Title: B modes for galaxy surveys

Date: Apr 04, 2014 02:00 PM

URL: http://pirsa.org/14040123

Abstract:

B modes for (future) galaxy surveys

Marc Kamionkowski (Johns Hopkins)
Perimeter Institute, 4 April 2014

1

Outline

- Lightning review of inflation
- Clustering fossils
- Tensor fossils from inflation
- How to get B modes from tensors with no T nor E

2

Who did most of the work

Donghui Jeong
(JHU->Penn State)

Liang Dai
(JHU)

3

Inflation!

Inflaton $\phi(\vec{x})$

curvature $\zeta(\vec{x})$

density perturbation $\delta(\vec{x})$

potential $\Phi(\vec{x})$

Are all <u>scalar fields</u>

(and interchangeable)

Inflationary Predictions

$$\tilde{\Phi}(\vec{k}) = \int d^3\vec{x}\, e^{i\vec{k}\cdot\vec{x}} \phi(\vec{x})$$

$$\langle \tilde{\Phi}(\vec{k}) \tilde{\Phi}^*(\vec{k}') \rangle = \delta_{\vec{k},\vec{k}'} P_\phi(k)$$

power spectrum

different Fourier modes are statistically independent

Gaussianity

$$\langle \tilde{\Phi}(\vec{k}_1) \tilde{\Phi}(\vec{k}_2) \cdots\cdots \tilde{\Phi}(\vec{k}_n) \rangle = 0 \qquad n \in \text{odd}$$

$$\langle \tilde{\Phi}(\vec{k}_1) \tilde{\Phi}(\vec{k}_2) \tilde{\Phi}(\vec{k}_3) \tilde{\Phi}(\vec{k}_4) \rangle = \delta_{\vec{k}_1,-\vec{k}_3} \delta_{\vec{k}_2,-\vec{k}_4} P_\phi(k_1) P_\phi(k_2) + 2 \text{ perms}$$

(= disconnected)

Because $S = \int d^4x \sqrt{-g} \left[-\frac{1}{2m_{pl}^2} R + \frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu \phi) - V(\phi) \right]$

$$\simeq \sum_{k} \left[k^2 |\tilde{\zeta}_{\vec{k}}|^2 + |\dot{\tilde{\zeta}}_{\vec{k}}|^2 \right] + \mathcal{O}(\tilde{\zeta}_{\vec{k}}^3)$$

$$\simeq \quad \underline{\qquad\qquad}$$

5

$\mathcal{O}(\zeta_{\vec{k}}^3)$ $\exists$! even if $V = 0$

from mixing of ϕ with scalar modes (Φ, Ψ, ζ) of $g_{\mu\nu}$

non-Gaussianity

$$\Rightarrow \langle \delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3)\rangle = \delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, B(k_1,k_2,k_3)$$

$\neq 0$ bispectrum

$B(k_1,k_2,k_3) =$ small in single-field slow-roll (SFSR) inflation (with standard kinetic term)

but may be big in beyond-SFSR inflation (e.g., DBI, curvaton, ... ; arXiv 2002-2012)

6

Inflationary Predictions

$$\tilde{\Phi}(\vec{k}) = \int d^3\vec{x}\, e^{i\vec{k}\cdot\vec{x}} \phi(\vec{x})$$

$$\langle \tilde{\Phi}(\vec{k}) \tilde{\Phi}^*(\vec{k}') \rangle = \delta_{\vec{k},\vec{k}'} P_\phi(k)$$

power spectrum

different Fourier modes are statistically independent

Gaussianity

$$\langle \tilde{\Phi}(\vec{k}_1) \tilde{\Phi}(\vec{k}_2) \cdots\cdots \tilde{\Phi}(\vec{k}_n) \rangle = 0 \quad n \in \text{odd}$$

$$\langle \tilde{\Phi}(\vec{k}_1) \tilde{\Phi}(\vec{k}_2) \tilde{\Phi}(\vec{k}_3) \tilde{\Phi}(\vec{k}_4) \rangle = \delta_{\vec{k}_1,-\vec{k}_3} \delta_{\vec{k}_2,-\vec{k}_4} P_\phi(k_1) P_\phi(k_2) + 2 \text{ perms}$$

(= disconnected)

Because $S = \int d^4x \sqrt{-g} \left[-\frac{1}{2m_{pl}^2} R + \frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu \phi) - V(\phi) \right]$

$$\simeq \sum_{k} \left[k^2 |\tilde{\zeta}_{\vec{k}}|^2 + |\dot{\tilde{\zeta}}_{\vec{k}}|^2 \right] + \mathcal{O}(\tilde{\zeta}_{\vec{k}}^3)$$

$$\simeq \quad \_\_\_\_\_\_$$

$\mathcal{O}(\zeta_{\vec{k}}^3)$ $\exists$! even if $V=0$

from mixing of ϕ with scalar modes (Φ, Ψ, ζ) of $g_{\mu\nu}$

non-Gaussianity

$$\Rightarrow \langle \delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3)\rangle = \delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, \underbrace{B(k_1,k_2,k_3)}_{\text{bispectrum}}$$

$\neq 0$

$B(k_1,k_2,k_3) =$ small in single-field slow-roll (SFSR) inflation (with standard kinetic term)

but may be big in beyond-SFSR inflation (e.g., DBI, curvaton,.....; arXiv 2002-2012)

