Title: On background-independent renormalization in state-sum model

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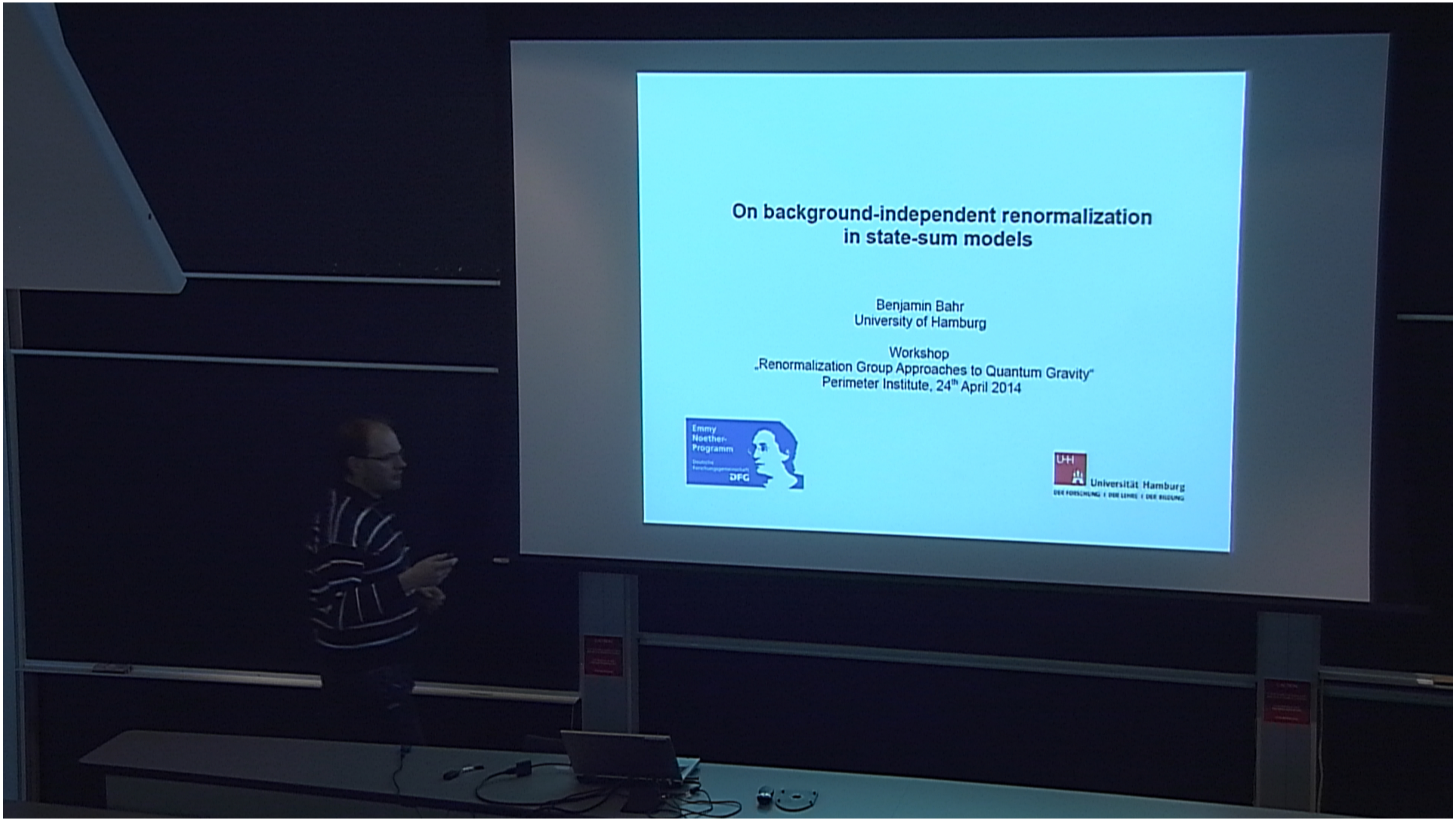
Abstract: In this talk we discuss some notion of coarse graining in state-sums, most notably a class of spin foam models in their holonomy representations. We discuss the notion of scale in this context, and how diffeomorphism-invariance ties into the existence of a continuum limit. We close with an example and muse about the interplay between diffeomorphism-invariance and non-renormalizability.

On background-independent renormalization in state-sum models

Benjamin Bahr
University of Hamburg

Workshop
„Renormalization Group Approaches to Quantum Gravity“
Perimeter Institute, 24th April 2014


On background-independent renormalization
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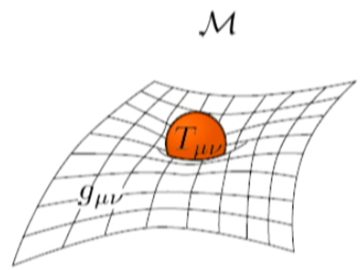
Emmy
Noether-
Programm
DFG
Universität Hamburg

- Introduction
- (Holonomy) Spin Foam Models
- Coarse Graining and continuum limit: Measure theory perspective
- Scale and RG flow in background-independent context
- Easy examples
- Summary

Introduction

Gravity $\Longleftrightarrow$ Curvature of space-time metric $g_{\mu\nu}$

$$S_{\rm EH}[g_{\mu\nu}] = \frac{1}{16\pi G_{\rm N}} \int d^4x \, \sqrt{-g} \, R$$



Quantum gravity: make sense of

$$Z \; = \; \int \mathcal{D}g_{\mu\nu} e^{iS_{\rm EH}[g_{\mu\nu}]}$$

or rather:

$$\langle \mathcal{O} \rangle \; = \; \frac{1}{Z} \int \mathcal{D}g_{\mu\nu} \; \mathcal{O}[g_{\mu\nu}] \, e^{iS_{\rm EH}[g_{\mu\nu}]}$$

Introduction

Perturbative ansatz: $g_{\mu\nu} = \eta_{\mu\nu} + \underline{h_{\mu\nu}} + \ldots$

