

Title: Gravitational RG flows on foliated spacetimes

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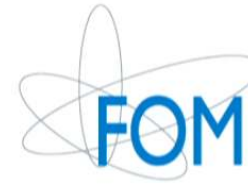
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Abstract: The role of time and a possible foliation structure of spacetime are longstanding questions which lately received a lot of renewed attention from the quantum gravity community. In this talk, I will review recent progress in formulating a Wetterich-type functional renormalization group equation on foliated spacetimes and outline its potential applications. In particular, I will discuss first results concerning the RG flow of  
Horava-Lifshitz gravity, highlighting a possible mechanism for a dynamical Lorentz-symmetry restoration at low energies.

# Gravitational RG flows from foliated spacetimes

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E. Manrique, S. Rechenberger, F.S., PRL 106 (2011) 251302

A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017

G. D'Odorico, F.S., in preparation

Renormalization group approaches to quantum gravity

Perimeter Institute, April 24, 2014

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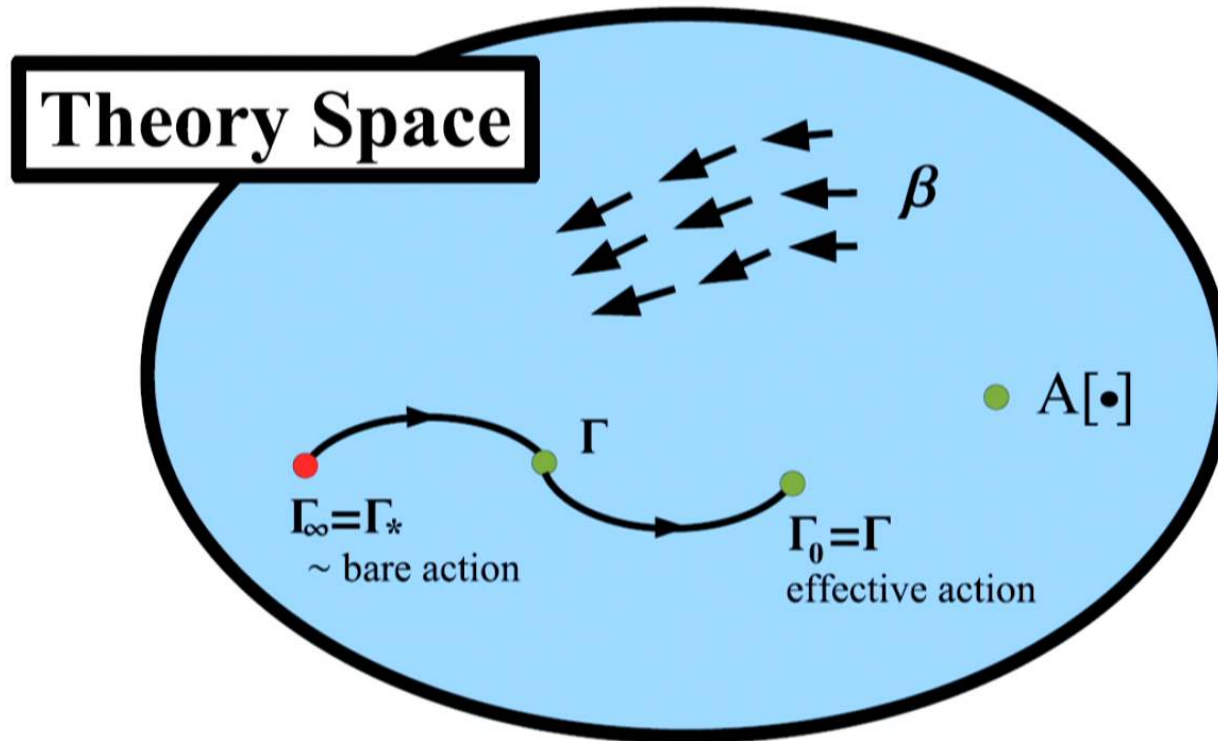
## Outline