6

$\mathcal{O}(\zeta_{\vec{k}}^3)$ $\exists$! even if $V = 0$

from mixing of ϕ with scalar modes (Φ, Ψ, ζ) of $g_{\mu\nu}$

non-Gaussianity

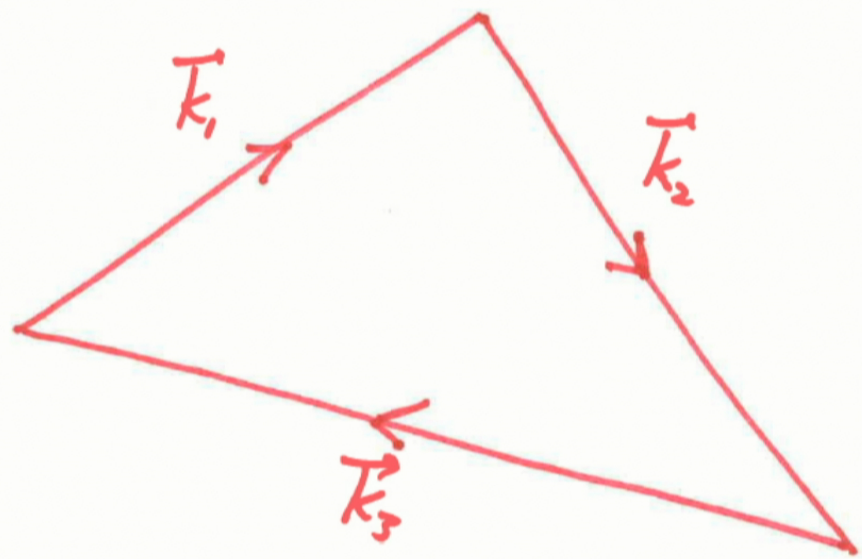
$$\Rightarrow \langle \delta(\vec{k}_1)\delta(\vec{k}_2)\delta(\vec{k}_3) \rangle = \delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, \underbrace{B(k_1,k_2,k_3)}_{\text{bispectrum}}$$

$\neq 0$

$B(k_1,k_2,k_3) =$ small in single-field slow-roll (SFSR) inflation (with standard kinetic term)

but may be big in beyond-SFSR inflation (e.g., DBI, curvaton, ... ; arXiv 2002-2012)

The Bispectrum in Cosmologese



7

But there is more!

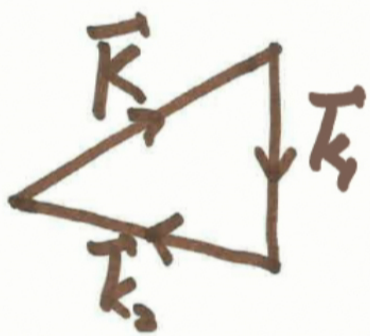
$$\mathcal{L} \supset g^{\mu\nu}(\nabla_\mu \phi)(\nabla_\nu \phi)$$

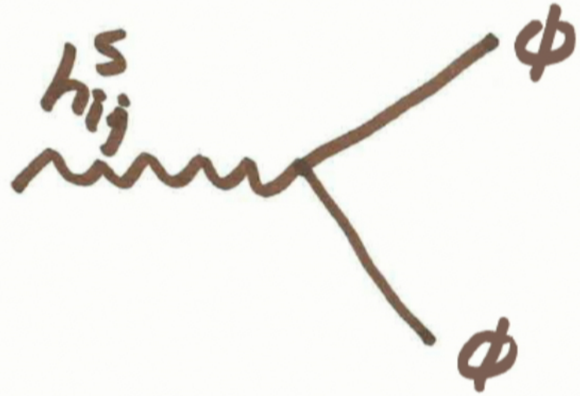
$$\Longrightarrow \quad h^{ij}(\partial_i \phi)(\partial_j \phi)$$

$\Longrightarrow$ tensor-scalar-scalar bispectrum (Maldacena 2002)
(GW)

$$\langle h^s_{\vec{K}}\, \delta_{\vec{k}_1}\, \delta_{\vec{k}_2} \rangle \propto \delta_D(\vec{k}_1 + \vec{k}_2 + \vec{k}_3)\, \epsilon^s_{ij}\, k_1^i k_2^j\, B_{tss}(K, k_1, k_2)$$

$s = +, \times$





But there is more!

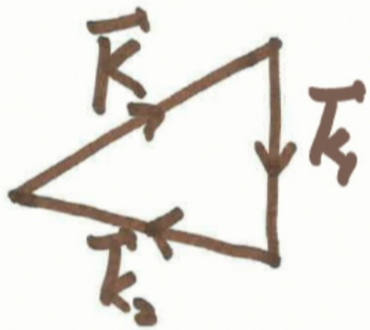
$$\mathcal{L} \supset g^{\mu\nu}(\nabla_\mu \phi)(\nabla_\nu \phi)$$

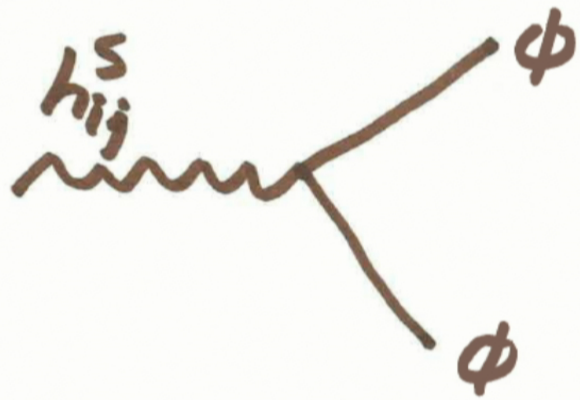
$$\Longrightarrow \quad h^{ij}(\partial_i \phi)(\partial_j \phi)$$

$\Longrightarrow$ tensor-scalar-scalar bispectrum (Maldacena 2002)
(GW)

$$\langle h^s_{\vec{K}} \, \delta_{\vec{k}_1} \delta_{\vec{k}_2} \rangle \propto \delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)\, \epsilon^s_{ij} k_1^i k_2^j \, B_{tss}(K, k_1, k_2)$$

$s = +, \times$





But there is more!