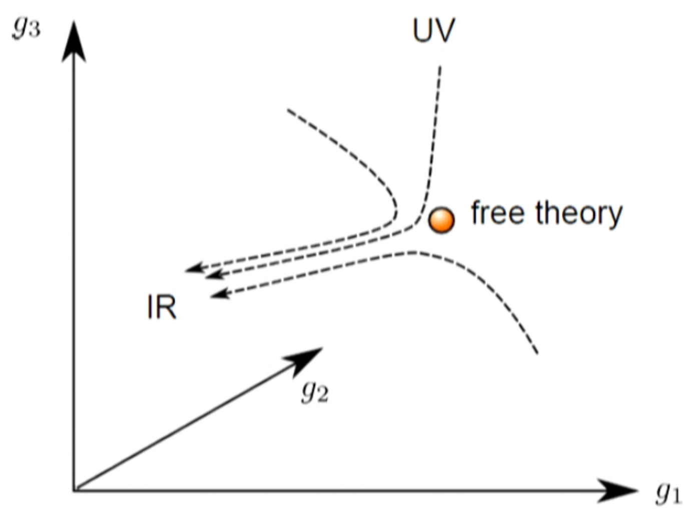
renormalization: scale-dependent couplings $g_i(k)$

scale-dependent effective action

$$S_{\rm EH} \;\rightarrow\; S^{(k)}_{\rm eff}[h_{\mu\nu}] = S_{\rm EH}[\eta_{\mu\nu} + h_{\mu\nu}] + \sum_i g_i(k)\mathcal{O}_i[h_{\mu\nu}]$$

non-renormalizable: need infinitely many coupling constants to UV complete theory

[Goroff, Sagnotti '85]



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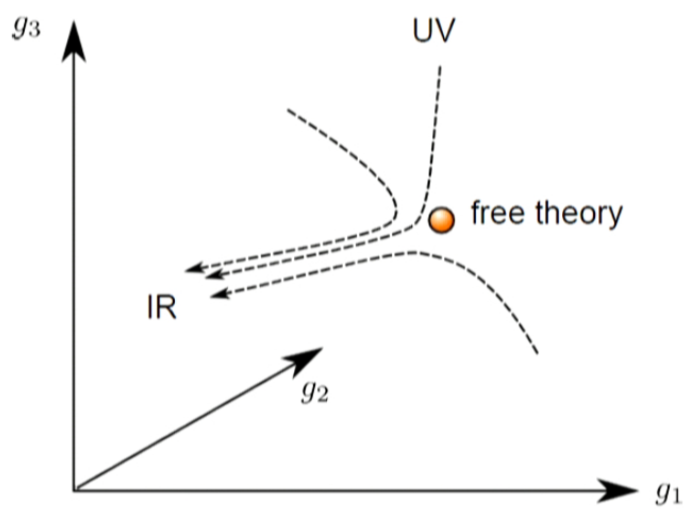
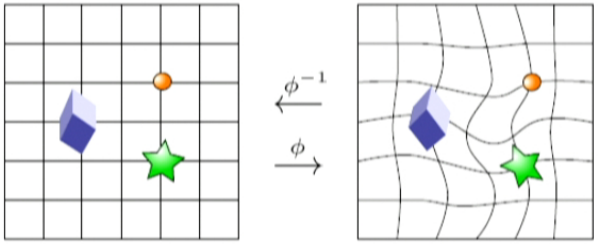


Diffeomorphism invariance

Diffeomorphism: $\phi : \mathcal{M} \to \mathcal{M}$

$$g_{\mu\nu}(x) \to \frac{\partial \phi^{\mu'}}{\partial x^{\mu}} \frac{\partial \phi^{\nu'}}{\partial x^{\nu}} g_{\mu'\nu'}(\phi(x))$$



Einsteins „hole argument“:

points in space-time have no meaning, just relations between them

diffeomorphisms = gauge symmetry of GR

$\Rightarrow$ background-independence $\Rightarrow$ no a priori scale available

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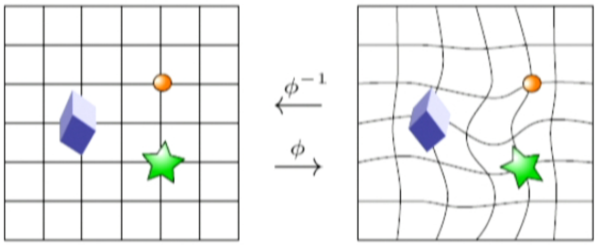

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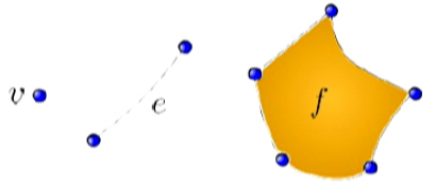
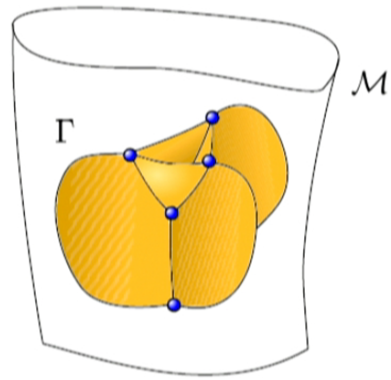
(Holonomy) spin foam models

[Pfeiffer '01, Pfeiffer, Oeckl '02, Magliaro, Perini, '10, BB, Dittrich, Hellmann, Kamiński '12]

Large class of models defined on an oriented, embedded 2-complex $\Gamma \subset \mathcal{M}$





For each Γ a configuration space $\mathcal{A}_\Gamma \simeq G^n$

and a measure („amplitude function") on it $\langle \cdot \rangle_\Gamma \, : \, C^0(\mathcal{A}_\Gamma) \to \mathbb{C}$

$$\langle \mathcal{O} \rangle_\Gamma = \int_{G^n} d\mu_\Gamma(h) \, \mathcal{O}(h)$$

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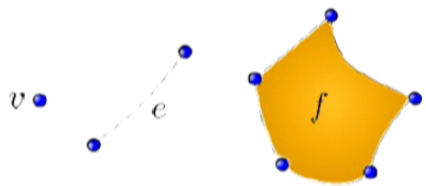

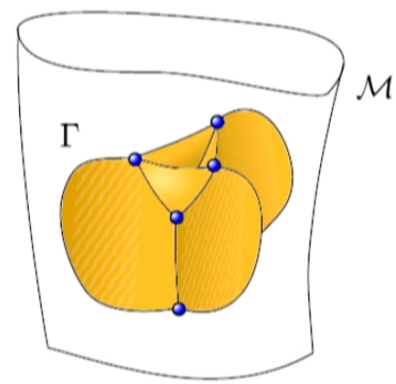

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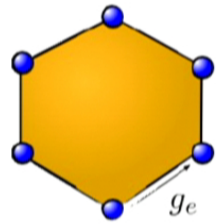
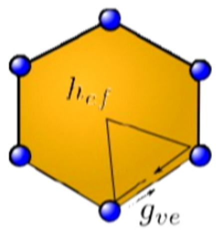
$\mathcal{A}_\Gamma$ and μ_Γ depend on your model of choice:



lattice gauge theory [Wilson '74, ...]

$$d\mu_\Gamma = \prod_f e^{-S[H_f]}\, dg_e$$



spin foam models [Reisenberger '94, Barrett, Crane '99, Livine, Speziale '07, Engle, Pereira, Rovelli, Livine '07, Freidel, Krasnov '07, Oriti Baratin '11, ...]

