Title: One-loop renormalization in a toy model of Horava-Lifshitz gravity

Date: Apr 24, 2014 03:10 PM

URL: http://pirsa.org/14040105

Abstract: I will present some recent results on the UV properties of a toy model of Horava-Lifshitz gravity in 2+1 dimensions. In particular, I will illustrate some details of a one-loopcalculation, leading to beta functions for the running couplings. The renormalization group flow obtained in such way shows that Newton's constant is asymptotically free. However, the DeWitt
supermetric approaches its Weyl invariant form with the same speed and the effective interaction coupling of the scalar degree of freedom remains constant along the flow. I will discuss some general lesson that we can learn from these results.

One-loop renormalization in a toy model of Hořava-Lifshitz gravity

Dario Benedetti

Albert Einstein Institute, Golm, Germany

April 24, 2014

based on JHEP 1403 (2014) 078 [arXiv:1311.6253] (with Filippo Guarnieri)

Motivations

Reconciling quantum field theory with gravity (beyond EFT)

Hořava-Lifshitz gravity [Hořava '09]:
privileged role of time

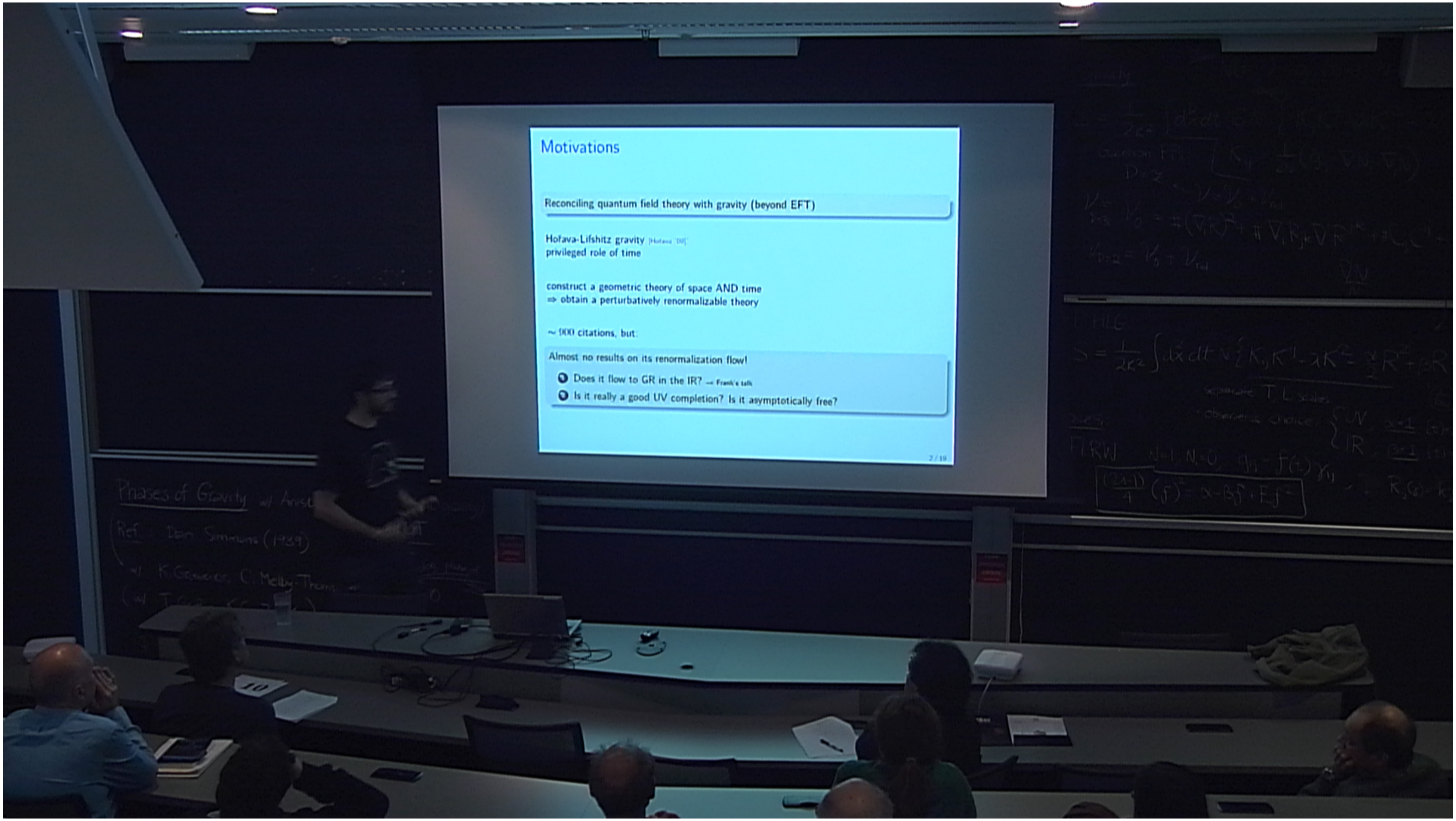
construct a geometric theory of space AND time
$\Rightarrow$ obtain a perturbatively renormalizable theory

