Title: Fibonacci Anyons From Abelian Bilayer Quantum Hall States

Date: Apr 01, 2014 03:30 PM

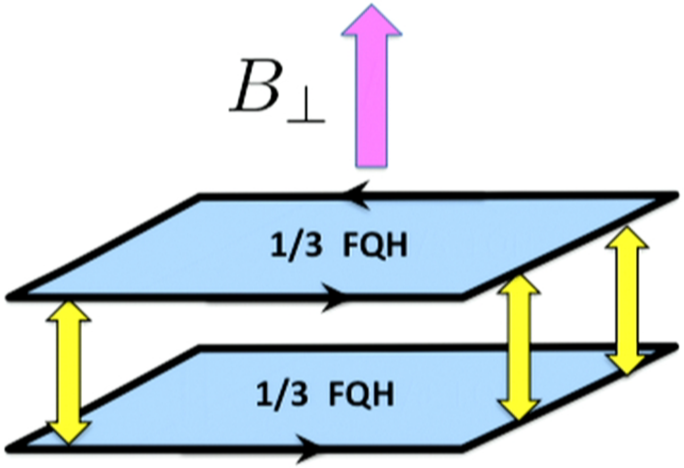
URL: http://pirsa.org/14040120

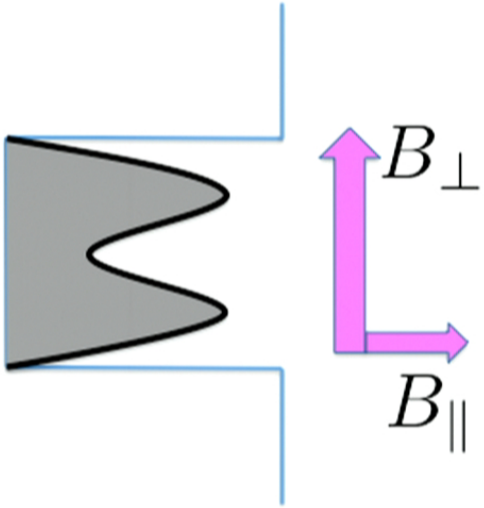
Abstract: The possibility of realizing non-Abelian statistics and utilizing it for topological quantum computation (TQC) has generated widespread interest. However, the non-Abelian statistics that can be realized in most accessible proposals is not powerful enough for universal TQC. In this talk, I consider a simple bilayer fractional quantum Hall (FQH) system with the 1/3 Laughlin state in each layer, in the presence of interlayer tunneling. I show that interlayer tunneling can drive a continuous phase transition to an exotic non-Abelian state that contains the famous `Fibonacci' anyon, whose non-Abelian statistics is powerful enough for universal TQC. The analysis that I will present towards this result rests on startling agreements from a variety of distinct methods, including thin torus limits, effective field theories, and coupled wire constructions.

 Next, I discuss the experimental aspects of our proposal and potential probes for the Fibonacci phase. I show that the charge gap remains open at the phase transition while the neutral gap closes. This raises the question of whether these exotic phases may have already been realized at nu=2/3 in bilayers, as past experiments may not have definitively ruled them out. Finally, I will discuss about the generalizations to multi-layer states as well as the duality between the interlayer pairing and interlayer tunneling problems.

 Reference: arXiv:1403.3383

Motivation: I

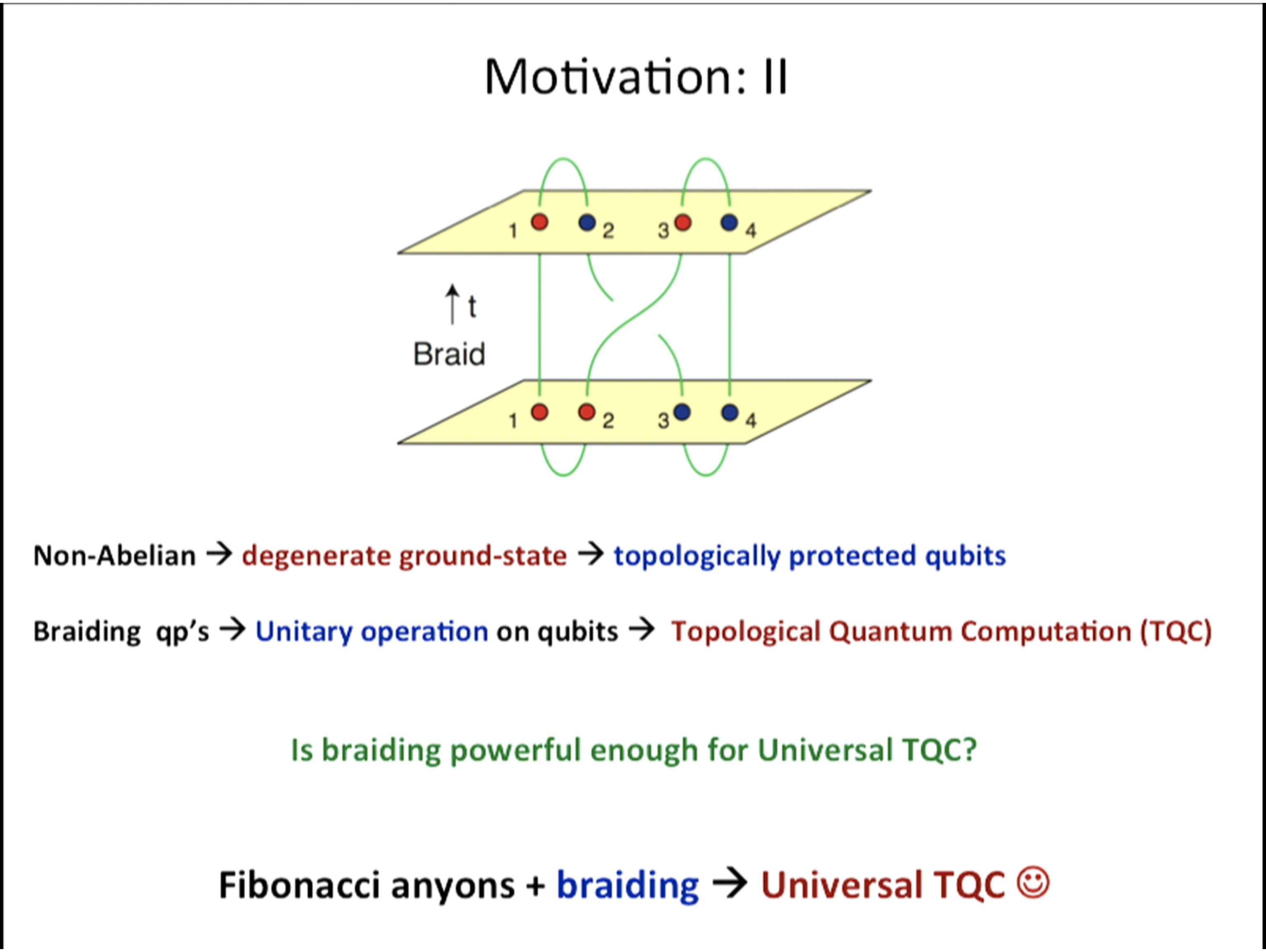




↕ **= interlayer tunneling**

Charge distribution of 2DEG
in a wide quantum well

t<< Eg :	(330) state	GSD=9	Abelian
t>> Eg :	PH conjugate of 1/3 state	GSD=3	Abelian
t ~ Eg :	?	GSD=?	?



Motivation: II

Non-Abelian → degenerate ground-state → topologically protected qubits

Braiding qp's → Unitary operation on qubits → Topological Quantum Computation (TQC)

Is braiding powerful enough for Universal TQC?

Fibonacci anyons + braiding → Universal TQC ☺

Previous studies

Symmetry of (330) : $U(1)_c \times U(1)_s \rtimes Z_2$

Gauging Z_2 → $U(1)_3/Z_2$ orbifold CFT = $\mathbf{Z_4}$ Read-Rezayi state

Barkeshli and Wen, PRB (2010)
Vaezi, PRB (2013)
Barkeshli et al, PRB (2013)

Wave-function:

$$\psi_{Z_4} = \mathcal{AS}\left[\psi_{\uparrow}^{3}(z_1, ..., z_{N/2})\psi_{\downarrow}^{3}(z_{N/2}, ..., z_N)\right]$$

Rezayi, Wen and Read, arXiv:1007.2022

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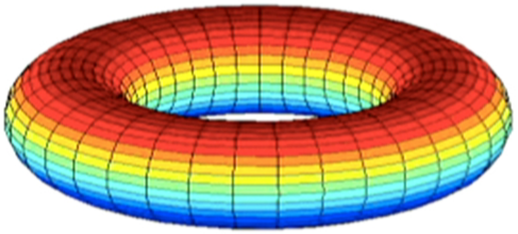
Overview

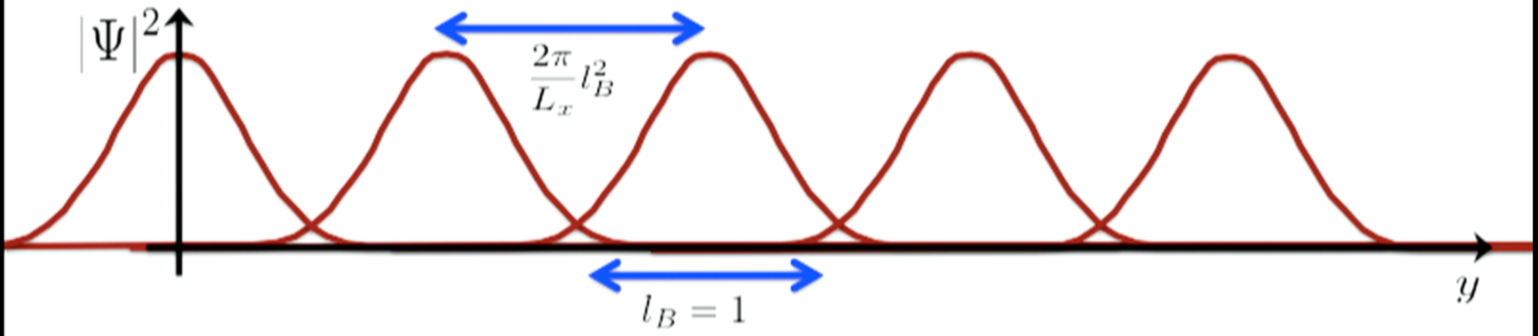
I. Thin torus limit approach
II. Chern-Simons gauge theory
III. Coupled wire construction
IV. Generalizations
V. Conclusion

FQH states on Torus

$$\Psi_m(x,y) = e^{-i\left(\frac{2\pi}{L_x}m\right)x} e^{-\left(y-\frac{2\pi m}{L_x}\right)^2/2}$$



$$A_x = -By, \quad A_y = 0$$



Ideal hamiltonian for 1/3 Laughlin state (in Landau gauge):

$$H = \sum_i \sum_{r>s} U_{r,s} c^\dagger_{i+s} c^\dagger_{i+r} c_{i+r+s} c_i \qquad U_{r,s} = (r^2 - s^2) e^{-2\pi^2 (r^2+s^2)/L_x^2}$$

Thin torus limit:

$$\boxed{L_x \ll l_B : \quad H \simeq \sum_i \left(U_{1,0} n_i n_{i+1} + U_{2,0} n_i n_{i+2}\right)}$$

