Title: Double scaling in tensor models

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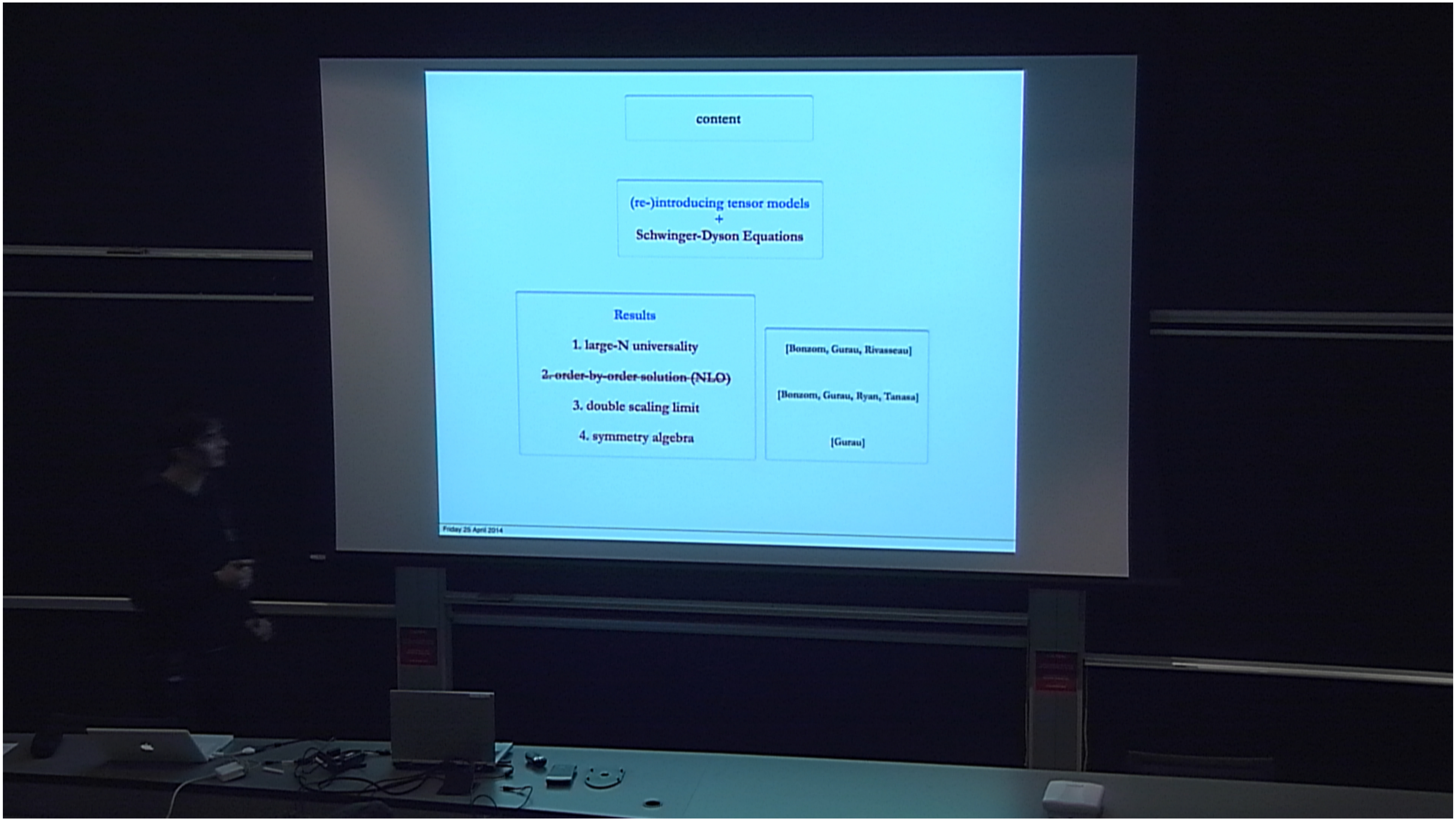
Abstract: I present recent work on the double scaling limit of random tensor models through the analysis of their Schwinger-Dyson equations. This study exemplifies their potential for probing the continuum phase structure of these theories.