Motivations

Reconciling quantum field theory with gravity (beyond EFT)

Hořava-Lifshitz gravity [Hořava '09]:
privileged role of time

construct a geometric theory of space AND time
$\Rightarrow$ obtain a perturbatively renormalizable theory


Motivations
Reconciling quantum field theory with gravity (beyond EFT)
Hořava-Lifshitz gravity
privileged role of time
construct a geometric theory of space AND time
⇒ obtain a perturbatively renormalizable theory
~ 900 citations, but:
Almost no results on its renormalization flow!
Does it flow to GR in the IR? ⇝ Frank's talk
Is it really a good UV completion? Is it asymptotically free?
2 / 19
Phases of Gravity

Outline

1. The toy model
2. The one-loop calculation
3. Discussion and prospects

Lifshitz scaling and renormalizability at the Lifshitz point

- Anisotropic scale invariance at the Lifshitz point

$$S = \int dt\, d^d x \left(\dot{\phi}^2 + \left((\nabla^2)^{\frac{z}{2}} \phi \right)^2 \right)$$

$$x \to \alpha\, x\,, \quad t \to \alpha^z t$$

- It is useful to introduce anisotropic scaling dimensions:

$$[x] = -1\,, \quad [t] = -z$$

$$\Rightarrow \quad [\phi] = \frac{d-z}{2}$$

- For $z = d$, $[\phi] = 0$ and interactions are renormalizable for any d
- Effectively shift critical dimension

Lifshitz scaling and renormalizability at the Lifshitz point

- Anisotropic scale invariance at the Lifshitz point

$$S = \int dt\, d^d x \left(\dot{\phi}^2 + \left((\nabla^2)^{\frac{z}{2}} \phi \right)^2 \right)$$

$$x \to \alpha\, x\,, \quad t \to \alpha^z t$$

- It is useful to introduce anisotropic scaling dimensions:

$$[x] = -1\,, \quad [t] = -z$$

$$\Rightarrow \quad [\phi] = \frac{d-z}{2}$$

- For $z = d$, $[\phi] = 0$ and interactions are renormalizable for any d
- Effectively shift critical dimension

Lifshitz scaling and renormalizability at the Lifshitz point

- Anisotropic scale invariance at the Lifshitz point

$$S = \int dt\, d^d x \left(\dot{\phi}^2 + \left((\nabla^2)^{\frac{z}{2}} \phi \right)^2 \right)$$

$$x \to \alpha\, x\,, \quad t \to \alpha^z t$$

- It is useful to introduce anisotropic scaling dimensions:

$$[x] = -1\,, \quad [t] = -z$$

$$\Rightarrow \quad [\phi] = \frac{d-z}{2}$$

- For $z = d$, $[\phi] = 0$ and interactions are renormalizable for any d
- Effectively shift critical dimension

Introducing a foliation for gravity

- Before applying this idea to gravity, we need to introduce a foliation
- $(d+1)$-dimensional manifold with topology $\mathcal{M} = \mathbb{R} \times \Sigma$, with given foliation $\mathcal{F}$
- **ADM decomposition** of (Euclidean) spacetime metric:

$$ds^2 = N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

- **Foliation-preserving diffeomorphisms** $\mathrm{Diff}_{\mathcal{F}}(\mathcal{M})$:

$$\begin{aligned} x^i &\to x^i + \zeta^i(\vec{x}, t) \\ t &\to t + \zeta(t) \end{aligned}$$

- Construct a theory of geometrodynamics invariant under $\mathrm{Diff}_{\mathcal{F}}(\mathcal{M})$

The general model

- Kinetic term

$$S_K = \frac{1}{16\pi G} \int dt\, d^d x N \sqrt{g}\, K_{ij}\, \mathcal{G}^{ijkl}\, K_{kl}$$

where

$$K_{ij} = \frac{1}{2N}(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i)$$

is the extrinsic curvature, and

$$\mathcal{G}^{ijkl} = \frac{1}{2}\left(g^{ik}g^{jl} + g^{il}g^{jk}\right) - \lambda\, g^{ij} g^{kl}$$

is the *DeWitt supermetric*

$\lambda = 1$: GR value
$\lambda = \frac{1}{d}$: (anisotropic) Weyl invariant value ($g_{ij} \to e^{2\,\Omega(x)} g_{ij}$, $N \to e^{d\,\Omega(x)} N$)

