Title: Gravitational RG flows on foliated spacetimes

Date: Apr 24, 2014 04:30 PM

URL: http://pirsa.org/14040106

Abstract: The role of time and a possible foliation structure of spacetime are longstanding questions which lately received a lot of renewed attention from the quantum gravity community. In this talk, I will review recent progress in formulating a Wetterich-type functional renormalization group equation on foliated spacetimes and outline its potential applications. In particular, I will discuss first results concerning the RG flow of
Horava-Lifshitz gravity, highlighting a possible mechanism for a dynamical Lorentz-symmetry restoration at low energies.

Gravitational RG flows from foliated spacetimes

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E. Manrique, S. Rechenberger, F.S., PRL 106 (2011) 251302

A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017

G. D'Odorico, F.S., in preparation

Renormalization group approaches to quantum gravity

Perimeter Institute, April 24, 2014

Outline

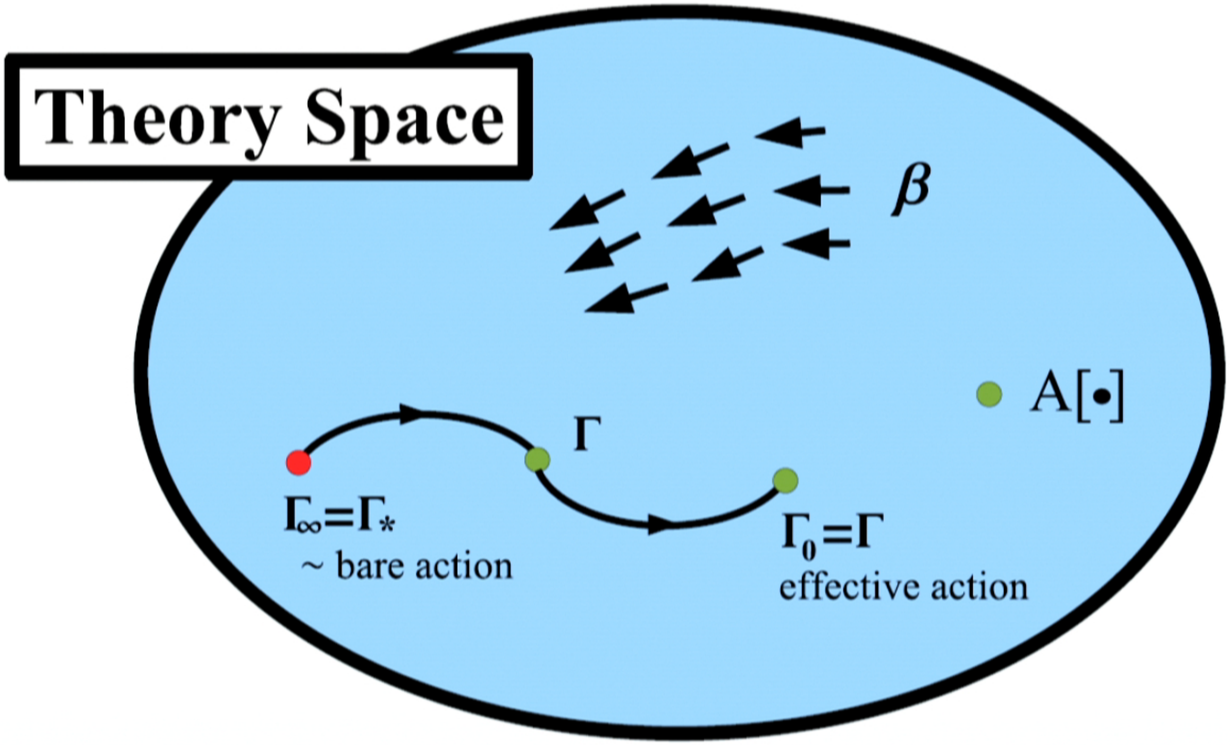
- Hořava-Lifshitz gravity from a Wilsonian viewpoint
- Wetterich equation for projective Hořava-Lifshitz gravity
- Constructing RG flows:
  - finite-temperature type computations
  - anisotropic heat-kernels
- Conclusions

