Title: Black holes without Lorentz symmetry

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Abstract: Our current definition of what a black hole is relies heavily on the assumption that there exists a finite maximum speed of propagation for any signal. Indeed, one is tempted to think that the notion of a black hole has no place in a world with infinitely fast signal propagation. I will use concrete examples from Lorentz-violating gravity theories to demonstrate that this naive expectation is not necessarily true.

Black holes without Lorentz symmetry

Thomas P. Sotiriou

with: Enrico Barausse, Ted Jacobson, Ian Vega and Daniele Vernieri

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LV and black hole structure

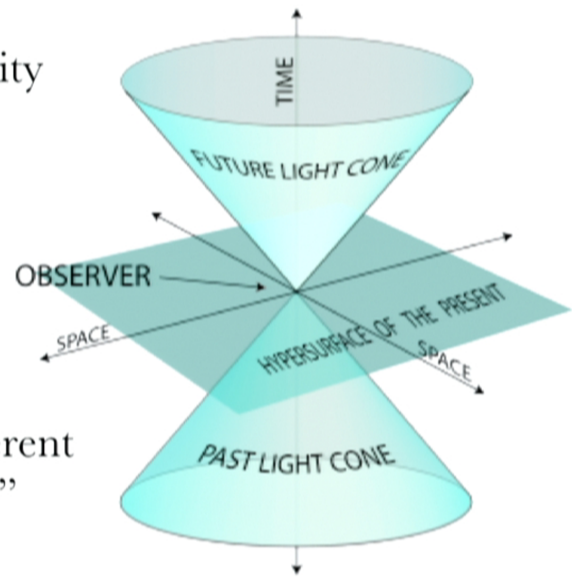
Causal structure in special relativity

- LV with linear dispersion relations

$$\omega \propto k$$

- Different modes have different speeds and different "light" cones

- But there are still "light" cones!



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What happens to black holes?

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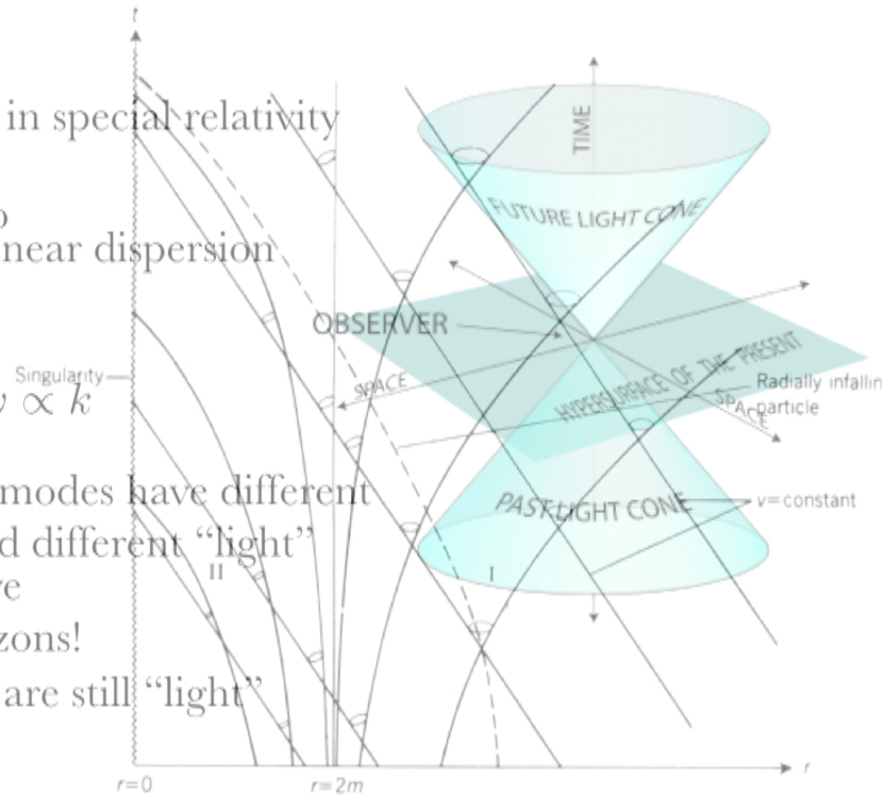
- Different modes have different speeds and different "light" cones
- They will have multiple horizons!
- But there are still "light" cones!