content
(re-)introducing tensor models
+
Schwinger-Dyson Equations
Results
1. large-N universality
2. order-by-order solution (NLO)
3. double scaling limit
4. symmetry algebra
[Bonzom, Gurau, Rivasseau]
[Bonzom, Gurau, Ryan, Tanasa]
[Gurau]
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content

(re-)introducing tensor models
+
Schwinger-Dyson Equations

Results

1. large-N universality — [Bonzom, Gurau, Rivasseau]

2. ~~order-by-order solution (NLO)~~ — [Bonzom, Gurau, Ryan, Tanasa]

3. double scaling limit

4. symmetry algebra — [Gurau]

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random tensor model

$$\mu = [\mathrm{d}T\,\mathrm{d}\overline{T}]\; e^{-N^{D-1} S(T,\overline{T})}$$

random tensor

$$T_{a_1 \dots a_D}$$

action

$$S(T,\overline{T}) = \mathcal{B}_2(T,\overline{T}) - \sum_{i \in I} \frac{z^{v_i - 1}}{v_i}\, t_i\, \mathcal{B}_i(T,\overline{T})$$

size: N

unitary covariance:

$$T'_{a_1 \dots a_D} = {}^{(1)}U_{a_1 b_1} \dots {}^{(D)}U_{a_D b_D}\; T_{b_1 \dots b_D}$$

tensor invariant

$$\mathcal{B}(T,\overline{T})$$

admissible contractions
no index mixing

extract information

partition function $$Z = \int [\mathrm{d}T\,\mathrm{d}\overline{T}]\; e^{-N^{D-1} S(T,\overline{T})}$$

moments $$\frac{\langle \mathcal{B} \rangle}{N} = \int [\mathrm{d}T\,\mathrm{d}\overline{T}]\; \frac{\mathcal{B}(T,\overline{T})}{N}\; e^{-N^{D-1} S(T,\overline{T})}$$

