Title: Global flows in quantum gravity

Date: Apr 24, 2014 09:40 AM

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Abstract: In this talk I present recent work on complete UV-IR flows for the fully momentum-dependent propagator, RG-consistent vertices, Newtons coupling and the cosmological constant. For the first time, a global phase diagram is obtained where the non-Gaussian ultraviolet fixed point of asymptotic safety is connected via smooth trajectories to an infrared fixed point with classical scaling. Physics implications as well as the extension to gauge-matter-gravity systems are discussed.

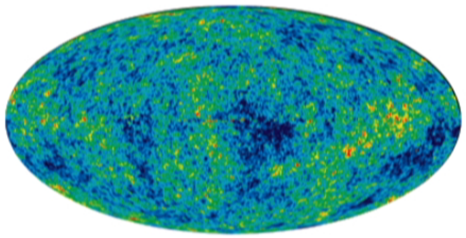
Global flows in quantum gravity

Jan M. Pawlowski
Universität Heidelberg & ExtreMe Matter Institute

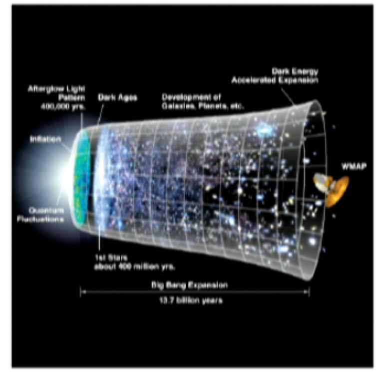
Perimeter Institute, April 24^{th} 2014

1

Phase diagram of quantum gravity

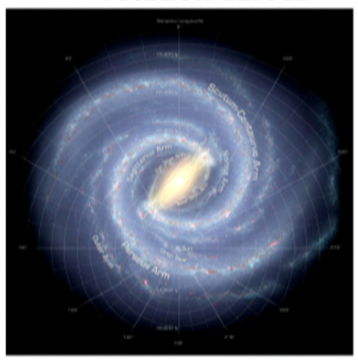


early universe

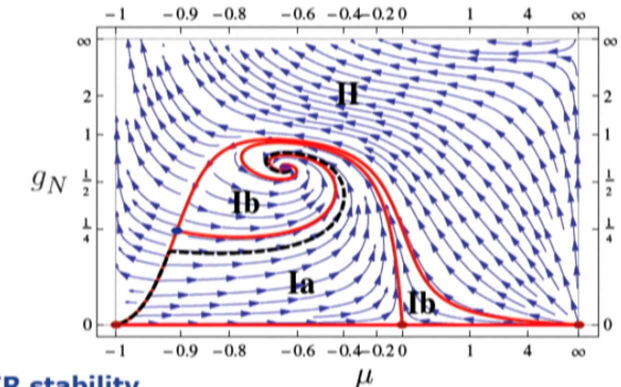


$$\boxed{g_N(E) \qquad \lambda(E)}$$

rotation curves



UV stability



IR stability

Functional approach to quantum gravity and diffeomorphism invariance

3

Functional approach to quantum gravity

Einstein-Hilbert action

Metric g **Cosmological constant** Λ

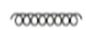
$$S[g] = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} \left(-R(g) + 2\Lambda \right)$$

Newton constant G_N **Ricci scalar** $R(g)$

Momentum dimension of couplings

$\dim G_N = -2$ $\dim \Lambda = 2$

perturbatively non-renormalisable

graviton propagator :  $\propto \frac{1}{p^2}$

3 – grav. vertex :  $\propto \sqrt{G}\, p^2$

4 – grav. vertex :  $\propto G\, p^2$

$\vdots$

Functional approach to quantum gravity

reparameterisation invariance

reparameterisation invariant path integral

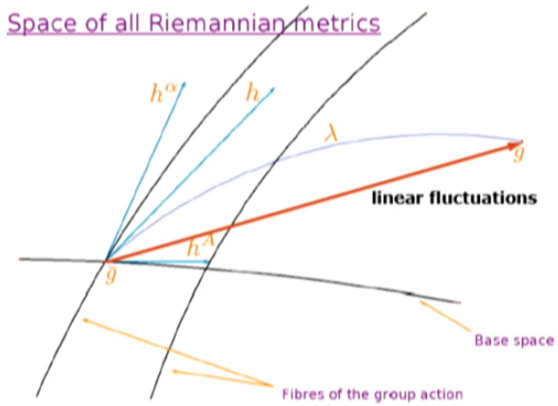
$$\int d\mu(\bar{g}, h)\, e^{-S[\bar{g},h]+\int_x J_h \cdot h}$$