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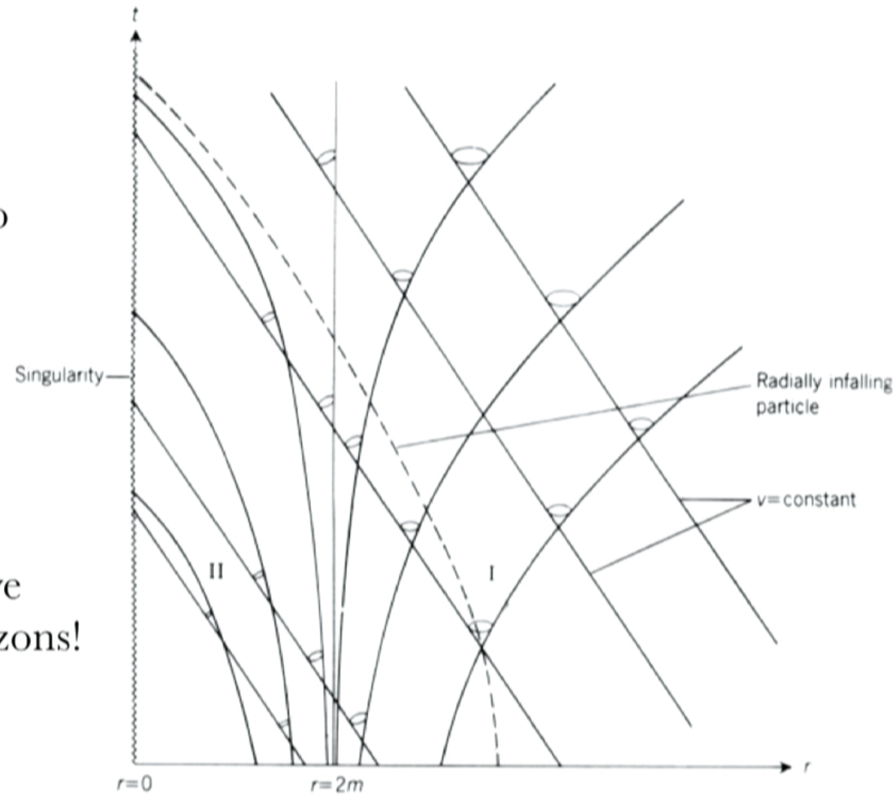

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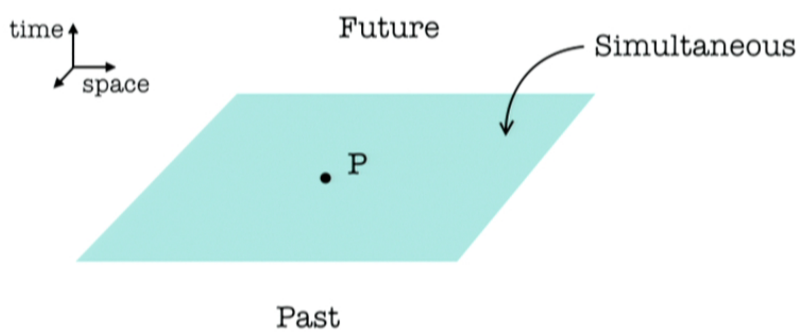
LV and black hole structure

- LV with non-linear dispersion relations

$$\omega^2 \propto k^2 + ak^4 + \dots$$

- No light cones!

Causal structure without relativity



No black holes at all??

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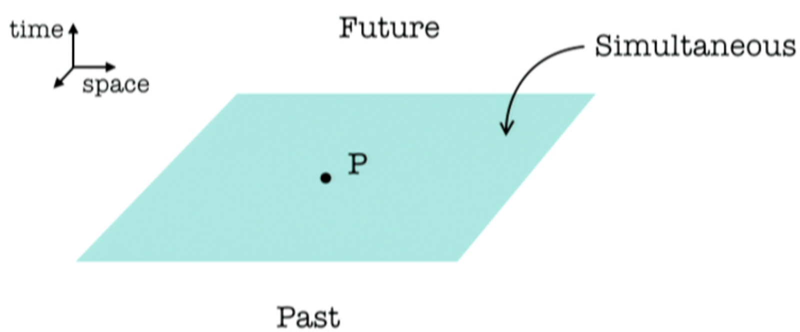
$$\omega^2 \propto k^2 + ak^4 + ...$$

- No light cones!

