Title: The Asymptotic Safety Program: New results and an inconvenient truth

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Abstract: We briefly review the various components and their conceptual status of the full Asymptotic Safety Program which aims at finding a nonperturbative infinite-cutoff limit of a regularized functional integral for a quantum field theory of gravity. It is explained why in the continuum formulation based on the Effective Average Action the key requirement of background independence unavoidably results in a "bi-metric" framework, and recent results on truncated RG flows of bi-metric actions are presented. They suggest that the next generation of truncations that must be explored should be of bi-metric type. As an application, a method of characterizing and counting physical states is shown to arise.


The Asymptotic Safety
Program
- New results and an inconvenient truth -
Martin Reuter

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2. **Can Asymptotic Safety coexist with Background Independence?**

(a) Background Independence, Split Symmetry, bi-metric character of the Effective Average Action

(b) New results from a bi-metric truncation

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(b) New results from a bi-metric truncation

The fundamental problem:

Give a meaning to ("define", "renormalize", "take the continuum limit of", ...) a functional integral over all metrics on a space time $\mathcal{M}$:

$$\int \mathcal{D}\hat{g}_{\mu\nu}\, e^{-S[\hat{g}_{\mu\nu}]}$$

S: diff($\mathcal{M}$)-invariant bare action, e.g. S_{EH} + counter terms

$$\mathcal{D}\hat{g}_{\mu\nu} \equiv \prod_{x\in\mathcal{M}} \prod_{\mu,\nu} dg_{\mu\nu}(x)$$

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The Asymptotic Safety program

- Pick (topological) manifold $\mathcal{M}$
- Pick a set of "carrier fields" Φ :

  $g_{\mu\nu}$, $e_\mu^{\ a}$, $(e_\mu^{\ a}, \omega_\mu^{\ ab})$, ...
- Pick symmetry / gauge group G :

  $\mathrm{Diff}(\mathcal{M})$, $\mathrm{Diff}(\mathcal{M}) \otimes O(4)_{loc}$,

  foliation preserving diffeo. s, ...

$\Rightarrow$ total set of fields $\Psi \equiv (\Phi, \bar{\Phi}, \text{FP ghosts})$,

$\bar{\Phi}$ = backgrd. field for Φ

- Pick space of fields :

$$\mathcal{F} = \{ \text{fields } \Psi \text{ on } \mathcal{M} \mid (\dots) \}$$

boundary conditions, degree of regularity, etc.

(N.B.: b.c. $\longleftrightarrow$ states !)

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- Pick space of action functionals ≡ theory space :

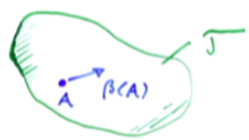
$$\mathcal{T} = \Big\{ A : \mathcal{F} \to \mathbb{R},\ \Psi \mapsto A[\Psi] \ \Big| \ \delta_G^B A = 0,\ (\cdots) \Big\}$$

degree of (non-) locality, regularity, etc.

- Pick coarse graining (averaging) scheme on $\mathcal{T}$: vector field β

$$A \xrightarrow[\text{RG step}]{\text{infinites.}} A + \underbrace{\beta(A)}_{\in T_A \mathcal{T}}$$