The general model

- Kinetic term

$$S_K = \frac{1}{16\pi G} \int dt\, d^d x N \sqrt{g}\, K_{ij}\, \mathcal{G}^{ijkl}\, K_{kl}$$

- Potential term:
  In the projectable version ($N = N(t)$) this takes the form of a higher derivative theory of gravity in d dimensions:

$$S_V = \frac{1}{16\pi G} \int dt\, d^d x N \sqrt{g}\, V[g_{ij}]$$

where $V[g_{ij}]$ contains spatial curvature invariants with up to $2z$ derivatives

$$\text{e.g.} \quad C_{ijkl} C^{ijkl}\,, \quad R^{ij} \nabla^2 R_{ij}\,, \quad R^3\,, \quad \ldots$$

In the non-projectable version, invariants include also the *acceleration vector*

$$a_i \equiv \frac{\nabla_i N}{N}$$

Lower dimension

- Lower dimensional theories are widely used as toy models (e.g. 3d gravity, even if it has no degrees of freedom [e.g. Carlip's book])
- Lower dimensional HL gravity:
  - [Hořava '08] (2+1)d HLG with detailed balance as quantum theory of membranes
  - [DB, Henson '09] (2+1)d HLG without detailed balance and CDT (spectral dimension)
  - [Sotiriou, Visser, Weinfurtner '11] (1+1)d and (2+1)d HLG as playgrounds for classical dynamics
  - [Anderson, Carlip, Cooperman, Hořava, Kommu, Zulkowski '11] (2+1)d HLG and extended CDT (phase diagram)
  - [Ambjorn, Glaser, Sato, Watabiki. '13] (1+1)d HLG and CDT (quantum Hamiltonian)

(2+1)-dimensional model

- Simplifications in $d = 2$:
  - no Weyl tensor, and $R_{ij} = \frac{1}{2} g_{ij} R$
  - $z = 2$ is sufficient for renormalizability

- Nontrivial: one scalar degree of freedom! (because of reduced symmetry)

(2+1)-dimensional model

- Simplifications in $d = 2$:
  - no Weyl tensor, and $R_{ij} = \frac{1}{2} g_{ij} R$
  - $z = 2$ is sufficient for renormalizability
- Nontrivial: one scalar degree of freedom! (because of reduced symmetry)
- Projectable model:

$$S = \frac{2}{\kappa^2} \int dt\, d^2x N \sqrt{g} \left\{ \lambda K^2 - K_{ij} K^{ij} - 2\Lambda + c\, R + \gamma\, R^2 \right\}$$

Note: $[t] = -2 \quad \Rightarrow \quad [\kappa] = [\lambda] = [\gamma] = 0,\ [\Lambda] = 4,\ [c] = 2$

- Non-projectable: 7 more couplings + other complications

(2+1)-dimensional model

- Simplifications in $d = 2$:
  - no Weyl tensor, and $R_{ij} = \frac{1}{2} g_{ij} R$
  - $z = 2$ is sufficient for renormalizability
- Nontrivial: one scalar degree of freedom! (because of reduced symmetry)
- Projectable model:

$$S = \frac{2}{\kappa^2} \int dt\, d^2x N \sqrt{g} \left\{ \lambda K^2 - K_{ij} K^{ij} - 2\Lambda + c\, R + \gamma\, R^2 \right\}$$

  Note: $[t] = -2 \quad \Rightarrow \quad [\kappa] = [\lambda] = [\gamma] = 0,\ [\Lambda] = 4,\ [c] = 2$

- Non-projectable: 7 more couplings + other complications

Gauge fixing

Fluctuation fields transform as

$$
\begin{aligned}
h_{ij} &\to h_{ij} + D_i\,\zeta_j + D_j\,\zeta_i + \zeta\,\dot{g}_{ij} \\
n_i &\to n_i + g_{ij}\,\dot{\zeta}^j \\
n &\to n + \dot{\zeta}
\end{aligned}
$$

- We can choose a proper-time gauge: $n = n_i = 0$ (only in projectable case!)
- Fadeev-Popov determinant: $\sqrt{\det(-\partial_t^2)}$
  $\Rightarrow$ No log divergences, only renormalization of cosmological term

Gauge fixing

Fluctuation fields transform as

$$
\begin{aligned}
h_{ij} &\to h_{ij} + D_i\,\zeta_j + D_j\,\zeta_i + \zeta\,\dot{g}_{ij} \\
n_i &\to n_i + g_{ij}\,\dot{\zeta}^j \\
n &\to n + \dot{\zeta}
\end{aligned}
$$