Causal structure without relativity



No black holes at all??

Einstein-aether theory

The action of the theory is

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int d^4x \sqrt{-g}(-R - M^{\alpha\beta\mu\nu}\nabla_\alpha u_\mu \nabla_\beta u_\nu)$$

where

$$M^{\alpha\beta\mu\nu} = c_1 g^{\alpha\beta} g^{\mu\nu} + c_2 g^{\alpha\mu} g^{\beta\nu} + c_3 g^{\alpha\nu} g^{\beta\mu} + c_4 u^\alpha u^\beta g_{\mu\nu}$$

and the aether is implicitly assumed to satisfy the constraint

$$u^\mu u_\mu = 1$$

- Most general theory with a unit timelike vector field which is second order in derivatives

T. Jacobson and D. Mattingly, Phys. Rev. D 64, 024028 (2001).

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Einstein-aether theory

- Extensively tested and still viable
- It propagates a spin-2, a spin-1 and spin-0 mode.
- Linear dispersion relations.
- These modes travel at different speeds.
- We expect multiple horizons!
- Requires a UV-completion (which would likely modify the dispersion relations and lead to arbitrarily higher speeds for all modes).

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Hypersurface orthogonality

Now assume $$u_\alpha = \frac{\partial_\alpha T}{\sqrt{g^{\mu\nu}\partial_\mu T \partial_\nu T}}$$

and choose T as the time coordinate

$$u_\alpha = \delta_{\alpha T}(g^{TT})^{-1/2} = N\delta_{\alpha T}$$

Replacing in the action and defining one gets

$$S^{ho}_{æ} = \frac{1}{16\pi G_H}\int dT d^3x N\sqrt{h}\left(K_{ij}K^{ij} - \lambda K^2 + \xi\, {}^{(3)}R + \eta a^i a_i\right)$$

with $a_i = \partial_i \ln N$ and the parameter correspondence

$$\frac{G_H}{G_{æ}} = \xi = \frac{1}{1-c_{13}} \qquad \lambda = \frac{1+c_2}{1-c_{13}} \qquad \eta = \frac{c_{14}}{1-c_{13}}$$

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Horava-Lifshitz gravity

The action of the theory is

$$S_{HL} = \frac{1}{16\pi G_H}\int dT d^3x\, N\sqrt{h}(\, L_2 + \frac{1}{M_\star^2}L_4 + \frac{1}{M_\star^4}L_6)$$

where

$$L_2 = K_{ij}K^{ij} - \lambda K^2 + \xi^{(3)}R + \eta a_i a^i$$

L_4 : contains all 4th order terms constructed with the induced metric h_{ij} and a_i

L_6 : contains all 6th order terms constructed in the same way

P. Hořava, Phys. Rev. D 79, 084008 (2009)
D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Let. 104, 181302 (2010)

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Our goal

We are interested in vacuum black hole solutions which are

- spherically symmetric (so, also h.o. aether)
- static
- asymptotically flat
- everywhere regular apart from the central singularity

Finding such solutions analytically seems unfeasible, so we find them numerically

- There is a one-parameter family of such solutions
- I suppress all the (complicated and challenging) details about how to prove that and about how to find these solutions

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Interior solution

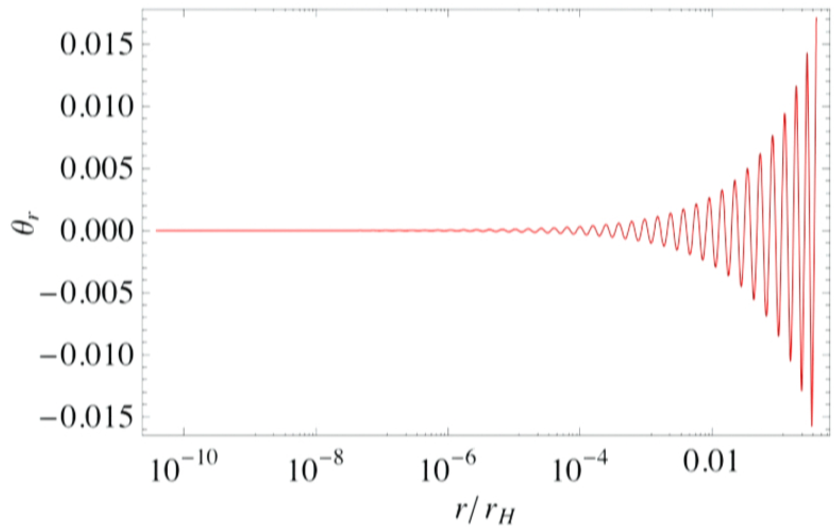
- Curvature singularity at the centre

Lorentz factor of the aether as measured by the future directed observer orthogonal to $r = $ const. hypersurfaces