Quantum Gravity

from a Wilsonian perspective

– p. 3/26

Theory space underlying the Functional Renormalization Group



Fixed points of the RG flow

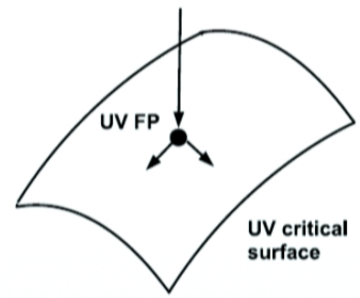
Fixed points $\{g_i^*\}$: Central ingredient in Wilsons picture of renormalization

- β-functions vanish: $\left(\beta_{g_i}(g_i)\right|_{g_i = g_i^*} = 0)$
  - RG-trajectories may “end” at a UV-fixed point
- dimensionless couplings remain finite
  - absence of unphysical UV divergences

Perturbations of fixed point theory controlled by stability matrix

$$B_{ij} \equiv \partial_{g_j} \beta_{g_i} \Big|_{g_i = g_i^*}$$

- 2 classes of scaling directions:
  - relevant = attracted to FP in UV
  - irrelevant = repelled from FP in UV
- predictivity:
  - finite number of relevant directions



Proposals for UV fixed points (incomplete. . .)

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in G_N

- isotropic Gaussian Fixed Point (GFP)
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling

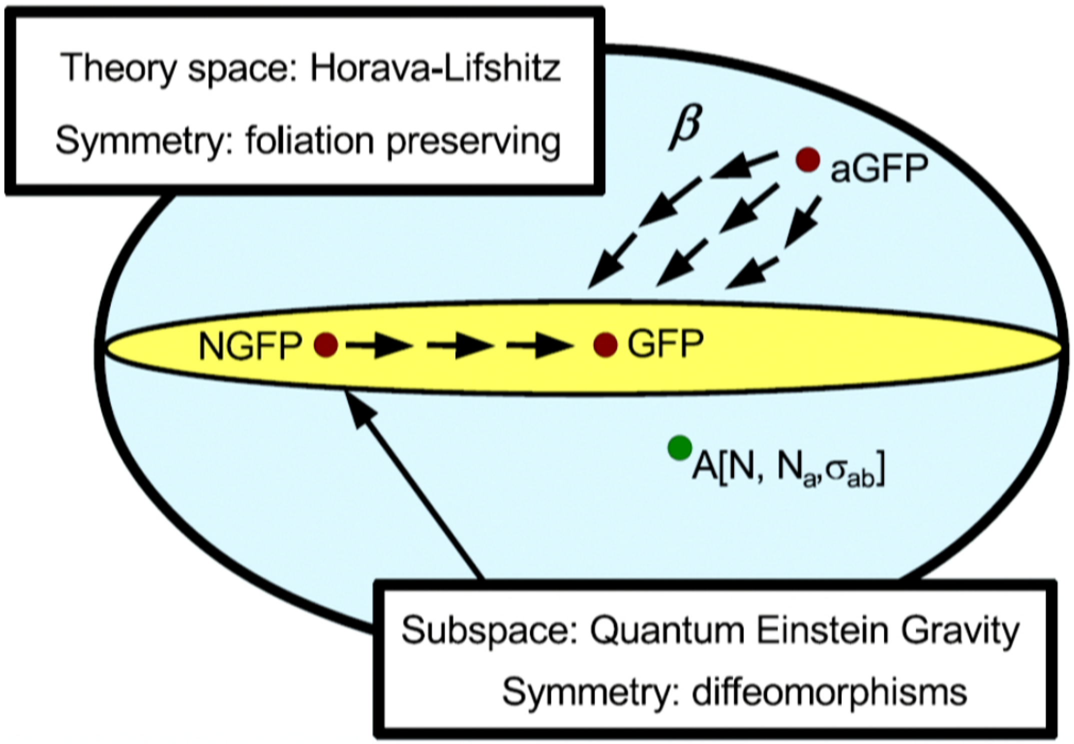
- non-Gaussian Fixed Point (NGFP)
  - fundamental theory: interacting
  - Lorentz-invariant, non-perturbatively renormalizable