FQH states on Torus

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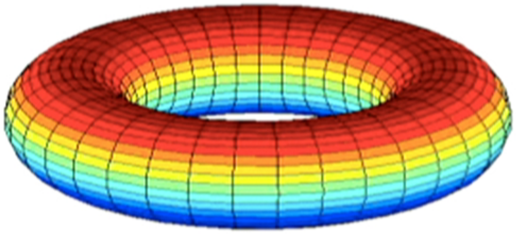

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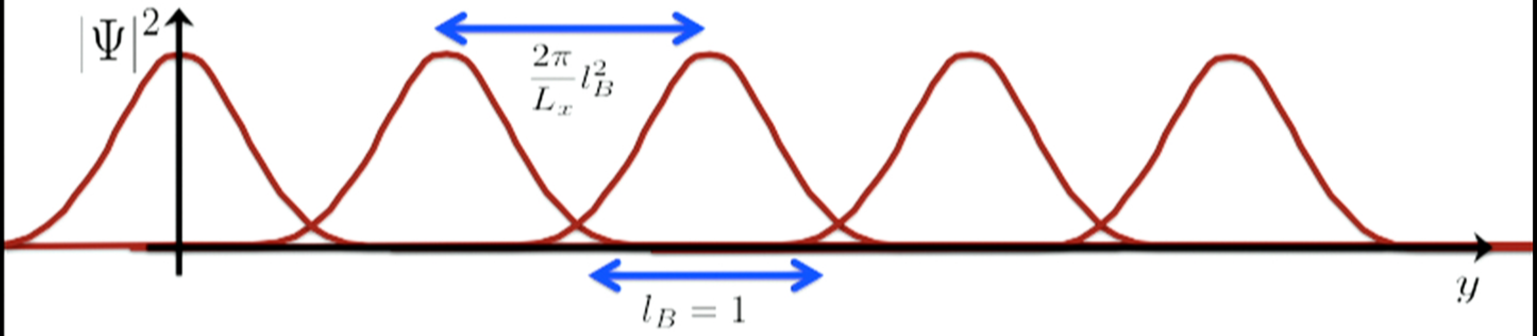

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I. Excitations: domain-walls

$$[100100100100|100100100100100|100100100]$$

Seidel et al, PRL (2005); Bergholtz and Karlhede, JSM (2006); Bergholtz and Karlhede, PRB (2008); Seidel and Yang, PRL (2008); Ardonne, PRL (2009)

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$$[100100100100|100100100100100|100100100]$$

$$\downarrow$$

$$[100100100100010010010010010100100100]$$

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I. Excitations: domain-walls

$$[100100100\underset{\overset{\downarrow}{i}}{}010010\ldots010\underset{\overset{\downarrow}{j}}{}100100\ldots] = V_1^\dagger(j)V_1(i)\,|g\rangle_1$$

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I. Bilayer (330) state: (negligible tunneling)

$$t \ll U_2 \ll U_1$$

Two identical and independent layers (t=0) :

$$|g\rangle_{a,b}, \quad a, b = 1, 2, 3$$

For example:

$$|g\rangle_{1,1} = \left| \begin{matrix} 100100100100100100100... \\ 100100100100100100100... \end{matrix} \right\rangle \equiv \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

$$|g\rangle_{1,2} = \left| \begin{matrix} 100100100100100100100... \\ 010010010010010010010... \end{matrix} \right\rangle \equiv \begin{bmatrix} 100 \\ 010 \end{bmatrix}$$

I. Excitations: domain-walls + layer index

$$q^*_\uparrow = ke/3 : \quad |g\rangle_{a,b} \to |g\rangle_{a+k,b}$$

$$\begin{bmatrix} 100 \\ 100 \end{bmatrix} \to \left| \begin{array}{l} 100100010010010010010\ldots \\ 100100100100100100100\ldots \end{array} \right\rangle$$

$$q^*_\downarrow = ke/3 : \quad |g\rangle_{a,b} \to |g\rangle_{a,b+k}$$

$$\begin{bmatrix} 100 \\ 010 \end{bmatrix} \to \left| \begin{array}{l} 100100100100100100100\ldots \\ 010010001001001001001\ldots \end{array} \right\rangle$$

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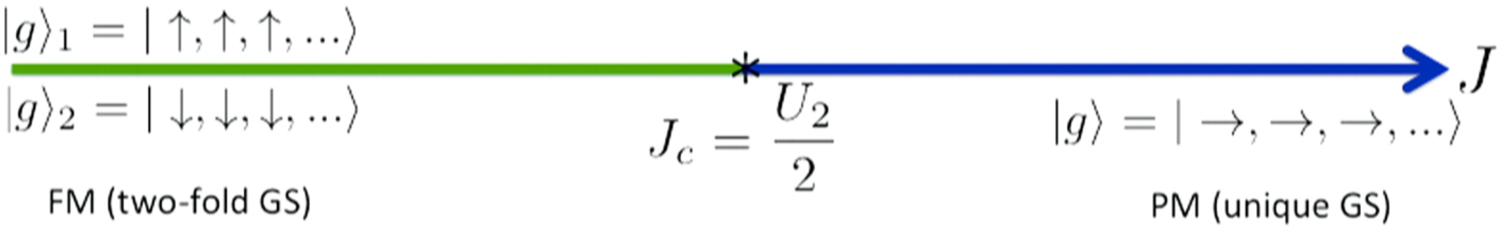
I. Bilayer (330) state: intermediate tunneling regime

$$U_2 \ll t \ll U_1$$

$$\begin{pmatrix} 100 \\ 010 \end{pmatrix}_i \equiv |\uparrow\rangle_i\,, \quad \begin{pmatrix} 010 \\ 100 \end{pmatrix}_i \equiv |\downarrow\rangle_i\,, \quad \begin{pmatrix} 110 \\ 000 \end{pmatrix}_i \equiv |d,1\rangle_i\,, \quad \begin{pmatrix} 000 \\ 110 \end{pmatrix}_i \equiv |d,2\rangle_i\,,$$

$$H_{U_1}|\uparrow>=0, \quad H_{U_1}|\downarrow>=0, \quad H_{U_1}|d,1>=U_1, \quad H_{U_1}|d,2>=U_1$$

$$H_{\rm eff} = -J\sum_i S_i^x - \frac{U_{2,0}}{2}\sum_i \left(S_i^z S_{i+1}^z + 1\right), \quad J = 4(t_0^{\perp})^2/U_{1,0}.$$



$|g\rangle_1 = |\uparrow,\uparrow,\uparrow,\ldots\rangle$

$|g\rangle_2 = |\downarrow,\downarrow,\downarrow,\ldots\rangle$

$J_c = \dfrac{U_2}{2}$

J

$|g\rangle = |\rightarrow,\rightarrow,\rightarrow,\ldots\rangle$

FM (two-fold GS)

PM (unique GS)

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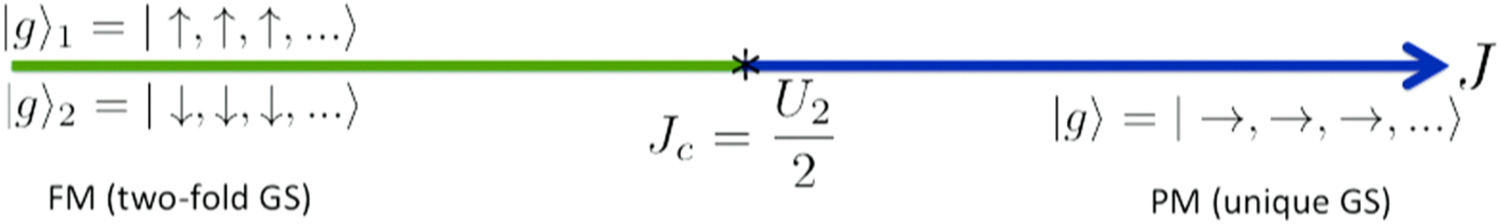

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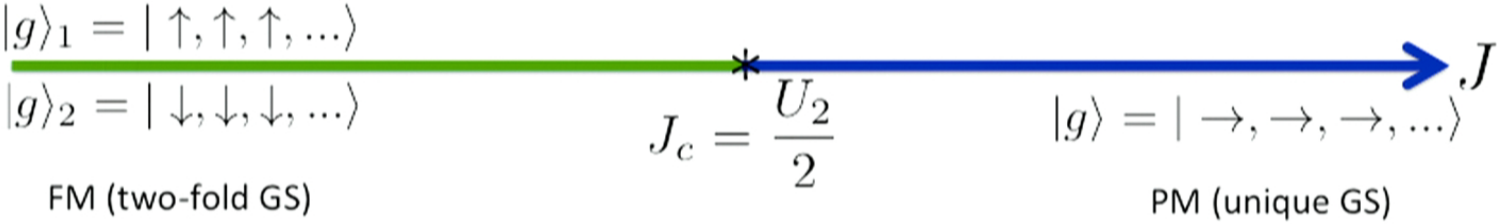

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$$\prod_i \begin{pmatrix} 100 \\ 100 \end{pmatrix}_i \equiv [200]$$

$$\prod_i \begin{pmatrix} 010 \\ 010 \end{pmatrix}_i \equiv [020]$$

$$\prod_i \begin{pmatrix} 001 \\ 001 \end{pmatrix}_i \equiv [002]$$

$$\prod_i \left(\begin{pmatrix} 100 \\ 010 \end{pmatrix}_i + \begin{pmatrix} 100 \\ 010 \end{pmatrix}_i \right) \equiv [110]$$

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← Z_2 **symmetric**

GSD = 6 =3 x 2 → Non-Abelian (NA)

“2” operators in the NA sector

(1, Fibonacci anyon)

$$X \times X = 1 + nX$$

$$\prod_i \left(\begin{pmatrix} 100 \\ 010 \end{pmatrix}_i - \begin{pmatrix} 010 \\ 100 \end{pmatrix}_i \right)$$

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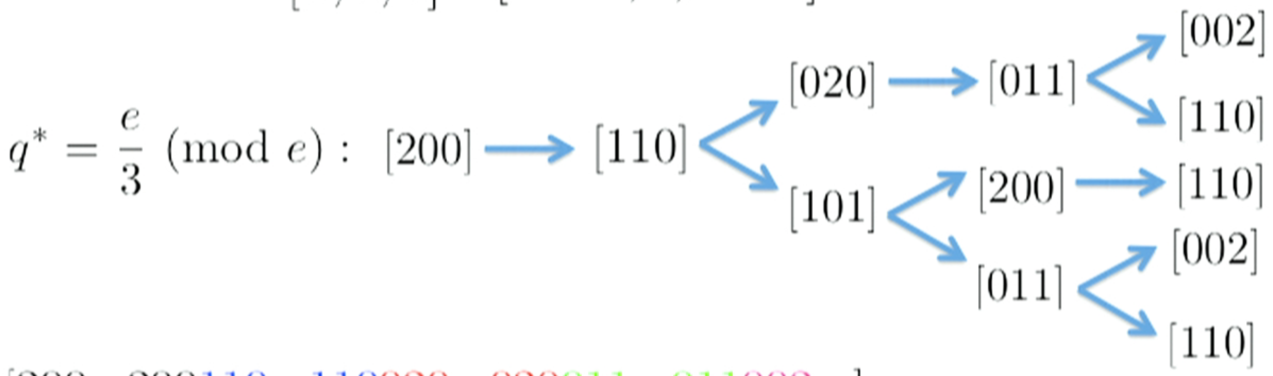
← Z_2 **Anti-symmetric**