$$\gamma_r \equiv u^\alpha_{\text{obs}} u_\alpha$$

and the boost angle

$$\theta_r = \text{arccosh}\gamma_r$$



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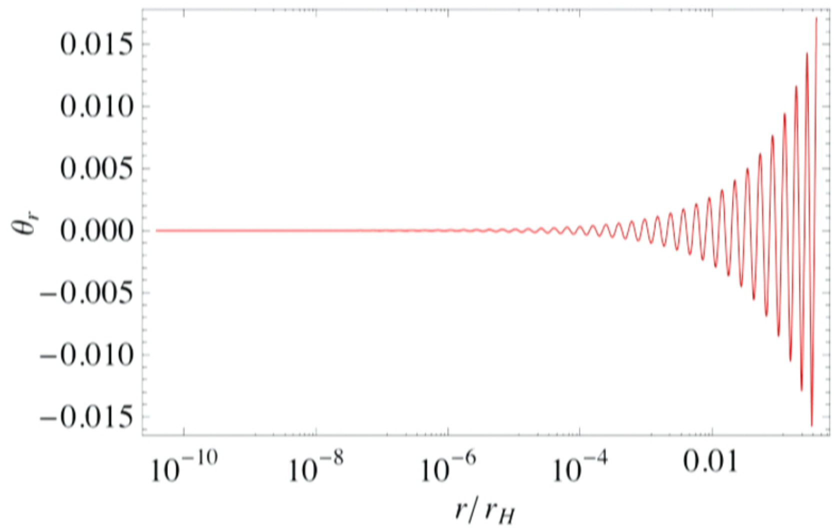


