Title: TBA

Date: Apr 25, 2014 11:00 AM

URL: http://pirsa.org/14040142

Abstract:

Condensed matter systems

Condensed matter systems
: systems with many ($\sim 10^{23}$) electrons and ions with Coulomb interaction

Weakly correlated systems

Many ≈ sum of individuals

$$H^e_{tot} \sim \sum_{i=1}^{N} h_{eff}(i) \left(= \sum_{i=1}^{N} \frac{p_i^2}{2m^*} + v_{eff}(x_i) \right)$$

Free electron,
Kohn-Sham equation,
...

If a Hamiltonian is separable, it is essentially a one-body problem.

$$\hat{h}_{eff}(i)|\psi_i >= \epsilon_i|\psi > \Longrightarrow |\Psi >\sim \prod_{i}^{N} |\psi_i > \quad E_{tot} \sim \sum_{i=1}^{N} \epsilon_i$$

Weakly correlated systems

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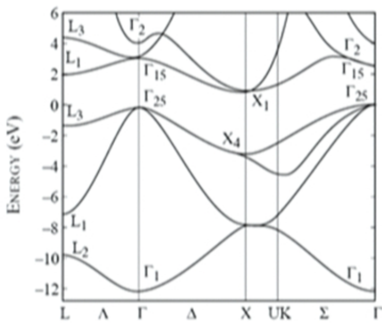
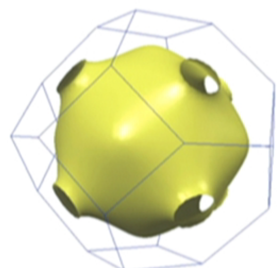
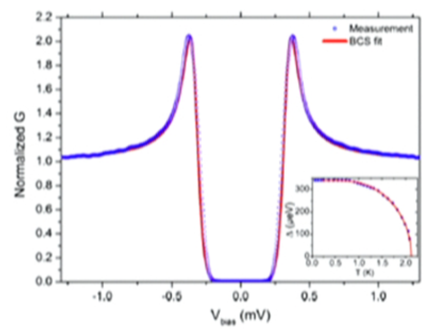
Triumph in 20^{th} century physics (mostly from DFT)



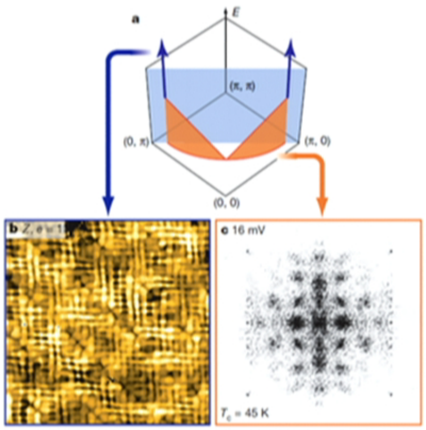


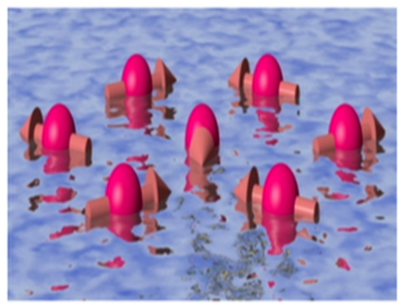


Strongly correlated systems

Many ≉ sum of individuals *(More is different)*

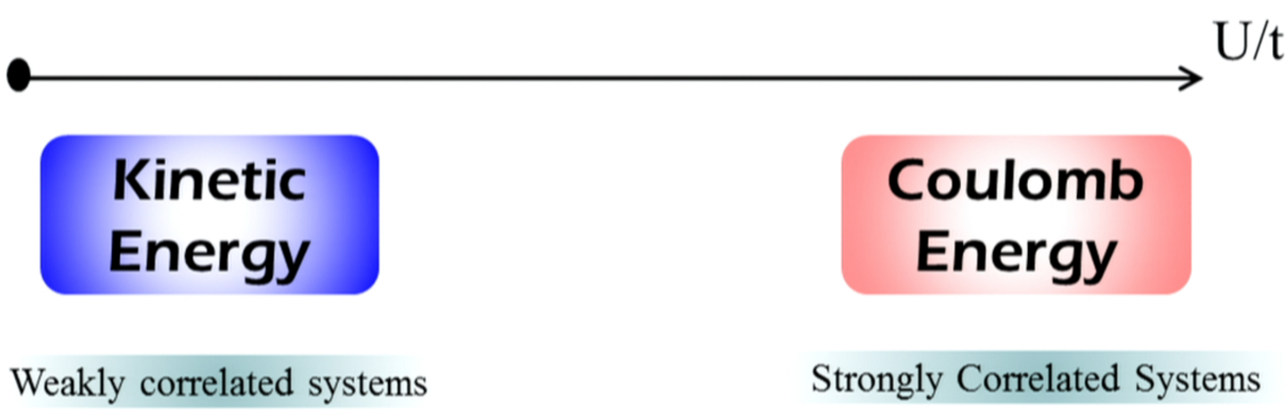
If a Hamiltonian is **not** separable, novel phenomena might happen!





Coulomb interaction is important.

Two regimes

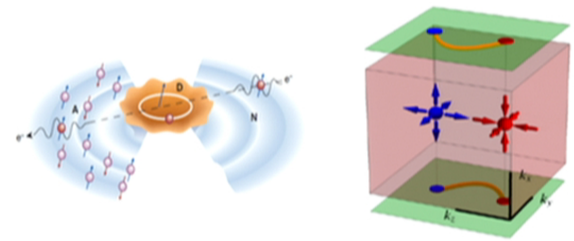

U/t
Kinetic Energy
Coulomb Energy
Weakly correlated systems
Strongly Correlated Systems

New regimes

λ/t

Spin-orbit

Topological insulator,…



Iridates,…

(see also Pesin and Balents)

U/t

Kinetic Energy

Coulomb Energy

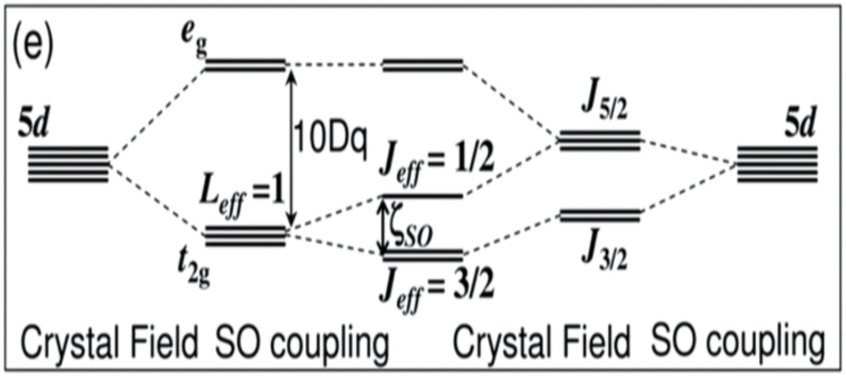
Weakly correlated systems

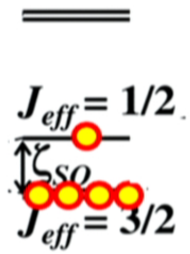
Strongly Correlated Systems

Iridates

- $Ir^{4+}=5d^5$

- Strong spin-orbit and coulomb interaction



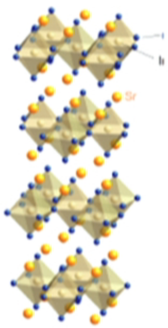


B.J. Kim. et.al. 2008

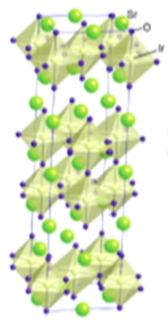
Local states : J_{eff} =1/2 states (half-filling)

