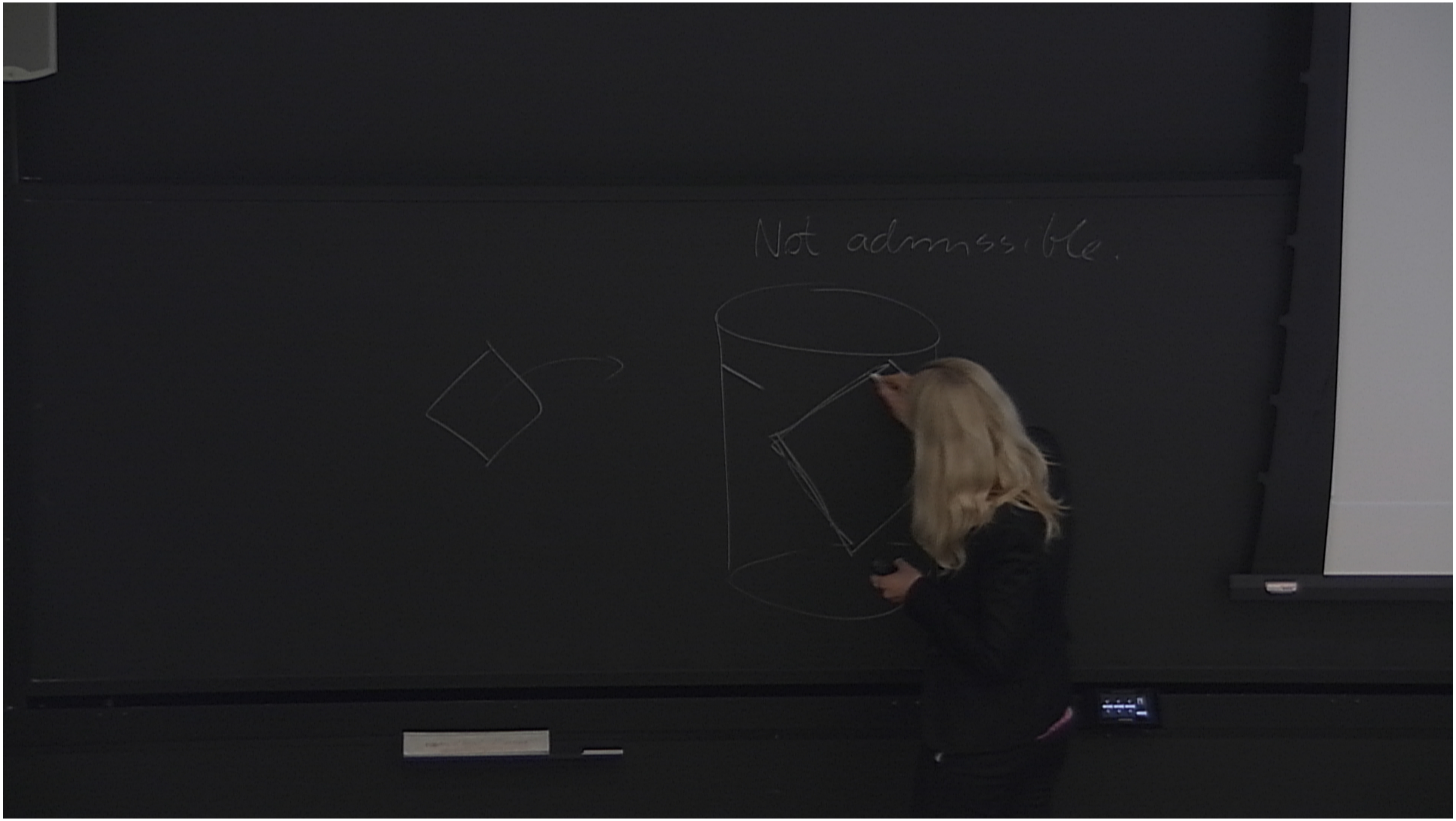
- Consider the class of all globally hyperbolic spacetimes $\mathcal{M} \doteq (M, g)$. An embedding $\psi : \mathcal{M} \to \mathcal{N}$ of such spacetimes is called admissable if it is isometric, orientations preserving and causal.




γ: [a, b] → ψ(M) causal
s.t. γ(a), γ(b) ∈ ψ(M)
then
ψ
M


Not admissible.

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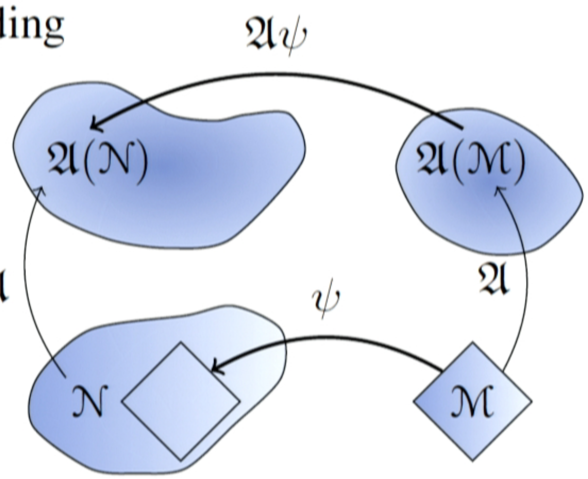
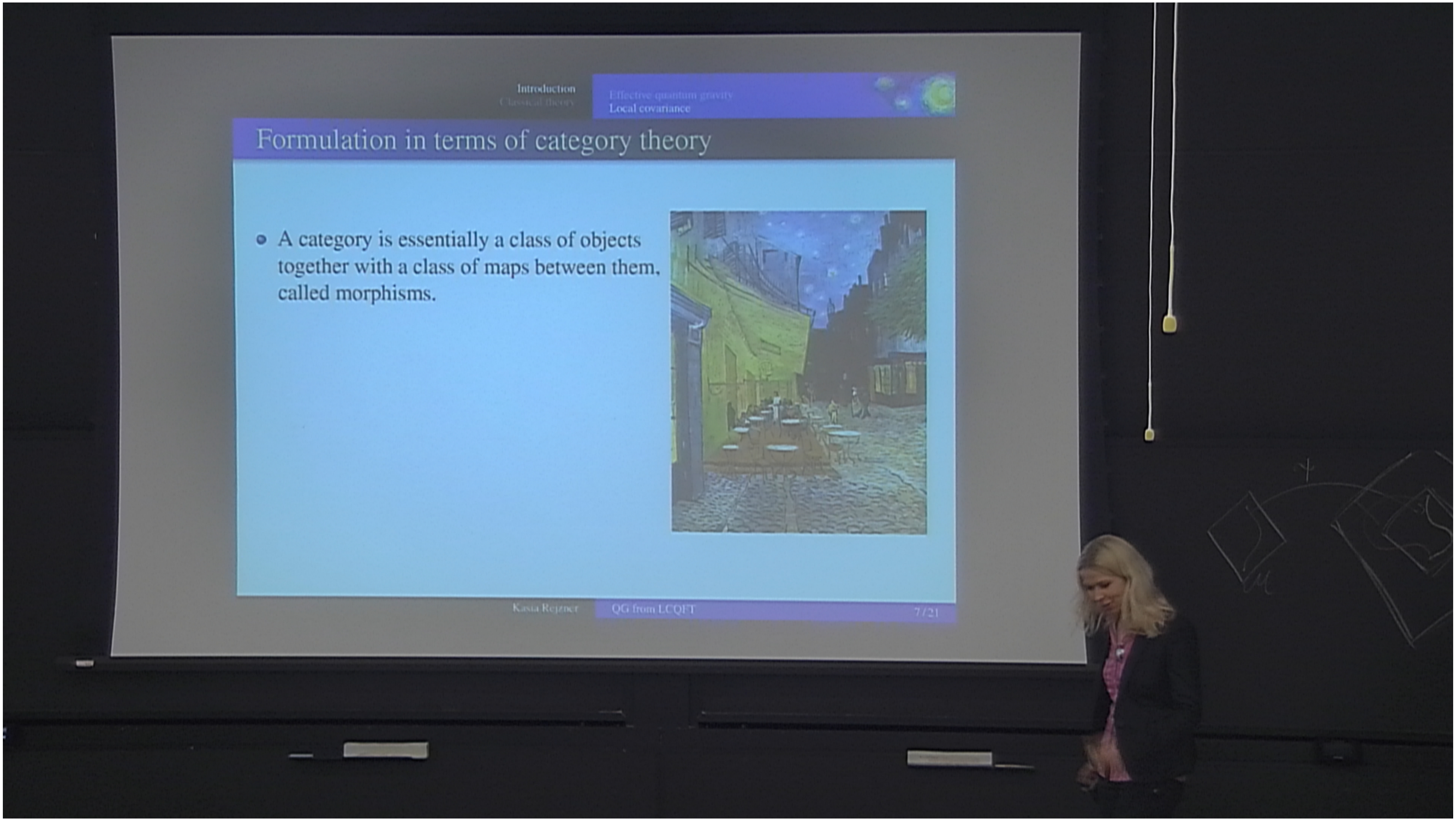
- Consider the class of all globally hyperbolic spacetimes $\mathcal{M} \doteq (M, g)$. An embedding $\psi : \mathcal{M} \to \mathcal{N}$ of such spacetimes is called admissable if it is isometric, orientations preserving and causal.
- A model in LCQFT is defined by assigning to each spacetime $\mathcal{M}$ an algebra $\mathfrak{A}(\mathcal{M})$ and to each admissible embedding ψ an inclusion of algebras α_ψ (notion of subsystems). This has to be done covariantly.





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Formulation in terms of category theory
A category is essentially a class of objects together with a class of maps between them, called morphisms.
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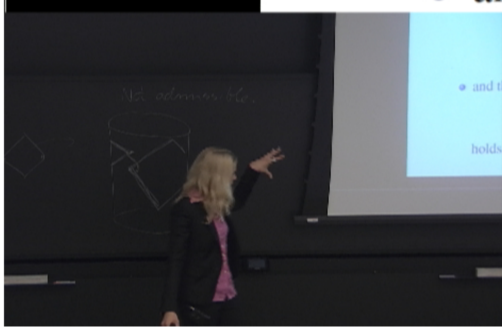
Formulation in terms of category theory

- An LCQFT model is a covariant functor $\mathfrak{A}$ between the category of spacetimes and the category of observables. This means that the following diagram commutes:

$$\begin{array}{ccc} M_1 & \xrightarrow{\psi} & M_2 \\ \mathfrak{A}\downarrow & & \downarrow\mathfrak{A} \\ \mathfrak{A}(M_1) & \xrightarrow{\mathfrak{A}(\psi)} & \mathfrak{A}(M_2) \end{array}$$

- and the covariance property,

$$\alpha_{\psi'} \circ \alpha_{\psi} = \alpha_{\psi' \circ \psi}\,, \quad \alpha_{\mathrm{id}_M} = \mathrm{id}_{\mathfrak{A}(M)}$$



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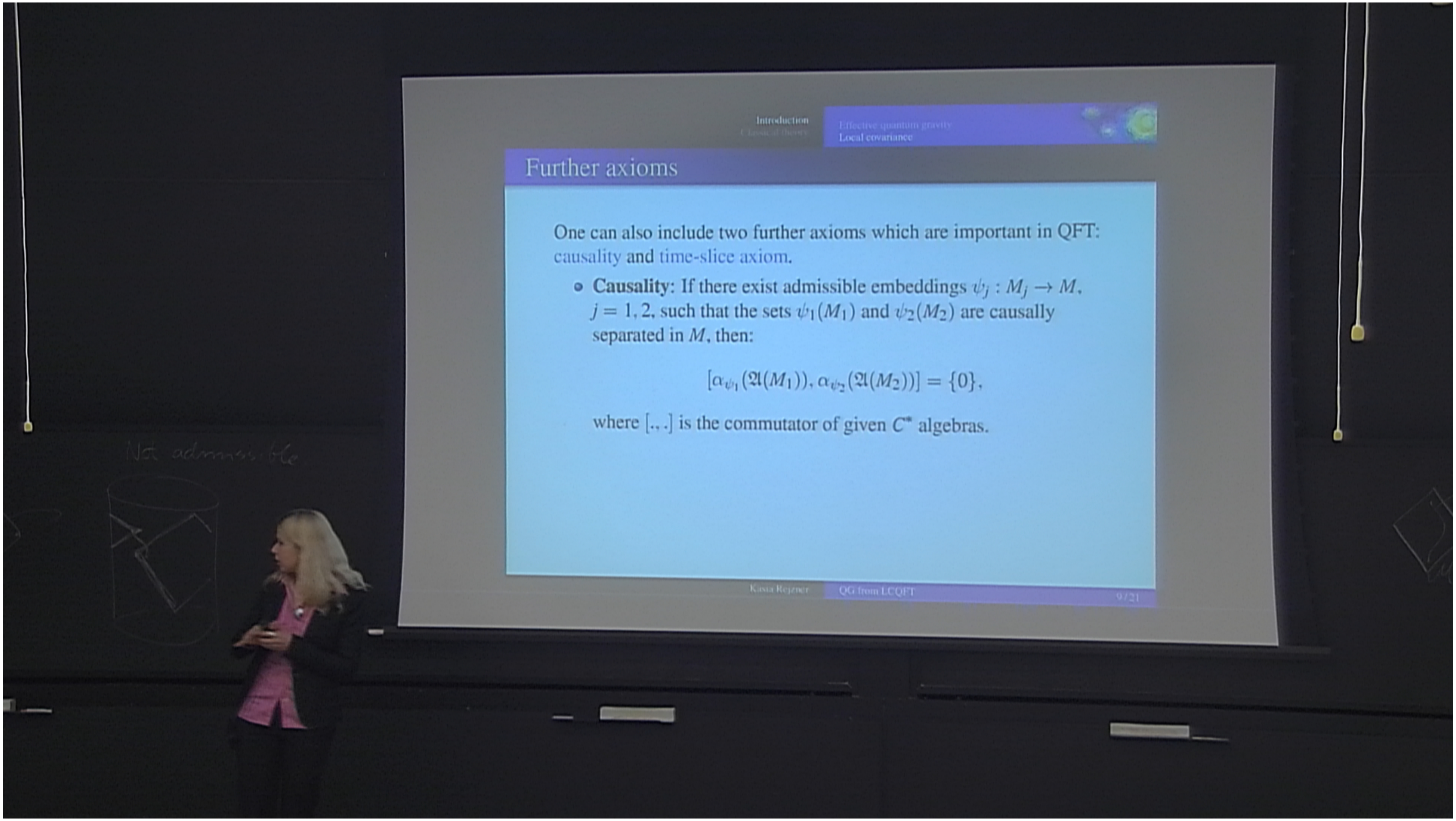
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Further axioms
One can also include two further axioms which are important in QFT: causality and time-slice axiom.
Causality: If there exist admissible embeddings $\psi_j : M_j \to M$, $j = 1, 2$, such that the sets $\psi_1(M_1)$ and $\psi_2(M_2)$ are causally separated in M, then:
$[\alpha_{\psi_1}(\mathfrak{A}(M_1)), \alpha_{\psi_2}(\mathfrak{A}(M_2))] = \{0\}$,
where [., .] is the commutator of given C^* algebras.
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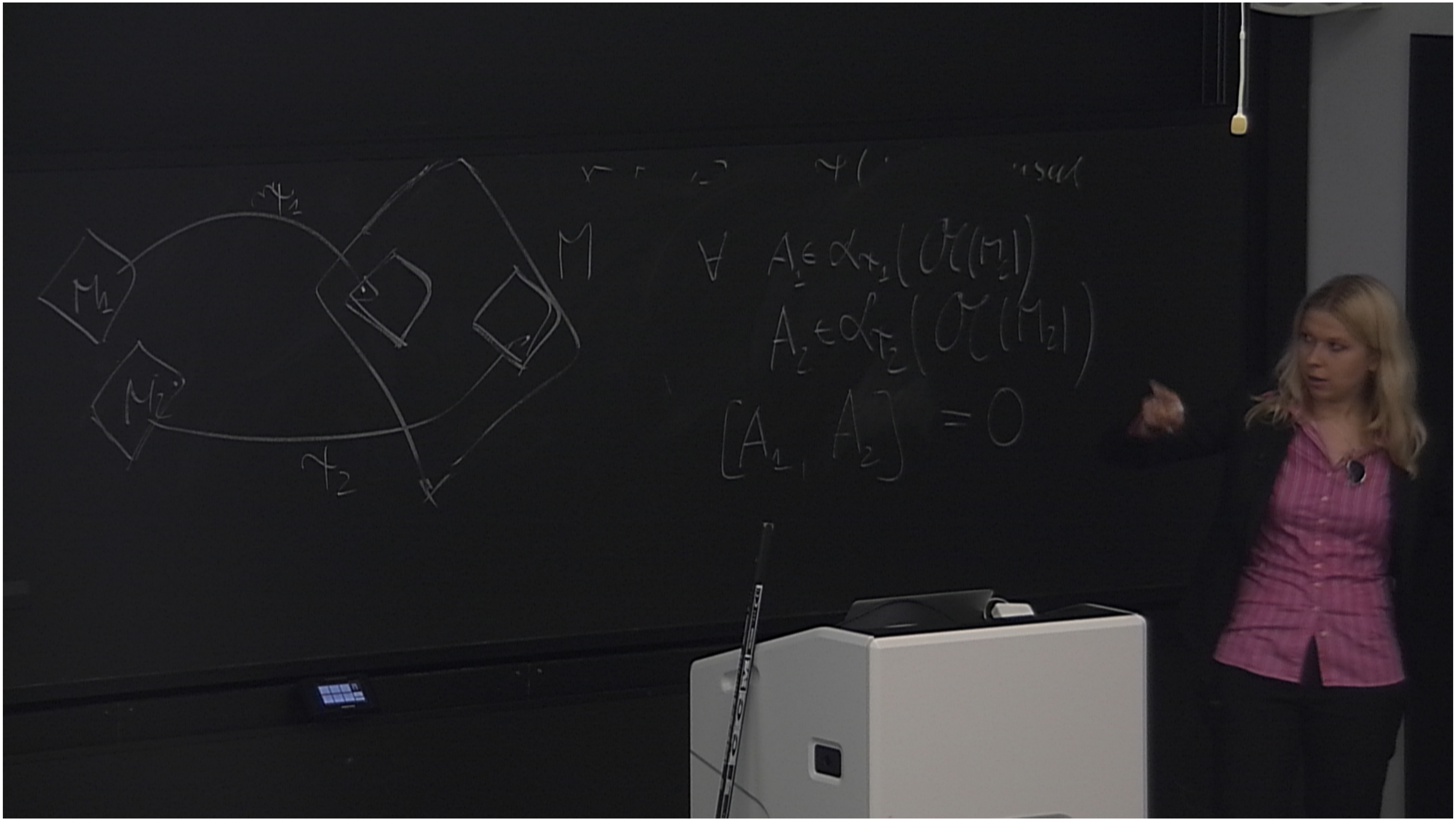
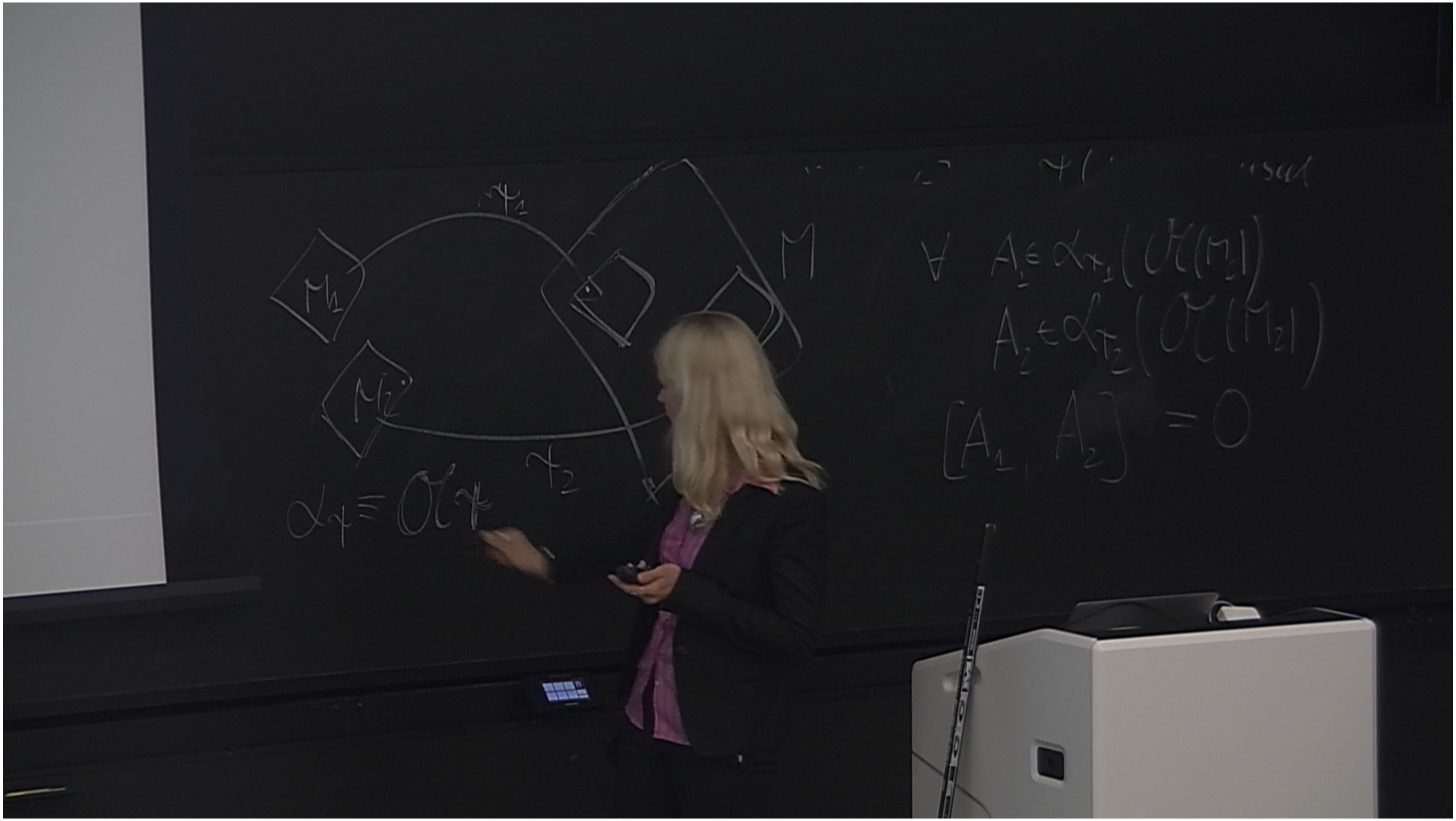