linear split (reminder)

$$g = \bar{g} + h$$

$\bar{h}$ **average of tangent vectors**

$$\bar{h}(x) = \langle h(x) \rangle$$



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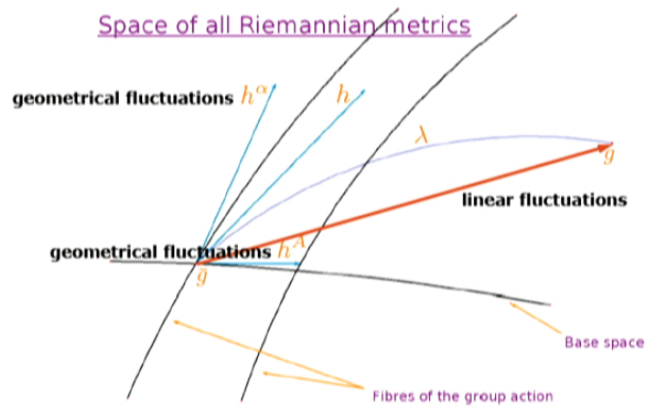
Geodesic normal fields

$$g = \bar{g} + h + \Delta g(\bar{g}, h)$$

$$\Delta g(h) = -\frac{1}{2}\Gamma_V * h^2 + O(h^3)$$

$$Dh = 1\!\!1 + O(h^2)$$

Γ_V**-covariant derivative**



Vilkovisky connection

$$\Gamma_V{}^A{}_{\beta C} = \Gamma_V{}^A{}_{B\gamma} = \Gamma_V{}^A{}_{\beta C} = 0$$

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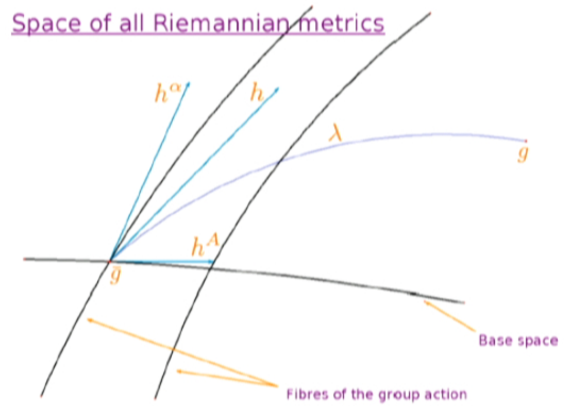
geometrical effective action

$$\Gamma = \Gamma[\bar{g}, \bar{h}^A]$$

$$Dh = \mathbb{1} + O(h^2)$$

Γ_V-**covariant derivative**



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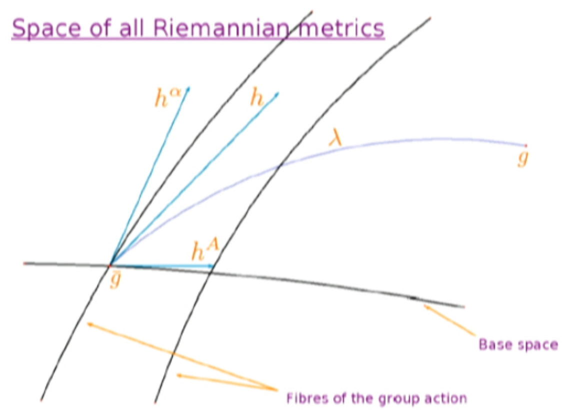
Γ_V-**covariant derivative**

background independence

$$\boxed{\frac{\delta\Gamma}{\delta\bar{g}} = \langle Dh \rangle * \frac{\delta\Gamma}{\delta\bar{h}}}$$