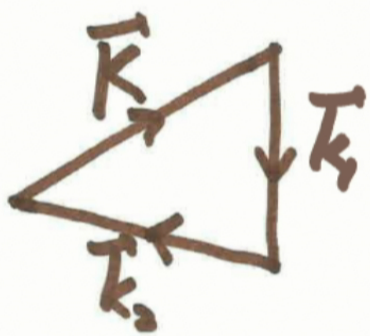
$$\mathcal{L} \subset g^{\mu\nu}(\nabla_\mu\phi)(\nabla_\nu\phi)$$

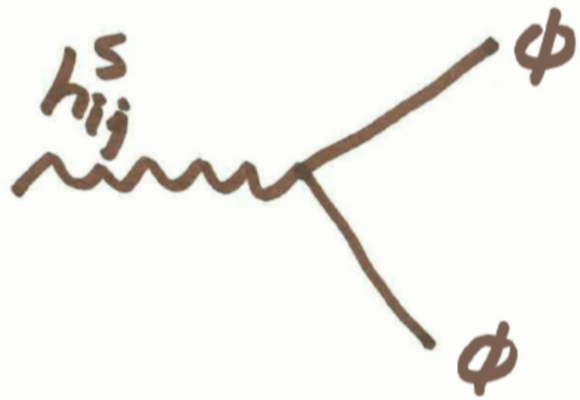
$$\Longrightarrow \quad h^{ij}(\partial_i\phi)(\partial_j\phi)$$

$\Longrightarrow$ tensor-scalar-scalar bispectrum (Maldacena 2002
(GW)

$$\langle h^s_{\vec{K}}\,\delta_{\vec{k}_1}\,\delta_{\vec{k}_2}\rangle \propto \delta_D(\vec{k}_1+\vec{k}_2+\vec{k}_3)\,\epsilon^s_{ij}\,k_1^i k_2^j\, B_{tss}(K,k_1,k_2)$$

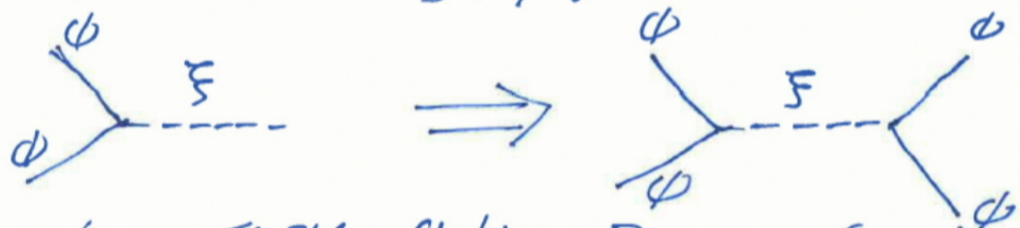
$s=+,\times$





A vector field V^μ? $(\partial_\mu \phi)(\partial_\nu \phi)\partial^{(\mu}V^{\nu)}$

A spectator scalar $\xi(\vec{x})$? $\xi\phi^2$



(e.g., SUSY inflation; Baumann-Green)

Some spectator spin-2 fields $T^{\mu\nu}$? $T^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi$

(e.g., gauge-, chromo-natural, or solid)

Jeong & Kamionkowski (2012)

Clustering Fossils (how to look with galaxy clustering for inflaton coupling to new scalar, vector, or tensor field)

If

$$\langle \delta_i(\boldsymbol{k}_1)\delta_i(\boldsymbol{k}_2)h_p(\boldsymbol{K})\rangle = (2\pi)^3 P_p(K) f_p(\boldsymbol{k}_1, \boldsymbol{k}_2)\epsilon^p_{ij} k^i_1 k^j_2 \delta^D(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{K})$$

power spectrum of new field

coupling amplitude

polarization basis (scalar, vector, tensor)

then

$$\langle \delta_i(\boldsymbol{k}_1)\delta_i(\boldsymbol{k}_2)\rangle|_{h_p(\boldsymbol{K})} = h_p(\boldsymbol{k}_1 + \boldsymbol{k}_2) f_p(\boldsymbol{k}_1, \boldsymbol{k}_2)\epsilon^p_{ij} k^i_1 k^j_2 \delta^D_{\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{K}}$$

two-point correlation for the density field in the presence of some realization of the fossil field

Jeong & Kamionkowski (2012)

Clustering Fossils (how to look with galaxy clustering for inflaton coupling to new scalar, vector, or tensor field)

If

$$\langle \delta_i(\boldsymbol{k}_1)\delta_i(\boldsymbol{k}_2)h_p(\boldsymbol{K})\rangle = (2\pi)^3 P_p(K) f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij}k_1^i k_2^j \delta^D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K})$$

power spectrum of new field

coupling amplitude

polarization basis (scalar, vector, tensor)

then

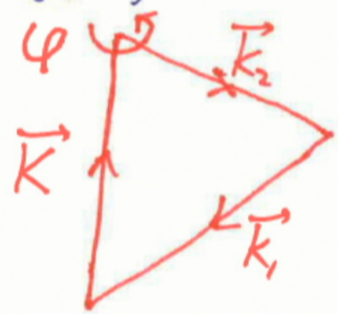
$$\langle \delta_i(\boldsymbol{k}_1)\delta_i(\boldsymbol{k}_2)\rangle|_{h_p(\boldsymbol{K})} = h_p(\boldsymbol{k}_1+\boldsymbol{k}_2) f_p(\boldsymbol{k}_1,\boldsymbol{k}_2)\epsilon^p_{ij}k_1^i k_2^j \delta^D_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{K}}$$

two-point correlation for the density field in the presence of some realization of the fossil field

The polarization tensor $\epsilon^p_{ij}(\vec{k})$

Symmetric 3×3 ⟹ 6 components

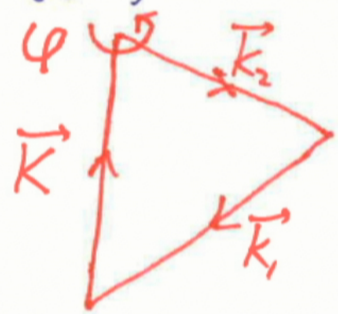
p	ϵ^p_{ij}		
0	δ_{ij}	scalar { trace	azimuthally symmetric about $\vec{k}$
z	$k^i k^j - k^2 \delta_{ij}/3$	scalar { longitudinal	azimuthally symmetric about $\vec{k}$
x,y	$k^{(i} w^{j)}$ $\partial^i w_i = 0$	vector	← $\sin\varphi$ $\cos\varphi$
$+$	$xx - yy$	transverse-traceless (GW)	$\cos 2\varphi$
$\times$	$2xy$	transverse-traceless (GW)	$\sin 2\varphi$



