Title: Asymptotic safety in a pure matrix model

Date: Apr 23, 2014 03:30 PM

URL: http://pirsa.org/14040096

Abstract: The connection between two-dimensional Euclidean gravity and pure matrix models has lead to may fundamental insights about quantum gravity and string theory. The pure matrix model is thus a blueprint for the connection between discrete models and Euclidean quantum gravity. I will report on work with Astrid Eichhorn in which this "blueprint" model is investigated with the functional renormalization group. In this model, I will discuss the questions: "What is a possible meaning of asymptotic safety in a discrete model?" and "Is it possible to apply the FRGE to tensor models?

Asymptotic Safety in a Pure Matrix Model

Renormalization Group Approaches to Quantum Gravity

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work with A. Sfondrini: IJMP A 26 (2011) 4009
and with A. Eichhorn: PRD 88 (2013) 084016

April 23, 2014

Are we there? – No.

How can we get there?

Why are we not there yet?

- Quantum Field Theory of General Relativity is unattainable
  $\Rightarrow$ redefine the goal
- Quantum Field Theory of GR is more subtle than we thought
  $\Rightarrow$ renormizability, unitarity, causality, classical limit, ...

Combine approaches:

each approach has built-in features and hard problems
$\Rightarrow$ combine approaches that solve each others hard problems

Personal example: LQG and RG (in 2008)

hard LQG problem: phase diagram
hard FRGE problem: unitary quantum theory

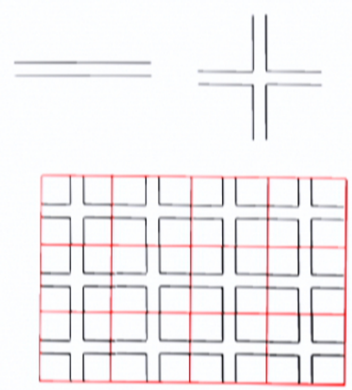
Unfortunately: there is no simple way to combine with real LQG!

A more compatible combination

Topic of talk: combine FRGE with tensor-inspired models

hard problem: phase diagram in tensor models
(or systematic search for bare actions)
⇒ FRGE allows a systematic search for bare actions

2D-Matrix models motivate tensor program



matrix Feynman graphs ⇒ 't Hooft expansion
⇒ large N-limit corresponds to Riemann surfaces
⇒ use matrix partition function to describe Riemann surfaces
⇒ generalization: use tensor partition function to describe Euclidean geometries

⇒ Investigate asymptotic safety in matrix models using the FRGE

Plan of the talk

Warm-up: Asymptotic Safety in Grosse-Wulkenhaar model

- GW-model = noncommutative ϕ_4^{*4} model
- Constructive program: GW model is asymptotically safe (bounded RG flow near GFP, no Landau pole)

$\Rightarrow$ Confirm this using the FRGE methods of AS program
(with A. Sfondrini: IJMPA 26 (2011) 4009)

Double Scaling Limit

- matrix model for 2D Euclidean gravity
- Constructive program: There is a nontrivial (critical) large N-limit

$\Rightarrow$ there is a non-Gaussian FP (if N is cut-off) $\Rightarrow$ find with FRGE
aim: extract recipe for tensor FRGE
(with A. Eichhorn: PRD 88 (2013) 084016)

General goals

(1) test FRGE as a tool for discrete geometry models
(2) test use of FRGE in AS by testing known results

Grosse-Wulkenhaar model

GW model is inbetween continuum and matrix models

- $S = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi) * (\partial^\mu \phi) + \frac{\Omega^2}{2}(\tilde{x}_\mu \phi) * (\tilde{x}^\mu \phi) + \frac{m^2}{2}\phi^{*2} + \frac{\lambda}{4!}\phi^{*4} \right)$

(where: $f * g := f \exp(\overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu) g$ with $\theta^{\mu\nu} = \theta(\epsilon^{\mu\nu 34} + \epsilon^{12\mu\nu})$)

- can be written as (tensor = matrix $\otimes$ matrix model)
- Langmann-Szabo transformation leaves $\Omega = 1$ invariant

$S = \nu \mathrm{Tr} \left(\frac{1}{2}\phi.XX.\phi + \frac{m^2}{2}\phi.\phi + \frac{\lambda}{4!}\phi.\phi.\phi.\phi \right)$