M_1
M_2
τ_1
τ_2
M
$\forall\ A_1 \in \alpha_{\tau_1}(\mathcal{O}(M_1))$
$A_2 \in \alpha_{\tau_2}(\mathcal{O}(M_2))$
$[A_1, A_2] = 0$


M
∀ A1 ∈ ατ1(O(M1))
A2 ∈ ατ2(O(M2))
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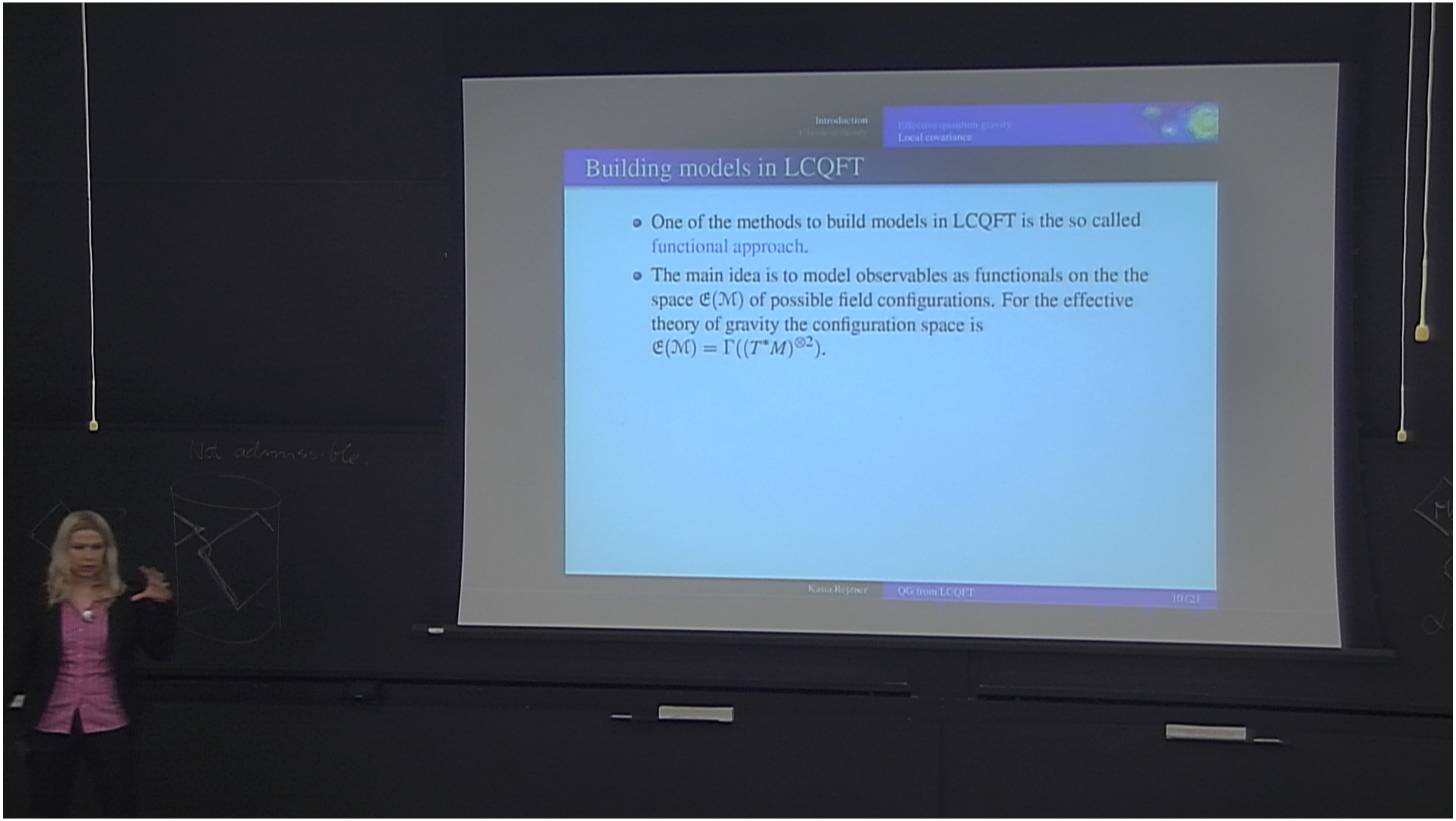
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Building models in LCQFT
One of the methods to build models in LCQFT is the so called functional approach.
The main idea is to model observables as functionals on the the space $\mathfrak{E}(\mathcal{M})$ of possible field configurations. For the effective theory of gravity the configuration space is $\mathfrak{E}(\mathcal{M}) = \Gamma((T^*M)^{\otimes 2})$.
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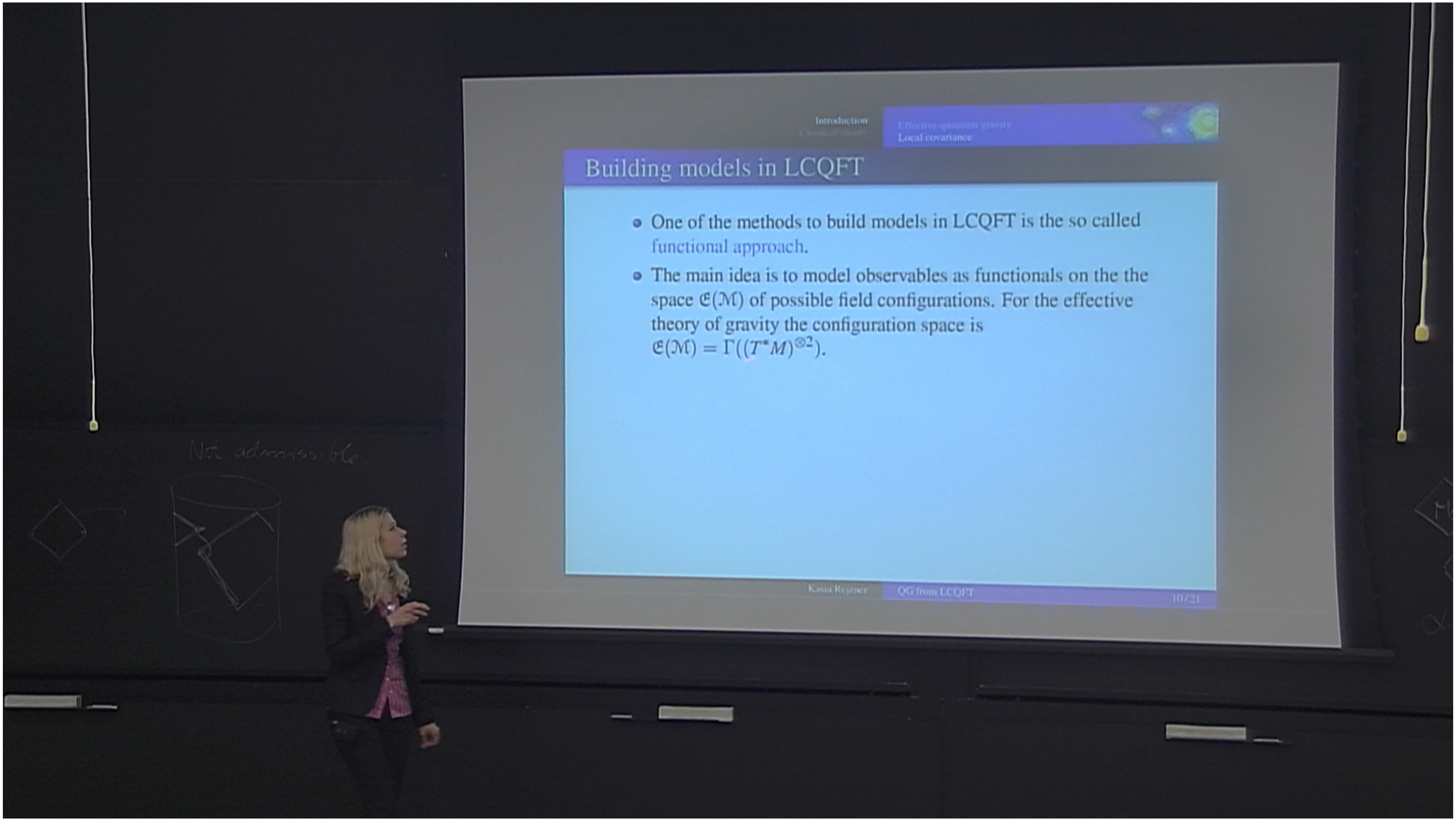
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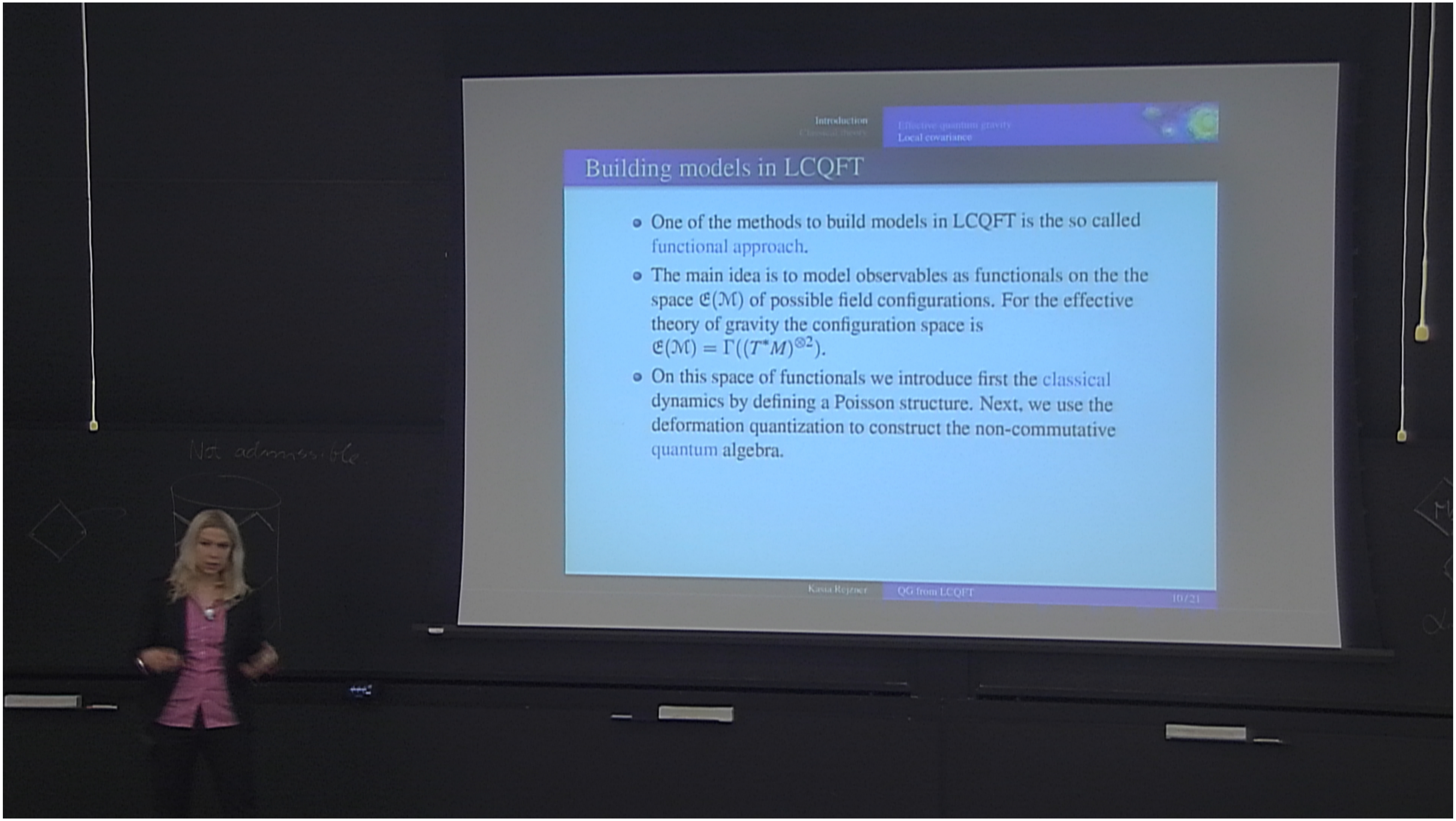
Not admissible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On this space of functionals we introduce first the classical dynamics by defining a Poisson structure. Next, we use the deformation quantization to construct the non-commutative quantum algebra.
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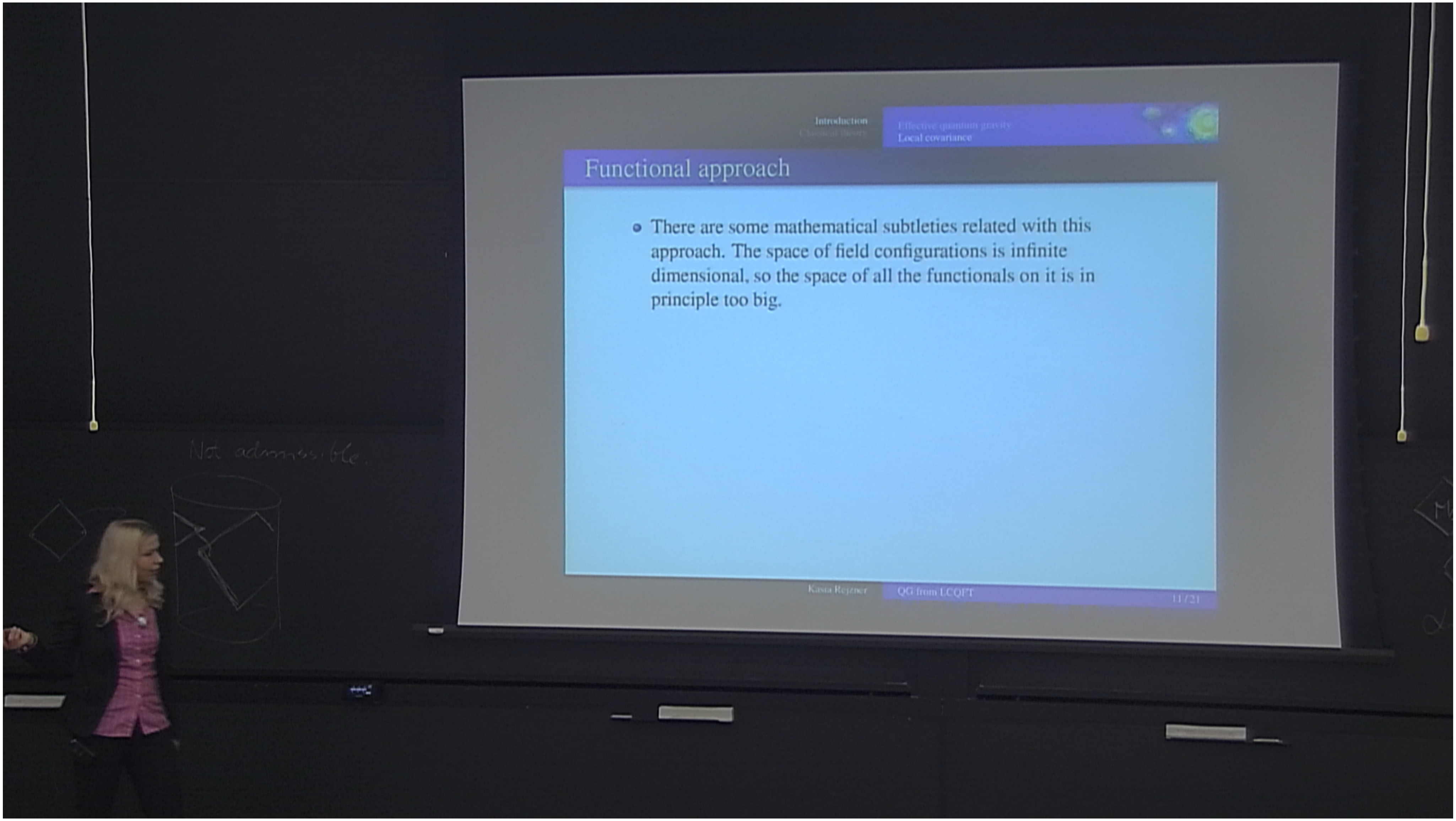
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Functional approach
There are some mathematical subtleties related with this approach. The space of field configurations is infinite dimensional, so the space of all the functionals on it is in principle too big.
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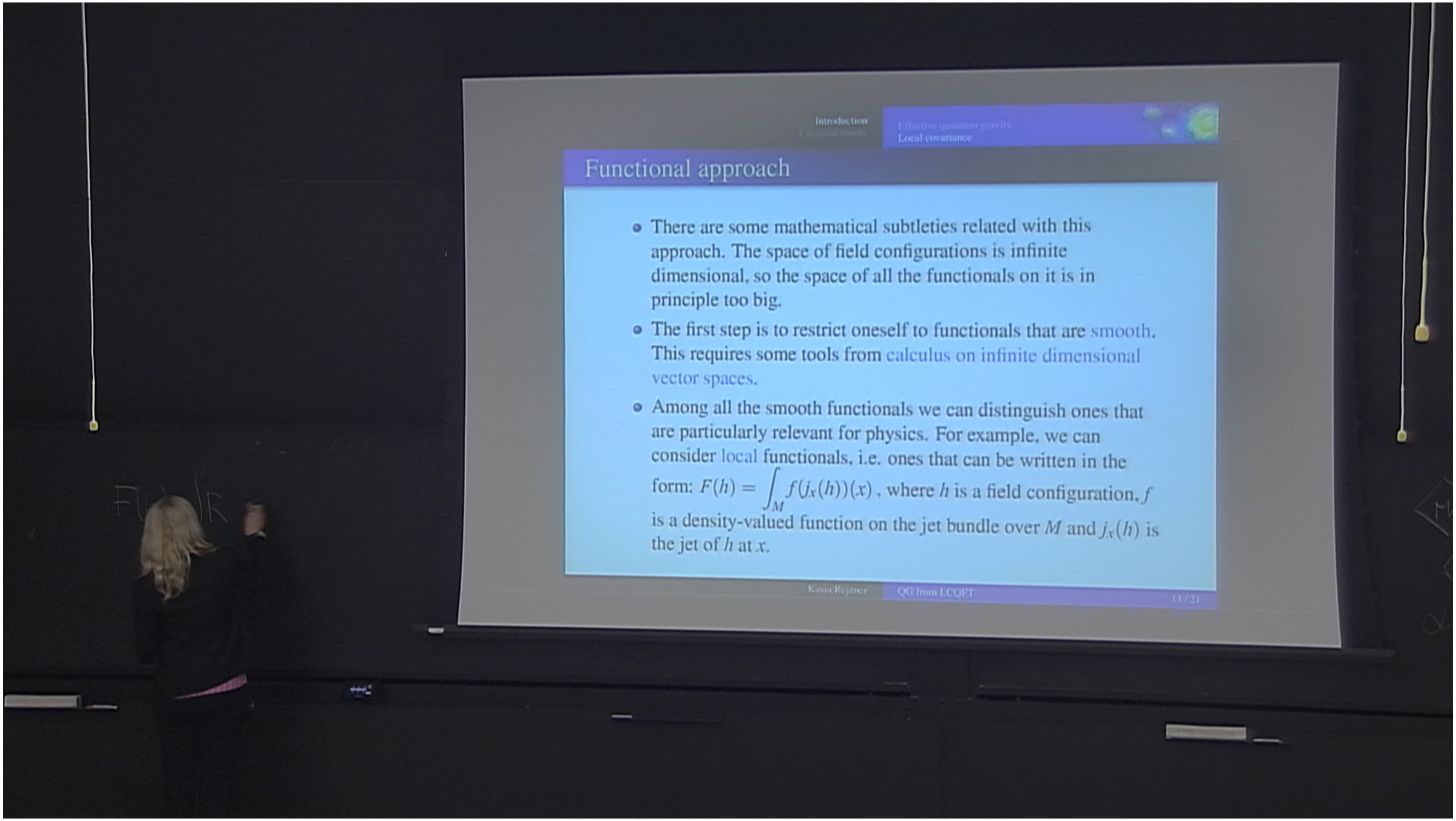
- The first step is to restrict oneself to functionals that are smooth. This requires some tools from calculus on infinite dimensional vector spaces.
- Among all the smooth functionals we can distinguish ones that are particularly relevant for physics. For example, we can consider local functionals, i.e. ones that can be written in the form: $F(h) = \int_M f(j_x(h))(x)$, where h is a field configuration, f is a density-valued function on the jet bundle over M and $j_x(h)$ is the jet of h at x.