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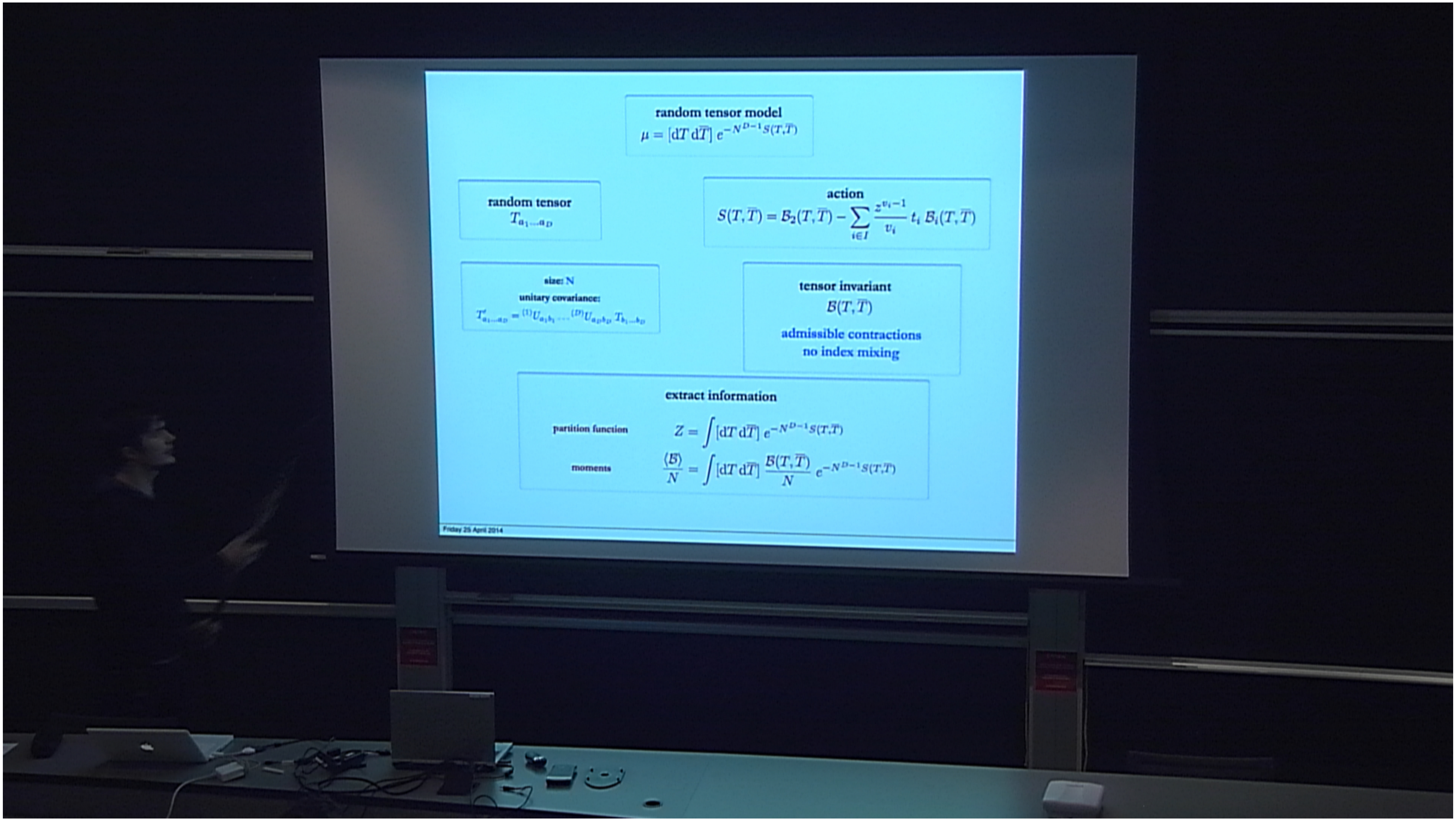
random tensor model
μ = [dT dT̄] e^{−N^{D−1} S(T,T̄)}
random tensor
T_{a_1…a_D}
action
S(T,T̄) = B_2(T,T̄) − Σ_{i∈I} (z^{v_i−1}/v_i) t_i B_i(T,T̄)
size: N
unitary covariance:
T′_{a_1…a_D} = ^{(1)}U_{a_1b_1} … ^{(D)}U_{a_Db_D} T_{b_1…b_D}
tensor invariant
B(T,T̄)
admissible contractions
no index mixing
extract information
partition function
Z = ∫[dT dT̄] e^{−N^{D−1} S(T,T̄)}
moments
⟨B⟩/N = ∫[dT dT̄] (B(T,T̄)/N) e^{−N^{D−1} S(T,T̄)}
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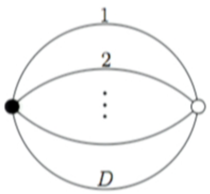
graphics - tensor invariants

$\mathcal{B}(T,\overline{T})$

catalogued by D-colored graphs

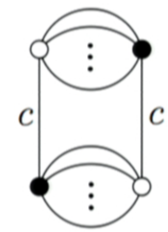
quadratic invariant

$$\mathcal{B}_2(T,\overline{T}) = T_{a_1\dots a_D}\ \overline{T}_{a_1\dots a_D}$$