Nielsen identity



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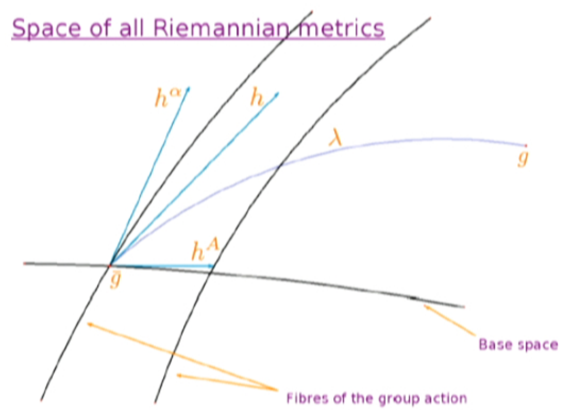
$$\Gamma_k = \Gamma_k[\bar{g}, \bar{h}^A]$$

Branchina, Meissner, Veneziano '03
JMP '03



background independence

$$\boxed{\frac{\delta \Gamma_k}{\delta \bar{g}} = \langle Dh \rangle * \frac{\delta \Gamma_k}{\delta \bar{h}} + R_k - \text{terms}}$$

Nielsen identity

JMP '03

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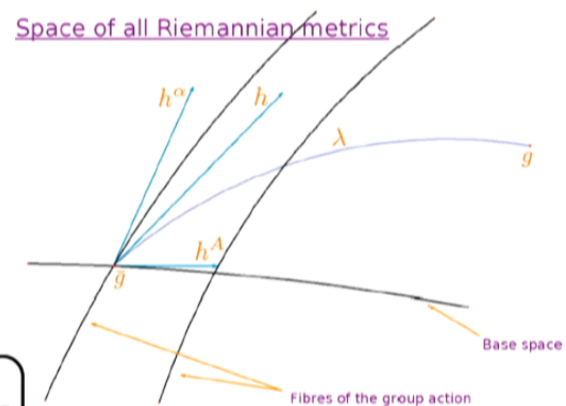

background independence

$$\frac{\delta\Gamma_k}{\delta\bar g} = \frac{\delta\Gamma_k}{\delta\bar h} + \left\langle \left.\frac{\delta(S_{\rm gf}+S_{\rm ghost})}{\delta\bar g}\right|_g \right\rangle_{\rm 1PI} + R_k - \text{terms}$$

linear split (reminder)

see talk of T. Morris

Vilkovisky connection

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symmetries and all that in the Wetterich RG

Wetterich '93
Ellwanger '94
Morris '94

QED/QCD:
Reuter, Wetterich '94
Ellwanger '94
D'Attanasio, Morris '96
Reuter, Wetterich '97
Litim, JMP '98
Igarashi, Itoh, So '99
Freire, Litim, JMP '00
JMP '02, '03
Litim, JMP '02
Braun, Gies, JMP '07
Lavrov, Shapiro '12
Fister, JMP '13

Gravity:
Reuter '96
JMP '03
Folkerts, Litim, JMP '11

Functional approach to quantum gravity

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Braun, Gies, JMP '07
Lavrov, Shapiro '12
Fister, JMP '13

Used in non-perturbative QCD
since '96
Ellwanger, Hirsch, Weber

Gravity:
Reuter '96
JMP '03
Folkerts, Litim, JMP '11

Used in non-perturbative QG
since '12
Donkin, JMP

Functional approach to quantum gravity

What is at stake?

background approximation

$$\frac{\delta^2\Gamma}{\delta\bar{g}^2} \simeq \frac{\delta^2\Gamma}{\delta\bar{h}^2}$$

aka split symmetry in the linear approx.

$$\Gamma[\bar{g}, h] = \Gamma[g] + S_{\text{gf}} + S_{\text{ghost}} + \Delta\Gamma_{\text{gauge}}[\bar{g}, h]$$

dropped as irrelevant

background independence

$$\frac{\delta\Gamma_k}{\delta\bar{g}} = \langle Dh\rangle * \frac{\delta\Gamma_k}{\delta\bar{h}} + R_k - \text{terms}$$