(where: $XX = \tilde{x}_\mu \tilde{x}^\nu$)

Known results (from constructive program)

1. perturbative UV-attractivity of $\Omega = 1$ (LS-self-dual model)
2. asymptotic safety near GFP for finite λ

$\Rightarrow$ can be tested with FRGE
$\Rightarrow$ provide good benchmark for FRGE in matrix model
$\Rightarrow$ LS symmetry makes theory space "small"

FRGE setup for GW model

FRGE from $e^{-W[j]} = \int [d\phi]_\Lambda e^{-S[\phi]-\Delta_k S[\chi]+\nu \mathrm{Tr}(j^T.\phi)}$ (Λ is UV cut-off in XX)

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}(\Gamma_k^{(2)}[\phi] + R_k)^{-1}_{m_1 \dots n_2, a_1 \dots b_2} (k\partial_k R_k)^{m_1 \dots n_2, a_1 \dots b_2}$$

IR suppression term

cut-off on $\Delta_{sd} = XX$ is effectively a cut-off on matrix size (finite dim. integrals!)

$\Rightarrow$ use standard cut-off $\Delta_k S[\phi] = \frac{k^2 \nu}{2} \mathrm{Tr}(\phi . r(\frac{XX}{k^2}).\phi)$

Theory space

$\Gamma_k[\phi] = \sum g^1_{a_1 b_1 \dots} \nu \mathrm{Tr}(\phi^{a_1} XX^{b_1} \dots) + \sum g^2_{a_1 b_1} \frac{\nu}{\theta^2} \mathrm{Tr}(\dots)\mathrm{Tr}(\dots) +$ "more traces"

Canonical dimensions (from position representation)

roughly: $[\phi] = 1, \quad [\nu \mathrm{Tr}] = -4, \quad [XX] = 2$

Summary of Grosse-Wulkenhaar FRGE

1. FRGE setup for tensor representation can be read-off from position rep.
2. UV attractivity of self-dual model can be found with significantly less work than in perturbation theory
3. λ is exactly marginal at GFP
4. flow of λ can be bounded in small region around GFP (asymptotically safe region)
5. calculations are comparatively simple

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Original Hermitian matrix model for Euclidean 2D quantum gravity

$\phi^2 + g\,\phi^3$ matrix model for 2D-QG

- Hermitian $N \times N$ matrices ϕ as fund. deg. of freed.
  model: $N^2 Z_{grav.} = \log\left(\int [d\phi]_N e^{-\frac{1}{2}\mathrm{Tr}(\phi^2)+N^{-1/2} g \mathrm{Tr}(\phi^3)}\right)$ take $N \to \infty$
- action generates random triangulations: $Z = \sum_h N^{2(1-h)} Z_h$
  $\Rightarrow$ planar scaling limit (i.e. only sphere, genus $h = 0$, remains)
- 2D QG = critical theory ("double scaling limit"):
  $N \to \infty$, $g \to g_c$ s.t. N^{-2h} is compensated
  $\Rightarrow$ keep $N(g - g_c)^{1-\gamma_{str.}/2} =$ const.
  $\Rightarrow$ continuum limit $N \to \infty$ corresponds to FP g_* (in large N scaling)
- analytic result: $\theta = N\partial_N \beta(g)|_{g=g_*} = 4/5$ (to reproduce with FRGE)
- **problem ϕ^3-interaction:**
  (1) is odd (large theory space)
  (2) unbounded (only formal path integral)

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Revised matrix models

Hermitian $\phi^2 + g\,\phi^4$

- corresponds to random tasselation by squares (pic. before)
- action has Z_2-symmetry $\Rightarrow$ smaller theory space
- analytic result $\theta = 4/5$ (what we actually want to reproduce)
- perturbative calculations available as benchmarks
  (Brezin, Zinn-Justin: PLB 288 (1992) 54; Ayala: PLB 311 (1993) 55)

Equivalent formulation:

- $N \to \infty$ turns out equivalent to real matrix model with $\phi \to O_1^T \phi O_2$ gauge symmetry
- correspondence: Hermitian matrices $\phi^2 \leftrightarrow$ real matrices $\rho := \phi^T \phi$
- generalization $\sum_i \mathrm{Tr}(\phi_i^T \phi_i) + g\mathrm{Tr}(\phi_1 \phi_2 \phi_3)$ close to colored tensor model
  (cf. Gurau: various publications in 2010,20111)

$\Rightarrow$ base FRGE setup on bare action $\phi^2 + g\,\phi^4$ and cut-off in matrix size

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FRGE setup for matrix model

Theory space

general invariant functional: $\Gamma_k[\phi] = F_k\left(\mathrm{Tr}(\phi^2), \mathrm{Tr}(\phi^4), ...\right)$
suitable expansion: number of traces (connected components) and powers of ϕ

Problem: no "Laplacian" in theory space

- "invent Laplacian" (s.t. cut-off on matrix size) $\Rightarrow$ larger theory space
- unfeasible Ward-id: $G_\epsilon \Gamma_k = \frac{1}{2} tr_{op}\left((\Gamma_k^{(2)} + R_k)^{-1} G_\epsilon R_k\right)$

  all regulators break $U(N)$ resp. $O(N)^2$ symmetry G_ϵ
- practically: good results on orig. theory sp. $\Rightarrow$ potential optimization

  (choice of Laplacian and regulator profile)

Canonical dimensions (from perturbative large N-limit):

- operator basis: $\Gamma_k[\phi] = \sum g^i_{n_1 \dots n_i} \mathrm{Tr}(\phi^{n_1}) \dots \mathrm{Tr}(\phi^{n_i})$

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Single trace truncation

extract β-functions using vertex expansion

- assume $[P, \phi] = 0$ and take large N-limit (reasonable propagator commutes w/h low-energy ϕ)

$\Rightarrow$ scheme dependence in $O(1)$ numbers $[\dot{R}P^n]$

Single trace truncation $\Gamma_k[\phi] = \frac{Z}{2}\mathrm{Tr}(\phi^2) + \sum_{n\geq 2} \frac{\bar{g}_{2n}}{2n}\mathrm{Tr}(\phi^{2n})$

- dimensionless couplings $\bar{g}_i = Z^{i/2} N^{i/2-1} g_i$
- β functions: $\eta = g_4[\dot{R}P^2]$

$$\beta(g_{2n}) = ((1+\eta)n - 1)g_{2n}$$
$$+2n \sum_{i;\{\vec{m}\}=\sum_i i m_i} (-1)^{\sum_i m_i} [\dot{R}P^{1+\sum_i m_i}] \begin{pmatrix} \sum m_i \\ m_1\, m_2 \dots \end{pmatrix} \Pi_i g_{2(i+1)}^{m_i}$$

Fixed point analysis:

- fixed point with **single** relevant direction appears in all truncations
- positive $\theta_1 \to\sim 1_+$ (reasonably aligned with g_4)
- all other exponents $\theta_i \to\sim -i+1$ (reasonably aligned with g_{2i+2})

Summary of double scaling limit:

FRGE is a tool to investigate phases of matrix/tensor QG models

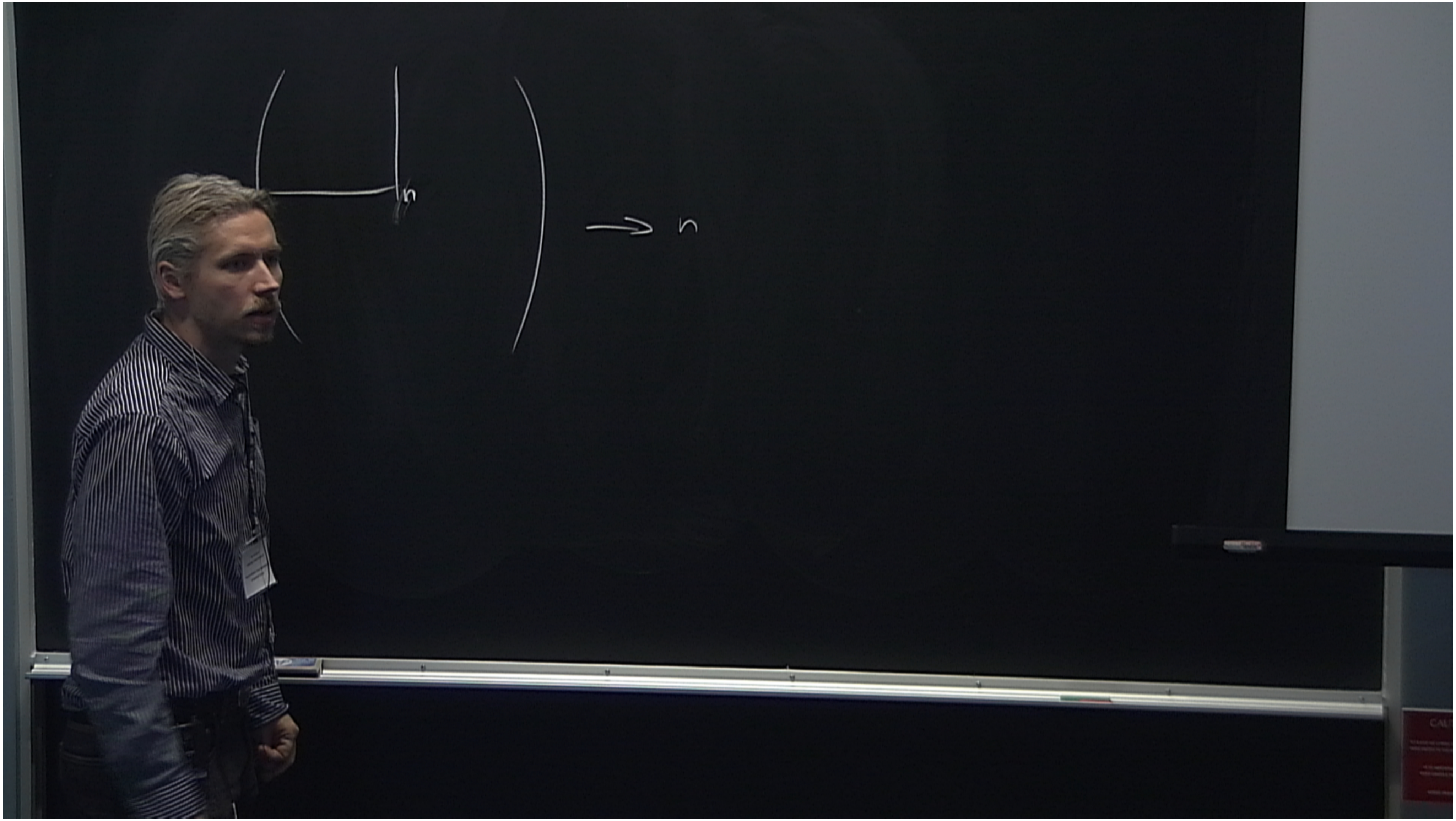
bottom-line: FRGE finds double scaling limit with reasonable accuracy.
(calculations appear simpler than pert. calc. !)