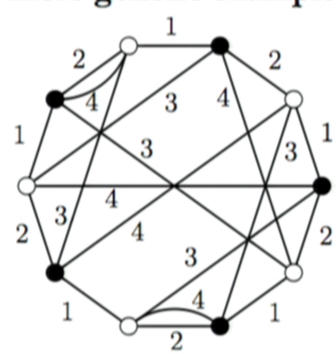


quartic invariant

$$\mathcal{B}_{4,c}(T,\overline{T})$$



more generic example



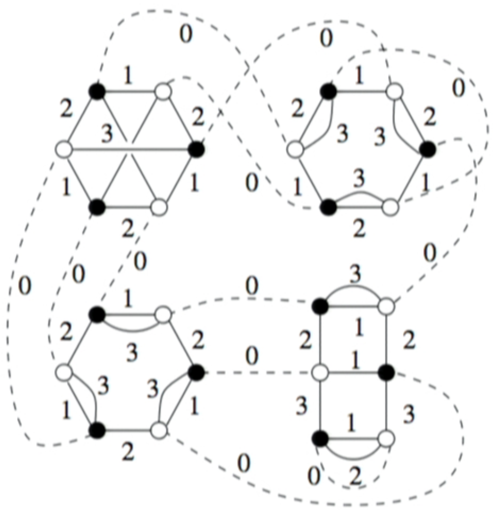
contact with QG - perturbative expansion

$$Z = \int [\mathrm{d}T\,\mathrm{d}\overline{T}]\; e^{-N^{D-1}S(T,\overline{T})} = \sum_{\mathcal{G}} A(\mathcal{G})$$

Taylor exp. + Wick's Thm.

"vertices" joined by propagators

(D+1)-colored graph



D-dimensional topology

$$A(\mathcal{G}) \propto N^{\sum_c f_{0c} - (D-1)(v-b)}\; z^{v-b} \prod_{i \in I} t_i^{b_i}$$

Dynamical Triangulations

reconstruct dual triangulation

triangulations equilateral

parameters

$(N, z) \longrightarrow (G, \Lambda)$ **bare**

$\{t_i\}$ **tunable for multicritical behaviours**

(matter on random geometry)

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graphics - covariant objects and bubble surgery

generic tensor invariant

Surgery 1:

$\mathcal{B}$ remove a $\overline{T}$ $(\mathcal{B}\backslash\overline{V})_{a_1\ldots a_D}$ remove a T $(\mathcal{B}\backslash\overline{V}\backslash V)_{a_1\ldots a_D;b_1\ldots b_D}$ contract $(\mathcal{B}\backslash\overline{V}\backslash V)$

Surgery 2:

$(\mathcal{B}\backslash\overline{V})_{a_1\ldots a_D}$ contract $(\mathcal{B}'\backslash V)_{b_1\ldots b_D}$

