Title: The characteristic structure of the Weyl curvature tensor and its utilization

Date: Apr 24, 2014 01:00 PM

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Abstract: The talk will summarize some results relating to clarifying the physical significance of the characteristic structure of the Weyl curvature tensor, and proposals for its utilization. I will begin by showing how null force-free or vacuum electrodynamic solutions experience reduced scattering by propagating along the principal null directions (GPNDs) of the spacetime, as if they were the flatter directions of the curvature tensor. Building on this observation, I will use complexified sectional curvatures to develop an analogy between the Weyl tensor and the second fundamental form of an immersed surface, and argue that, despite their name, the GPNDs are more akin to the asymptotic directions rather than the principal directions. I will also identify the ``true'' principal directions and curvatures, which has so far been under-utilized. I will then move on to propose usage for these quantities in topics related to the peeling theorem, the large scale geometry of the spacetime, the transition from the merger phase of binary black hole merger into the quasinormal mode ringdown phase, and perturbation of generic background spacetimes.

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GaugeItems.input                     Profiler.1.txt                 Profiler.42.txt  WaveExtraction.input
zwicky$ cd GW2/
zwicky$ ls
CceR0100.h5  CceR0380.h5                        RawStrahlkorperIntegrals.h5
CceR0193.h5  PhiMinus_FiniteRadii_CodeUnits.h5  rh_FiniteRadii_CodeUnits.h5
CceR0287.h5  PhiPlus_FiniteRadii_CodeUnits.h5   rPsi4_FiniteRadii_CodeUnits.h5
zwicky$ ExtractFromH5 rPsi4_FiniteRadii_CodeUnits.h5
zwicky$ pdd
-bash: pdd: command not found
zwicky$ pwd
/home/fanz/ds06/RecursiveID/WITHENVELOP/NewSource/Test0/Ev/Lev5_I/Run/GW2
zwicky$ cd ../../../Lev5_
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zwicky$ cd ../../../Lev5_J/Run/GW2/
zwicky$ ls
CceR0100.h5  CceR0380.h5                        RawStrahlkorperIntegrals.h5
CceR0193.h5  PhiMinus_FiniteRadii_CodeUnits.h5  rh_FiniteRadii_CodeUnits.h5
CceR0287.h5  PhiPlus_FiniteRadii_CodeUnits.h5   rPsi4_FiniteRadii_CodeUnits.h5
zwicky$ ExtractFromH5 rPsi4_FiniteRadii_CodeUnits.h5
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CceR0100.h5  extracted-rPsi4_FiniteRadii_CodeUnits  rh_FiniteRadii_CodeUnits.h5
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R0103.dir/ R0110.dir/ R0119.dir/ R0128.dir/ R0140.dir/ R0154.dir/ R0171.dir/ R0192.dir/
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zwicky$
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Previous Next 1 (1 of 37) Fit Page Width

Index

The characteristic structure of the Weyl curvature tensor and its utilization

Fan Zhang

PI, Strong-Gravity seminar, Apr 24, 2014

Why study the characteristic structure

- Principal null directions (GPNDs) underlie proof of peeling theorem and Petrov classification. Also important for scattering of waves etc. Thus an intuitive feel for them is beneficial.
- Characteristic structure is gauge invariant, providing infrastructure to unambiguously measure spacetime geometry.
- Analogy with second fundamental form allows to borrow ideas from surface theory for use in gravitational studies.

Outline of sections

1. GPNDs are (despite the name) more akin to asymptotic directions.
   - 1.1 Role of GPND in scattering.
   - 1.2 Analogy with second fundamental form.
2. The "true" principal curvatures and directions.
3. Utilities for the true principal quantities
   - 3.1 For wave extraction.
   - 3.2 As visualization tool.
   - 3.3 For studying large scale geometry.

Role of GPNDs in scattering

- Robinson (1961) studied null solutions to Maxwell equations in vacuum.
- Brennan, Gralla, and Jacobson (2013) found null solutions for force-free electrodynamics.
- Null solution defined by (equivalently):
  - Coinciding principal null directions (EPNDs) of the EM field tensor
  - Invariants $F^{ab}F_{ab} = 0 = F_{ab}{}^{*}F^{ab}$
  - In field vector terms $B^2 - E^2 = 0 = \mathbf{E} \cdot \mathbf{B}$
- Locally similar to plane waves. Purely radiative solutions, no Coulomb background or oppositely moving wave

- Mathematically important in their relation with twistors etc.
- Astrophysically important as they can be used to model outer magnetosphere of a pulsar. (movie for stability)
- Simplicity (closed form solution) owing to ability to avoid being strongly scattered by spacetime curvature.