Gravity

- anisotropic Gaussian Fixed Point (aGFP)
  - fundamental theory: Hořava-Lifshitz gravity
  - Lorentz-violating, perturbatively renormalizable

Gravity

Embedding of QEG in Hořava-Lifshitz gravity



Wetterich equation

for projective Hořava-Lifshitz gravity

projective Hořava-Lifshitz gravity in a nutshell

P. Hořava, Phys. Rev. D79 (2009) 084008, arXiv:0901.3775

central idea: find a perturbatively renormalizable quantum theory of gravity

fundamental fields: $\{N(\tau), N_i(\tau, x), \sigma_{ij}(\tau, x)\}$

symmetry: Diff($\mathcal{M}, \Sigma$) $\subset$ Diff($\mathcal{M}$)

- breaks Lorentz-invariance at high energies

Can construct the effective average action for projective HL-gravity

S. Rechenberger and F.S., JHEP 03 (2013) 010, arXiv:1212.5114

- scale-dependence governed by functional renormalization group equation

$$k\partial_k \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

  - Complication: anisotropic models have two correlation lengths

RG flows for projective HL gravity

finite temperature type computations

Foliated functional renormalization group equation

Flow equation: formally the same as in covariant construction

$$k\partial_k \Gamma_k[h, h_i, h_{ij}; \bar{\sigma}_{ij}] = \frac{1}{2}\mathrm{STr}\left[\left(\Gamma_k^{(2)} + \mathcal{R}_k\right)^{-1} k\partial_k \mathcal{R}_k\right]$$

- covariant: $\mathcal{M}^4$

$$\mathrm{STr} \approx \sum_{\text{fields}} \int d^4p$$

- foliated: $S^1 \times \mathcal{M}^3$

$$\mathrm{STr} \approx \sqrt{\epsilon} \sum_{\text{component fields}} \sum_{\text{KK-modes}} \int d^3p$$

  - structure resembles: quantum field theory at finite temperature!

Advantages of the foliated flow equation:

- captures RG-flow on theory space of Hořava-Lifshitz gravity
- same structure as CDT
- ϵ-dependence: keep track of signature effects

– p. 11/26

ADM-decomposed Einstein-Hilbert truncation

fundamental fields: $\{N(\tau), N_i(\tau, x), \sigma_{ij}(\tau, x)\}$

ADM-decomposed Einstein-Hilbert action:

$$\Gamma_k^{\rm ADM} = \frac{\sqrt{\epsilon}}{16\pi G_k} \int d\tau d^3x N \sqrt{\sigma} \Big\{ \epsilon^{-1} \underbrace{K_{ij}}_{\text{extrinsic curvature}} \left[\sigma^{ik}\sigma^{jl} - \sigma^{ij}\sigma^{kl}\right] K_{kl} - \underbrace{R^{(3)}}_{\text{intrinsic curvature}} + 2\Lambda_k \Big\}$$