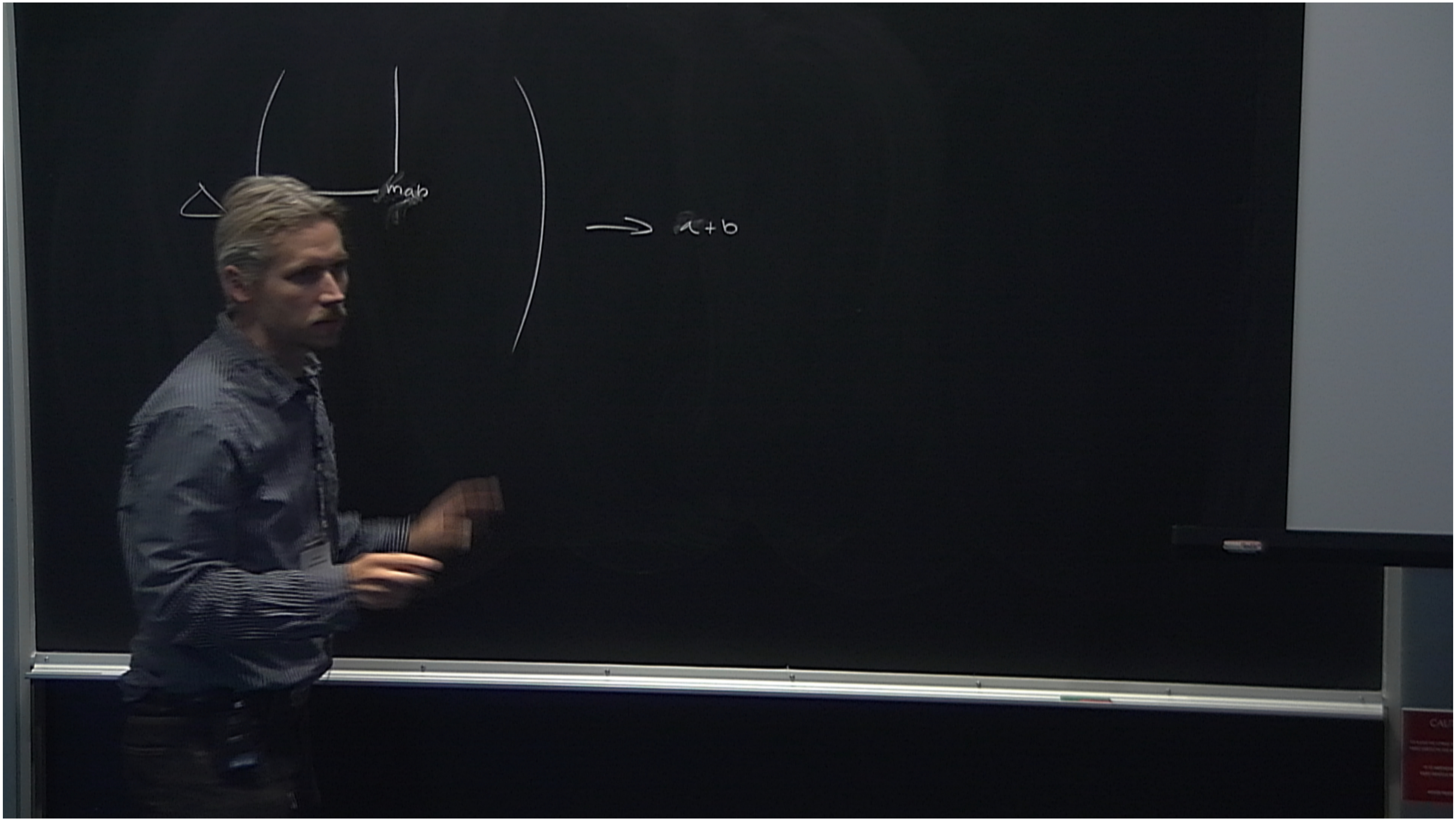
General recipe for matrix/tensor models

- Theory space from field content and symmetries of bare action
- Standard regulator (invent Laplacian; min. gauge sym. breaking)
- Canonical dim. partly determined by requiring $1/N$ expansion of β-fkt.
- β-fkt. found from vertex expansion, $[P, \phi] \approx 0$ for large N