- F_{ab} is tensorial, couples to spacetime curvature by Ricci identity. Thus generically scattered, can travel inside null cone and does not satisfy Huygens' principal.
- Wave equation

$$\nabla^c \nabla_c F_{ab} = -2R_{acbd}F^{cd} + R_a{}^c F_{cb} + F_a{}^c R_{cb} + \nabla_b J_a - \nabla_a J_b$$

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- Mathematically important in their relation with twistors etc.
- Astrophysically important as they can be used to model outer magnetosphere of a pulsar. (movie for stability)
- Simplicity (closed form solution) owing to ability to avoid being strongly scattered by spacetime curvature.
- Important to know how they avoid scattering.

- F_{ab} is tensorial, couples to spacetime curvature by Ricci identity. Thus generically scattered, can travel inside null cone and does not satisfy Huygens' principal.
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  RHS shows scattering terms.
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- Use spinor formalism, "square root" of null vectors $l^a = \iota^A \bar{\iota}^{A'} \sigma^a_{AA'}$.
- Spinor dyad $(\o_A, \iota_A)$ related to Newman-Penrose null tetrad by

$$l^a = \sigma^a_{AA'} \o^A \bar{\o}^{A'}, \quad n^a = \sigma^a_{AA'} \iota^A \bar{\iota}^{A'}, \quad m^a = \sigma^a_{AA'} \o^A \bar{\iota}^{A'}.$$

  Soldering form σ essentially vierbein from orthonormal tetrad to coordinate tetrad.
- Metric maps to antisymmetric bispinor

$$g^{ab} = -\sigma^a_{AA'} \sigma^b_{BB'} \epsilon^{AB} \epsilon^{A'B'}.$$

  Self-contraction vanishes $\iota_A \iota^A = 0$ just like null vectors.
- Break **F** (and similarly **C**) into antiself-dual $\mathbf{F} + \mathbf{i}^*\mathbf{F}$ and self-dual $\mathbf{F} - \mathbf{i}^*\mathbf{F}$ parts

$$F_{ab} = (\phi_{AB} \epsilon_{A'B'} + \bar{\phi}_{AB} \epsilon_{AB}) \sigma_a^{AA'} \sigma_b^{BB'}$$

- Spacetime scattering term is

$$\Psi_{ACBD}\phi^{CD} = \frac{1}{6}\sum_{i>j|k>l}\left(\alpha^{(i)}_{C}\o^{C}\right)\left(\alpha^{(j)}_{D}\iota^{D}\right)\alpha^{(k)}_{(A}\alpha^{(l)}_{B)}$$

where $\Psi_{ABCD} = \alpha^{(1)}_{(A}\alpha^{(2)}_{B}\alpha^{(3)}_{C}\alpha^{(4)}_{D)}$ and $\phi_{AB} = \o_{(A}\iota_{B)}$ (GPNDs and EPNDs).

- More coinciding GPNDs and EPNDs, more terms vanish, scattering is simpler in spinorial/tensorial structure.

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where $\Psi_{ABCD} = \alpha_{(A}^{(1)}\alpha_B^{(2)}\alpha_C^{(3)}\alpha_{D)}^{(4)}$ and $\phi_{AB} = \varnothing_{(A}\iota_{B)}$ (GPNDs and EPNDs).
- More coinciding GPNDs and EPNDs, more terms vanish, scattering is simpler in spinorial/tensorial structure.

- Null solution in Type III or N spacetimes experience no scattering at all.
- In type D Kerr spacetime, $\alpha^{(1)} = \alpha^{(2)} = \o = \iota = \tilde{\o}$ and $\alpha^{(3)} = \alpha^{(4)} = \tilde{\iota}$.
- Scattering simplified to

$$C_{acbd}F^{cd} = \phi_0 \Psi_2 \tilde{\o}_A \tilde{\o}_B \epsilon_{A'B'} + c.c. \quad .$$

- No ι contribution on the right, helps avoid contamination by Coulomb and oppositely moving waves, as the spinor counterpart of F_{ab} is given by

$$\phi_{AB} = \phi_0 \tilde{\o}_A \tilde{\o}_B - 2\phi_1 \tilde{\o}_{(A} \tilde{\iota}_{B)} + \phi_2 \tilde{\iota}_A \tilde{\iota}_B,$$

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Analogy with second fundamental form

- Let $M \hookrightarrow N$, $N = \mathbb{R}^3$ and M is a two dimensional surface. Second fundamental form

$$\mathcal{K}(\mathbf{w}, \mathbf{v}) = S(\mathbf{w}) \cdot \mathbf{v},$$

S is

$$S(\mathbf{w}) = \mathbb{P}\left(\nabla_{\mathbf{W}} \mathbf{N}\right),$$

- Curvature of $\exp(\mathbf{w})$ is

$$\kappa(\mathbf{w}) = \frac{\mathcal{K}(\mathbf{w}, \mathbf{w})}{\mathbf{w} \cdot \mathbf{w}}.$$