I. Quasi-hole excitations

For arbitrary **|abcabcabc…>:=[abc]** pattern, the following DWs lead to e/3 charge excitation

$$\begin{tikzcd} & {[\overset{\curvearrowright}{a, b}, c] = [a-1, b+1, c]} \\ {[a, b, c]} \arrow[ur] \arrow[r] \arrow[dr] & {[a, \overset{\curvearrowright}{b, c}] = [a, b-1, c+1]} \\ & {[a, b, \overset{\curvearrowright}{c}] = [a+1, b, c-1]} \end{tikzcd}$$



$$q^* = \frac{e}{3} \pmod{e} :$$

$$\begin{tikzcd} & & & & {[002]} \\ & & {[020]} \arrow[r] & {[011]} \arrow[ur] \arrow[dr] & \\ & & & & {[110]} \\ {[200]} \arrow[r] & {[110]} \arrow[uur] \arrow[ddr] & & & \\ & & & {[200]} \arrow[r] & {[110]} \\ & & {[101]} \arrow[ur] \arrow[ddr] & & \\ & & & & {[002]} \\ & & & {[011]} \arrow[ur] \arrow[dr] & \\ & & & & {[110]} \end{tikzcd}$$

$[200\ldots200110\ldots110020\ldots020011\ldots011002\ldots]$

$[200\ldots200110\ldots110020\ldots020011\ldots011110\ldots]$

$[200\ldots200110\ldots110101\ldots101200\ldots200110\ldots]$

$[200\ldots200110\ldots110101\ldots101200\ldots200002\ldots]$

I. GSD: Adjacency matrix

$$A_{e/3} = \begin{array}{cccccc|c} [200] & [020] & [002] & [110] & [101] & [011] & \\ 0 & 0 & 0 & 1 & 0 & 0 & [200] \\ 0 & 0 & 0 & 0 & 0 & 1 & [020] \\ 0 & 0 & 0 & 0 & 1 & 0 & [002] \\ 0 & 1 & 0 & 0 & 1 & 0 & [110] \\ 1 & 0 & 0 & 0 & 0 & 1 & [101] \\ 0 & 0 & 1 & 1 & 0 & 0 & [011] \end{array}$$

1st row: $[200] \longrightarrow [110]$

4th row:

$$\begin{tikzcd} & {[020]} \\ {[110]} \arrow[ur] \arrow[dr] & \\ & {[101]} \end{tikzcd}$$

$$GSD(n_{qh}) \simeq Tr\left(A^{n_{qh}}\right) \sim \lambda_1^{n_{qh}}$$

$$\boxed{d_{e/3} = \lambda_1 = \frac{1+\sqrt{5}}{2}}$$

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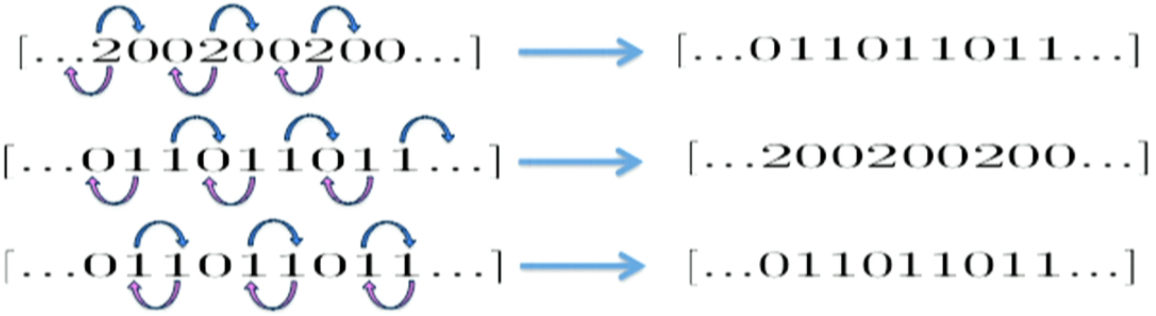
I. Neutral excitations (Fibonacci anyons)



$[\ldots 200200200 \ldots] \longrightarrow [\ldots 011011011 \ldots]$

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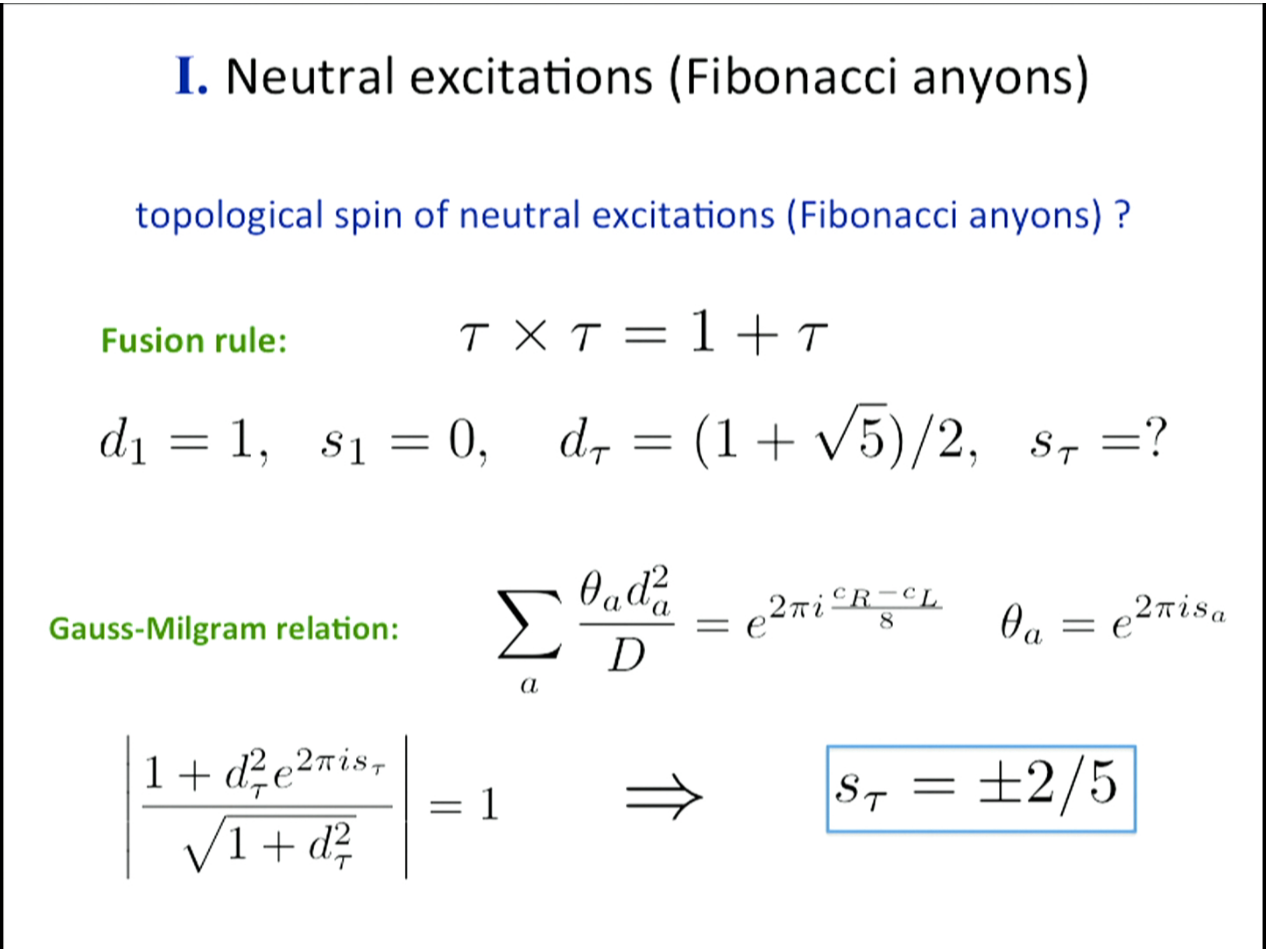
$$A_{e/3,-e/3} = \begin{array}{c} \begin{array}{cccccc} [200] & [011] & [020] & [101] & [002] & [110] \end{array} \\ \left(\begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right) \end{array} \begin{array}{c} \\ [200] \\ [011] \\ [020] \\ [101] \\ [002] \\ [110] \end{array}$$

$$GSD(n_{qh}) \simeq Tr\left(A^{n_{qh}}\right) \sim \lambda_1^{n_{qh}} \qquad \boxed{d_{e/3,-e/3} = \lambda_1 = \frac{1+\sqrt{5}}{2}}$$



I. Neutral excitations (Fibonacci anyons)

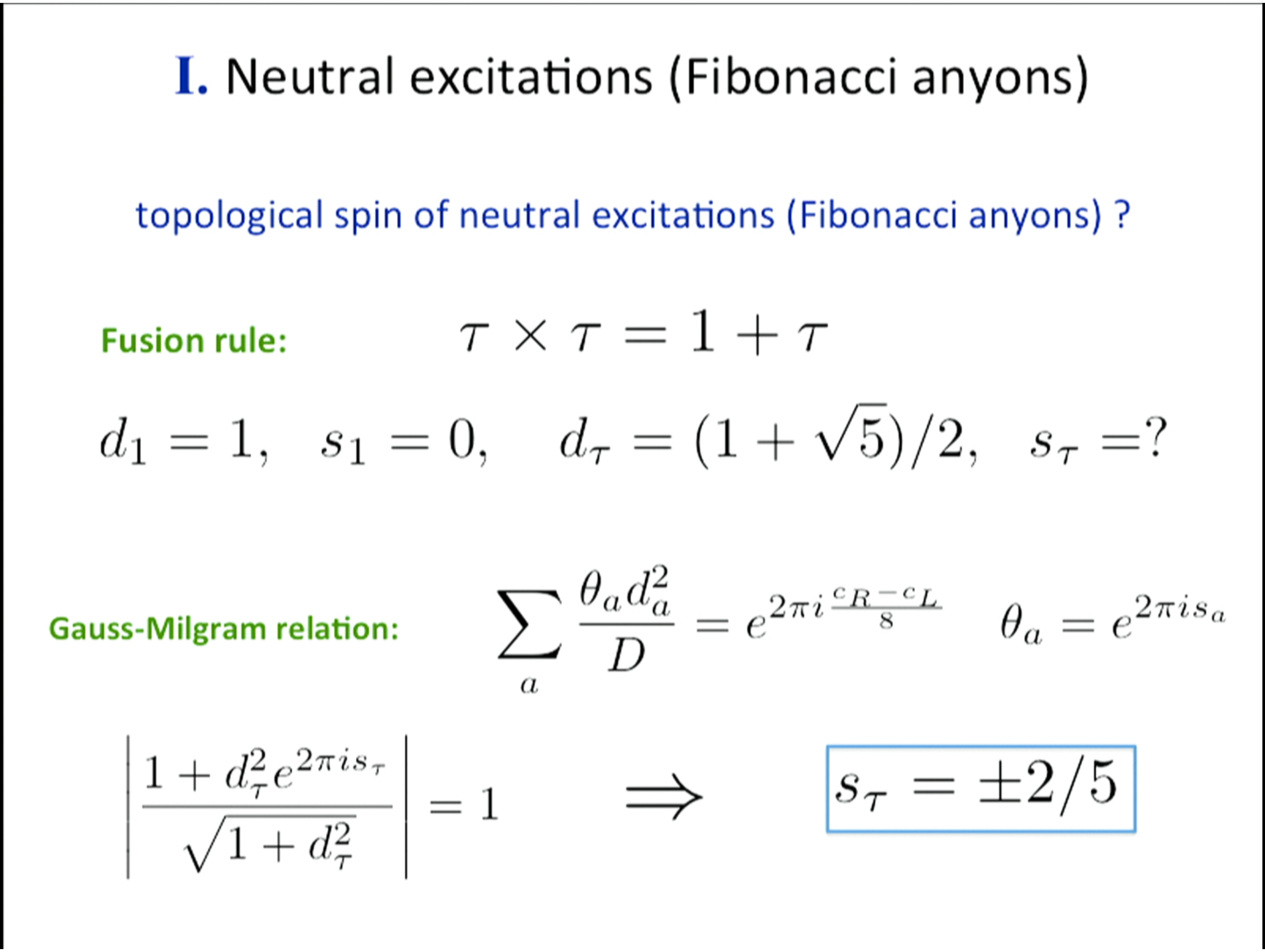
topological spin of neutral excitations (Fibonacci anyons) ?