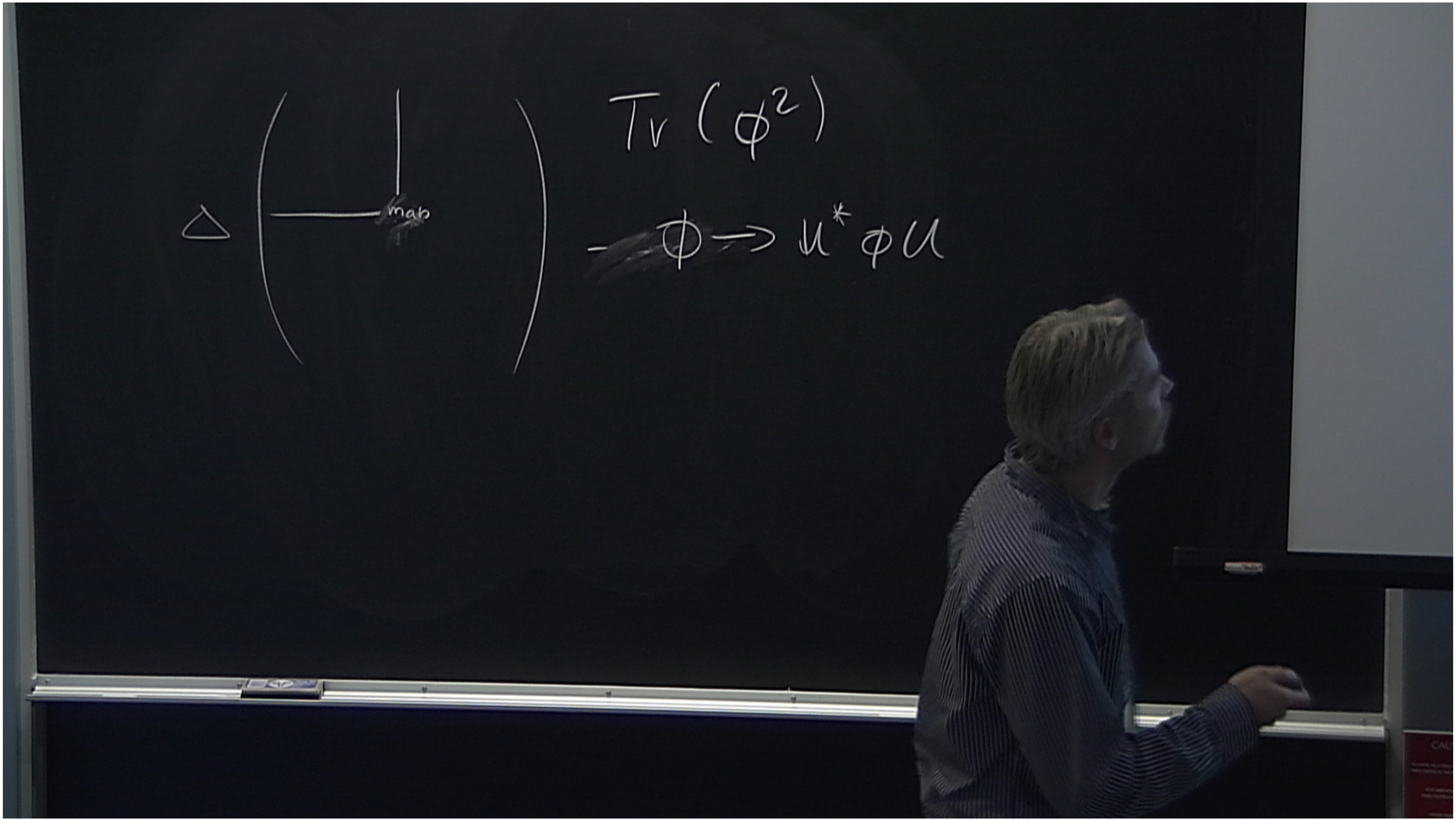
Summary

- Starting point: desire to explore continuum limit in LQG related models
  $\Rightarrow$ combine FRGE with discrete geometry models
- First step: investigate matrix models
- Matrix model results:
  1. AS of GW model can be confirmed with FRGE, but method does not straightforwardly generalize to NGFP
  2. FRGE finds double scaling limit
     $\Rightarrow$ establish FRGE as tool in matrix models
     $\Rightarrow$ do optimization and precision calculations to benchmark FRGE
  3. practical experience leads to general FRGE "recipe" in tensor model (and GFT) framework
- countless applications of FRGE in discrete approaches to QG
  in particular: FRGE allows systematic investigation of theory space
  $\Rightarrow$ phase diagram