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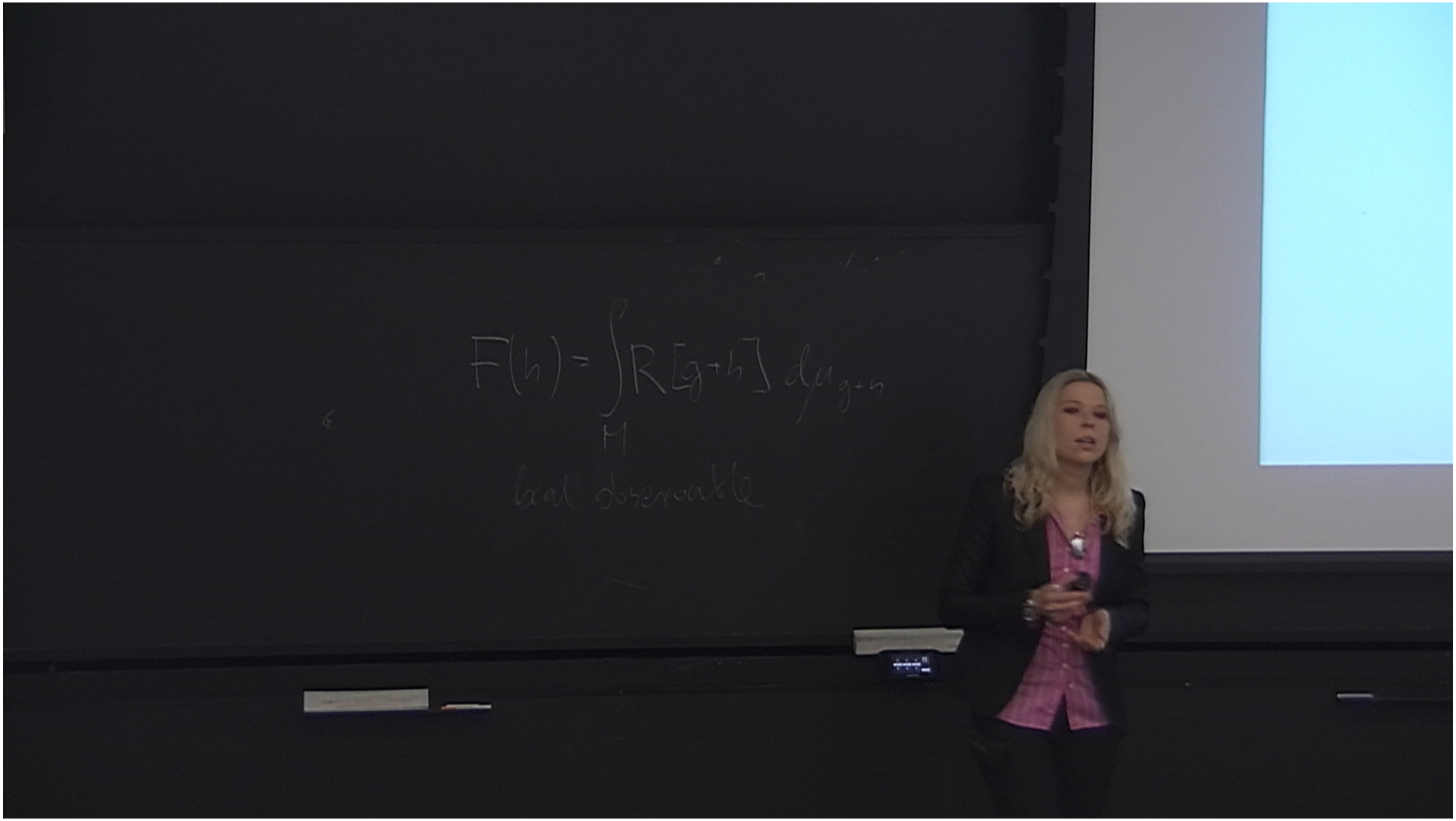
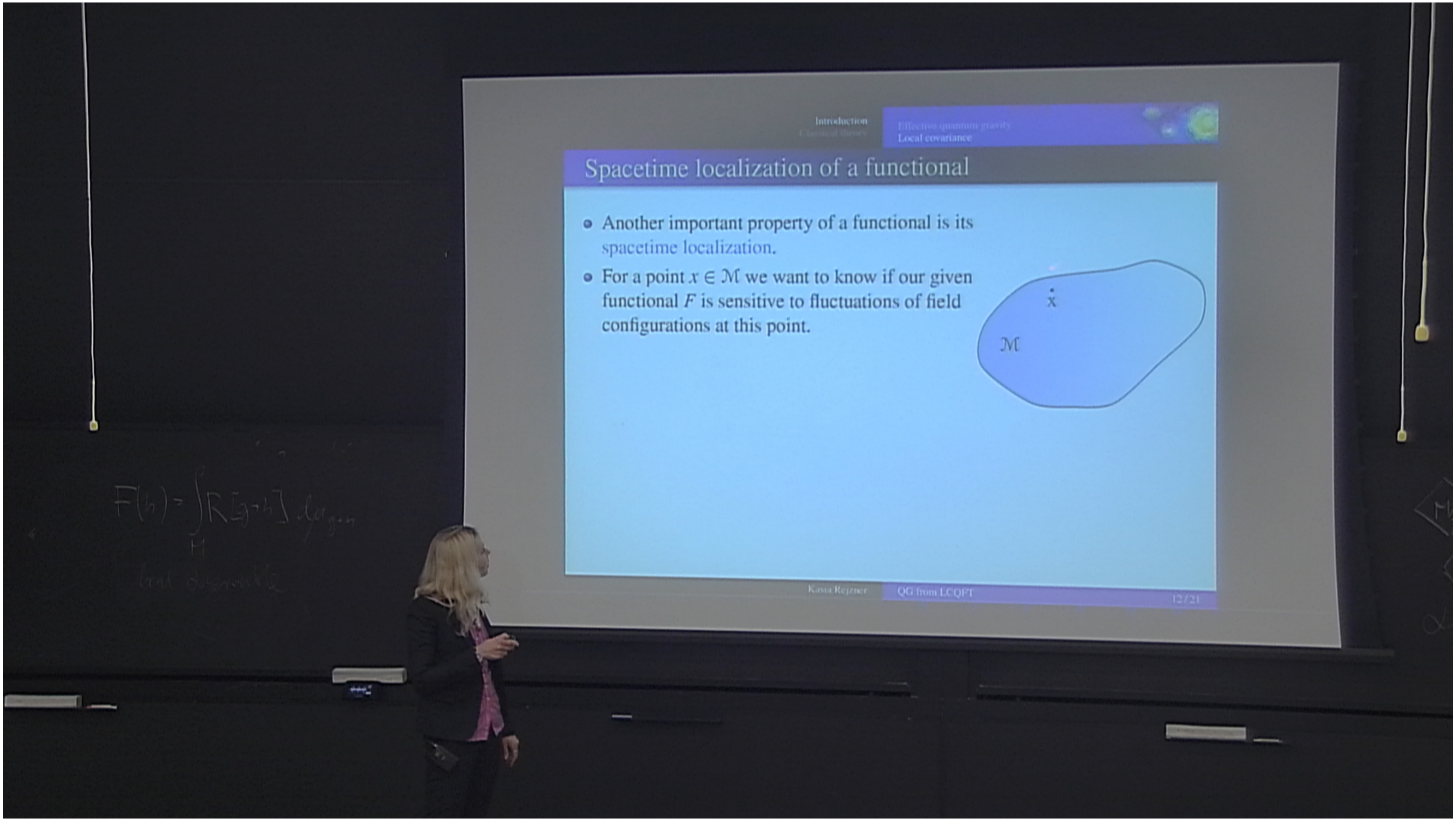

$F(h) = \int_M R[g+h]\, d\mu_{g+h}$
local observable



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Spacetime localization of a functional
Another important property of a functional is its spacetime localization.
For a point $x \in \mathcal{M}$ we want to know if our given functional F is sensitive to fluctuations of field configurations at this point.
x
$\mathcal{M}$
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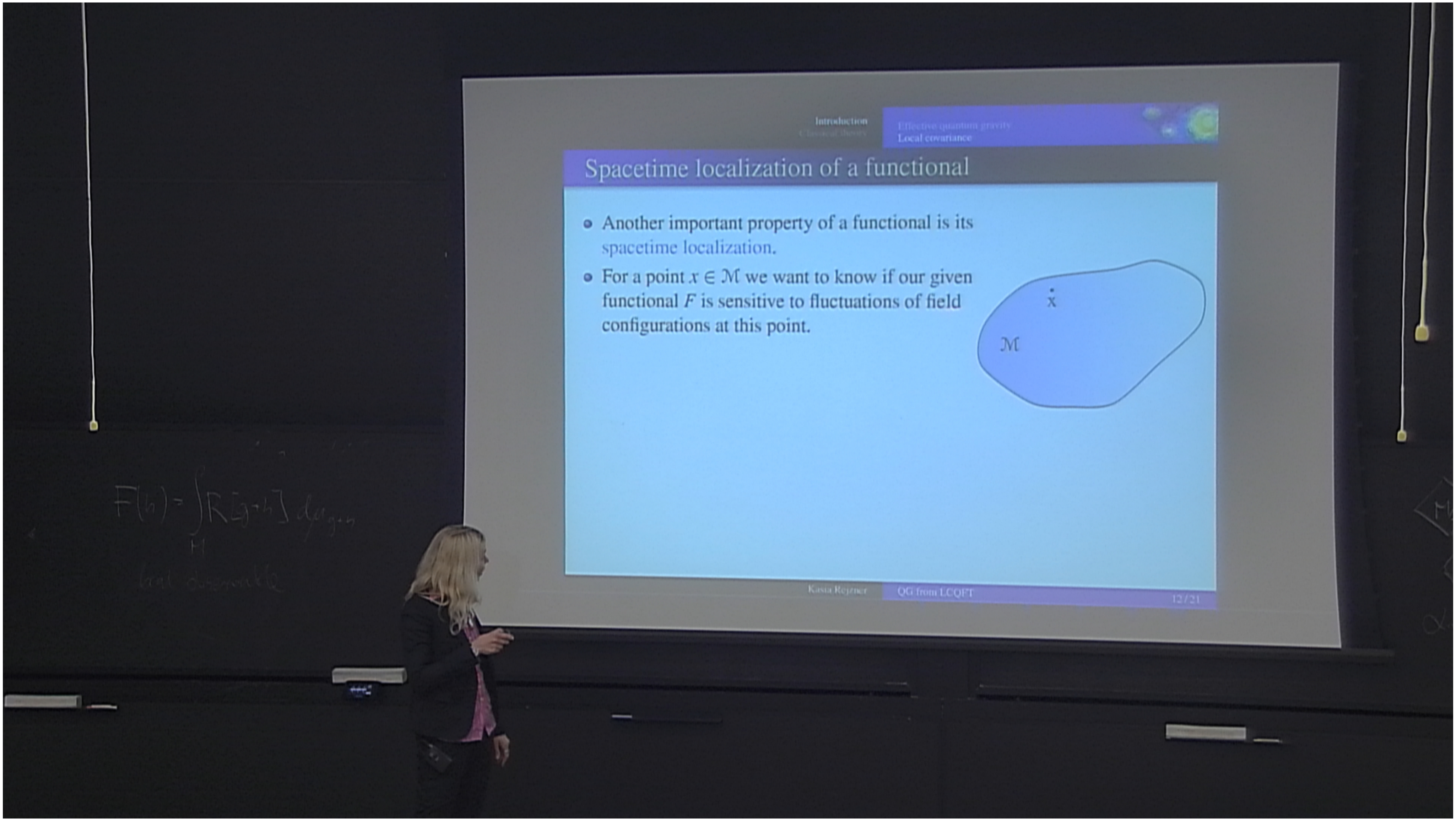
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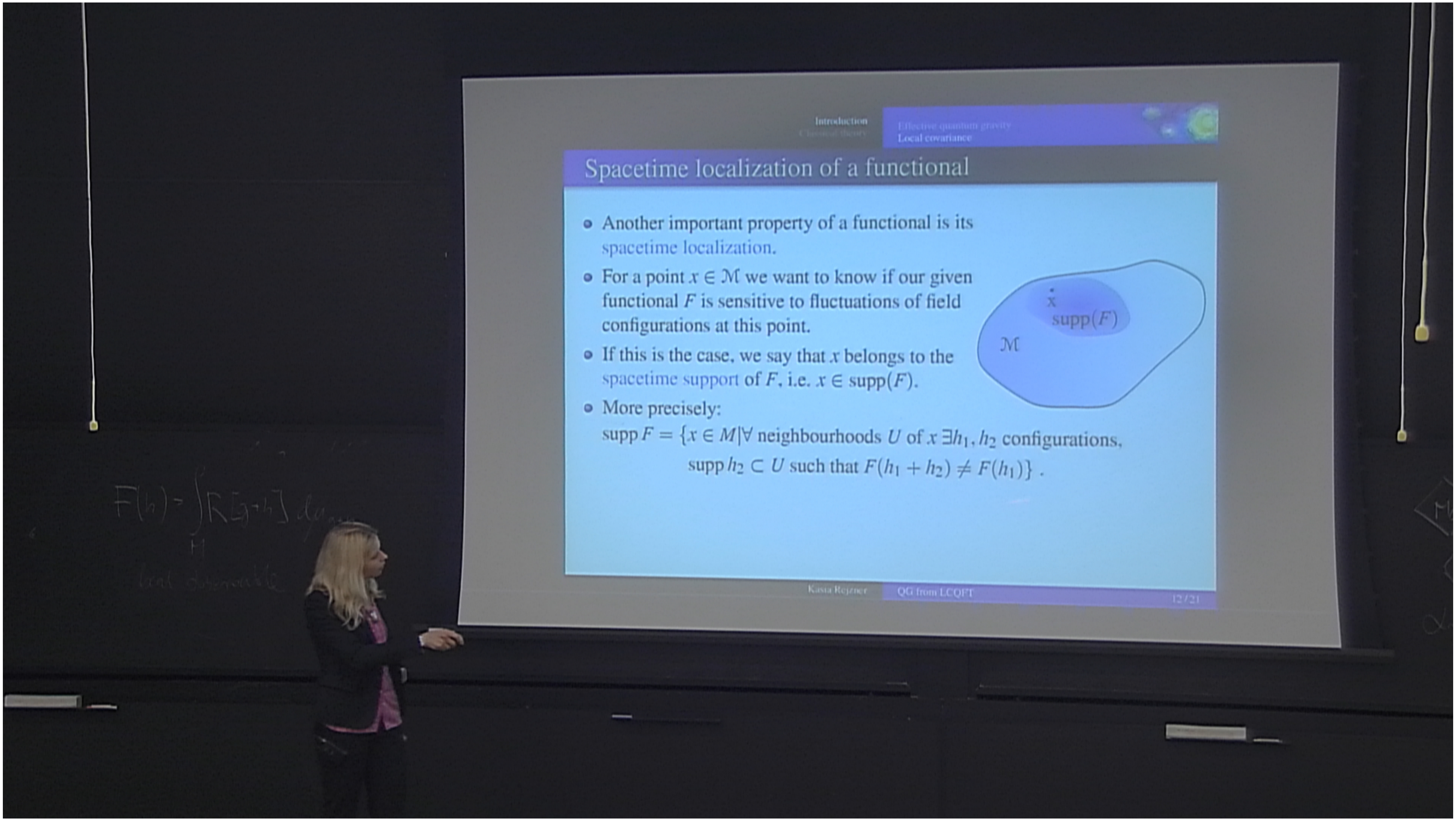
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For a point $x \in \mathcal{M}$ we want to know if our given functional F is sensitive to fluctuations of field configurations at this point.
If this is the case, we say that x belongs to the spacetime support of F, i.e. $x \in \mathrm{supp}(F)$.
More precisely:
$\mathrm{supp}\, F = \{x \in M | \forall$ neighbourhoods U of x $\exists h_1, h_2$ configurations, $\mathrm{supp}\, h_2 \subset U$ such that $F(h_1 + h_2) \neq F(h_1)\}$.
x
supp(F)
$\mathcal{M}$
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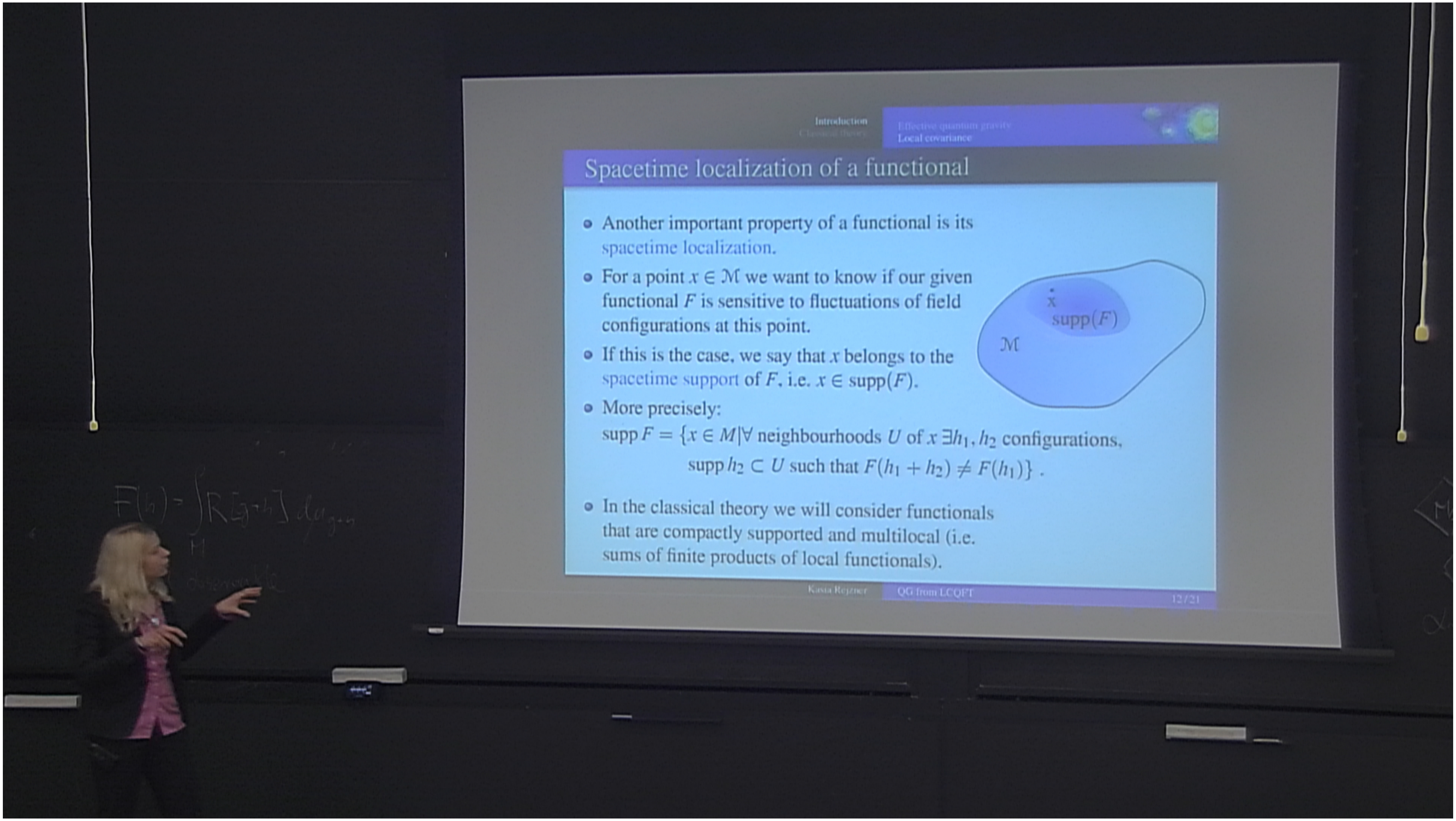
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More precisely:
supp F = {x ∈ M|∀ neighbourhoods U of x ∃h1, h2 configurations, supp h2 ⊂ U such that F(h1 + h2) ≠ F(h1)} .
In the classical theory we will consider functionals that are compactly supported and multilocal (i.e. sums of finite products of local functionals).
x
supp(F)
𝓜
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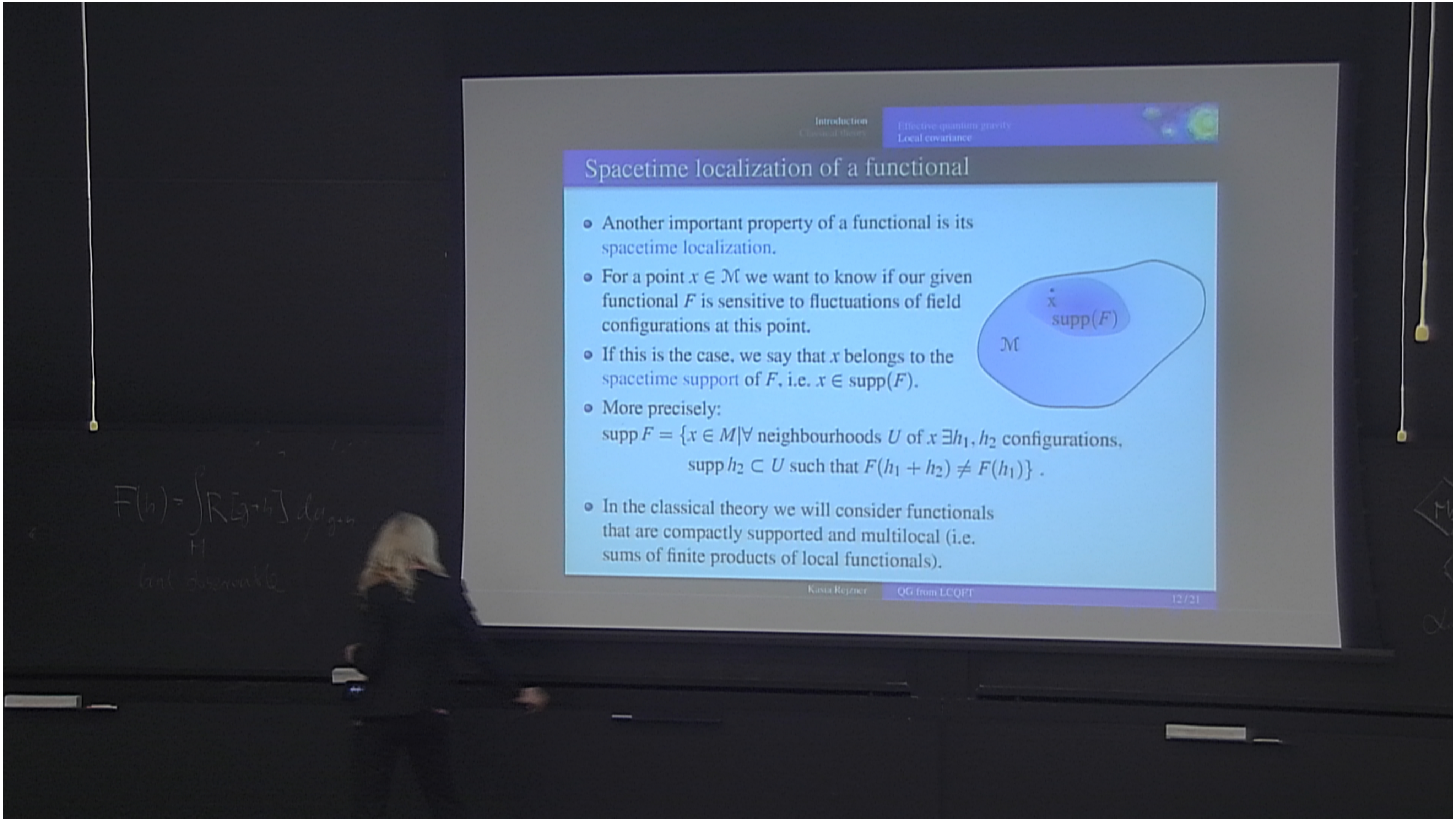
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supp(F)
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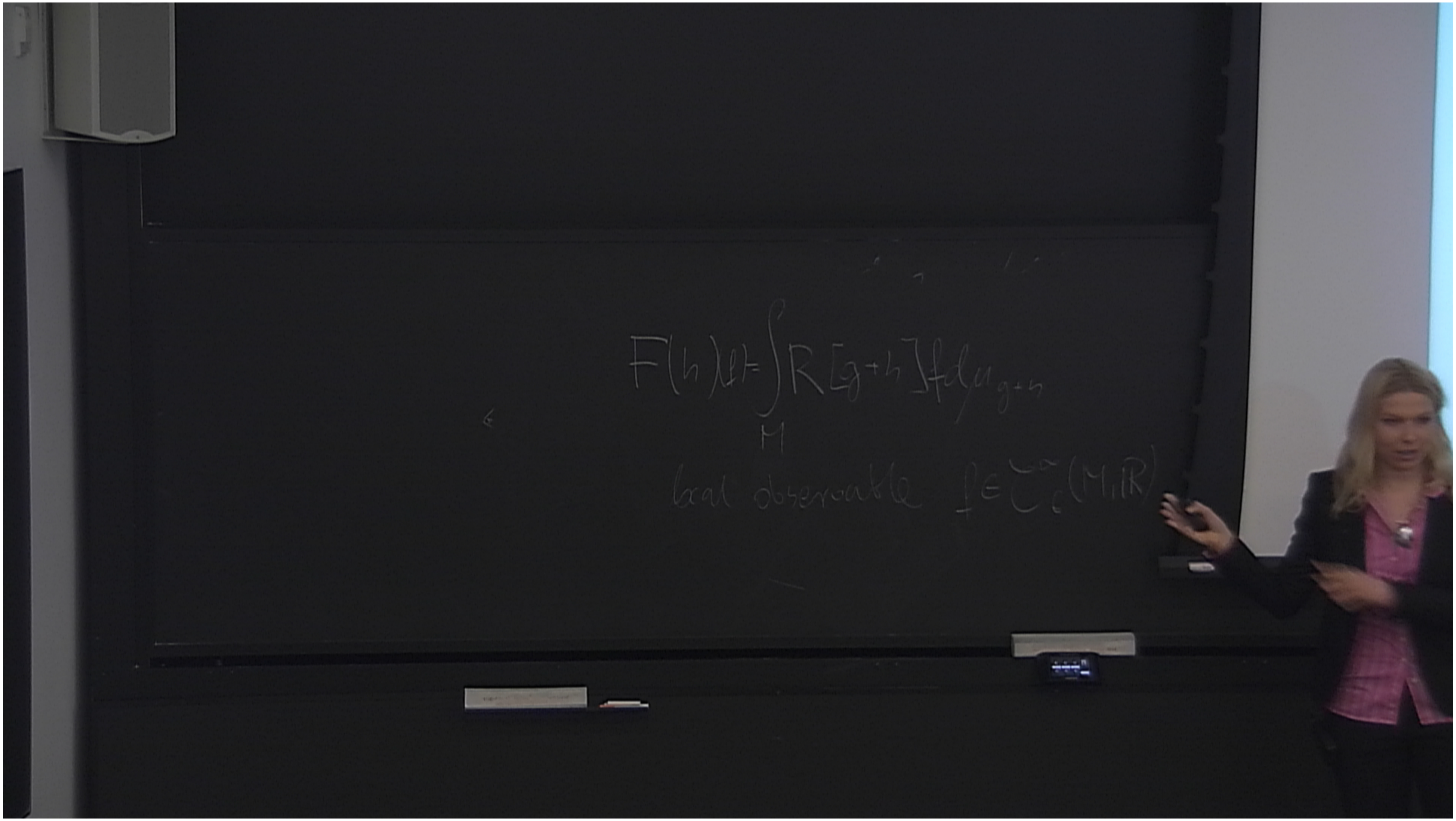
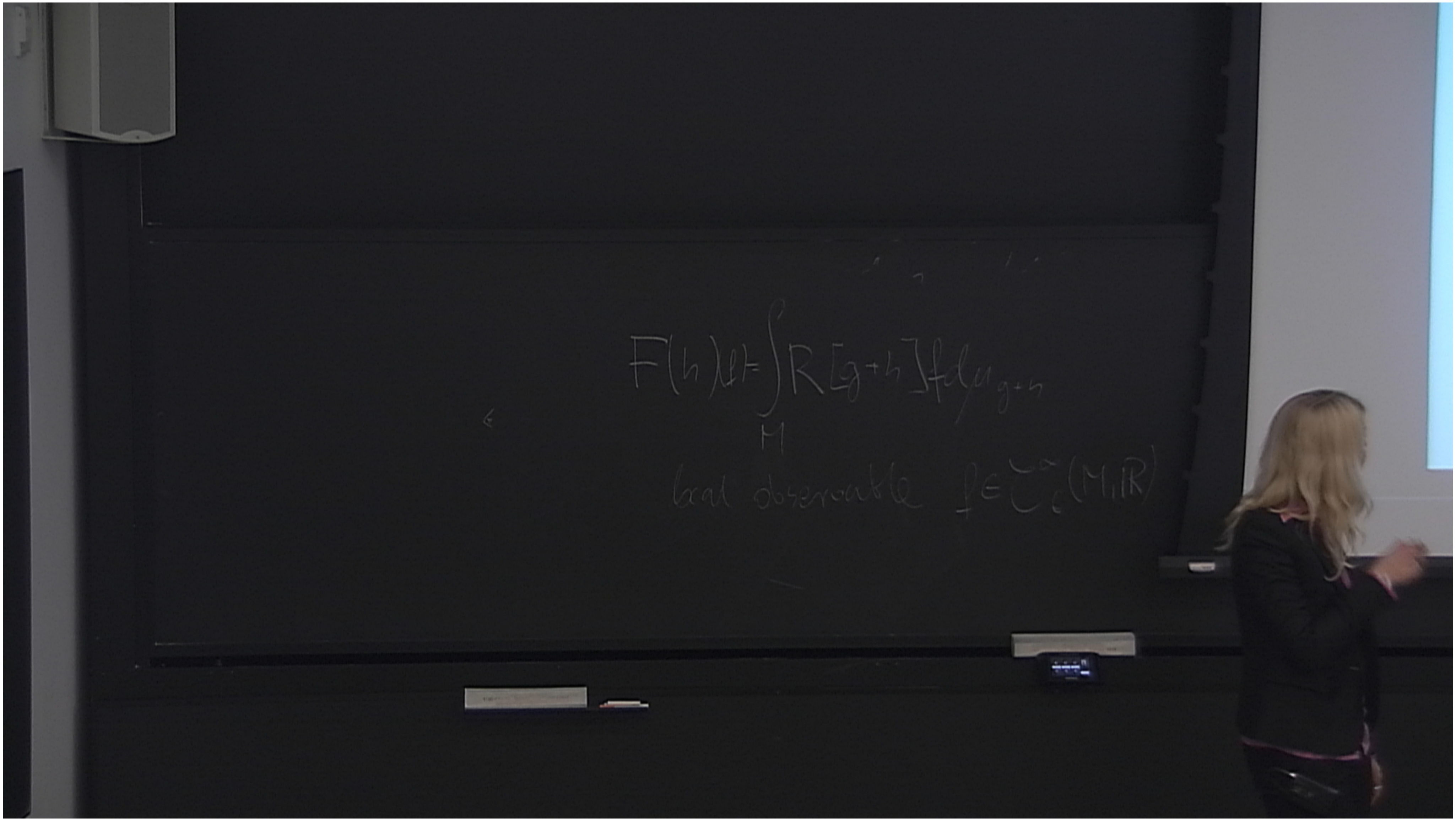