The polarization tensor $\epsilon^p_{ij}(\vec{K})$

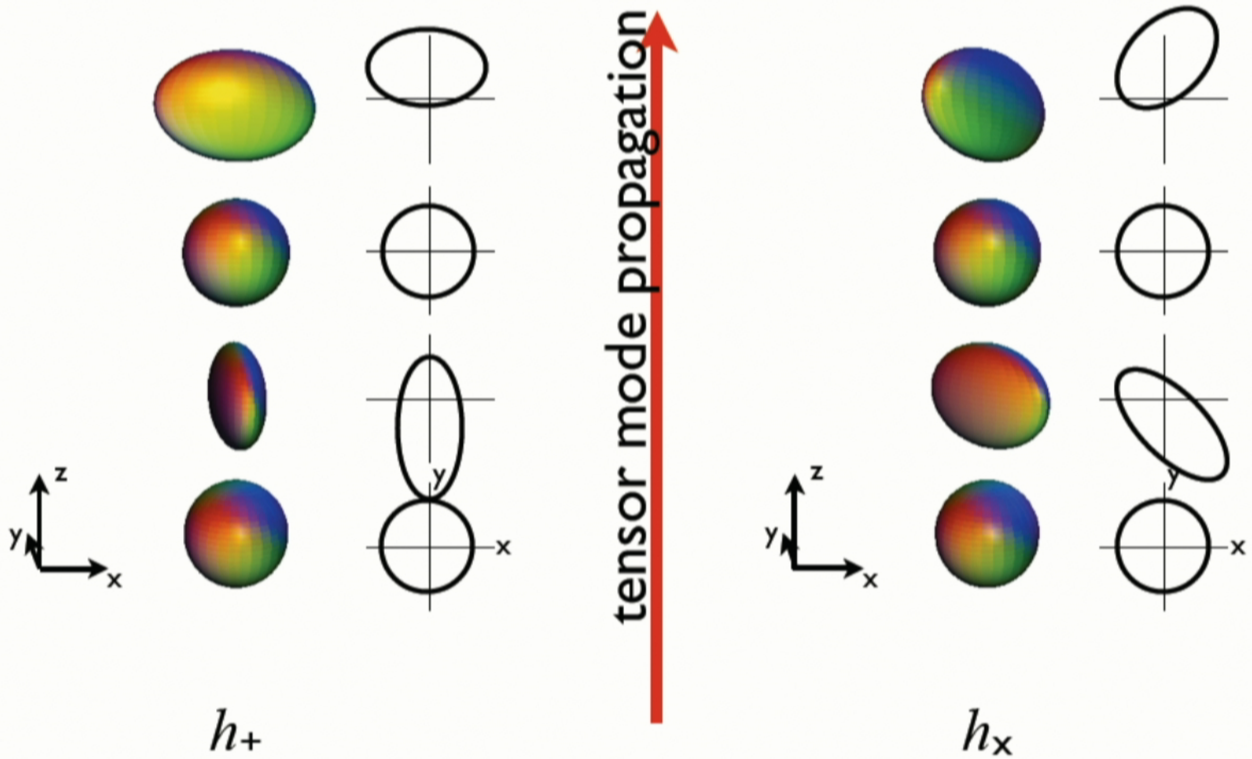
Symmetric 3×3 ⇒ 6 components

p	ϵ^p_{ij}			
0	δ_{ij}	scalar	trace	azimuthally symmetric about $\hat{K}$
z	$K^i K^j - K^2 \delta_{ij}/3$	scalar	longitudinal	azimuthally symmetric about $\hat{K}$
x, y	$K^{(i} w^{j)}$ $\partial^i w_i = 0$	vector	←	$\sin\varphi$ $\cos\varphi$
+	xx − yy	transverse-traceless (GW)		~~$\sin 2\varphi$~~ $\cos 2\varphi$
×	2xy	transverse-traceless (GW)		$\sin 2\varphi$



ξ(r) with single tensor mode

(p=+,x)



13

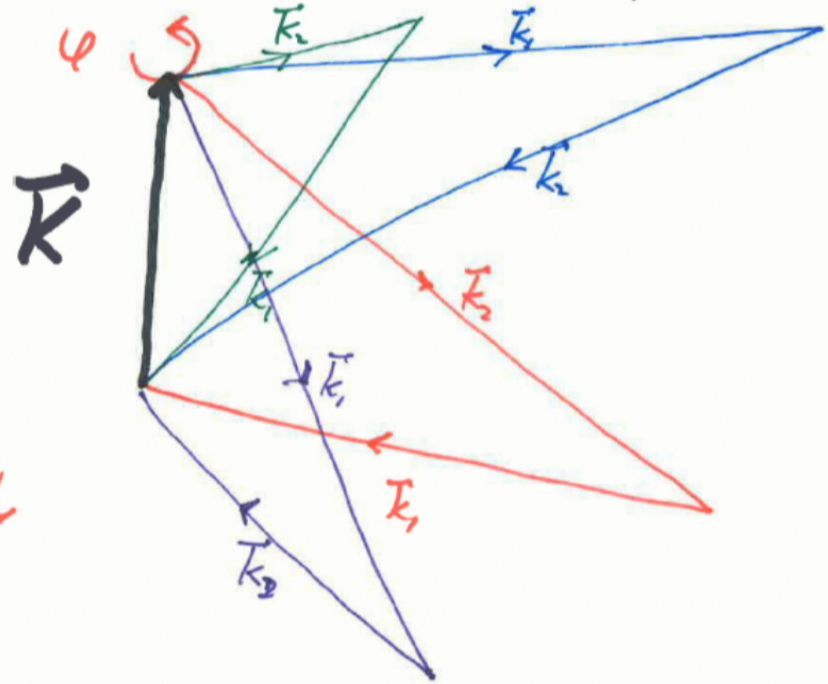
minimum-variance

$\wedge$ Estimator for $h_p(\vec{K})$

$$\sum_{\vec{k}_1+\vec{k}_2=\vec{K}} \delta(\vec{k}_1)\delta(\vec{k}_2)\left[f_p(\vec{k}_1,\vec{k}_2)\,\epsilon^p_{ij}\,k_1^i k_2^j\right]$$