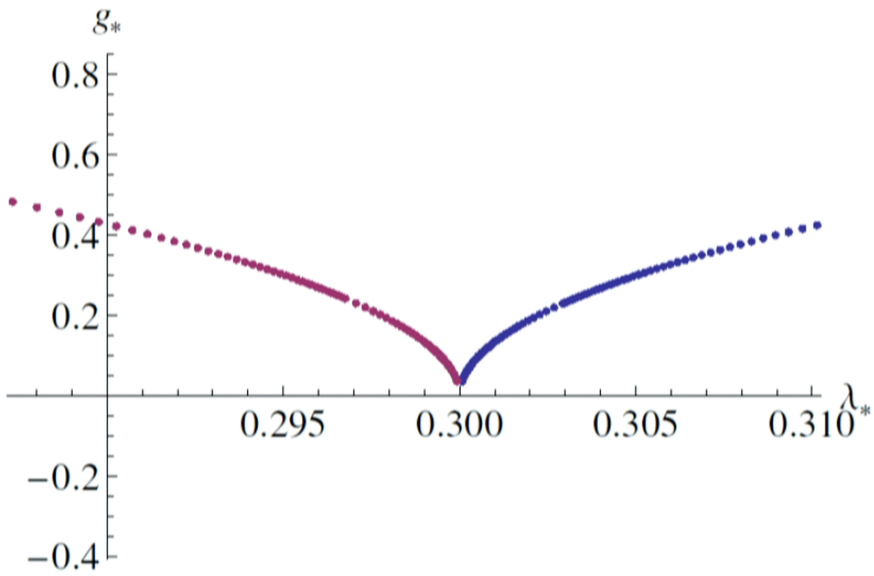
- lives on foliation $S^1_T \times \mathcal{M}^{(3)}$
- running couplings: G_k, Λ_k
- signature parameter ϵ

β-functions depend parametrically on $m = \frac{2\pi}{Tk}$:

$$k\partial_k g_k = \beta_g(g, \lambda; m), \qquad k\partial_k \lambda_k = \beta_\lambda(g, \lambda; m)$$

- m: anisotropy between cutoff in spatial/time direction

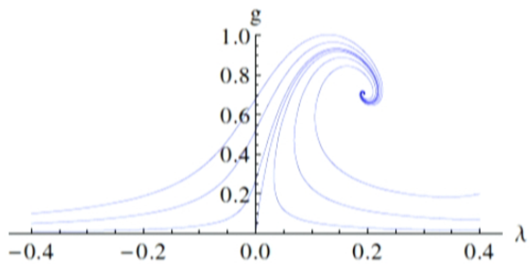
result: signature dependence of NGFP



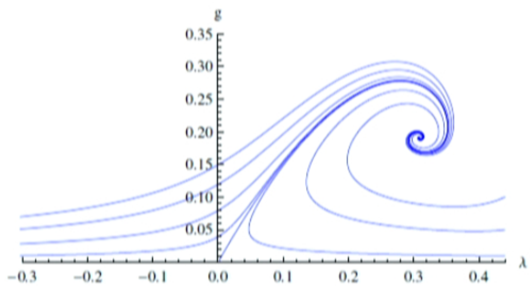
for m finite NGFPs separate:

- $\epsilon = +1$: Euclidean signature (blue)
- $\epsilon = -1$: Lorentzian signature (magenta)

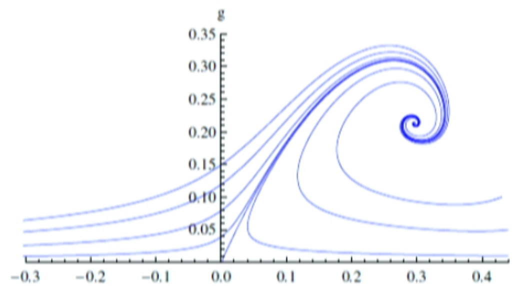
result: phase diagrams



covariant computation



Euclidean



Lorentzian

RG-flows of HL-gravity in the IR

A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017

RG-flow of anisotropic Einstein-Hilbert truncation

$$\Gamma_k^{\rm grav}[N, N_i, \sigma_{ij}] = \frac{1}{16\pi G_k} \int d\tau d^3x N\sqrt{g}\left[K_{ij}K^{ij} - \lambda_k K^2 -^{(3)} R + 2\Lambda_k\right]$$

Fixed points of the beta functions:

- Wheeler-de Witt metric ⇒ line of GFPs

$$\tilde{G}_* = 0\,, \qquad \tilde{\Lambda}_* = 0\,, \qquad \lambda = \lambda_*$$