F(h)(f) = ∫_M R[g+h] f dμ_{g+h}
local observable f ∈ C_c^∞(M,ℝ)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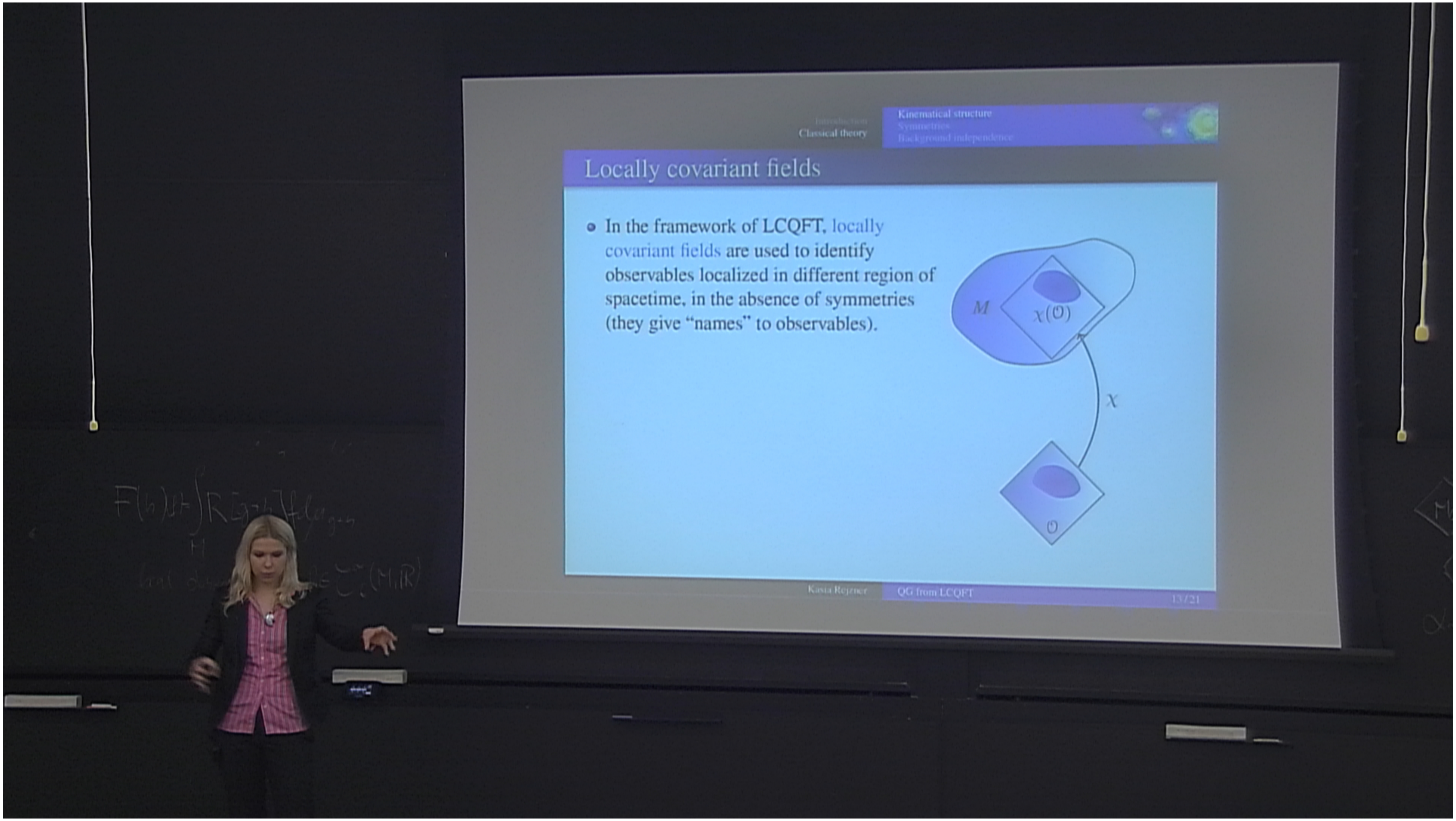
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In the framework of LCQFT, locally covariant fields are used to identify observables localized in different region of spacetime, in the absence of symmetries (they give "names" to observables).
M
χ(O)
χ
O
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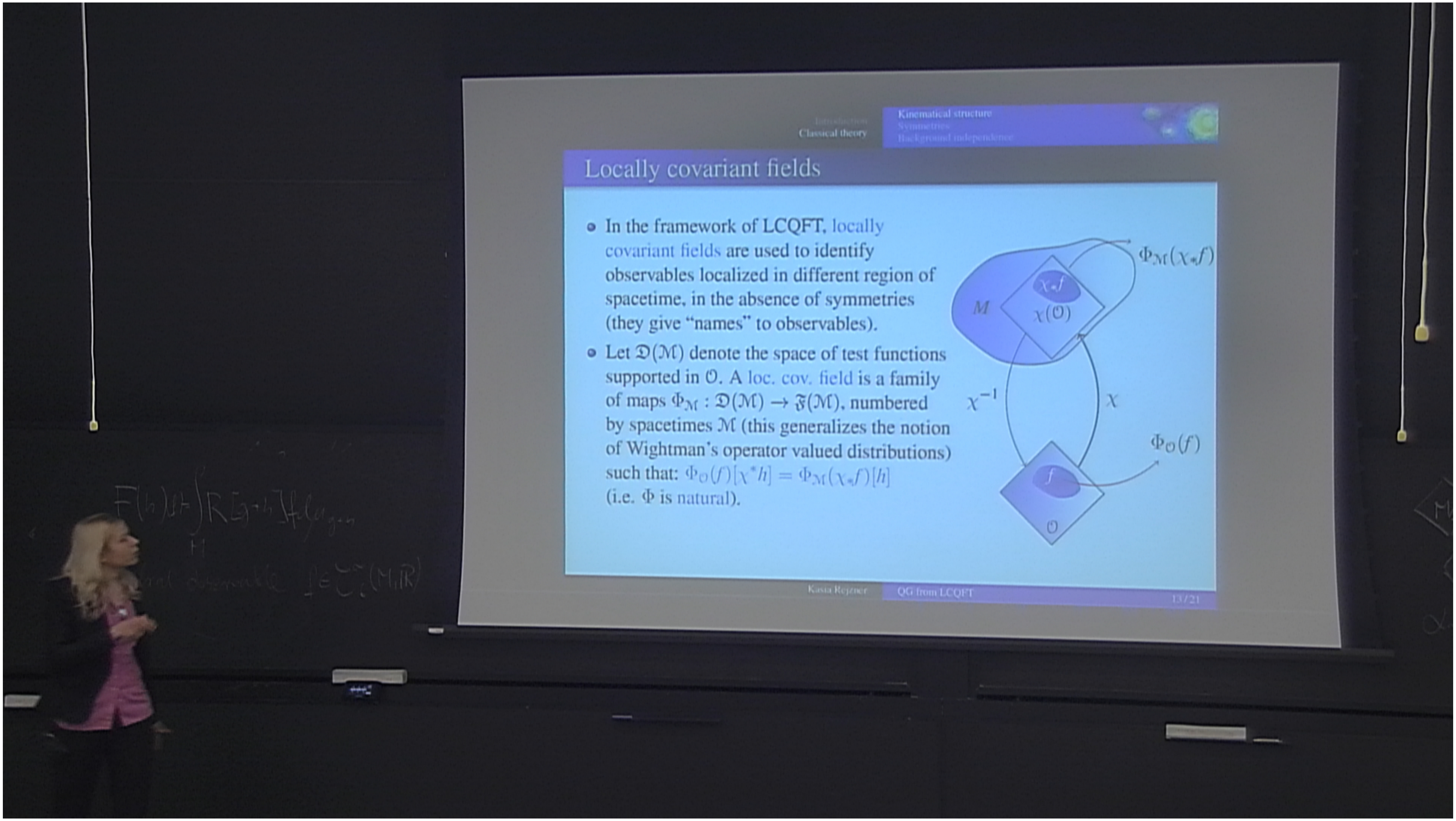
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Φ_𝓜(χ_*f)
M
χ(𝒪)
χ^{-1}
χ
Φ_𝒪(f)
f
𝒪
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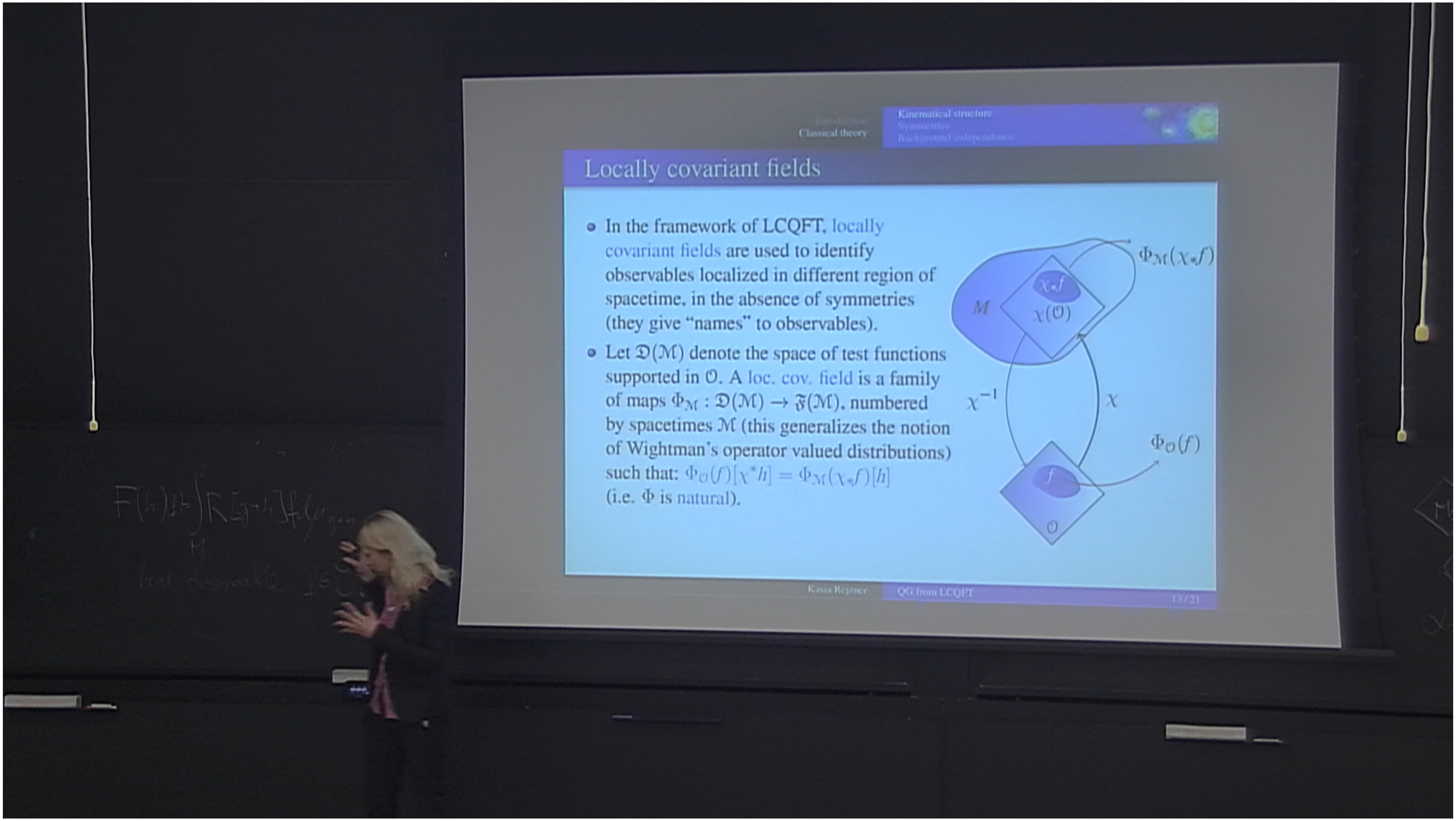

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$\Phi_{\mathcal{M}}(\chi_* f)$
$\chi_* f$
M
$\chi(\mathcal{O})$
χ^{-1}
χ
$\Phi_{\mathcal{O}}(f)$
f
$\mathcal{O}$
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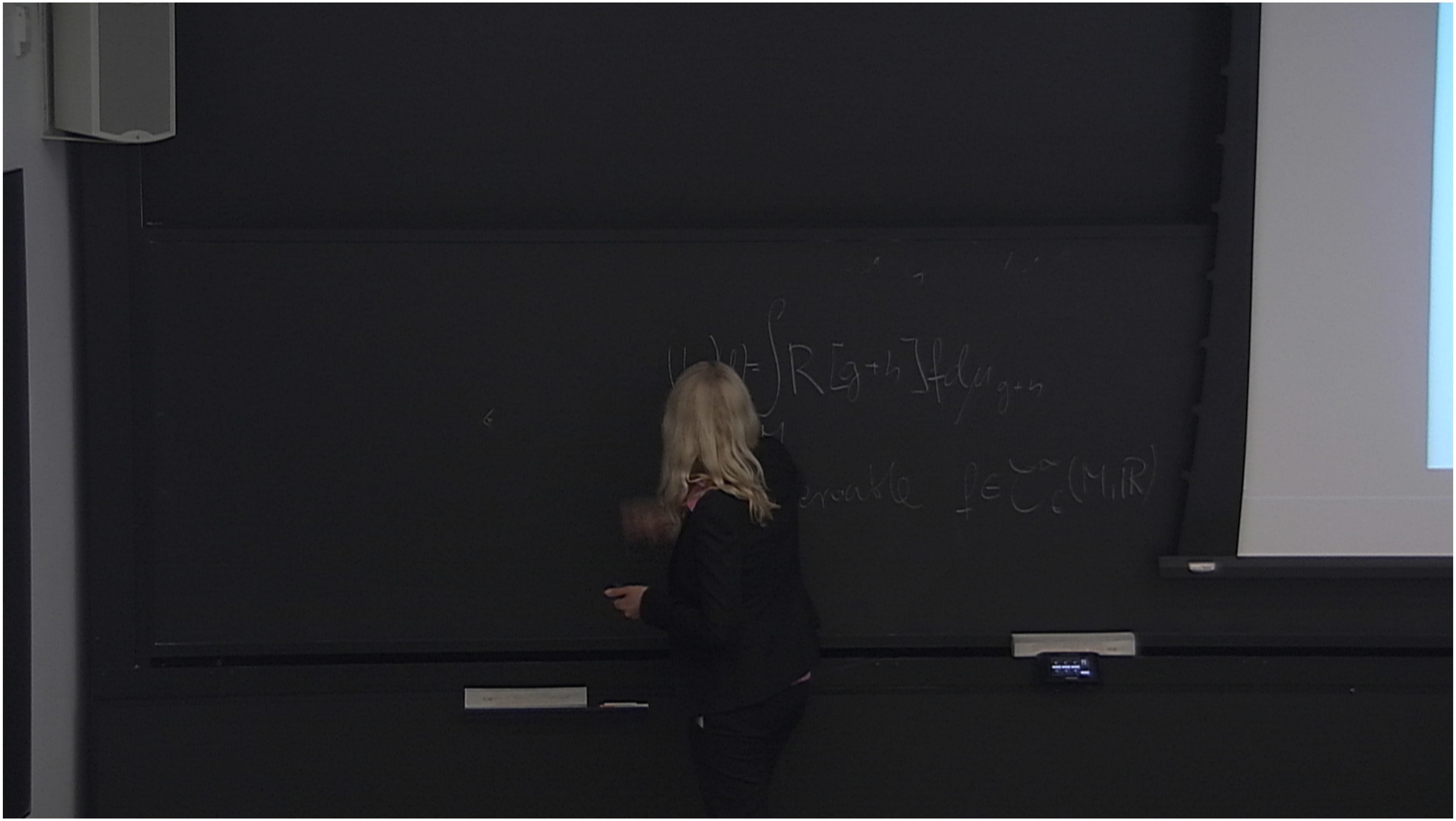