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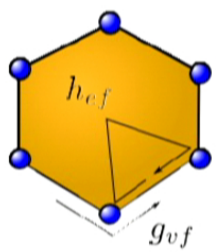

tensor field theories [DePietri, Freidel, Kransov, Rovelli '00, Oriti '06, Gurau '09, Gurau, Rivasseau '11, ...]

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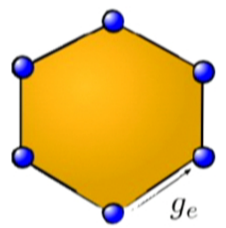

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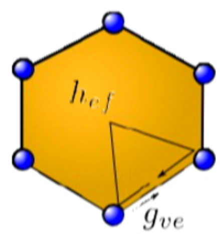

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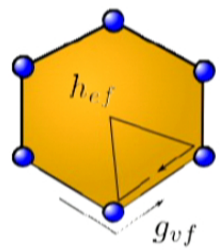

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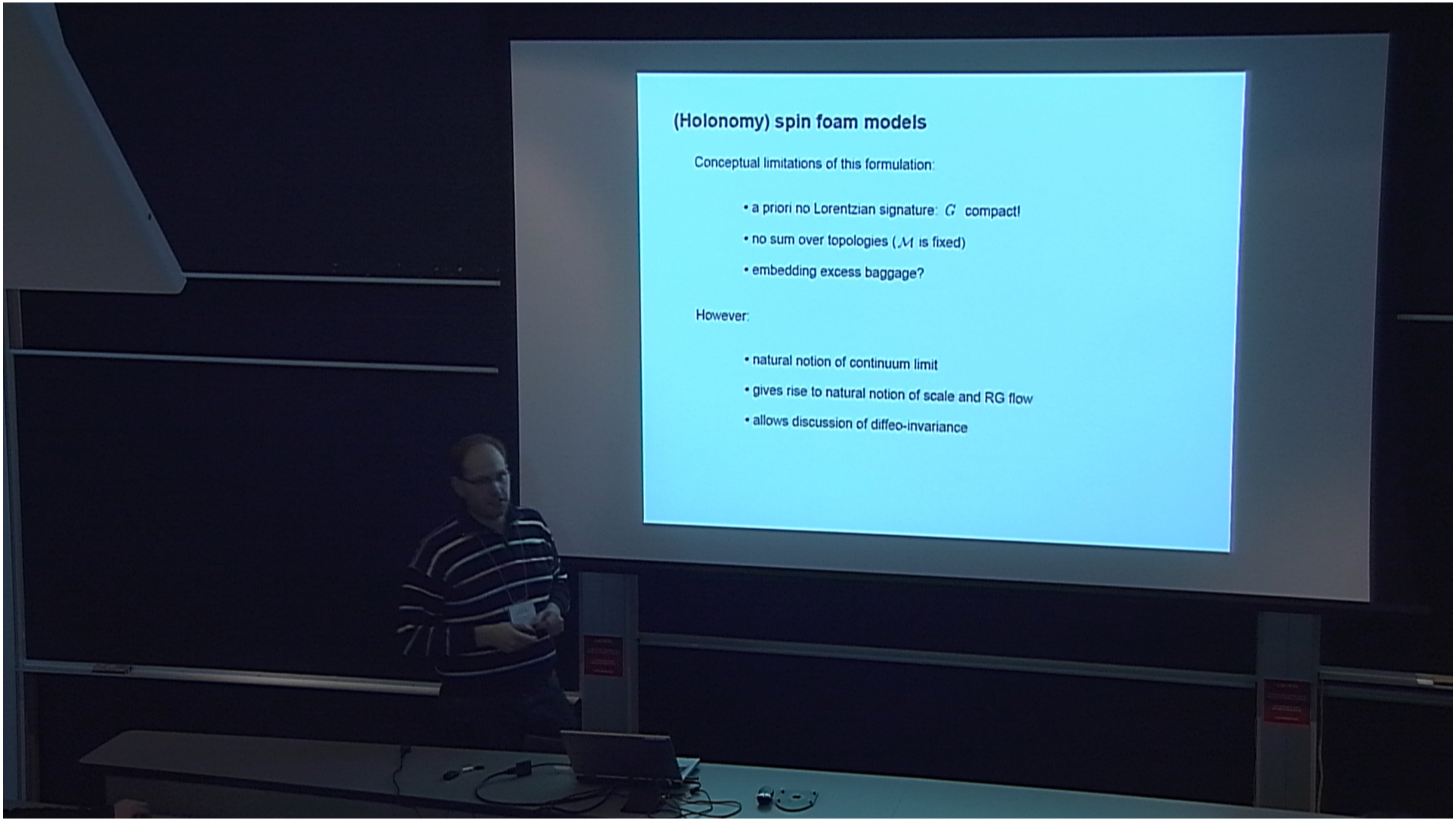
(Holonomy) spin foam models

Conceptual limitations of this formulation:

- a priori no Lorentzian signature: G compact!
- no sum over topologies ($\mathcal{M}$ is fixed)
- embedding excess baggage?

However:

- natural notion of continuum limit
- gives rise to natural notion of scale and RG flow
- allows discussion of diffeo-invariance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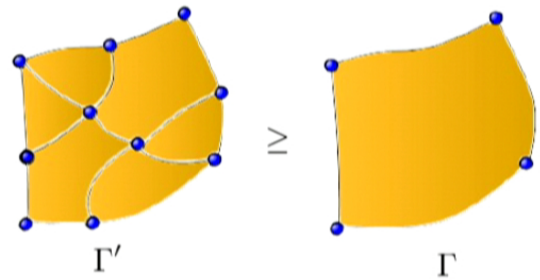
Continuum limit

Natural partial ordering of Γ's (in semi-analytic category) [Łojasiewicz, '64, Fleischhack '08]

$$\Gamma \leq \Gamma'$$

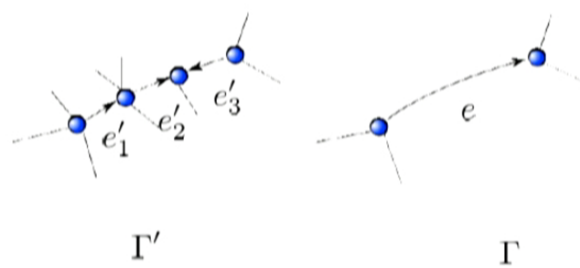


Not every two Γ's can be compared, but: for each two Γ, Γ' there is a finer one:

input: projection („coarse graining map") $\pi_{\Gamma'\Gamma} \,:\, \mathcal{A}_{\Gamma'} \to \mathcal{A}_{\Gamma}$

example from LGT:

$$\pi_{\Gamma'\Gamma}(h_{e'}) = (\cdots, \underbrace{h_{e'_3}^{-1} h_{e'_2} h_{e'_1}}_{=h_e}, \cdots)$$