Fusion rule: $\tau \times \tau = 1 + \tau$

$$d_1 = 1, \quad s_1 = 0, \quad d_\tau = (1+\sqrt{5})/2, \quad s_\tau = ?$$

Gauss-Milgram relation: $\sum_a \frac{\theta_a d_a^2}{D} = e^{2\pi i \frac{c_R - c_L}{8}} \qquad \theta_a = e^{2\pi i s_a}$

$$\left| \frac{1 + d_\tau^2 e^{2\pi i s_\tau}}{\sqrt{1+d_\tau^2}} \right| = 1 \qquad \Rightarrow \qquad \boxed{s_\tau = \pm 2/5}$$



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	Label	Charge (mod e)	Topological Spin	Quantum Dim.
1	V_0	0	0	1
2	V_1	$2e/3$	$1/3$	1
3	V_2	$e/3$	$1/3$	1
4	τ	0	$\pm 2/5$	F
5	$V_1\tau$	$2e/3$	$1/3 \pm 2/5$	F
6	$V_2\tau$	$e/3$	$1/3 \pm 2/5$	F

$$V_a = e^{ia\phi_c/R_c} \quad R_c = \sqrt{3/2} \quad \theta_a = a^2/3$$

Plus sign yields a fermionic theory, and corresponds to

$$SU(2)_3 \times U(1)_6 = U(6)_1/SU(3)_2$$

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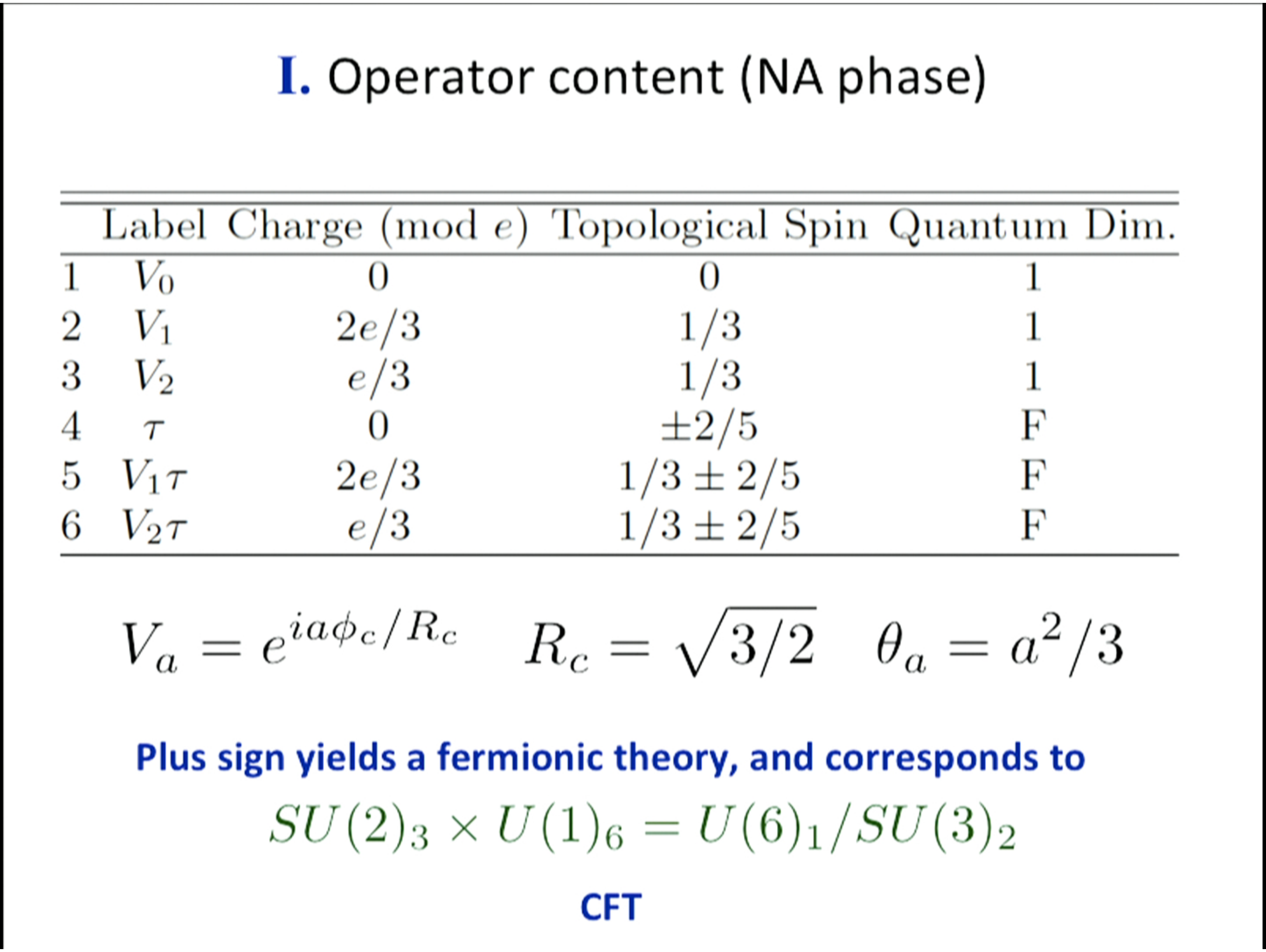

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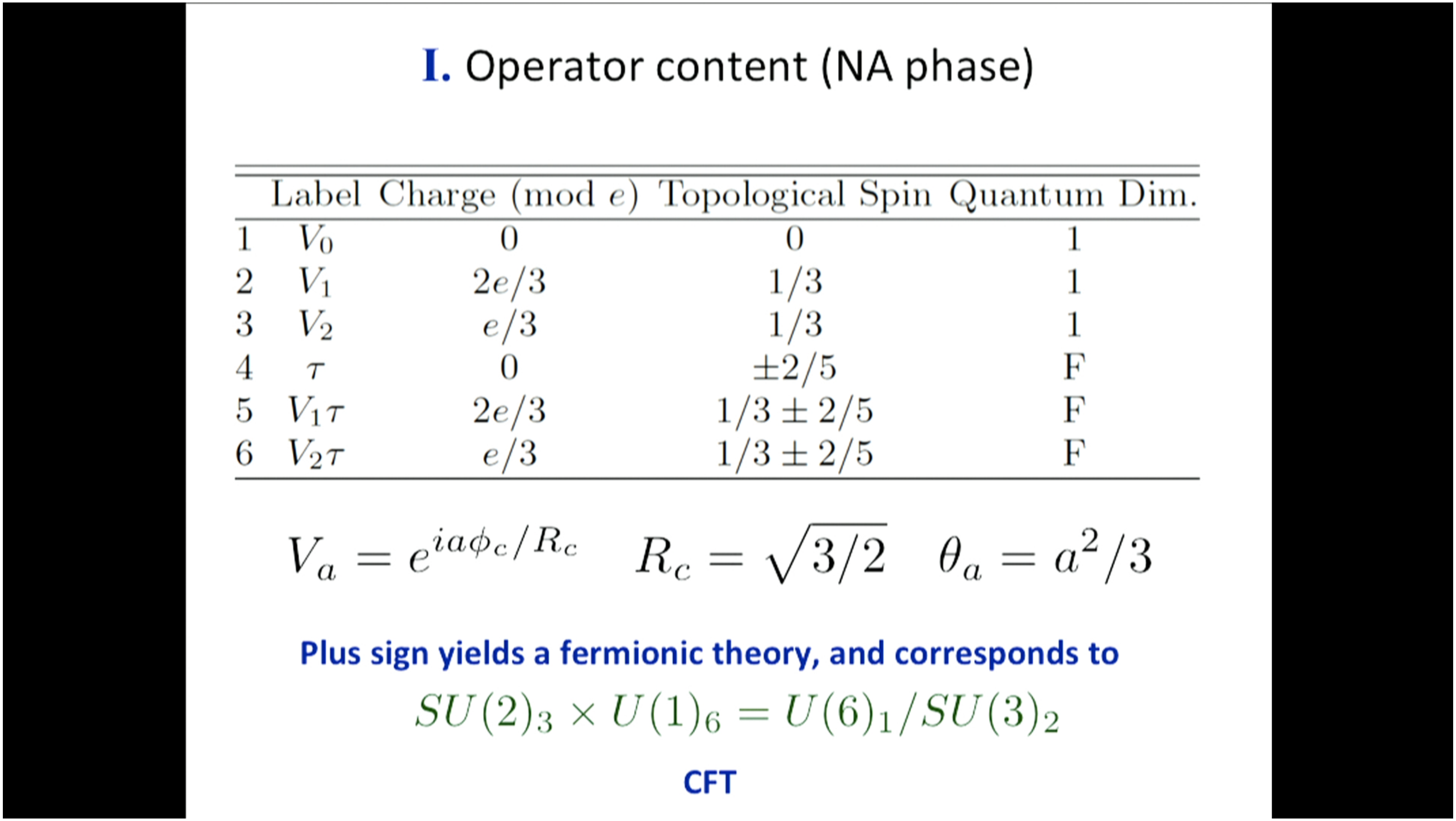

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I. Bilayer (330) state: intermediate tunneling regime

$$U_2 \ll t \ll U_1$$

$$\prod_i \begin{pmatrix} 100 \\ 100 \end{pmatrix}_i \equiv [200]$$

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← Z_2 **symmetric**

GSD = 6 =3 x 2 → Non-Abelian (NA)

“2” operators in the NA sector

(1, Fibonacci anyon)

$$X \times X = 1 + nX$$

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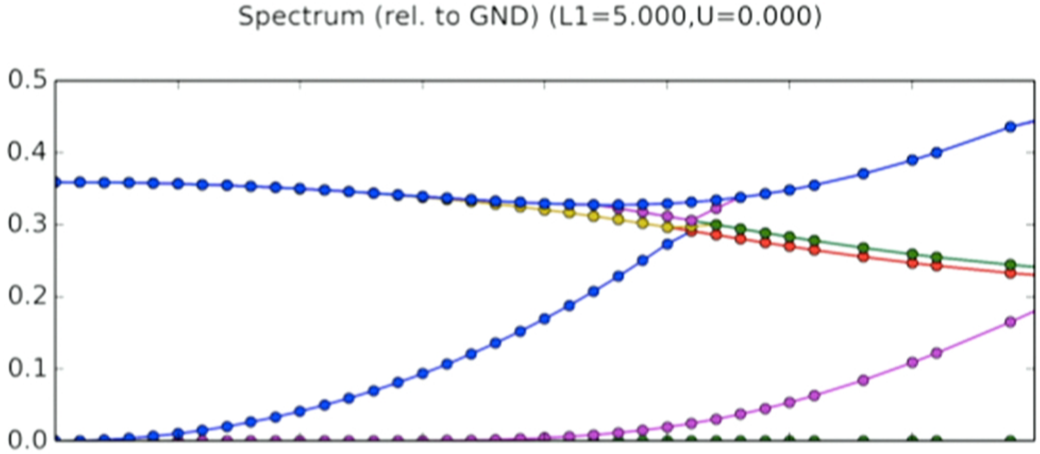