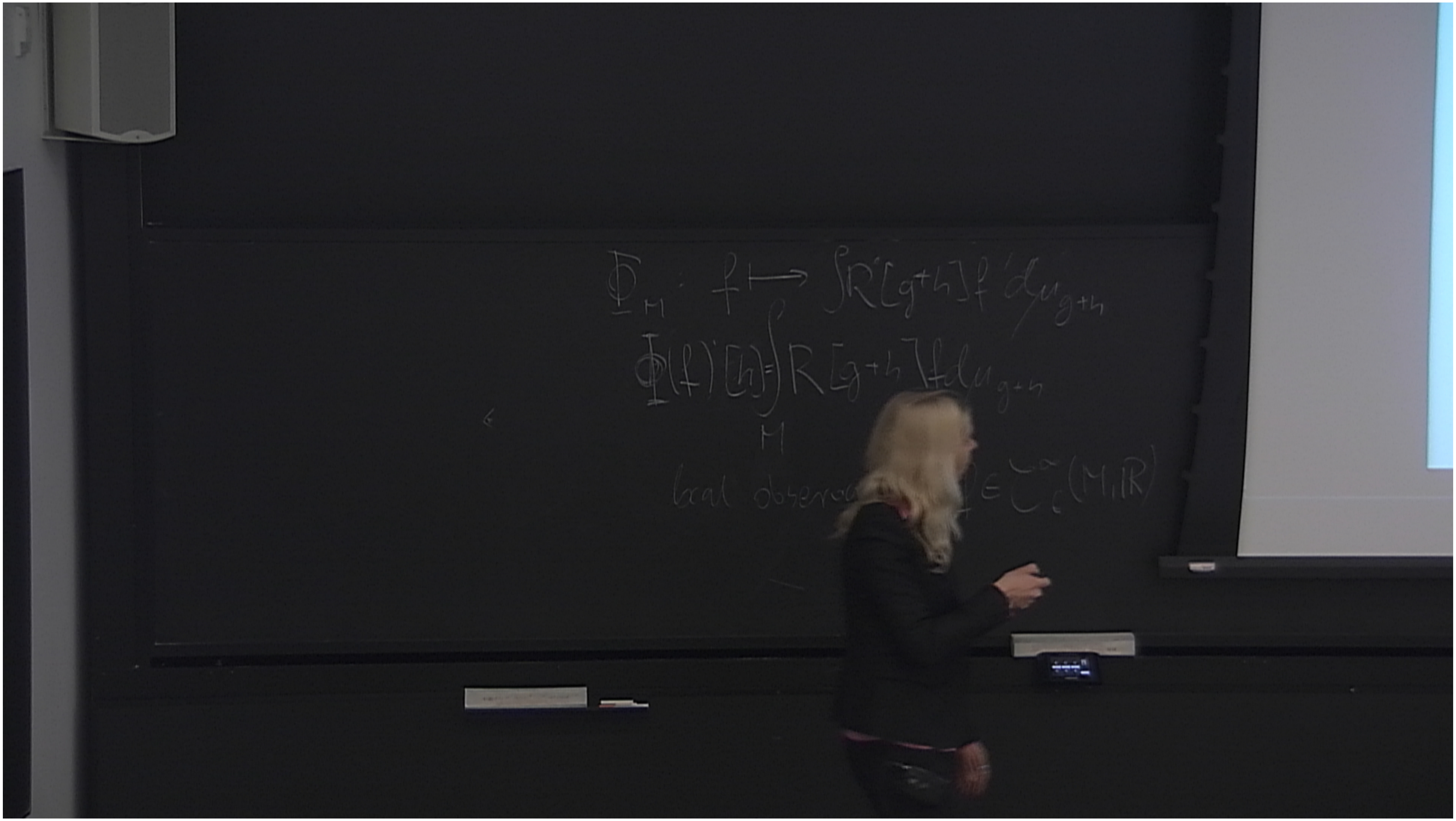

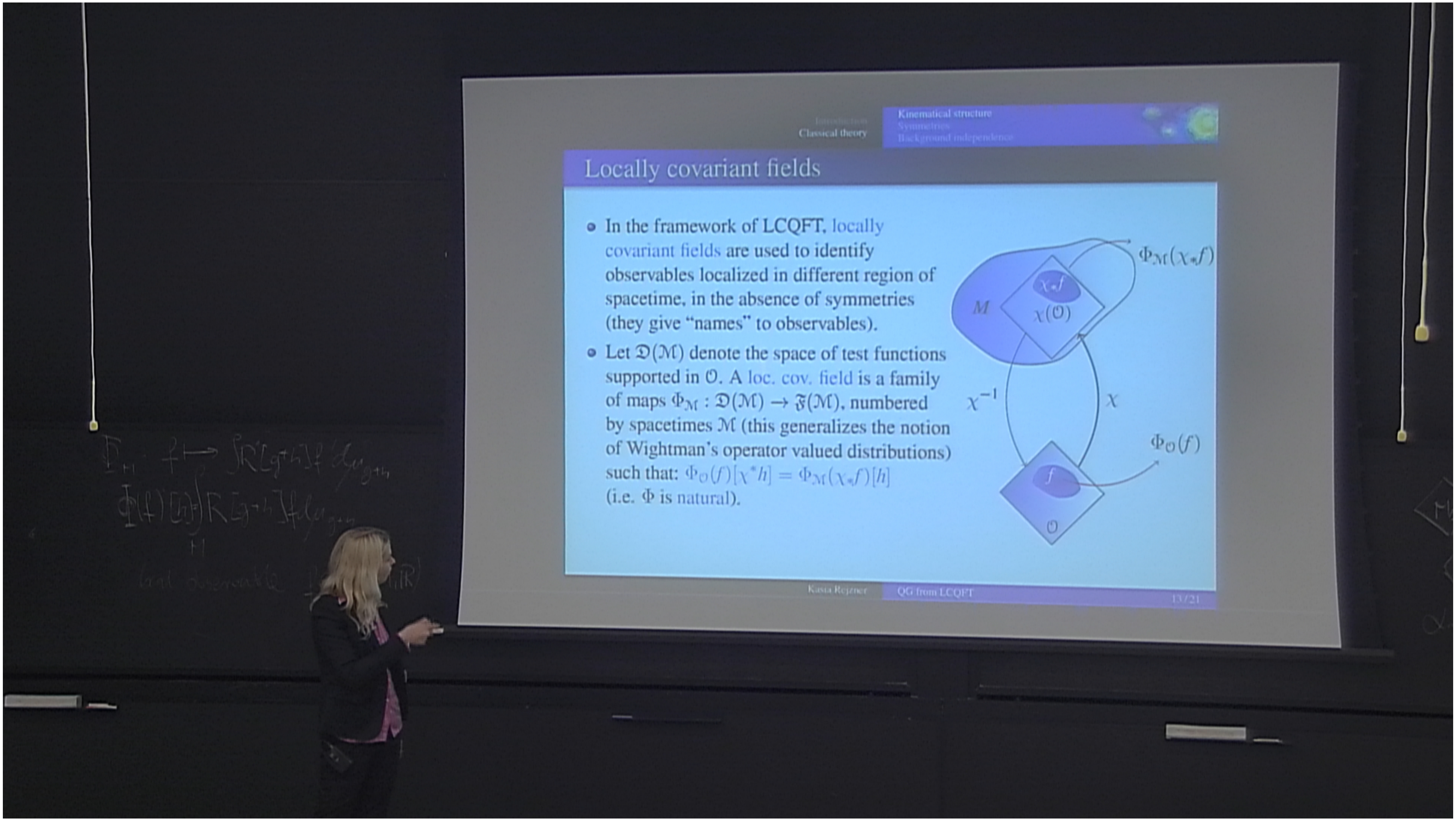
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$\chi_* f$
M
$\chi(\mathcal{O})$
χ^{-1}
χ
$\Phi_{\mathcal{O}}(f)$
f
$\mathcal{O}$
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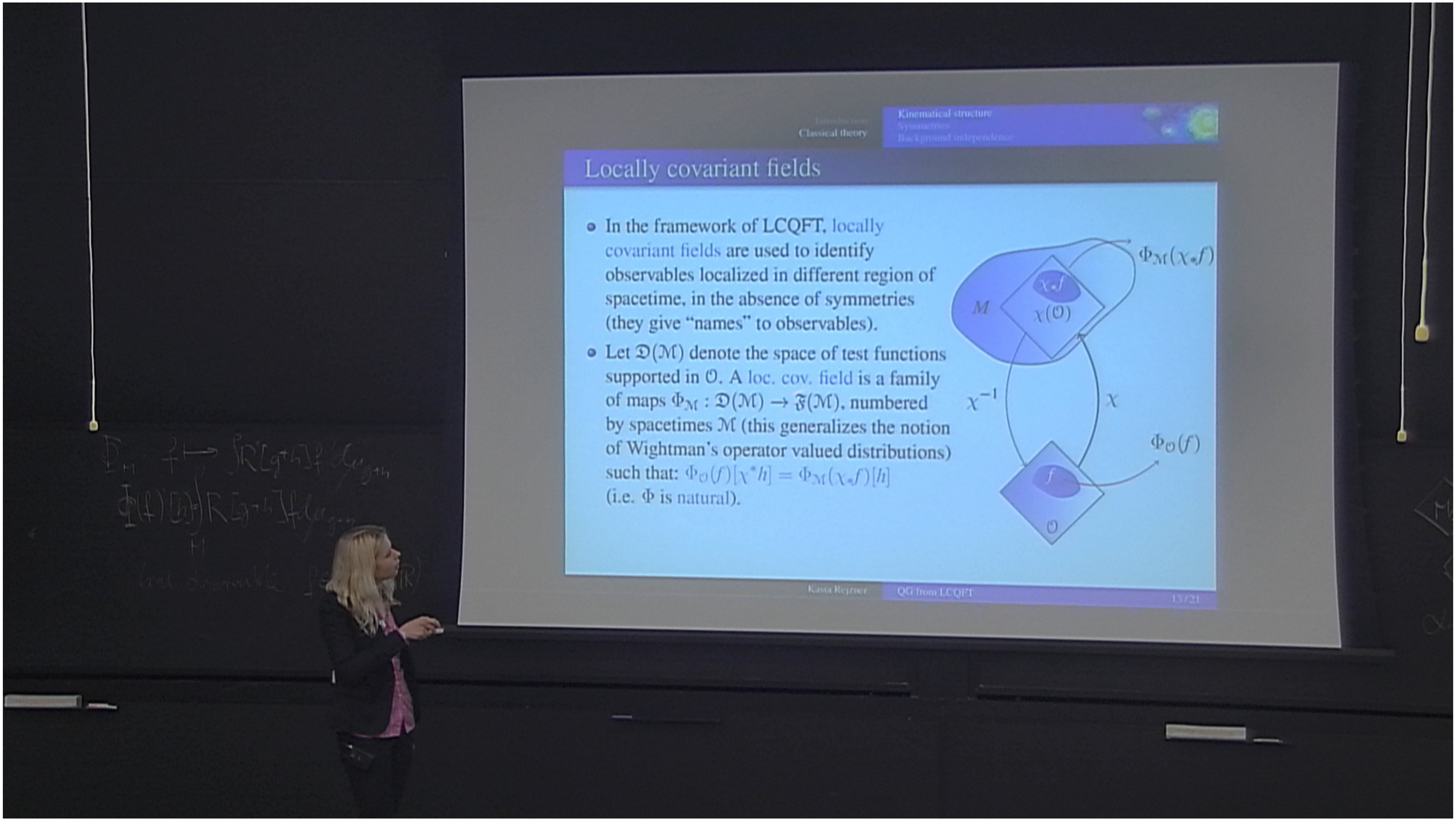
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Φ_𝓜(χ_*f)
M
χ(𝒪)
χ^{-1}
χ
Φ_𝒪(f)
f
𝒪
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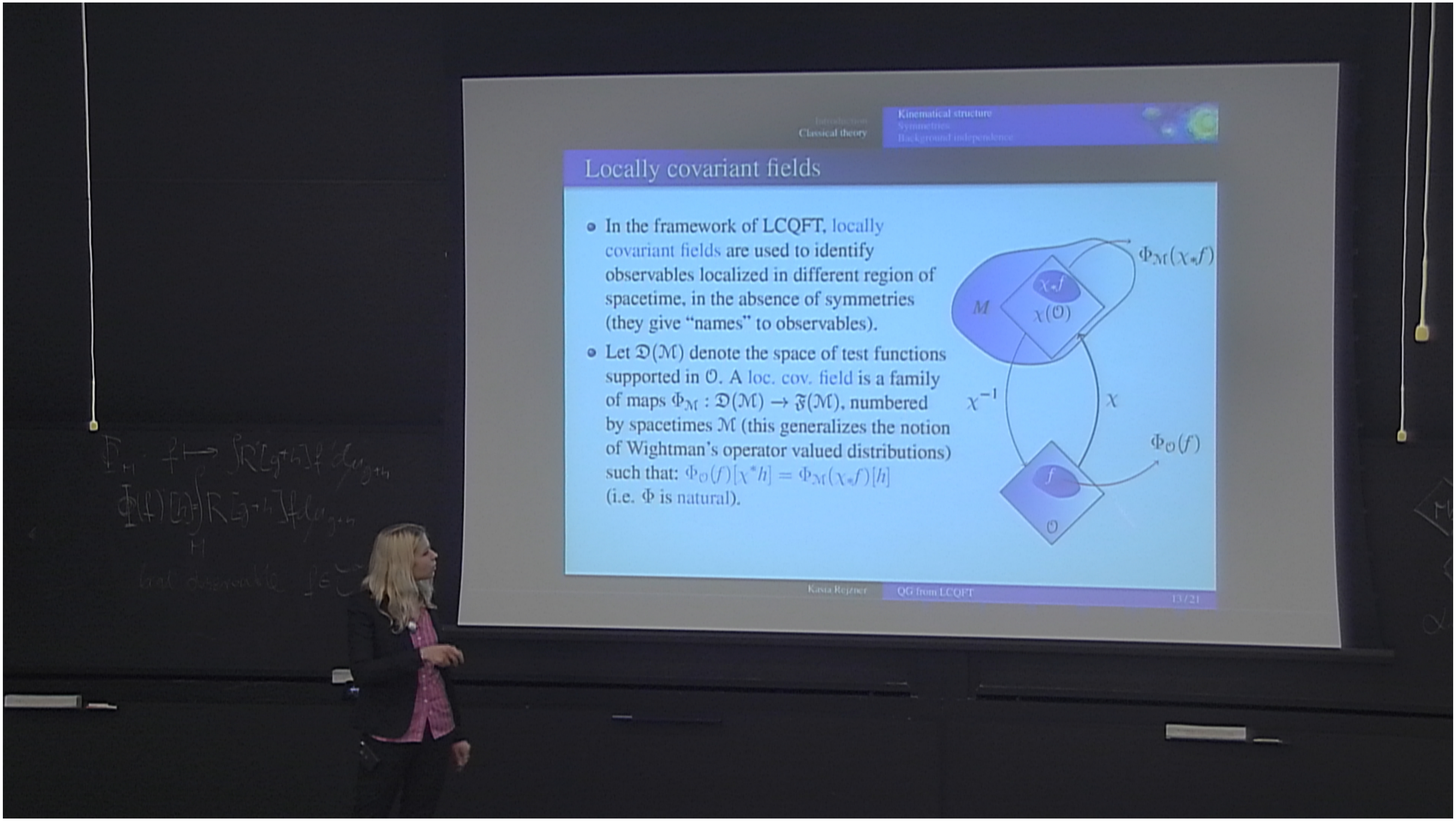
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$\Phi_{\mathcal{M}}(\chi_* f)$
$\chi_* f$
M
$\chi(\mathcal{O})$
χ^{-1}
χ
$\Phi_{\mathcal{O}}(f)$
f
$\mathcal{O}$
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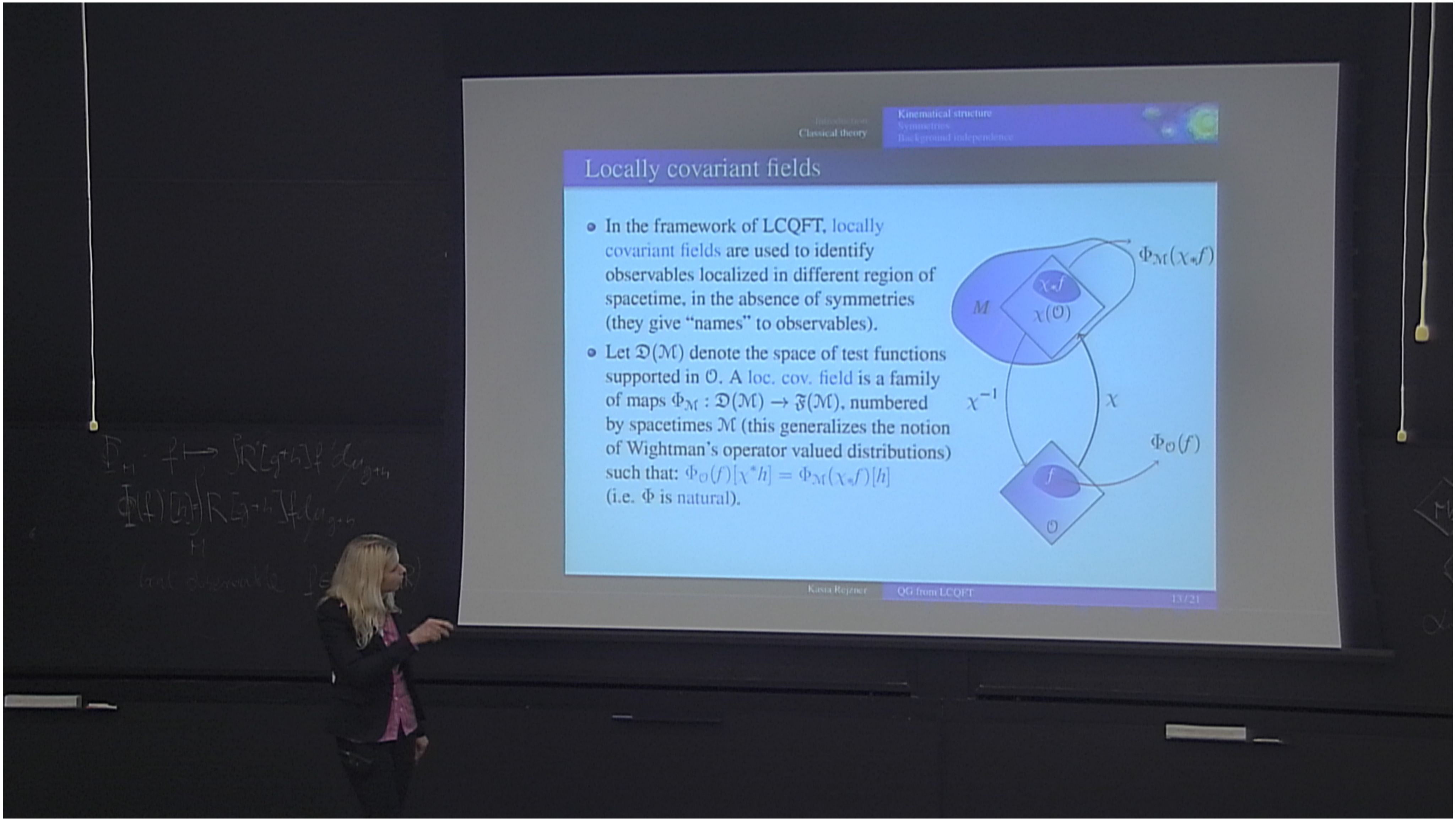
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M
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χ(𝒪)
χ^{-1}
χ
Φ_𝒪(f)
f
𝒪
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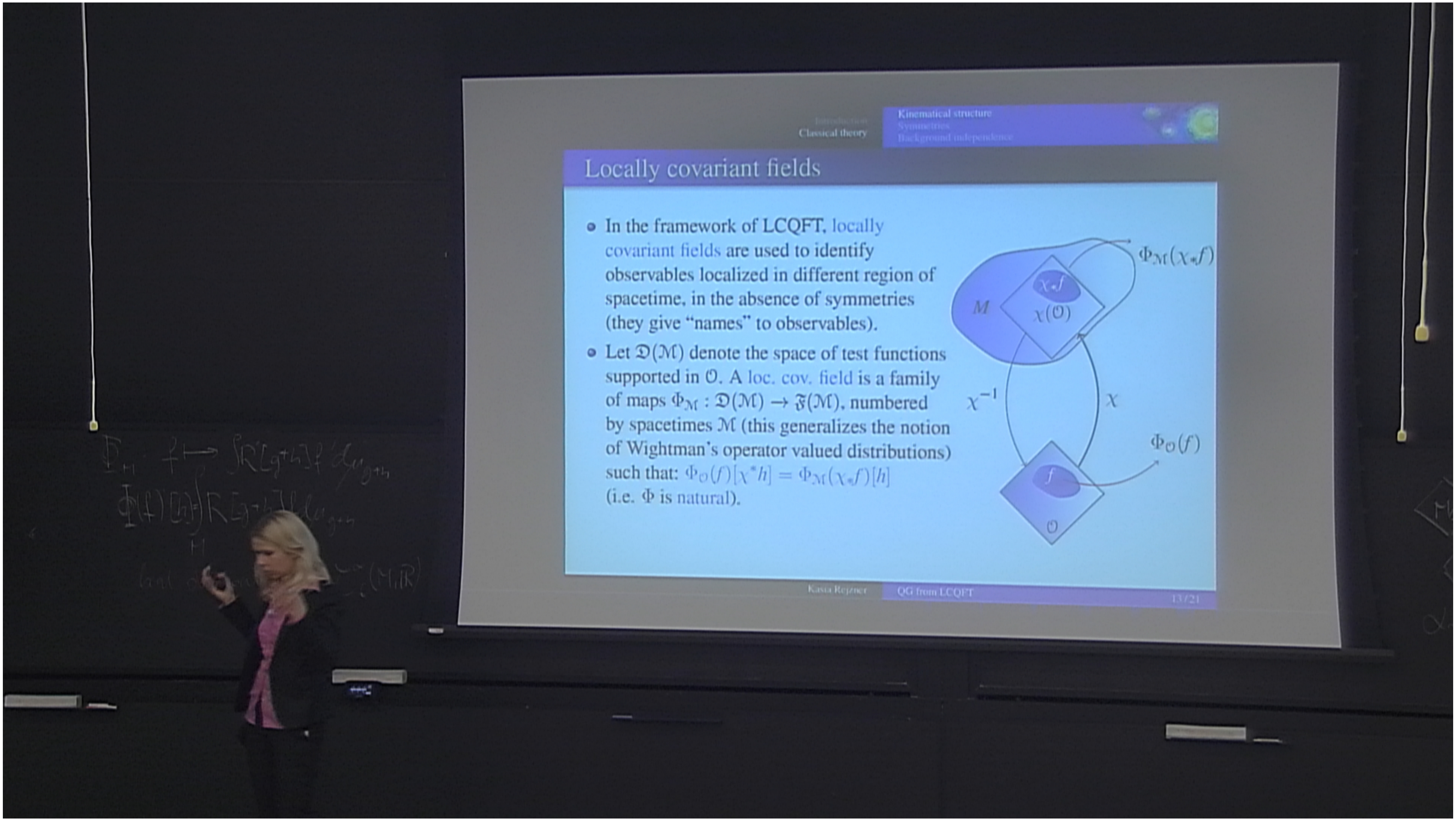
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M
$\chi_* f$
$\chi(\mathcal{O})$
χ^{-1}
χ
$\Phi_{\mathcal{O}}(f)$
f
$\mathcal{O}$
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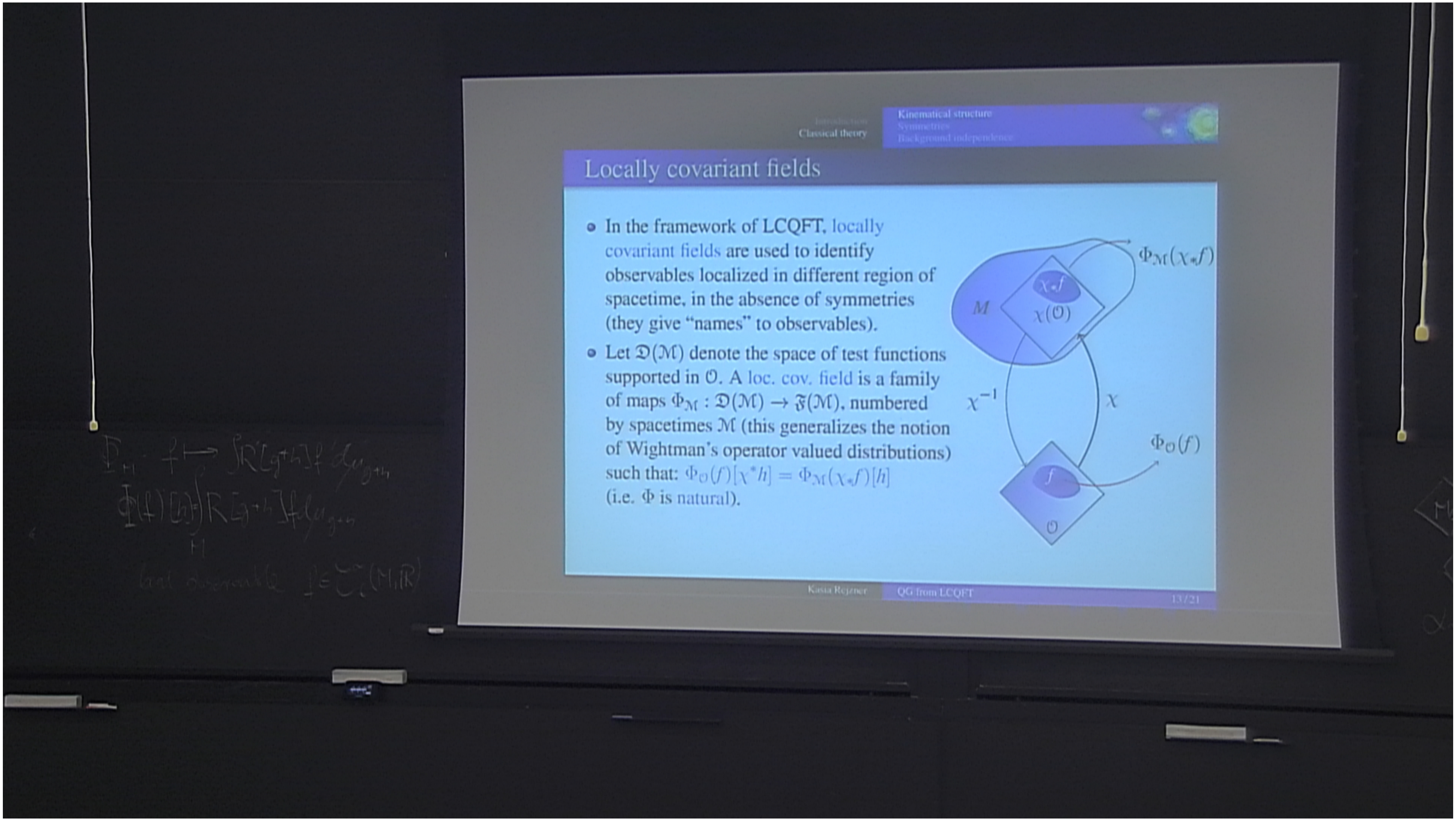