- Hořava-Lifshitz gravity from a Wilsonian viewpoint
- Wetterich equation for projective Hořava-Lifshitz gravity
- Constructing RG flows:
  - finite-temperature type computations
  - anisotropic heat-kernels
- Conclusions

Quantum Gravity  
from a Wilsonian perspective

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## Theory space underlying the Functional Renormalization Group



## Fixed points of the RG flow

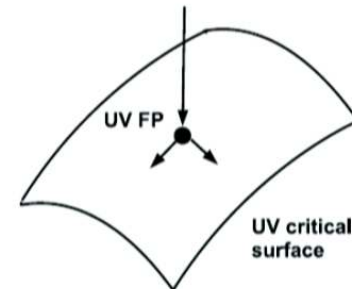
Fixed points  $\{g_i^*\}$ : Central ingredient in Wilsons picture of renormalization

- $\beta$ -functions vanish:  $(\beta_{g_i}(g_i)|_{g_i=g_i^*} = 0)$ 
  - RG-trajectories may “end” at a UV-fixed point
- dimensionless couplings remain finite
  - absence of unphysical UV divergences

Perturbations of fixed point theory controlled by stability matrix

$$B_{ij} \equiv \partial_{g_j} \beta_{g_i} |_{g_i=g_i^*}$$

- 2 classes of scaling directions:
  - relevant = attracted to FP in UV
  - irrelevant = repelled from FP in UV
- predictivity:
  - finite number of relevant directions



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## Proposals for UV fixed points (incomplete...)

- **isotropic Gaussian Fixed Point (GFP)**
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in  $G_N$
- **isotropic Gaussian Fixed Point (GFP)**
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling
- **non-Gaussian Fixed Point (NGFP)**
  - fundamental theory: interacting
  - **Lorentz-invariant**, non-perturbatively renormalizable
- **anisotropic Gaussian Fixed Point (aGFP)**
  - fundamental theory: Hořava-Lifshitz gravity
  - **Lorentz-violating**, perturbatively renormalizable