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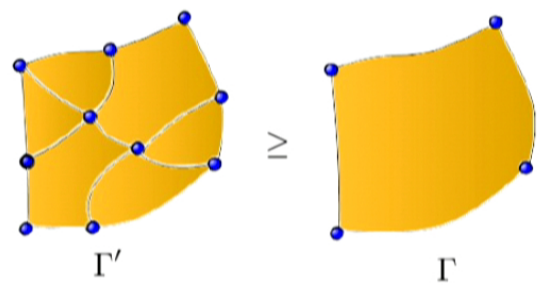

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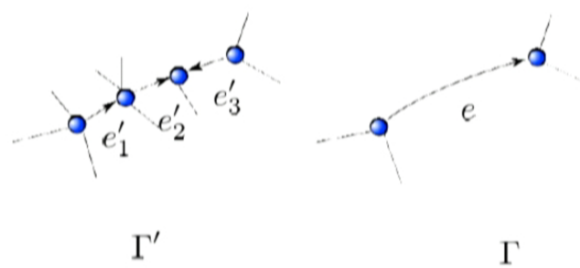

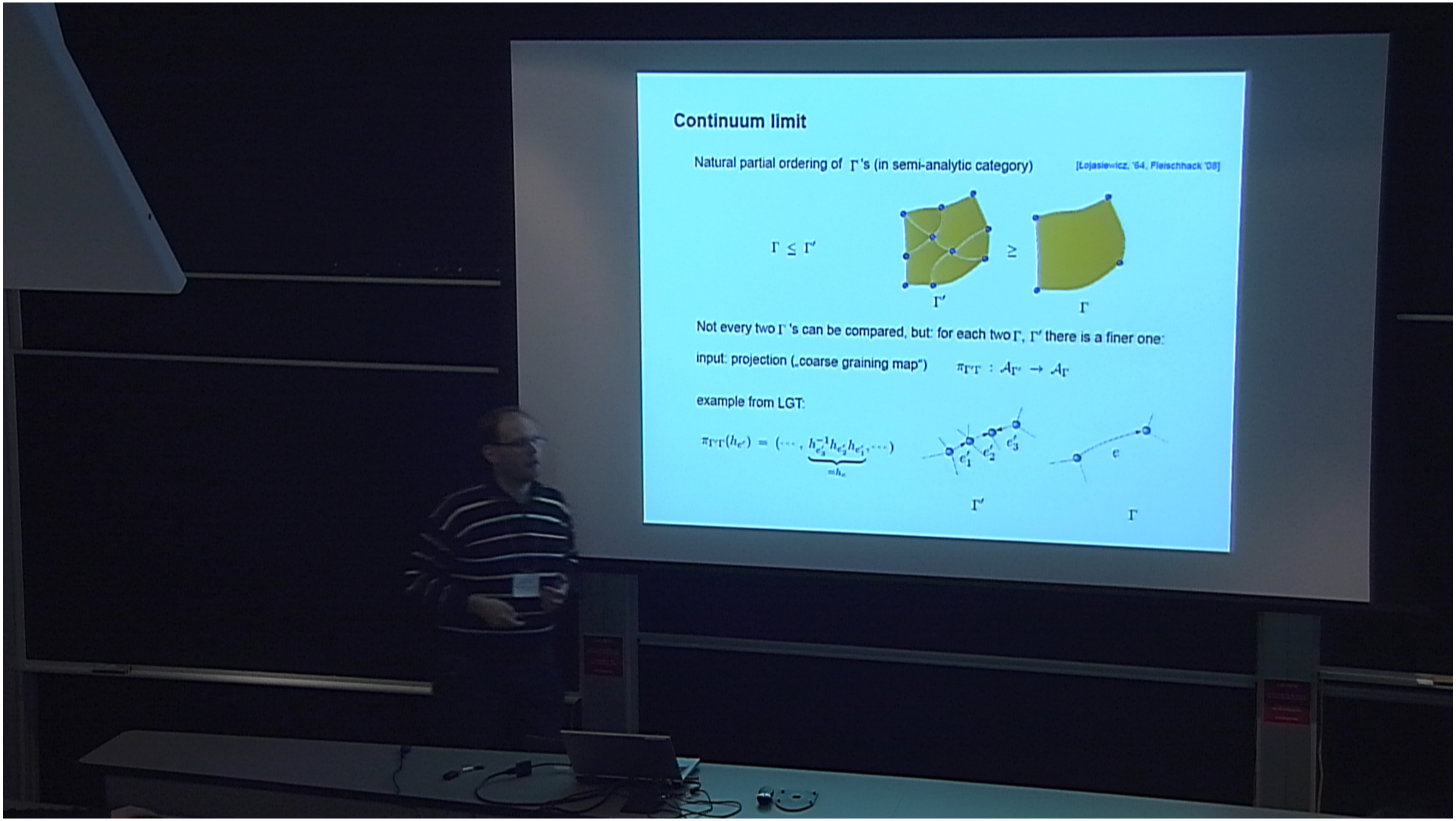
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[Ashtekar, Isham '92, Ashtekar, Lewandowski '94]

Consider all (or sufficiently many) Γ at the same time:

projective limit: $\overline{\mathcal{A}} := \lim_{\Gamma \leftarrow} \mathcal{A}_\Gamma = \left\{ \{a_\Gamma\}_\Gamma \,\middle|\, a_\Gamma \in \mathcal{A}_\Gamma, \pi_{\Gamma'\Gamma} a_{\Gamma'} = a_\Gamma \right\}$

space of (generalized) continuum connections: compact Hausdorff space

$$\mathcal{A}_\mathcal{M} \subset \overline{\mathcal{A}}$$

[crf. Wiener measure]

condition for continuum measure μ on $\overline{\mathcal{A}}$:

$$\langle \mathcal{O} \rangle_\Gamma = \langle \mathcal{O} \circ \pi_{\Gamma'\Gamma} \rangle_{\Gamma'}$$

„cylindrical consistency"