Iridates

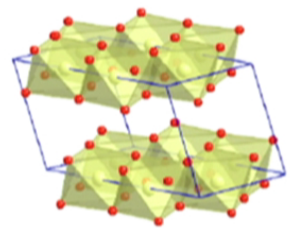
Structures



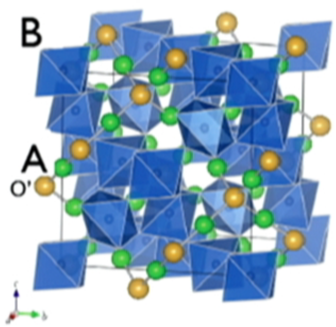
Sr_2IrO_4



$Sr_3Ir_2O_7$



Na_2IrO_3

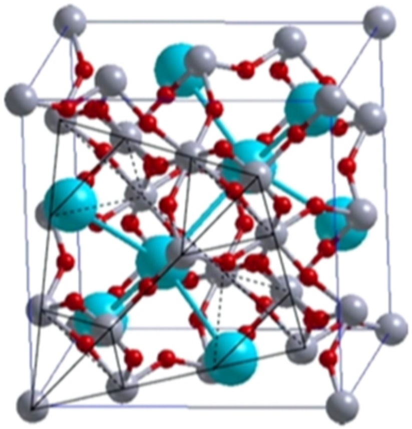


$R_2Ir_2O_7$

pyrochlores

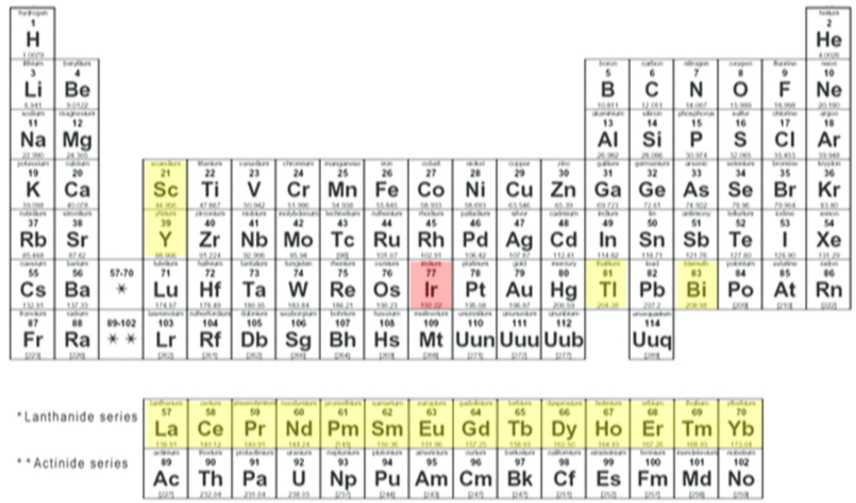
$Na_4Ir_3O_8$

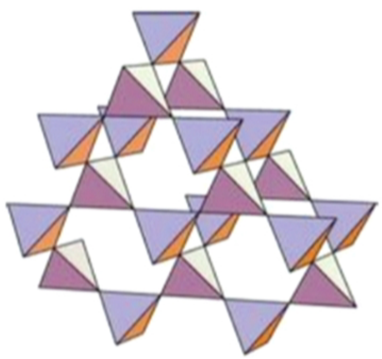
Pyrochlore structure



$A_2B_2O_7$

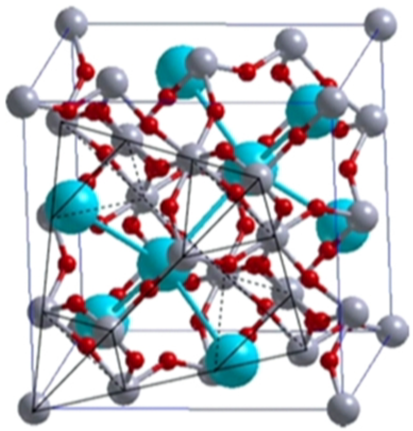
A : rare earth B : Ir^{4+}





A (B) sublattice

Pyrochlore structure



$A_2B_2O_7$

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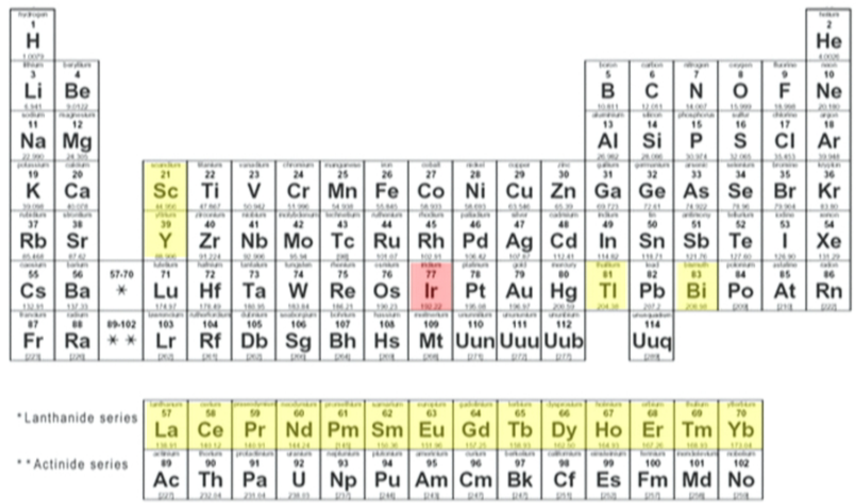
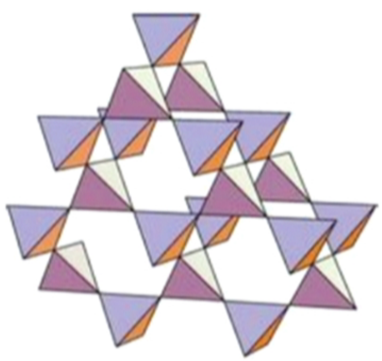
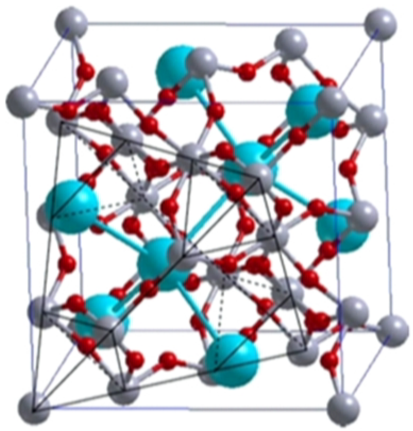




A (B) sublattice

Pyrochlore iridates



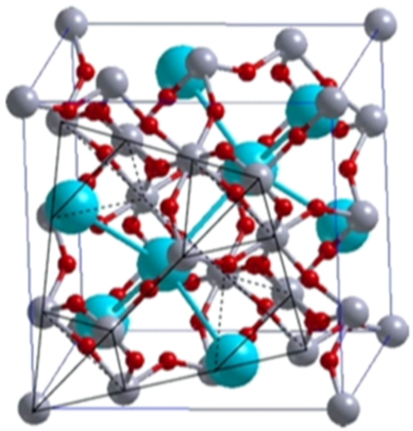
- Cubic symmetry
- Time reversal symmetry (paramagnetic phase)
- Strong spin-orbit
- Half-filling with sub-lattice structure

Representation of quantum states

O	E	$\mathcal{R}$	$3C_4^2 + 3\mathcal{R}C_4^2$	$6C_4$	$6\mathcal{R}C_4$	$6C_2 + 6\mathcal{R}C_2$	$8C_3$	$8\mathcal{R}C_3$
Γ_1	1	1	1	1	1	1	1	1
Γ_2	1	1	1	-1	-1	-1	1	1
Γ_{12}	2	2	2	0	0	0	-1	-1
$\Gamma_{15'}$	3	3	-1	1	1	-1	0	0
$\Gamma_{25'}$	3	3	-1	-1	-1	1	0	0
Γ_6	2	-2	0	$\sqrt{2}$	$-\sqrt{2}$	0	1	-1
Γ_7	2	-2	0	$-\sqrt{2}$	$\sqrt{2}$	0	1	-1
Γ_8	4	-4	0	0	0	0	-1	1

Tinkham, group theory

Pyrochlore iridates



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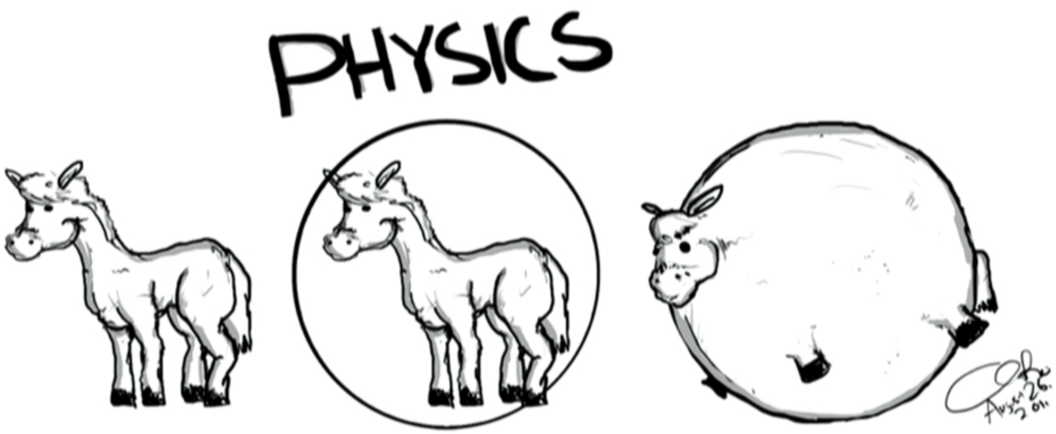
Phases in pyrochlore iridates ?

How to look at the world

Spherical horse in a vacuum



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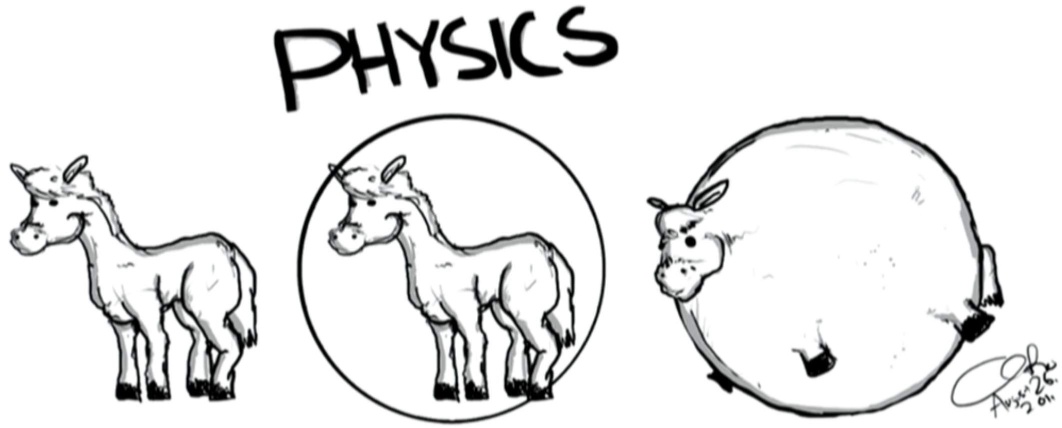

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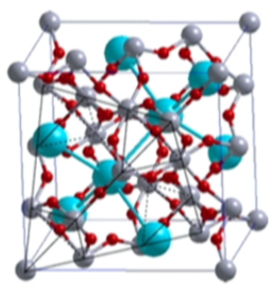
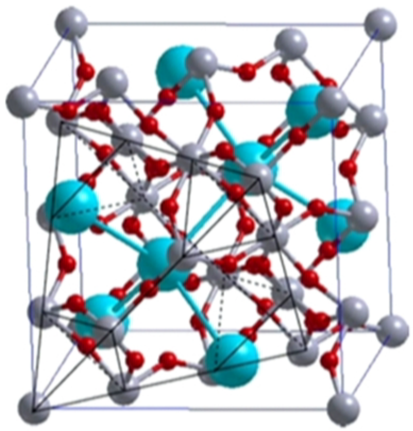


→ ?