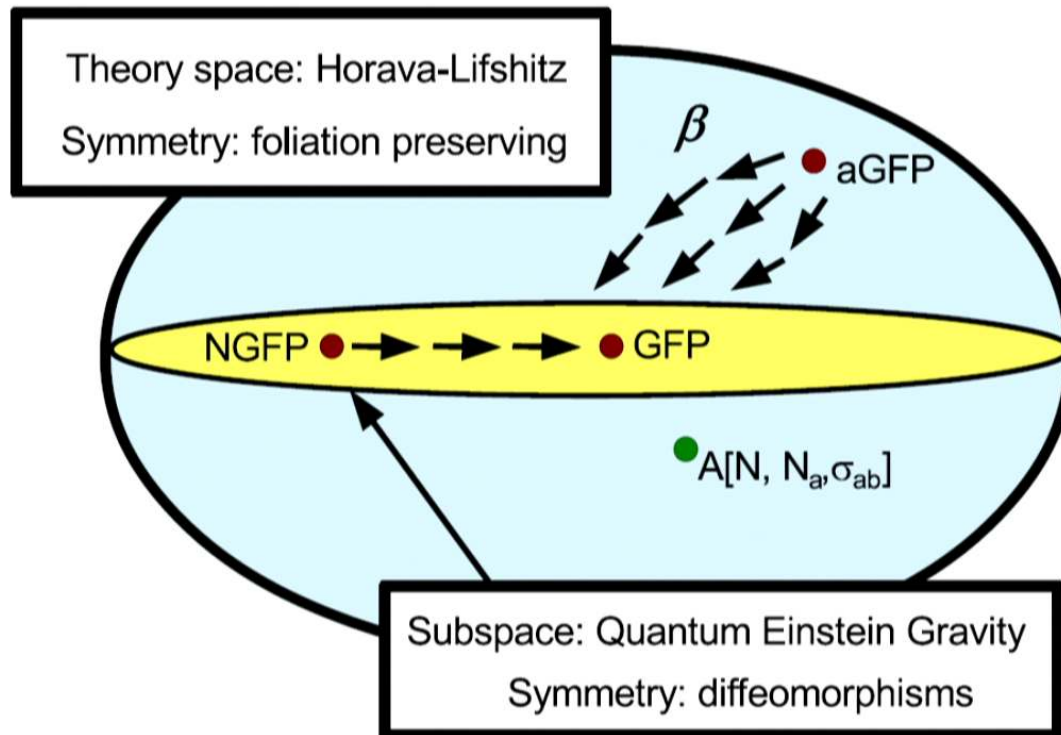


Gravity

Gravity

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## Embedding of QEG in Hořava-Lifshitz gravity



Wetterich equation  
for projective Hořava-Lifshitz gravity

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## projective Hořava-Lifshitz gravity in a nutshell

P. Hořava, Phys. Rev. D79 (2009) 084008, arXiv:0901.3775

central idea: find a perturbatively renormalizable quantum theory of gravity

fundamental fields:  $\{N(\tau), N_i(\tau, x), \sigma_{ij}(\tau, x)\}$

symmetry:  $\text{Diff}(\mathcal{M}, \Sigma) \subset \text{Diff}(\mathcal{M})$

- breaks Lorentz-invariance at high energies

Can construct the effective average action for projective HL-gravity

S. Rechenberger and F.S., JHEP 03 (2013) 010, arXiv:1212.5114

- scale-dependence governed by functional renormalization group equation

$$k\partial_k \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2} \text{STr} \left[ \left( \Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right]$$

- Complication: anisotropic models have two correlation lengths

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RG flows for projective HL gravity  
finite temperature type computations

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## Foliated functional renormalization group equation

Flow equation: formally the same as in covariant construction

$$k\partial_k\Gamma_k[h, h_i, h_{ij}; \bar{\sigma}_{ij}] = \frac{1}{2}\text{STr} \left[ \left( \Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k\mathcal{R}_k \right]$$

- covariant:  $\mathcal{M}^4$

$$\text{STr} \approx \sum_{\text{fields}} \int d^4p$$

- foliated:  $S^1 \times \mathcal{M}^3$

$$\text{STr} \approx \sqrt{\epsilon} \sum_{\text{component fields}} \sum_{\text{KK-modes}} \int d^3p$$

- structure resembles: quantum field theory at finite temperature!

Advantages of the foliated flow equation:

- captures RG-flow on theory space of Hořava-Lifshitz gravity
- same structure as CDT
- $\epsilon$ -dependence: keep track of signature effects

## ADM-decomposed Einstein-Hilbert truncation

fundamental fields:  $\{N(\tau), N_i(\tau, x), \sigma_{ij}(\tau, x)\}$

ADM-decomposed Einstein-Hilbert action:

$$\Gamma_k^{\text{ADM}} = \frac{\sqrt{\epsilon}}{16\pi G_k} \int d\tau d^3x N \sqrt{\sigma} \left\{ \underbrace{\epsilon^{-1} K_{ij}}_{\text{extrinsic curvature}} [\sigma^{ik} \sigma^{jl} - \sigma^{ij} \sigma^{kl}] K_{kl} - \underbrace{R^{(3)}}_{\text{intrinsic curvature}} + 2\Lambda_k \right\}$$

