Title: Playing with a Bouncing Universe

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Abstract:

Playing with a Bouncing Universe

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Based on work with Itzhak Bars, Paul Steinhardt
New work with Latham Boyle, Kieran Finn
Steffen Gielen

Wow!

Inflation has turned out to be an amazing phenomenological model: e.g

Inflation versus Cyclic Predictions for Spectral Tilt

Justin Khoury[1], Paul J. Steinhardt[2] and Neil Turok[3]

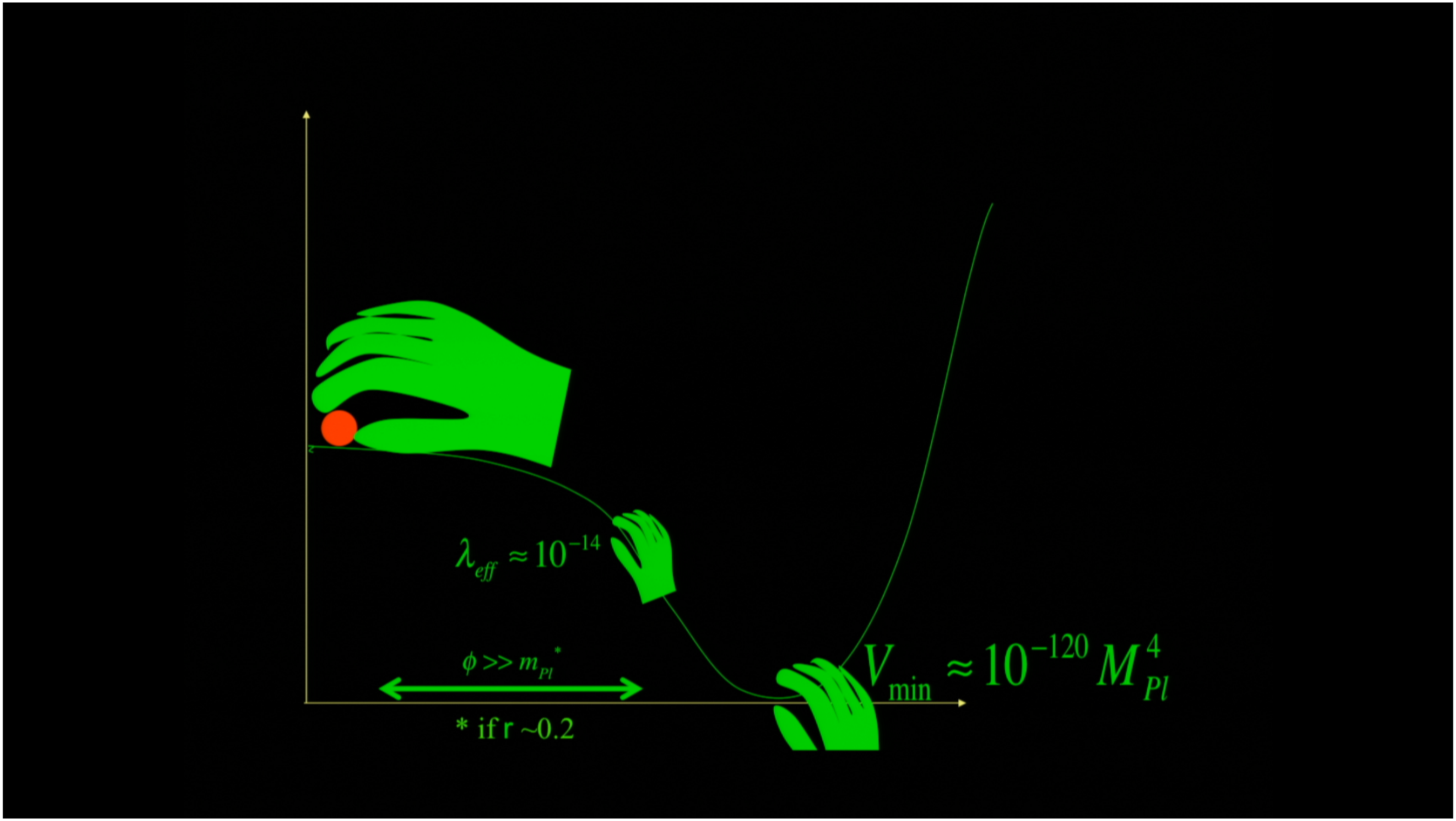
Sep 2003

$$(n_s - 1)_{inf} \approx -\frac{3}{\mathcal{N}} \quad T/S \approx 13.8\bar{\epsilon} \approx \frac{13.8}{\mathcal{N}} \approx 23\%$$

If BICEP2 is confirmed, I'll be delighted to pay up:
pair of roller-blades for Eva and $200 (CDN) to Stephen H

But let's not forget…

It still has some pretty big problems



$\lambda_{eff} \approx 10^{-14}$
$\phi >> m_{Pl}{}^{*}$
* if r ~0.2
$V_{min} \approx 10^{-120} M_{Pl}^{4}$

With all of inflation's successes, we should still ask:

might there be a simpler and more predictive explanation?

a bouncing universe is a natural possibility to explore

A cyclic scenario is attractive because it might provide a dynamical solution to various tuning problems including the mass hierarchies (though I won't discuss this here).

Let's pursue a minimalist approach

Curiosity 1: flat FRW radiation dominated universe

$$ds^2 = t^2(-dt^2 + d\vec{x}^2), \ -\infty < t < \infty$$

- an analytic bounce!

$$\text{Perturbations}: \delta_r \sim e^{\pm ikt/\sqrt{3}}, \quad h_{ij} \sim \frac{e^{\pm ikt}}{t}$$

- single-valued in the complex $t-$plane around $t=0$
  (in a suitable gauge)

Interesting to explore at higher order

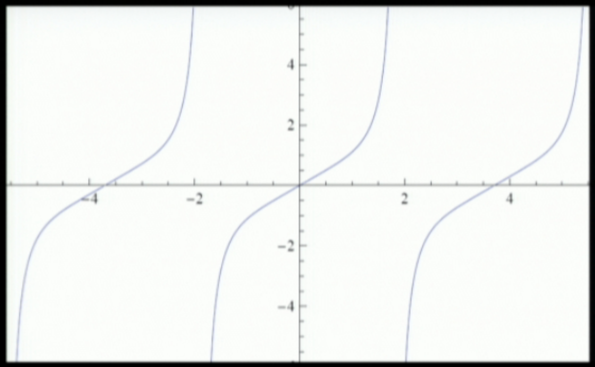
w/ S. Gielen

Curiosity 2: include Λ

$$ds^2 = a(t)^2(-dt^2 + d\vec{x}^2); \quad a'^2 = \tfrac{8\pi G}{3}\left(\rho_r + \rho_\Lambda a^4\right)$$

$$\Rightarrow a = -e^{i\pi/4}(\rho_r / \rho_\Lambda)^{1/4} sn(e^{3i\pi/4}(\rho_r \rho_\Lambda)^{1/4} t, -1)$$