$\Rightarrow$ RG flow $(\mathcal{T}, \beta)$

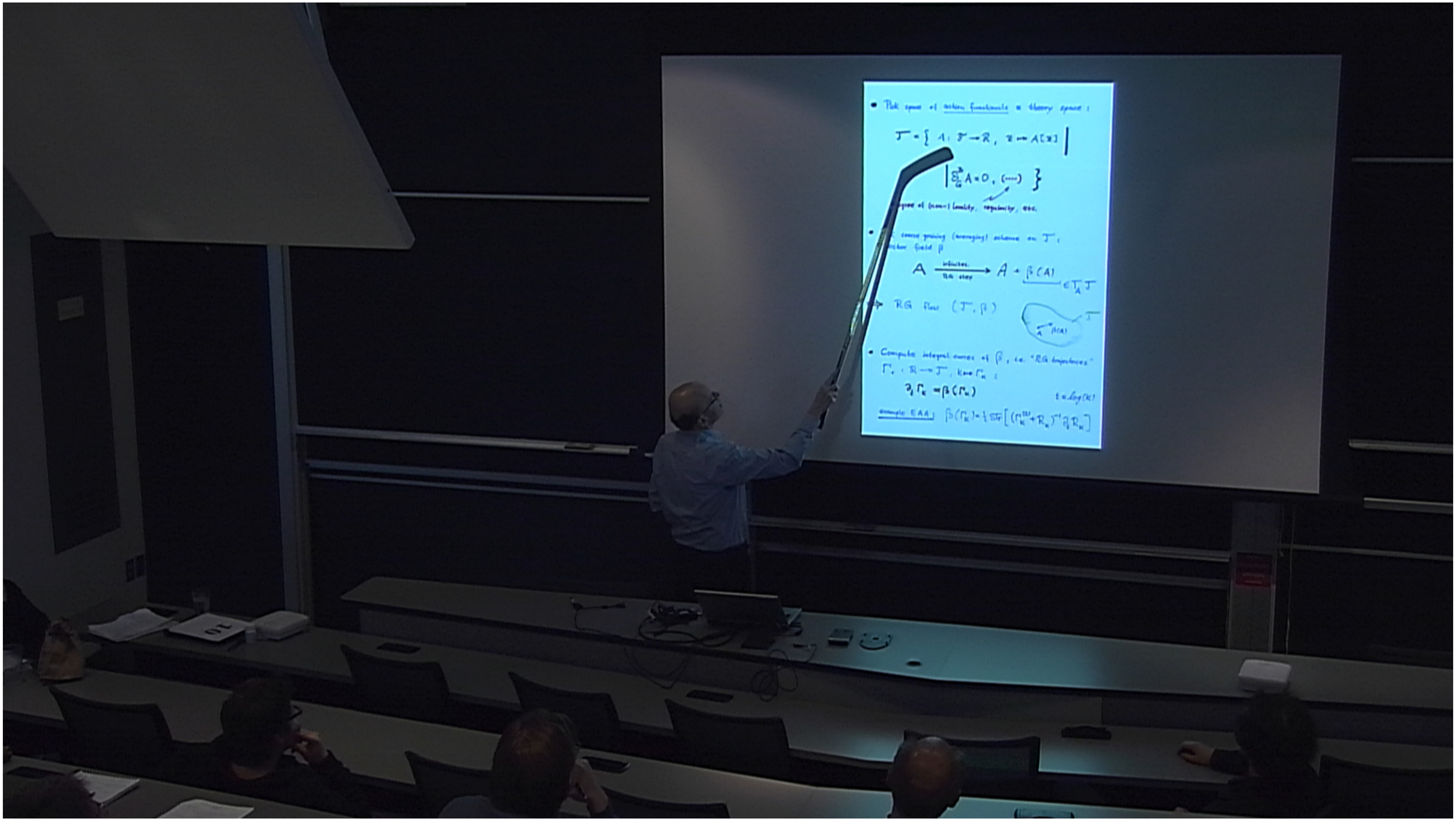


- Compute integral curves of β, i.e. "RG trajectories"

$\Gamma_\bullet : \mathbb{R} \to \mathcal{T}$, $k \mapsto \Gamma_k$:

$$\partial_t \Gamma_k = \beta(\Gamma_k) \qquad t \equiv \log(k)$$

example EAA: $\beta(\Gamma_k) = \frac{1}{2} \mathrm{STr}\left[(\Gamma_k^{(2)} + R_k)^{-1} \partial_t R_k \right]$



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RG flow (T, β)
Compute integral curves of β, i.e. "RG trajectories"
t = log(k)

- Search for fixed points : $\beta(A_*) = 0$
- Determine linearized flow near A_* :

  eigenvectors $\hat{=}$ scaling fields
  eigenvalues $\hat{=}$ critical exponents

- Determine UV critical hypersurface $\mathcal{S}_{UV}$

- Try to find <u>complete</u> RG trajectories, having well-behaved limits $k \to \infty$ and $k \to 0$.