  - one IR attractive, one IR repulsive, one marginal direction

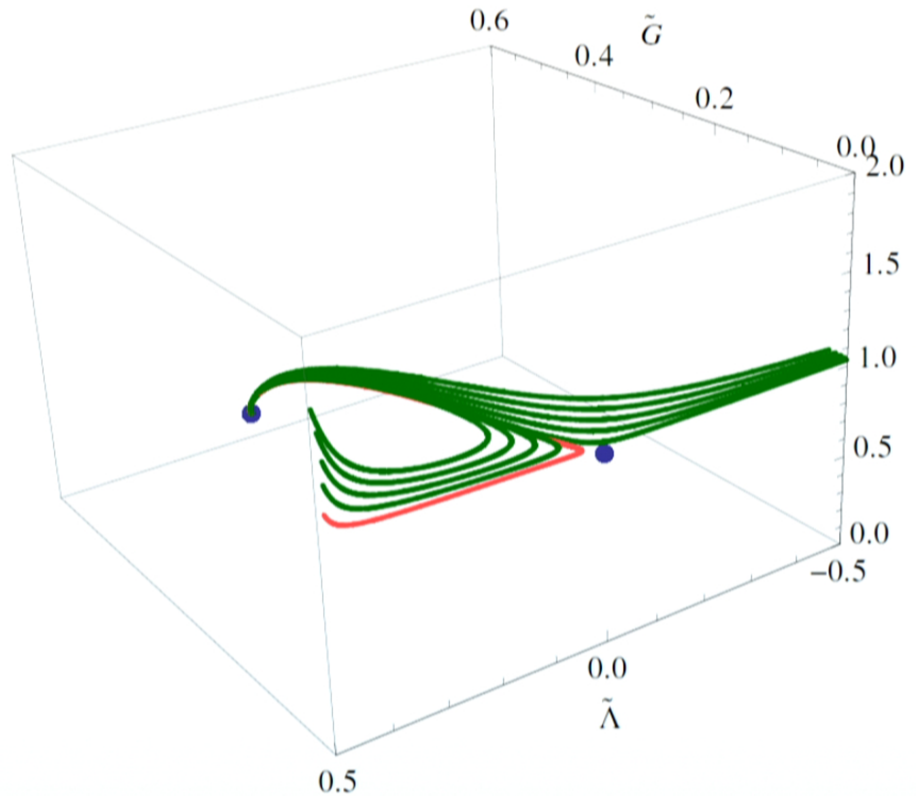
- NGFP:

$$\tilde{G}_* = 0.49\,, \qquad \tilde{\Lambda}_* = 0.17\,, \qquad \lambda = 0.44$$

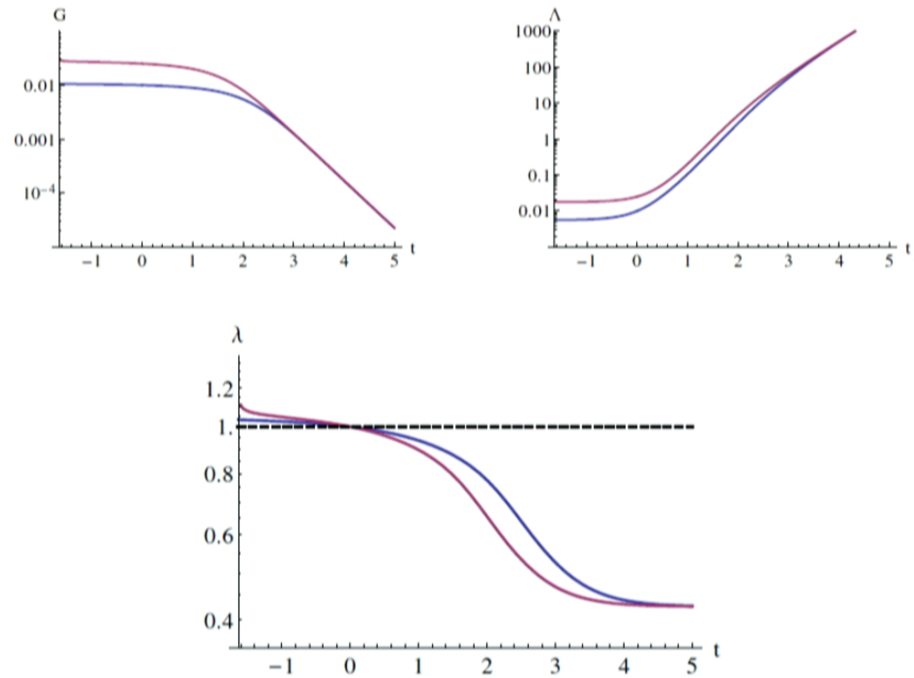
  - three UV-attractive eigen-directions
  - imprint of Asymptotic Safety

