Title: Perturbative quantum gravity calculations and running couplings

Date: Apr 23, 2014 09:00 AM

URL: http://pirsa.org/14040091

Abstract: <span>We know how to make perturbative calculations in quantum gravity using the framework of effective field theory. I will describe the basics of the effective field theory treatment and look at several calculations. There are obstacles to describing these with running coupling constants. Finally, I will do my best to try to connect these with the Asymptotic Safety program.</span>

# Perturbative quantum gravity and running couplings

**Motivation – EFT background looking at AS**

Euclidean AS running makes sense (within its limitations) – defining a theory as $k \to 0$

EFT shifts focus to IR

Lorentzian applications violate EFT – e.g. large low energy running of $\Lambda$

Example cosmology with a running $\Lambda$

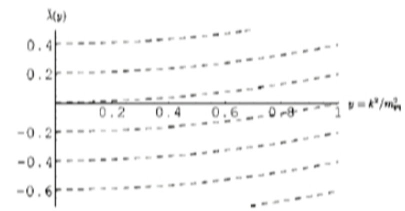


FIG. 3. Solution (4.5) to the naive flow equation for different initial values $\tilde{\lambda}(y^i)$ and $\tilde{G}(0) = 1$.

**Proposal to match the two – consistent with each**
**- but still different**


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# Perturbative quantum gravity and running couplings

## Outline:

A) **Running in an EFT**
- QED
- Chiral EFT

B) **Gravity EFT running**
- what runs
- what does not
- no useful definition of G(E)

C) **Reconciling EFT and AS practice**
- matching to EFT structure, but left-over running from Euclidean

## Aside: QED Trace anomaly:

Tree Lagrangian has no scale

$$A_\mu(x) \to \lambda A_\mu(\lambda x),\ \psi(x) \to \lambda^{3/2}\psi(\lambda x),\ \phi(x) \to \lambda\phi(\lambda x)$$

Such that $J^\mu_{\rm scale} = x_\nu \theta^{\mu\nu}$ , $\partial_\mu J^\mu_{\rm scale} = \theta^\nu{}_\nu = 0$

But loops introduce scale dependence in the derivatives

$$S = \int d^4x \ -\frac{1}{4} F_{\rho\sigma}\left[\frac{1}{e^2(\mu)} - b\log\left(\nabla^2/\mu^2\right)\right] F^{\rho\sigma}$$

Now:

$$\partial_\mu J^\mu_{\rm scale} = \theta^\nu{}_\nu = \frac{\partial \hat{\mathcal{L}}_\lambda}{\delta\lambda}|_{\lambda=1} = \frac{b}{4} F_{\rho\sigma}F^{\rho\sigma}$$

$$\hat{\mathcal{L}}_\lambda = \lambda^{-4}\mathcal{L}[\lambda A(\lambda x)]$$

**Anomaly not derivable from any local Lagrangian, but does come from a non-local action**

# Chiral Effective Field Theory

QCD at low energy – pions, kaons, photons…..

Symmetry requires a non-linear interaction

$$U = e^{i\tau^a \pi^a / F} \qquad \text{with} \qquad U \to L U R^\dagger$$

Plus derivative interactions

$$\Gamma_\mu = U^\dagger \partial_\mu U$$

With the low energy Lagrangian **

$$\mathcal{L}_2 = -\frac{F^2}{4}\, Tr(\Gamma_\mu \Gamma^\mu) \qquad\qquad F = 92.2\text{MeV}$$

** For presentation purposes, I display only certain terms and use $m_\pi$=0.

## Summary:

EFT running is often different than usual in renomalizable field theories

Power counting says that one renormalizes different operators

Many operators imply non-universal quantum effects

Running of cutoff in Euclidean is different from running w.r.t. momenta in Lorentzian

Can have a relation that respects both forms of running in their own realms

Importance of non-local effective actions.