- Eigenvectors of $\mathcal{K}$

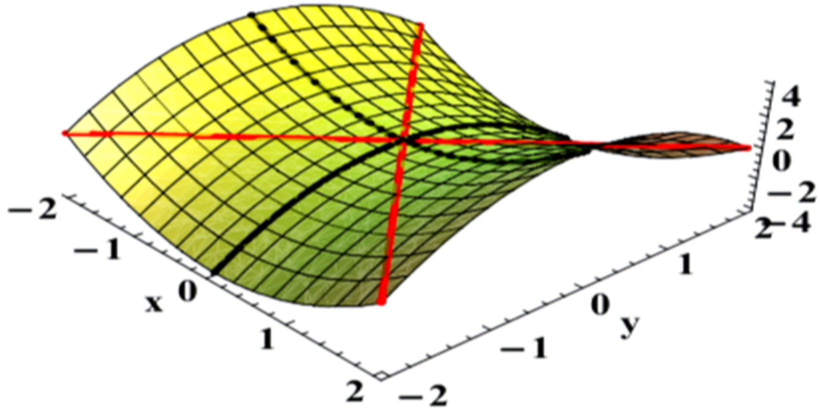
$$\mathbf{v}^{(a)}, \quad (a) \in \{1, \cdots, \dim(M)\},$$

are the principal directions. The eigenvalues $\lambda^{(a)}$ are the principal curvatures.

- Asymptotic direction is $\mathbf{r}$ s.t.

$$\mathcal{K}(\mathbf{r}, \mathbf{r}) = 0,$$

i.e. vanishing normal curvature for the geodesic $\exp(\mathbf{r})$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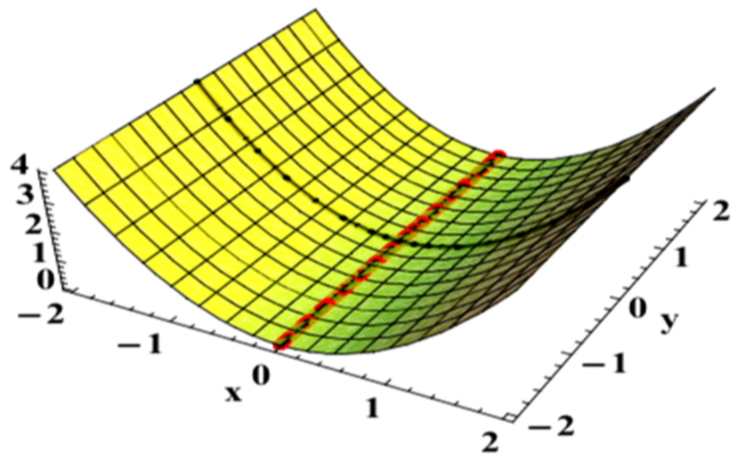
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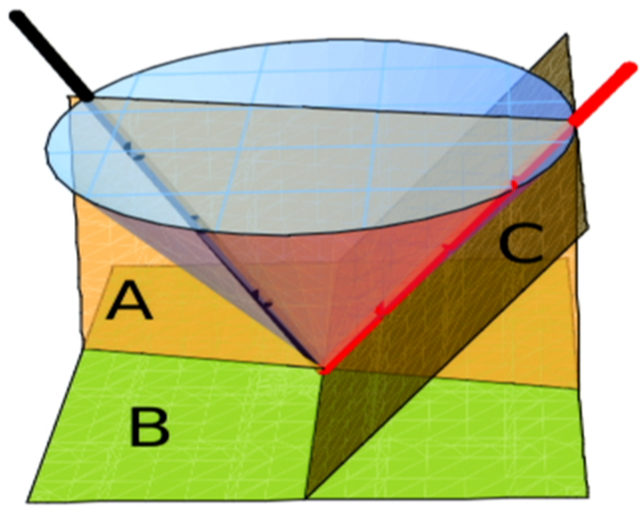
- Define complex sections

$$\mathcal{V}_{ab} = \frac{1}{2}\left(V_{ab} + i^{\perp}V_{ab}\right) = \varphi_{AB}\epsilon_{A'B'},$$



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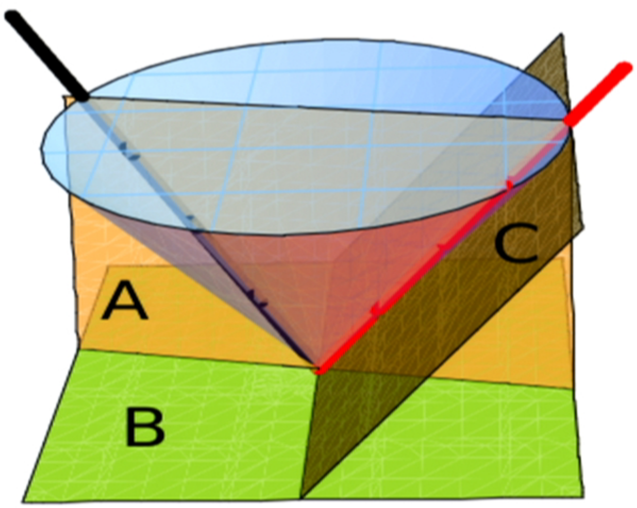

- The complex sectional curvature is given by

$$
\begin{aligned}
\mathcal{R} + i\mathcal{X} &= \Psi_{ABCD}\varphi^{AB}\varphi^{CD} \\
&= -2\alpha^{(1)}_{(A}\alpha^{(2)}_{B}\alpha^{(3)}_{C}\alpha^{(4)}_{D)}\xi^{(A}\zeta^{B)}\xi^{(C}\zeta^{D)},
\end{aligned}
$$

- When the sections are spanned by GPNDs, many contractions vanish. These sections are not the eigenvectors of ${\Psi_{AB}}^{CD}$.
- GPNDs are more closely analogous to the asymptotic directions, flat sectional curvatures in type III or N – matching scattering intuition.