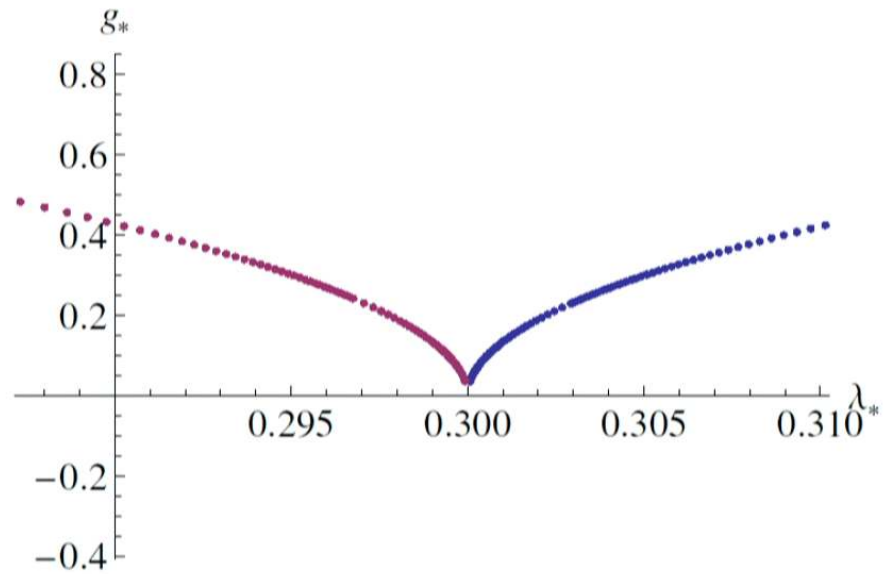
- lives on foliation  $S_T^1 \times \mathcal{M}^{(3)}$
- running couplings:  $G_k, \Lambda_k$
- signature parameter  $\epsilon$

$\beta$ -functions depend parametrically on  $m = \frac{2\pi}{Tk}$ :

$$k \partial_k g_k = \beta_g(g, \lambda; m), \quad k \partial_k \lambda_k = \beta_\lambda(g, \lambda; m)$$

- $m$ : anisotropy between cutoff in spatial/time direction

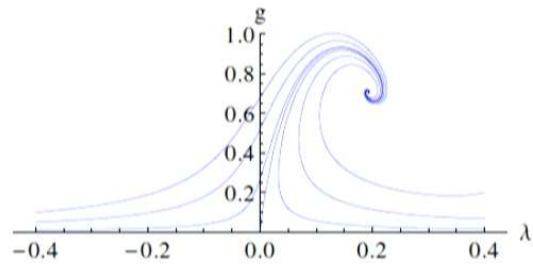
## result: signature dependence of NGFP



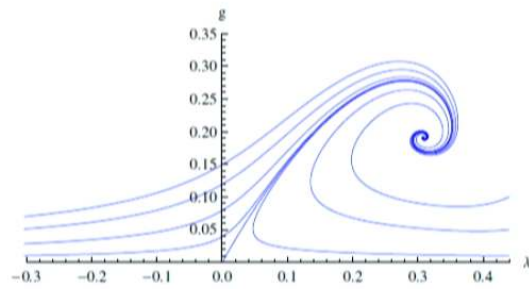
for  $m$  finite NGFPs separate:

- $\epsilon = +1$ : Euclidean signature (blue)
- $\epsilon = -1$ : Lorentzian signature (magenta)

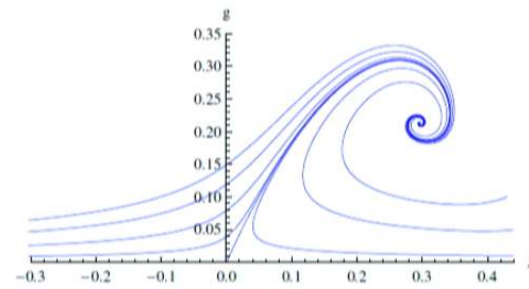
## result: phase diagrams



covariant computation



Euclidean



Lorentzian

## RG-flows of HL-gravity in the IR

A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017

RG-flow of anisotropic Einstein-Hilbert truncation

$$\Gamma_k^{\text{grav}}[N, N_i, \sigma_{ij}] = \frac{1}{16\pi G_k} \int d\tau d^3x N \sqrt{g} \left[ K_{ij} K^{ij} - \lambda_k K^2 - {}^{(3)}R + 2\Lambda_k \right]$$

Fixed points of the beta functions:

- Wheeler-de Witt metric  $\Rightarrow$  line of GFPs

$$\tilde{G}_* = 0, \quad \tilde{\Lambda}_* = 0, \quad \lambda = \lambda_*$$

- one IR attractive, one IR repulsive, one marginal direction

- NGFP:

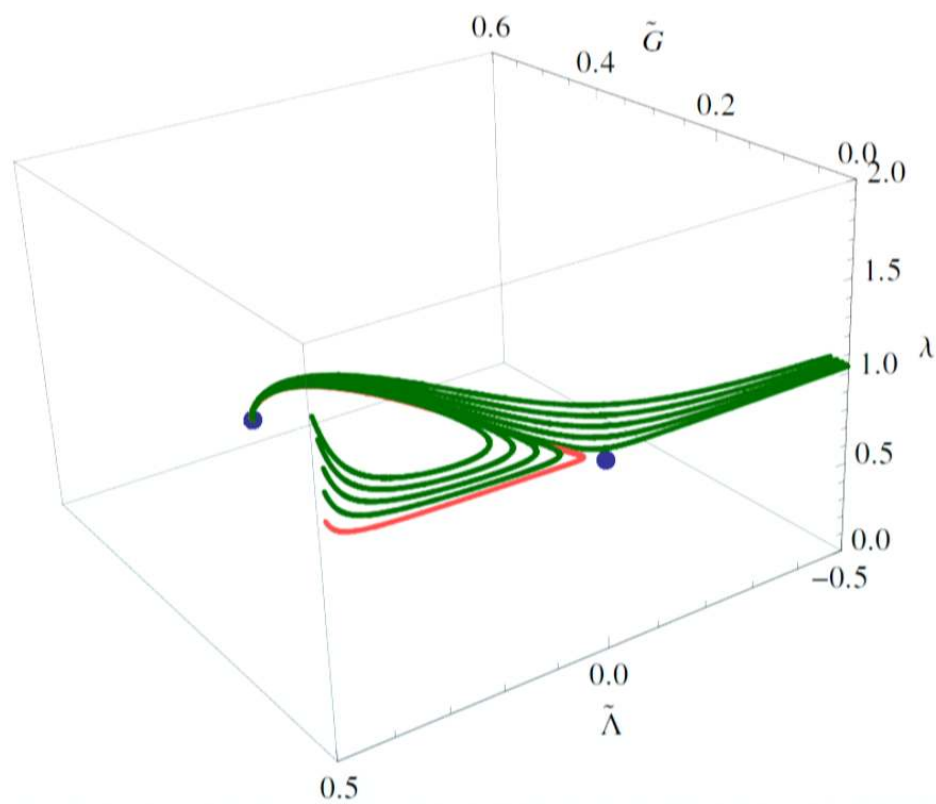
$$\tilde{G}_* = 0.49, \quad \tilde{\Lambda}_* = 0.17, \quad \lambda = 0.44$$

- three UV-attractive eigen-directions
- imprint of Asymptotic Safety

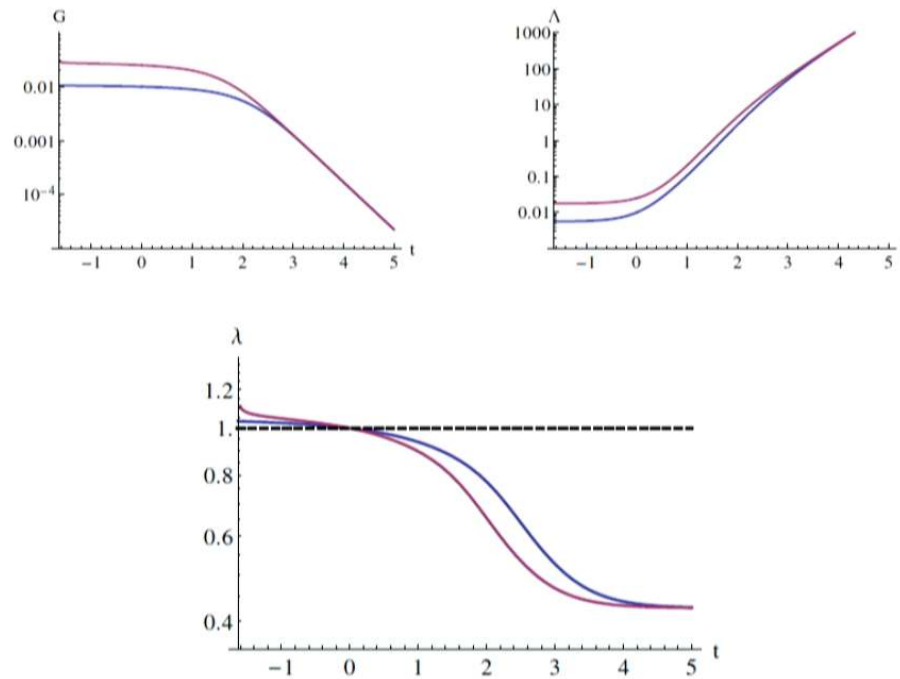
- aGFP providing UV-limit of HL-gravity not in truncation