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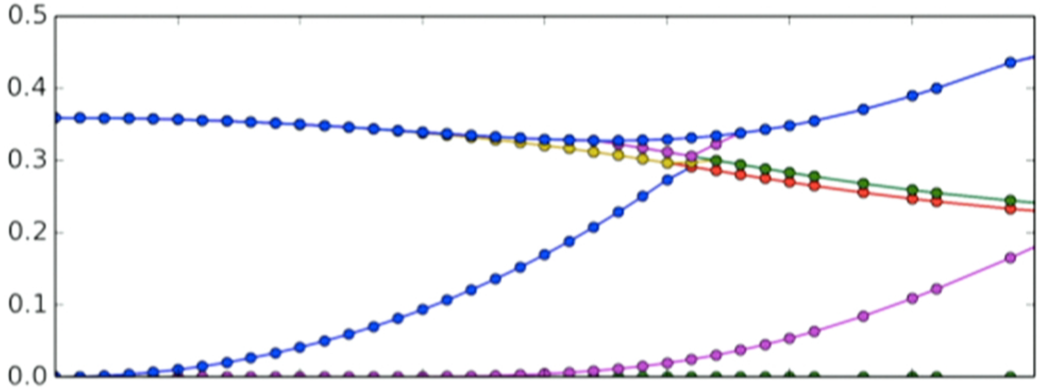
Spectrum (rel. to GND) (L1=5.000,U=0.000)



II. Chern-Simons Approach (mono-layer)

$$c_i = \frac{\epsilon^{abc} f_{a,i} f_{b,i} f_{c,i}}{6!}$$

f_1, f_2, f_3 define 3D complex space → *c* = volume of the complex space→ *c* invariant under **SU(3)** gauge symmetry

$$\nu_q = \nu_1/q \qquad \nu_1 = 1/3 \Rightarrow \nu_{1/3} = 1$$

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Integrating out fermions:
$SU(3)_1$ CS action

$$\mathcal{L}_{\rm CS} = \frac{1}{4\pi}\epsilon^{\mu\nu\rho}{\rm Tr}\left(\mathbf{A}_\mu\partial_\nu\mathbf{A}_\rho + \frac{2}{3}\mathbf{A}_\mu\mathbf{A}_\nu\mathbf{A}_\rho\right)$$

X.-G. Wen, PRB (1999).

3 chiral fermions generates $U(3)_1$ symmetry. $SU(3)_1$ subgroup is redundant (gauge symmetry)

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Interlayer tunneling: condensation of $\chi_{ab} = f^\dagger_{a,\uparrow} f_{b,\downarrow}$ operator that carries charge under $A^\uparrow - A^\downarrow$ → Higgs mechanism: $A^\uparrow = A^\downarrow = A$

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→ $SU(3)_2$ CS

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$$\tilde{c}_i = f_{i,1,+} f_{i,2,+} f_{i,3,+} \qquad f_{i,a,+} = \frac{f_{i,a,\uparrow} + f_{i,a,\downarrow}}{\sqrt{2}} \qquad C = 2$$

Integrating out partons → SU(3)$_C$ Chern Simons action

Parton's wf:

$$\psi_{f_+} = \mathcal{AS}\left[\psi_\uparrow(z_1, ..., z_{N/2})\psi_\downarrow(z_{N/2+1}, ..., z_N)\right]$$

Electron's wf:

$$\psi_{\text{Fib}} = \psi_{f_+}^3 = \psi_{C=2}^3$$

Topological entanglement entropy: $D = \sqrt{3(1+F^2)}$

Zhang and Vishwanath, PRB (2013)

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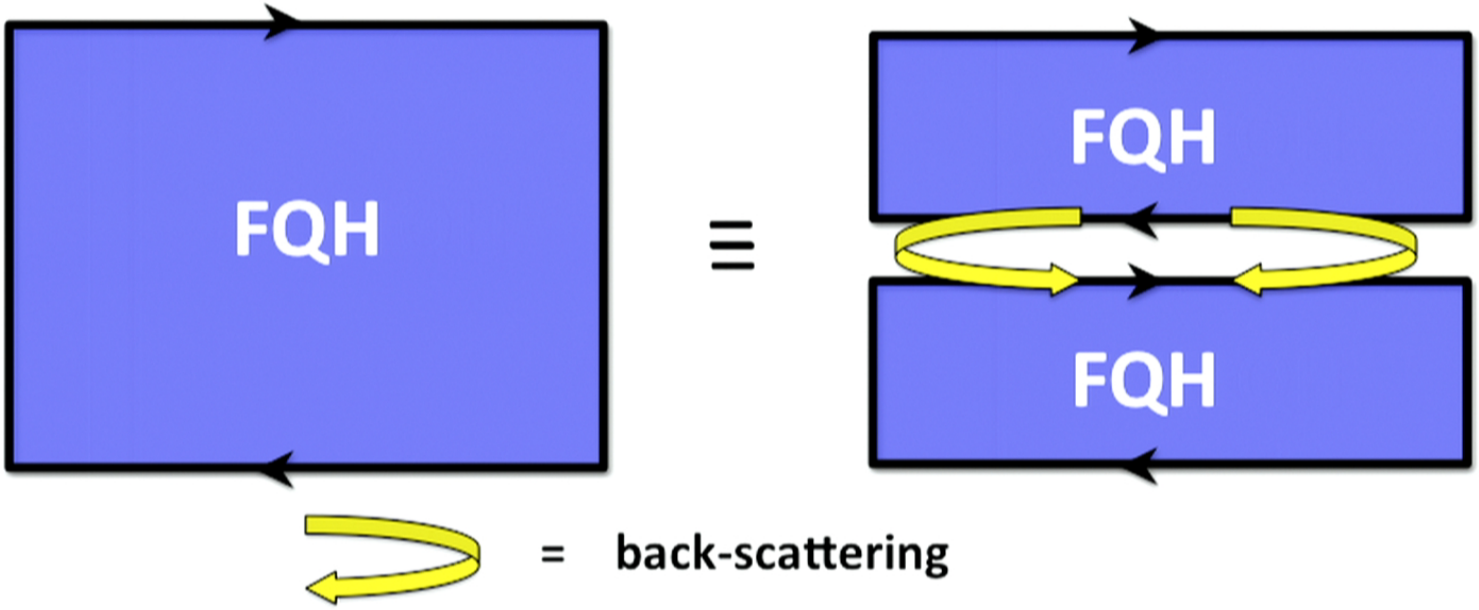
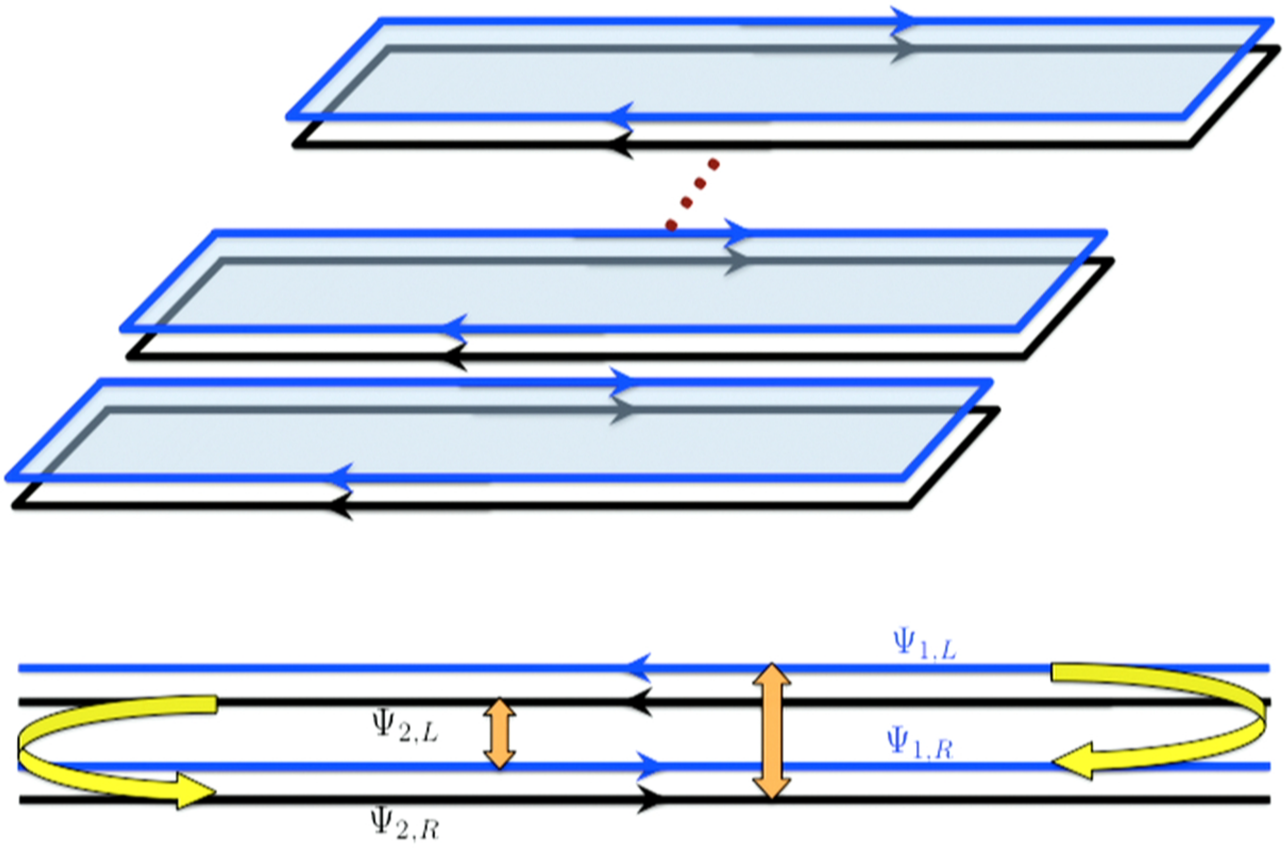


= **back-scattering**

Vaezi, arXiv:1307.8069, PRX (accepted)