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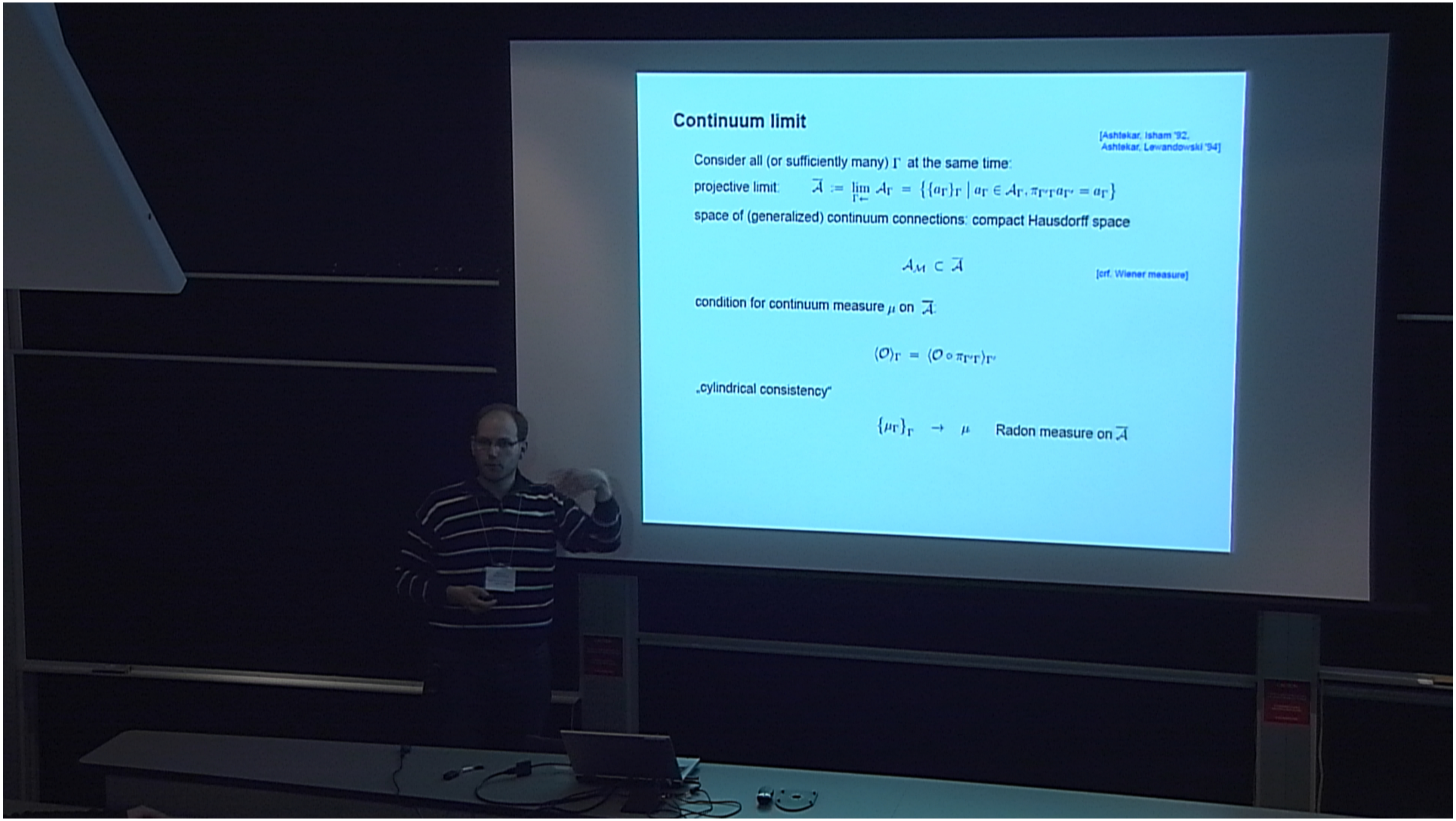
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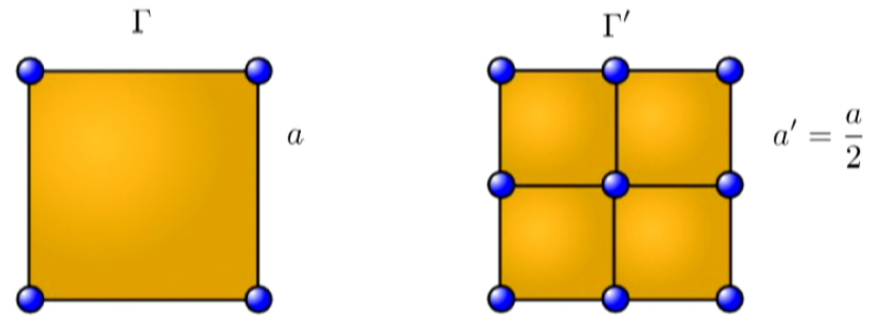
Where is the physical intuition?

cylindrical consistency: $\langle \mathcal{O} \rangle_\Gamma = \langle \mathcal{O} \circ \pi_{\Gamma'\Gamma} \rangle_{\Gamma'}$

is precisely the idea of Wilsonian RG flow!



$$d\mu = e^{-S[h_e, g_i(a)]} \, dh_e$$

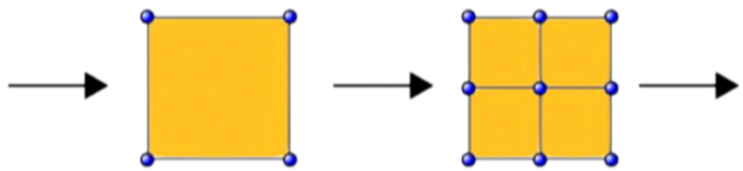
measure („action“) parametrised by parameters $g_i(a)$ which depend on a

$$e^{-S[h_e,\, g_i(a)]} = \int dh_{e'} \; e^{-S[h_{e'},\, g_i(a')]} \, \delta\left(h_e = \overrightarrow{\prod}_{e' \subset e} h_{e'}\right)$$

„scale“ with background:

$g_i(a)$

convergence
in the sense of sequence:



„scale“ without background:

[Manrique, Oeckl, Weber, Zapata '95
Smerlak, Rovelli '11]