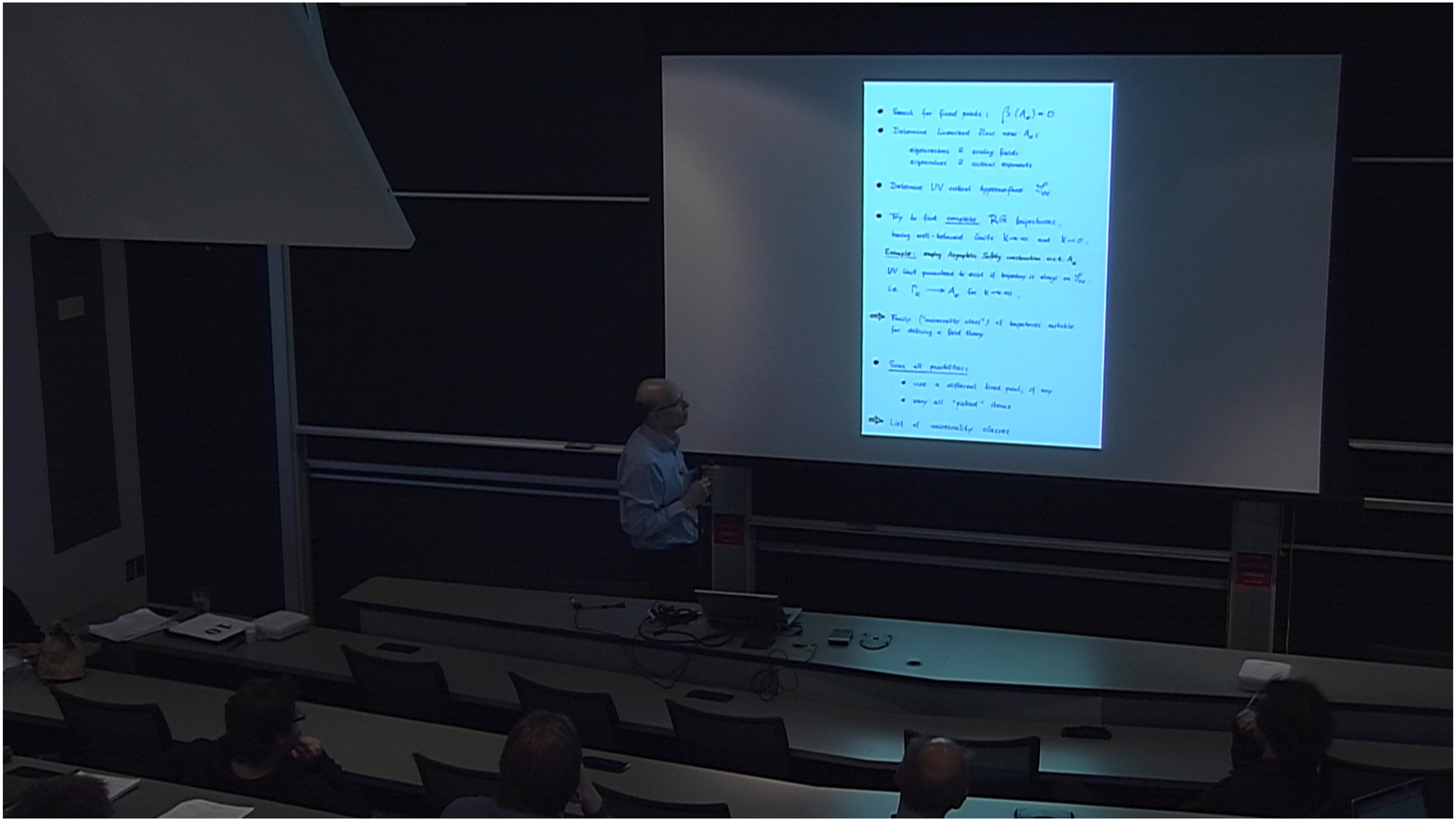
<u>Example:</u> employ Asymptotic Safety construction w.r.t. A_*

UV limit guaranteed to exist if trajectory is always on $\mathcal{S}_{UV}$, i.e. $\Gamma_k \longrightarrow A_*$ for $k \to \infty$.

$\Rightarrow$ Family ("universality class") of trajectories suitable for defining a field theory

- <u>Scan all possibilities:</u>
  - use a different fixed point, if any
  - vary all "picked" items