III. Coupled wire construction (bilayer)



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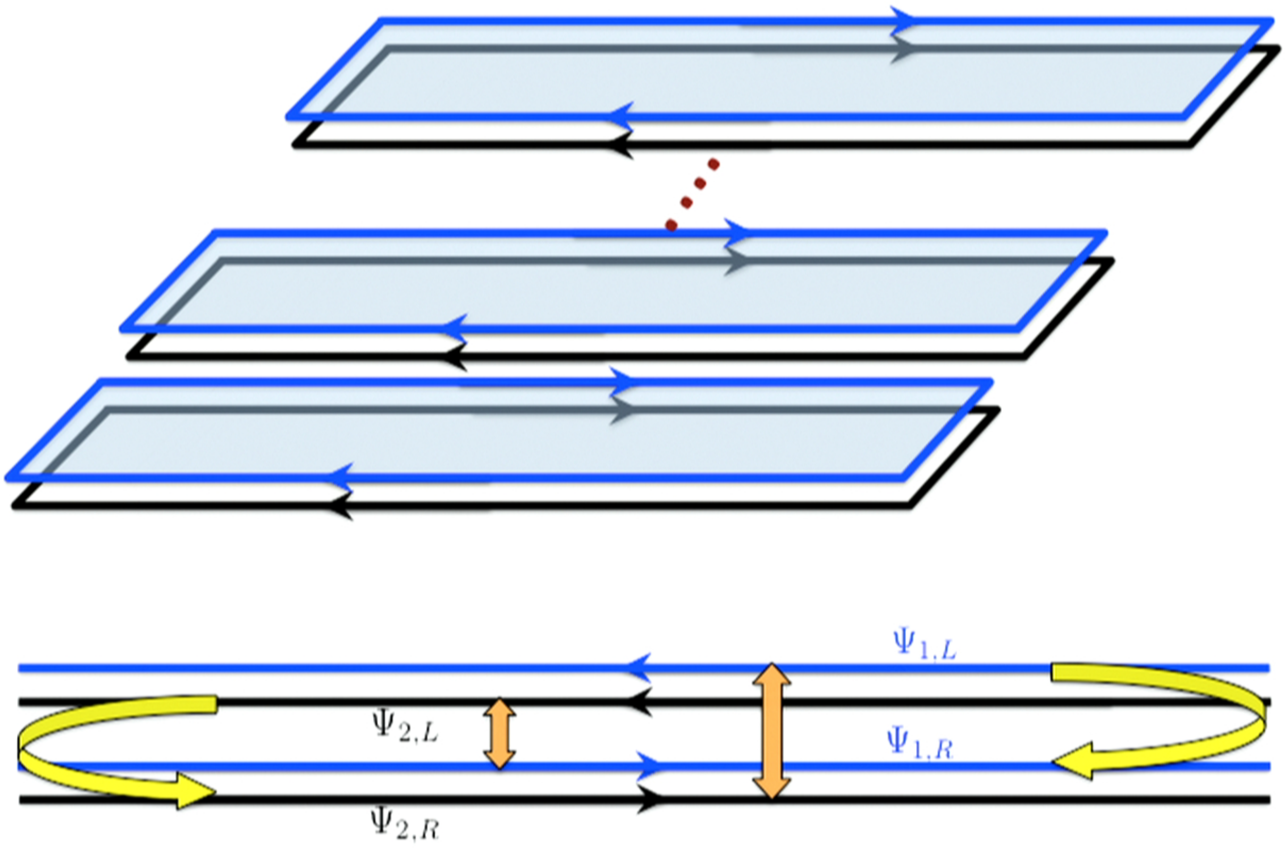

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Intra-layer BS only → Gapped

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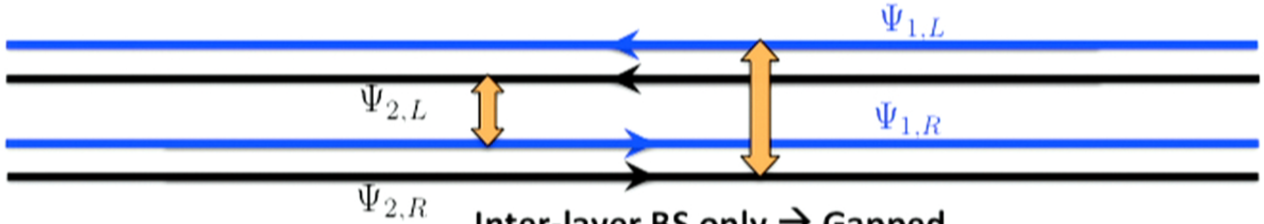

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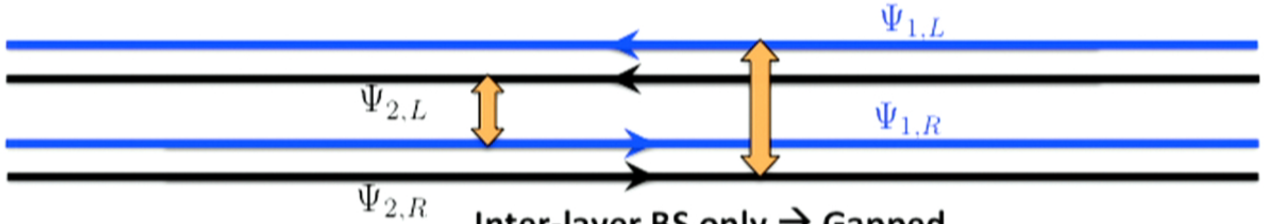

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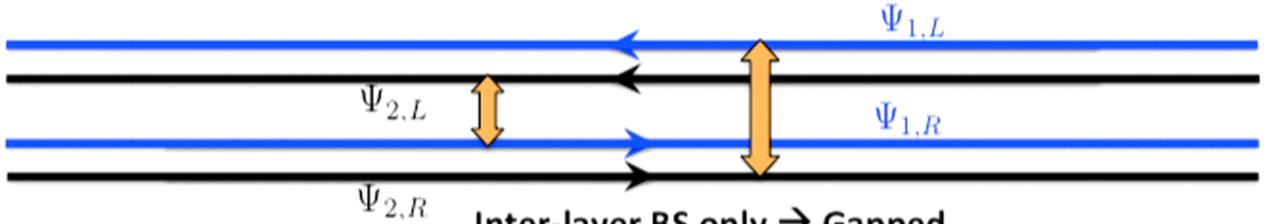

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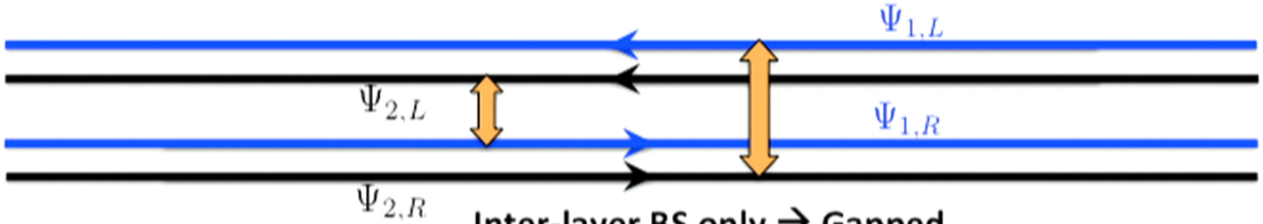

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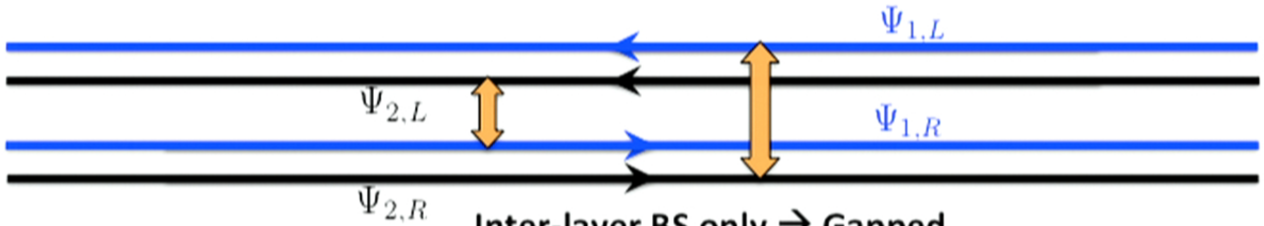

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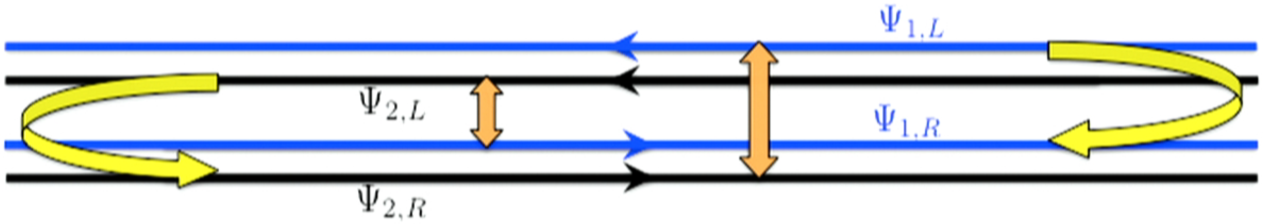


Both intra-layer & inter-layer → ???

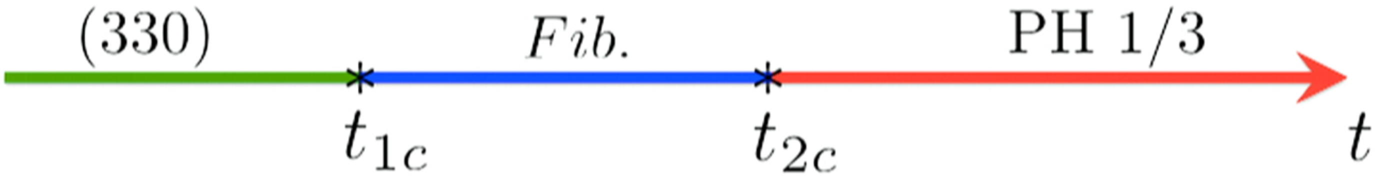
III. Self-dual sine-Gordon model

$$\mathcal{H}_{\text{SG}} = \frac{1}{4\pi}\int dx \left[(\partial_x\varphi_s)^2 + (\partial_x\theta_s)^2\right]$$
$$- \int dx \ \left[g_1 \cos(\sqrt{3}\varphi_s) + g_2 \cos(\sqrt{3}\theta_s)\right]$$

$g_1 = g_2$ **: self-dual point → critical → CFT description?**

P. Lecheminant et al, Nucl. Phys. B639, 502 (2002).

Experimental probes



(1) Detecting topological phase transition

Past Exp. : **charge gap** does NOT close by varying **"t"**

Phase transition happens in the **neutral** sector

Neutral Excitons carry electric **dipole**

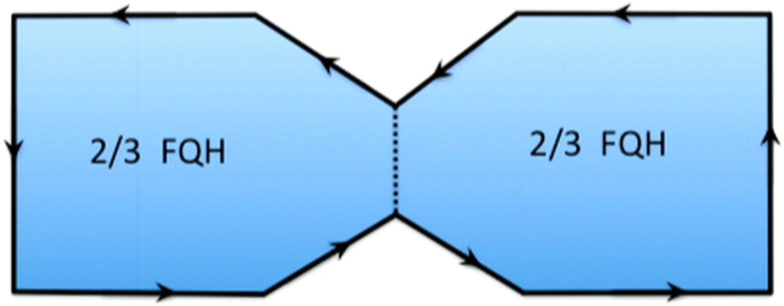
dipole-dipole correlation diverges at critical point

Surface acoustic phonon measurment?