- aGFP providing UV-limit of HL-gravity not in truncation

Hořava-Lifshitz gravity: recovering general relativity in the IR



Scale-dependence of dimensionful couplings



GFP governs IR-behavior of HL-gravity

small value of cosmological constant makes λ compatible with experiments

RG flows for projectable HL gravity

anisotropic heat-kernels

Zooming into the aGFP in $D = 3 + 1$

Compute matter-induced gravitational β-functions

$$\Gamma_k = \Gamma_k^{\rm HL} + S^{\rm LM}$$

where

$$\Gamma_k^{\rm HL} = \frac{1}{16\pi G_k} \int dt d^3x \sqrt{\sigma} \left[\left(K_{ij} K^{ij} - \lambda_k K^2 \right) - g_7 R \Delta_x R - g_8 R_{ij} \Delta_x R^{ij} + \ldots \right]$$

$$S^{\rm LM} = \frac{1}{2} \int dt d^3x \sqrt{\sigma} \left[\phi \left(\Delta_t + (\Delta_x)^z \right) \phi \right]$$

- 8 running couplings including two wave-function renormalizations

key ingredient: anisotropic Laplace operator

$$D = \Delta_t + (\Delta_x)^z$$

$$\Delta_t = -\sqrt{\sigma}^{-1} \partial_t \sqrt{\sigma} \partial_t \,, \qquad \Delta_x = -\sigma^{ij}(t, x) D_i D_j$$

Zooming into the aGFP in $d = 4$

Compute matter-induced gravitational β-functions

$$\Gamma_k = \Gamma_k^{\mathrm{HL}} + S^{\mathrm{LM}}$$

where

$$\Gamma_k^{\mathrm{HL}} = \frac{1}{16\pi G_k} \int dt d^3x \sqrt{\sigma} \left[\left(K_{ij} K^{ij} - \lambda_k K^2 \right) - g_7 R \Delta_x R - g_8 R_{ij} \Delta_x R^{ij} + \ldots \right]$$

$$S^{\mathrm{LM}} = \frac{1}{2} \int dt d^3x \sqrt{\sigma} \left[\phi \left(\Delta_t + (\Delta_x)^z \right) \phi \right]$$

- expansion: $\sigma_{ij} = \delta_{ij} + \sqrt{16\pi G_k}\, h_{ij}$

Gravitational propagators (flat space):

$$[\mathcal{G}_{s=2}(\omega, \vec{p})] \propto \omega^2 - g_8\, \vec{p}^{\,6}$$

$$[\mathcal{G}_{s=0}(\omega, \vec{p})] \propto \left(\tfrac{1}{3} - \lambda_k \right) \left(\omega^2 - \frac{1}{\frac{1}{3} - \lambda_k} \left(\tfrac{8}{9}\, g_7 + \tfrac{1}{3} g_8 \right) \vec{p}^{\,6} \right)$$

Heat kernel expansion of anisotropic operators

FRGE computations use heat-kernel expansion of Laplacian $\Delta \equiv -g^{\mu\nu} D_\mu D_\nu$

$$\mathrm{Tr} e^{-s\Delta} \simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \sum_{n\geq 0} s^n \, a_{2n}$$
$$\simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \, \left[1 + \tfrac{s}{6} R + \ldots\right]$$

Heat kernel expansion of anisotropic operators

$$D \equiv \Delta_t + (\Delta_x)^z$$

- apply the "Universal Renormalization Group Machine"