$\Rightarrow$ List of universality classes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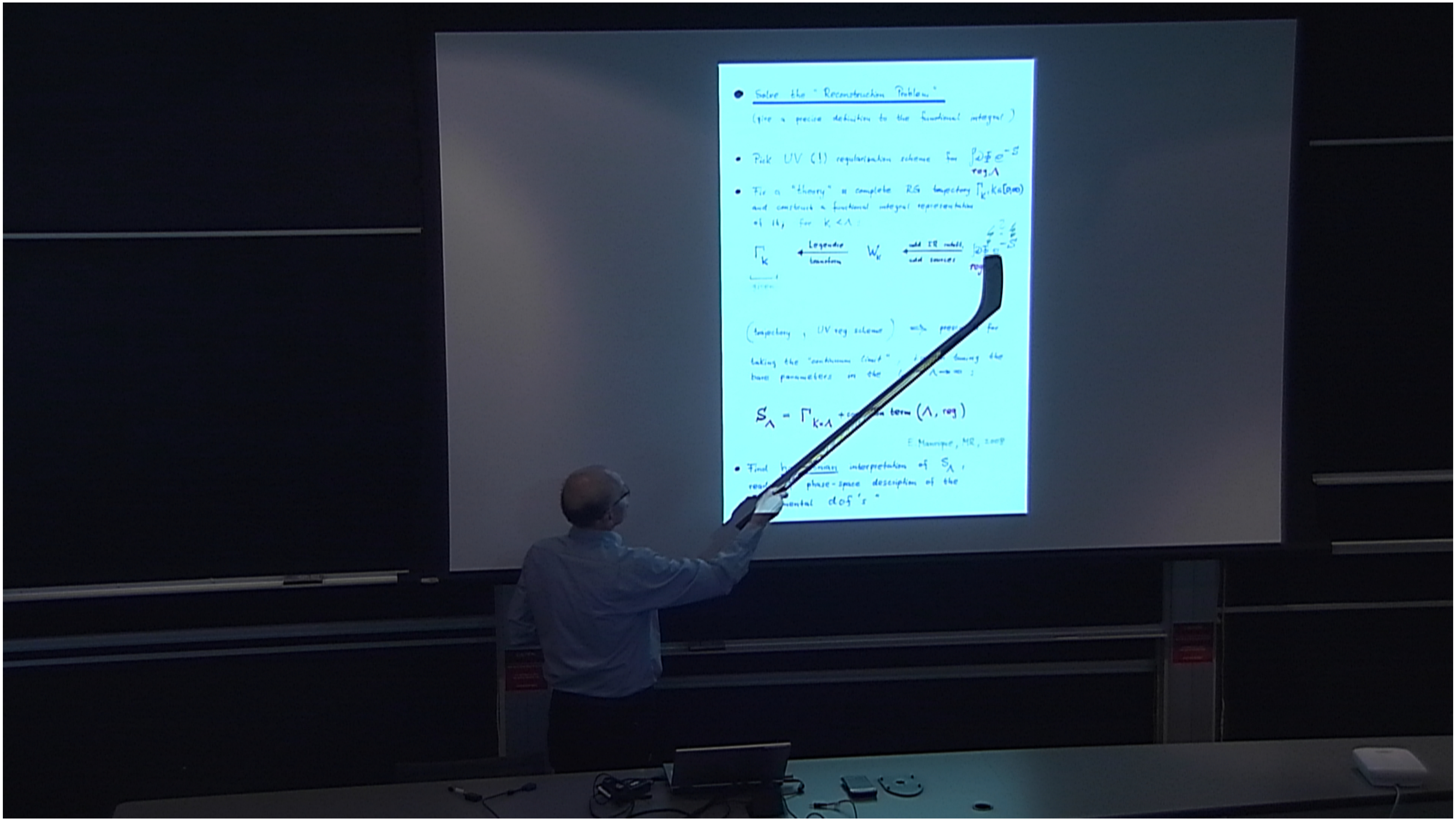
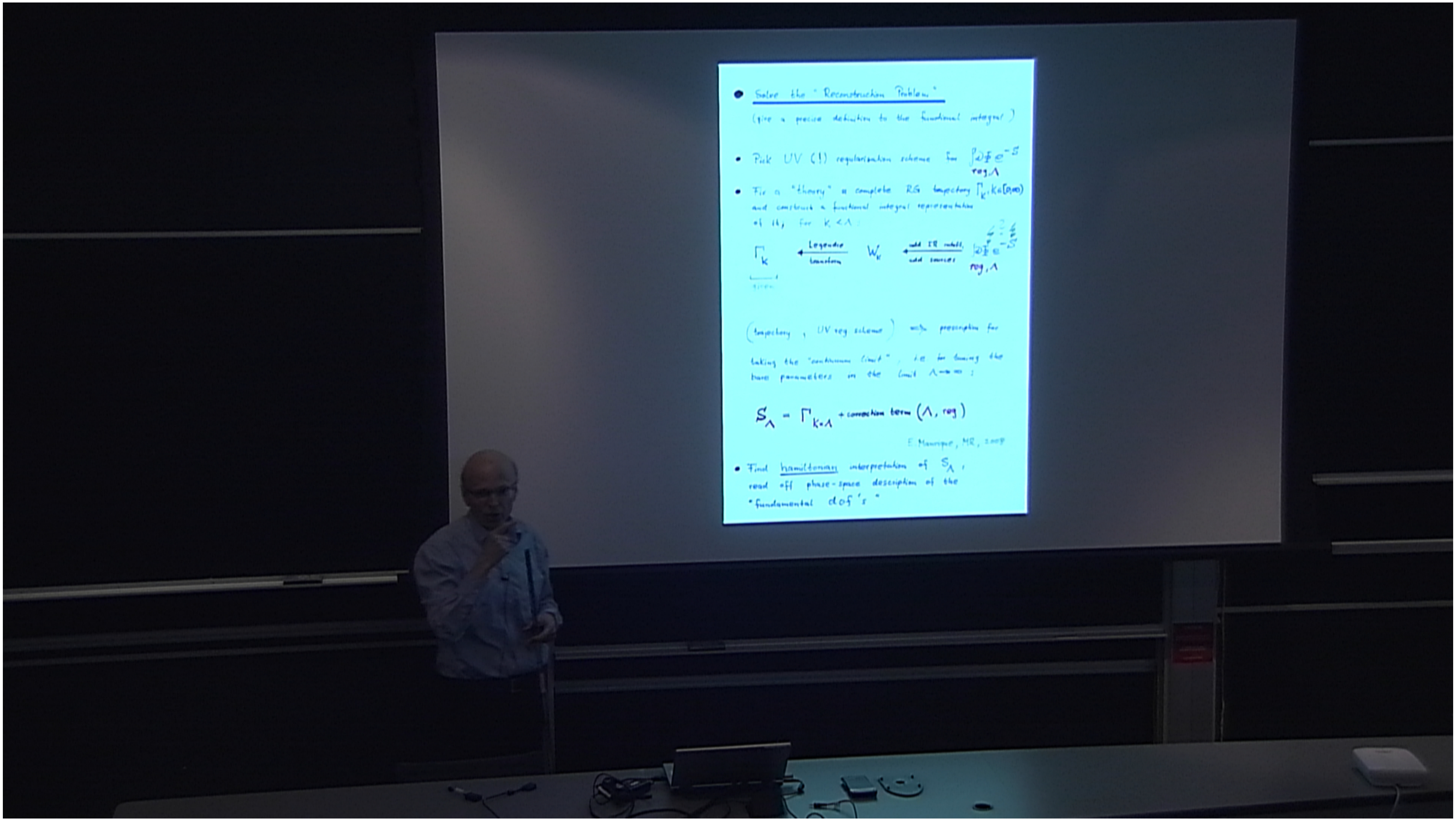