Pyrochlore iridates



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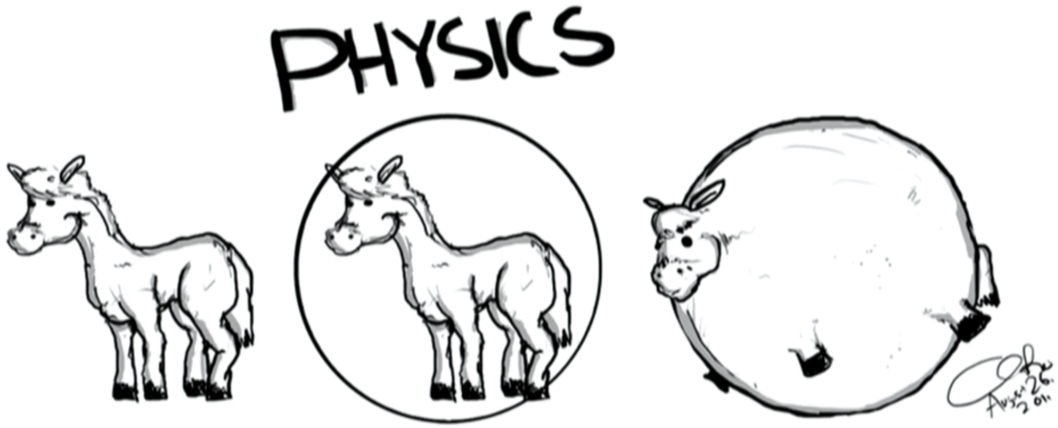

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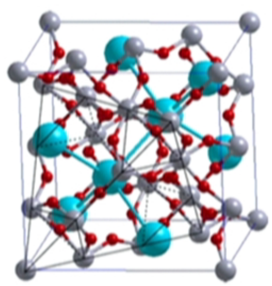


Band Insulator, Metal,

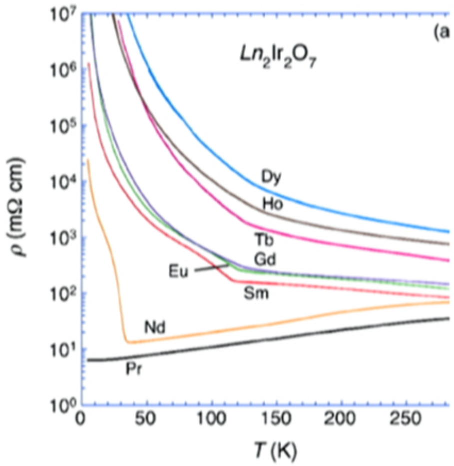
:

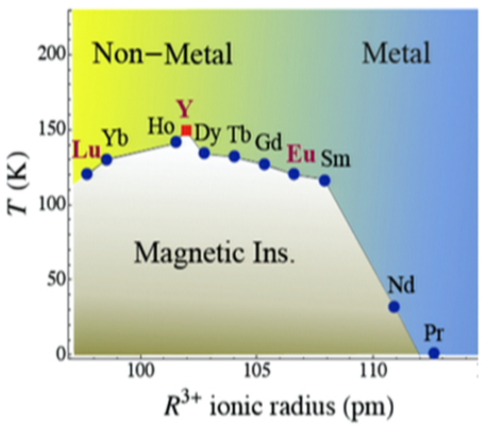
:

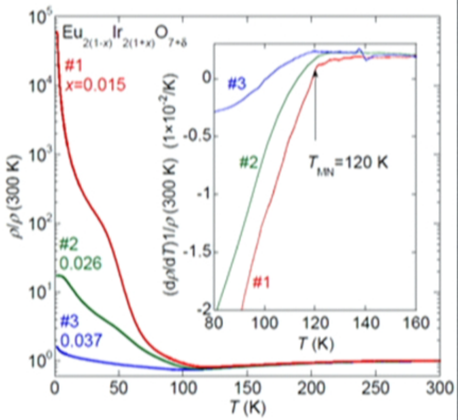
Non-Fermi liquid,

Topological insulator, Weyl SM….

Experiments







Matsuhira. et.al. 2011 *Witzak-Krempa, et. al., 2013* *Ishikawa. et.al. 2012*

2nd order metal-insulator transition (~magnetism)

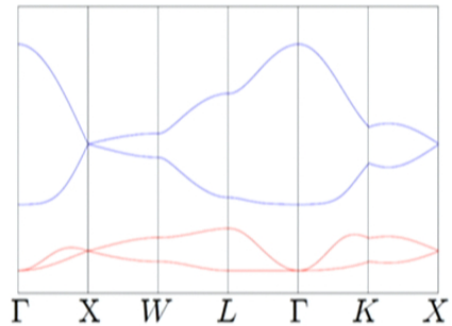
Ionic radius dependence

Sensitive doping dependence

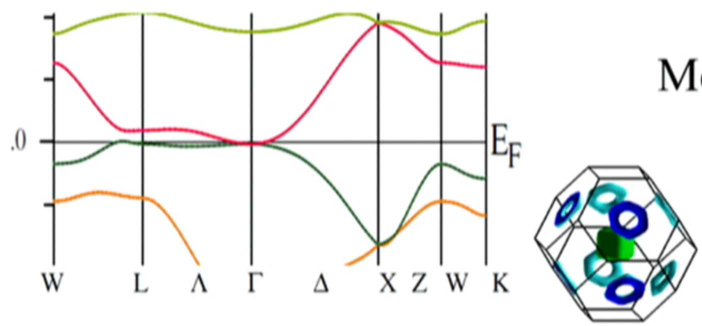
Neither good metal nor good insulator

Conventional phases (theory)

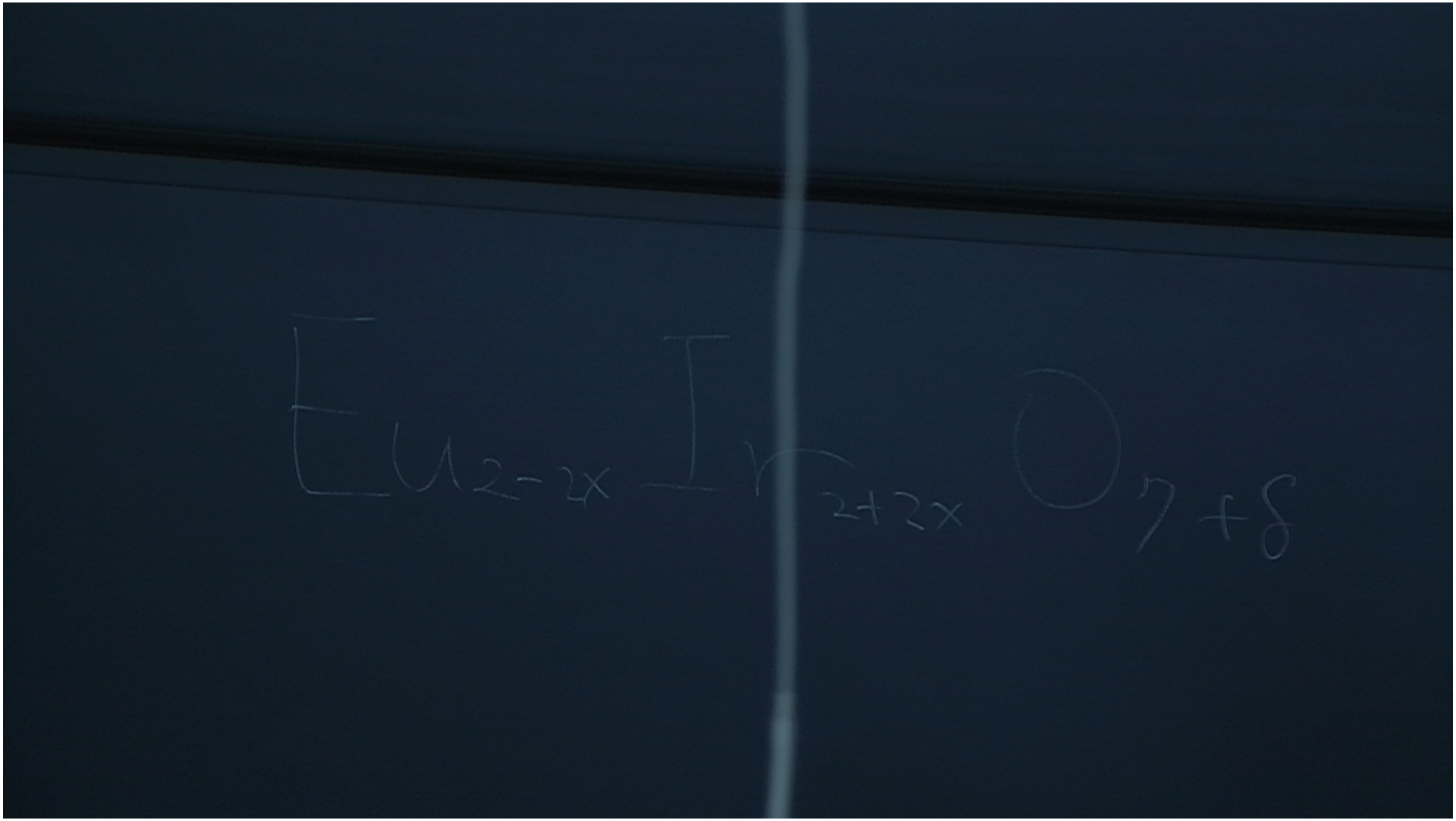
Examples of band structure calculation



(trivial) band Insulator

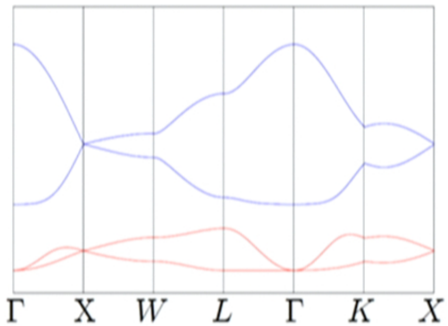


Metal

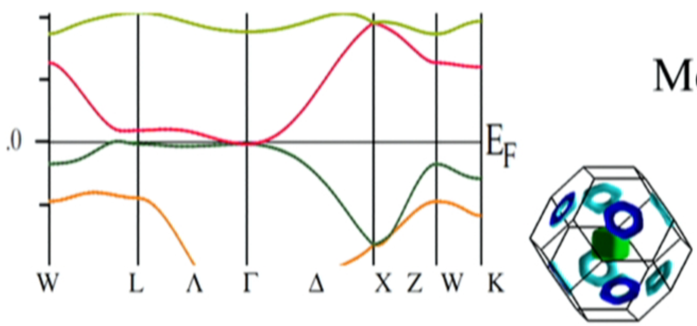

$Eu_{2-2x} Ir_{2+2x} O_{7+\delta}$

Conventional phases (theory)