$\{\mu_\Gamma\}_\Gamma$

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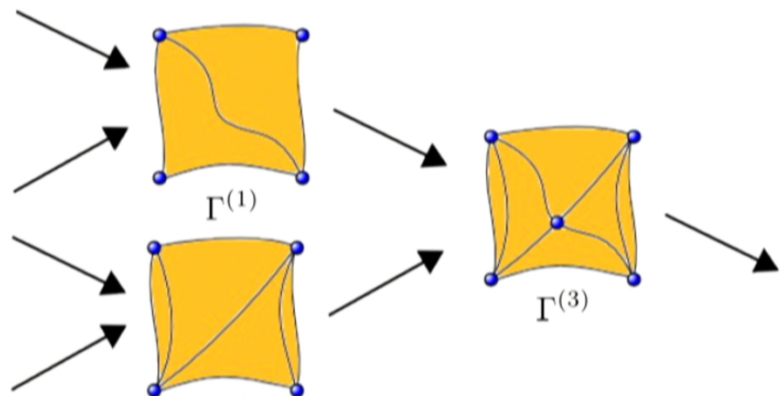
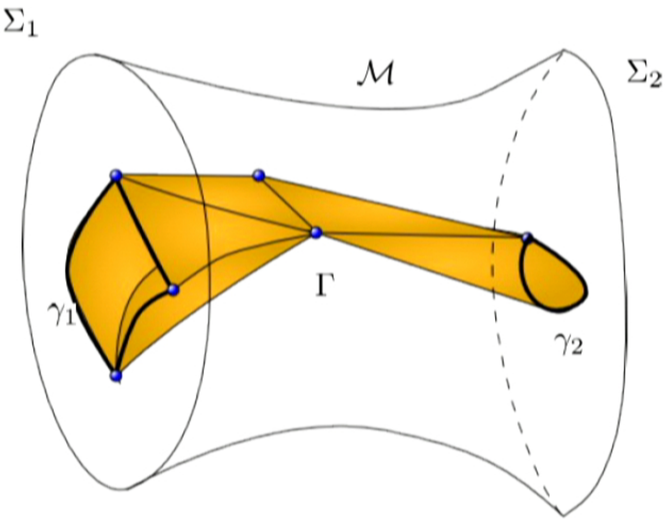
in sense of filters:



Hamiltonian formulation



Manifold with boundary, e.g.

$$\partial\mathcal{M} = \overline{\Sigma_1} \sqcup \Sigma_2$$

boundary graph $\partial\Gamma = \overline{\gamma_1} \sqcup \gamma_2$

boundary holonomies $\mathcal{B}_\gamma = G^{m_\gamma}$

kinematical boundary Hilbert space:

$$\mathcal{H}_\gamma = L^2(\mathcal{B}_\gamma)$$

boundary observables: $\psi \in C^0(\mathcal{B}_\gamma)$

$$\langle \psi_1 \,|\, \psi_2 \rangle_{\Gamma,\,\mathrm{phys}} := \langle \overline{\psi_1} \otimes \psi_2 \rangle_\Gamma$$

[Ashtekar et al '95, Marolf, Guilini '98]

provides physical inner product in sense of rigging map in RAQ

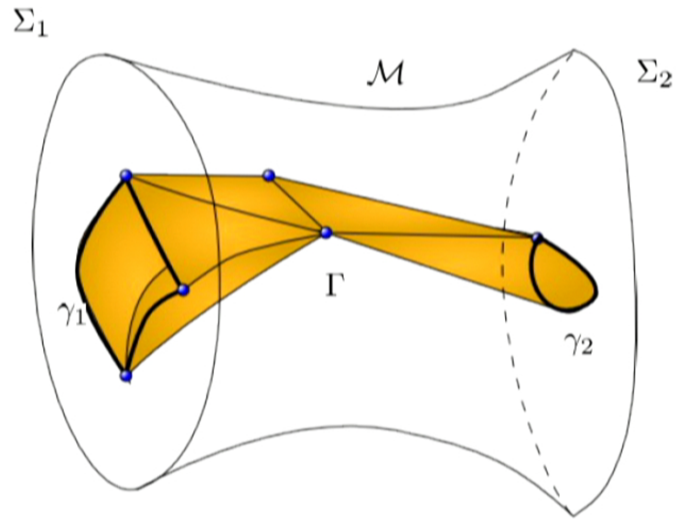
cylindrical consistency of $\langle \cdot \rangle_\Gamma$ guarantees extension to continuum boundary HS

$$\gamma \leq \gamma' \qquad \iota_{\gamma\gamma'} : \mathcal{H}_\gamma \hookrightarrow \mathcal{H}_{\gamma'} \qquad \langle \psi_1 \,|\, \psi_2 \rangle_{\Gamma,\mathrm{phys}} = \langle \iota_{\gamma\gamma'}\psi_1 \,|\, \iota_{\gamma\gamma'}\psi_2 \rangle_{\Gamma',\mathrm{phys}}$$

$$\mathcal{H}_\Sigma = \lim_{\gamma\rightarrow} \mathcal{H}_\gamma$$

[Dittrich, Steinhaus '12]

Hamiltonian formulation



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[Dittrich, Steinhaus '12]

Diffeomorphism group action

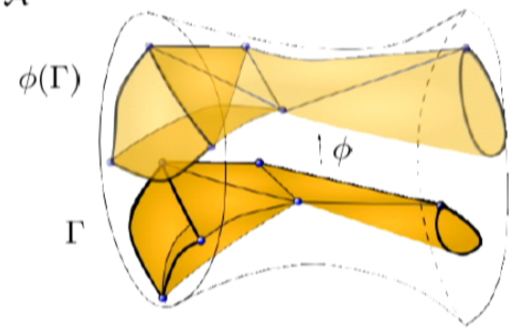
$\phi : \mathcal{M} \to \mathcal{M}$

Group of (semi-analytic) diffeomorphisms act on classical (continuum) connections $\mathcal{A} \subset \overline{\mathcal{A}}$

$$A \longmapsto (\phi^{-1})^* A$$

induced action on generalized connections via action on 2-complexes.

$$\phi : \overline{\mathcal{A}} \longrightarrow \overline{\mathcal{A}}$$



invariance of continuum measure under ϕ equivalent to

$$\langle \mathcal{O} \rangle_\Gamma = \langle \phi^* \mathcal{O} \rangle_{\phi(\Gamma)}$$

for all Γ supporting $\mathcal{O}$

$$\Leftrightarrow \quad \phi_* \mu = \mu$$

Diffeomorphism group action

Group of (semi-analytic) diffeomorphisms act on classical (continuum) connections $\mathcal{A} \subset \overline{\mathcal{A}}$

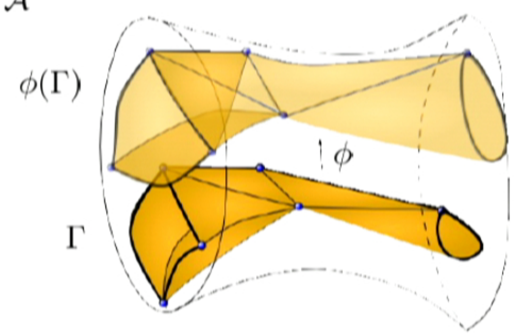
$$A \longmapsto (\phi^{-1})^* A$$

induced action on generalized connections via action on 2-complexes.