- Three sets of eigen-bispinors.
- Each factorizes into two principal spinors ⇔ forming a dyad ⇔ mapping into two null vectors ⇔ then into a tetrad.
- Three tetrads spanning six real sections ⇔ intersect into four orthogonal directions ⇔ form orthonormal tetrad $\{T_a, E_a^2, E_a^3, N_a\}$ ⇔ related to three null tetrads by

$$l^a = \frac{1}{\sqrt{2}}(T^a + N^a), \qquad n^a = \frac{1}{\sqrt{2}}(T^a - N^a),$$
$$E_a^3 = \Re(m_a), \qquad E_a^4 = \Im(m_a)$$

and a (handedness preserving) permutation of spatial triad.

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- The GPNDs and six null directions from principal spinors of eigen-bispinors can be represented on the anti-celestial sphere. Form disphenoid, with symmetry similar to asymptotic and principal directions of $\mathcal{K}$.
- Coincidence of GPNDs as in Petrov classification signals reduction in the number of independent non-vanishing principal curvatures (eigenvalues of $\Psi_{AB}{}^{CD}$), similar to $\mathcal{K}$.

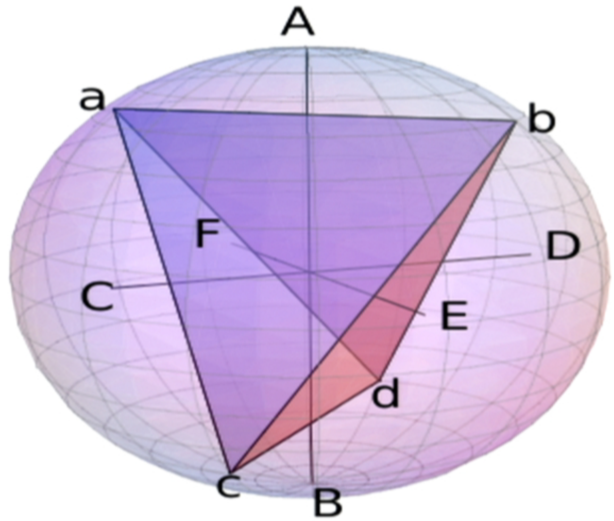

Figure: Reproduced from Spinors and Spacetime Vol II.

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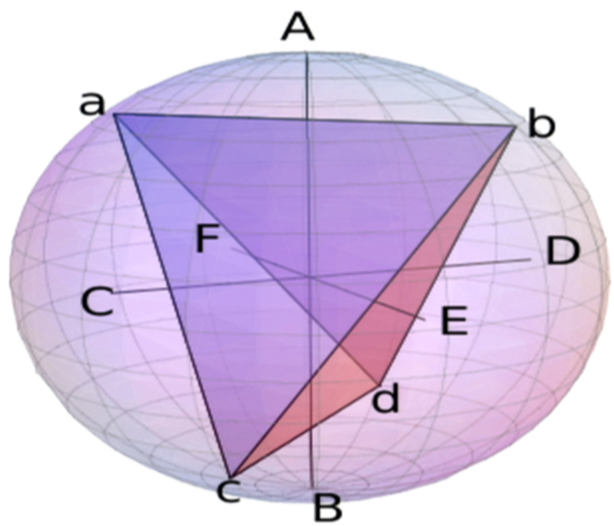

Figure: Reproduced from Spinors and Spacetime Vol II.

Utility of true principal directions: wave extraction

- GPNDs have been given more attention, true principal directions are hardly utilized.
- First, usage in wave extraction.
- Wave Ψ_4 is a component of Weyl tensor, thus tetrad dependent, need to fix unique tetrad optimized for peeling.
- Defn. of gravitational wave depend on peeling.

- If tetrad compatible with the peel-off rates

$$\Psi_0 \sim \tau^{-5},\ \Psi_1 \sim \tau^{-4},\ \Psi_2 \sim \tau^{-3},\ \Psi_3 \sim \tau^{-2},\ \Psi_4 \sim \tau^{-1}$$

  can expand

$$\Psi_4 = \frac{\Psi_4^{(1)}}{\tau} + \frac{\Psi_4^{(2)}}{\tau^2} + \frac{\Psi_4^{(3)}}{\tau^3} + \frac{\Psi_4^{(4)}}{\tau^4} + O\left(\frac{1}{\tau^5}\right)$$

- At $\mathscr{I}^+$, $\Psi_\cdot^{(1)}$ are components of conformal Weyl tensor. $\mathscr{I}^+$ is type O in physical spacetime and type I in conformal spacetime.
- "Gravitational wave well-defined on $\mathscr{I}^+$" not strictly true.