Examples of band structure calculation



(trivial) band Insulator

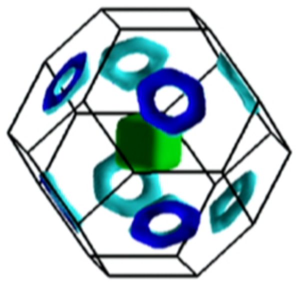


Metal

Conventional phases (theory)

Examples of band structure calculation

Metal



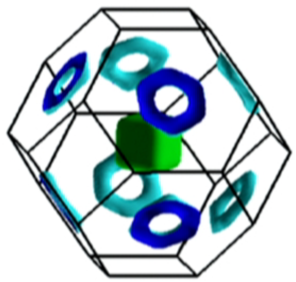
Coulomb interaction is rather weak.
(Thomas-Fermi screening)

$$E_F \gg E_{Coulomb} \sim \frac{1}{r} \qquad (r \to \infty)$$

Zero Fermi Energy

Examples of band structure calculation

Metal



Coulomb interaction is rather weak.
(Thomas-Fermi screening)

$$E_F \gg E_{Coulomb} \sim \frac{1}{r} \qquad (r \to \infty)$$

What happen if E_F is tiny or even zero?

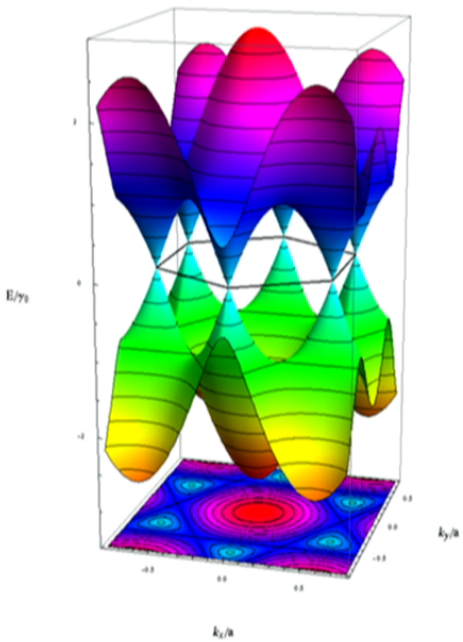
$$E_F = 0 \ll E_{Coulomb} \sim \frac{1}{r} \qquad (r \to \infty)$$

Coulomb interaction is dominant! Exotic phase?

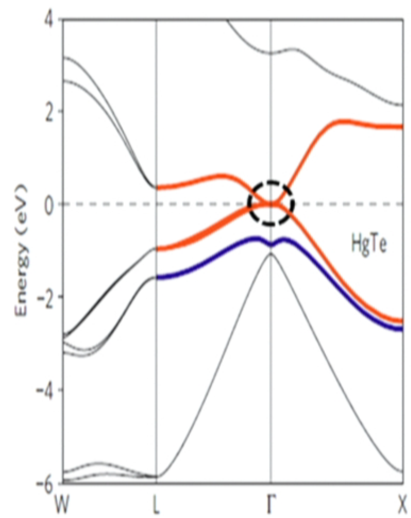
Zero Fermi Energy

Examples of zero E_F states

Graphene

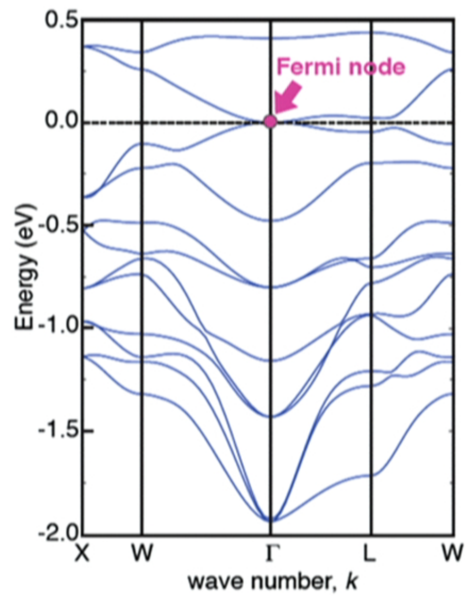


HgTe



Chadov et. al., (2010)

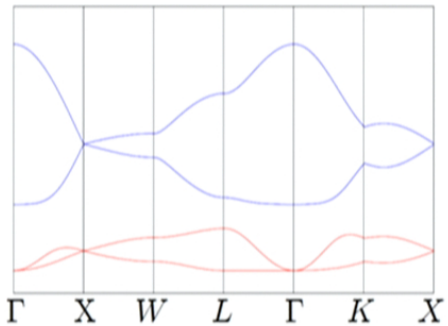
$Pr_2Ir_2O_7$ (LDA + mBJ)



T. Kondo, et. al. (to appear)

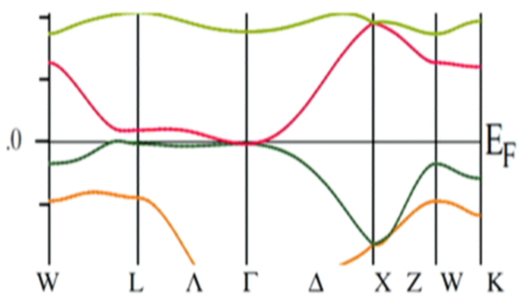
Conventional phases (theory)

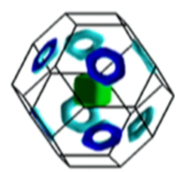
Examples of band structure calculation



(trivial) band Insulator

the most well understood phase
(Kittel, Ashcroft-Mermin)

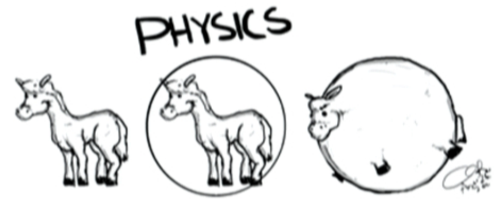




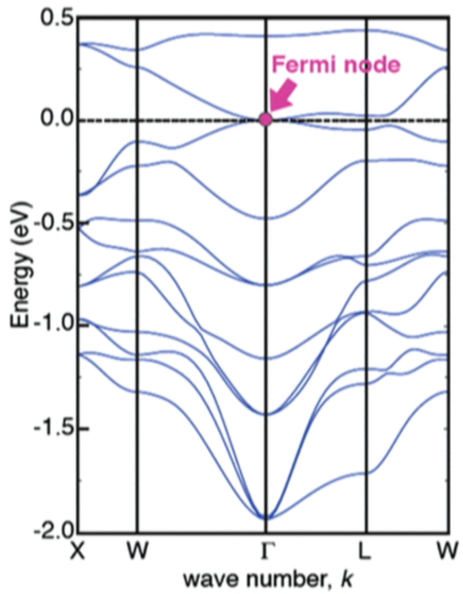
Metal

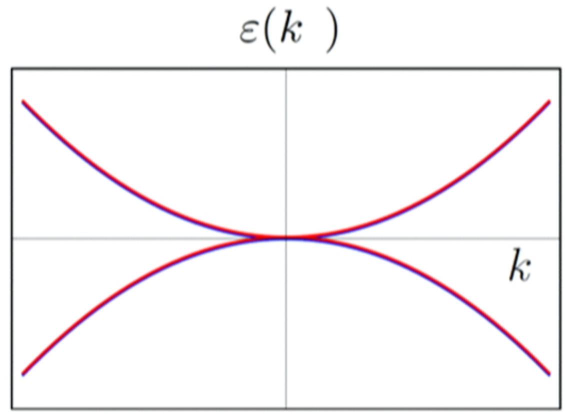
Gapless excitations from Fermi surfaces

Band structure and minimal model



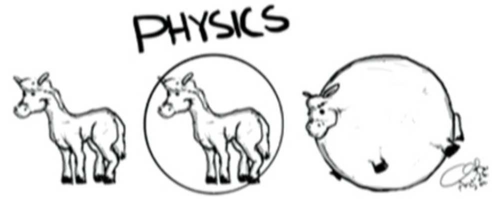
- Minimal model
  - Four degenerate states, quadratic band touching
  - Similar to the Luttinger model in semi-conductors





Γ_8 band

Band structure and minimal model



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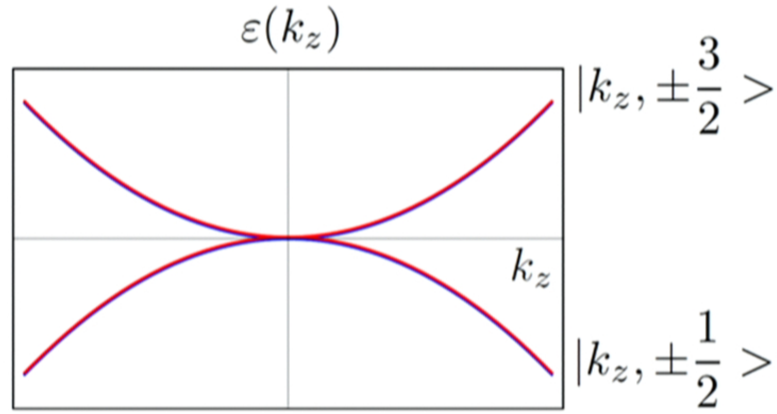
$$\mathcal{H}_0(k) = \alpha_1 k^2 + \alpha_2(\vec{k}\cdot\vec{J})^2 + \alpha_3(k_x^2 J_x^2 + k_y^2 J_y^2 + k_z^2 J_z^2)$$