$a(t)$



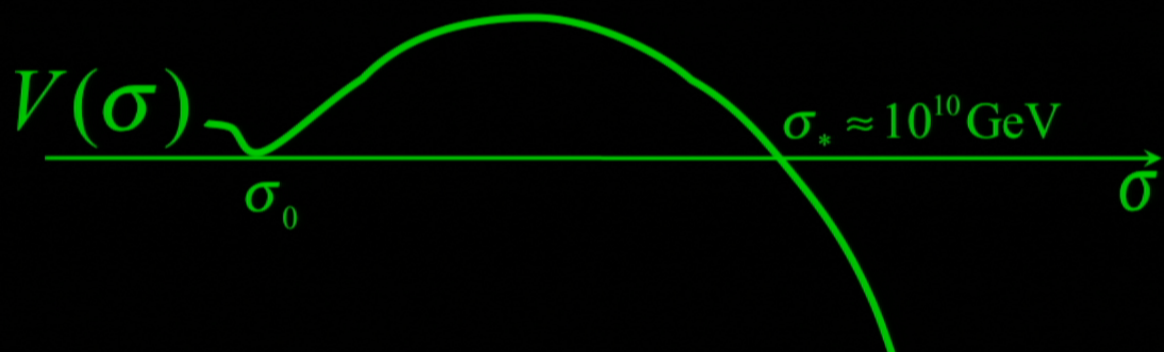
t

- a cyclic universe!

Only singularities are poles, unique analytic continuation to complex t-plane, doubly periodic (elliptic) function

Now, lets try to a bit more realistic

Assuming no new physics, we can extrapolate MSM to near-Planckian Higgs vevs:

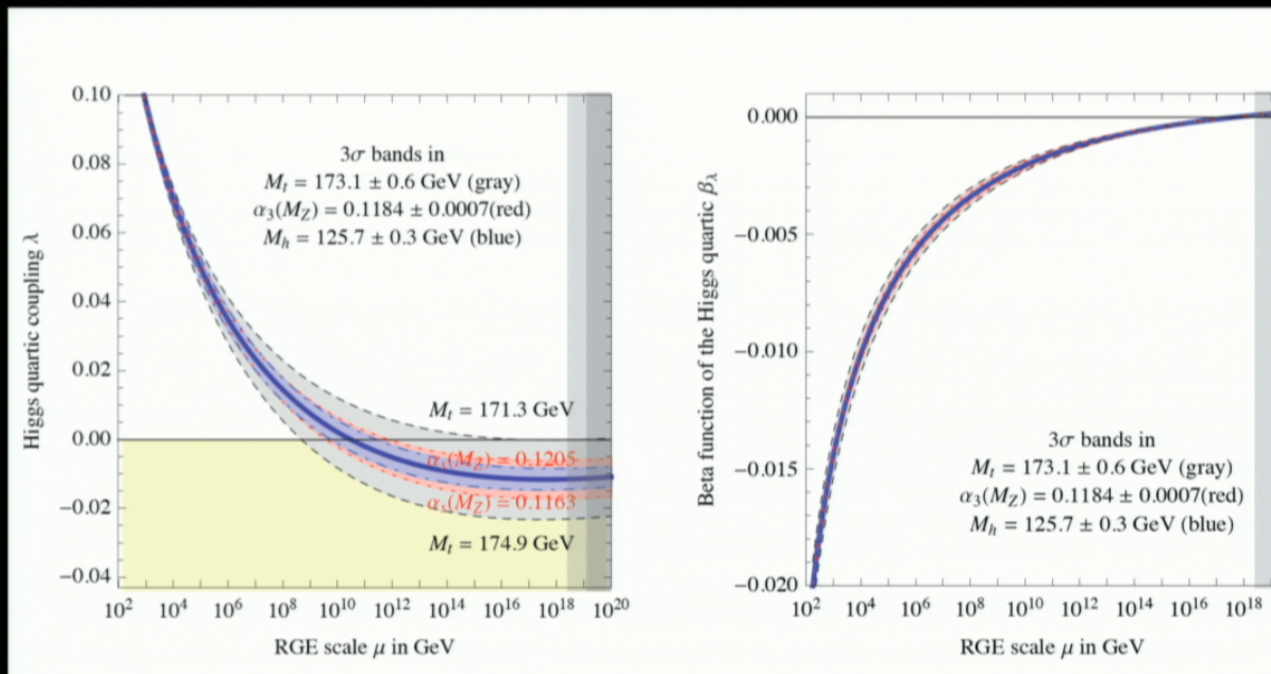


Conclusions from LHC:

1) SM vac "on the edge" of stability

2) As $\varphi \to m_{Pl}$, evidence for scale invariance

$$V = \lambda(\sigma)\sigma^4$$



Buttazzo et al 1307.3536

The Standard Model:

$$\int\left[m_{Pl}^2 R + \left|D\sigma\right|^2 - V(\sigma, m_H, \Lambda) - F^2 + \bar{\psi} i D\psi - y\bar{\psi}\psi\sigma - Y\nu_R C\nu_R M \right]$$

Content:

Gravity, Higgs
Gauge $SU_3 x SU_2 x U_1$
Fermions

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{B-L}$
q_L	3	2	$+1/6$	$+1/3$
u_R	3	1	$+2/3$	$+1/3$
d_R	3	1	$-1/3$	$+1/3$
l_L	1	2	$-1/2$	-1
ν_R	1	1	0	-1
e_R	1	1	-1	-1
h	1	2	$+1/2$	0
φ	1	1	0	$+2$

A tight and consistent structure:

- chiral anomaly cancellation
- gauging one U1 requires one "miracle"
- gauging two (Y, B-L) requires two "miracles"
  and rh neutrinos

- just 4 dimensionful numbers – potentially, all are a result of spontaneously broken scale symmetry

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“Lift” SM to conformal (Weyl) symmetry)

$$\int\left[m_{Pl}^2 R + |Dh|^2 + V(h, m_H, \Lambda)\right]$$

$$\Rightarrow \int\left[\tfrac{1}{6}\left(\varphi^2 - |h|^2\right)R - |D\varphi|^2 + |Dh|^2 + V_4(h,\varphi)\right]$$

φ : looks like ghost but can be locally gauged away

vev $\langle\varphi\rangle \rightarrow m_{Pl}, M, m_H, m_\Lambda$

Nevertheless, nontrivial dynamics of φ can generate RH neutrinos -> baryogenesis, dark matter