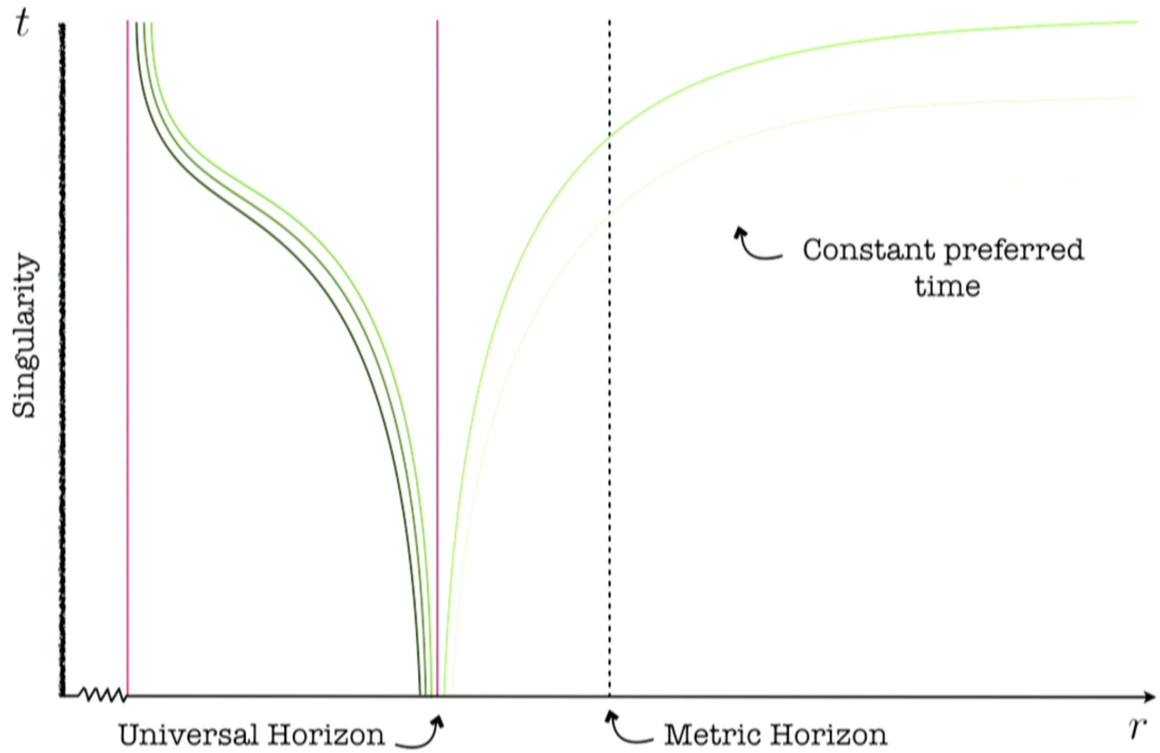
Interior solution

- Signals cannot travel backwards in time
- Future and past direction are locally defined by the aether
- The aether is orthogonal to constant time hypersurfaces in the preferred foliation
- When the boost angle vanishes the aether is orthogonal to constant r hypersurfaces as well!
- Ultimate causal boundary for all signals!

The same result found at decoupling in Horava gravity. However, this horizons seems to be unstable!

D. Blas and S. Sibiryakov, Phys. Rev. D 84, 124043 (2011)

Spacetime diagram



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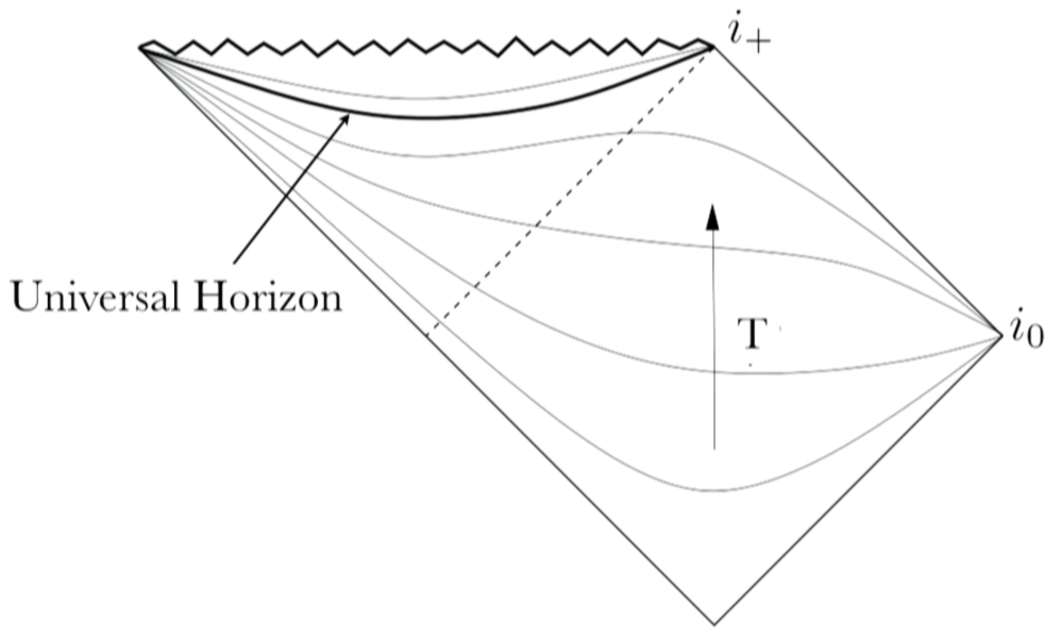
Penrose diagram