with inverse-variance weighting

Azimuthal (φ) dependence distinguishes scalar, from vector from tensor geometrically



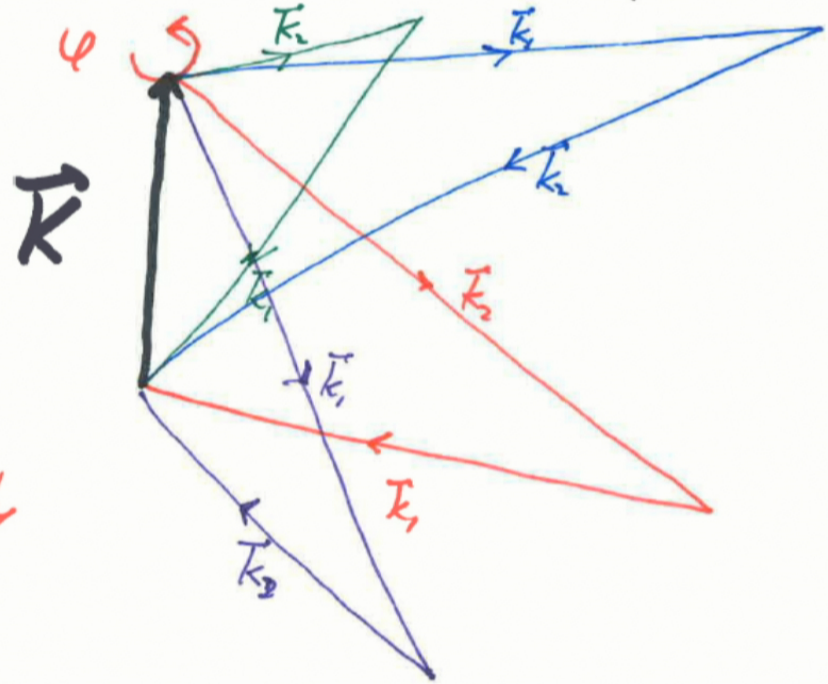
minimum-variance

$\hat{}$ Estimator for $h_p(\vec{K})$

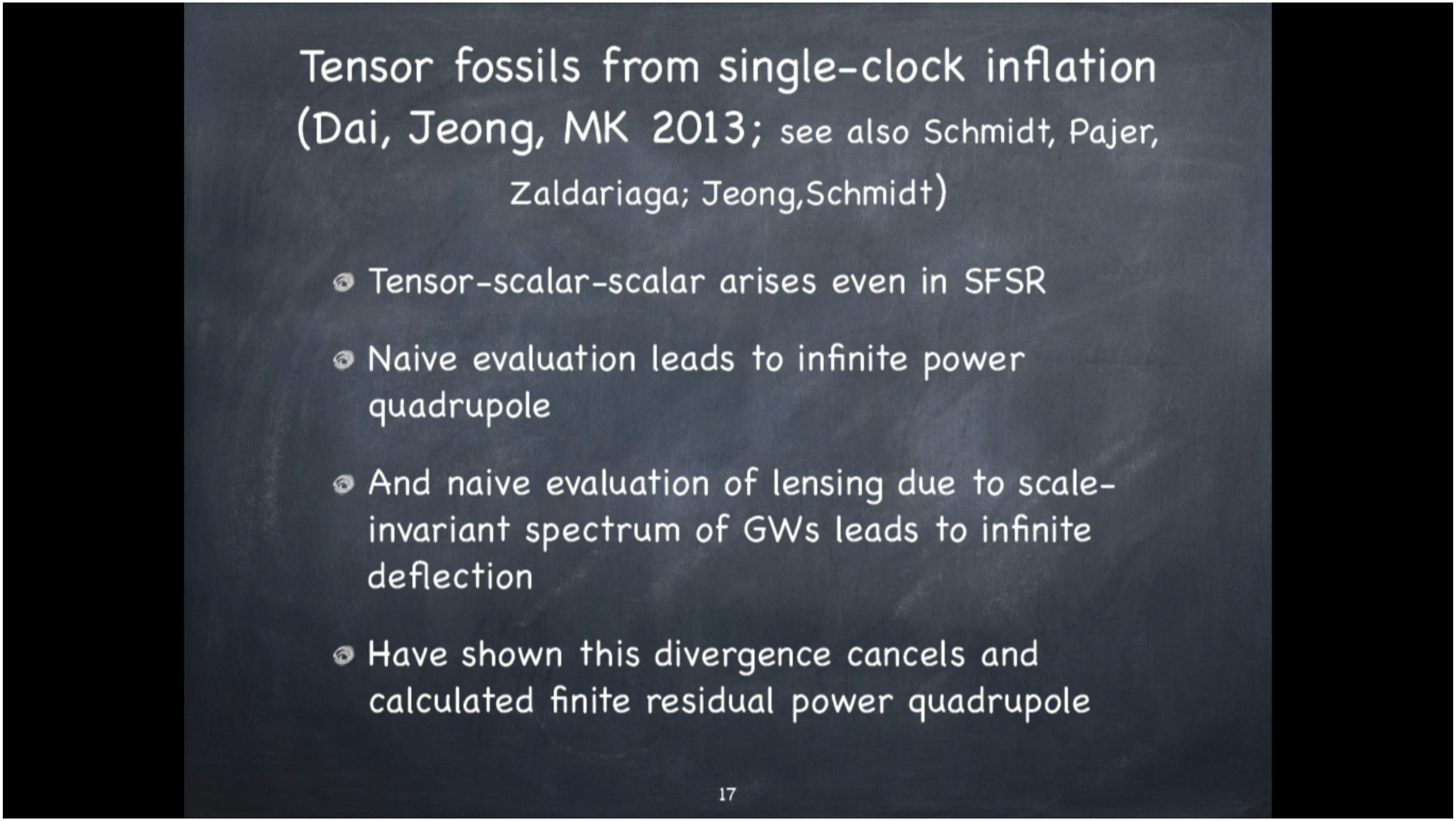
$$\sum_{\vec{k}_1+\vec{k}_2=\vec{K}} \delta(\vec{k}_1)\,\delta(\vec{k}_2)\left[f_p(\vec{k}_1,\vec{k}_2)\,\epsilon^p_{ij}\,k_1^i k_2^j\right]$$