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Study FRW dynamics in various Weyl gauges

Let $\varphi_{\pm} \equiv \varphi \pm h$

$\varphi_{+} = 1$ "*M theory*" *gauge*

$\varphi_{+} = 1/\varphi_{-}$ "*Einstein*" *gauge*

$\varphi_{+} = 1/\varphi_{-}^{2}$ "*string*" *gauge*

For resolving FRW-type singularities,

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FRW cosmology

$$ds^2 = a(t)^2(-dt^2 + d\vec{x}^2)$$

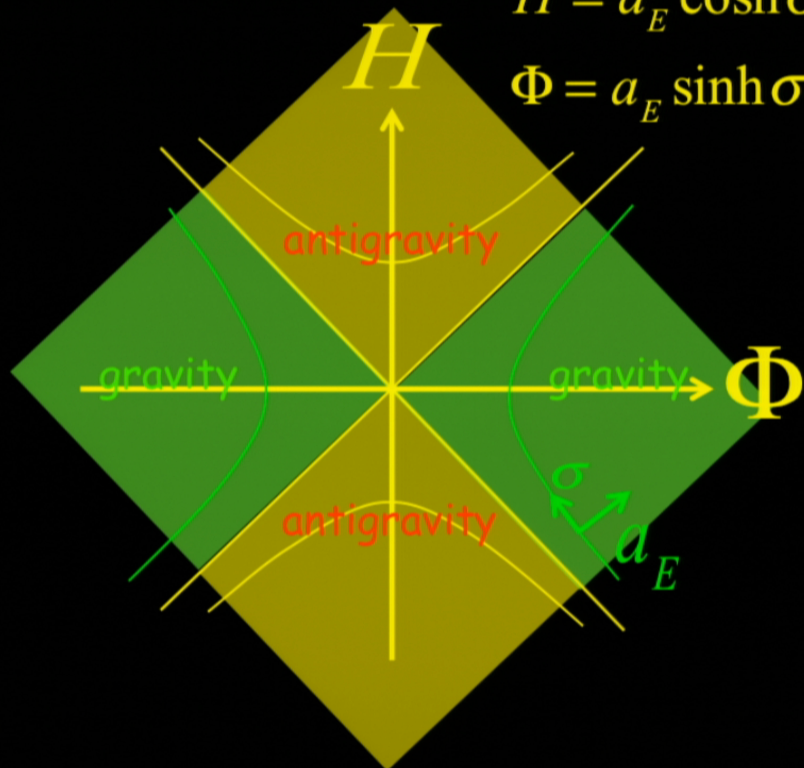
Define Weyl-invariant $\Phi \equiv a\varphi,\ H = ah$

($i.e.$, φ and h in unimodular gauge)

Then $\Phi^2 - H^2 = a^2(\varphi^2 - h^2) = a_E^2$ in Einstein gauge

$$S = \int dt\left(-\Phi'^2 + H'^2 - (\Phi^2 - H^2)^2 V(H/\Phi) - \rho_r\right)$$

Kinetic-dominated dynamics is regular across $a_E = 0$



$$H = a_E \cosh\sigma$$
$$\Phi = a_E \sinh\sigma$$

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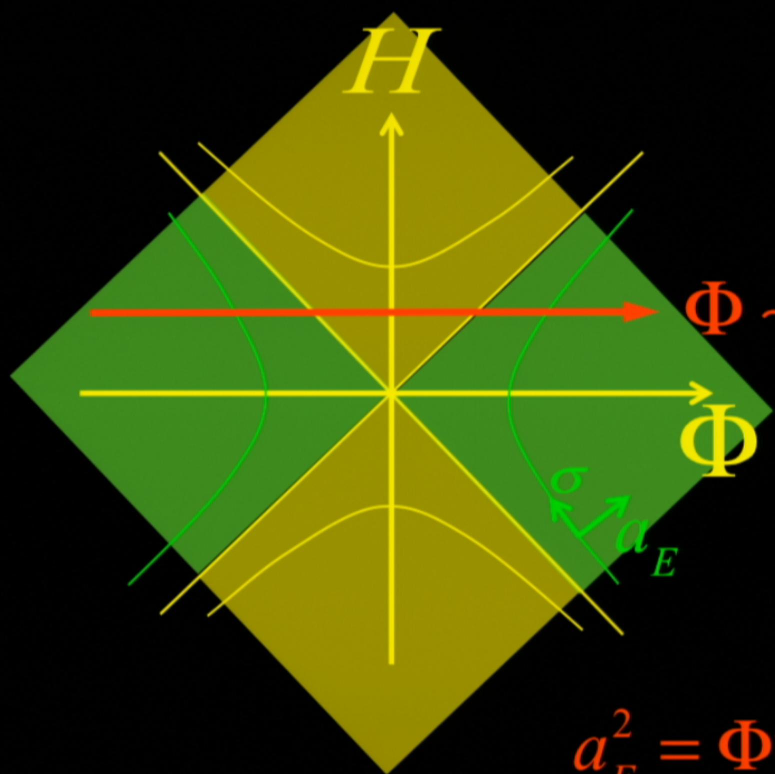
$$H = a_E \sinh\sigma$$
$$0 < a_E < \infty,$$
$$-\infty < \sigma < \infty$$



$$\Phi \sim \rho_r^{\,1/2} t$$

$$a_E^2 = \Phi^2 - H^2 \sim t^2 - t_*^2$$

A deeper and more powerful perspective is obtained by analytically continuing the solution into the complex t-plane.

$$a^2(t) = t^2 - t_*^2$$

$$\sigma \sim m_{Pl} \ln(\tfrac{t-t_*}{t+t_*})$$

The solution is complex but *a priori* it seems a reasonable candidate saddle point in a semi-classical approximation to Feynman's path integral