[D. Benedetti, K. Groh, P. F. Machado and F. Saueressig, arXiv:1012.3081]

$$\mathrm{Tr} e^{-sD} \simeq (4\pi)^{-(d+1)/2} \, s^{-\frac{1}{2}(1+d/z)} \int dt d^d x \sqrt{\sigma}$$
$$\left[\frac{s}{6} \frac{\Gamma(\frac{d}{2z})}{z\,\Gamma(\frac{d}{2})} \left(\frac{d-z+3}{d+2} K^2 - \frac{d+2z}{d+2} K_{ij} K^{ij}\right) + \sum_{n\geq 0} s^{n/z} \, b_n \, a_{2n}\right]$$

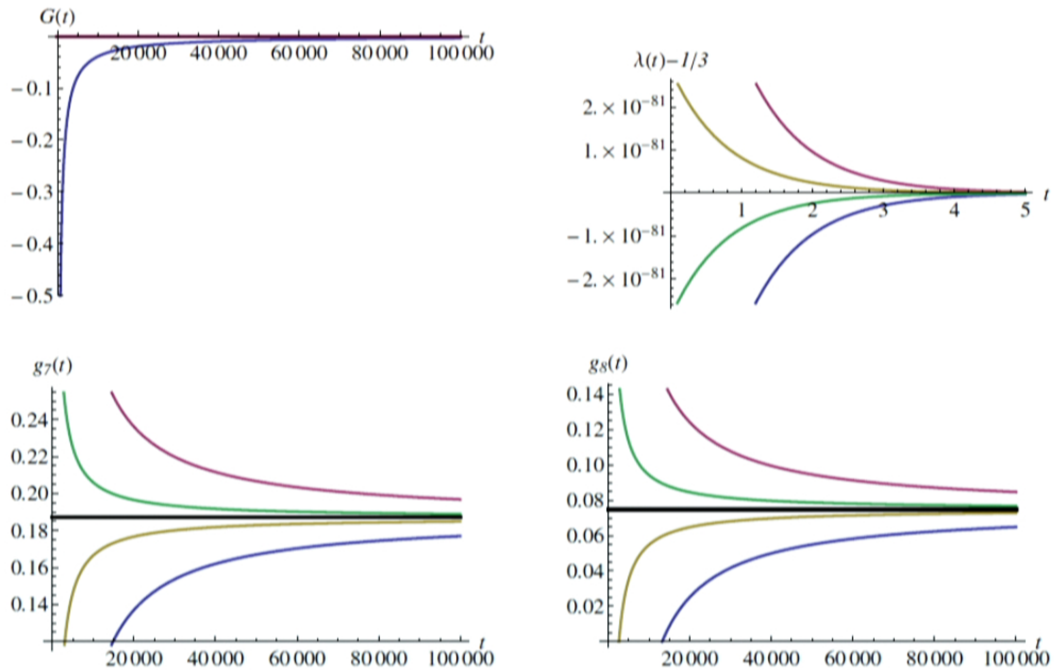
Heat kernel coefficients for anisotropic operators

	$d=2$			$d=3$			
	$z=1$	$z=2$	$z=3$	$z=1$	$z=2$	$z=3$	$z=4$
b_0	1	$\frac{\sqrt{\pi}}{2}$	$\Gamma(\frac{4}{3})$	1	$\frac{4}{3\sqrt{\pi}}\Gamma(\frac{7}{4})$	$\frac{2}{3}$	$\frac{4}{3\sqrt{\pi}}\Gamma(\frac{11}{8})$
b_1	1	1	1	1	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{5}{4})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{7}{6})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{9}{8})$
b_2	1	0	0	1	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{3}{4})$	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{5}{6})$	$\frac{1}{\sqrt{\pi}}\Gamma(\frac{7}{8})$
b_3	1	-2	0	1	$-\frac{2}{\sqrt{\pi}}\Gamma(\frac{5}{4})$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{\pi}}\Gamma(\frac{5}{8})$
b_4	1	0	6	1	$-\frac{4}{\sqrt{\pi}}\Gamma(\frac{7}{4})$	$\frac{9}{2\sqrt{\pi}}\Gamma(\frac{7}{6})$	$\frac{2}{\sqrt{\pi}}\Gamma(\frac{11}{8})$