with inverse-variance weighting

Azimuthal (φ) dependence distinguishes scalar, from vector from tensor geometrically



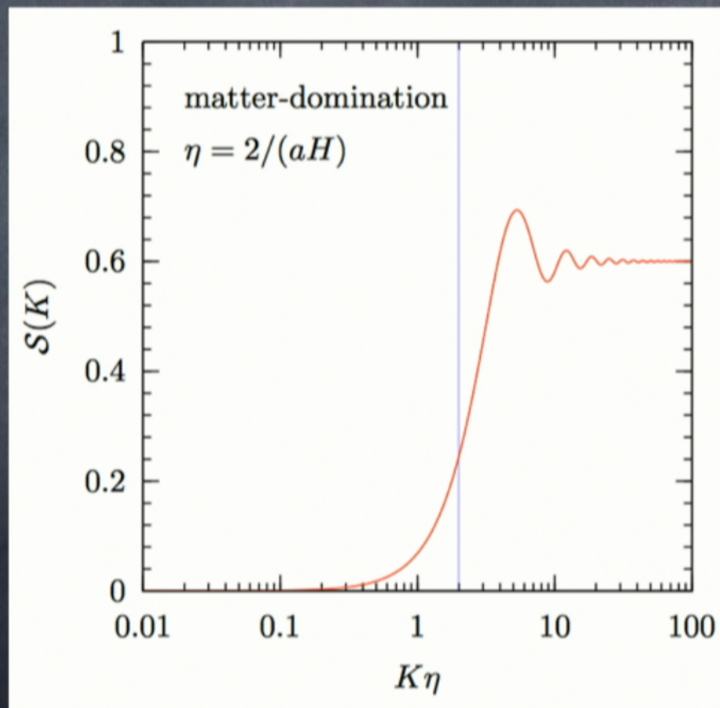
16


Tensor fossils from single-clock inflation (Dai, Jeong, MK 2013; see also Schmidt, Pajer, Zaldariaga; Jeong,Schmidt)
Tensor-scalar-scalar arises even in SFSR
Naive evaluation leads to infinite power quadrupole
And naive evaluation of lensing due to scale-invariant spectrum of GWs leads to infinite deflection
Have shown this divergence cancels and calculated finite residual power quadrupole
17

Are these signals detectable?

- Not any time soon with SFSR inflation
- But signals may be larger (maybe even far larger) with beyond-SFSR inflation?
- SFSR signals conceivably detectable with far-future 21-cm maps of neutral hydrogen during dark ages

$$\mathcal{Q}_{ij} = -\frac{1}{2}\frac{d\ln P_\delta}{d\ln k}\left(1-\mathcal{T}_\gamma\right)\gamma_{p,ij} + 2\mathcal{S}_N(K)\gamma_{p,ij}$$
$$-\frac{d\ln P_\delta}{d\ln k}\left(\frac{1}{2}\gamma_{ij} + \partial_{(i}\Delta x_{j)} - \frac{1}{3}\delta_{ij}\partial\cdot\Delta x\right),$$



19

- In squeezed limit,

$$B_p^{\rm obs}(k_L, k_2, k_3) = B_p(k_L, k_2, k_3) + \frac{1}{2} P_\gamma^p(k_L) P_\zeta(k_S) \epsilon_{ij}^p k_S^i k_S^j \frac{\partial \ln P_\zeta(k_S)}{\partial \ln k_S}$$

(SPZ;DJK;Brahma,Nelson,Shandera)

- In single-clock inflation, two contributions cancel
- But may have observable power quadrupole in other scenarios (gaugeflation; chromo-natural; solid; non-attractor; particle/string sources; etc) (Dimastrogiovanni, Fasiello, Dai, Jeong, J. Wang, MK, in prep)

The CMB predictions for tensor-scalar-scalar fossils

(Dai, Jeong, MK)

- tensor-scalar-scalar induces correlations between different a_{lm}'s, parametrized in terms of bipolar spherical harmonics (BiPoSHs):

$$\langle a^{T}_{l_1 m_1} a^{T*}_{l_2 m_2} \rangle_h = C^{TT}_{l_1} \delta_{l_1 l_2} \delta_{m_1 m_2} + \sum_{JM} (-1)^{m_2} \langle l_1 m_1 l_2, -m_2 | JM \rangle A^{JM}_{l_1 l_2}$$

with l_1+l_2+J=odd (as opposed to scalar-scalar-scalar, which produces only l_1+l_2+J=even)

21

The CMB predictions for tensor-scalar-scalar fossils

(Dai, Jeong, MK)