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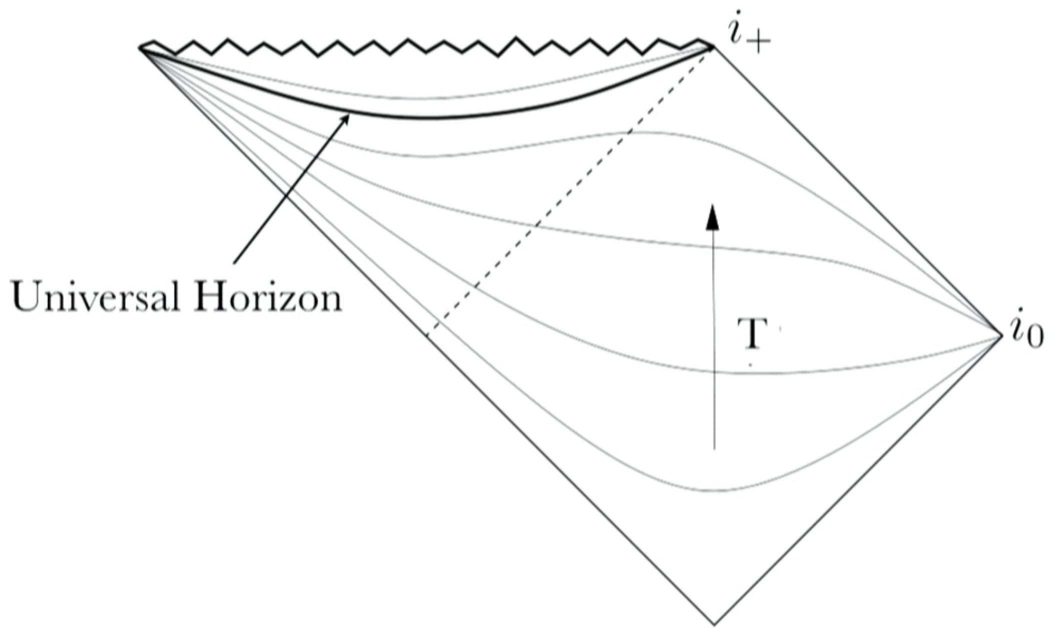

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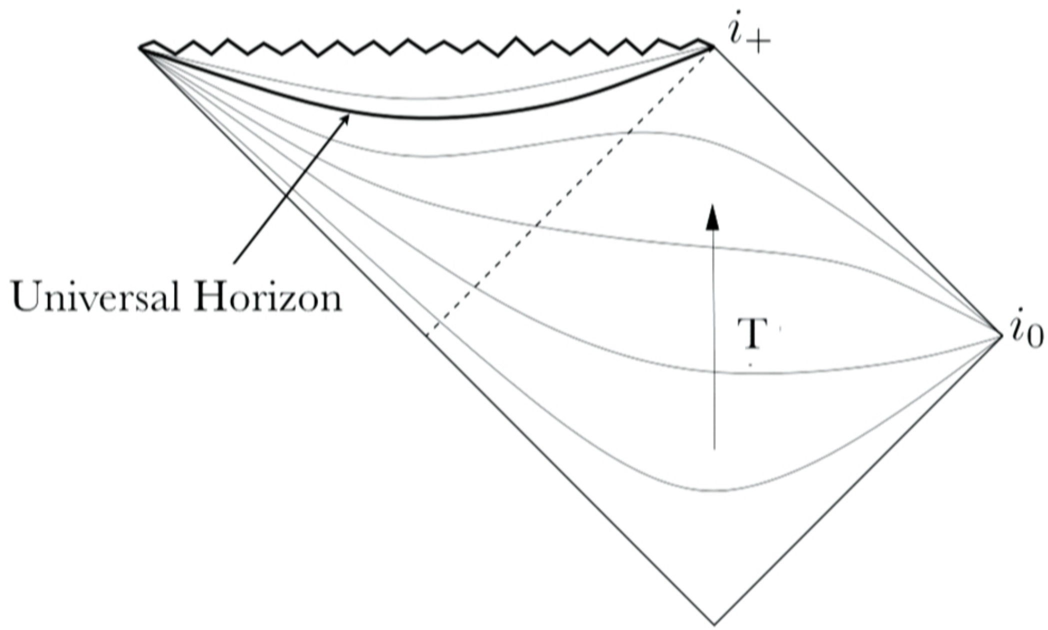

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Slowly rotating BHs

- What about rotating black holes?
- Difficult to find them, easier to focus on slow rotation

Most general slowly rotating, stationary, axisymmetric metric

$$\begin{aligned} ds^2 &= f(r)dt^2 - \frac{B(r)^2}{f(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) \\ &\quad + \epsilon r^2 \sin^2\theta\, \Omega(r,\theta) dt d\varphi + \mathcal{O}(\epsilon^2) \end{aligned}$$

- $f(r)$ and $B(r)$ are the "seed" solutions, so they are known
- $\Omega(r,\theta)$ is to be determined at the next order in ϵ

Slow rotation and the aether

Symmetries require that

$$\partial_t u_\mu = \partial_\phi u_\mu = 0$$

<u>If</u> the aether is hypersurface orthogonal then

$$\epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma = 0$$

and then one obtains

$$u_\phi = 0$$

So, one has for the aether at

$$\boldsymbol{u} = \frac{1 + f A^2}{2A} dt + \frac{B}{2A}\left(\frac{1}{f} - A^2\right) dr + \mathcal{O}(\epsilon)^2$$

i.e. nothing to determine at $\mathcal{O}(\epsilon)$!