$(\mathcal{B}\backslash\overline{V})\cdot(\mathcal{B}'\backslash V)$

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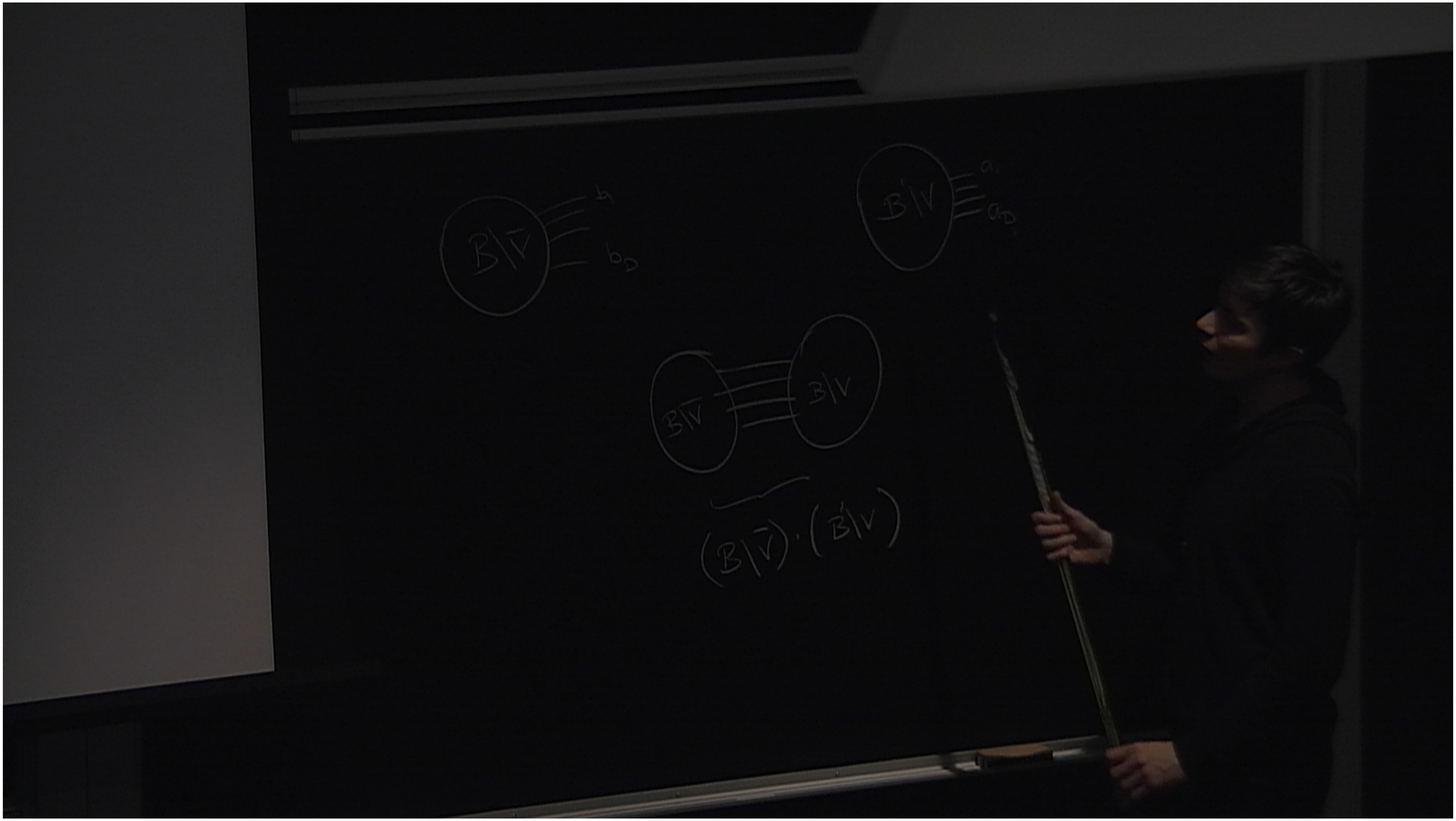
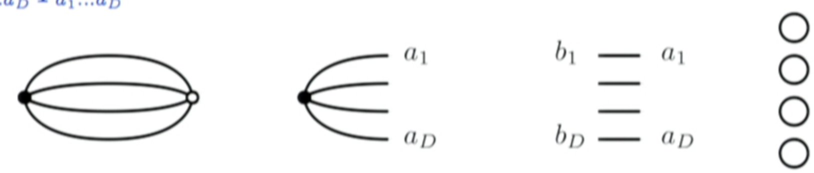

B\V̄
b1
bD
B'\V
a1
aD
(B\V̄)·(B'\V)

graphics - covariant objects and bubble surgery

quadratic and quartic invariants

Surgery 1:

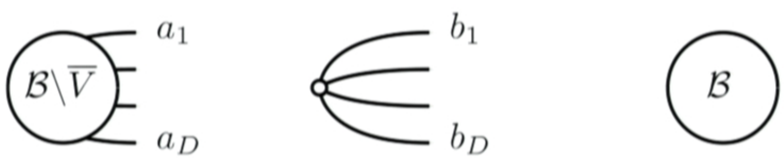
$\mathcal{B} = \mathcal{B}_2 = T_{a_1 \dots a_D} \overline{T}_{a_1 \dots a_D}$