Result starts of close to $R^2$ and ends up similar to R

In this pathway, net effect is different from AS expectations

**Graviton propagator becomes**:

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left[L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right]}{2q^2\left(1 - \frac{N}{10\pi}\frac{Gq^2}{1+\frac{NGq^2}{3\pi}}\log\left(\frac{-NGq^2}{3\pi}\right)\right)}$$

And graviton exchange involves:

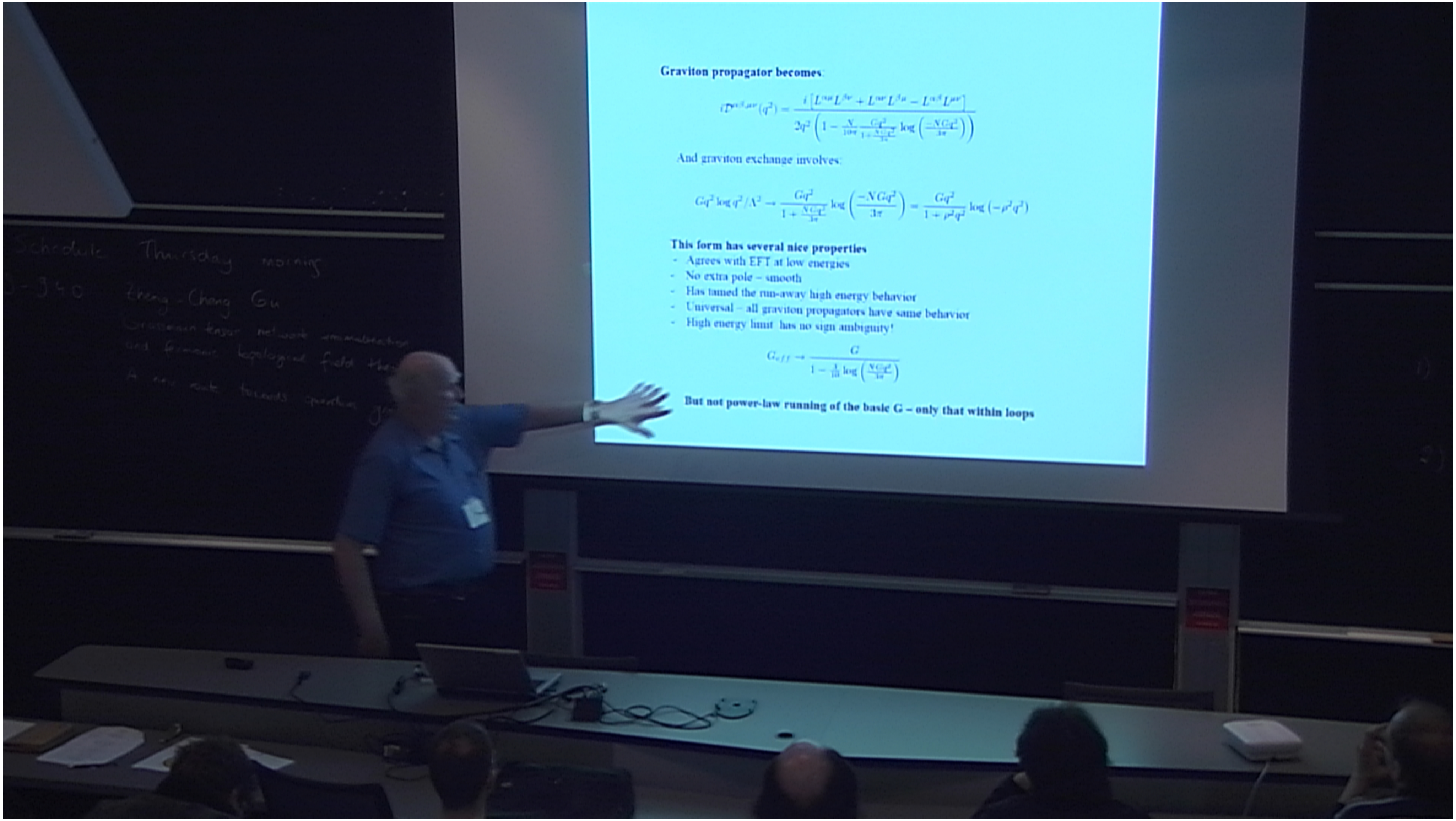
$$Gq^2 \log q^2/\Lambda^2 \to \frac{Gq^2}{1+\frac{NGq^2}{3\pi}}\log\left(\frac{-NGq^2}{3\pi}\right) = \frac{Gq^2}{1+\rho^2 q^2}\log\left(-\rho^2 q^2\right)$$

**This form has several nice properties**

- Agrees with EFT at low energies
- No extra pole – smooth
- Has tamed the run-away high energy behavior
- Universal – all graviton propagators have same behavior
- High energy limit has no sign ambiguity!