Nielsen identity

aka split symmetry

aka mSTI

JMP '03

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scalar theories
Litim, JMP '02
Bridle, Dietz, Morris '13

at finite cutoff: change of universal quantities in the FRG even at one loop

e.g. $\beta_{\text{1loop,YM}}$ **Litim, JMP '02**
JMP '02

cured by use of Nielsen identity

e.g. $\text{sign}(\Delta\beta_{\text{gravity,YM}})$ **Folkerts, Litim, JMP '11**

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cured by use of Nielsen identity

e.g. $\text{sign}(\Delta\beta_{\text{gravity},\text{YM}})$ — **Folkerts, Litim, JMP '11**

at vanishing cutoff: loss of the confining property of the order parameter potential

$$\frac{\delta^2\Gamma}{\delta\bar{A}^2}(p\to 0) \propto p^2 \qquad \frac{\delta^2\Gamma}{\delta\bar{a}^2}(p\to 0) \propto \text{mass gap}$$

Braun, Gies, JMP '07
Braun, Eichhorn, Gies, JMP '10
Fister, JMP '13

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aka split symmetry in the linear approx.

relevance for gravity

Folkerts, Litim, JMP '11
Donkin, JMP '12
Christiansen, Litim, JMP, Rodigast '12
Christiansen, Knorr, JMP, Rodigast '14

the simpler **the** **merrier**

$$\Gamma[\bar{g}, h] = \Gamma[g] + S_{\text{gf}} + S_{\text{ghost}} \boxed{+ \Delta\Gamma_{\text{gauge}}[\bar{g}, h]}$$

dropped as irrelevant

Functional approach to quantum gravity

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Folkerts, Litim, JMP '11
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Christiansen, Litim, JMP, Rodigast '12
Christiansen, Knorr, JMP, Rodigast '14

power counting

the more relevant

the un-merrier

qualitative difference

cosmological constant $\neq$ graviton mass parameter

semi-qualitative/quantitative difference

Newton constant ren. $\neq$ graviton wave function

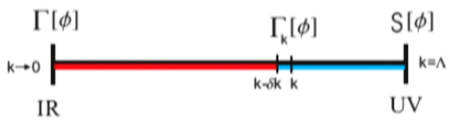
Global phase structure of quantum gravity

Christiansen, JMP, Knorr, Rodigast, arXiv:1403.1232

Functional approach to quantum gravity

Functional RG

free energy at momentum scale k



RG-scale k: $t = \ln k$

gravity quantum fluctuations

bosonic quantum fluctuations

free energy

fermionic quantum fluctuations

Geometrical approach: fully diffeomorphism invariant

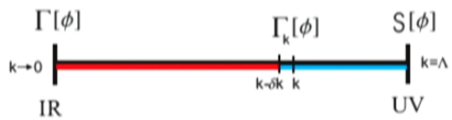
1st global (UV-IR) phase structure: Donkin, JMP '12

$$g = \bar{g} + h + O(h^2)$$

Functional approach to quantum gravity

Functional RG

free energy at momentum scale k



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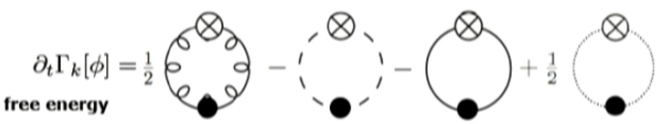

Geometrical approach: fully diffeomorphism invariant

1^{st} global (UV-IR) phase structure: Donkin, JMP '12

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Flat expansion about Minkowski background

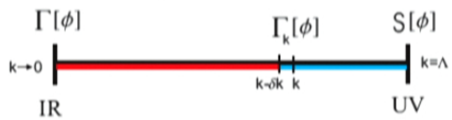
1^{st} smooth global phase structure

Christiansen, Litim, JMP, Rodigast '12

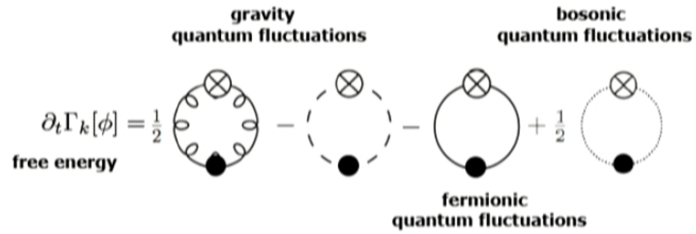
Functional approach to quantum gravity

Functional RG

free energy at momentum scale k



$\boxed{\text{RG-scale k: } t = \ln k}$



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Flat expansion about Minkowski background