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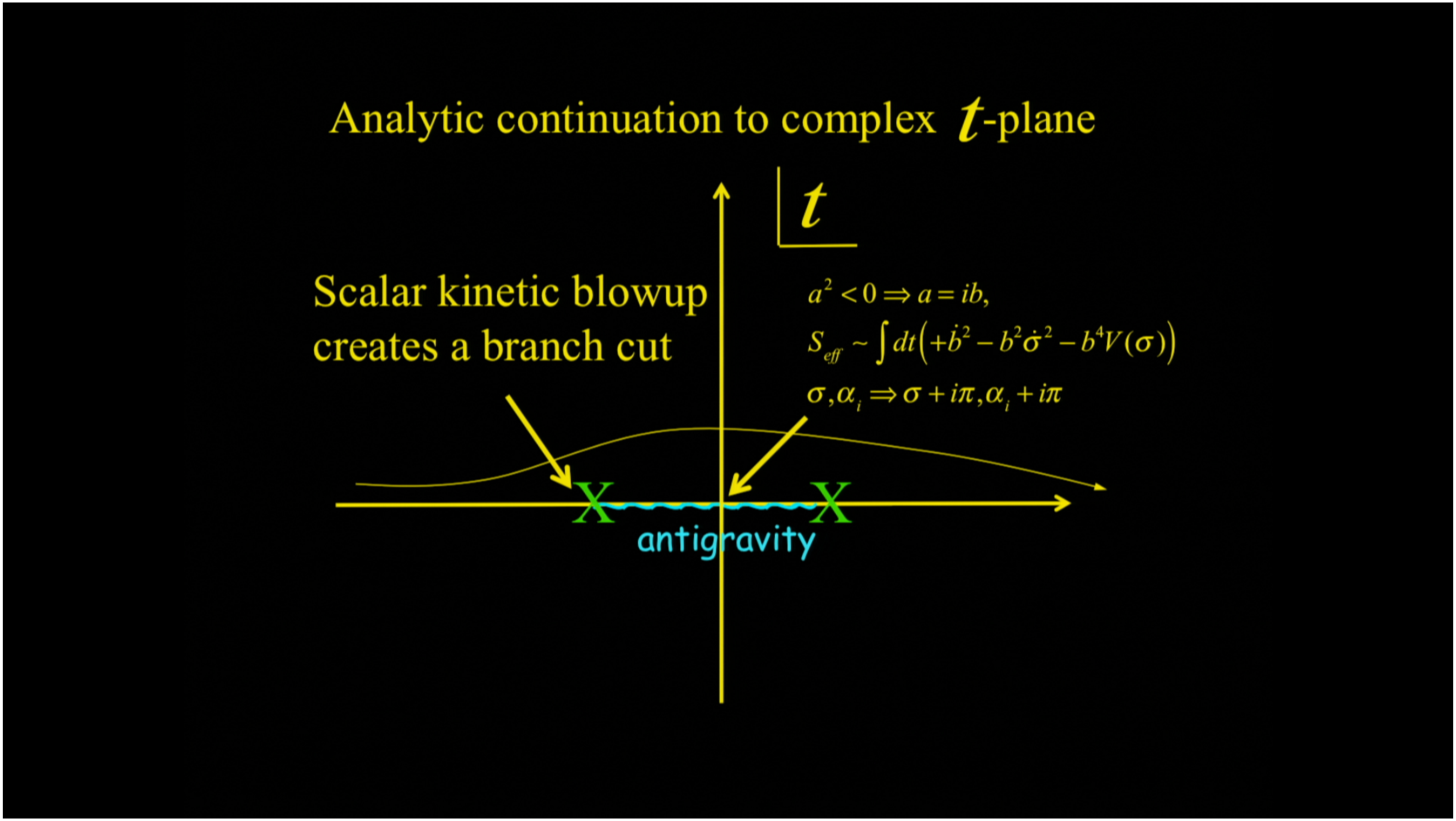

Analytic continuation to complex t-plane
t
Scalar kinetic blowup
creates a branch cut
$a^2 < 0 \Rightarrow a = ib,$
$S_{eff} \sim \int dt\left(+\dot{b}^2 - b^2\dot{\sigma}^2 - b^4 V(\sigma)\right)$
$\sigma, \alpha_i \Rightarrow \sigma + i\pi, \alpha_i + i\pi$
X
X
antigravity

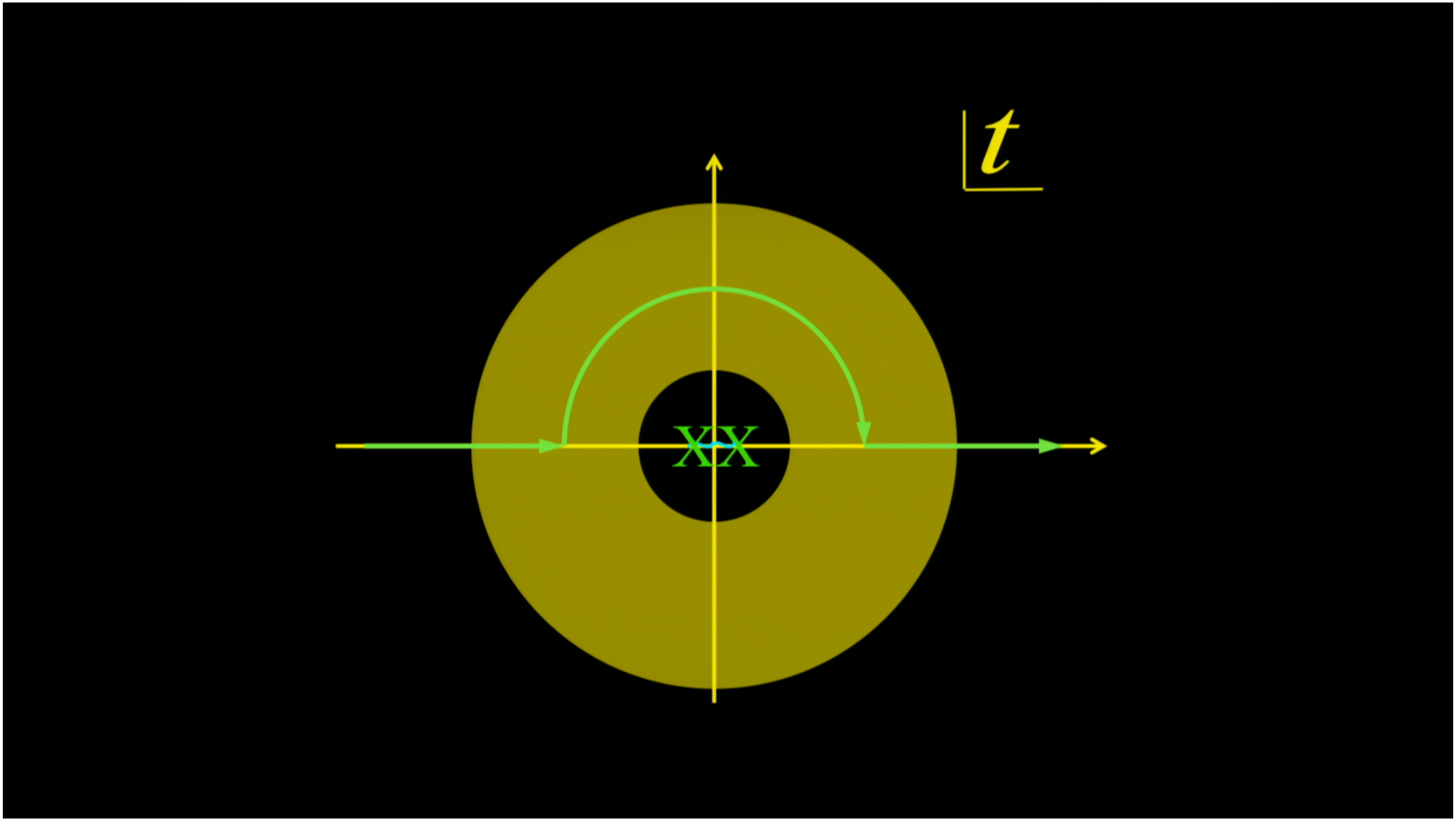
Production of tensor modes in kinetic-dominated regime