Solve the "Reconstruction Problem"
(give a precise definition to the functional integral)
Pick UV (!) regularization scheme for $\int_{reg,\Lambda} \mathcal{D}\Phi\, e^{-S}$
Fix a "theory" = complete RG trajectory Γ_k, $k \in [0,\infty)$
and construct a functional integral representation
of it, for $k < \Lambda$:
Γ_k
Legendre transform
W_k
add IR cutoff, add sources
(trajectory , UV reg scheme)
taking the "continuum limit"
bare parameters in the
$S_\Lambda = \Gamma_{k=\Lambda}$
term (Λ, reg)
E. Manrique, MR, 2008
Find interpretation of S_Λ,
phase-space description of the
dof's"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(trajectory, UV reg scheme) ⟹ prescription for
taking the "continuum limit", i.e. for tuning the
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$S_\Lambda = \Gamma_{k=\Lambda}$ + correction term (Λ, reg)
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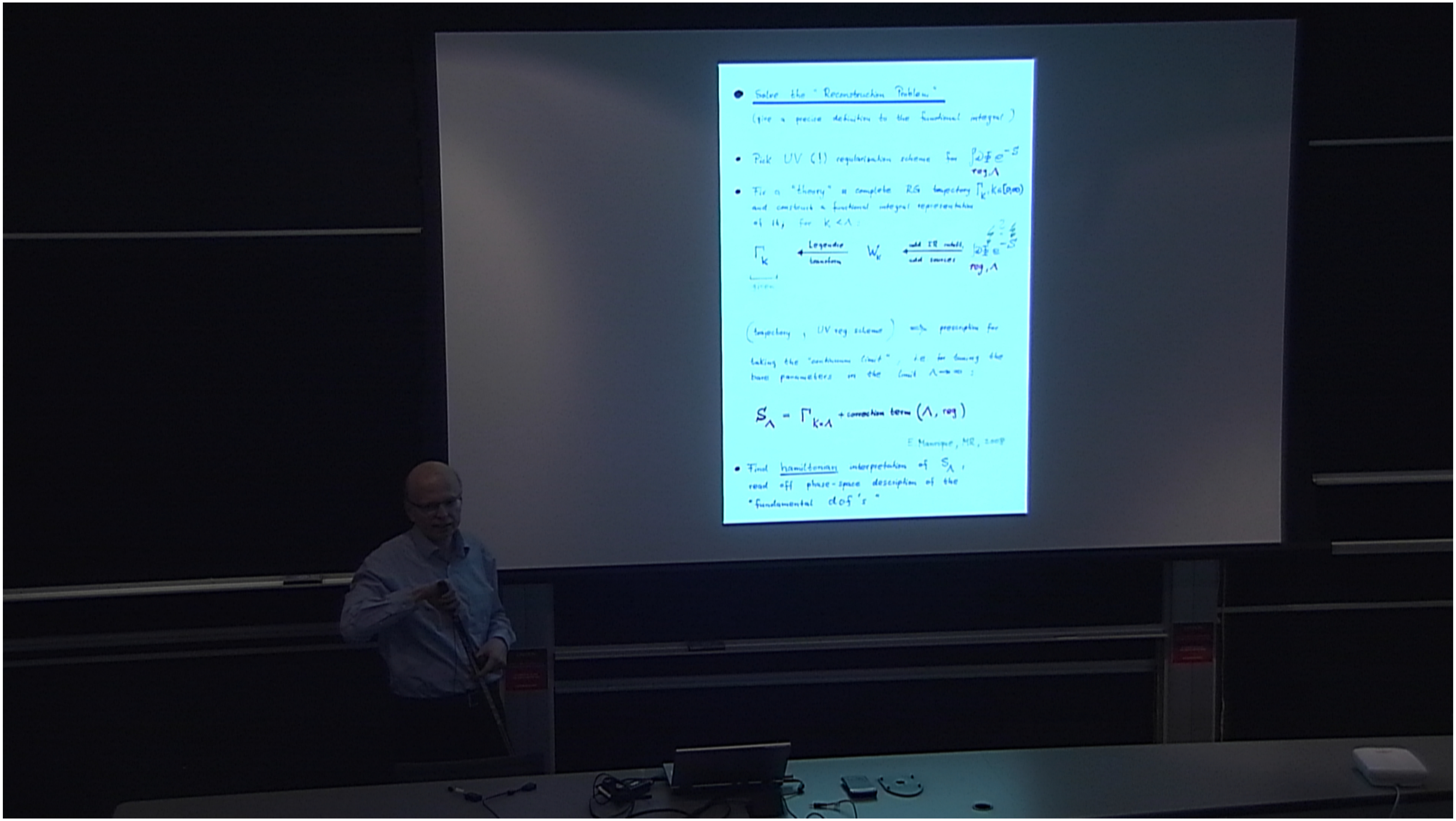
• Find hamiltonian interpretation of S_Λ,
read off phase-space description of the
"fundamental dof's"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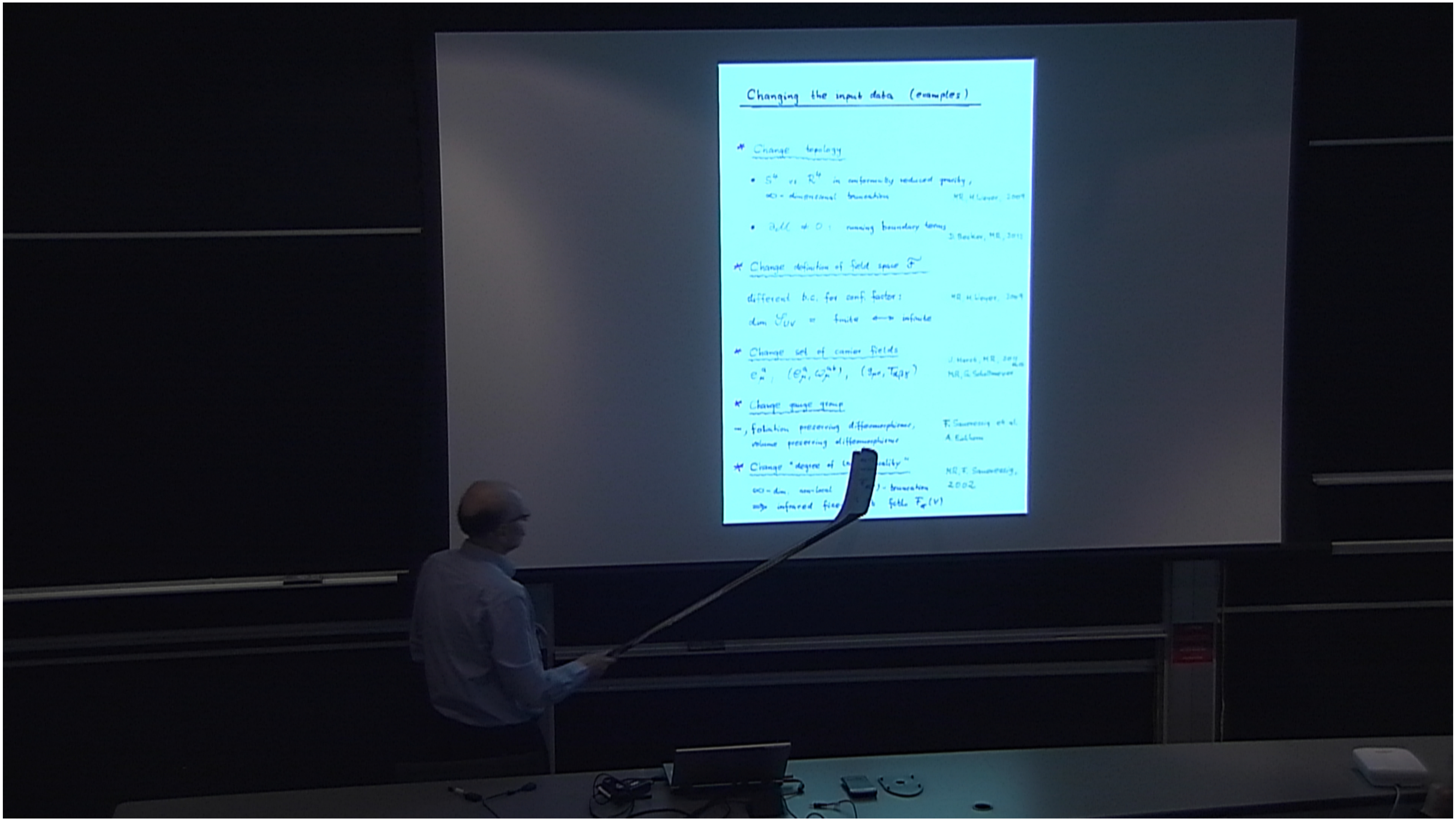
Changing the input data (examples)
Change topology
different b.c. for conf. factor:
dim $\mathcal{S}_{UV}$ = finite ⟷ infinite
Change set of carrier fields
Change gauge group
Change "degree of
ality"
infrared
2002