1st smooth global phase structure

Christiansen, Litim, JMP, Rodigast '12

Flows

$$\partial_t g_{i,\text{fluc}} = \text{Flow}_{g_{i,\text{fluc}}}(\vec{g}_{\text{fluc}})$$

$$\partial_t g_{i,\text{back}} = \text{Flow}_{g_{i,\text{back}}}(\vec{g}_{\text{fluc}}, \vec{g}_{\text{back}})$$

dynamical flow

Nielsen ID

background flow

Donkin, JMP '12

Functional approach to quantum gravity

approximation scheme

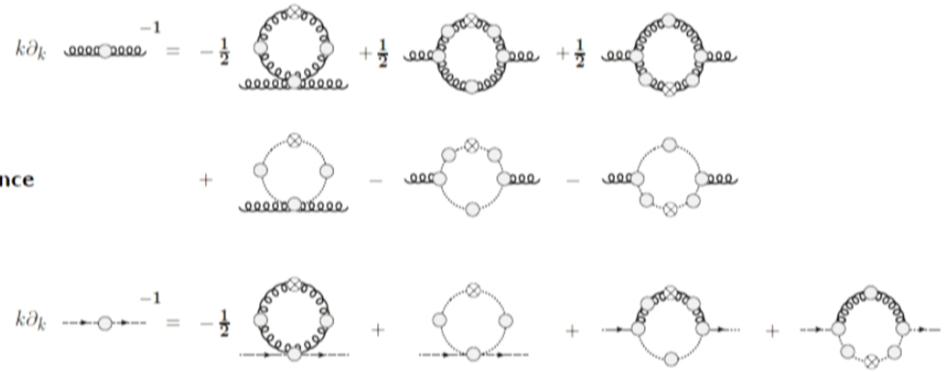
Christiansen, JMP, Knorr, Rodigast, arXiv:1403.1232

Propagators

graviton

full momentum dependence

ghost



Vertices

$$Z_{\text{graviton}} \neq Z_{g_N}$$

$$M^2_{\text{graviton}} \neq -2\Lambda$$

consistent momentum-dependent RG-dressing

a la Fischer, JMP '09
Donkin, JMP '12

similar: Codello, D'Odorico, Pagani '13

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full momentum dependence New

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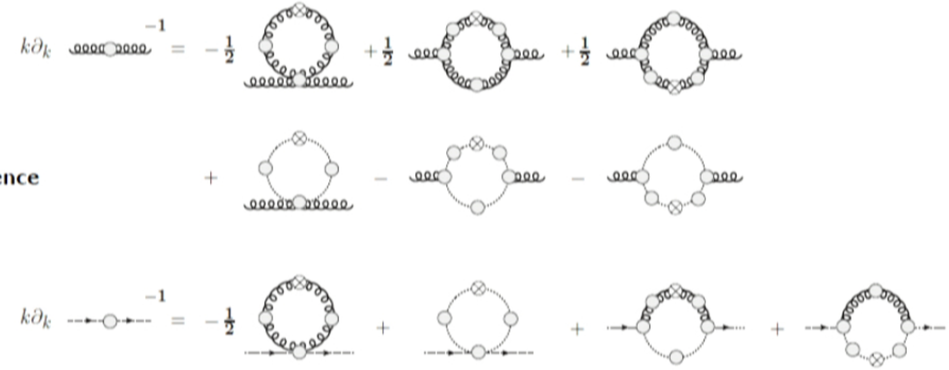