$$a^2 = t^2 - t_*^2$$

$$\ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 0;\ h_{ij} = a^{-1}\chi_{ij}$$

$$\ddot{\chi}_{ij} + k^2\chi_{ij} = \frac{\ddot{a}}{a}\chi_{ij} = -\frac{t_*^2}{(t^2 - t_*^2)^2}\chi_{ij}$$

No analytic solution, but can see that graviton production vanishes by analytic continuation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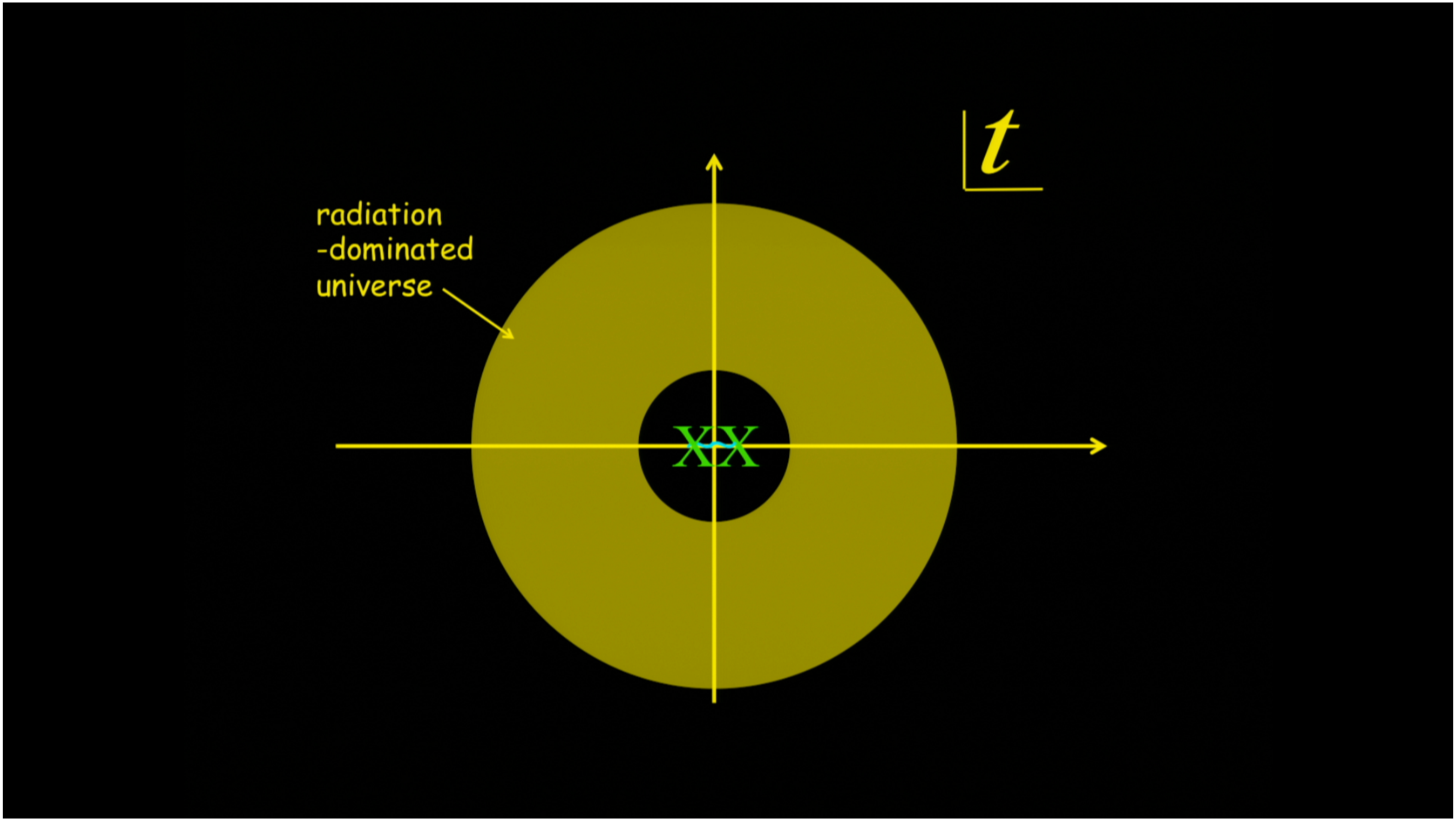
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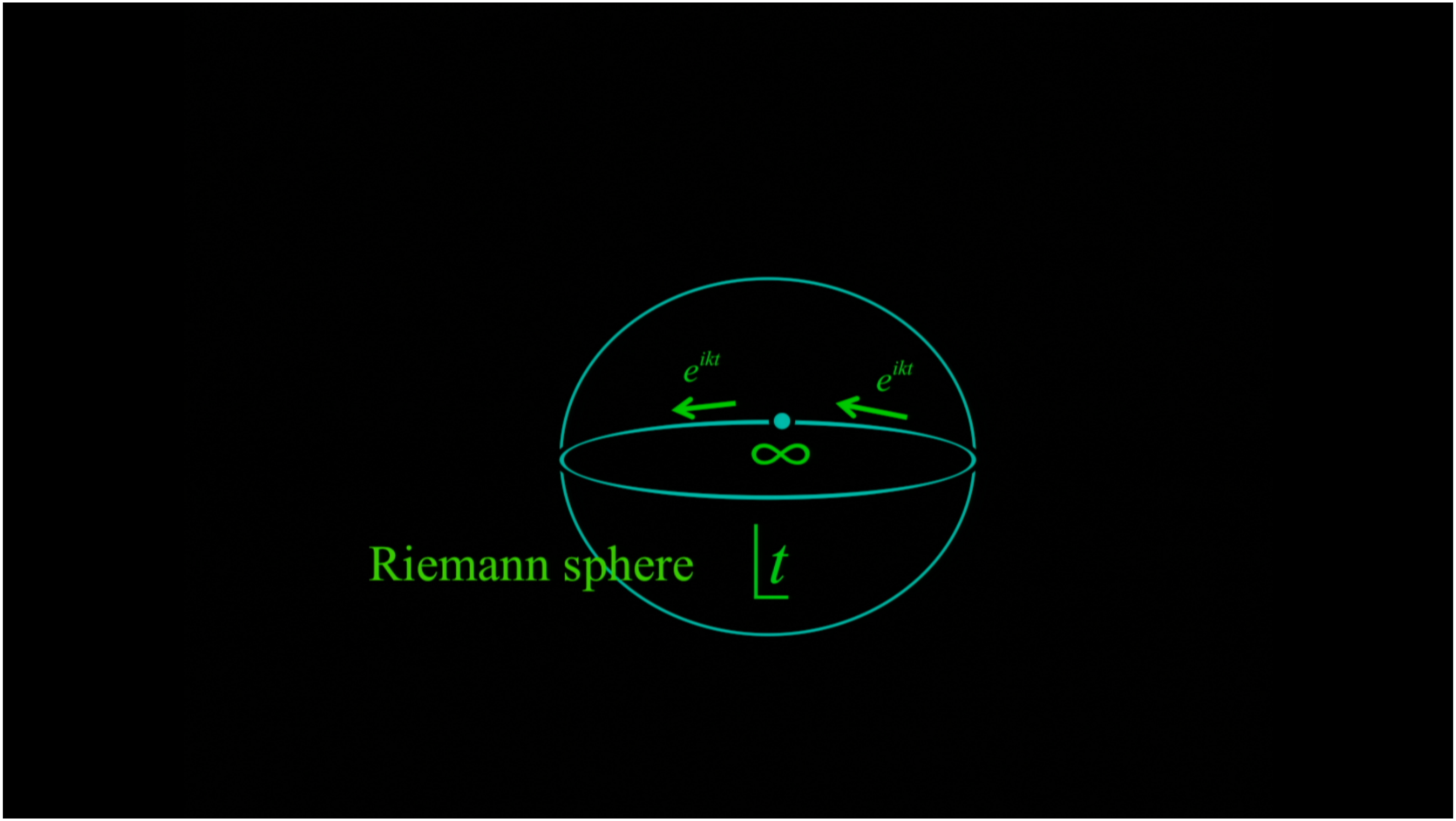