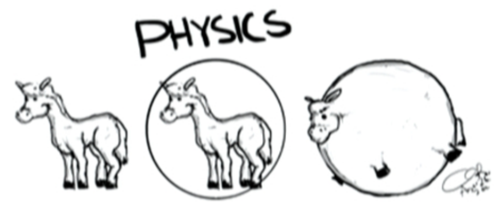
$J_{x,y,z}$: 4x4 angular momentum matrices

$$\mathcal{H}(k_z\hat{z}) = \frac{k_z^2}{2m}(J_z^2 - \frac{5}{4})$$

$$J_z = \begin{pmatrix} +\frac{3}{2} & 0 & 0 & 0 \\ 0 & +\frac{1}{2} & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -\frac{3}{2} \end{pmatrix}$$



Band structure and minimal model



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$J_{x,y,z}$: 4x4 angular momentum matrices

  - Coulomb interaction effect

"kinetic" term

Coulomb

$$H_{eff} = \int d^3x\, \psi^\dagger \mathcal{H}_0(-i\nabla)\psi + \frac{e^2}{2}\int \frac{n(x)n(y)}{|x-y|} d^3x d^3y$$

$$\{\psi^\dagger(\vec{k})|\text{vac}>\} = \{|\vec{k}, J_z = \pm\frac{3}{2}>, |\vec{k}, J_z = \pm\frac{1}{2}>\}$$

Outline

1. Introduction : *bird's eye-view of correlated electron systems*

2. Band structure and minimal model *: theoretical starting point*

3. Strong Coulomb interaction : *correlation, correlation, and correlation*

4. Symmetry breaking and topological phases

5. Conclusion

Strong Coulomb interaction

- Estimation of Coulomb interaction

  - Coulomb interaction : $E_C \sim \frac{e^2}{r}$ - kinetic energy: $E_{kin}(k) \sim k^2$

  - Heisenberg uncertainty relation : $E_{kin} \sim \frac{1}{mr^2}$, $E_C \sim \frac{e^2}{r}$

$$E_{kin} \ll E_C \quad r \to \infty$$

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Coulomb interaction is dominant!
Exotic phases?

Strong Coulomb interaction

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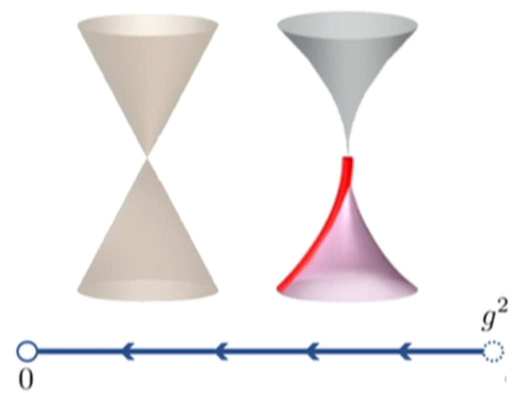
$$E_{kin} \ll E_C \quad r \to \infty$$

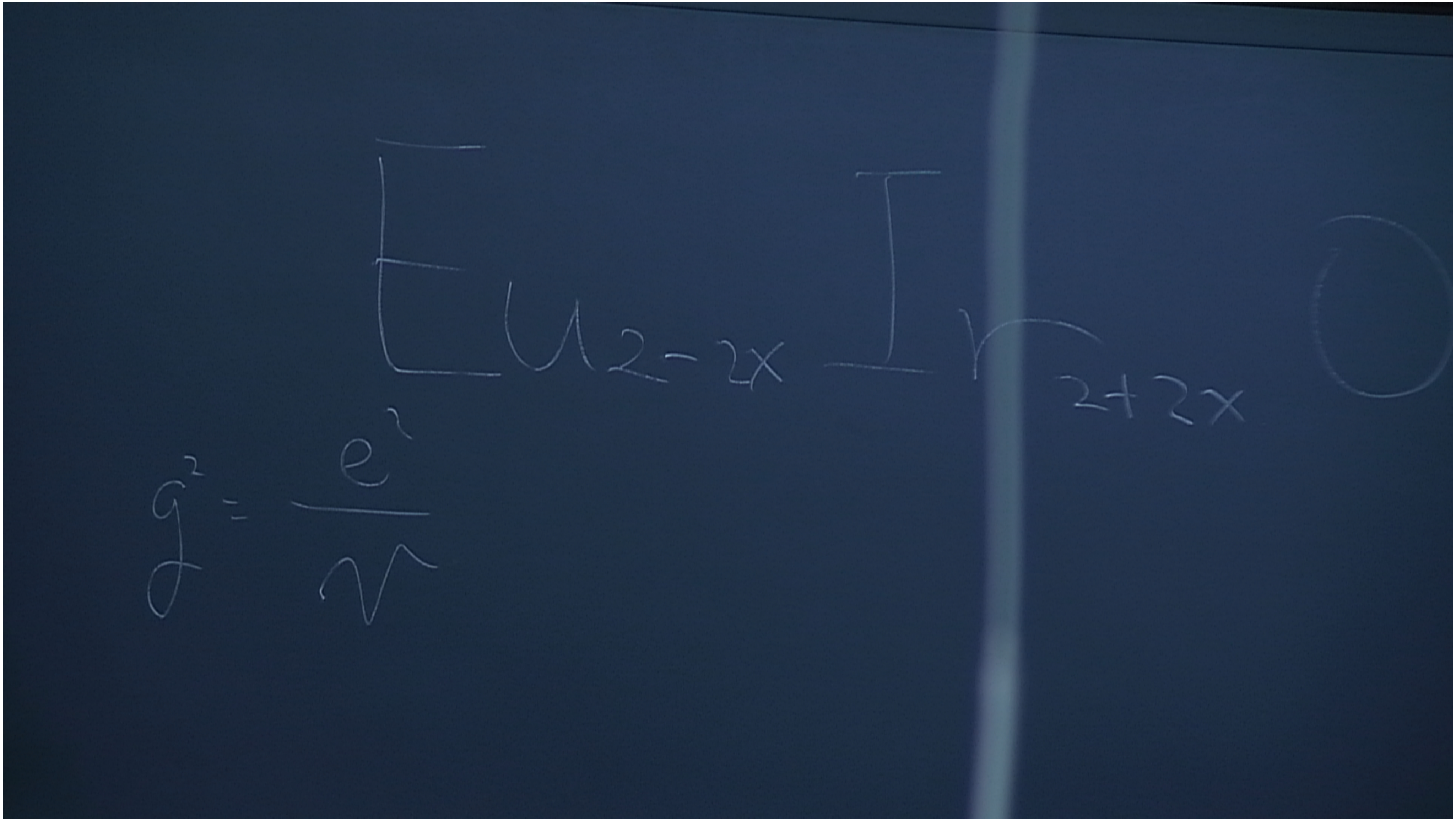
cf) graphene at zero doping

- kinetic energy : $E_{kin}(k) \sim k$
- Coulomb interaction : $E_C \sim \frac{e^2}{r}$

$$E_{kin} \sim E_C$$

marginal Coulomb interaction
→ velocity renormalization




Eu2-2x Ir2+2x O
g2 = e2 / ν

Strong Coulomb interaction

- Renormalization group

$$\mathcal{H}_0(k) = \alpha_1 k^2 + \alpha_2(\vec{k}\cdot\vec{J})^2 + \alpha_3(k_x^2 J_x^2 + k_y^2 J_y^2 + k_z^2 J_z^2)$$

$J_{x,y,z}$: 4x4 matrices

$$S_L = \int d\tau d^d x \left\{ \psi^\dagger \left[\partial_\tau - ie\,\varphi + \mathcal{H}_0 \right] \psi + \frac{c_0}{2}(\partial_i \varphi)^2 \right\}$$

Scaling analysis (z=2, d=3) :

$$x \rightarrow x\, e^{-l} \qquad \tau \rightarrow \tau\, e^{-zl}$$

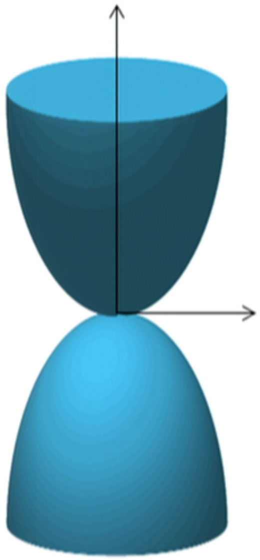
$$[\psi^\dagger \psi] = d \quad [\varphi^2] = d$$

$[e^2] = 1$ **relevant!**

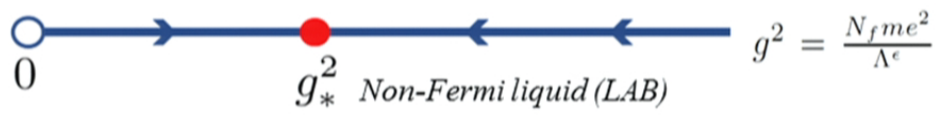
Two methods : ε(=4-d) expansion, Large N_f expansion (d=3)

Strong Coulomb interaction

- Renormalization group



1. ε(=4-d) : $$\frac{d\tilde{g}}{dl} = \epsilon\tilde{g} - N_f\tilde{g}^2 \qquad \eta_b = \epsilon + O(\frac{\epsilon^2}{N_f})$$



Strongly interacting **quantum critical phase** in d=3!