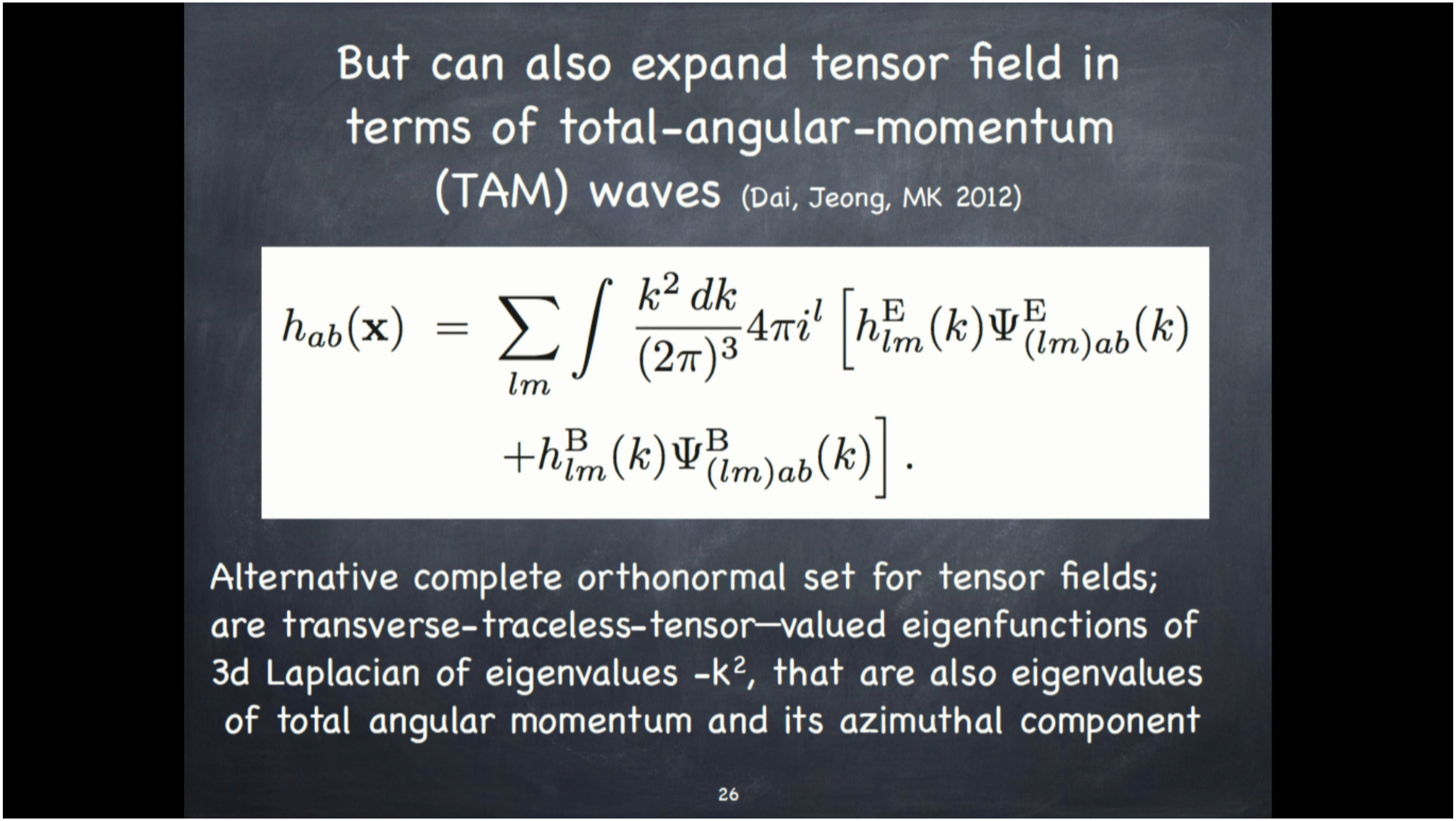
- SFSR signal probably too small to see
- But perhaps bigger in beyond-SFSR models?
- but canNOT distinguish vectors from tensors geometrically with 2d CMB T map

Back to CMB B modes: How to get B modes with no T nor E

(Dai, Jeong, MK, arXiv:1404.nextweek)

E/B tensor spherical harmonics obtained from application of derivative operators on scalar spherical harmonics:

$$\begin{aligned} Y^{E}_{(lm)AB}(\hat{\mathbf{n}}) &= \sqrt{\frac{2(l-2)!}{(l+2)!}}\left(-\nabla_A\nabla_B + \frac{1}{2}g_{AB}\nabla^C\nabla_C\right)Y_{(lm)}(\hat{\mathbf{n}}) \\ &\equiv -\sqrt{\frac{(l-2)!}{2(l+2)!}}W^{\mathrm{E}}_{AB}Y_{(lm)}(\hat{\mathbf{n}}), \\ Y^{B}_{(lm)AB}(\hat{\mathbf{n}}) &= \sqrt{\frac{2(l-2)!}{(l+2)!}}\left(\epsilon_B{}^C\nabla_C\nabla_A + \epsilon_A{}^C\nabla_C\nabla_B\right)Y_{(lm)}(\hat{\mathbf{n}}) \\ &\equiv -\sqrt{\frac{(l-2)!}{2(l+2)!}}W^{\mathrm{B}}_{AB}Y_{(lm)}(\hat{\mathbf{n}}), \qquad (1) \end{aligned}$$


But can also expand tensor field in terms of total-angular-momentum (TAM) waves (Dai, Jeong, MK 2012)
$$h_{ab}(\mathbf{x}) = \sum_{lm} \int \frac{k^2\,dk}{(2\pi)^3} 4\pi i^l \left[h^{\rm E}_{lm}(k) \Psi^{\rm E}_{(lm)ab}(k) + h^{\rm B}_{lm}(k) \Psi^{\rm B}_{(lm)ab}(k) \right].$$
Alternative complete orthonormal set for tensor fields; are transverse-traceless-tensor—valued eigenfunctions of 3d Laplacian of eigenvalues -k^2, that are also eigenvalues of total angular momentum and its azimuthal component
26