n
→ n



mab
→ a+b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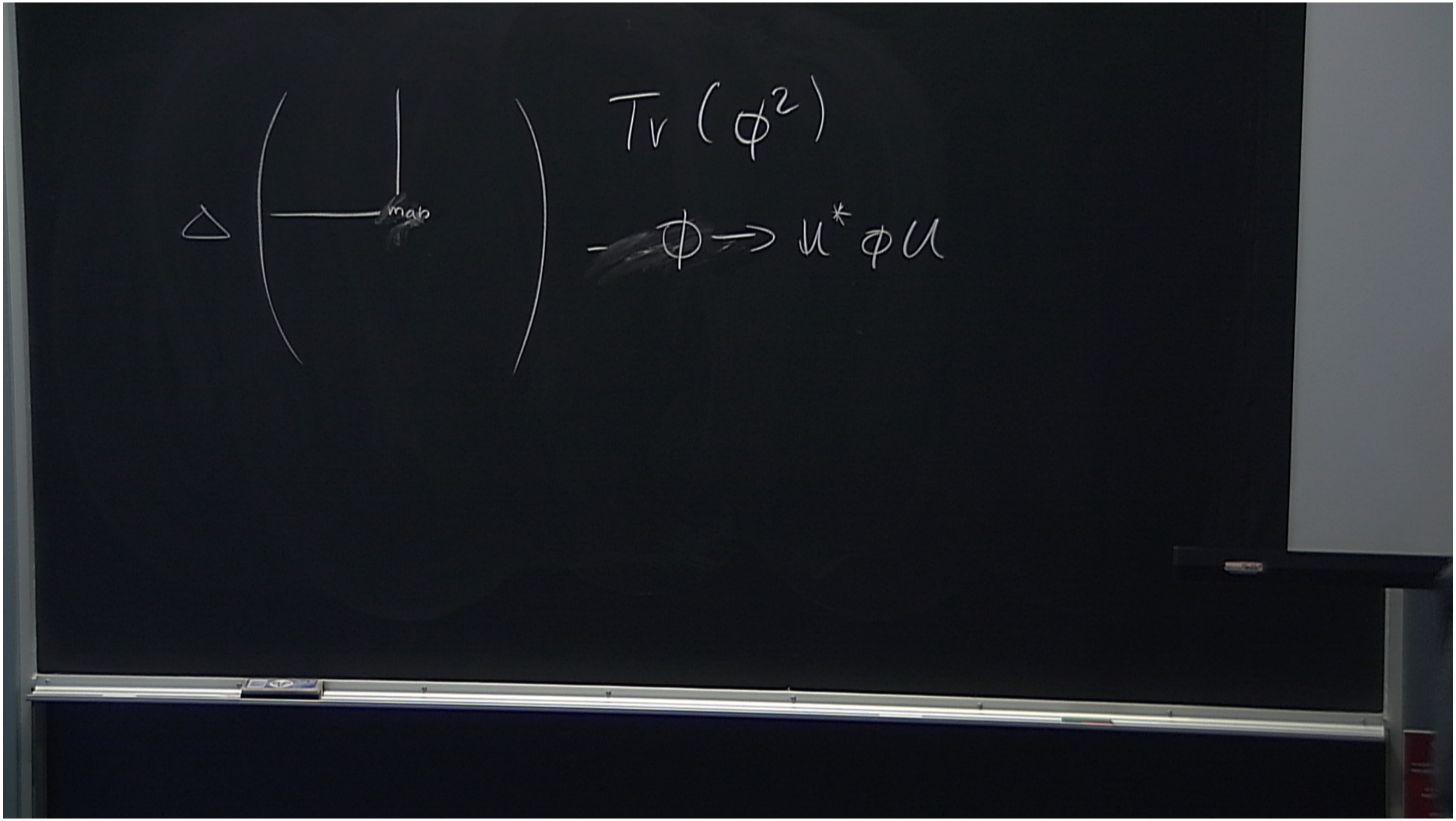
φ → U* φ U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ϕ^6 double trace truncation

Operators that directly renormalize bare action $O(\phi^4)$

- only tadpole over $\mathrm{Tr}(\phi^2)\mathrm{Tr}...$ receives N^2 (necessary to overcome dimensional suppression)

$\Rightarrow$ include $\frac{1}{4}\bar{g}_{2,2}(\mathrm{Tr}(\phi^2))^2$ and $\frac{1}{2}\bar{g}_{2,4}\mathrm{Tr}(\phi^2)\mathrm{Tr}(\phi^4)$

β-functions

- η receives $+2g_{2,2}[\dot{R}P^2]$ contribution
- $\beta(g_4)$ receives $-2g_{2,4}[\dot{R}P^2]$ contribution
- double trace β-fkt. become messy

Fixed point:

- FP with single relevant direction

$$\theta_1 = 1.21, \quad \theta_2 = -0.69, \quad \theta_3 = -1.01, \quad \theta_4 = -1.88$$

- slight improvement over pert. calculation (but possibility for optimization not yet explored)


Tr (φ²)
φ → u⁺φu