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$\Phi_{\mathcal{O}}(f)$
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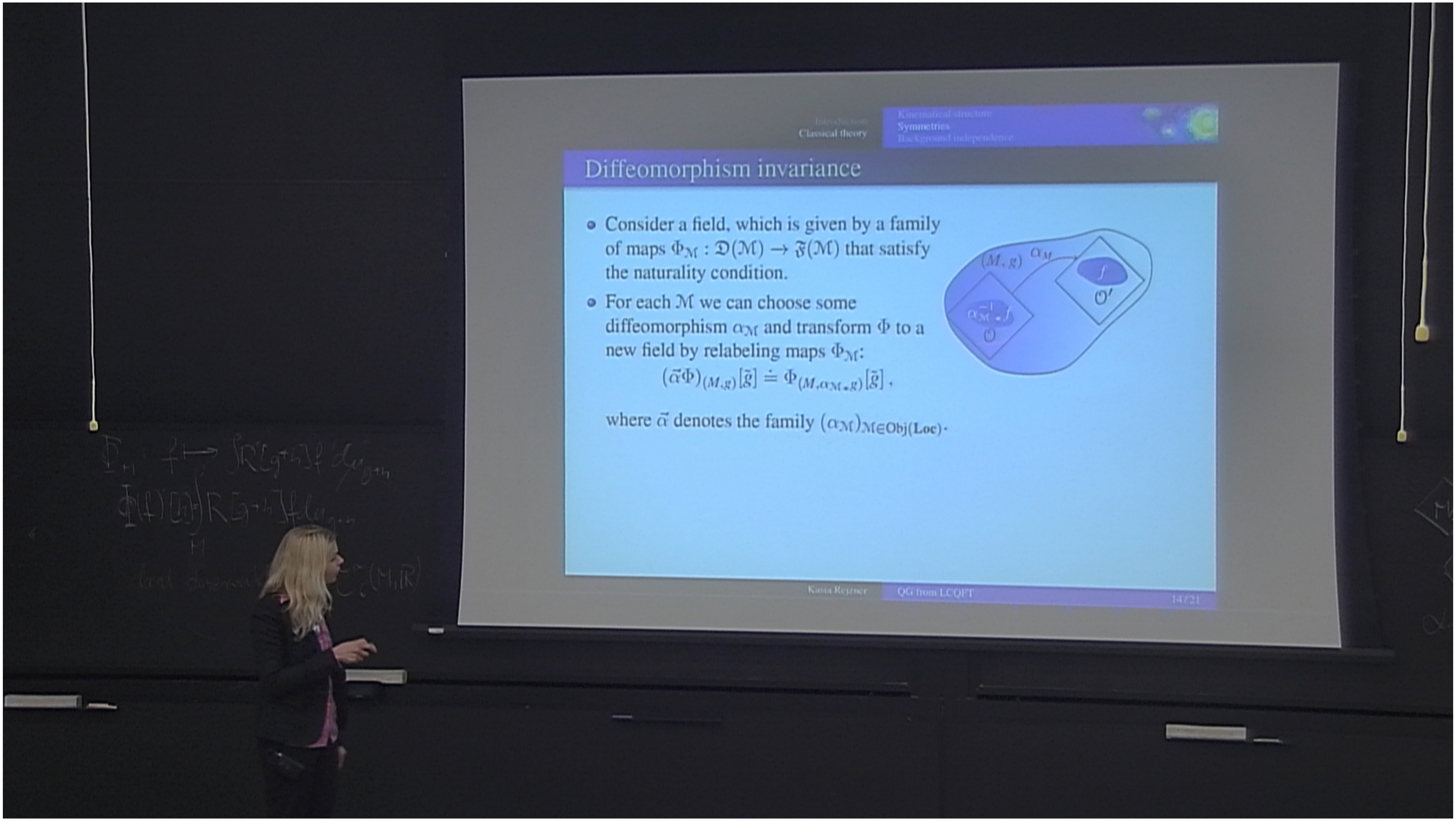

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Consider a field, which is given by a family of maps $\Phi_{\mathcal{M}} : \mathfrak{D}(\mathcal{M}) \to \mathfrak{F}(\mathcal{M})$ that satisfy the naturality condition.
For each $\mathcal{M}$ we can choose some diffeomorphism $\alpha_{\mathcal{M}}$ and transform Φ to a new field by relabeling maps $\Phi_{\mathcal{M}}$:
$(\vec{\alpha}\Phi)_{(M,g)}[\tilde{g}] \doteq \Phi_{(M,\alpha_{\mathcal{M}*}g)}[\tilde{g}]$,
where $\vec{\alpha}$ denotes the family $(\alpha_{\mathcal{M}})_{\mathcal{M}\in\mathrm{Obj}(\mathbf{Loc})}$.
(M, g)
α_M
f
$\mathcal{O}'$
$\alpha_{\mathcal{M}*}^{-1}f$
$\mathcal{O}$
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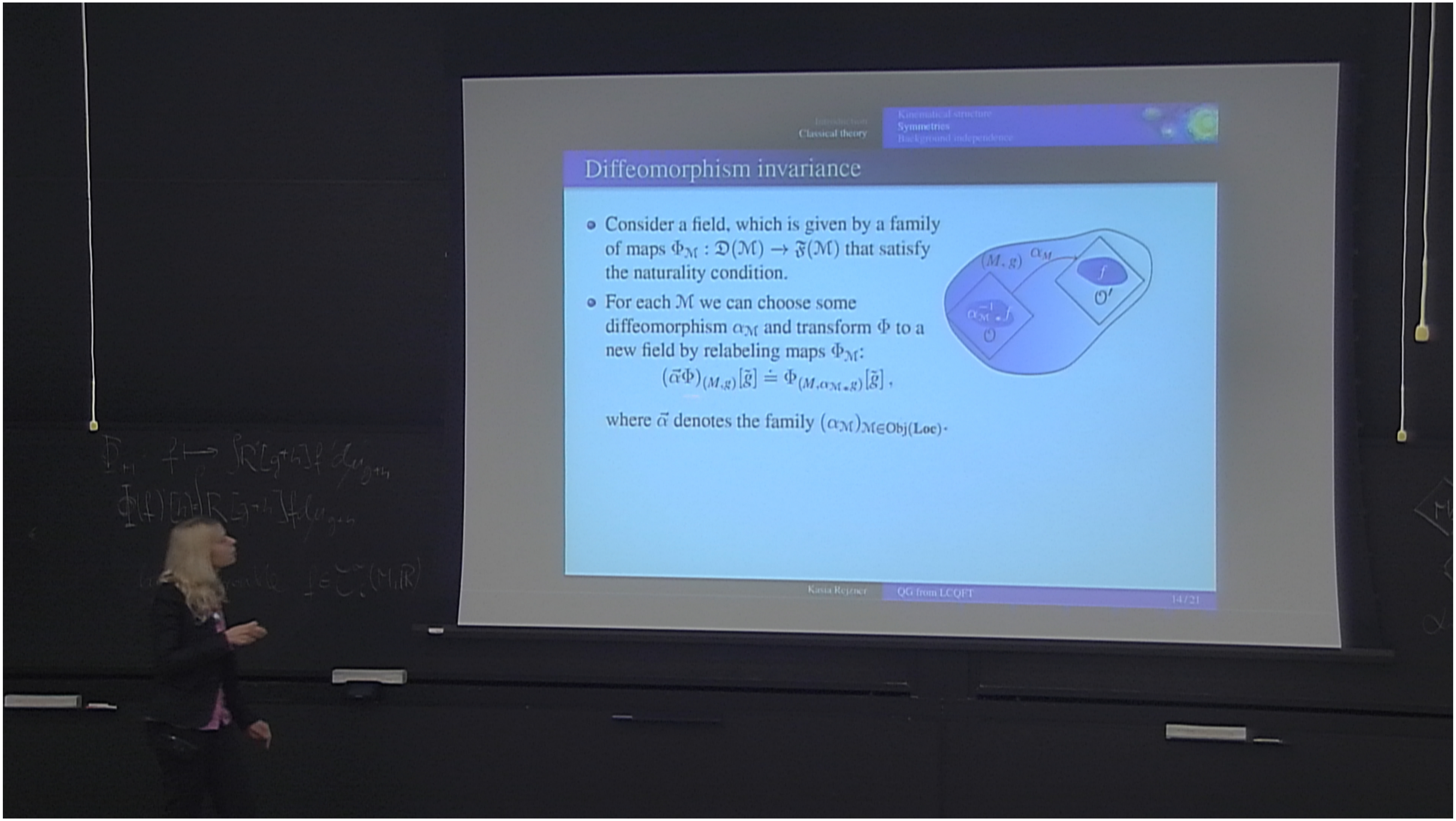
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(M,g)
α_M
f
$\mathcal{O}'$
$\mathcal{O}$
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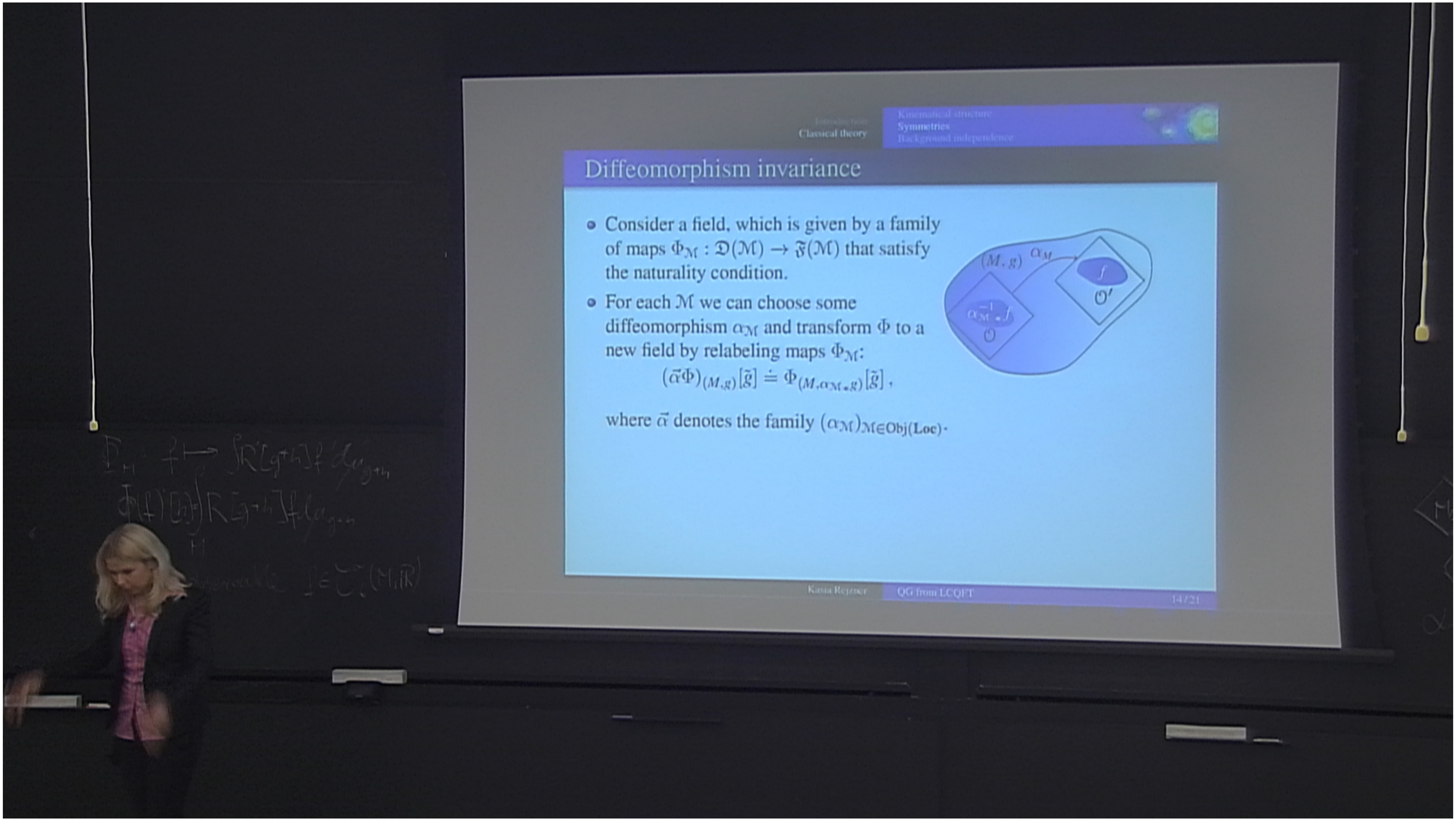
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$\mathcal{O}'$
$\alpha_{\mathcal{M}}^{-1}{}_*f$
$\mathcal{O}$
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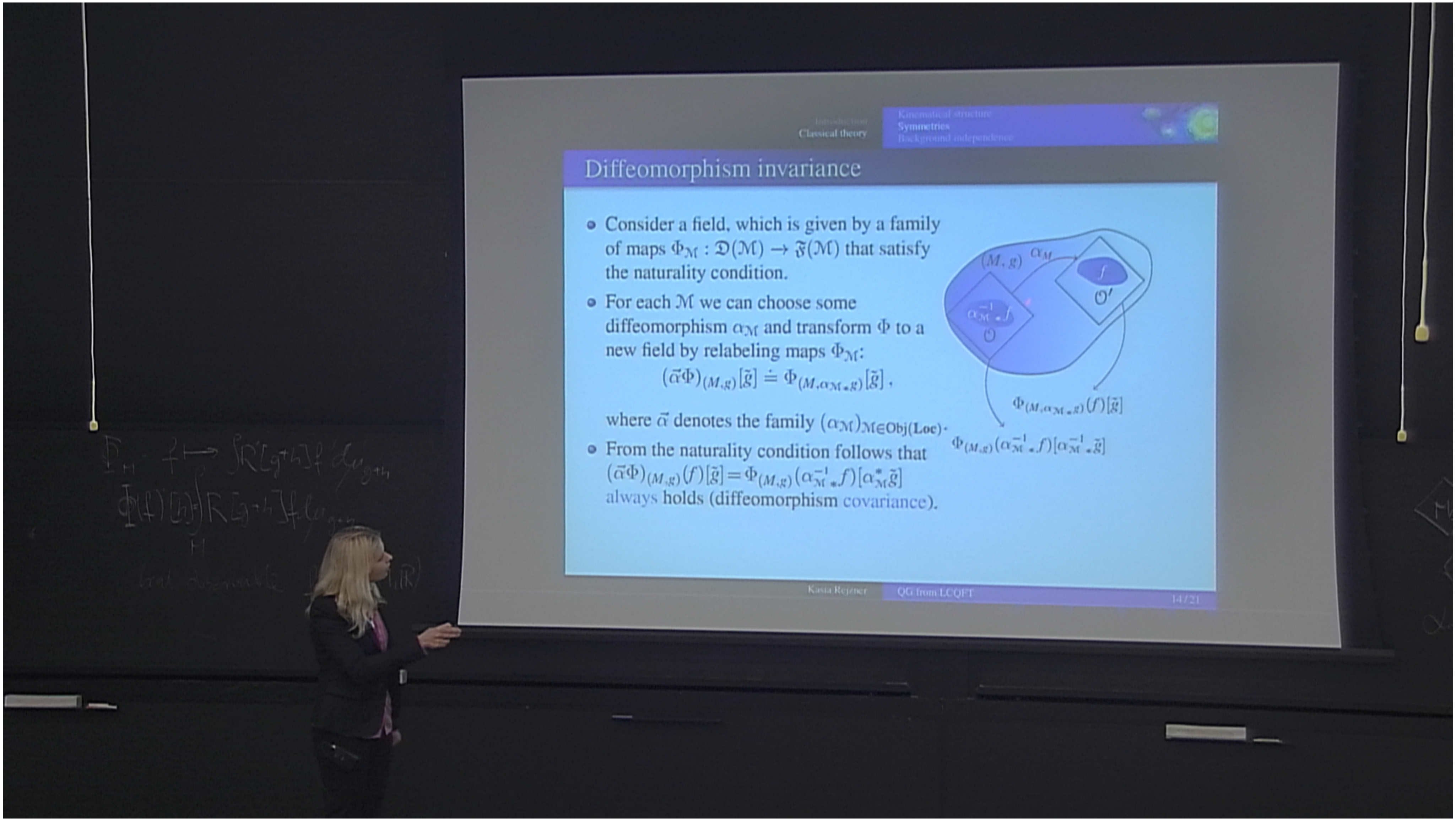
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(M,g)
α_M
f
$\mathcal{O}'$
$\alpha^{-1}_{\mathcal{M}*}f$
$\mathcal{O}$
$\Phi_{(M,\alpha_{\mathcal{M}*}g)}(f)[\tilde{g}]$
$\Phi_{(M,g)}(\alpha^{-1}_{\mathcal{M}*}f)[\alpha^{-1}_{\mathcal{M}*}\tilde{g}]$
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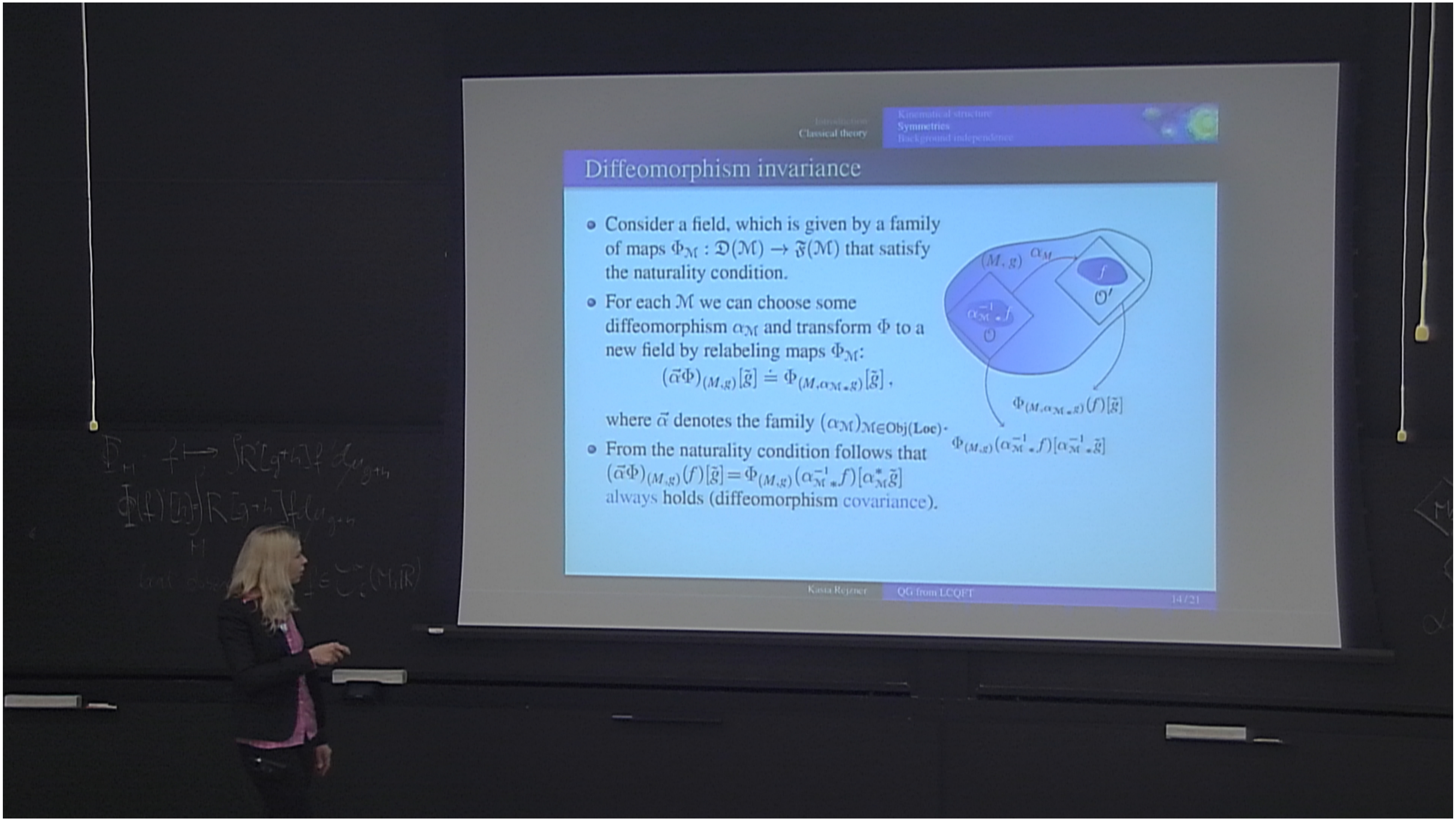
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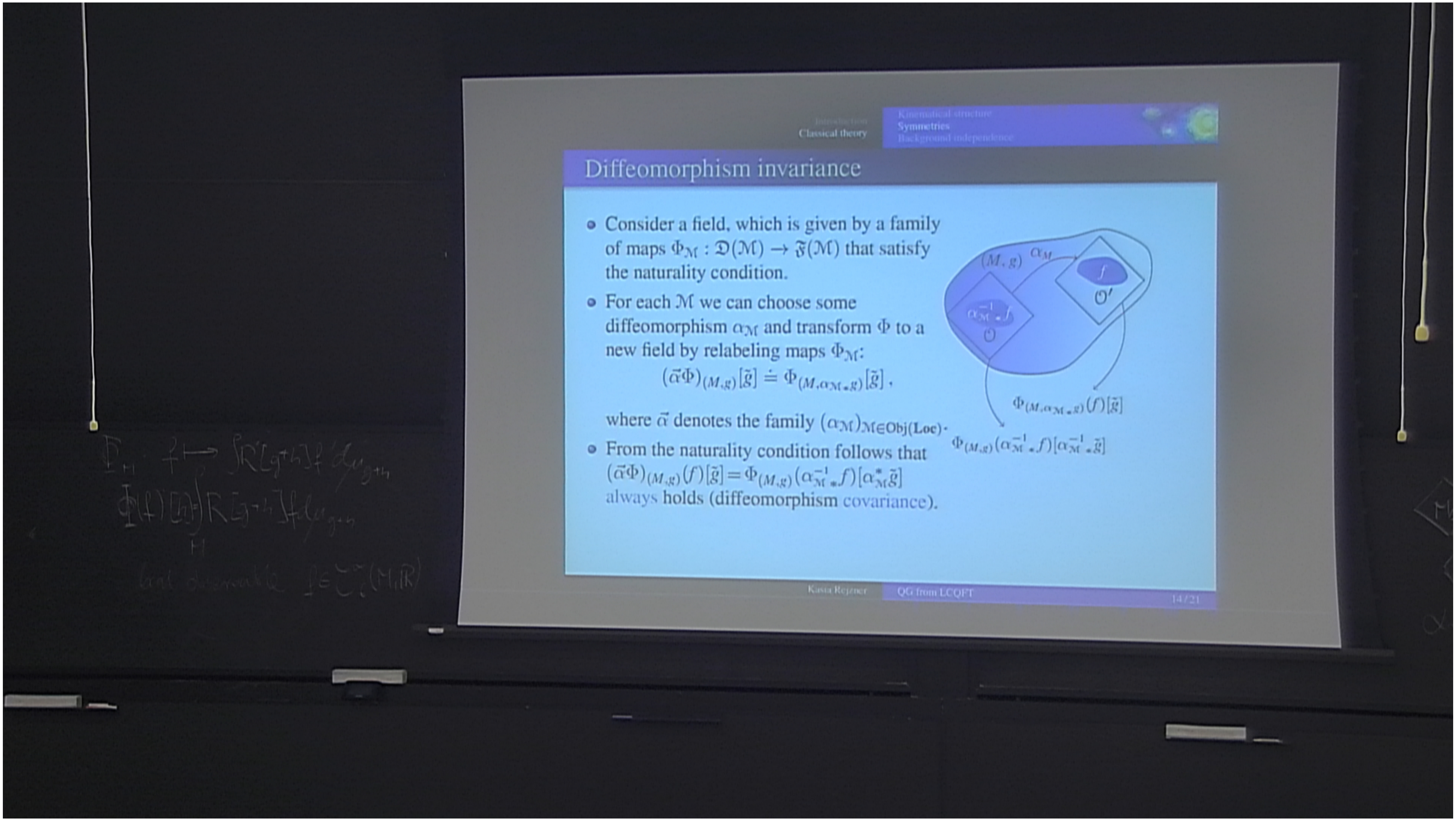
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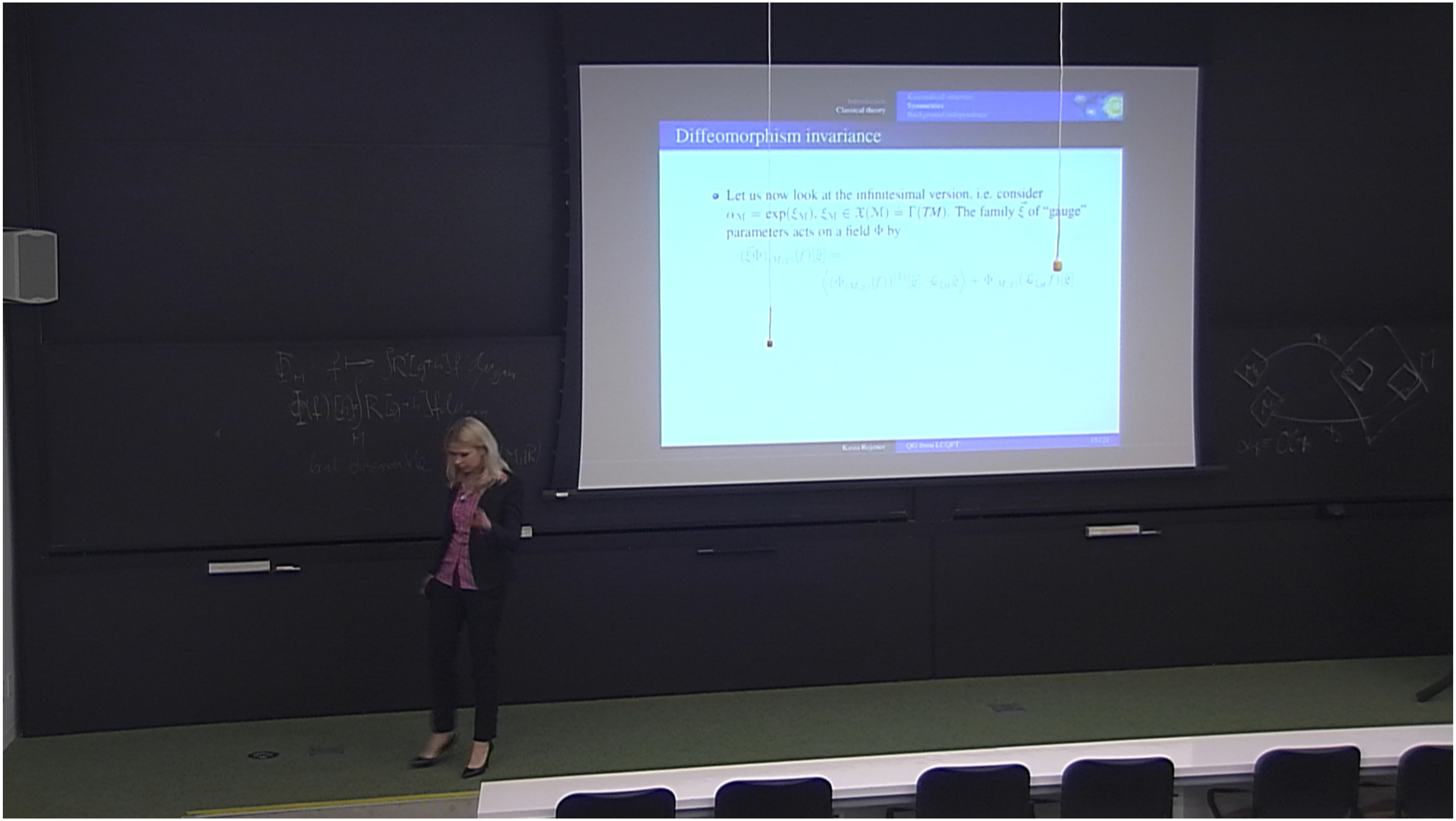
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Let us now look at the infinitesimal version, i.e. consider $\alpha_M = \exp(\xi_M)$, $\xi_M \in \mathfrak{X}(M) \doteq \Gamma(TM)$. The family $\vec{\xi}$ of "gauge" parameters acts on a field Φ by