N^D

Surgery 2:

$$(\mathcal{B} \backslash \overline{V}) \cdot (\mathcal{B}_2 \backslash V) = \mathcal{B}$$



Schwinger-Dyson equations

$$0 = \frac{1}{N^D Z} \int [\mathrm{d}T\, \mathrm{d}\overline{T}]\ \frac{\partial}{\partial T_{a_1 \dots a_D}} \left[(\mathcal{B} \backslash \overline{V})_{a_1 \dots a_D}\ e^{-N^{D-1} S(T, \overline{T})} \right]$$

$$S(T, \overline{T}) = \mathcal{B}_2(T, \overline{T}) - \sum_{i \in I} \frac{z^{v_i - 1}}{v_i}\, t_i\, \mathcal{B}_i(T, \overline{T})$$

$$0 = \sum_{v \in \mathcal{B}} \frac{\langle \mathcal{B} \backslash \overline{V} \backslash V \rangle}{N^D} - \frac{\langle \mathcal{B} \rangle}{N} + \sum_{i \in I} \frac{z^{v_i - 1}}{v_i}\, t_i \sum_{V \in \mathcal{B}_i} \frac{\langle \mathcal{B} \backslash \overline{V} \cdot \mathcal{B}_i \backslash V \rangle}{N}$$

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prime example

$$0 = \sum_{v \in \mathcal{B}} \frac{\langle \mathcal{B} \backslash \overline{V} \backslash V \rangle}{N^D} - \frac{\langle \mathcal{B} \rangle}{N} + \sum_{i \in I} \frac{z^{v_i - 1}}{v_i} t_i \sum_{V \in \mathcal{B}_i} \frac{\langle \mathcal{B} \backslash \overline{V} \cdot \mathcal{B}_i \backslash V \rangle}{N}$$

quadratic $\mathcal{B} = \mathcal{B}_2$

$$0 = 1 - \frac{\langle \mathcal{B}_2 \rangle}{N} + \sum_{i \in I} z^{v_i - 1} t_i \frac{\langle \mathcal{B}_i \rangle}{N}$$

$$(\mathcal{B}_2 \backslash \overline{V} \backslash V) = N^D$$

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prime example

$$0 = \sum_{v\in\mathcal{B}} \frac{\langle \mathcal{B}\backslash\overline{V}\backslash V\rangle}{N^D} - \frac{\langle\mathcal{B}\rangle}{N} + \sum_{i\in I} \frac{z^{v_i-1}}{v_i}\, t_i \sum_{V\in\mathcal{B}_i} \frac{\langle \mathcal{B}\backslash\overline{V}\cdot\mathcal{B}_i\backslash V\rangle}{N}$$

quadratic $\mathcal{B} = \mathcal{B}_2$

$$0 = 1 - \frac{\langle\mathcal{B}_2\rangle}{N} + \sum_{i\in I} z^{v_i-1}\, t_i\, \frac{\langle\mathcal{B}_i\rangle}{N}$$

$$(\mathcal{B}_2\backslash\overline{V}\backslash V) = N^D$$

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Case study 1: Large-N universality

Scaling Input

1. Factorization

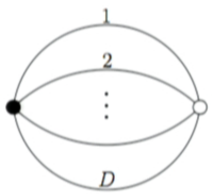
$$\langle \mathcal{B}_1 \mathcal{B}_2 \rangle = \langle \mathcal{B}_1 \rangle \langle \mathcal{B}_2 \rangle$$

$$N^{\sum_c f_{0c} - (D-1)(v-b)}$$

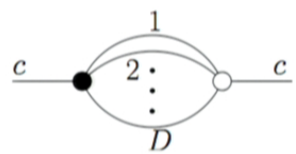
2. Relevant observables

$$\lim_{N\to\infty} \frac{\langle \mathcal{B} \rangle}{N} = \begin{cases} K_{\mathcal{B}}(z,\{t_i\}) & \mathcal{B} \text{ melonic} \\ 0 & \text{otherwise} \end{cases}$$