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Solutions

- The two theories share the spherical solutions, but not the slowly rotating one!
- There are no slowly rotating solutions with a preferred foliation in Einstein-aether theory. Can there be a universal horizon then?
- In HL gravity the foliation remain the same in slow rotation

E. Barausse and T.P.S., Class. Quant. Grav. 30, 244010 (2013)

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Lower-dimensional model

- 1+1 dimensions: No dynamics, identical to 1+1 ae-theory at all energies
- 2+1 dimensions: 1 scalar dof with same behaviour!

Most general action is 2+1 dimensions:

$$S = \frac{M_{\rm pl}^2}{2}\int d^2x\, dt\, N\sqrt{g}\Big\{K^{ij}K_{ij} - \lambda K^2 + \xi R + \eta\, a_i a^i + g_1 R^2 + g_2 \nabla^2 R + g_3 (a^i a_i)^2 + g_4 R a^i a_i + g_5 a^2(\nabla\cdot a) + g_6(\nabla\cdot a)^2 + g_7(\nabla_i a_j)(\nabla^i a^j)\Big\}$$

A good toy model for some of the open questions

T.P.S., M. Visser and S. Weinfurtner, Phys. Rev. D 83, 124021 (2011)

3d black holes

- No solutions with AdS asymptotics unless $\eta = 0$

Most general solution when $\eta = 0$

$$ds^2 = F(r)^2 dt^2 - \frac{1}{F(r)^2} dr^2 - r^2 (d\phi + \Omega(r) dt)^2$$

$$u_t = \sqrt{F^2 + \left(\frac{a}{r} + br\right)^2} \qquad u_r = \frac{1}{F^2}\left(\frac{a}{r} + br\right)$$

$$F^2 = -\mathcal{M} + \frac{\bar{\mathcal{J}}^2}{4r^2} - \bar{\Lambda} r^2 \qquad \Omega = -\frac{\mathcal{J}}{2r^2}$$

$$\bar{\mathcal{J}}^2 = \frac{\mathcal{J}^2 + 4a^2(1-\xi)}{\xi} \qquad \bar{\Lambda} = \Lambda - \frac{b^2(2\lambda - \xi - 1)}{\xi}$$

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Singularities and horizons

$$R = -6\bar{\Lambda} + \frac{1}{2r^4}\left(\bar{\mathcal{J}}^2 - \mathcal{J}^2\right)$$

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = 12\bar{\Lambda}^2 - \frac{2\bar{\Lambda}}{r^4}\left(\bar{\mathcal{J}}^2 - \mathcal{J}^2\right) + \frac{11}{4r^8}\left(\bar{\mathcal{J}}^2 - \mathcal{J}^2\right)^2$$

Non-flat asymptotics:

Case		$\bar{\mathcal{J}}^2\bar{\Lambda} \geq 0$	$\bar{\mathcal{J}}^2 = 0$	$0 > \bar{\mathcal{J}}^2\bar{\Lambda} > -\mathcal{M}^2$	$\bar{\mathcal{J}}^2\bar{\Lambda} = -\mathcal{M}^2$
$\mathcal{M} > 0,\ \bar{\Lambda} < 0$	horizons	$\tilde{r}_+$ **(b)**	$2r_{(1/2)}$ **(b)**	$r_\pm$ **(b)**	$r_{(1/2)}$ **(b)**
	singularity	spacelike	spacelike	timelike	timelike
$\mathcal{M} > 0,\ \bar{\Lambda} > 0$	horizons	$\tilde{r}_-$ **(c)**	—	—	—
	singularity	timelike	spacelike	spacelike	spacelike
$\mathcal{M} < 0,\ \bar{\Lambda} < 0$	horizons	$\tilde{r}_-$ **(b)**	—	—	—
	singularity	spacelike	timelike	timelike	timelike
$\mathcal{M} < 0,\ \bar{\Lambda} > 0$	horizons	$\tilde{r}_+$ **(c)**	$2r_{(1/2)}$ **(c)**	r_+ **(c)**, r_- **(b)**	$r_{(1/2)}$ **(c)**
	singularity	timelike	timelike	spacelike	spacelike