Experimental probes

(2) Thermal conductivity: $\kappa_{xy}/T = c\frac{\pi^2 k_B^2}{3h}$

(3) Edge tunneling:



(a) **I-V** curve $I \propto V^{2g_{qh}-1}$

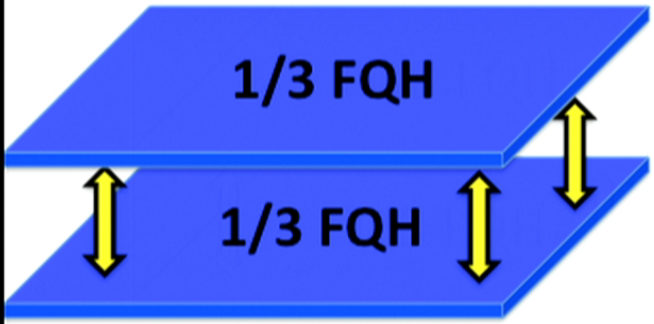
(b) Zero bias conductance: $\sigma_{xy} \propto T^{2g_{qh}-2}$

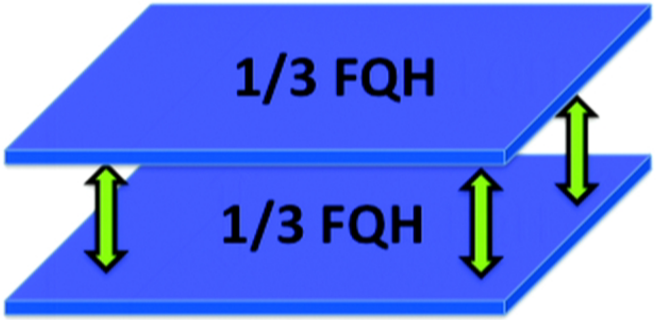
$$\boxed{\text{Fib}: \quad g_{qh} = 7/15 \quad c = 14/5}$$

(4) Interferometry experiments

Duality between pairing and tunneling

Symmetry of (330) : $U(1)_c \times U(1)_s \rtimes Z_2$





Interlayer tunneling :

$$\psi = \psi_\uparrow + \psi_\downarrow$$
$$= 2e^{i\sqrt{\frac{3}{2}}\phi_c} \cos\left(\sqrt{\frac{3}{2}}\phi_s\right)$$

$U(1)_s$ broken

Interlayer pairing :

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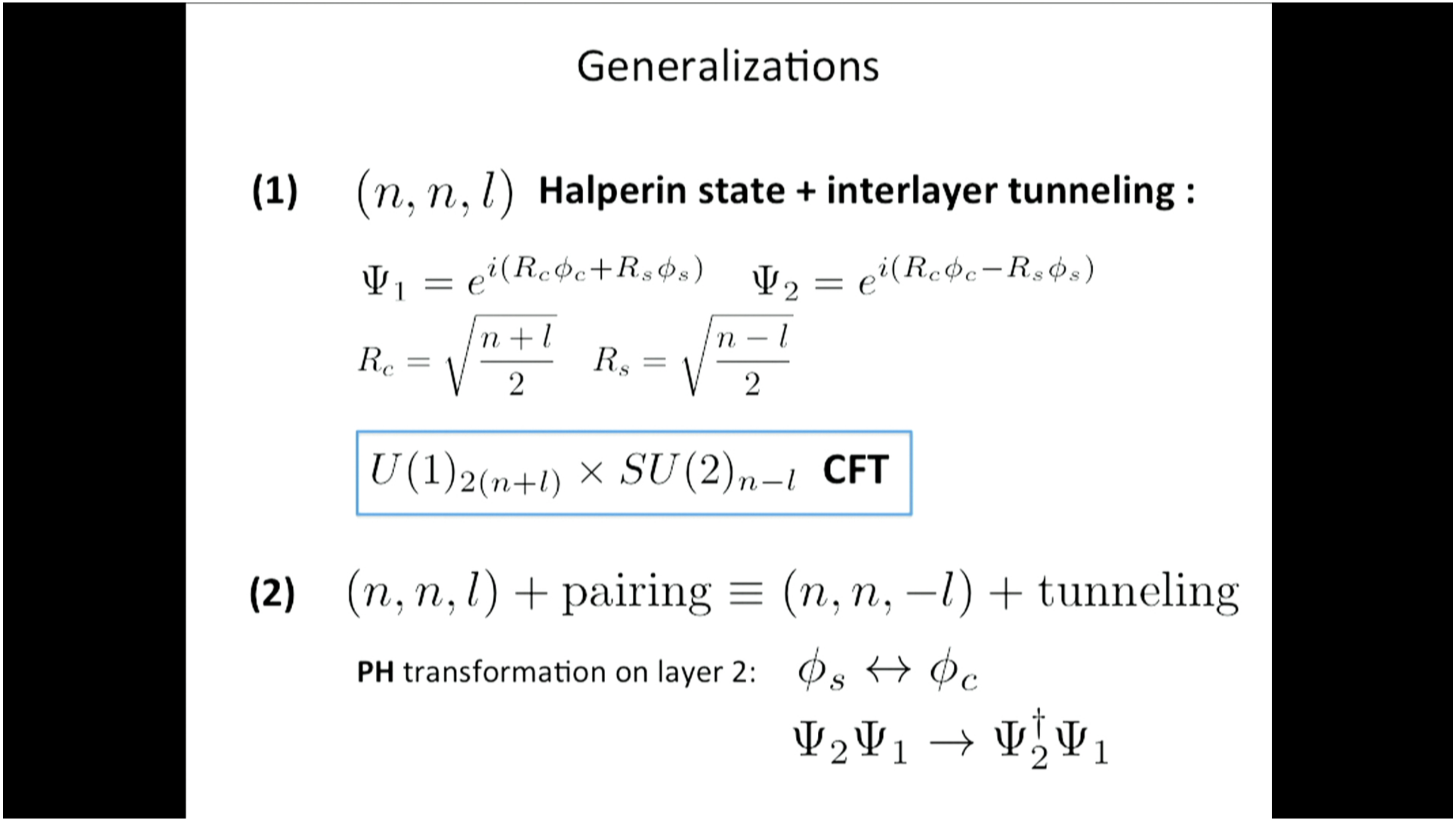
Mong et al. , (2013); Vaezi, (2013);
Vaezi, (2012); Vaezi, Barkeshli (2014);



Generalizations

(1) (n,n,l) **Halperin state + interlayer tunneling :**

$$\Psi_1 = e^{i(R_c\phi_c + R_s\phi_s)} \quad \Psi_2 = e^{i(R_c\phi_c - R_s\phi_s)}$$

$$R_c = \sqrt{\frac{n+l}{2}} \quad R_s = \sqrt{\frac{n-l}{2}}$$

$$\boxed{U(1)_{2(n+l)} \times SU(2)_{n-l} \text{ CFT}}$$

(2) $(n,n,l) + \text{pairing} \equiv (n,n,-l) + \text{tunneling}$

PH transformation on layer 2: $\phi_s \leftrightarrow \phi_c$

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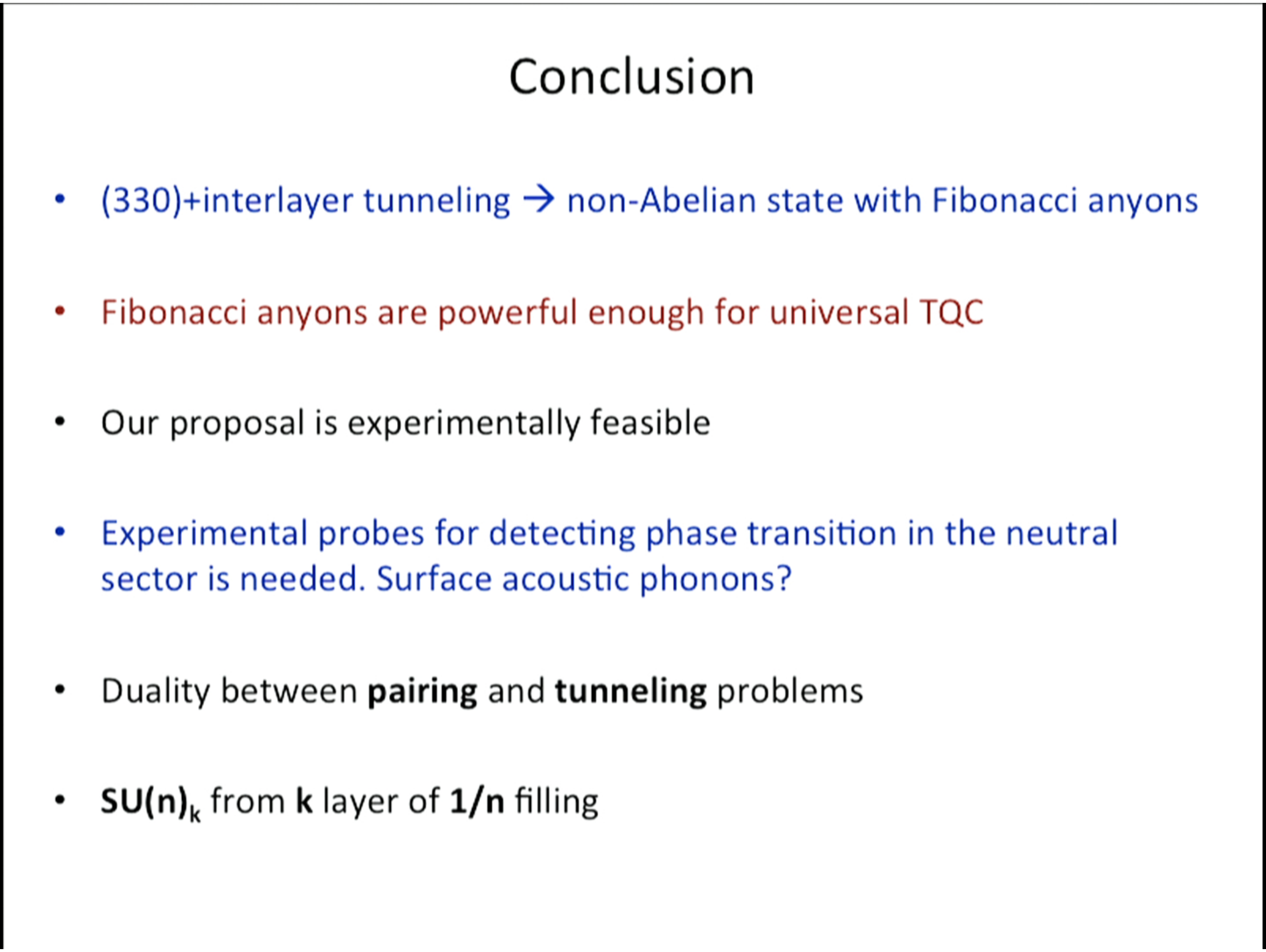
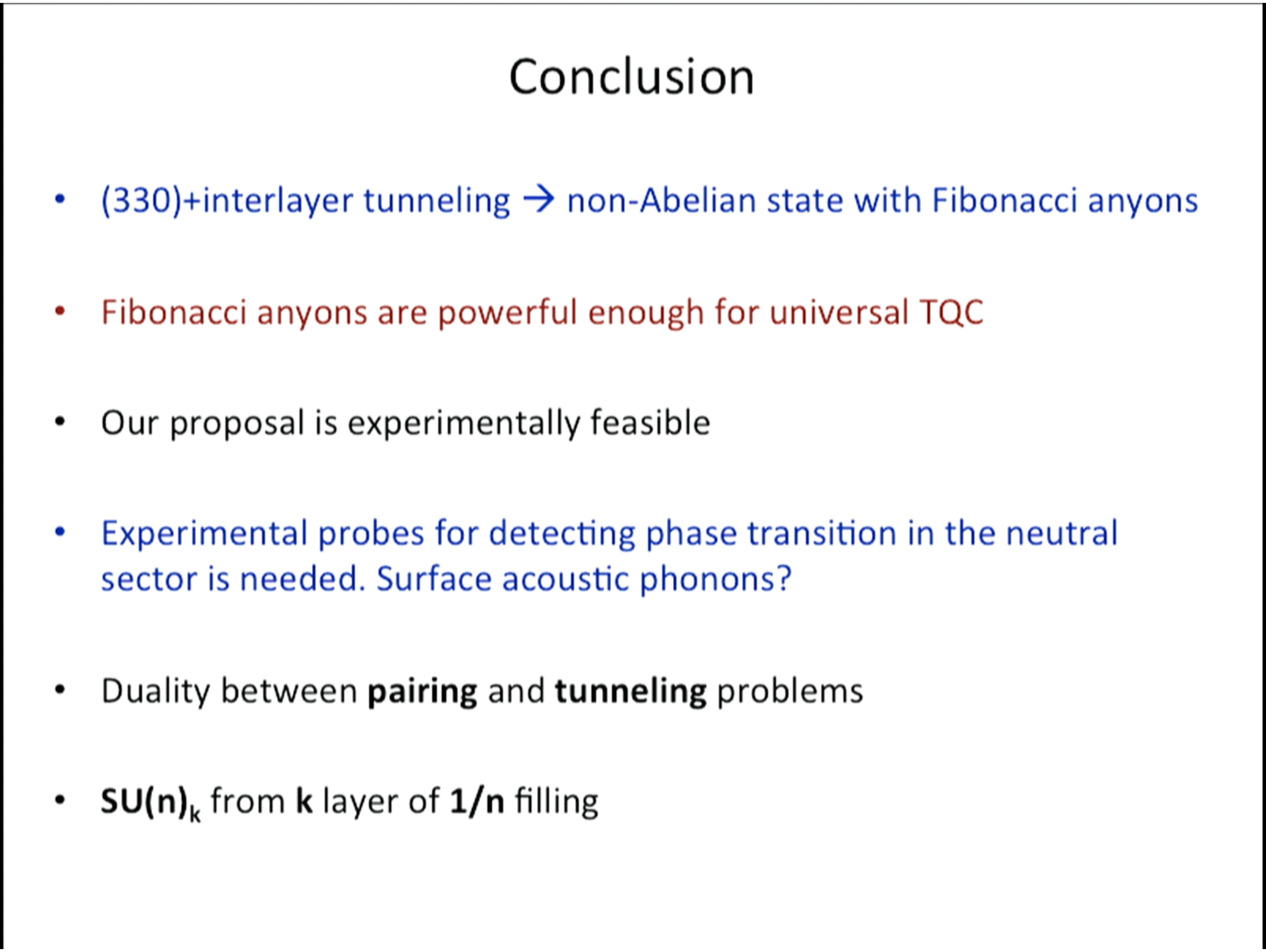