Strong Coulomb interaction

- Renormalization group

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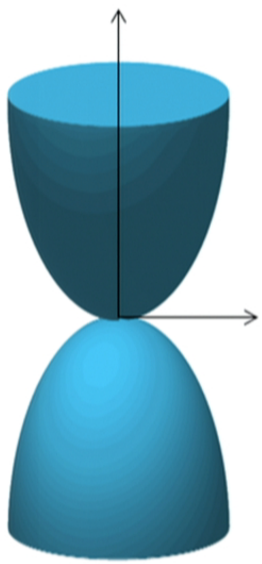

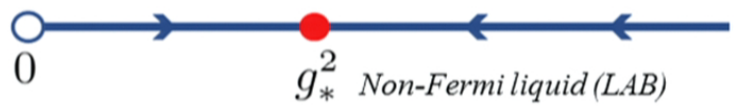

$g^2 = \frac{N_f m e^2}{\Lambda^\epsilon}$

Strongly interacting **quantum critical phase** in d=3!

Strong Coulomb interaction

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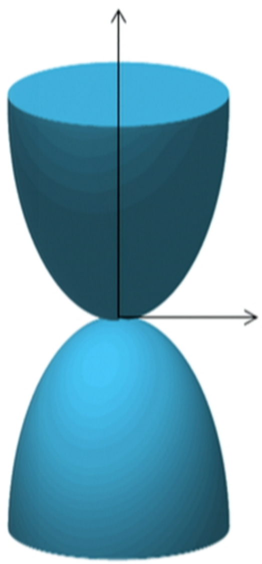
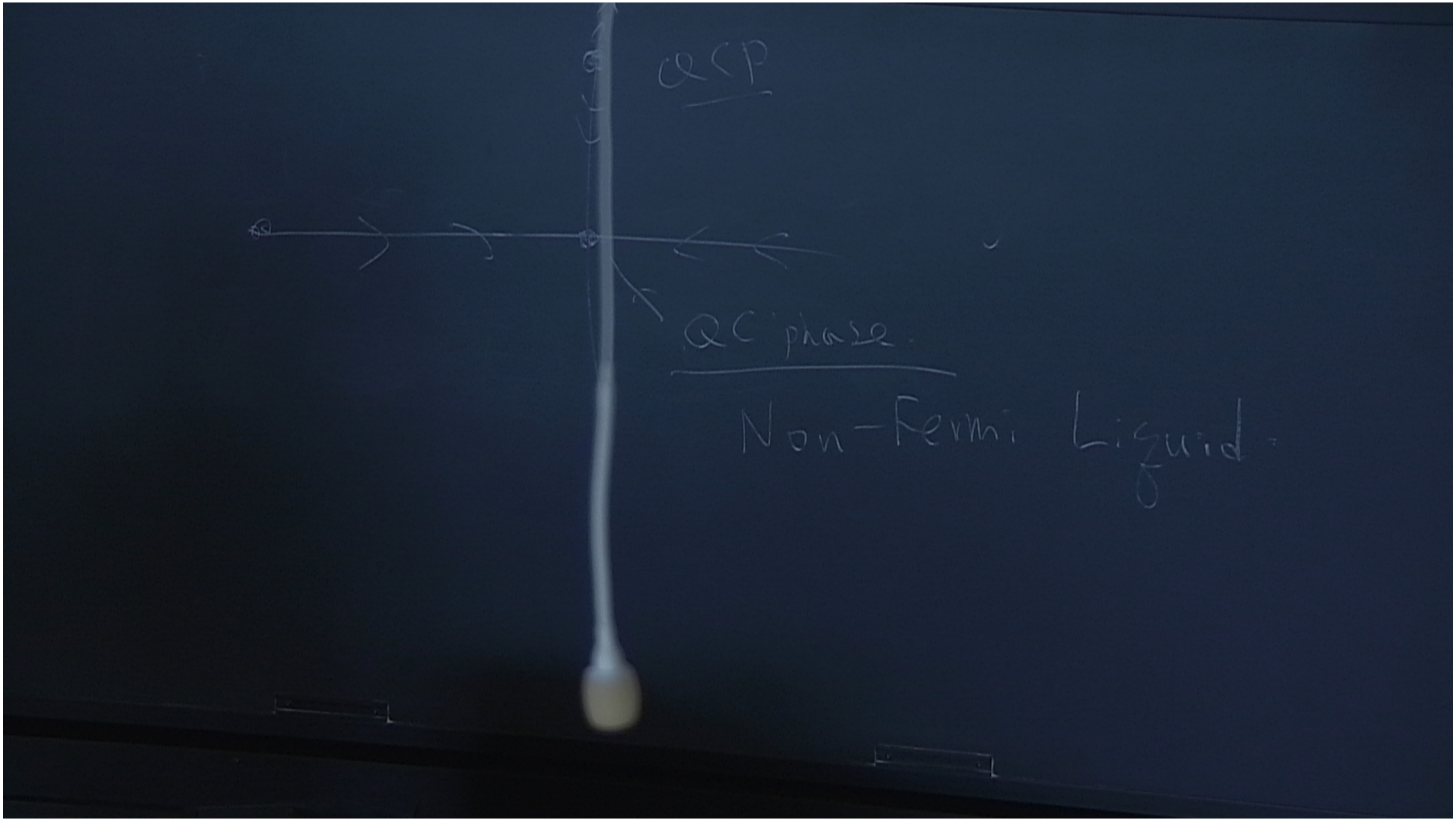


1. ε(=4-d) : $$\frac{d\tilde{g}}{dl} = \epsilon\tilde{g} - N_f\tilde{g}^2 \qquad \eta_b = \epsilon + O(\frac{\epsilon^2}{N_f})$$

2. Large N_f: $$\eta_b = 1 \qquad \eta_f = \frac{0.17}{N_f} \qquad z = 2 - \frac{0.069}{N_f}$$

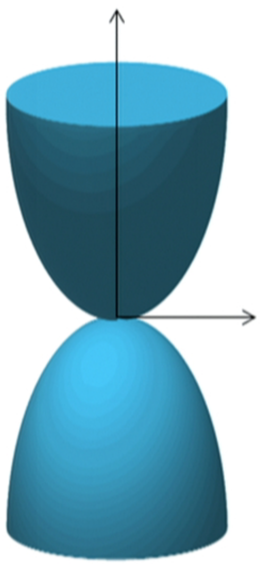
Two methods are consistent! $$V_c(q) \sim \frac{1}{q^2} \rightarrow \frac{1}{q^{2-\eta_b}}$$

Strongly interacting **quantum critical phase** in d=3!


QCP
QC phase
Non-Fermi Liquid

Strong Coulomb interaction

- Renormalization group



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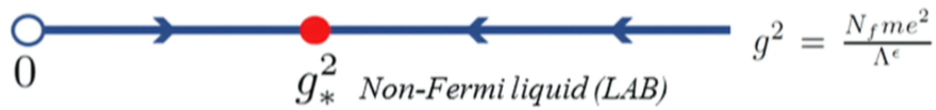

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Strongly interacting **quantum critical phase** in d=3!

Implication

$$S_L = \int d\tau d^d x \left\{ \psi^\dagger \left[\partial_\tau - ie\,\varphi + \mathcal{H}_0 \right] \psi + \frac{c_0}{2} (\partial_i \varphi)^2 \right\}$$

Symmetry broken states

$$E_{1,2} = \pm\sqrt{k_x^2 + k_y^4}$$

$$E_{1,1,2} = \pm\sqrt{k_x^2 + k_y^2 + k_z^4}$$

$$E_{2,2,1} = \pm\sqrt{(k_x^2 + k_y^2)^2 + k_z^2}$$

Physical realization?
Coulomb interaction effect?

Non Fermi liquid

- LAB (Luttinger-Abrikosov-Beneslaevski) phase

- **Anomalous dimensions** in physical quantities
- Symmetry protection (cubic+ time reversal)
- Emergent symmetries (particle-hole and full rotation)

Dispersion relation : $\omega \sim k^z \approx k^{1.7}$

Specific Heat : $c_v \sim T^{d/z} \approx T^{1.7}$

Susceptibility : $\chi(T) \sim a + b\,T^{(d-z+2\eta_{12})/z} \approx a + b\,T^{0.5}$

Non linear susceptibility : $\chi_3 = \partial^3 M/\partial H^3\big|_{H=0} \approx T^{-1.7}$

Anomalous Hall : $\sigma_{xy} \sim H^{1/(z-\eta_{12})} \approx H^{0.51}$

Outline

1. Introduction : *bird's eye-view of correlated electron systems*

2. Band structure and minimal model *: theoretical starting point*

3. Strong Coulomb interaction : *Correlation, correlation, and correlation*

4. Symmetry breaking and topological phases : *World without symmetry*

5. Conclusion

Implication

$$S_L = \int d\tau d^d x \left\{ \psi^\dagger \left[\partial_\tau - ie\,\varphi + \mathcal{H}_0 \right] \psi + \frac{c_0}{2} (\partial_i \varphi)^2 \right\}$$

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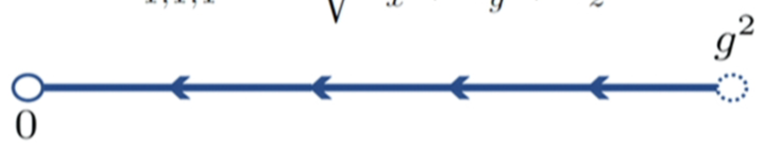
$$S_L = \int d\tau d^d x \left\{ \psi^\dagger \left[\partial_\tau - ie\,\varphi + \mathcal{H}_0 \right] \psi + \frac{c_0}{2} (\partial_i \varphi)^2 \right\}$$