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## Hořava-Lifshitz gravity: recovering general relativity in the IR



## Scale-dependence of dimensionful couplings



GFP governs IR-behavior of HL-gravity  
small value of cosmological constant makes  $\lambda$  compatible with experiments

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RG flows for projectable HL gravity  
anisotropic heat-kernels

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## Zooming into the aGFP in $D = 3 + 1$

Compute matter-induced gravitational  $\beta$ -functions

$$\Gamma_k = \Gamma_k^{\text{HL}} + S^{\text{LM}}$$

where

$$\Gamma_k^{\text{HL}} = \frac{1}{16\pi G_k} \int dt d^3x \sqrt{\sigma} [(K_{ij} K^{ij} - \lambda_k K^2) - g_7 R \Delta_x R - g_8 R_{ij} \Delta_x R^{ij} + \dots]$$

$$S^{\text{LM}} = \frac{1}{2} \int dt d^3x \sqrt{\sigma} [\phi (\Delta_t + (\Delta_x)^z) \phi]$$

- 8 running couplings including two wave-function renormalizations

key ingredient: anisotropic Laplace operator

$$D = \Delta_t + (\Delta_x)^z$$

$$\Delta_t = -\sqrt{\sigma}^{-1} \partial_t \sqrt{\sigma} \partial_t, \quad \Delta_x = -\sigma^{ij}(t, x) D_i D_j$$

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## Zooming into the aGFP in $d = 4$

Compute matter-induced gravitational  $\beta$ -functions

$$\Gamma_k = \Gamma_k^{\text{HL}} + S^{\text{LM}}$$

where

$$\Gamma_k^{\text{HL}} = \frac{1}{16\pi G_k} \int dt d^3x \sqrt{\sigma} [(K_{ij} K^{ij} - \lambda_k K^2) - g_7 R \Delta_x R - g_8 R_{ij} \Delta_x R^{ij} + \dots]$$

$$S^{\text{LM}} = \frac{1}{2} \int dt d^3x \sqrt{\sigma} [\phi (\Delta_t + (\Delta_x)^z) \phi]$$

- expansion:  $\sigma_{ij} = \delta_{ij} + \sqrt{16\pi G_k} h_{ij}$

Gravitational propagators (flat space):

$$[\mathcal{G}_{s=2}(\omega, \vec{p})] \propto \omega^2 - g_8 \vec{p}^6$$

$$[\mathcal{G}_{s=0}(\omega, \vec{p})] \propto \left(\frac{1}{3} - \lambda_k\right) \left(\omega^2 - \frac{1}{\frac{1}{3} - \lambda_k} \left(\frac{8}{9} g_7 + \frac{1}{3} g_8\right) \vec{p}^6\right)$$

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## Heat kernel expansion of anisotropic operators

FRGE computations use heat-kernel expansion of Laplacian  $\Delta \equiv -g^{\mu\nu} D_\mu D_\nu$

$$\begin{aligned} \text{Tre}^{-s\Delta} &\simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \sum_{n \geq 0} s^n a_{2n} \\ &\simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \left[ 1 + \frac{s}{6} R + \dots \right] \end{aligned}$$