Melonic observables



supermelon



melonic building block

re-iterative insertion

Case study 1: Large-N universality

Argument

1. Melonic dominance + Factorization

$$0 = \frac{\langle \mathcal{B} \backslash \mathcal{M} \rangle}{N} - \frac{\langle \mathcal{B} \rangle}{N} + \sum_{i \in I} \frac{z^{v_i - 1}}{v_i} t_i \sum_{V \in \mathcal{B}_i} \frac{\langle \mathcal{B} \backslash \overline{V} \cdot \mathcal{B}_i \backslash V \rangle}{N}$$

melonic annihilation

2. Inductive argument $\langle \mathcal{B} \rangle = E_{v_{\mathcal{B}}}(z, \{t_i\})$

$$E_n - E_{n+1} - \sum_{i \in I} z^{v_i - 1} t_i E_{n+v_i} = 0$$

3. Characteristic polynomial with unique physical solution

$$E_{v_{\mathcal{B}}} = \langle \mathcal{B} \rangle = N \left[T(z, \{t_i\}) \right]^{v_{\mathcal{B}}}$$

full 2-point function at large-N

universal Gaussian behaviour at large-N

$$1 - T + \sum_{i \in I} z^{v_i - 1} t_i T^{v_i} = 0$$

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Case study 3: Double-scaling limit

Were I to have presented Case Study 2

1. Restrict to melonic actions

2. NLO arises at $\frac{1}{N^{D-2}}$

3. Non-Gaussianity appears

$$\frac{\langle \mathcal{B}_2 \rangle}{N} = T(z, \{t_i\}) + \frac{1}{N^{D-2}} K_2^{NLO}(z, \{t_i\}) + \dots$$

$$\frac{\langle \mathcal{B}_i \rangle}{N} = \left(1 + \frac{\alpha_i}{N^{D-2}} + \dots\right) \left(\frac{\langle \mathcal{B}_2 \rangle}{N}\right)^{v_i} + \frac{1}{N^{D-2}} K_4^{LO}(z, \{t_i\}) + \dots$$

4. SDE for quadratic and quartic invariants form closed coupled equations

$$\frac{\langle \mathcal{B}_2 \rangle}{N} \sim T(z, \{t_i\}) + \frac{1}{N^{D-2}} \frac{1}{\sqrt{z_{critical} - z}} (\dots) + \dots$$

5. Coupled equations are linear in higher order terms

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5. Coupled equations are linear in higher order terms

Case study 3: Double-scaling limit

1. Change expansion $$\frac{\langle \mathcal{B}_2 \rangle}{N} = T(z_{critical}, \{t_i\}) + \frac{1}{N^a} K_2^{DS}(x, \{t_i\}) + \dots$$

2. SDE $$0 = 1 - \frac{\langle \mathcal{B}_2 \rangle}{N} + \sum_{i \in I} z^{v_i - 1} t_i \frac{\langle \mathcal{B}_i \rangle}{N}$$

3. Substituting the ansatz $a = \frac{D-2}{2}$ $\quad x = (z_{critical} - z) N^{D-2}$

$O(1/N^{\frac{D-2}{2}})$ **linear term** $K_2^{DS}(x, \{t_i\})$ **no information**

$O(1/N^{D-2})$ **determines** $K_2^{DS}(x, \{t_i\})$ **branched polymers?**

$\sim \sqrt{x_{critical} - x}$

critical dimension D = 6

4. Input relevant observables $\mathcal{B}_2 \quad \mathcal{B}_{4,c}$

Outlook

improve scaling arguments

derive multi-scaling limits

relationship between symmetry (sub)-algebras
and
higher dimensional diffeomorphism symmetries

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