$$G_{eff} \to \frac{G}{1 - \frac{3}{10}\log\left(\frac{NGq^2}{3\pi}\right)}$$

**But not power-law running of the basic G – only that within loops**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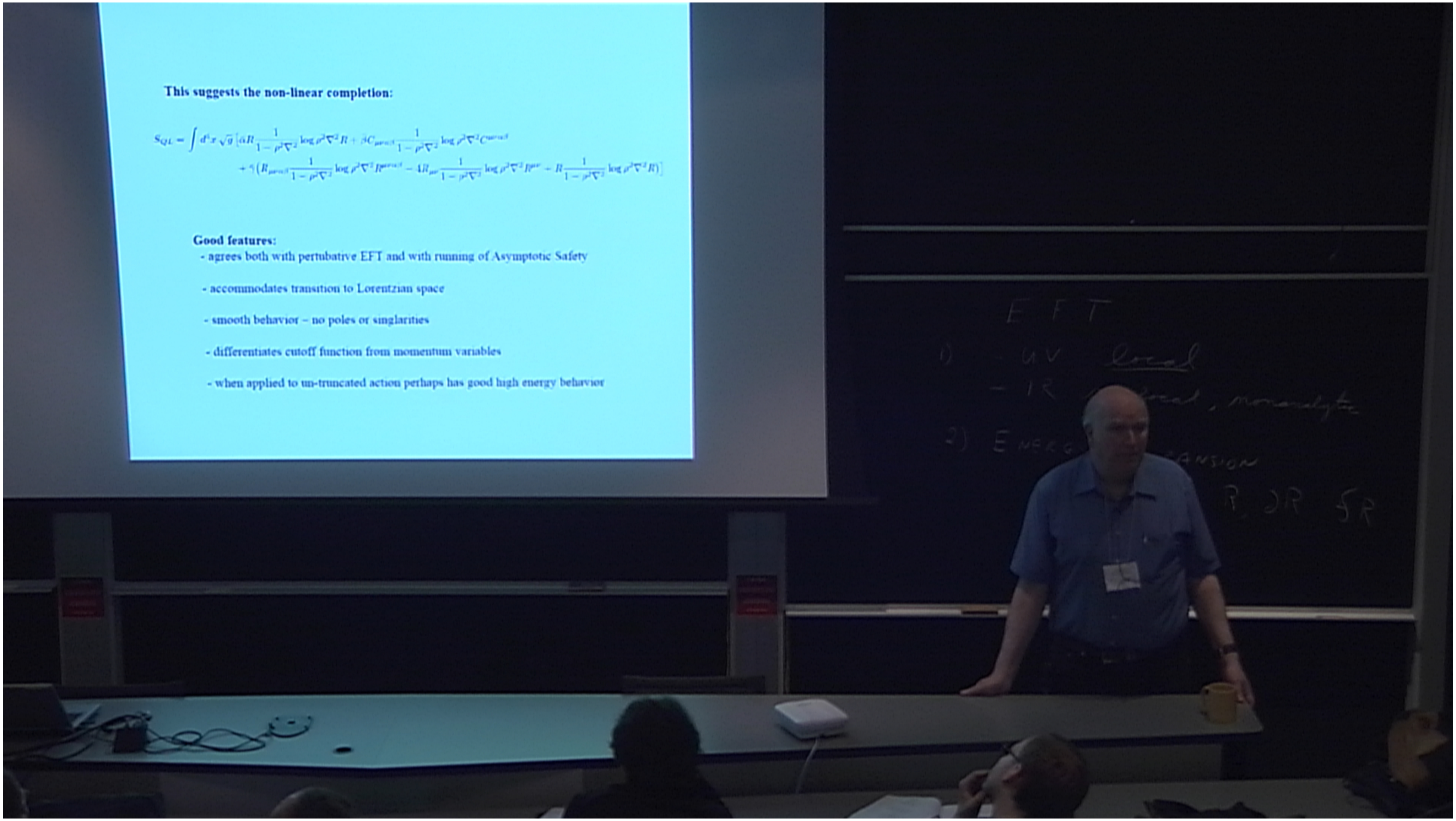
**This suggests the non-linear completion:**

$$S_{QL} = \int d^4x \sqrt{g}\, \big[\bar{\alpha} R \frac{1}{1-\rho^2\nabla^2} \log \rho^2\nabla^2 R + \bar{\beta} C_{\mu\nu\alpha\beta} \frac{1}{1-\rho^2\nabla^2} \log \rho^2\nabla^2 C^{\mu\nu\alpha\beta}$$
$$+ \bar{\gamma}\big(R_{\mu\nu\alpha\beta} \frac{1}{1-\rho^2\nabla^2} \log \rho^2\nabla^2 R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} \frac{1}{1-\rho^2\nabla^2} \log \rho^2\nabla^2 R^{\mu\nu} + R \frac{1}{1-\rho^2\nabla^2} \log \rho^2\nabla^2 R\big)\big]$$