Diffeomorphism invariance

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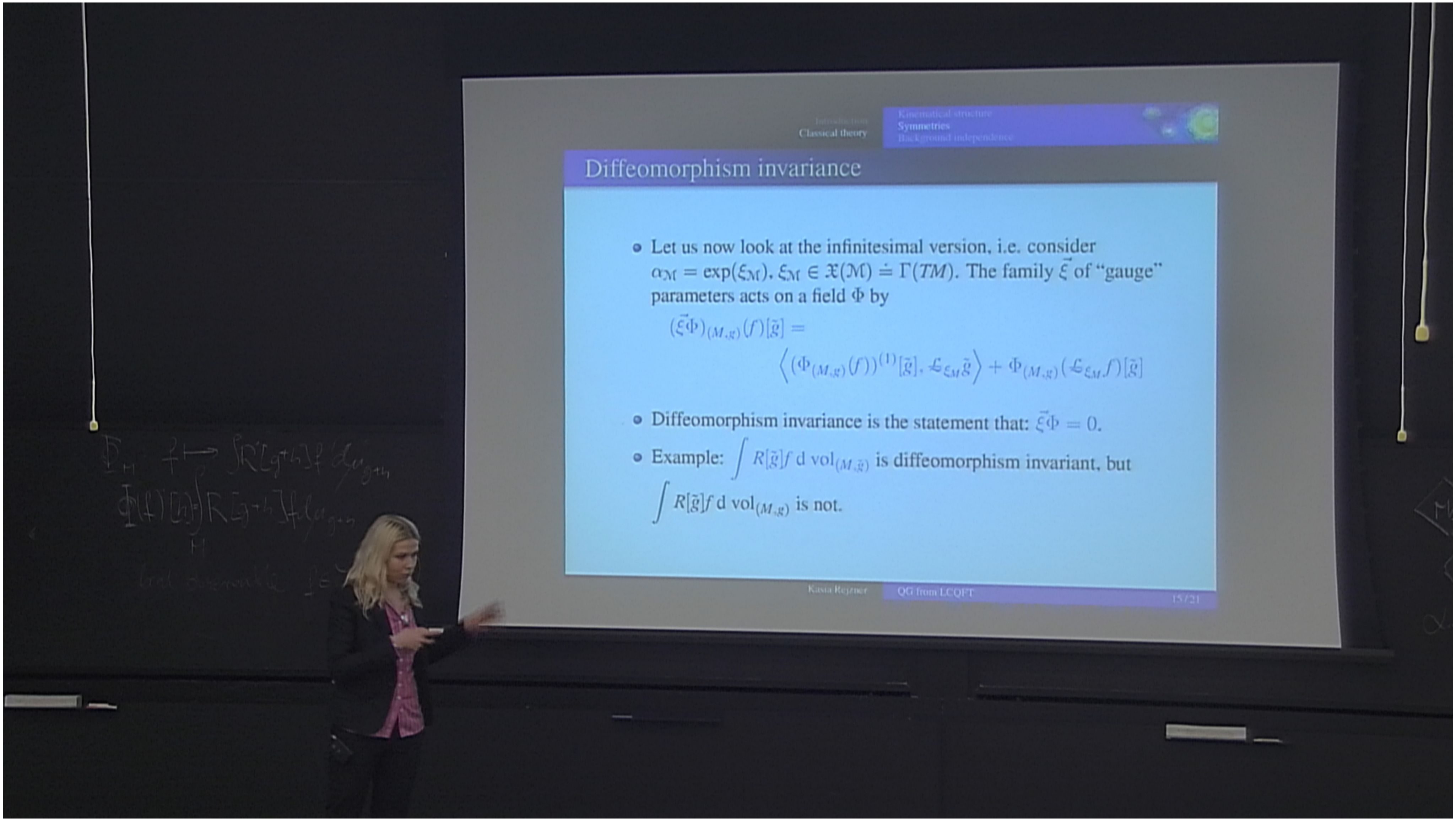
$$(\vec{\xi}\Phi)_{(M,g)}(f)[\tilde{g}] = \left\langle (\Phi_{(M,g)}(f))^{(1)}[\tilde{g}], \pounds_{\xi_M}\tilde{g} \right\rangle + \Phi_{(M,g)}(\pounds_{\xi_M} f)[\tilde{g}]$$

- Diffeomorphism invariance is the statement that: $\vec{\xi}\Phi = 0$.
- Example: $\int R[\tilde{g}] f \, \mathrm{d}\,\mathrm{vol}_{(M,\tilde{g})}$ is diffeomorphism invariant, but $\int R[\tilde{g}] f \, \mathrm{d}\,\mathrm{vol}_{(M,g)}$ is not.