- We can choose a proper-time gauge: $n = n_i = 0$ (only in projectable case!)
- Fadeev-Popov determinant: $\sqrt{\det(-\partial_t^2)}$
  $\Rightarrow$ No log divergences, only renormalization of cosmological term
- Residual symmetry: $\zeta^i = \zeta^i(x)$, time-independent
- Canonical analysis: gauge-fix residual symmetry on one slice
  $\Rightarrow$ constraints preserve gauge fixing on all subsequent slices
  $\Rightarrow$ longitudinal modes of $\hat{h}_{ij}$ are not propagating

Here: no path integral over longitudinal modes

Gauge fixing

Fluctuation fields transform as

$$\begin{aligned} h_{ij} &\to h_{ij} + D_i\,\zeta_j + D_j\,\zeta_i + \zeta\,\dot{g}_{ij} \\ n_i &\to n_i + g_{ij}\,\dot{\zeta}^j \\ n &\to n + \dot{\zeta} \end{aligned}$$

- We can choose a proper-time gauge: $n = n_i = 0$ (only in projectable case!)
- Fadeev-Popov determinant: $\sqrt{\det(-\partial_t^2)}$
  $\Rightarrow$ No log divergences, only renormalization of cosmological term
- Residual symmetry: $\zeta^i = \zeta^i(x)$, time-independent
- Canonical analysis: gauge-fix residual symmetry on one slice
  $\Rightarrow$ constraints preserve gauge fixing on all subsequent slices
  $\Rightarrow$ longitudinal modes of $\hat{h}_{ij}$ are not propagating

Here: no path integral over longitudinal modes

One loop effective action

- The one-loop effective action can be written as

$$\Gamma = S_{tot} + \hbar S^{1-loop} + \mathcal{O}(\hbar^2)$$

where

$$S^{1-loop} = \frac{1}{2} \mathrm{STr} \ln(S^{(2)}_{tot})$$

and

$$S_{tot} = S + S_{gf} + S_{gh}$$

- The functional trace contains divergences $\Rightarrow$ renormalization

Effective coupling

- Expand the action

$$S[g_{ij} + \epsilon\, h_{ij}] = S[g_{ij}] + \epsilon\, \delta S[g_{ij}; h_{ij}] + \epsilon^2\, \delta^2 S[g_{ij}; h_{ij}] + \mathcal{O}(\epsilon^3)$$

- Normalizing to $1/2$ the coefficient of the kinetic term in $\delta^2 S$ we obtain

$$\epsilon^2\, \delta^2 S = \frac{1}{2} \int dt\, d^2x \sqrt{g}\, h\, \mathcal{D}\, h$$

where

$$\mathcal{D} = -\frac{1}{\sqrt{g}}\, \partial_t \sqrt{g}\, \partial_t + \frac{\gamma}{\lambda - \frac{1}{2}}\, (D^2 + R)^2$$

- Canonical normalization defines the effective coupling

$$\epsilon = \frac{\kappa}{(\lambda - \frac{1}{2})^{1/2}}$$

Heat kernel

- The first three coefficients are found to be (using in part [Baggio, de Boer, Holsheimer '11])

$$a_0 = \frac{1}{16\,\pi}\left(\frac{\lambda-\frac{1}{2}}{\gamma}\right)^{\frac{1}{2}} \;, \quad a_1 = \frac{7}{48\,\pi^{3/2}} R$$

$$a_2 = -\frac{1}{64\,\pi}\left(\frac{\lambda-\frac{1}{2}}{\gamma}\right)^{\frac{1}{2}}\left(K_{ij}\,K^{ij} - \frac{1}{2}\,K^2\right)$$

- Plugging into the trace:

$$\begin{aligned}\frac{1}{2}\,\mathrm{Tr}\ln(\mathcal{D}) = -\frac{1}{2}\int dt\,d^2x\sqrt{g}\,\Big\{&(\Lambda^4-\mu^4)\,\frac{1}{16\,\pi}\left(\frac{\lambda-\frac{1}{2}}{\gamma}\right)^{\frac{1}{2}} \\ &+ (\Lambda^2-\mu^2)\,\frac{14}{48\,\pi^{3/2}} R \\ &- \ln\left(\frac{\Lambda}{\mu}\right)\frac{1}{16\,\pi}\left(\frac{\lambda-\frac{1}{2}}{\gamma}\right)^{\frac{1}{2}}\left(K_{ij}\,K^{ij} - \frac{1}{2}\,K^2\right) + \mathcal{O}\left(\frac{1}{\Lambda^2}\right)\Big\}\end{aligned}$$

- Note: $[dtd^2x] = -4 \quad \Rightarrow$ same divergence degree as in 4d
- Note also the absence of an R^2 term