Singularities and horizons

Flat asymptotics:

$$ds^2 = \left(1 + \frac{\bar{\mathcal{J}}^2}{4r^2}\right) dt^2 - \left(1 + \frac{\bar{\mathcal{J}}^2}{4r^2}\right)^{-1} dr^2 - r^2 \left(d\phi - \frac{\mathcal{J}}{2r^2} dt\right)^2$$

$$u_t = \sqrt{1 + \frac{\bar{\mathcal{J}}^2}{4r^2} + \left(\frac{a}{r} + br\right)^2} \qquad u_r = \frac{4r^2}{4r^2 + \bar{\mathcal{J}}^2} \left(\frac{a}{r} + br\right)$$

- It's a black hole even for $J = 0$
- Similar to foliated 5D Schwarzschild black hole

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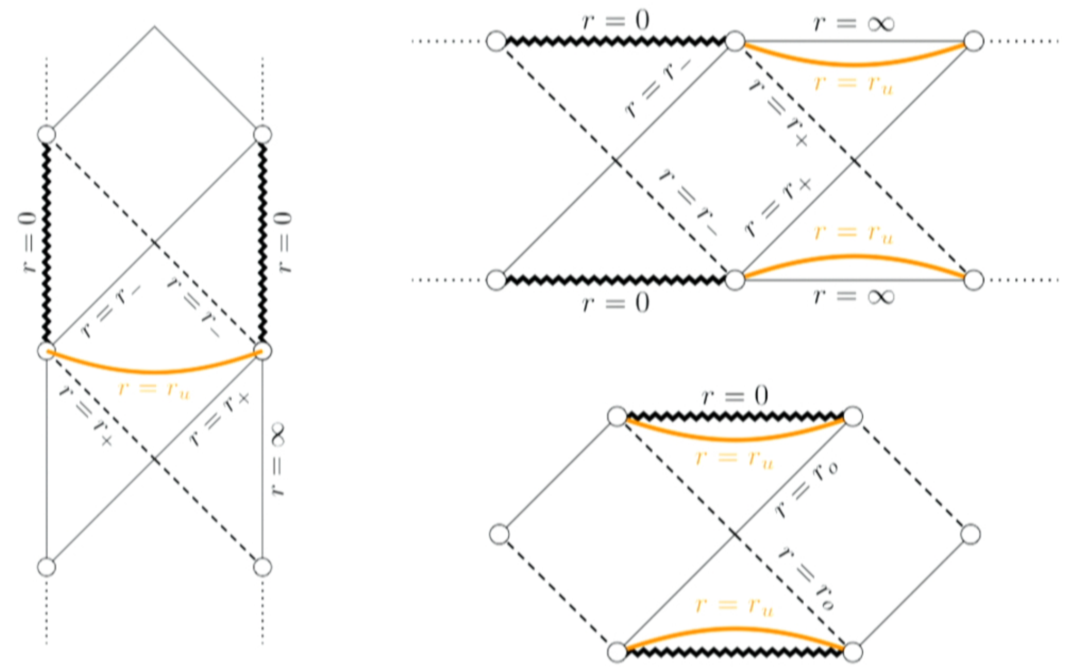
Universal horizons

- They don't always exist even without rotation. E.g. there is no UH for the flat case when $b = 0$ and $J = 0$!
- They are not always regular when they exist - a and b are not independent if regularity is imposed.
- They are not always behind the event horizon - they can be beyond the de Sitter horizon.

Penrose diagrams



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Conclusions and Perspectives

- Black holes are of great interest in Lorentz-violating theories. New notion: "universal horizon"
- Examples show that the UH survives rotation when there is a preferred foliation and that it is not specific to black hole spacetimes.
- But it is not always there when there are Killing horizons.
- What about rapidly rotating black holes in 4D?
- Is this horizon stable?
- Will this horizon (and black holes) exist if one has less symmetry? Does it form from collapse?

M. Saravani, N. Afshordi and R. B. Mann, Phys. Rev. D 89, 084029 (2014)

Thomas P. Sotiriou - Perimeter Institute, May 8th 2014