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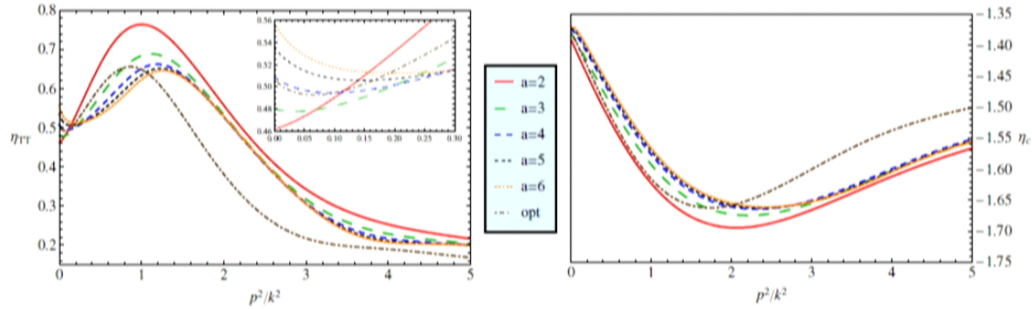
Phase diagram of quantum gravity

Propagators

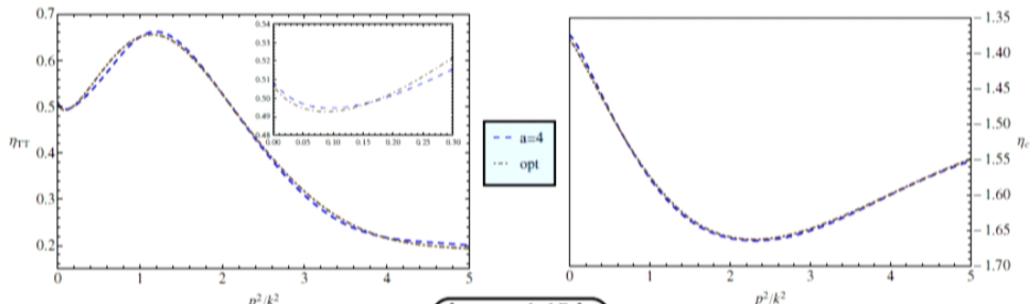


graviton **anomalous dimensions** ghost

regulators

$$R_{k,a}(p^2) = p^2\, r_a(x)$$

$$r_a(x) = \frac{1}{x(2e^{x^a} - 1)}$$

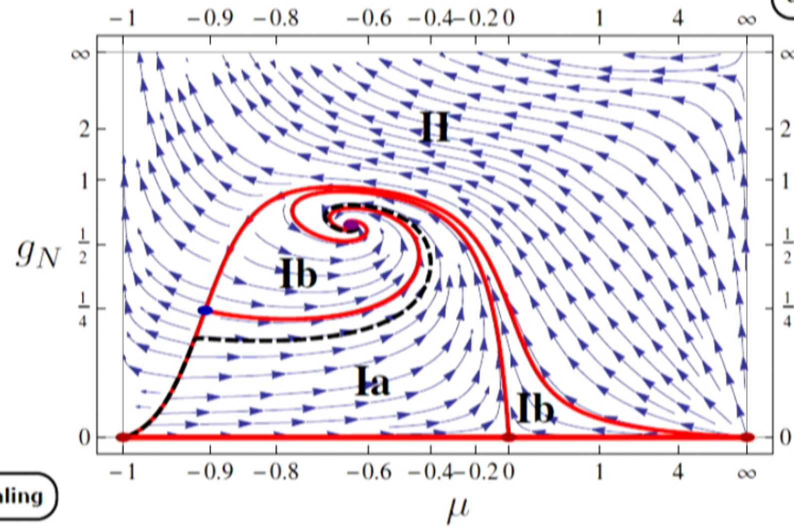


$k_{\text{opt}} = 1.15\, k_4$

Phase diagram of quantum gravity

global phase diagram

asymptotically safe UV scaling



$a = 4$

classical IR scaling

$g_N = G_N k^2$

$\mu = \frac{M^2_{\text{graviton}}}{k^2}$

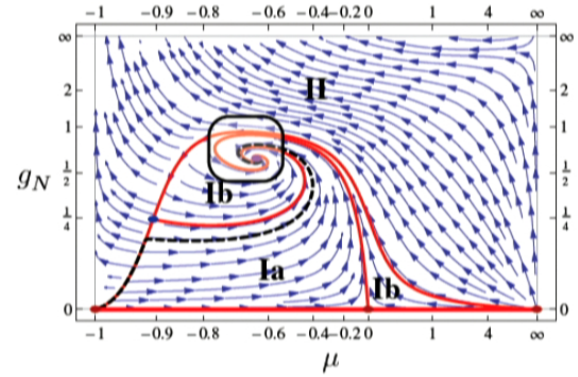
Phase diagram of quantum gravity

global phase diagram

UV-fixed point

regulator-dependence

a	2	3	4	5	6	opt
μ_*	-0.637	-0.641	-0.645	-0.649	-0.651	-0.489
g_*	0.621	0.622	0.614	0.606	0.600	0.831
$\overline{g}_*$	0.574	0.573	0.567	0.559	0.553	0.763
λ_*	0.319	0.316	0.316	0.318	0.319	0.248
EVs	-1.284 ±3.247i	-1.284 ±3.076i	-1.268 ±3.009i	-1.255 ±2.986i	-1.244 ±2.974i	-1.876 ±2.971i
	-2	-2	-2	-2	-2	-2
	-1.358	-1.360	-1.360	-1.358	-1.356	-1.370



comparison with other results

	here	Litim03	Christiansen12	Donkin12	Manrique10	Becker14	Codello13	here mixed
$\overline{g}_*$	0.763	1.178	2.03	0.966	1.055	0.703	1.617	1.684