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Diffeomorphism invariance
Let us now look at the infinitesimal version, i.e. consider $\alpha_{\mathcal{M}} = \exp(\xi_{\mathcal{M}})$, $\xi_{\mathcal{M}} \in \mathfrak{X}(\mathcal{M}) \doteq \Gamma(T\mathcal{M})$. The family $\vec{\xi}$ of "gauge" parameters acts on a field Φ by
$(\vec{\xi}\Phi)_{(M,g)}(f)[\tilde{g}] = \left\langle (\Phi_{(M,g)}(f))^{(1)}[\tilde{g}], \pounds_{\xi_M}\tilde{g} \right\rangle + \Phi_{(M,g)}(\pounds_{\xi_M} f)[\tilde{g}]$
Diffeomorphism invariance is the statement that: $\vec{\xi}\Phi = 0$.
Example: $\int R[\tilde{g}] f \, d\,\mathrm{vol}_{(M,\tilde{g})}$ is diffeomorphism invariant, but $\int R[\tilde{g}] f \, d\,\mathrm{vol}_{(M,g)}$ is not.
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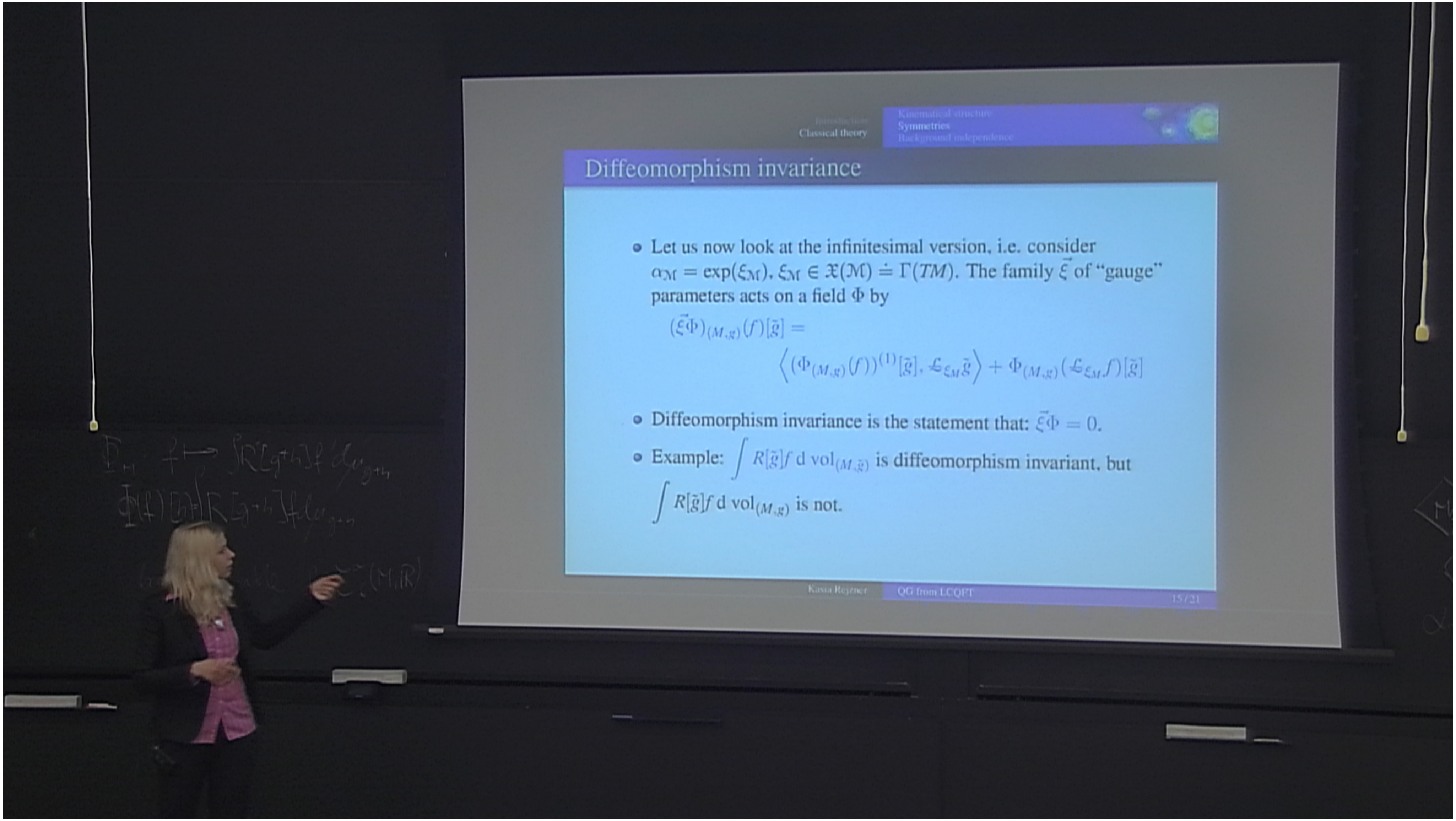
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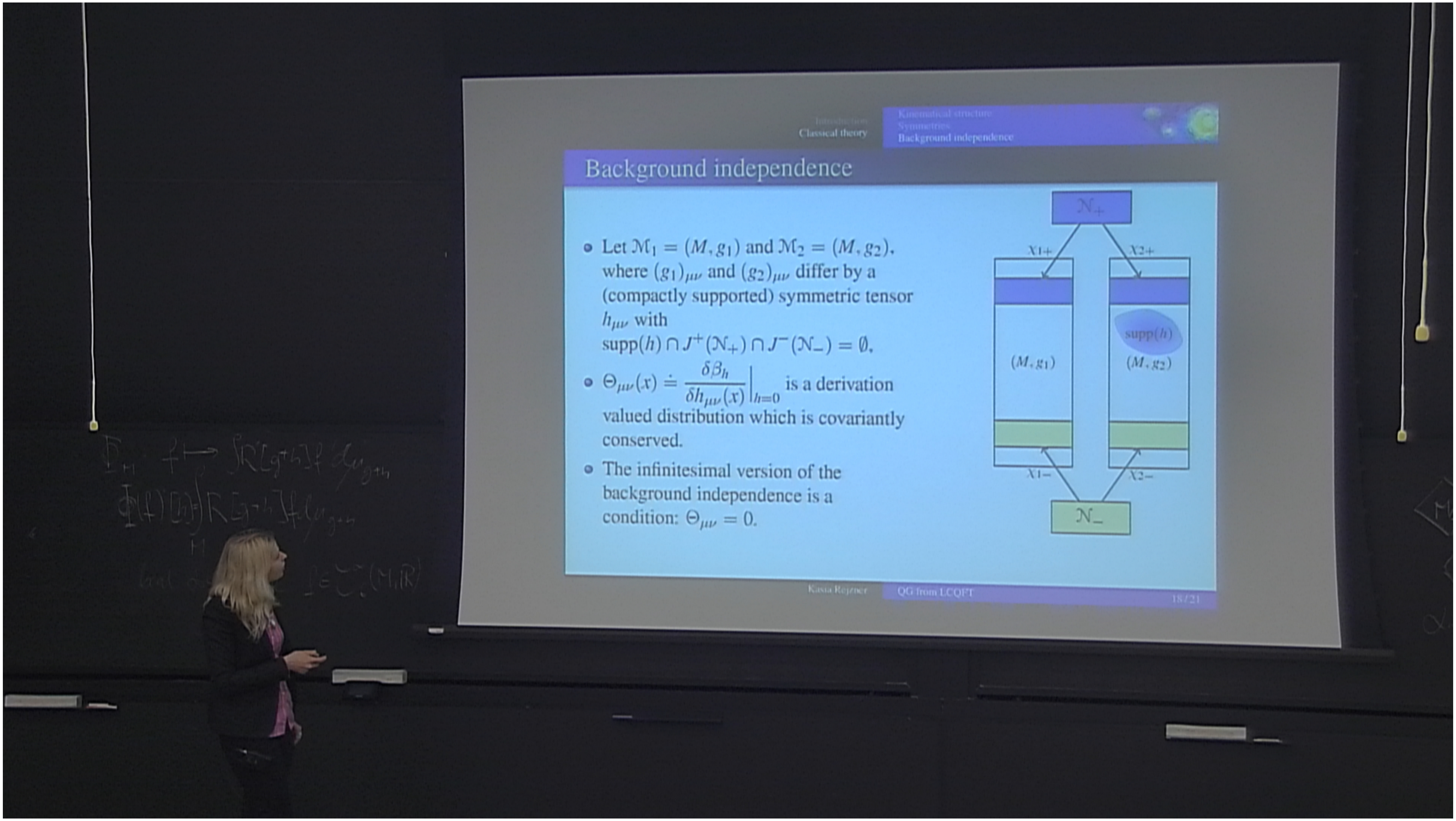
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Let $\mathcal{M}_1 = (M, g_1)$ and $\mathcal{M}_2 = (M, g_2)$, where $(g_1)_{\mu\nu}$ and $(g_2)_{\mu\nu}$ differ by a (compactly supported) symmetric tensor $h_{\mu\nu}$ with $\text{supp}(h) \cap J^+(\mathcal{N}_+) \cap J^-(\mathcal{N}_-) = \emptyset$.
$\Theta_{\mu\nu}(x) \doteq \frac{\delta\beta_h}{\delta h_{\mu\nu}(x)}\Big|_{h=0}$ is a derivation valued distribution which is covariantly conserved.
The infinitesimal version of the background independence is a condition: $\Theta_{\mu\nu} = 0$.
$\mathcal{N}_+$
χ_{1+}
χ_{2+}
supp(h)
(M, g_1)
(M, g_2)
χ_{1-}
χ_{2-}
$\mathcal{N}_-$
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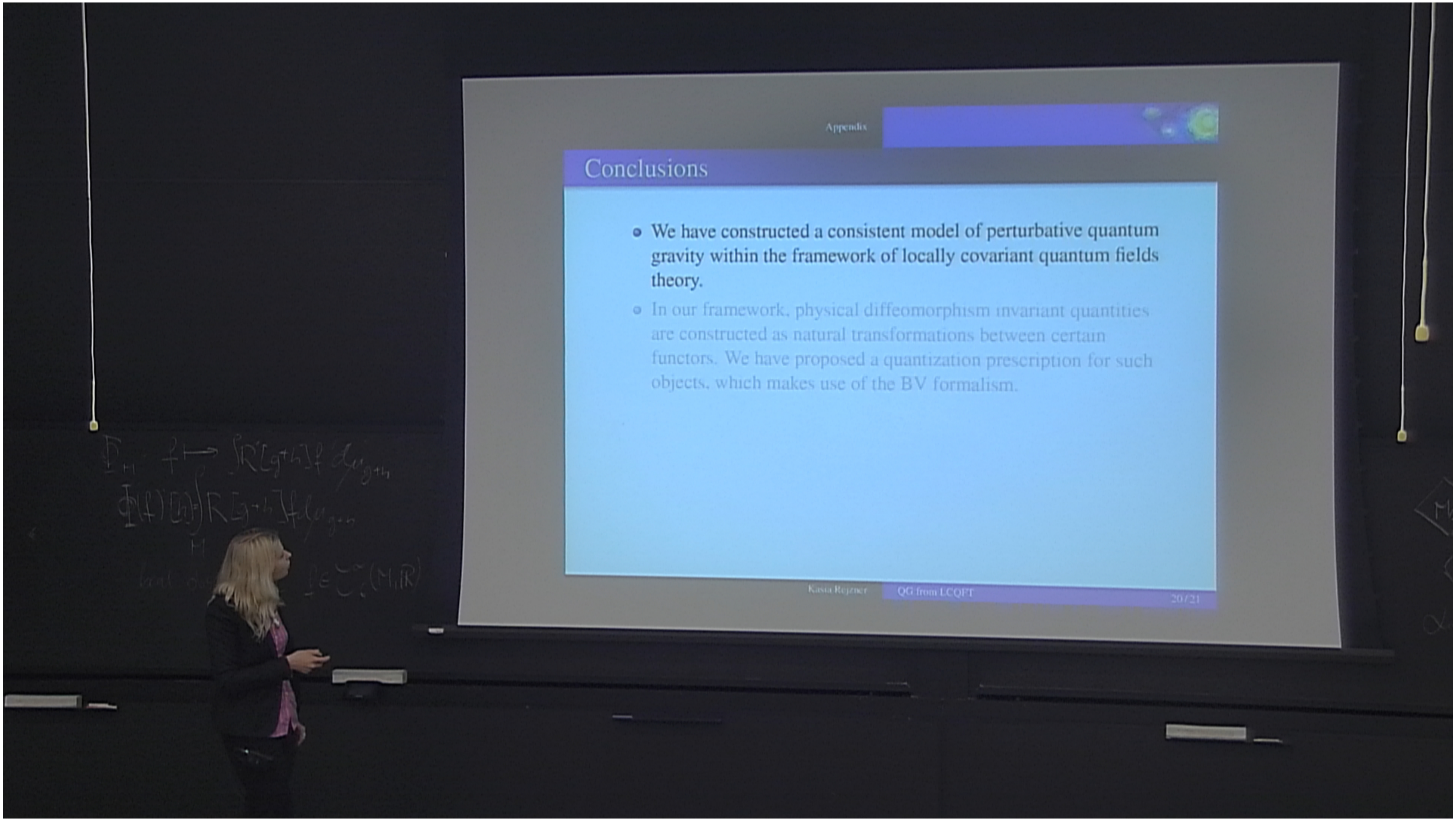
Appendix
Conclusions
We have constructed a consistent model of perturbative quantum gravity within the framework of locally covariant quantum fields theory.
In our framework, physical diffeomorphism invariant quantities are constructed as natural transformations between certain functors. We have proposed a quantization prescription for such objects, which makes use of the BV formalism.
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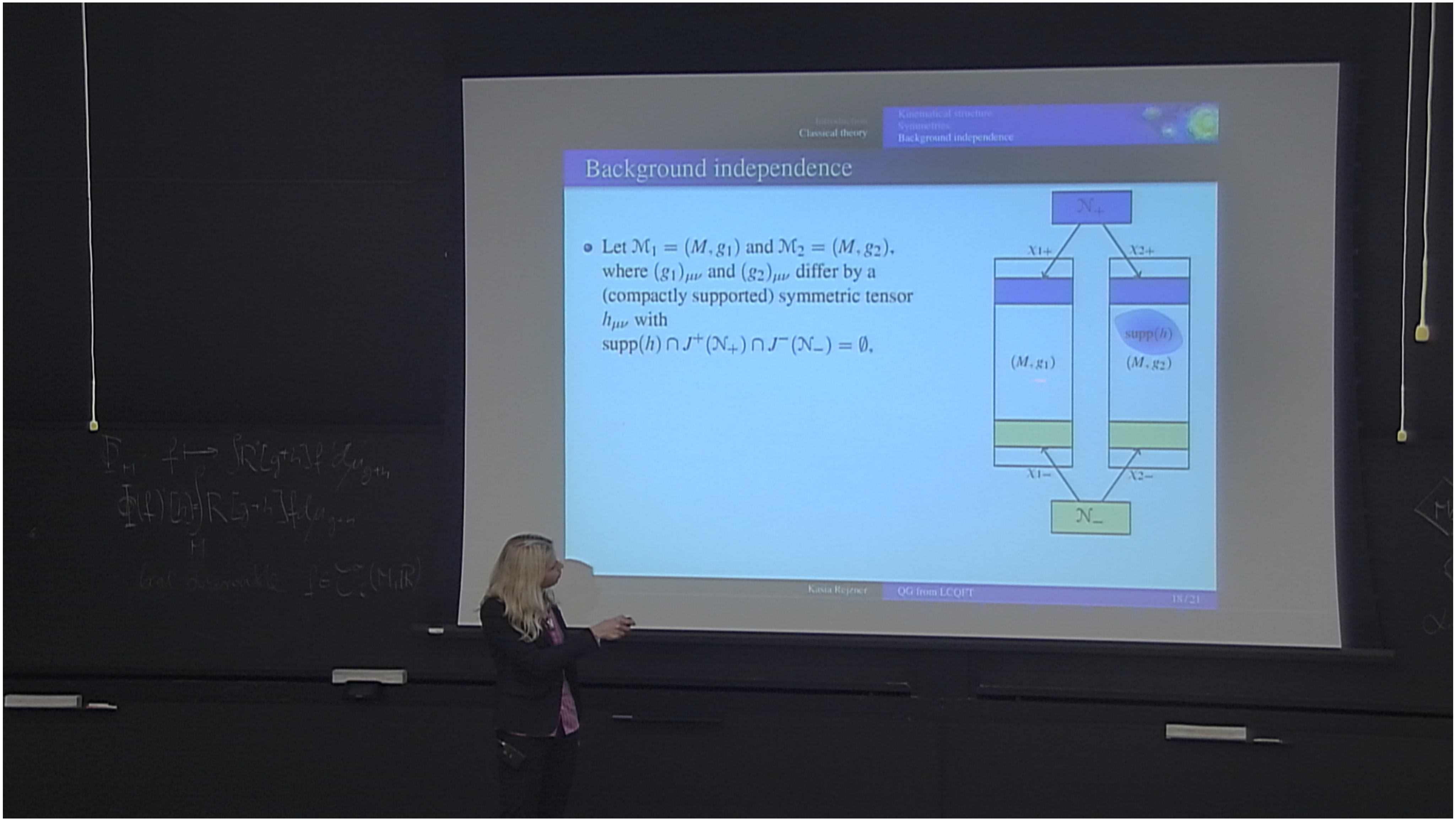

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χ_{2+}
supp(h)
(M, g_1)
(M, g_2)
χ_{1-}
χ_{2-}
$\mathcal{N}_-$
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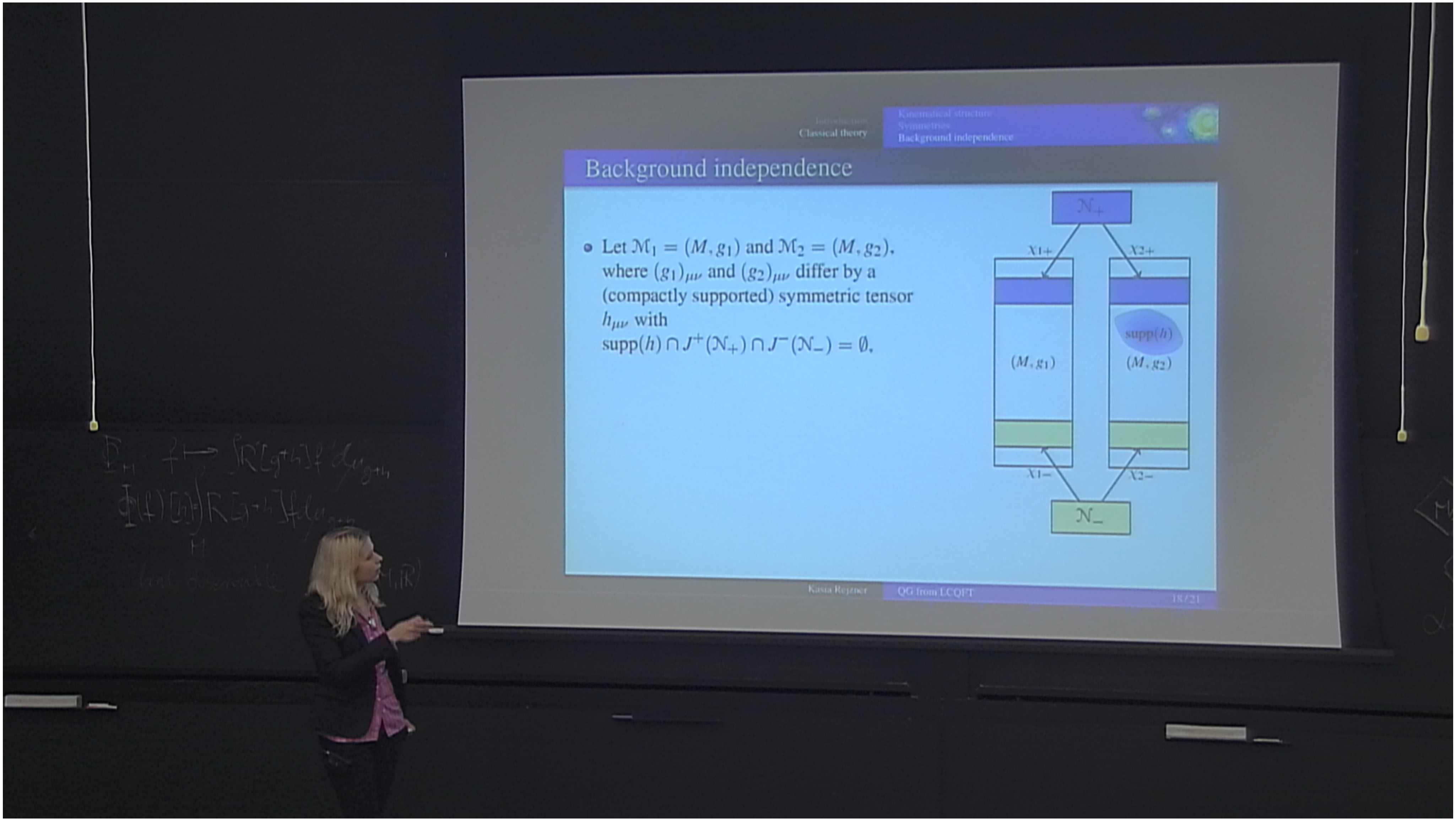

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χ_{2-}
$\mathcal{N}_-$
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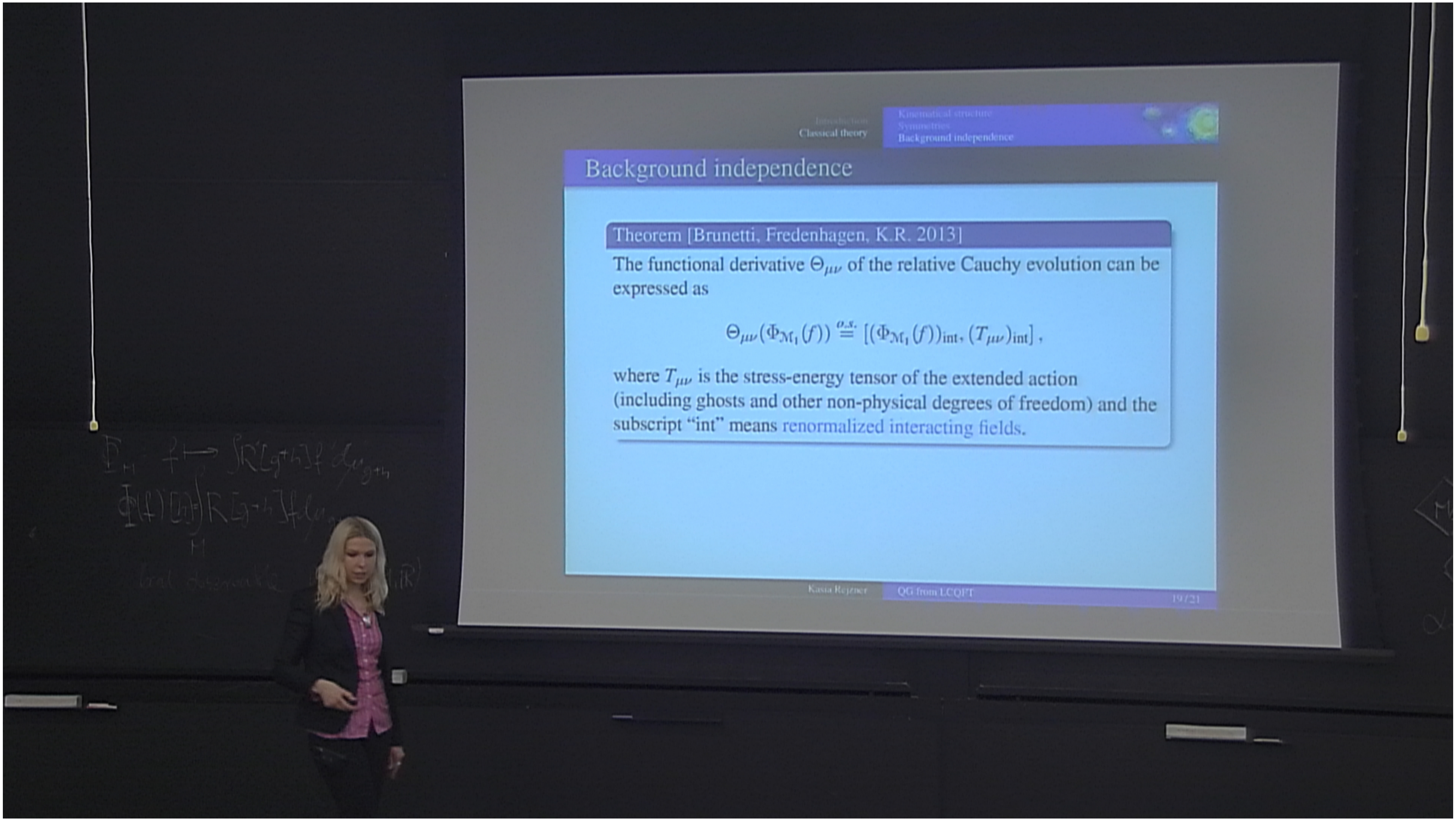
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Theorem [Brunetti, Fredenhagen, K.R. 2013]
The functional derivative $\Theta_{\mu\nu}$ of the relative Cauchy evolution can be expressed as
$$\Theta_{\mu\nu}(\Phi_{\mathcal{M}_1}(f)) \overset{o.s.}{=} [(\Phi_{\mathcal{M}_1}(f))_{\text{int}}, (T_{\mu\nu})_{\text{int}}] ,$$
where $T_{\mu\nu}$ is the stress-energy tensor of the extended action (including ghosts and other non-physical degrees of freedom) and the subscript "int" means renormalized interacting fields.
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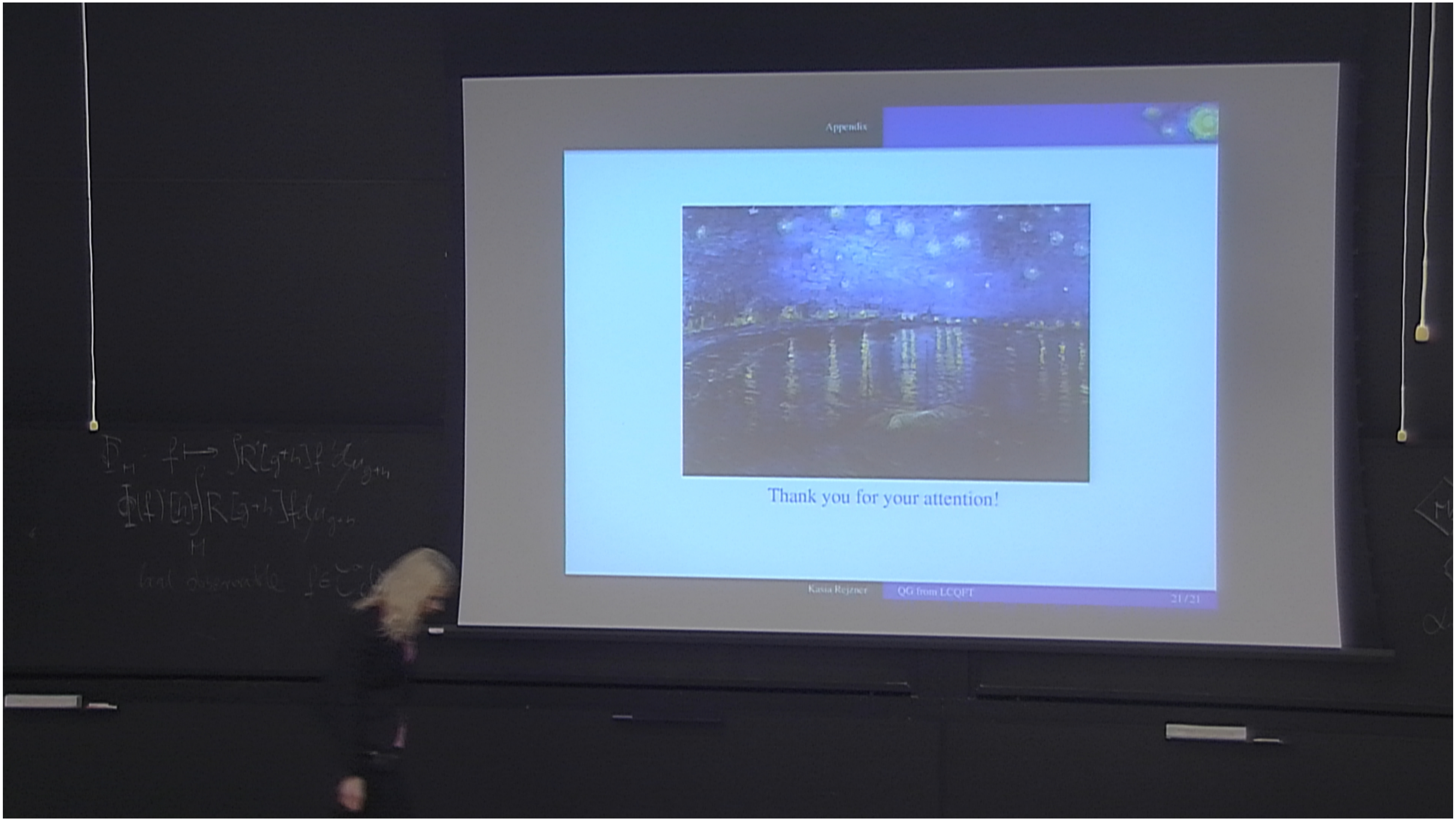
Appendix
Thank you for your attention!
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