$$\phi : \overline{\mathcal{A}} \longrightarrow \overline{\mathcal{A}}$$

invariance of continuum measure under ϕ equivalent to

$\phi : \mathcal{M} \to \mathcal{M}$



$$\langle \mathcal{O} \rangle_\Gamma = \langle \phi^* \mathcal{O} \rangle_{\phi(\Gamma)}$$ for all Γ supporting $\mathcal{O}$

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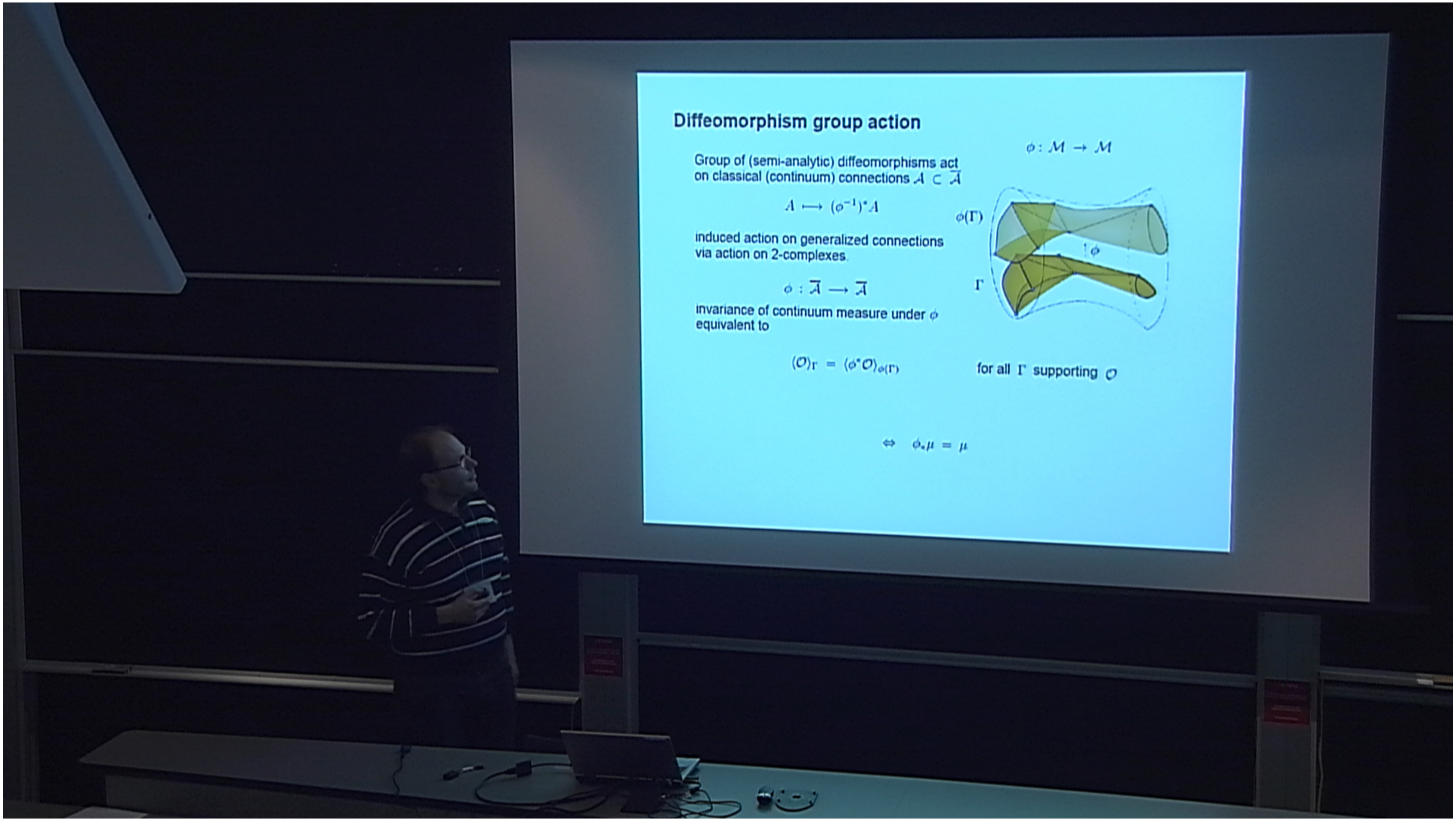

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$\phi(\Gamma)$
ϕ
Γ

Example:

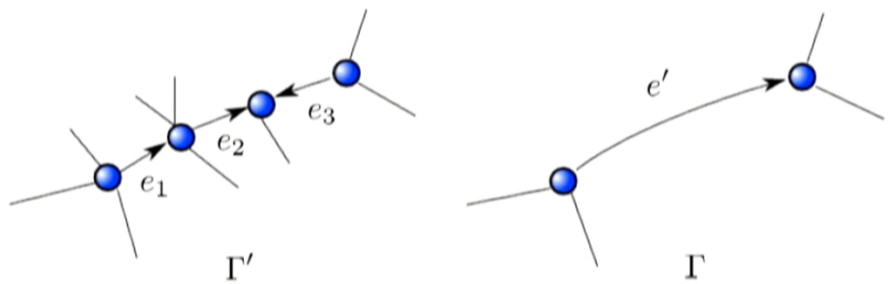
Near trivial example: $\dim \mathcal{M} \;=\; 2, \qquad G = U(1) \qquad \mathcal{A}_\Gamma \;=\; U(1)^E$

„charge-network functions" $\mathcal{O}(h_e) \;=\; \prod_e h_e^{m_e}$

$$\langle \mathcal{O} \rangle_\Gamma \;:=\; \frac{1}{Z} \int_{U(1)^E} dh_e \, \mathcal{O}(h_e) \prod_f \sum_{n_f \in \mathbb{Z}} \exp\left(-n_f^2/2a_f + i\theta_f n_f\right) \left(\prod_{e \subset f} h_e^{[e,f]} \right)^{n_f}$$

$$= \int_{U(1)^E} d\mu_\Gamma^{\vec{a},\vec{\theta}}(h_e)\, \mathcal{O}(h_e) \qquad a_f > 0,\ \theta_f \in \mathbb{R}$$

coarse graining map:

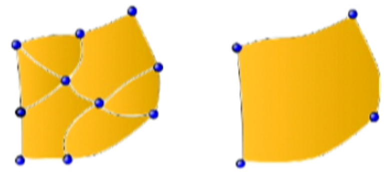


$$\pi_{\Gamma'\Gamma}(h_e) \;=\; (\cdots,\; \underbrace{h_{e_1} h_{e_2} h_{e_3}^{-1}}_{=h_{e'}}, \cdots)$$