λ_*	0.248	0.250	0.22	0.132	0.222	0.207	-0.062	-0.035
$\overline{g}_*\lambda_*$	0.189	0.295	0.45	0.128	0.234	0.146	-0.100	-0.059

Litim '03 — background approximation
Christiansen, Litim, JMP, Rodigast '12 — flat expansion, bi-local
Donkin, JMP '12 — geometrical
Manrique, Reuter, Saueressig '10 — bi-metric
Becker, Reuter '14 — bi-metric
Codello, D'Odorico, Pagani '13 — flat expansion, mixed approach

mixed approach: $\mu = -2\lambda$

bi-metric: see talk of M. Reuter

Phase diagram of quantum gravity

global phase diagram

UV-fixed point

regulator-dependence

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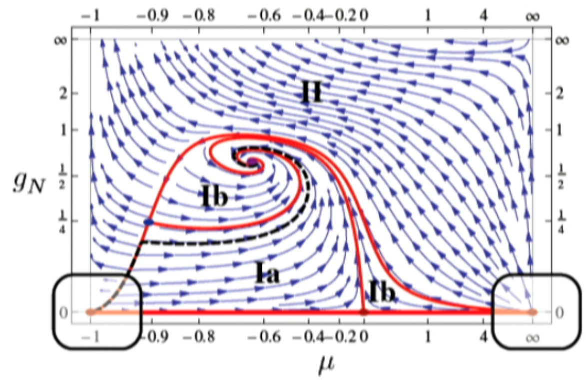
	-1.358	-1.360	-1.360	-1.358	-1.356	-1.370



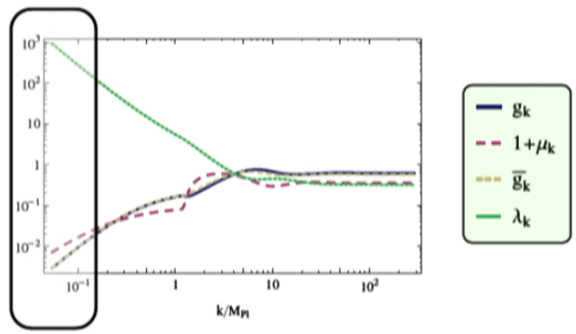
IR-fixed points

$$g, \overline{g} \sim k^2$$

$$\lambda \sim \frac{1}{k^2}$$

$$\eta_h \to 0$$

$$\eta_c \to 0$$



Summary & outlook

- **Phase diagram of quantum gravity**
  - **first smooth global flow diagram with classical IR regime**
    in agreement with experimental observations
  - **IR-stability of quantum gravity**
  - **UV-stability of the gauge-gravity system**
- **Outlook**
  - **fully-coupled matter-gauge-gravity systems in the UV**
  - **long & short distance physics**