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Riemann sphere
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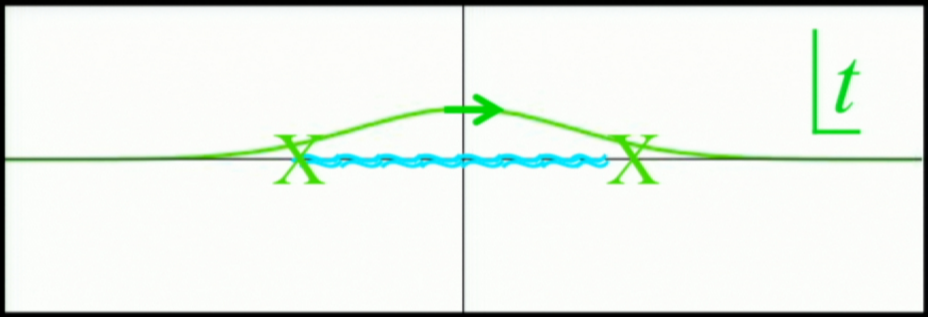
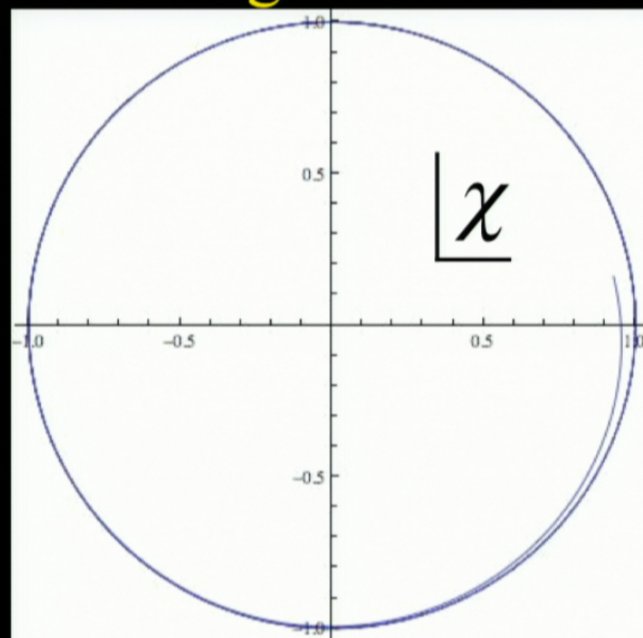
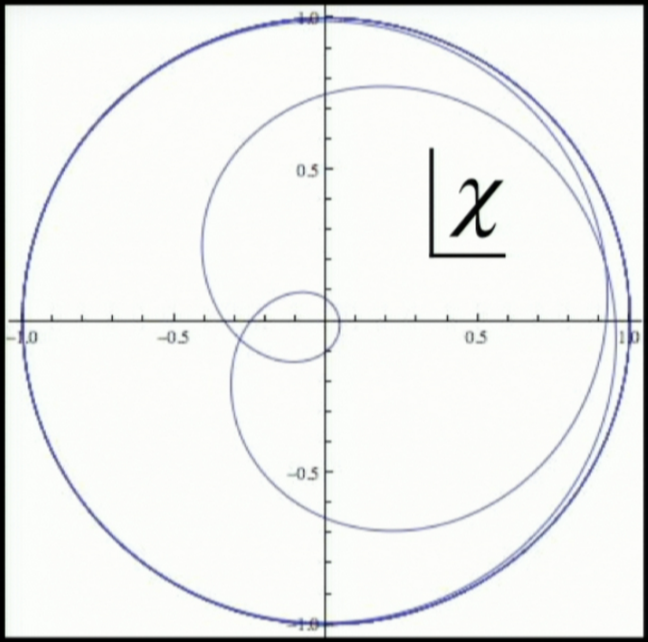
Integrate along contour above singularities



Incoming



Outgoing: no graviton production!