Heat kernel expansion of anisotropic operators

$$D \equiv \Delta_t + (\Delta_x)^z$$

- apply the “Universal Renormalization Group Machine”

[D. Benedetti, K. Groh, P. F. Machado and F. Saueressig, arXiv:1012.3081]

$$\begin{aligned} \text{Tre}^{-sD} &\simeq (4\pi)^{-(d+1)/2} s^{-\frac{1}{2}(1+d/z)} \int dt d^d x \sqrt{\sigma} \\ &\left[ \frac{s}{6} \frac{\Gamma(\frac{d}{2z})}{z \Gamma(\frac{d}{2})} \left( \frac{d-z+3}{d+2} K^2 - \frac{d+2z}{d+2} K_{ij} K^{ij} \right) + \sum_{n \geq 0} s^{n/z} b_n a_{2n} \right] \end{aligned}$$

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## Heat kernel coefficients for anisotropic operators

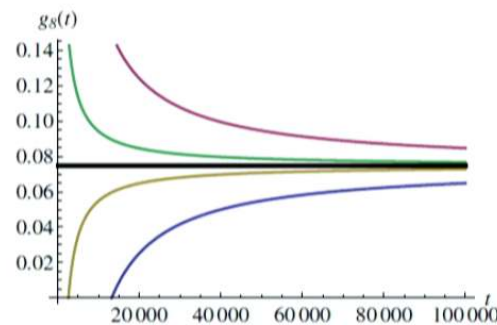
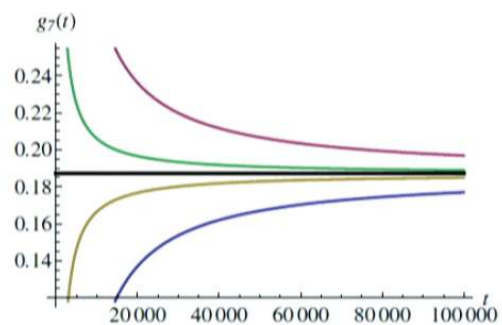
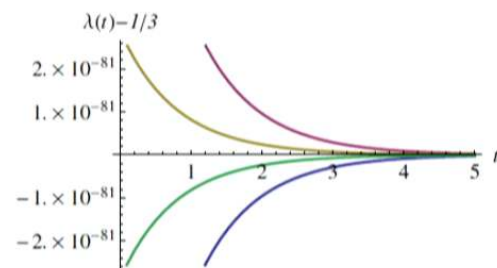
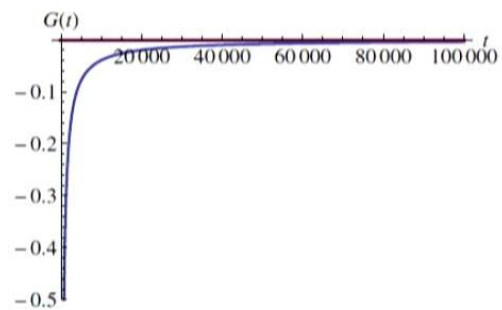
	$d = 2$			$d = 3$			
	$z = 1$	$z = 2$	$z = 3$	$z = 1$	$z = 2$	$z = 3$	$z = 4$
$b_0$	1	$\frac{\sqrt{\pi}}{2}$	$\Gamma(\frac{4}{3})$	1	$\frac{4}{3\sqrt{\pi}}\Gamma(\frac{7}{4})$	$\frac{2}{3}$	$\frac{4}{3\sqrt{\pi}}\Gamma(\frac{11}{8})$
$b_1$	1	1	1	1	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{5}{4})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{7}{6})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{9}{8})$
$b_2$	1	0	0	1	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{3}{4})$	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{5}{6})$	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{7}{8})$
$b_3$	1	-2	0	1	$-\frac{2}{\sqrt{\pi}}\Gamma(\frac{5}{4})$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{\pi}}\Gamma(\frac{5}{8})$
$b_4$	1	0	6	1	$-\frac{4}{\sqrt{\pi}}\Gamma(\frac{7}{4})$	$\frac{9}{2\sqrt{\pi}}\Gamma(\frac{7}{6})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{11}{8})$