Background Independence

The hallmark of classical GR and Quantum Gravity !

- Treat all possible spacetimes on an equal footing
- no "vacuum" singled out a priori
- derive rather than put in "by hand" the arena of all non-gravitational physics (Minkowski space, ...)

Even more fundamental and momentous than the renormalizability issue !

Truncating theory space (metric gravity)

$\mathcal{J}_{QEG}$: $\Gamma_k[h, C, \bar{C}; \bar{g}] \equiv \Gamma_k[g, \bar{g}, C, \bar{C}] \Big|_{g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}}$

"extra $\bar{g}$ - dependence"

"single metric truncations": MR, 1996

$$\Gamma_k = \underbrace{\Gamma_k[g]}_{1996 \,\cdots\, 2009} + \int (\mathcal{F}(\bar{g}) h)^2 + \int \bar{C} M(\bar{g}) C$$

Manrique, MR, 2009

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p = "level"

single metric approx.: retain only level $p = 0$, $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ not disentangled

- **6 fixed points on the total theory space:**

$G^{B} \oplus G^{Dyn}$-FP , $G^{B} \oplus NG_{-}^{Dyn}$-FP , $G^{B} \oplus NG_{+}^{Dyn}$-FP

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candidate for the A.S. construction:

$$\left(g_*^{Dyn} = 0.703 ,\ \lambda_*^{Dyn} = 0.207 ;\ g_*^{B} = 8.2 ,\ \lambda_*^{B} = -0.01 \right)$$

$$\Theta = (3.6 \pm 3.0\, i ,\ 2 ,\ 4) \rightsquigarrow \dim \mathcal{S}_{UV} = 4$$

- **Background Independence and Asymptotic Safety can be achieved simultaneously:**

There exists a 2-parameter subset of RG trajectories on the 4-dimensional theory space which are asymptotically safe w.r.t. the $NG^{B} \oplus NG_{+}^{Dyn}$-FP, and which restore split-symmetry in the physical limit $(k \to 0)$.

Predictivity higher than expected:

$$\dim \mathcal{S}_{UV} - 2 = 4 - 2 = \underline{2} \text{ free param.s}$$

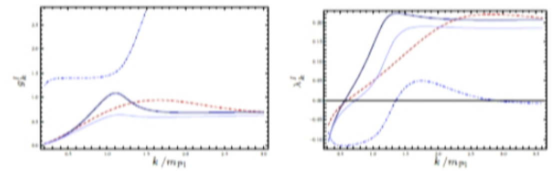
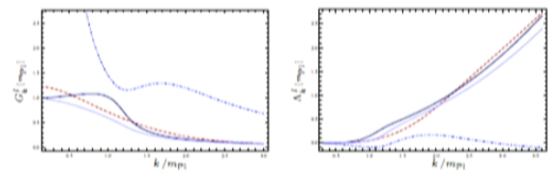


Figure 6. Type $(\text{Ia})^{\text{Dyn}}$-$\mathbf{(Attr)^B}$trajectory: dimensionless couplings.



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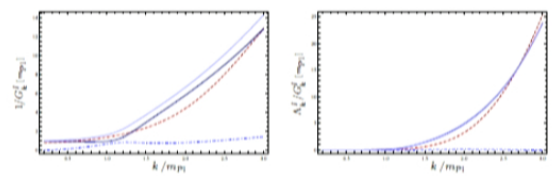

Figure 8. Type $(\text{Ia})^{\text{Dyn}}$-$\mathbf{(Attr)^B}$trajectory: the coefficients as they appear in the EAA. Note the perfect split-symmetry restoration in the IR: $1/G_k^{\text{B}}$ and $\Lambda_k^{\text{B}}/G_k^{\text{B}}$ vanish for $k \to 0$, implying that Γ_k^{grav} looses its extra $\bar{g}_{\mu\nu}$ dependence.

dashed (red):	‒ ‒ ‒ ‒ ‒	$I = \text{sm}$ (single-metric)
solid (dark-blue):	———	$I = \text{Dyn} \equiv (p)$ for $p \geq 1$
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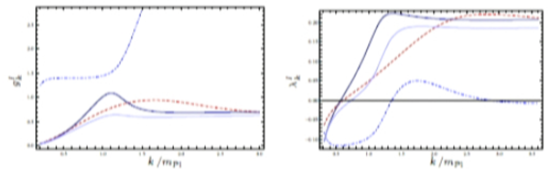

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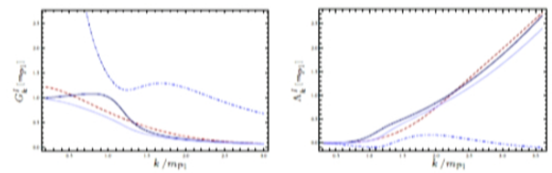

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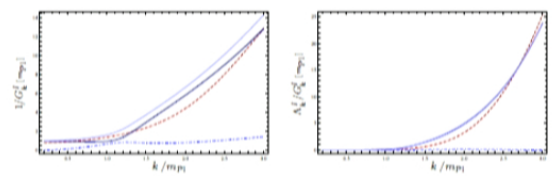

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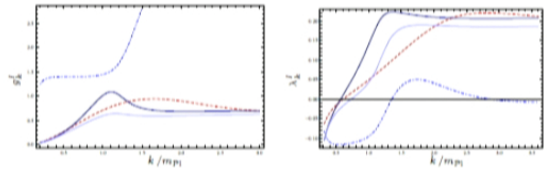