- Slightly off $\mathscr{I}^+$, Ψ_4 dominates, thus nearly type N in physical spacetime.
- Radiation defined in leading order expansion sense as $\Psi_4^{(1)}$. Thus depend on expansion parameter $\Leftrightarrow$ affine parameter $\Leftrightarrow$ conformal factor $\Omega \approx 1/\tau$.
- Dependence on conformal factor expected as $\Psi_4^{(1)}$ is a component of conformal Weyl tensor on $\mathscr{I}^+$.
- Numerically obtain $\Psi_4^{(1)}$ by fitting Ψ_4 in the computational domain to the peeling polynomial.
- Need unique tetrad in the computational domain to be consistent with peeling theorem, ideally minimize higher order coefficients, to avoid fitting instability.

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- Recall relationship between algebraic speciality and flatness ($\mathcal{K}$ example). Peeling is gradual process of achieving flatness (we externally imposed asymptotic flatness), manifesting in terms of GPNDs becoming closer in direction.
- The spacetime does not need to go through whole chain of specialities. Peeling figure commonly misinterpreted.

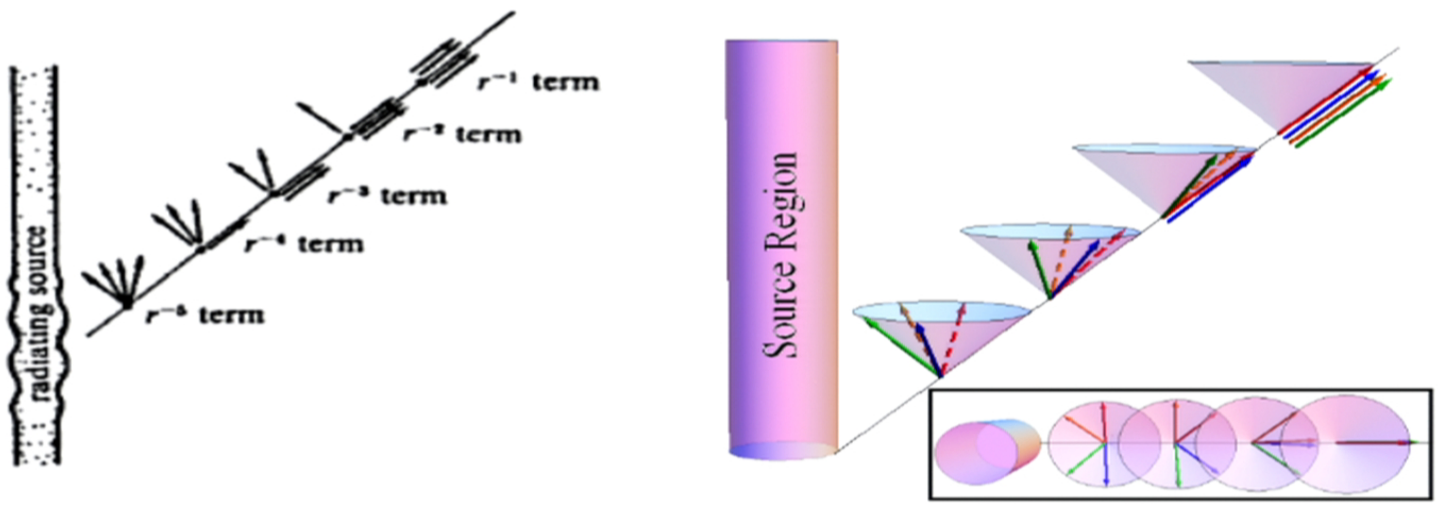


Figure: Left: reproduced from Spinors and Spacetime VII.