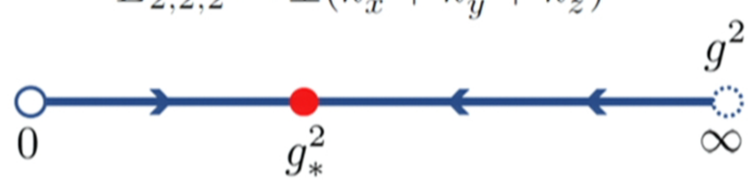
Mono-layer graphene $E_{1,1} = \pm\sqrt{k_x^2 + k_y^2}$

Bi-layer graphene $E_{2,2} = \pm(k_x^2 + k_y^2)$

Quantum electrodynamics (linear Weyl semi-metal) $E_{1,1,1} = \pm\sqrt{k_x^2 + k_y^2 + k_z^2}$

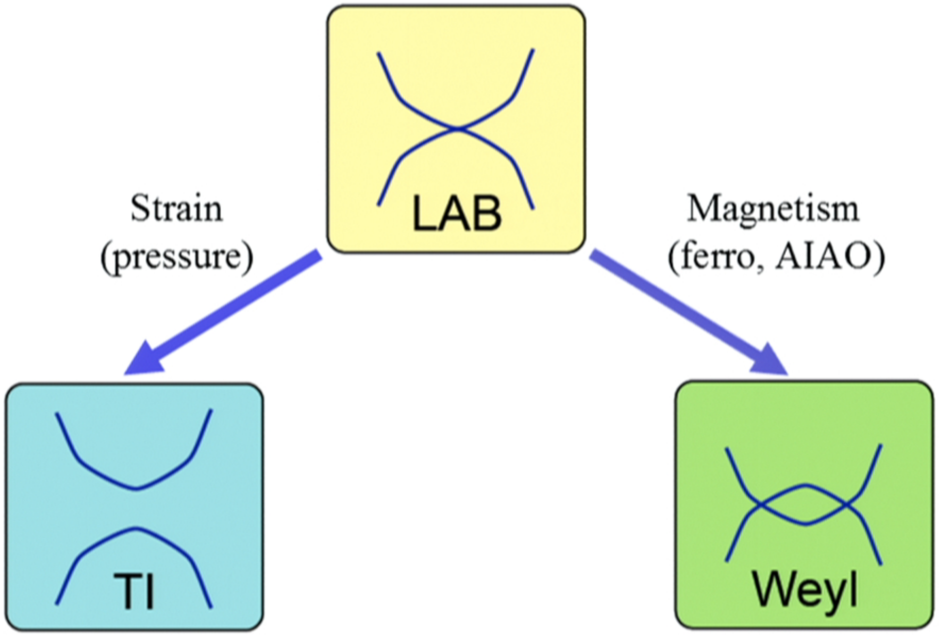


$Pr_2Ir_2O_7$ $E_{2,2,2} = \pm(k_x^2 + k_y^2 + k_z^2)$



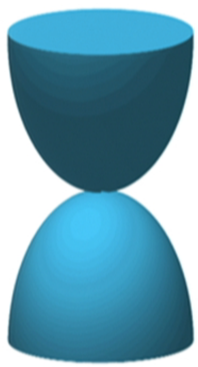
Symmetry breaking

- Topological phases around the NFL



Coulomb interaction in Weyl semimetals

Spin-orbit + cubic + time reversal : quadratic band touching (QBT)



Break some symmetry
: QBT becomes unstable.
ex) linear Weyl (LW) semimetal

Coulomb interaction in LW

(~quantum electrodynamics)

$$\mathcal{H}_{LW} = k_x\sigma_x + k_y\sigma_y + k_z\sigma_z$$

Hosor et. al. , PRL 2012

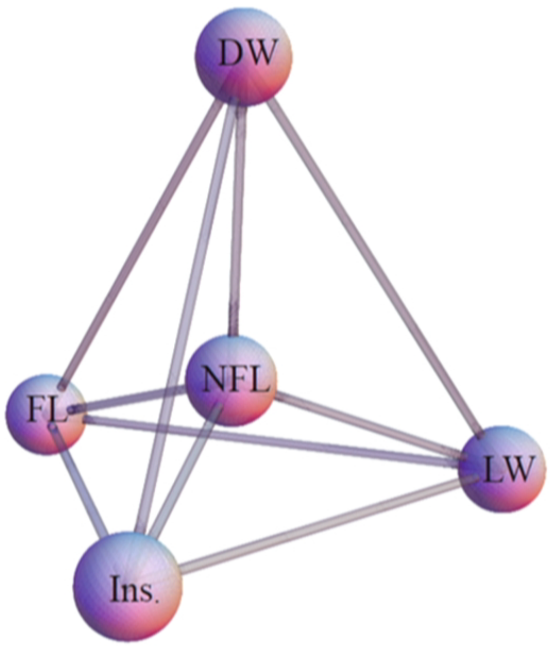
Current Situation (Exp)

Exotic phases in iridates

NFL	Maybe? Nakatusji group, Nature 2010	Clean sample, bulk and thin film, Charge probe (transport, optical, ARPES, q-oscillation,…), Theory and Exp,…
Weyl SM	Maybe? Tokura group, PRL 2012	
TI	Not yet!	
…		

Summary of phases

Phases and phase transitions in pyrochlore iridates



FL : Fermi liquids
DOS = Constant $C_v \sim T$

Ins : Insulator (band, magnetism….)
DOS = zero $C_v \sim e^{-\frac{\Delta}{T}}$

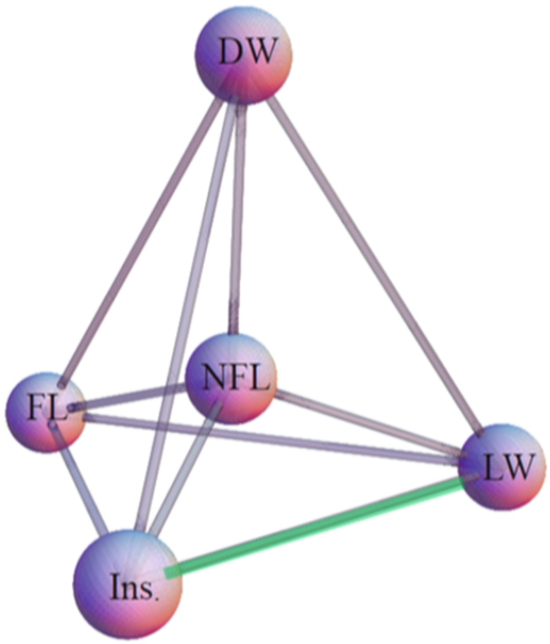
LW : Linear Weyl semi-metal
DOS(E) ~ E^2 $C_v \sim T^3$

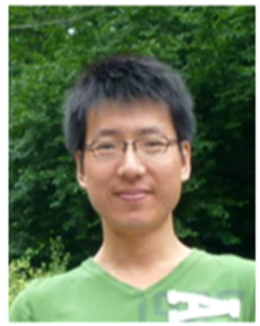
DW : Double Weyl semi-metal
DOS(E) ~ E $C_v \sim T^2$

NFL : Non-Fermi liquids
(Luttinger-Abrikosov-Beneslavskii)
$C_v \sim T^{1.5+\delta}$ $\delta \sim 0.2 + O(\epsilon^2)$

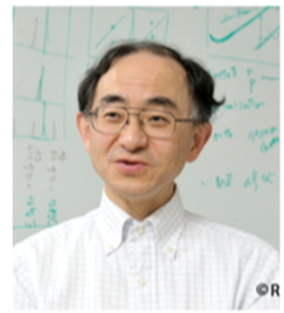
Collaborators

Phases and phase transitions in pyrochlore iridates





B.J. Yang
Riken



N. Nagaosa,
Riken

Quantum criticality of topological phase transitions in 3d interacting electrons
Yang, *EGM, Isobe, and Nagaosa (to appear)*

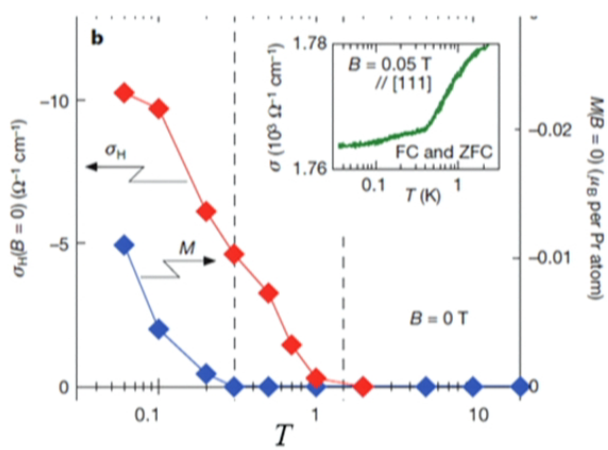
Thank you for your attention!

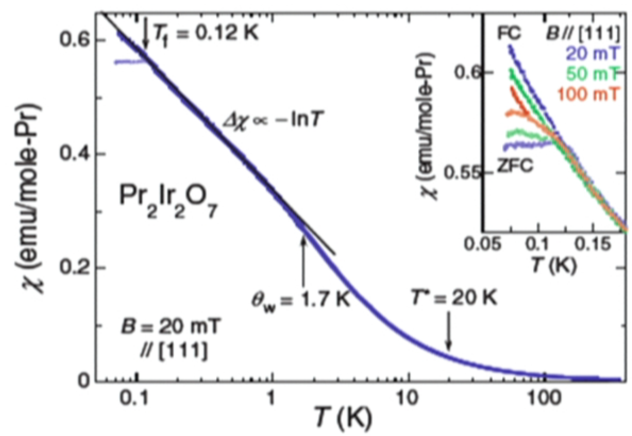
Symmetry breaking

- Observed non-Fermi liquid

Non Fermi liquid behavior in $Pr_2Ir_2O_7$

Anomalous Hall effect, large non-linear susceptibility, logarithmic susceptibility, etc.