**Good features:**

- agrees both with pertubative EFT and with running of Asymptotic Safety
- accommodates transition to Lorentzian space
- smooth behavior – no poles or singlarities
- differentiates cutoff function from momentum variables
- when applied to un-truncated action perhaps has good high energy behavior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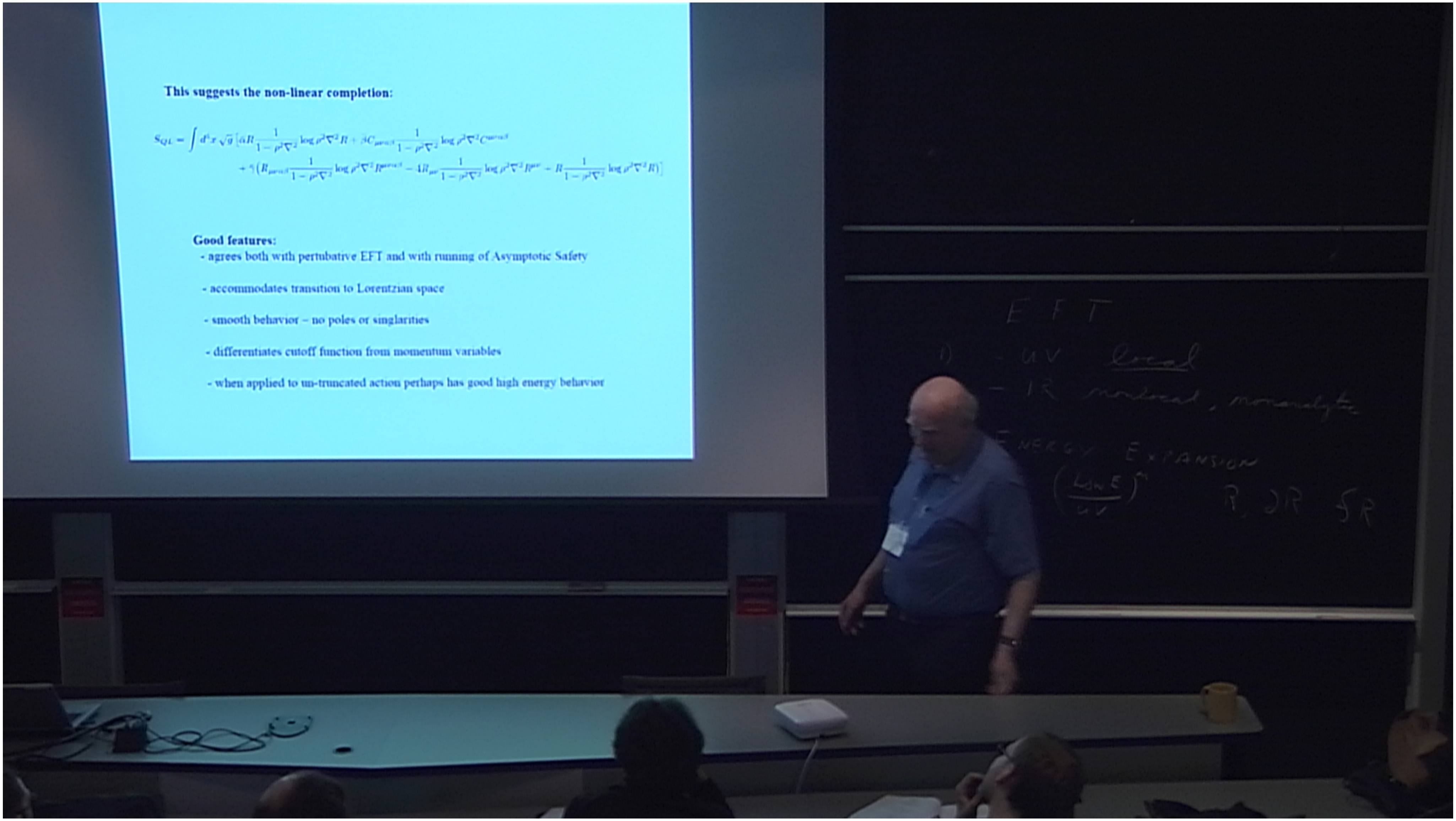
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EFT
UV local
IR nonlocal, nonanalytic
ENERGY EXPANSION