- $z=1$: reproduces standard heat-kernel
- $z=2, d=2$: reproduces

  [M. Baggio, J. de Boer and K. Holsheimer, arXiv:1112.6416]
- d even: zero coefficients in heat kernel expansion

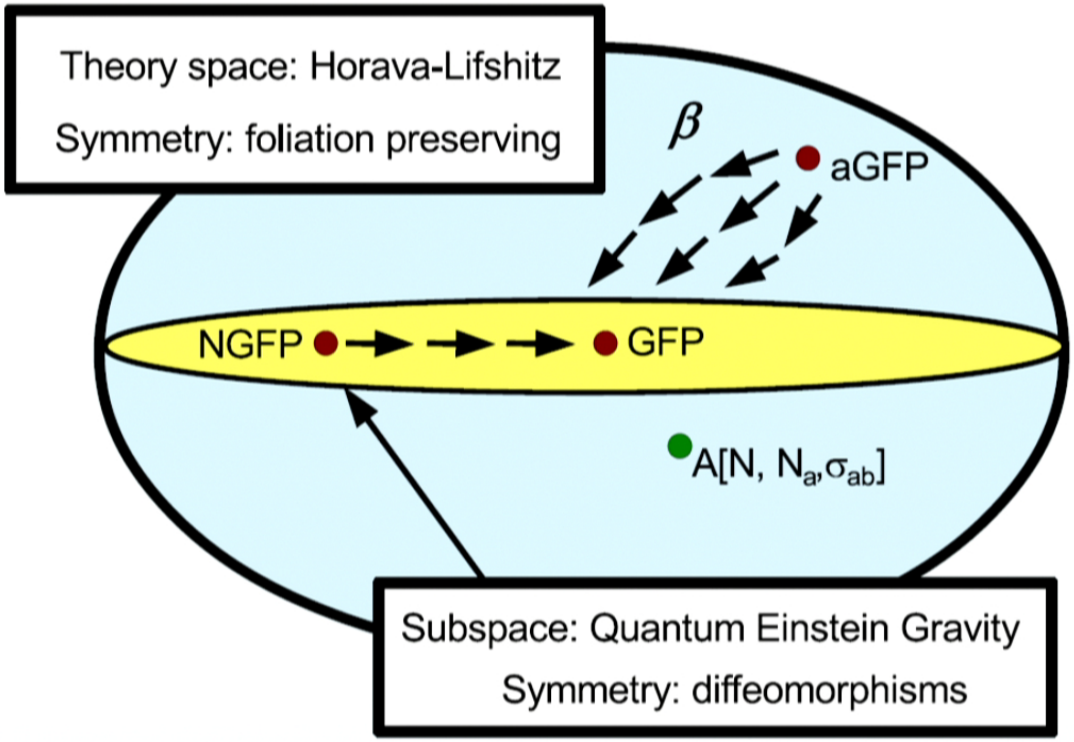
matter-induced RG flows in $d = 4$



UV attractive anisotropic GFP

$$G^* = 0, \quad \lambda^* = 1/3, \quad g_7^* = \frac{5\pi}{84}, \quad g_8^* = \frac{\pi}{42}$$

Embedding of QEG in Hořava-Lifshitz gravity



Conclusions

Conclusions

Wetterich equation for projectable HL gravity

- powertool for constructing RG flows in anisotropic gravity
- two correlation lengths

theory space of projective HL gravity

- contains NGFP from asymptotic safety
- GFP capable of providing IR completion

matter-induced RG flow possesses anisotropic GFP:

$$G^* = 0\,, \qquad \lambda^* = 1/3\,, \qquad g_7^* = \frac{5\pi}{84}\,, \qquad g_8^* = \frac{\pi}{42}$$

- anisotropic GFP is UV attractive in G_k

projective HL gravity is asymptotically free at large N