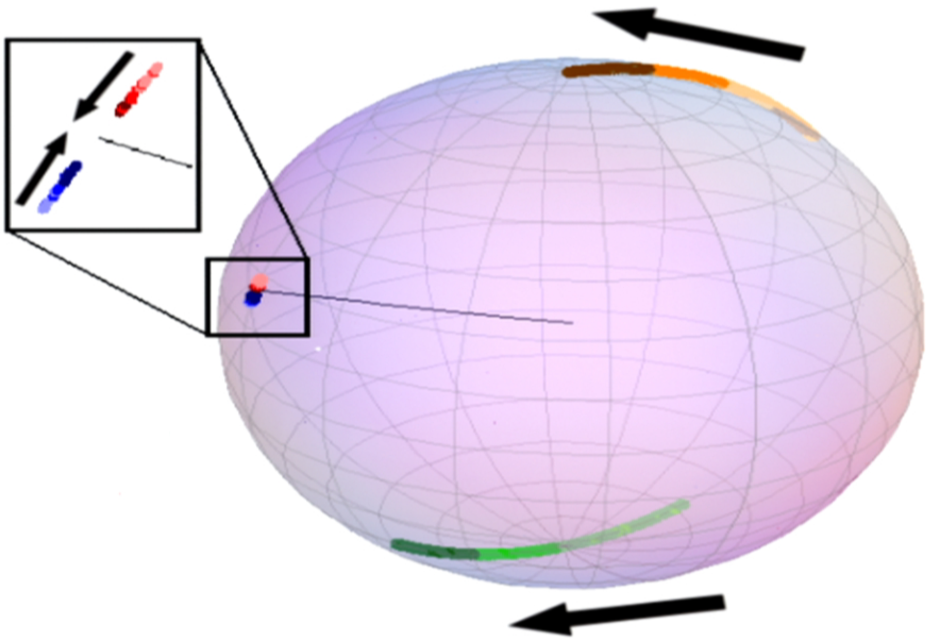
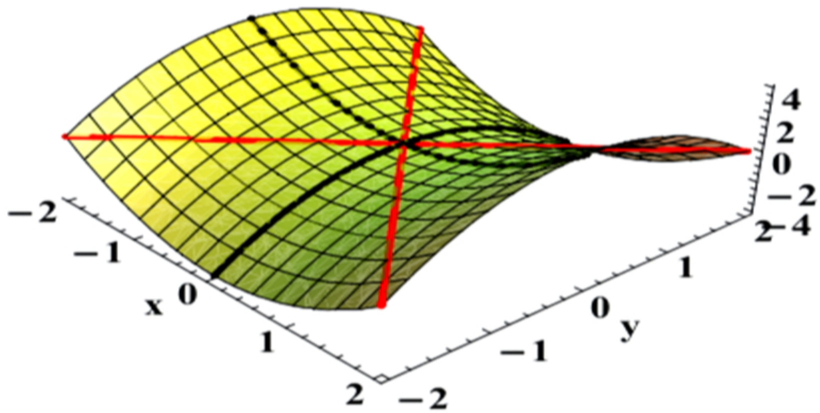


Figure: Actual motion of GPNDs.

- To translate from GPND closeness into expansion of NP Ψ's:
  1. Null vectors close in direction $\Rightarrow$ small inner product.
  2. Take mutual product GPNDs and averge over them as measure of their closeness.
  3. Or instead use a reference null vector l (a NP null basis), take inner product of GPNDs with it and multiply the results together.
  4. Translates into spinor contraction. By definition, Ψ_α with larger α contains more innerproduct of GPND with l, thus decays faster.

$$\Psi_0 = \Psi_{ABCD} \o^A \o^B \o^C \o^D,$$
$$\Psi_1 = \Psi_{ABCD} \o^A \o^B \o^C \iota^D,$$
$$\Psi_2 = \Psi_{ABCD} \o^A \o^B \iota^C \iota^D,$$
$$\Psi_3 = \Psi_{ABCD} \o^A \iota^B \iota^C \iota^D,$$
$$\Psi_4 = \Psi_{ABCD} \iota^A \iota^B \iota^C \iota^D,$$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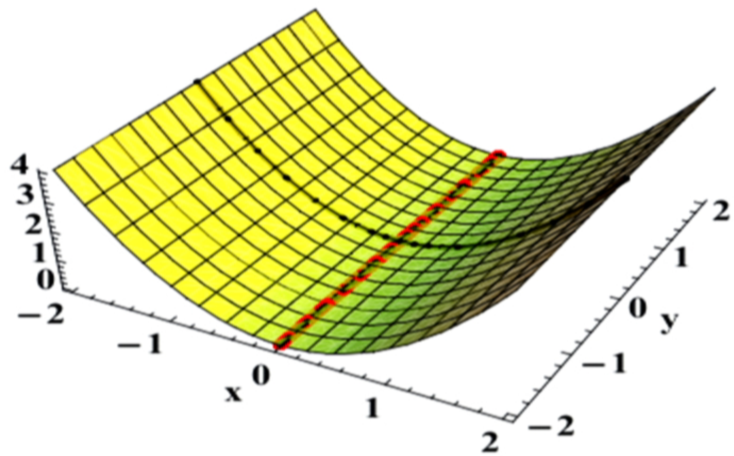
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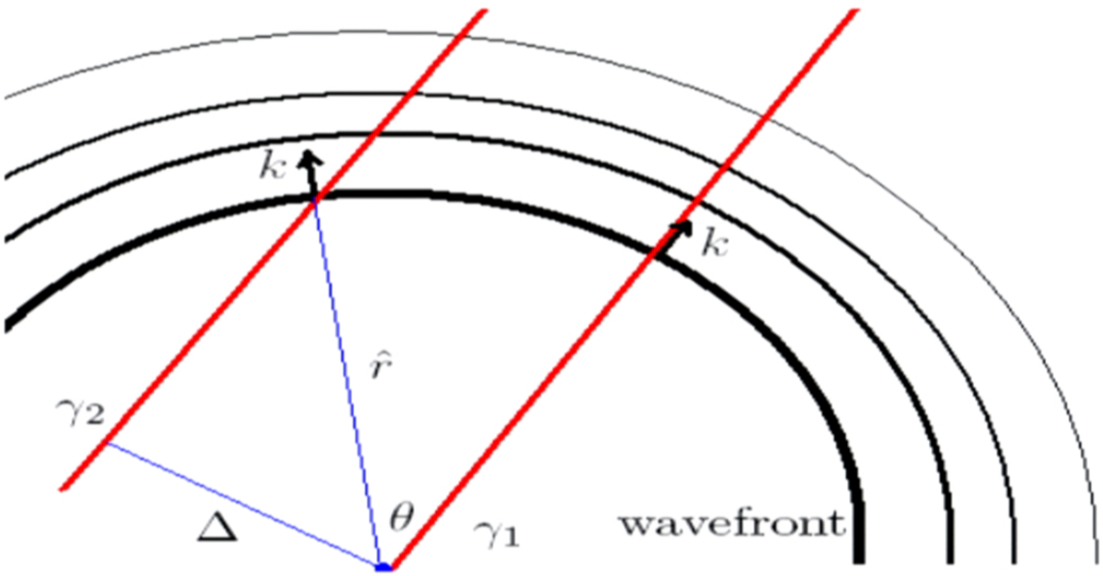
- Sampling angular variation of wavefront (nothing to do with peeling) also contributes to higher coefficients.