Machida. et.al. 2010

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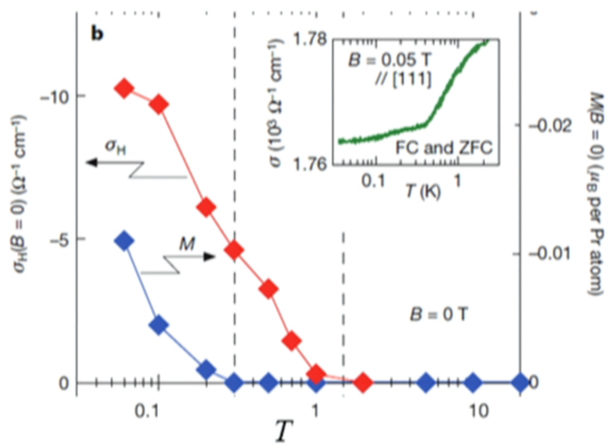

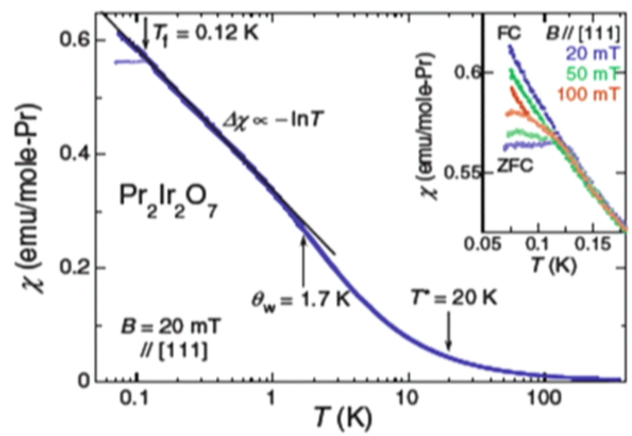

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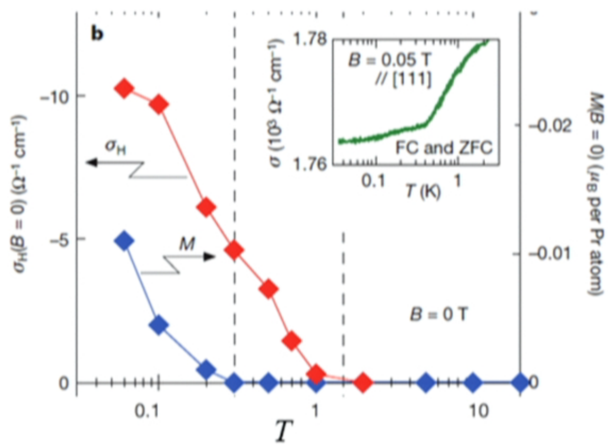
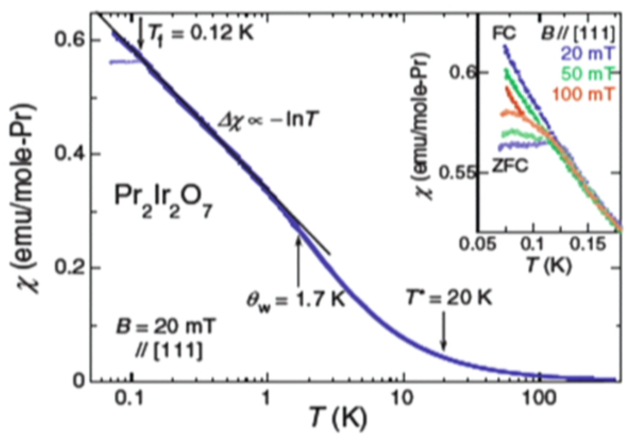


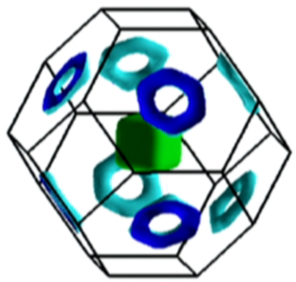


Machida. et.al. 2010

Conventional phases (theory)

Examples of band structure calculation

Metal



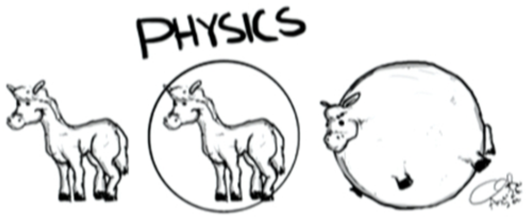
Coulomb interaction is rather weak.
(Thomas-Fermi screening)

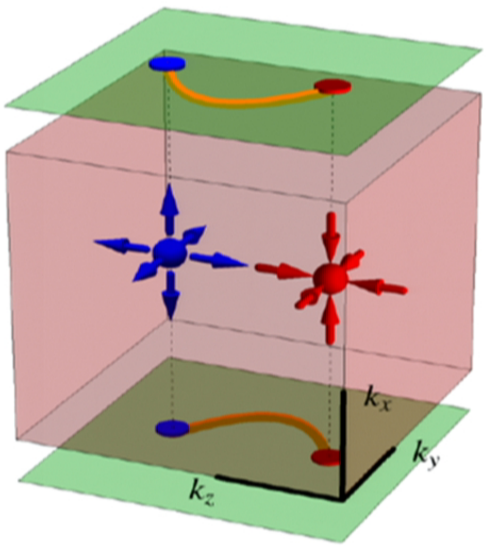
$$E_F \gg E_{Coulomb} \sim \frac{1}{r} \qquad (r \to \infty)$$

Symmetry breaking

- Weyl semi-metal

$$\mathcal{H}_2^{\pm} = \pm k_z \tau^z + \frac{\sqrt{3}}{2}(k_x^2 - k_y^2)\tau^x + \sqrt{3} k_x k_y \tau^y$$





Surface :
Fermi “arc”

Bulk :
Weyl points

Anomalous Hall effect (3d) :

$$\sigma_{\mu\nu}^H = \frac{e^2}{h} \epsilon_{\mu\nu\lambda} K_\lambda$$

Surface contribution

$$\frac{K_w^2}{2m} \sim \mu_B H \qquad K_w \sim H^{0.5}$$

$$\sigma_H = \frac{e^2}{h} K_w \sim H^{0.5} \quad \gg H$$

(for small H)

Witzak-Krempa, et. al., 2013

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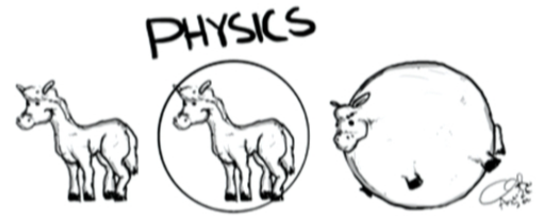

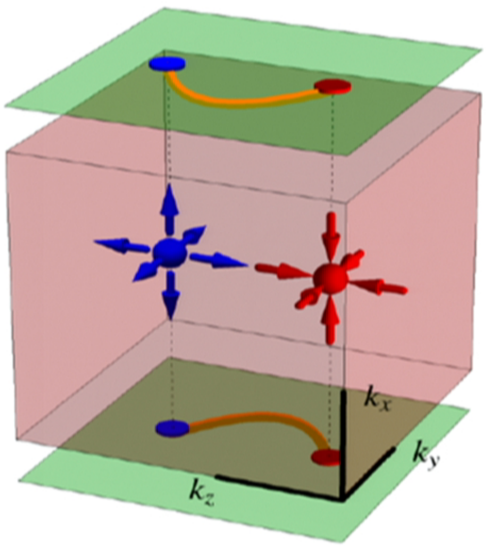

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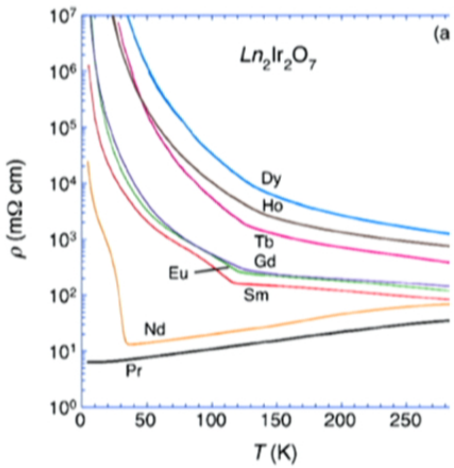
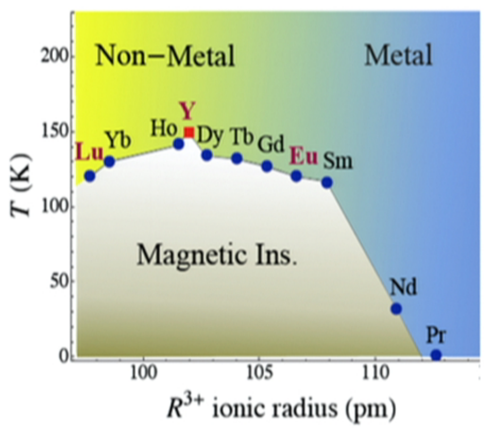
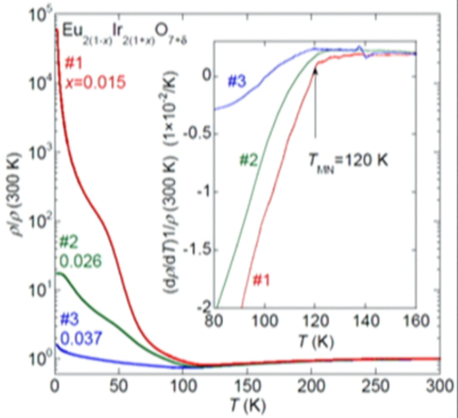
Witzak-Krempa, et. al., 2013

Experiments







Matsuhira. et.al. 2011 *Witzak-Krempa, et. al., 2013* *Ishikawa. et.al. 2012*

2nd order metal-insulator transition (~magnetism)

Ionic radius dependence

Sensitive doping dependence

Neither good metal nor good insulator