Now add anisotropy to the background

$$ds^2 = -dt^2 + a^2(t)\, ds_3^2 = a^2(\tau)\left(-d\tau^2 + ds_3^2\right),$$

anisotropic Bianchi IX metric with curvature K

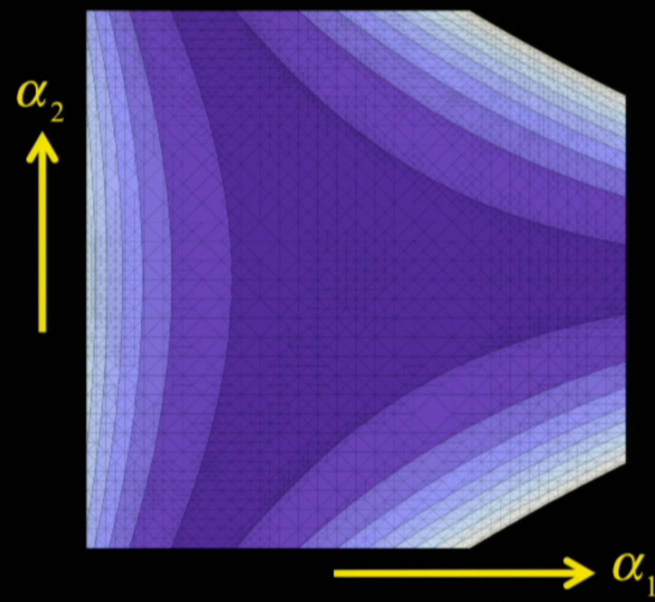
$$\left(ds_3^2\right)_{IX} = e^{-2\sqrt{2/3}\kappa\alpha_1}\left(d\sigma_z\right)^2 + e^{\sqrt{2/3}\kappa\alpha_1}\left(e^{\sqrt{2}\kappa\alpha_2}\left(d\sigma_x\right)^2 + e^{-\sqrt{2}\kappa\alpha_2}\left(d\sigma_y\right)^2\right),$$

Einstein-frame effective action

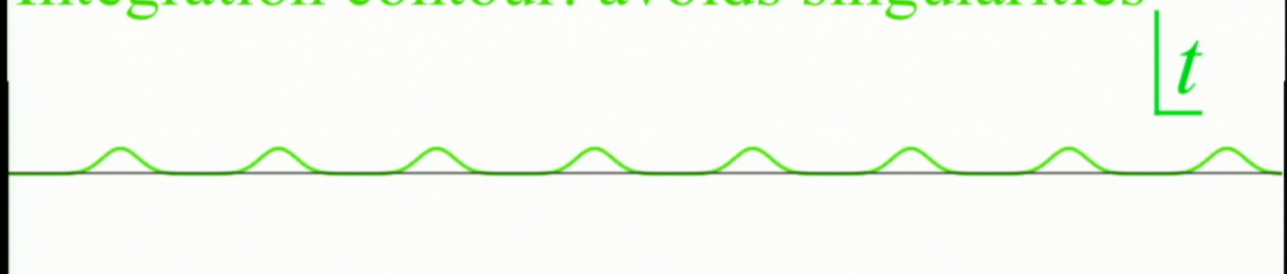
$$S_{\text{eff}}^{E} = \int d\tau \left\{ \begin{array}{c} \frac{1}{e}\left[-\frac{6}{2\kappa^2}\dot{a}_E^2 + \frac{1}{2}a_E^2\dot{\sigma}^2 + \frac{1}{2}a_E^2\dot{\alpha}_1^2 + \frac{1}{2}a_E^2\dot{\alpha}_2^2\right] \\ -e\left[a_E^4 V(\sigma) + \rho_0 - \frac{6K}{2\kappa^2}a_E^2 v(\alpha_1,\alpha_2)\right] \end{array} \right\}.$$

$$v(\alpha_1,\alpha_2) = \frac{1}{3}\left(-e^{-4\sqrt{2/3}\kappa\alpha_1} - 4e^{2\sqrt{2/3}\kappa\alpha_1}\sinh^2\left(\sqrt{2}\kappa\alpha_2\right) + 4e^{-\sqrt{2/3}\kappa\alpha_1}\cosh\left(\sqrt{2}\kappa\alpha_2\right)\right)$$

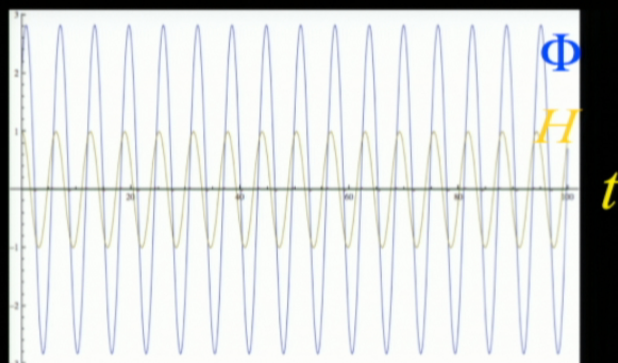
Anisotropy Potential $V_{aniso}(\alpha_1,\alpha_2)$

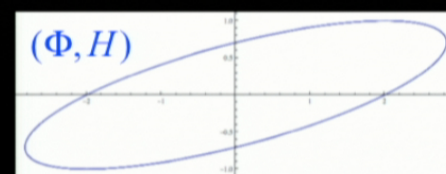


Integration contour: avoids singularities

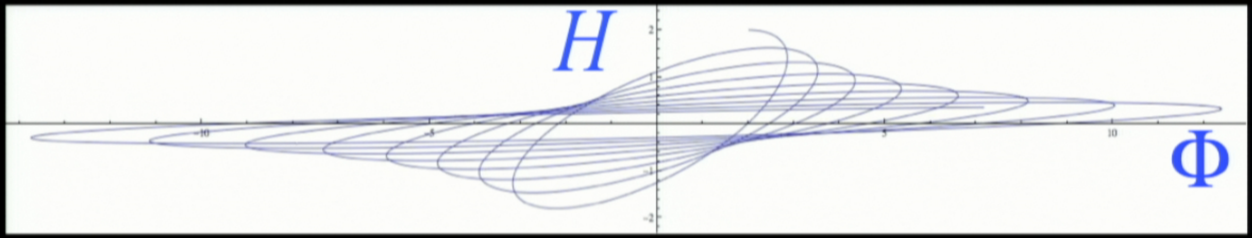


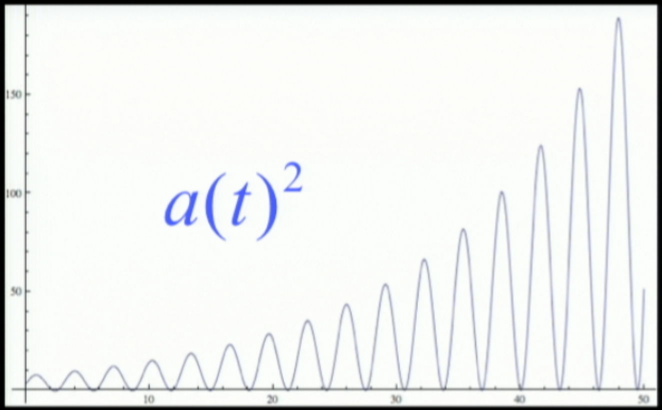
Evolution with small anisotropy $\alpha_1 \sim \alpha_2 \sim 10^{-3}$