- Discussion on the PNDs suggest QKT l pointed along wave propagation direction defined asymptotically. Even locally, it matches Bel-Robinson vector.
- Bel-Robinson vector $\mathcal{P}_i$ analogous to Poynting vector. Identify energy flow direction

$$\frac{dE}{dt} = \frac{1}{4\pi} \oint \left[\int_{-\infty}^{t} dt' \left(\pm \sqrt{\frac{|\mathcal{P}^\alpha r_\alpha|}{2}} \right) \right]^2 r^2 d\Omega$$

- Under transverse tetrad,

$$\mathcal{P}_i = \frac{1}{2} \left(|\Psi_4|^2 - |\Psi_0|^2 \right) N_i$$

- Reduce higher coefficients by launching geodesics initially in true principal direction, or use integral curve of it instead.

The "true" principal directions and curvatures

- Anti-selfdual complexified Weyl tensor $(C + i^{*}C)_{ab}{}^{cd}$, seen as matrix in complexified bivectors space $\mathcal{V}_{ab} \equiv V_{ab} + i^{*}V_{ab}$
- Equivalently $\Psi_{AB}{}^{CD}$ in space of symmetric bispinors

$$\begin{bmatrix} -\Psi_2 + (\Psi_0 + \Psi_4)/2 & -i(\Psi_0 - \Psi_4)/2 & \Psi_3 - \Psi_1 \\ -i(\Psi_0 - \Psi_4)/2 & -\Psi_2 - (\Psi_0 + \Psi_4)/2 & i(\Psi_1 + \Psi_3) \\ \Psi_3 - \Psi_1 & i(\Psi_1 + \Psi_3) & 2\Psi_2 \end{bmatrix}.$$

where given a dyad (NP null tetrad)

$$\Psi_0 = \Psi_{ABCD}o^A o^B o^C o^D,$$
$$\Psi_1 = \Psi_{ABCD}o^A o^B o^C \iota^D,$$
$$\Psi_2 = \Psi_{ABCD}o^A o^B \iota^C \iota^D,$$
$$\Psi_3 = \Psi_{ABCD}o^A \iota^B \iota^C \iota^D,$$
$$\Psi_4 = \Psi_{ABCD}\iota^A \iota^B \iota^C \iota^D,$$

- Discussion on the PNDs suggest QKT l pointed along wave propagation direction defined asymptotically. Even locally, it matches Bel-Robinson vector.
- Bel-Robinson vector $\mathcal{P}_i$ analogous to Poynting vector. Identify energy flow direction

$$\frac{dE}{dt} = \frac{1}{4\pi} \oint \left[\int_{-\infty}^{t} dt' \left(\pm \sqrt{\frac{|\mathcal{P}^\alpha r_\alpha|}{2}} \right) \right]^2 r^2 d\Omega$$

- Under transverse tetrad,

$$\mathcal{P}_i = \frac{1}{2} \left(|\Psi_4|^2 - |\Psi_0|^2 \right) N_i$$

- Reduce higher coefficients by launching geodesics initially in true principal direction, or use integral curve of it instead.