Conclusion

- (330)+interlayer tunneling → non-Abelian state with Fibonacci anyons
- Fibonacci anyons are powerful enough for universal TQC
- Our proposal is experimentally feasible
- Experimental probes for detecting phase transition in the neutral sector is needed. Surface acoustic phonons?
- Duality between **pairing** and **tunneling** problems
- $\mathbf{SU(n)}_k$ from **k** layer of **1/n** filling



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II. Comparison with previous studies

Symmetry of (330) : $U(1)_c \times U(1)_s \rtimes Z_2$

Gauging $\mathbf{Z_2}$ → $U(1)_3/Z_2$ orbifold CFT = $\mathbf{Z_4}$ Read-Rezayi state

$$\psi_{Z_4} = \mathcal{AS}\left[\psi_\uparrow^3(z_1, \ldots, z_{N/2})\psi_\downarrow^3(z_{N/2}, \ldots, z_N)\right]$$

Rezayi, Wen and Read, arXiv:1007.2022

Global Z_2 (condensing current operator J)→ Fibonacci state

$$\psi_{\text{Fib}} = \left(\mathcal{AS}\left[\psi_\uparrow(z_1, \ldots, z_{N/2})\psi_\downarrow(z_{N/2}, \ldots, z_N)\right]\right)^3 = \Psi_{C=2}^3$$

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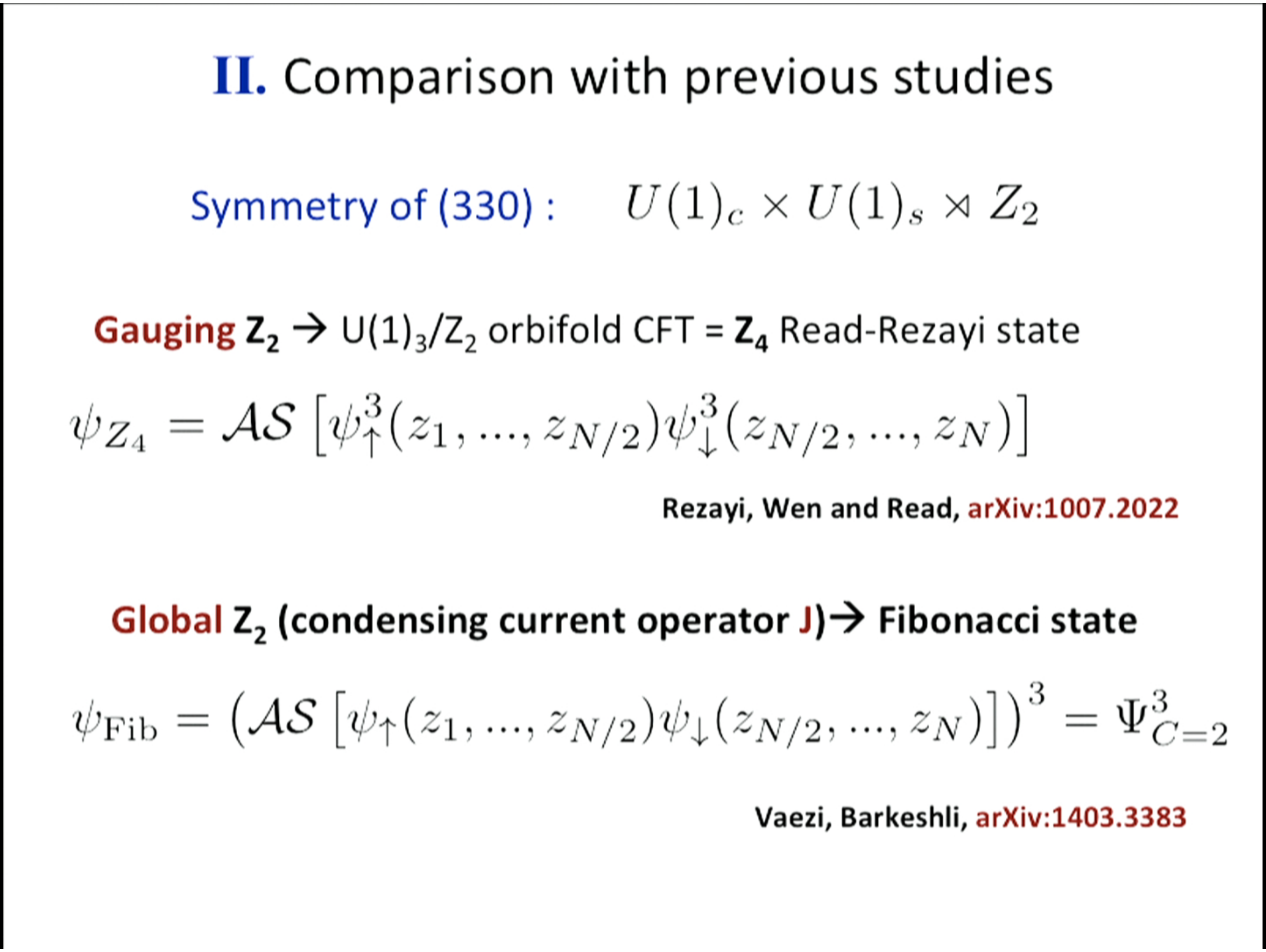

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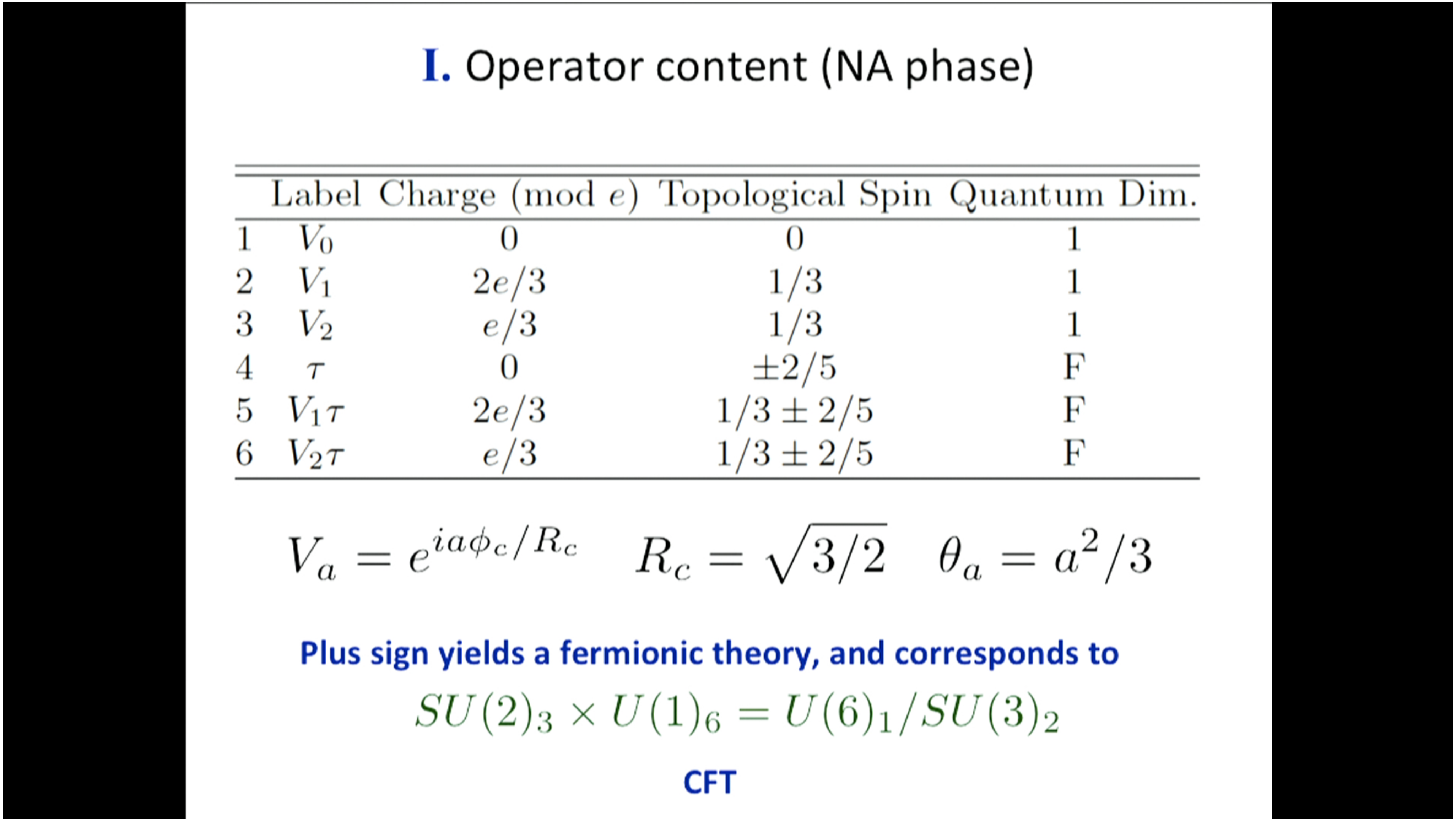

I. Operator content (NA phase)

	Label	Charge (mod e)	Topological Spin	Quantum Dim.
1	V_0	0	0	1
2	V_1	$2e/3$	$1/3$	1
3	V_2	$e/3$	$1/3$	1
4	τ	0	$\pm 2/5$	F
5	$V_1\tau$	$2e/3$	$1/3 \pm 2/5$	F
6	$V_2\tau$	$e/3$	$1/3 \pm 2/5$	F

$$V_a = e^{ia\phi_c/R_c} \quad R_c = \sqrt{3/2} \quad \theta_a = a^2/3$$

Plus sign yields a fermionic theory, and corresponds to

$$SU(2)_3 \times U(1)_6 = U(6)_1/SU(3)_2$$

CFT