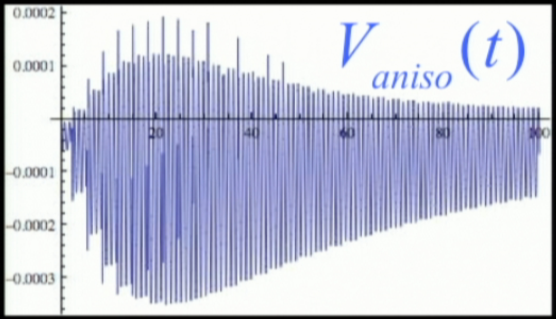


Solution with entropy production





Anisotropy damps away



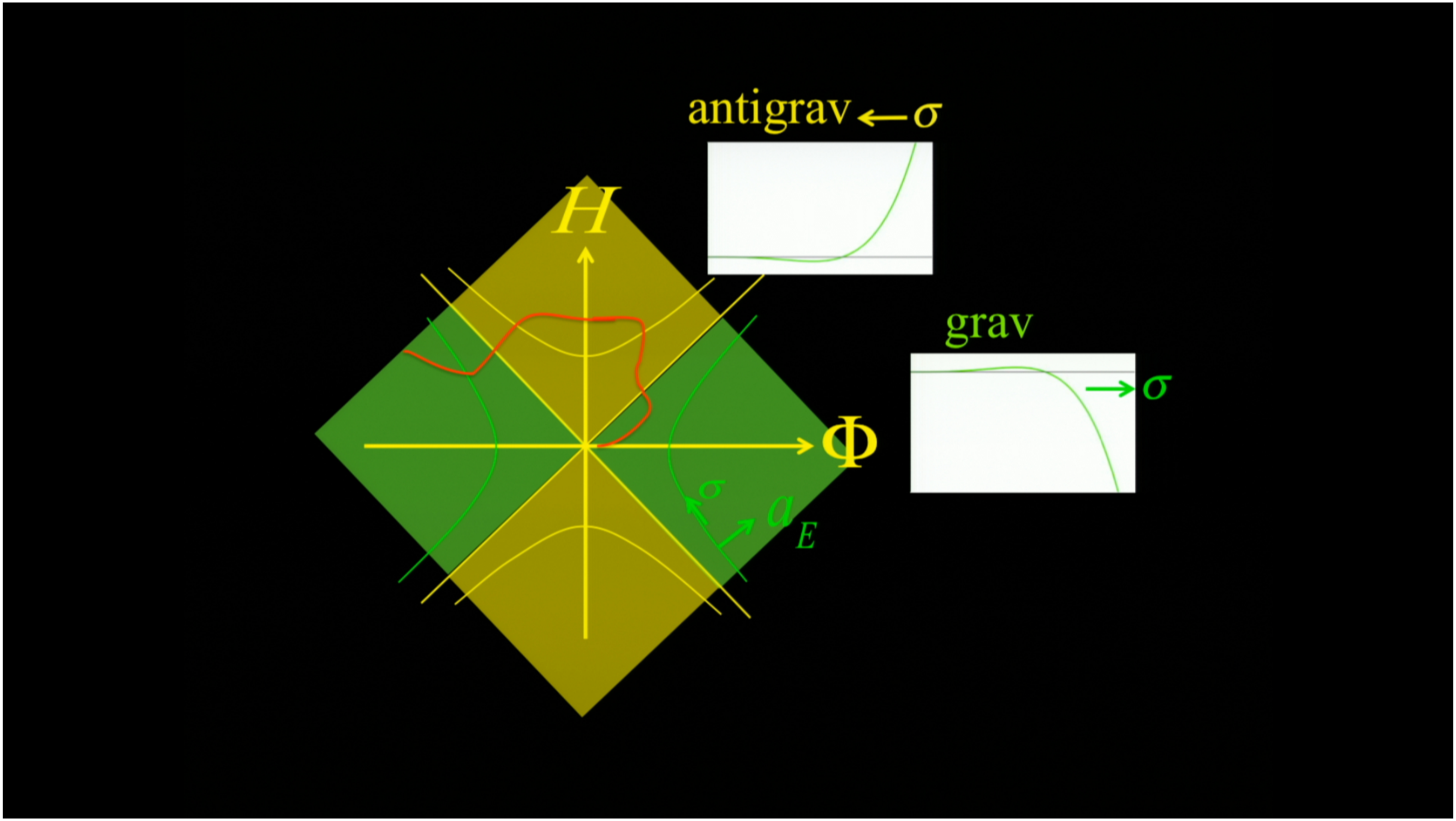
Note: presence of radiation speeds passage through antigravity phase

Can we get something that looks like inflation in the "antigravity" region?

It seems the answer is yes! a negative Higgs potential behaves like positive potential in that region (since scalar KE reverses)

$V_{eff}(\sigma) \sim -\lambda\sigma^4,\ \lambda \approx 10^{-14}$ is allowed by LHC,

I am just extrapolating this to σ beyond m_{Pl}


antigrav ← σ
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σ
a_E

A (super)-economical version of leptogenesis becomes feasible

w/ L Boyle and K Finn

Recall: Majorana mass term becomes $\varphi \nu_R C \nu_R$

$$[\tfrac{i}{a}\left(\gamma^0 \partial_t + \gamma^i \partial_i\right) + \tfrac{3}{2}\tfrac{a'}{a^2}\gamma^0 - m(t)]\Psi = 0$$

Define $\psi = a^{3/2}\Psi$, $\mu = am$

$$\Rightarrow \left(i\gamma^\mu \partial_\mu - \mu(t)\right)\psi = 0.$$

In the simplest bounce solution, $\mu = Y\rho_r^{1/2} t \equiv \alpha t$

Modes obey $\ddot{\chi}_k^\pm + (k^2 + \alpha^2 t^2 \mp i\alpha)\chi_k^\pm = 0$

Particle production across bounce

$$n_k = |\beta_k|^2 = e^{-k^2/\alpha}$$

$$\Rightarrow n = \int \frac{d^3\vec{k}}{(2\pi)^3} |\beta_k|^2 \sim \alpha^{3/2} \sim Y^{3/2} \rho_r^{3/4} \theta_{CP,\nu}^2$$

$$\Rightarrow \frac{n_L}{s} \sim Y^{3/2} \theta_{CP,\nu}^2,$$

converted to $\frac{n_L}{s}$ via EW B+L anomaly

Conclusion

A simple approach to bounces (but there are virtues in that!)

Still need to include (or parameterize) entropy generation at bounce resulting from microphysics

Perhaps this can become a competitor to inflation