- $z = 1$ : reproduces standard heat-kernel
- $z = 2, d = 2$ : reproduces
- $d$  even: zero coefficients in heat kernel expansion

[M. Baggio, J. de Boer and K. Holsheimer, arXiv:1112.6416]

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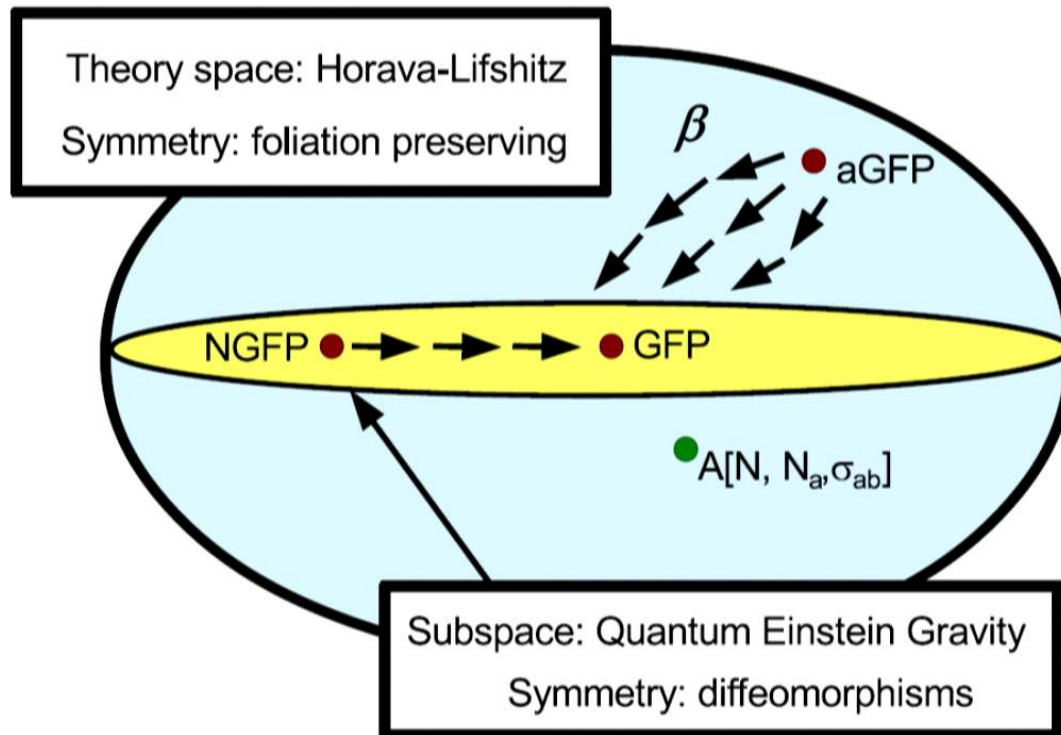
## matter-induced RG flows in $d = 4$



UV attractive anisotropic GFP

$$G^* = 0, \quad \lambda^* = 1/3, \quad g_7^* = \frac{5\pi}{84}, \quad g_8^* = \frac{\pi}{42}$$

## Embedding of QEG in Hořava-Lifshitz gravity



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# Conclusions

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## Conclusions

Wetterich equation for projectable HL gravity

- power tool for constructing RG flows in anisotropic gravity
- two correlation lengths

theory space of projective HL gravity

- contains NGFP from asymptotic safety
- GFP capable of providing IR completion

matter-induced RG flow possesses anisotropic GFP:

$$G^* = 0, \quad \lambda^* = 1/3, \quad g_7^* = \frac{5\pi}{84}, \quad g_8^* = \frac{\pi}{42}$$

- anisotropic GFP is UV attractive in  $G_k$

projective HL gravity is asymptotically free at large  $N$

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