Example:

RG equations:

$$a_f^{\Gamma} = \sum_{f' \subset f} a_{f'}^{\Gamma'}$$
$$\theta_f^{\Gamma} = \sum_{f' \subset f} [f', f]\theta_{f'}^{\Gamma'}$$



$\Gamma' \geq \Gamma$

Obvious solution:

$g \in \mathrm{Sym}^2 T^* M$ (area) metric

$\theta \in \Omega^2(M)$ 2-form

$$a_f = \int_f \mathrm{dvol}_g \qquad \theta_f = \int_f \theta$$

Limit solutions:

$\theta = 0$ 2d Yang-Mills

$g \to 0$ $\displaystyle\int_{\mathcal{A}} \mathcal{D}A\, \delta\big(F[A] - \theta\big)$

Example 2:

RG flow equations:

$$\omega^{(\Gamma)}_{(n_1,m_1,p_1),\ldots,(n_E,m_E,p_E)} := \Omega_\Gamma\left(E^{n_1}F^{m_1}K^{p_1}\otimes\ldots E^{n_E}F^{m_E}K^{p_E}\right)$$

e.g. edge subdivision:

$$\omega^{(\Gamma)}_{\ldots,(0,m,p),\ldots} = \sum_{k=0}^{m}\left[\begin{pmatrix} m \\ k \end{pmatrix}\right]_q \omega^{(\Gamma')}_{\ldots,(0,k,p-m+k),\,(0,m-k,p+k)\ldots}$$

Most obvious diffeo-invariant solution: $\Omega_\Gamma = \epsilon\otimes\ldots\otimes\epsilon$

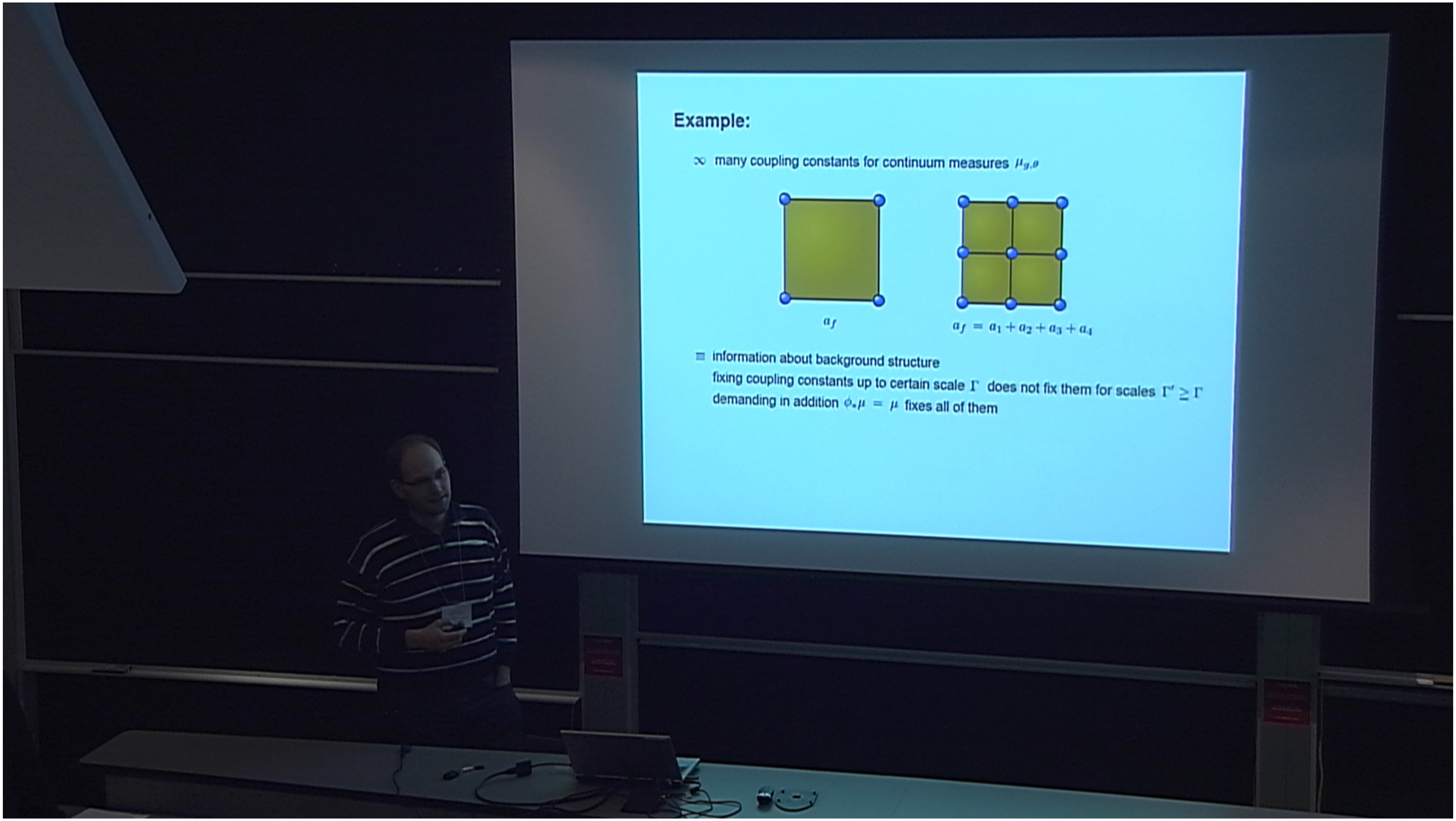
counit: $\epsilon : U_q\mathfrak{sl}_2 \to \mathbb{C}$

$$\epsilon(E) = \epsilon(F) = 0, \quad \epsilon(K^{\pm 1}) = 1, \quad \epsilon(ab) = \epsilon(a)\epsilon(b)$$

$$\Rightarrow \phi^*\Omega = \Omega$$

Summary:

- There is a clear notion of background-independent Wilsonian RG flow
- Scale = discretization of manifold $\mathcal{M}$ given by 2-complex Γ
- RG flow equation result from coarse graining scales: cylindrical consistency flow along partially ordered set!
- Solution to RG flow equations $\Leftrightarrow$ continuum limit
- Clear notion of diff-invariance: strong extra conditions on solutions!


Example:
∞ many coupling constants for continuum measures $\mu_{g,\theta}$
a_f
$a_f = a_1 + a_2 + a_3 + a_4$
≡ information about background structure
fixing coupling constants up to certain scale Γ does not fix them for scales $\Gamma' \geq \Gamma$
demanding in addition $\phi_* \mu = \mu$ fixes all of them