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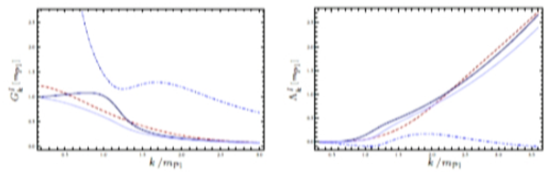

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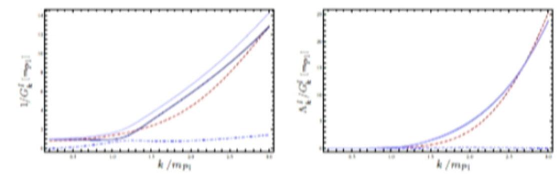

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The Main Result :

The 2-parameter family of RG trajectories restoring split symmetry in the IR consists of precisely those which approach $\text{Attr}^{B}(k \to 0)$ in the IR.

Summary :

(Potentially) acceptable QFTs can be based on RG trajectories with

initial point $(k \to \infty)$: $NG^{B} \oplus NG_{+}^{Dyn}$-FP

final point $(k \to 0)$: $\text{Attr}^{B}(k \to 0)$

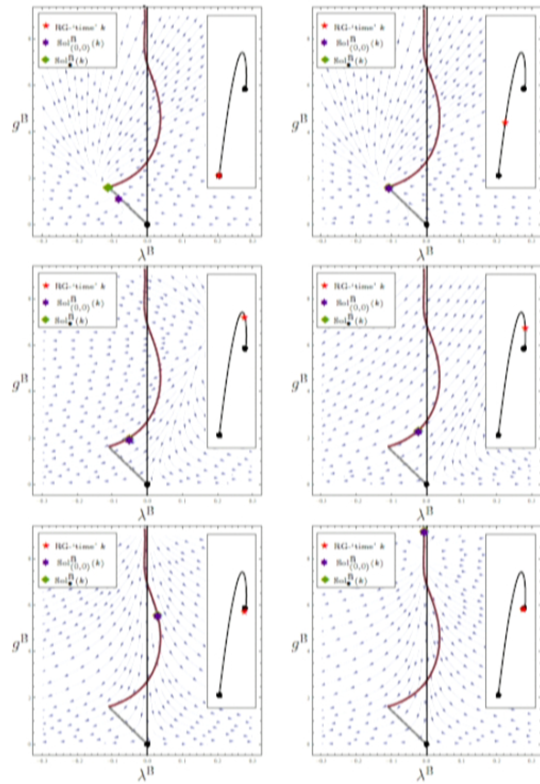

Figure 3. The B-phase portraits at increasing scales k. The underlying separatrix in the Dyn-sector is shown in the inset on the right, and the current RG time is marked with a star therein. The arrows point towards the IR and picture the instantaneous vector field in the B-sector. The (red) solid and the (gray) dashed curve highlight two important solutions in the B-sector, namely $\mathrm{Sol}^{\mathrm{B}}_{\bullet}(k)$ and $\mathrm{Sol}^{\mathrm{B}}_{(0,0)}(k)$, respectively. Their current position is indicated by the (green) diamond and the (violet) six-pointed star, respectively.

Towards an EAA-based "C-Theorem"

D. Becker, MR, 2012 ; 2014

$$e^{-\Gamma_k[\langle\phi\rangle_k]} = \int \mathcal{D}\phi\, e^{-S[\phi]} e^{-\Delta S_k[\phi]} \sim \int_{p>k} \mathcal{D}\phi\, e^{-S}$$

Γ_k evaluated "on-shell" counts field modes integrated out above the scale k.

gravity: $\Gamma_k[h=0;\, \bar{g}_k^{\text{self consistent}}] \equiv C_k$

$$\left(\frac{\delta}{\delta h_{\mu\nu}} \Gamma_k\right)\Big|_{h=0,\, \bar{g}=\bar{g}_k^{\text{self cons.}}} = 0 \quad \left(\begin{matrix}\text{tadpole}\\ \text{equation}\end{matrix}\right)$$

bi-metric EH truncation:

$$\Gamma_k[h=0;\bar{g}] = -\frac{1}{16\pi\, G_k^{(0)}} \int dx\sqrt{\bar{g}}\, \{R(\bar{g}) - 2\Lambda_k^{(0)}\}$$

$$G_{\mu\nu}(\bar{g}_k^{\text{self con.}}) = -\Lambda_k^{(1)} [\bar{g}_k^{\text{self con}}]_{\mu\nu}$$

$\rightsquigarrow$ S^4 with radius $\sim [\Lambda_k^{(1)}]^{-1/2}$

Conclusion

Split-symmetry ≡ Background Independence

- is crucial to select the correct universality class
- is restrictive, but not too restrictive
- can indeed coexist with Asymptotic Safety, at least in the truncation considered

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"extra $\bar{g}$-dependence" (pointing to $\bar{g}$)

"single metric truncations":

MR, 1996

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