Are <u>not</u> tensor spherical harmonics, which are basis for tensors on the two-sphere

Are moreover <u>not</u> products of tensor spherical harmonics and radial eigenfunctions, although they can be projected onto spherical harmonics

27

Explicitly,

$$\Psi^{B}_{(lm)ab}(\mathbf{x}) = -\sqrt{\frac{(l-2)!}{2(l+2)!}} T^{B}_{ab} \Psi_{(lm)}(\mathbf{x})$$

$$\Psi^{E}_{(lm)ab}(\mathbf{x}) = -\sqrt{\frac{(l-2)!}{2(l+2)!}} T^{E}_{ab} \Psi_{(lm)}(\mathbf{x})$$

$$T^{\mathrm{B}}_{ab} = K_{(a}M_{b)} + M_{(a}K_{b)} + 2D_{(a}K_{b)},$$

$$T^{\mathrm{E}}_{ab} = M_{(a}M_{b)} - K_{(a}K_{b)} + 2D_{(a}M_{b)},$$

where

$$D_a \equiv \frac{i}{k}\nabla_a, \quad K_a \equiv -iL_a, \quad M_a \equiv \epsilon_{abc}D^b K^c$$

Scalar TAM wave → $\Psi^{k}_{(lm)}(\mathbf{x}) \equiv j_l(kr)Y_{(lm)}(\hat{\mathbf{n}})$

28

With this formalism, B mode of TAM wave gives rise to B-mode CMB polarization, and E mode of TAM wave gives rise to CMB T and E

So, can have B modes from tensor fluctuations with no T nor E!

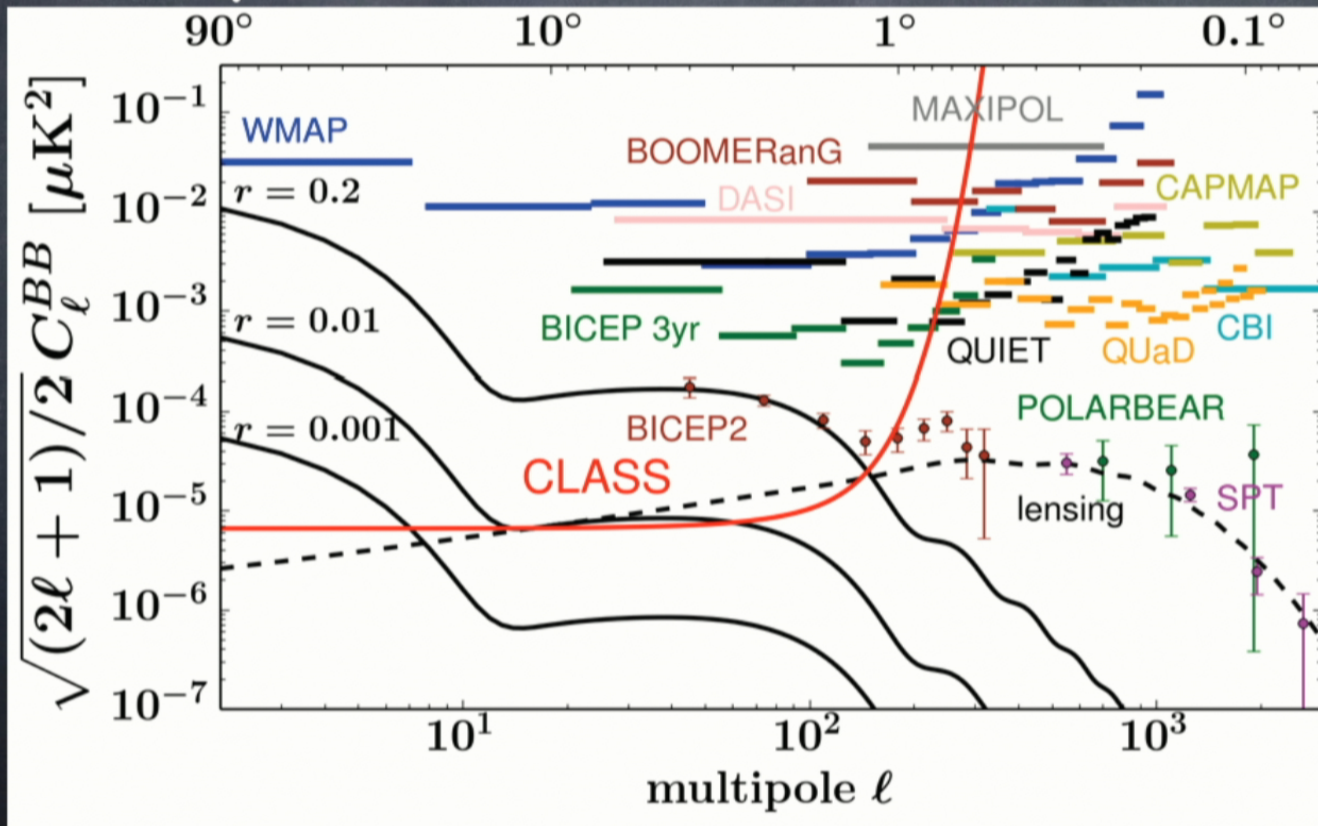
29

But.....

E and B mix under translation of origin, and so difference in power between E and B tensor TAM waves implies that we occupy a preferred place in the Universe

Still....perhaps may be of interest as new ingredient if B/TE tension continues???

Advertisement: CLASS will complement BICEP/Keck/etc



To close

- (If the BICEP2 results are confirmed) The era of tensor science has arrived
- Obvious short-term goals:
  - Verify signal, and if verified, measure BB to cosmic-variance limit; need large-angle experiment (e.g., CLASS/PIPER to complement BICEP2 at l~80)
- But its time to think about longer-term goals!! (cf. B-mode theory->detection = 18 years):
  - large-scale structure probes, for SFSR and other models
  - novel CMB tests
  - and let's go BBO and DECIGO!!!

32