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- During ringdown: electric and magnetic parity QNMs share the same frequency as determined by the Teukolsky equation.
- During inspiral: electric parity mode generated by mass quadrupole moment, magnetic parity mode by current quadrupole moment
- Mass quadrupole at twice orbital frequency, current quadrupole at orbital frequency plus spin precession frequency.
- Need to synchronize the two before QNM ringdown can begin.
- Spin precession frequency can increase towards orbital frequency but will not lock. Only momentary coincidence, imply ringdown has to begin precisely at that point if no additional dynamics.
- Precession frequency independent of spin magnitude to leading order, so infinitesimal spin determines start of QNMs. Does not seem right.

- Use either $\lambda^{(a)}$, I, J or geometrical coordinates to visualize highly dynamical spacetimes where analytical description is not available.
- Example using eigenvalue of spatial projection of imaginary part of ${\Psi_{AB}}^{CD}$, can also use fully invariant quantities above.
- A mystery of how to transition into quasinormal modes (QNMs) during binary black hole merger. Concentrate on dominant $(2, 2)$ mode.

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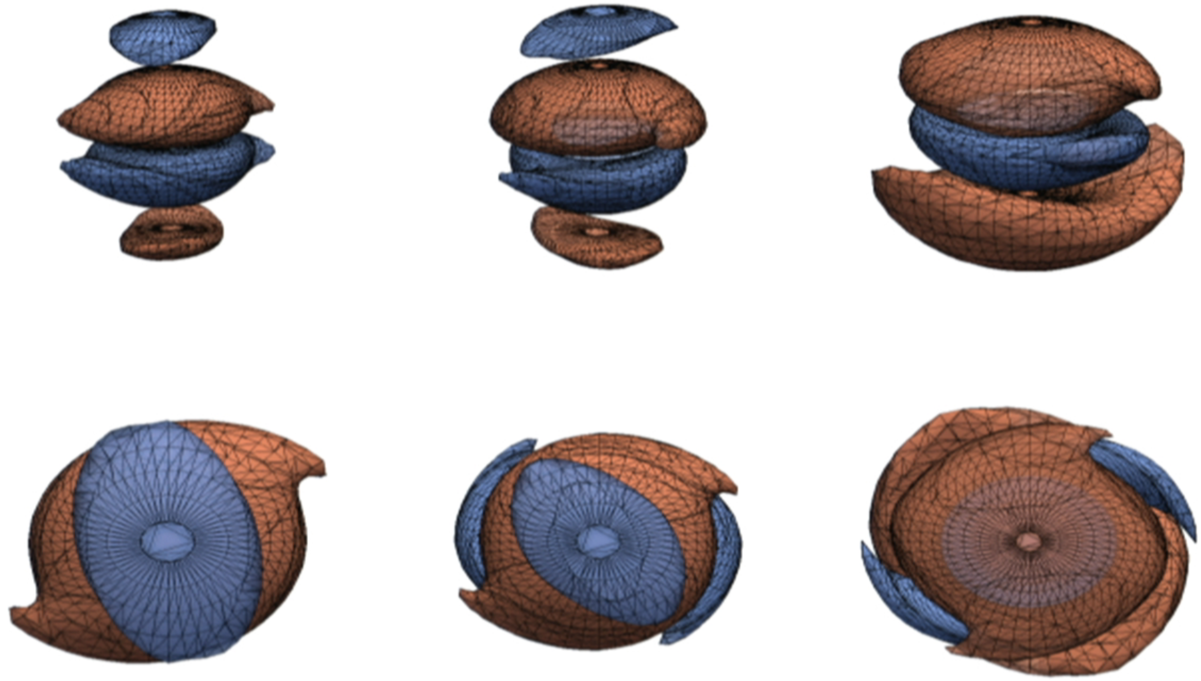
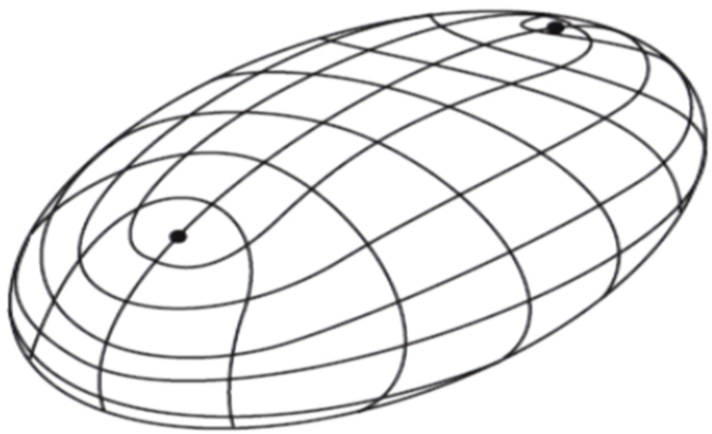
1. Current quadruple: refl. symm., skew-refl. anti-symm.
2. Mass quadruple: refl. anti-symm., skew-refl. symm.
3. Current dipole: refl. anti-symm., skew-refl. anti-symm.



Utilizing the integral curves of true principal directions

- Integral curves of true principal directions are useful for studying large scale geometry of spacetimes.
- e.g. number and type of umbilics determined by topology via Poincare-Hopf theorem.



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